Introduction into Theory of Direction Finding

Introduction

Applications of direction finding

While direction finding for navigation purposes (referred to as cooperative direction finding) is becoming less important due to the availability of satellite navigation systems, there is a growing requirement for determining the location of emitters as the mobility of communications equipment increases:

- In radiomonitoring in line with ITU guidelines
  - Searching for sources of interference
  - Localization of non-authorized transmitters
- In security services
  - Reconnaissance of radiocommunications of criminal organizations
- In military intelligence [1]
  - Detecting activities of potential enemies
  - Gaining information on enemy's communications order of battle
- In intelligent communications systems
  - Space division multiple access (SDMA) requiring knowledge of the direction of incident waves [2]
- In research
  - Radioastronomy
  - Earth remote sensing

Another reason for the importance of direction finding lies in the fact that spread-spectrum techniques are increasingly used for wireless communications. This means that the spectral components can only be allocated to a specific emitter if the direction is known. Direction finding is therefore an indispensable first step in radiodetection, particularly since reading the contents of such emissions is usually impossible.

The localization of emitters is often a multistage process. Direction finders distributed across a country allow an emitter to be located to within a few kilometers (typ. 1% to 3% of the DF distances) by means of triangulation. The emitter location can more precisely be determined with the aid of direction finders installed in vehicles. Portable direction finders moreover allow searching within the last 100 m, for instance in buildings.

Historical development

The DF technique has existed for as long as electromagnetic waves have been known. It was Heinrich Hertz who in 1888 found out about the directivity of antennas when conducting experiments in the decimetric wave range. A specific application of this discovery for determining the direction of incidence of electromagnetic waves was proposed in 1906 in a patent obtained by Scheller on a homing DF method.

The initial DF units were polarization direction finders. They consisted of a rotatable electric or magnetic dipole whose axis was brought to coincidence with the direction of the electric or magnetic field. From the direction of polarization, the direction of incidence was then deduced. The rotating-loop direction finder is one of the best known direction finders of this type. In 1907, Bellini and Tosi discovered the DF principle that was named after them: a combination of two crossed directional antennas (e.g. loop antennas) with a moving-coil goniometer for determining the direction [1]. Despite this invention, rotating-loop direction finders were often used in the First World War (Fig. 1).

The invention made by Adcock meant a great leap forward in the improvement of the DF accuracy with respect to sky waves in the shortwave range. The pharmacist by profession realized in 1917 that with the aid of vertical linear antennas (rod antennas or dipoles) directional patterns can be generated that correspond to those of loop antennas but do not pick up any interfering horizontally polarized field components (G. Eckard proved in 1972 that this is not true in all cases [3]). It was not until 1931 that Adcock antennas were first employed in Great Britain and Germany.

In 1925/26, Sir Watson-Watt took the step from the mechanically moved goniometer direction finder to the electronic visual direction finder. As from 1943, British naval vessels were equipped with crossed loops and three-channel Watson-Watt direction finders for the shortwave range ("huff-duff" for detecting German submarines).

As from 1931, camouflaged direction finders were available for use in vehicles and as portable direction finders for detecting spies.

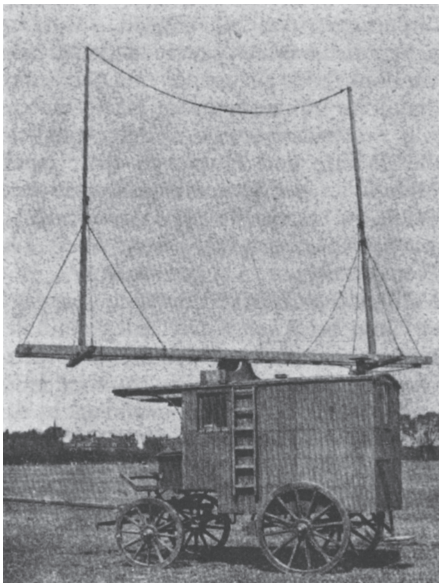


Fig. 1: Mobile rotating-loop direction finder for military use (about 1918) [1].

The first shortwave direction finder operating on the Doppler principle was built in 1941. The rapid progress in the development of radar in Great Britain made it necessary to cover higher frequency ranges: In 1943, the first direction finders for "radar observation" at around 3000 MHz were delivered [1].

As from 1943, wide-aperture circular-array direction finders (Wullenweber) were built for use as remote direction finders. Since the 1950s, airports all over the world have been equipped with VHF/UHF Doppler direction finding systems for air traffic control.

In the early 1970s, digital technology made its way into direction finding and radiolocation; digital bearing generation and digital remote control are major outcomes of this development.

Since 1980, digital signal processing has been increasingly used in direction finding. It permits the implementation of the interferometer direction finder and initial approaches toward the implementation of multiwave direction finders (super-resolution). The first theoretical considerations were made much earlier, e.g. in [4].

Another important impetus for development came from the requirement for the direction finding of frequency-agile emissions such as frequency-hopping and spread-spectrum signals. The main result of this was the broadband direction finder, which is able to simultaneously perform searching and direction finding based on digital filter banks (usually with the aid of fast Fourier transform (FFT)) [5].

Tasks of direction finding

The task of a radio direction finder is to estimate the direction of an emitter by measuring and evaluating electromagnetic field parameters.

Usually, the azimuth α is sufficient to determine the direction; the measurement of elevation ε is of interest for emitters installed on flying platforms and especially for the direction finding of shortwave signals (Fig. 2).

Only in the case of undisturbed wave propagation is the direction of the emitter identical with the direction of incidence of the radio waves. Usually, there is a large number of partial waves arriving from different directions and making up a more or less scattered field. The direction finder takes spatial and temporal samples from this wavefront and, in the ideal case, delivers the estimated values $\hat{\alpha}$ and $\hat{\varepsilon}$ as the most probable direction of the emitter observed.

Bearings can be taken using the following reference directions (Fig. 3) (see also EN 3312 [6]):

- Geographic north (true north) ▷ true radio bearing
- Magnetic north
- Vehicle axis ▷ relative or direct radio bearing

DF principles

Generation and characterization of electromagnetic waves

Electromagnetic waves are caused by charging and discharging processes on electrical conductors that can be represented in the form of alternating currents [7], [8].

The first assumption is based on the undisturbed propagation of a harmonic wave of wavelength λ. At a sufficiently large distance, the radial field components are largely decayed so that, limited to a small area, the wave can be considered to be plane: Electric and magnetic field components are orthogonal to and in phase with one another and are perpendicular to the direction of propagation, which is defined by the radiation density vector (Poynting vector) $\vec{S}$

$$\vec{S} = \vec{E} \times \vec{H} = \overline{e_0} \frac{|E|^2}{Z_0}$$

where E = RMS value of electric field strength
Z_0 = characteristic impedance of free space;
$Z_0 \cong 120 \cdot \pi\,\Omega$

or by the wave number vector $\vec{k}$ (Fig. 4).

$$\vec{k} = \overline{e_0} \frac{2\pi}{\lambda}$$

Overview of main DF principles

Direction finding relies on the basic characteristics of electromagnetic waves, which are:

- Transversality, i.e. field vectors are perpendicular to the direction of propagation
- Orthogonality of phase surfaces and direction of propagation

Every DF process essentially employs one of the following methods (Table 1, on next page):

- Method A: measures the direction of electric and/or magnetic field vectors ▷ polarization direction finders
- Method B: measures the orientation of surfaces of equal phase (or lines of equal phase if the elevation is not of interest) ▷ phase direction finder

Fig. 2: Definition of emitter direction

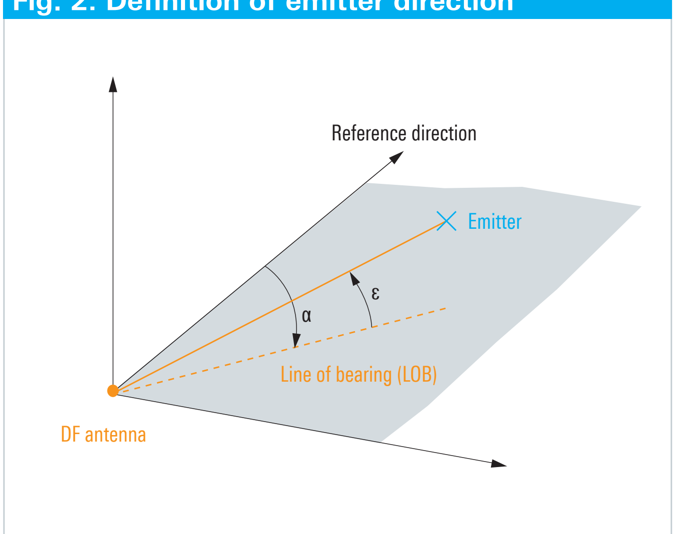
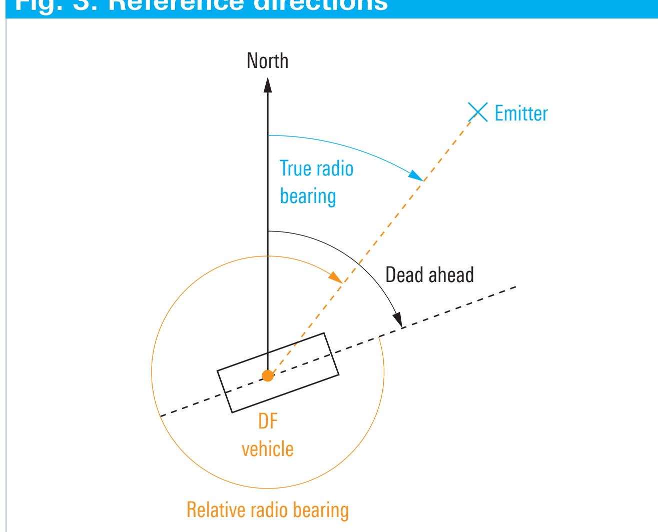
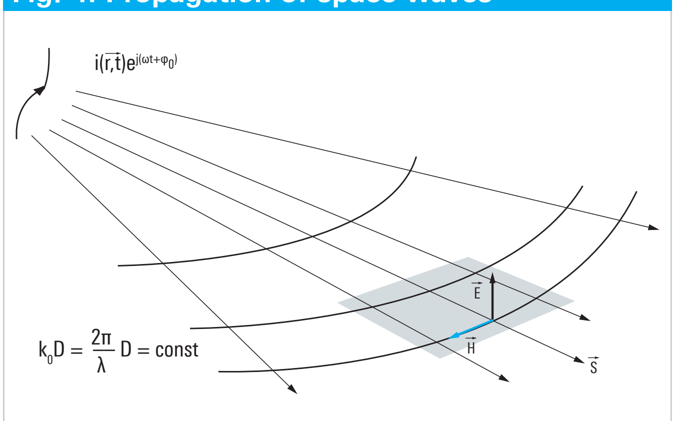


Fig. 3: Reference directions



Polarization direction finders are implemented by means of dipole and loop antennas. The classic rotating-loop direction finder also belongs to this category (rotation of loop to minimum of received signal ▷ direction of incident wave perpendicular to loop). Today, polarization direction finders are used in applications where there is sufficient space only for small antennas, e.g. in vehicles and on board ships for direction finding in the HF band. Evaluation is usually based on the Watson-Watt method (see section "Classic DF methods"). To obtain unambiguous DF results, however, the phase information must be evaluated in addition.

Fig. 4: Propagation of space waves



Phase direction finders obtain the direction information (bearing information) from the spatial orientation of lines or surfaces of equal phase. There are two basic methods:

- Direction finding based on directional patterns:
  With this method, partial waves are coupled out at various points of the antenna system and combined at one point to form a sum signal. The maximum of the sum signal occurs at the antenna angle at which the phase differences between the partial waves are at a minimum. The sum signal is thus always orthogonal to the phase surfaces of the incident wave (maximum-signal direction finding). For minimum-signal direction finding, the partial waves are combined so that the phase differences in the direction of the incident wave become maximal and there is a distinct minimum of the received signal
- Direction finding by aperture sampling:
  With this method, samples are taken at various points of the field and applied to evaluation circuits sequentially or in parallel. These circuits determine the bearing by linking the samples, which is today mostly done by mathematical operations

Typical examples are interferometers and Doppler direction finders.

The DF methods mentioned so far are suitable only to a limited extent for determining the directions of incidence of several waves that overlap in the frequency domain.

With the progress made in digital signal processing, the methods known from the theory of spectral estimation have been applied to the analysis of wavefront and developed. The term "sensor array processing" describes the

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Table 1: DF principles.

Wave characteristic	Transversality	Phase surfaces	⊥ Direction of propagation	
DF principle	**Polarization direction finder**	**Phase direction finder**		
Examples		**Direction finding with directional patterns**	**Aperture sampling**	
		Conversion phase ▷ amplitude	Direct evaluation	Sensor array processing
	Rotating loop	Directional antenna Maximum-signal direction finder Minimum-signal direction finder	Interferometer	Correlation direction finder
	Dipole	Adcock with Watson-Watt evaluation	Rotating-field direction finder	Adaptive beam former
	Loaded loop		Doppler direction finder	MUSIC
	Crossed loop with Watson-Watt evaluation			ESPRIT

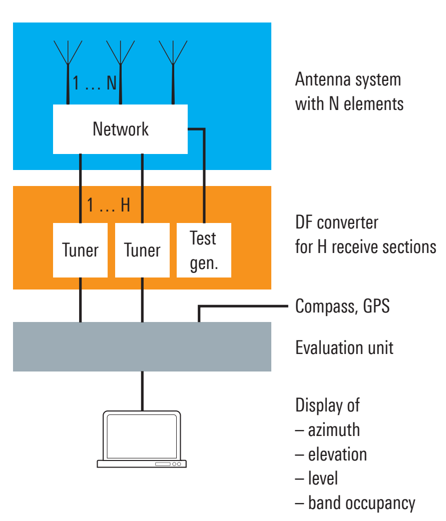
technique of gaining information about the parameters of incident waves from the signals derived from the elements, or sensors, of a sensor array (antenna array in direction finding, hydrophone array for sonar).

There are basically three different methods:
- Beamforming methods, e.g. correlation direction finder, spatial Fourier analysis, adaptive antenna
- Maximum likelihood method as the most general model-based method
- Subspace methods, e.g. MUSIC, ESPRIT

Main requirements on DF systems
- High accuracy
- High sensitivity
- Sufficient large-signal immunity
- Immunity to field distortion caused by multipath propagation
- Immunity to polarization errors
- Determination of elevation in shortwave range
- Stable response in case of non-coherent co-channel interferers
- Short minimum required signal duration
- Scanning direction finders: high scanning speed, and high probability of intercept (POI)

Components of a DF system
A DF system (Fig. 5) consists of the following components:
- Antenna system
- DF converter
- Evaluation unit
- Display unit

Fig. 5: DF system components



Depending on the configuration, systems for determining the direction finder's own coordinates/orientation (GPS, compass), remote-control units (LAN, WAN), antenna control units, etc., can be added.

The achievable DF speed mainly depends on the number H of receive sections, as this parameter determines the number of antenna outputs that can be measured in parallel.

To achieve maximum speed, it must be possible to generate a bearing in a single time step, i.e. from one set of samples (monopulse direction finding). For unambiguous direction finding over the total azimuth range, at least three antenna outputs are required. If there are also three receive sections, multiplexing of the measurement channel is not necessary.

Typical examples of monopulse DF antennas:
- Multimode antenna for amplitude comparison direction finders, e.g. Adcock antenna
- Interferometer and rotating-field (phase) direction finder

For high DF accuracy (e.g. 1°) and large bandwidth (e.g. 1 MHz to 30 MHz or 20 MHz to 1000 MHz), five to nine aperture samples are usually required. Since monopulse solutions would then be very complex, one fixed and two sequentially switched receive sections are frequently used.

The DF converter converts the carrier-frequency antenna signals to a fixed IF. Since this conversion must be performed with equal phase and amplitude in all receive sections, the use of a common synthesizer is indispensable. Moreover, with most multipath direction finders, the receive sections are calibrated in order to ensure equal amplitude and phase. Calibration is performed with the aid of a test generator at defined intervals and prior to the actual DF operation.

The evaluation unit determines the bearing from the amplitudes and/or phases of the IF signal.

Classic DF methods

Using directional antennas
Evaluating the receive voltage of a mechanically rotated directional antenna with reference to the direction is the simplest way of direction finding. With this method, the bearing is derived from the characteristic of the receive voltage as a function of the antenna rotation angle: When a wave arrives, the receive voltage yields the directional pattern of the antenna. The pattern position relative to the antenna rotation angle is the measured bearing [1].

This type of direction finder is a phase direction finder since the directivity of the receive antenna is achieved by superimposing partial waves whose phase differences depend on the angle of incidence. In the simplest case, the antenna is rotated and the bearing determined by the operator. The antenna is rotated until the receiver output voltage assumes an extreme value. The antenna direction thus found is read from a scale, and the bearing is determined from it. If the directional antenna (with maximum or minimum pattern) is permanently rotated with the aid of a motor, and the receive voltage is displayed graphically as a function of the angle of rotation, a rotating direction finder is obtained (Fig. 6). Using suitable automatic evaluation of the receive voltage characteristic, e.g. by means of a maximum detector, a fully automatic direction finder is obtained.

The following benefits are common to all variations of this DF method:

- High sensitivity due to the directivity of the antenna
- Simple and inexpensive implementation (only one receiver required (single-channel principle))
- Resolution of multiwavefronts is possible (prerequisite: different angles of incidence and high-directivity antenna system)
- Same antenna can be used for direction finding and monitoring

The drawbacks of this method result from the restricted angular detection range, which is due to the directivity of the antenna, and the antenna's limited rotating speed, which is mainly due to the use of a mechanical rotator:

- Probability of intercept is reciprocal to directivity
- Method fails in case of short-duration signals, i.e. with signal dwell times that are short compared to a scanning cycle of the antenna

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Despite these drawbacks, DF methods using mechanically rotated directional antennas are still in use today since achieving the described advantages with other methods involves in part considerably higher cost and effort. In the microwave range, in particular, the mechanical DF method is often the only justifiable compromise between gain, low noise and expenditure.

If, in addition to a directional pattern with a maximum in the direction of the incident wave, a directional pattern with a minimum is used, a monopulse direction finder is obtained that even with a slowly rotating or fixed antenna delivers bearings as long as waves arrive in the main receiving direction of the antenna. Fig. 7 shows a typical implementation using log-periodic dipole antennas that are connected by means of a 0/180° hybrid. This results in the directional patterns shown below.

The quotient of the difference and the sum signal yields a dimensionless, time-independent function, i.e. the DF function:

$$PF(\alpha) = \frac{V_{\Delta}(\alpha)}{V_{\Sigma}(\alpha)}$$

After forming the quotient of the two test voltages, the DF function immediately delivers the bearing α.

Fig. 6: DF using a directional antenna

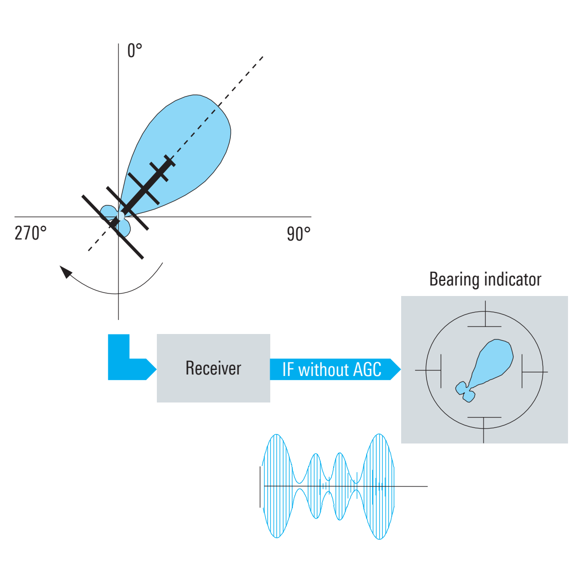
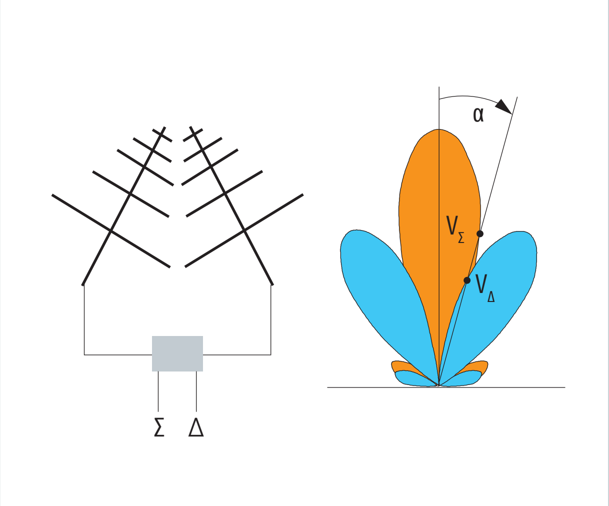


Fig. 7: DF using sum-difference method



Typical implementation on the left, directional patterns for sum (Σ) and difference (Δ) outputs on the right.

Watson-Watt principle

If the amplified and filtered signals of a receiving antenna with outputs for a sine-shaped and a cosine-shaped directional pattern are applied to the x and y deflection plates of a cathode-ray tube (CRT), a line Lissajous figure is obtained in the ideal case, whose inclination $\hat{\alpha}$ corresponds to the wave angle but exhibits an ambiguity of 180°. The indicated angle is obtained from the ratio of the two signals as follows:

$$\hat{\alpha} = \arctan\frac{V_x}{V_y}$$

An unambiguous bearing indication is obtained (Fig. 8) if a blanking signal is additionally used in this DF method, which was first implemented by Watson-Watt in 1926. The blanking signal is derived from an omnidirectional receiving antenna with an unambiguous phase relationship.

If there is a phase difference δ between the two voltages V_x and V_y, which may be due to ambient interference (e.g. reflections), the displayed figure is an ellipse. The position of the main axis yields the bearing $\hat{\alpha}$, which is calculated from the two voltages by means of the equation below [9].

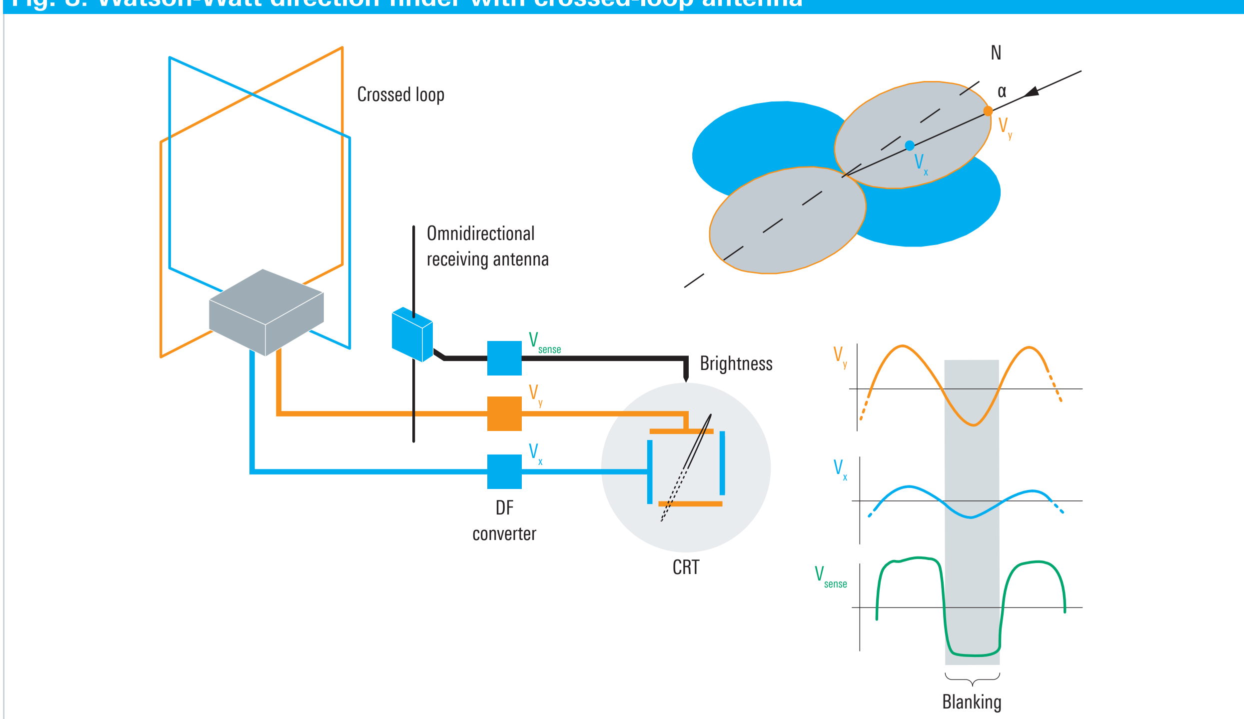
$$\hat{\alpha} = \mathrm{Re}\left(\arctan\frac{V_x}{V_y}\right) = \frac{1}{2}\arctan\frac{2|V_x||V_y|\cos\delta}{|V_y|^2 - |V_x|^2}$$

The principal advantage of this method is that the bearing is indicated without delay, which means that it is capable of monopulse direction finding over the entire azimuth range.

Suitable antennas (Fig. 9) with sine-shaped or cosine-shaped directional patterns are in particular the following:
- Loop antennas (or ferrite antennas)
- Adcock antennas (monopole or dipole arrays)

Crossed-loop antennas with Watson-Watt evaluation are mainly suitable for mobile applications due to their compact size. They feature the following benefits and drawbacks:
- Benefits:
  - Extremely short signal duration is sufficient
  - Implementation is simple
  - Minimum space is required
- Drawbacks:
  - Small-aperture system (D/λ < 0.2) causing errors in case of multipath propagation
  - Large DF errors when receiving sky waves with steep elevation angles

Fig. 8: Watson-Watt direction finder with crossed-loop antenna



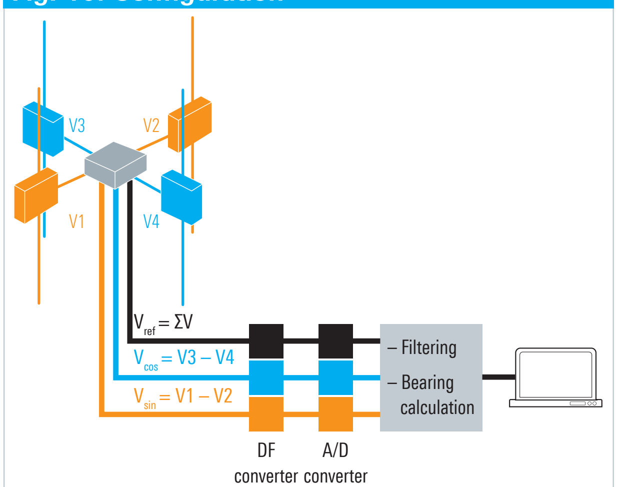
Adcock antennas feature the following advantages over crossed-loop antennas:

- Improved error tolerance for sky wave reception
- Wider apertures can be implemented to reduce errors in case of multipath reception (e.g. $D/\lambda < 1$ for 8-fold Adcock)

Modern direction finders no longer display the IF voltages of antenna signals on a CRT but digitally process the signals after converting them into a relatively wide IF band (Fig. 10).

Selection is mainly effected by means of digital filters; bearings are calculated numerically, e.g. using the last equation above, and displayed on a computer (workstation, PC) with a graphical user interface (GUI).

Fig. 10: Configuration



Configuration of a modern direction finder operating on the Watson-Watt principle.

A number of disadvantages of analog direction finders are avoided, yielding the following effects:

- Synchronous operation of channels also on filter edges
- Simple procedure of taking into account correction values for antenna networks, cables, etc.
- No temperature drift in digital section
- Bearings available as numeric values for evaluation, especially for easy transmission to remote evaluation stations

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Doppler direction finder

If an antenna element rotates on a circle with radius R, the received signal with frequency ω_0 is frequency-modulated with the rotational frequency ω_r of the antenna due to the Doppler effect. If the antenna element moves toward the radiation source, the receive frequency increases; if the antenna element moves away from the radiation source, the receive frequency decreases.

From the instantaneous amplitude

$$u(t) = a\cos(\phi(t)) = a\cos\left(\omega_0 t + \frac{2\pi R}{\lambda_0}\cos(\omega_r t - \alpha) + \varphi\right)$$

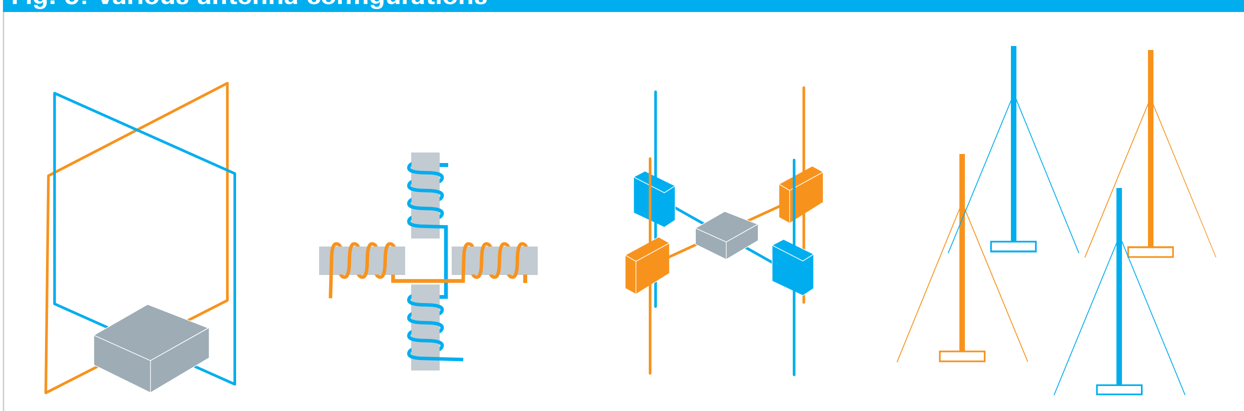
the instantaneous frequency is derived by differentiation of the phase

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_0 - \frac{2\pi R}{\lambda_0}\omega_r \sin(\omega_r t - \alpha)$$

After filtering out the DC component ω_0, the demodulated Doppler signal is obtained as

$$S_D = \frac{2\pi R}{\lambda_0}\omega_r \sin(\omega_r t - \alpha)$$

Fig. 9: Various antenna configurations



For generating sine-shaped or cosine-shaped directional patterns: crossed-loop antenna, ferrite antenna, H Adcock, U Adcock (from left).

The phase of the demodulated signal is compared with a reference voltage of equal center frequency derived from the antenna rotation

$$S_r = -\sin\omega_r t$$

which yields the bearing α [1].

Since rotating an antenna element mechanically is neither practically possible nor desirable, several elements (dipoles, monopoles, crossed loops) are arranged on a circle (Fig. 11) and electronically sampled by means of diode switches (cyclic scanning).

To obtain unambiguous DF results, the spacing between the individual antenna elements must be less than half the operating wavelength; in practice, a spacing of about one third of the minimum operating wavelength is selected.

If this rule is adhered to, Doppler DF antennas of any size are possible, i.e. wide-aperture systems with the following features can easily be implemented:

- High immunity to multipath reception
- High sensitivity

A drawback of the Doppler method is the amount of time it requires. At least one antenna scanning cycle is required in order to obtain a bearing. At a typical rotational frequency of 170 Hz in the VHF/UHF range, one cycle takes approx. 6 ms.

Interferometer

The interferometer direction finder was first used in radio astronomy [10]. The objective was to increase the resolution power and the sensitivity of the DF system by superimposing the signals of only a few antenna elements that were however spaced many wavelengths apart (Fig. 12).

The ambiguous interference pattern is weighted with the directional pattern of the antenna element, which yields an unambiguous bearing (see Fig.12, diagrams on the right).

Using the minimum of three omnidirectional receiving elements, unambiguous determination of the azimuth and elevation is possible only if the spacing a between the antennas is no greater than half the wavelength. With Φ_1, Φ_2, Φ_3 as the phases measured at the outputs of the antenna elements, the azimuth is calculated as

$$\hat{\alpha} = \arctan\frac{\Phi_2 - \Phi_1}{\Phi_3 - \Phi_1}$$

The elevation angle is obtained as

$$\hat{\varepsilon} = \arccos\frac{\sqrt{(\Phi_2 - \Phi_1)^2 + (\Phi_3 - \Phi_1)^2}}{2\pi a/\lambda}$$

In practice, the three-antenna configuration (Fig. 13) is enhanced by additional antenna elements so that the spacings between the antennas can be optimally adapted to the operating frequency range, and antenna spacings of a > λ/2 can be used to increase the accuracy of small-aperture DF systems [1]. Frequently used antenna arrangements include the isosceles right triangle and the circular array (Fig. 14).

Triangular configurations are usually restricted to frequencies below 30 MHz. At higher frequencies, circular arrays are preferred for the following reasons:

- They ensure equal radiation coupling between the antenna elements
- They ensure minimum coupling with the antenna mast
- They favor direction-independent characteristics at different positions due to the symmetry around the center point

Fig. 11: Principle of Doppler direction finder

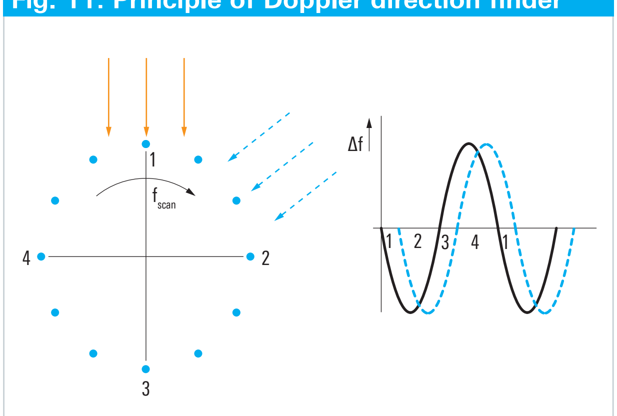
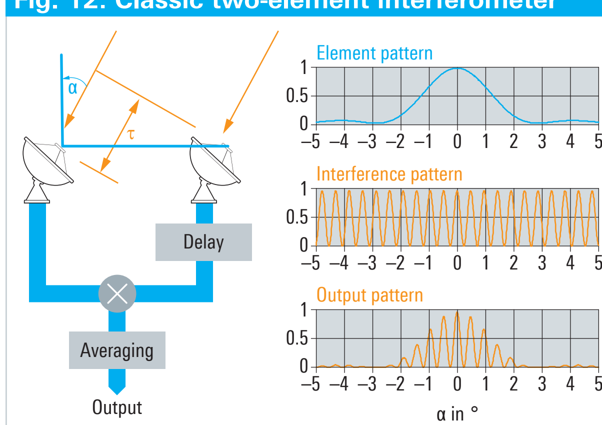


Fig. 12: Classic two-element interferometer



Used in radio astronomy; block diagram (left) and antenna patterns (right).

It is essential to avoid ambiguities, which result from the fact that unambiguous measurement of the phase is possible only in the range of ±180°. As already mentioned, this condition is met in the case of the three-element (small-aperture) interferometer by limiting the spacing between the elements to half the minimum operating wavelength. With multi-element interferometers, there are the following possibilities:

- Use of "filled" antenna arrays: Phase differences between neighboring elements are always < 180°; ambiguities are avoided.
- Use of "thinned" antenna arrays: There is a phase difference of > 180° between at least one pair of neighboring elements. Such ambiguities in antenna subarrays are today usually eliminated by subjecting the signals from all elements simultaneously to a pattern comparison by way of correlation ▷ correlative interferometer.

The basic principle of the correlative interferometer consists in comparing the measured phase differences with the phase differences obtained for a DF antenna system of known configuration at a known wave angle. The comparison is performed by calculating the quadratic error or the correlation coefficient of the two data sets. If the comparison is made for different azimuth values of the reference data set, the bearing is obtained from the data for which the correlation coefficient is at a maximum.

Fig. 13: Three-element interferometer

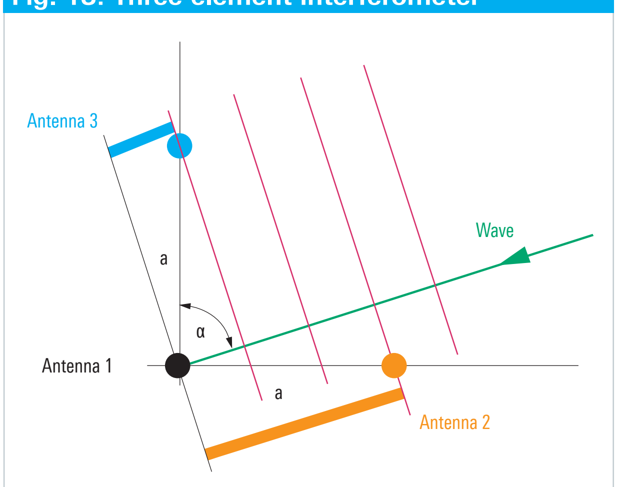
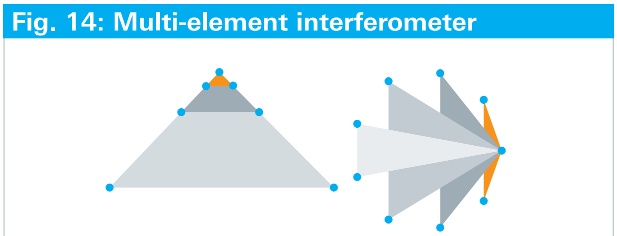


Fig. 14: Multi-element interferometer



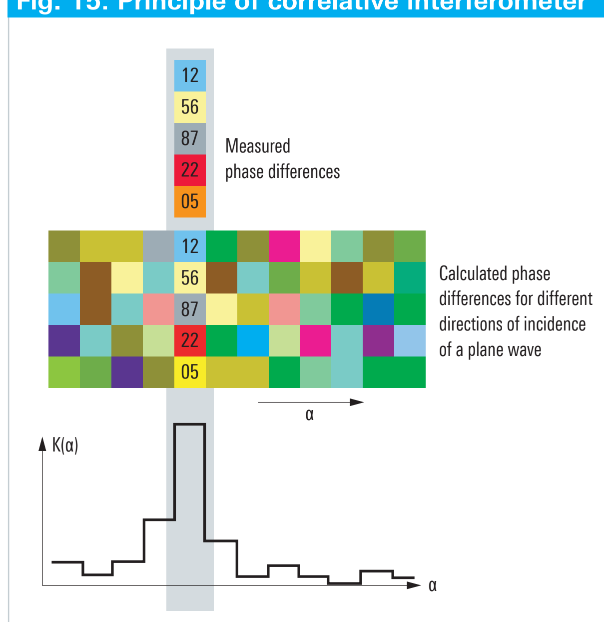
Three-antenna configuration enhanced to form a multi-element interferometer.

This is demonstrated by the example of a five-element antenna as shown in Fig. 15. Each column of the lower data matrix corresponds to a wave angle a and forms a reference vector. The elements of the reference vectors represent the expected phase differences between the antenna elements for that wave angle. The upper 5 × 1 data matrix contains the actual measured phase differences (measured vector).

To determine the unknown wave angle (direction of incidence), each column of the reference matrix is correlated with the measured vector by multiplying and adding the vectors element by element. This process results in the correlation function K(a) , which reaches its maximum at the point of optimum coincidence of reference vector and measured vector. The angle represented by that specific reference vector is the wanted bearing.

This method constitutes a special form of a beamforming algorithm [11], which will be discussed in greater detail in the following section.

Fig. 15: Principle of correlative interferometer



Direction finding using sensor array processing

General
The development of the classic DF methods was aimed at designing antenna configurations that allowed bearings to be determined using a circuit design as simple as possible. It was important to establish a simple mathematical relationship between the antenna signals and the direction of the incident wave largely independent of frequency, polarization and environment.

With the development of digital signal processing, new approaches have become possible:
- With high-speed signal-processing chips now available, the requirement for a simple and frequency-independent relationship between the antenna signals and the bearing no longer applies. Even highly complex mathematical relationships can be evaluated in a reasonably short period of time for determining the bearing, or handled quickly and economically by means of search routines
- Numeric methods allow the separation of several waves arriving from different directions even with limited antenna apertures (high-resolution methods, superresolution, multiwave resolution)

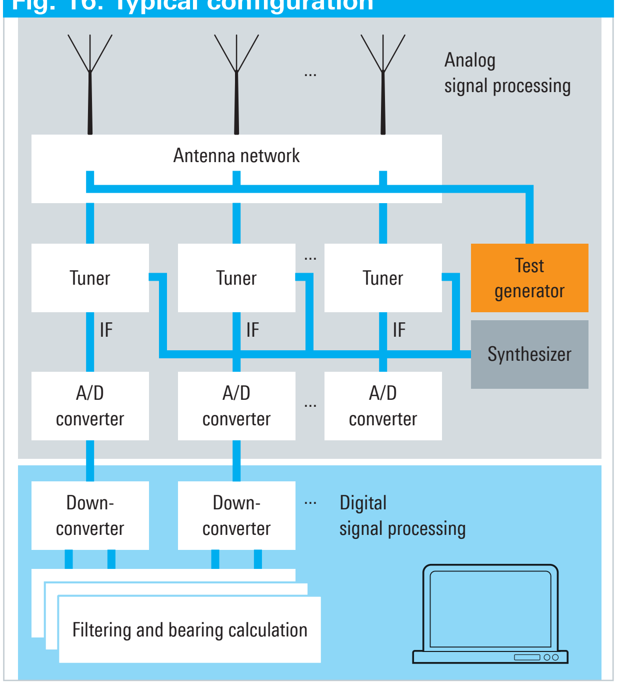
Basic design
Fig. 16 shows a typical hardware configuration of a DSP-based direction finder [12].

The outputs of the individual antenna elements are usually first taken to a network that contains the following, for instance:
- Test signal inputs
- Multiplexers if the number N of antenna outputs to be measured is higher than the number H of receive sections (tuners and A/D converters) in the direction finder

The signals are then converted to an intermediate frequency that is appropriate for the selected sampling rate of the A/D converters and digitized. To reduce the data volume, the data is digitally downconverted into the baseband. The complex samples $x_i(t)$ (i = 1, 2, to N) of the baseband signals are filtered for the desired evaluation bandwidth and applied to the bearing calculation section.

Fig. 17 shows a typical implementation including a nine-element circular array antenna and a three-path receiver. The signals of the antenna elements are measured sequentially based on three-element subarrays.

Fig. 16: Typical configuration



Typical configuration of a DSP-based direction finder.

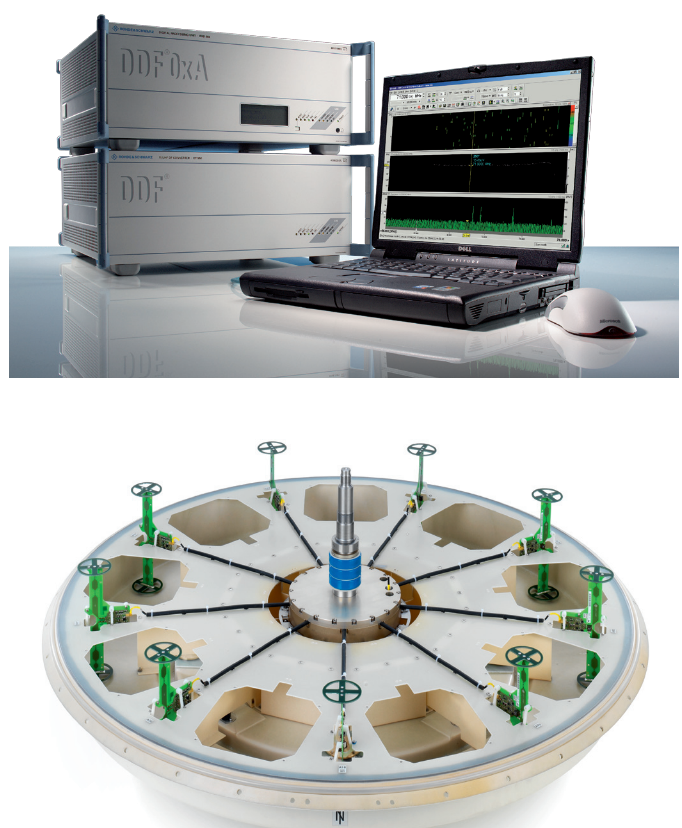


Fig. 17: DSP-based R&S®DDF06A broadband direction finder for the frequency range from 0.3 MHz to 3000 MHz and circular array antenna (R&S®ADD153SR, without cover) for the 20 MHz to 1300 MHz range.

General DF task

For an antenna array of N elements positioned in an unknown wave field, the general DF task (Fig. 18) is to estimate – from the measured data $x_1(t)$, $x_2(t)$, to $x_N(t)$ – the parameters listed below:

- Number M of waves
- Directions of incidence of the waves
- Amplitudes of the waves

Procedure

The task is performed in two steps:

- Determine the relationships between the measured data $x_1(t)$ to $x_N(t)$ and the number, directions and amplitudes of the waves involved
- Develop methods for determining the number, directions, and amplitudes of the main waves involved based on the relationships determined in the first step

Relationship between the measured data and the parameters of the incoming waves

Data model for a single wave

For the sake of simplicity, it is assumed that the emitter and the receiving antennas are located in the same plane, i.e. the elevation angle of the waves can be ignored. It is also assumed that the waves and the antenna elements are vertically polarized.

For the model discussed here, it is assumed that a signal is radiated by an emitter with a carrier frequency of f_0 (wavelength λ_0) and modulated with the function s(t). The wave is considered to be plane in the far field of the emitter and to arrive at an angle α_0 (Fig. 19).

For signal bandwidths that are small in comparison with the reciprocal of the signal delay between the elements spaced at the maximum distance (narrowband approximation), the baseband signal at the output of the i-th sensor can be modeled in accordance with the following equation:

$$\begin{aligned} x_i(t) &= s(t)c_i(\alpha_0)e^{j\frac{2\pi}{\lambda_0}|r_i|\cos(\alpha_0\beta_i)} + n(t) \\ &= s(t)a_i(\alpha_0) + n_i(t) \end{aligned}$$

The term $s(t) = r(t)e^{jp(t)}$ describes the waveform and the amplitude of the signal in the form of the complex envelope.

The term $n_i(t)$ describes the inherent noise of the sensor channel.

The term $c_i(\alpha_0)$ describes the characteristic of the antenna element.

The term $e^{j\frac{2\pi}{\lambda_0}|r_i|\cos(\alpha_0-\beta_i)}$ describes the phase displacement due to the delay between the reference point O and the position r_i of the i-th antenna element. The phase displacement solely depends on the position of the element (normalized to the wavelength λ_0) and the direction of incidence of the wave.

3

Fig. 18: General DF task

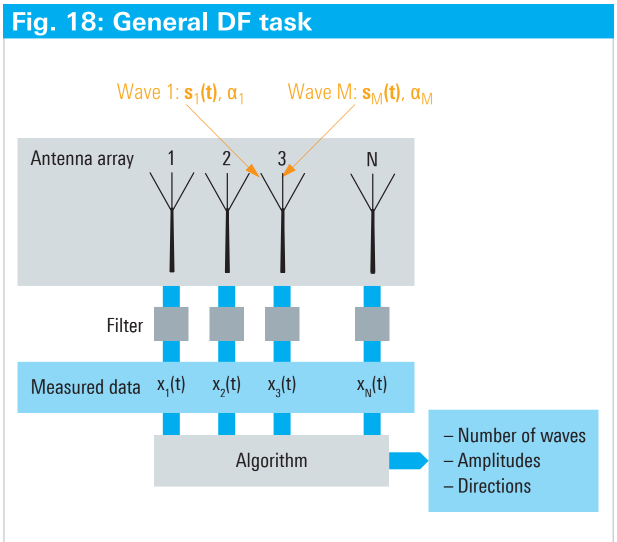
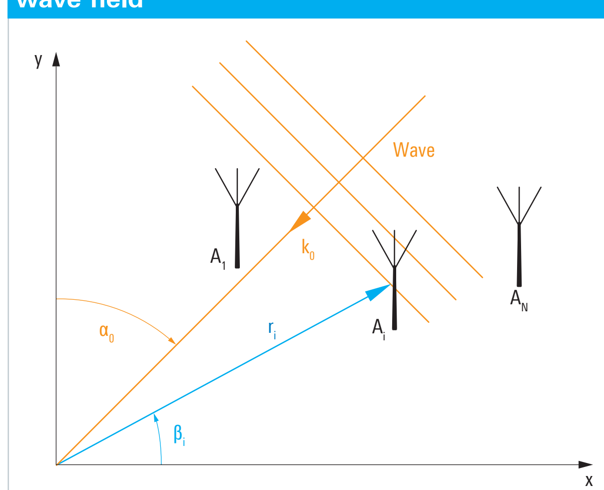


Fig. 19: Characterization of incoming wave field



The antenna- and direction-dependent quantities are then combined in the direction component:

$$a_i(\alpha_0) = c_i(\alpha_0) e^{j\frac{2\pi}{\lambda_0}|r_i|\cos(\alpha_0 - \beta_i)}$$

In the interest of simplified notation and straightforward geometrical interpretation, the signals $x_i(t)$ at the element outputs are assumed to be components of a vector x(t) in the observation space:

$$\mathbf{x}(t) = s(t)\mathbf{a}(\alpha_0) + \mathbf{n}(t)$$

$$\mathbf{x}(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ \dots \\ x_N(t) \end{pmatrix}, \quad \mathbf{n}(t) = \begin{pmatrix} n_1(t) \\ n_2(t) \\ \dots \\ n_N(t) \end{pmatrix}$$

$$\mathbf{a}(\alpha_0) = \begin{pmatrix} e^{j\frac{2\pi}{\lambda_0}|r_1|\cos(\alpha_0 - \beta_1)} \\ e^{j\frac{2\pi}{\lambda_0}|r_2|\cos(\alpha_0 - \beta_2)} \\ \dots \\ e^{j\frac{2\pi}{\lambda_0}|r_N|\cos(\alpha_0 - \beta_N)} \end{pmatrix}$$

a(α) designates a specific direction α and is referred to as direction vector. The set of all direction vectors forms the array manifold, a parameter that plays a vital role in every aspect of array processing [13].

Characterization of antenna array

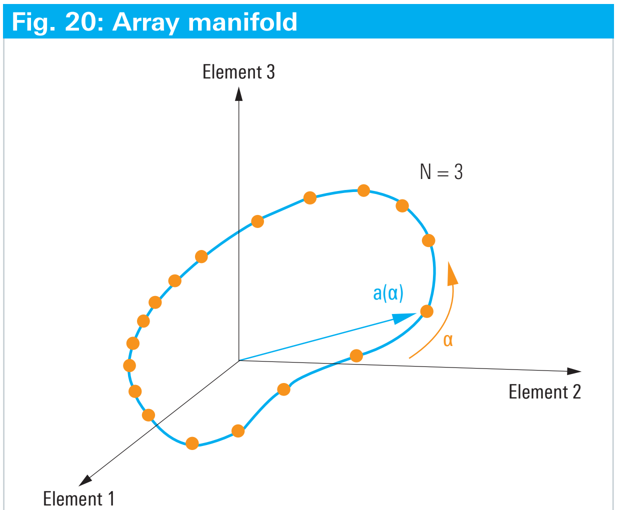
If the wave angle α is continuously varied across the range of interest, the tip of the antenna vector a(α) describes a curve in the N-dimensional space (Fig. 20). The curve is referred to as an array manifold and fully characterizes the antenna for the parameter α, except for any loss or gain factors [13].

It is obtained by measurement or by calculation.

Example: For an ideal crossed-loop antenna (N = 2), it is assumed that one element has a sine-shaped and the other a cosine-shaped directional pattern. The array manifold is expressed by the equation

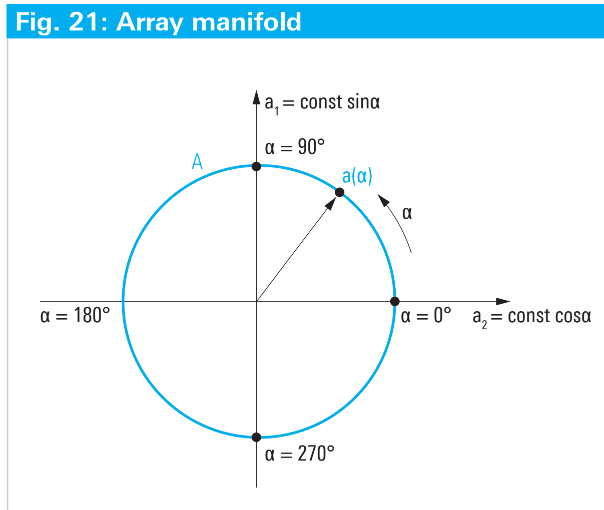
$$\mathbf{a}(\alpha) = \begin{pmatrix} \sin\alpha \\ \cos\alpha \end{pmatrix}$$ and describes a circle (Fig. 21).

Fig. 20: Array manifold



Array manifold characterizing an antenna array.

Fig. 21: Array manifold



Array manifold of a crossed-loop antenna.

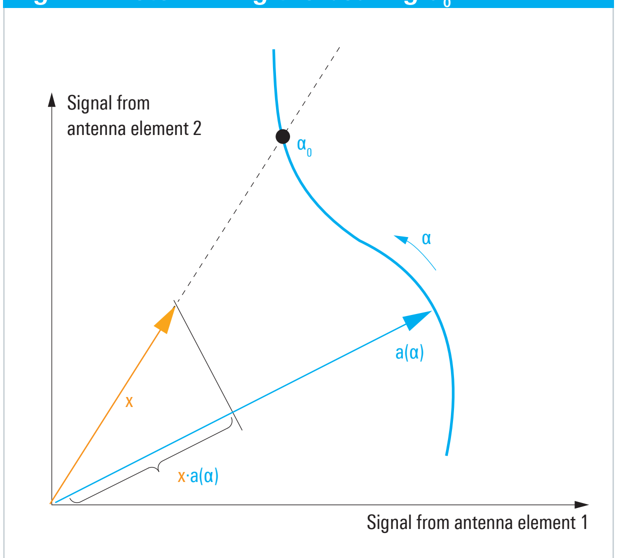
Solution for single-wave model

Assuming there is no noise, i.e. $\mathbf{n}(t) = 0$, the measured vector $\mathbf{x}(t)$ differs from the direction vector $a(\alpha_0)$ only with respect to length. To solve the DF task, the direction vector has to be found that is parallel to the measured vector. The degree of parallelism can be determined from the direction cosine between the two vectors, which is proportional to the scalar product (Fig. 22).

$$\mathbf{x} \cdot \mathbf{a}(\alpha) = \mathbf{x}^H \mathbf{a}(\alpha)$$

$\mathbf{x}^H$ is the vector whose conjugate has been taken and that has been transposed relative to x.

Fig. 22: Determining the bearing α_0



Determining the bearing α_0 from the direction vector a, which is parallel to the measured vector **x**.

Assuming a measured vector with noise superimposed on it, which is the case in practice, x and $a(\alpha_0)$ are no longer located on a straight line. The bearing can only be estimated as the best approximation between x and $a(\alpha_0)$:

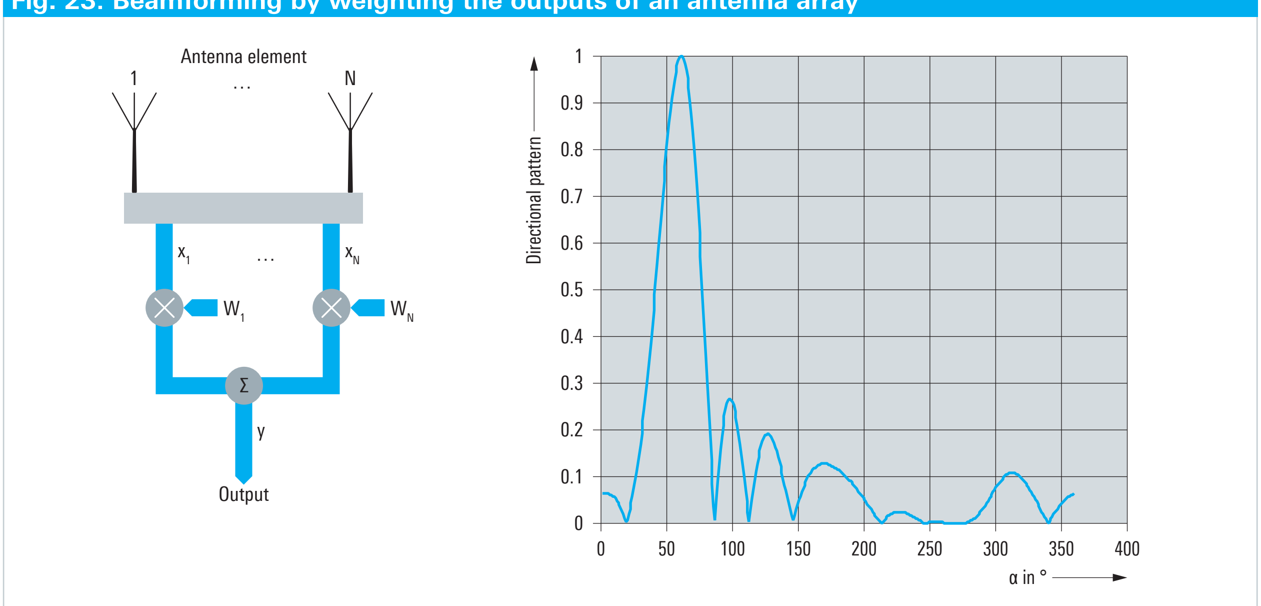
$$\hat{\alpha}_0 = \underset{\alpha}{\operatorname{argmax}} \left| \mathbf{x}^H \mathbf{a}(\alpha) \right|$$

This approach by estimation corresponds to the beamforming method. Like in conventional antenna arrays, the element signals x_i are multiplied by complex weighting factors $w_i = a_i^*(\alpha)$ and added together (Fig. 23). This yields a sum signal which, corresponding to the resulting directional pattern, depends on the direction of incidence α_0 of the wave and the look direction α. The asterisk (*) in the term above means conjugate complex.

The response of the output signal y to the variation of the weighting factors w_i is used for direction finding same as with a classic rotating or goniometer direction finder. The only difference is that – with numeric generation of the antenna pattern – the DF speed is limited only by the computing power.

3

Fig. 23: Beamforming by weighting the outputs of an antenna array



If antenna arrays with largely the same elements and an array geometry that can be described by analytical means are used, the weighting factors can in most cases directly be calculated from the array geometry. If multiport antennas are used (Fig. 24), the variation of the port voltages V_i as a function of the wave angle is as a rule determined by measurements.

Since beamforming using general multiport antennas often does not yield a distinct directivity of the (synthetic) antenna pattern, the following terms are used in this case as well:

- Correlation method
- Vector matching

Multiwave direction finding and super-resolution DF methods

If unwanted waves are received in addition to the wanted wave in the frequency channel of interest, the conventional beamforming method will lead to bearing errors as a function of the antenna geometry. There are two approaches to solve this problem:

- If the power of the unwanted wave component is lower than that of the wanted wave component, the direction finder can be dimensioned to minimize bearing errors, in particular by choosing a sufficiently wide antenna aperture (see [1], chapter on multiwave direction finding)
- If the unwanted wave component is greater than or equal to the wanted wave component, the unwanted waves must also be determined in order to eliminate them. When using conventional beamforming algorithms, this means that the secondary maxima of the DF function must also be evaluated. The limits are reached if either of the following occurs:
  - The ratio between the primary maximum and the secondary maxima of the directional pattern becomes too small
  - The angle difference between the wanted and the unwanted wave is less than the width of the main lobe

By optimizing the weighting factors, the level of the secondary maxima can be lowered, but the width of the main maximum is increased at the same time.

The super-resolution (SR) DF methods are to solve this problem.

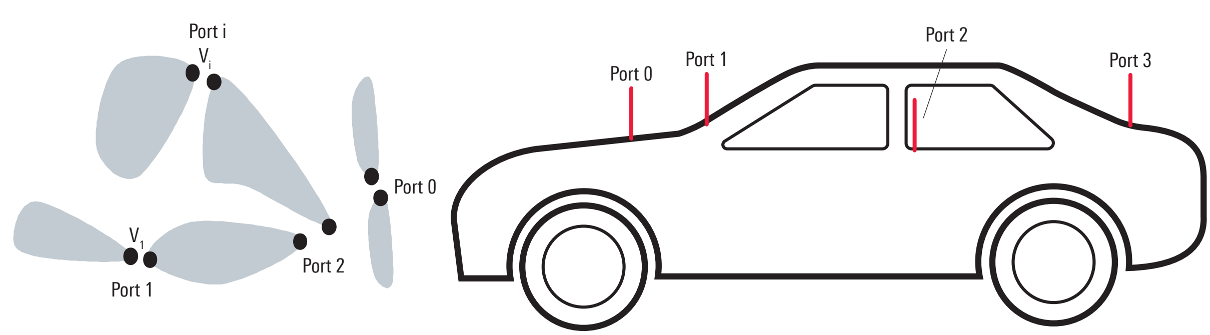
Minimum-signal direction finders are considered the grandfathers of the SR direction finders. In the early days of direction finding, bearings of co-channel signals were taken by alternately suppressing the waves by means of a rotating loop [1]. It is worth mentioning that signals can only be separated by audiomonitoring of the modulation, i.e. that correlation with acoustic patterns is required in order to determine the loop null.

Beamforming methods

Adaptive antennas are antenna arrays with beamformers that allow the automatic spatial suppression of unwanted waves [13], [14], [15]. In communications systems, this is done in order to optimize the signal-to-noise ratio; in direction finding, the weighting applied in order to suppress signals is used to determine the directions of incidence of the incoming waves.

The weighting of the beamformer is selected such that the output power is minimized under certain secondary conditions. In the case of the Capon beamformer [16], the secondary condition for setting the weighting is defined such that the antenna gain remains constant for a given direction α_r.

Fig. 24: General multiport antenna with typical application



If the incoming waves are uncorrelated, the beamformer is adjusted for nulls to occur in all signal directions except for direction α_r (Fig. 25).

If the direction of an incident wave coincides with the given direction α_r, there is a distinct maximum in the output power. Fig. 26 shows an example of the angular spectrum of a Capon beamformer with a nine-element circular array ($D/\lambda = 1.4$) and five uncorrelated waves.

As with a minimum-signal direction finder, the resolution highly depends on the signal-to-noise ratio.

Fig. 27 shows the same receiving scenario with noise increased by a factor of 10. The resolution of waves arriving at angles of 5° and 10° is no longer possible.

Subspace methods

Subspace methods are intended as a means of eliminating the effect of noise. The N-dimensional space opened up by the element outputs is split up into subspaces. The common MUSIC (multiple signal classification) algorithm makes use of the fact that signals lie perpendicular to the noise subspace. If the direction vectors are projected to the noise subspace, nulls that are independent of the noise level are obtained if signals are present [13], [17]. The reciprocal value is normally used as the DF function, so that distinct peaks occur at the directions of incidence of the signals (Fig. 28 on next page).

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Fig. 25: Super-resolution direction finding through nulling

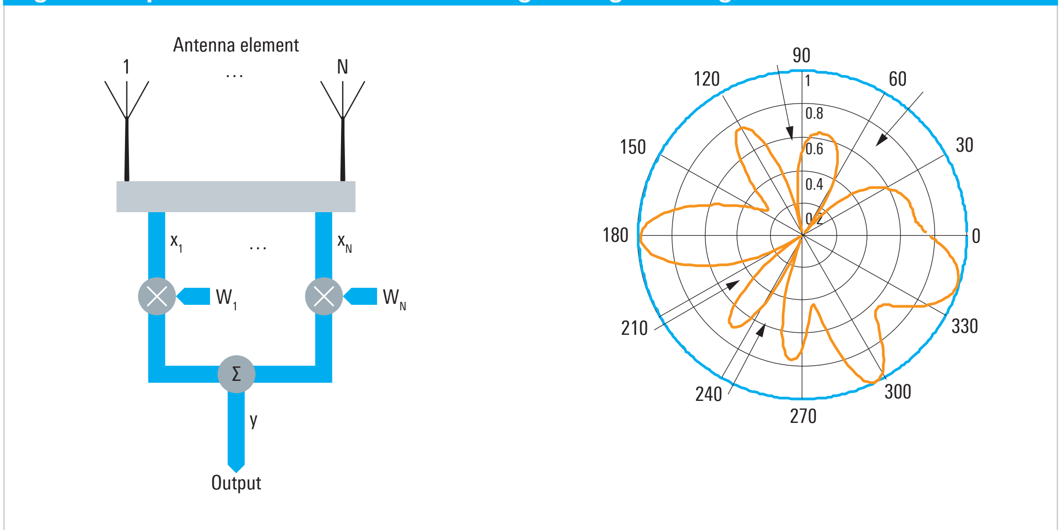
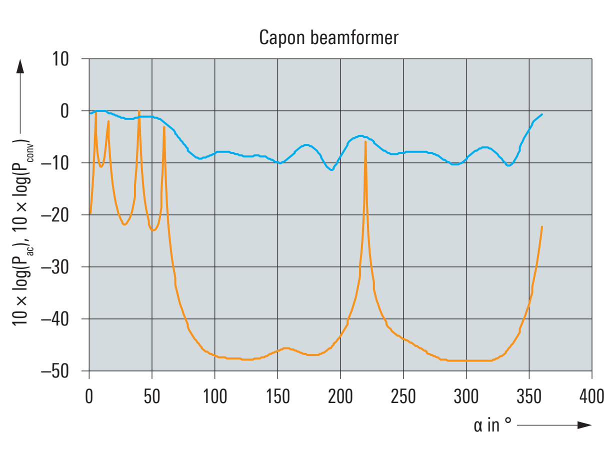
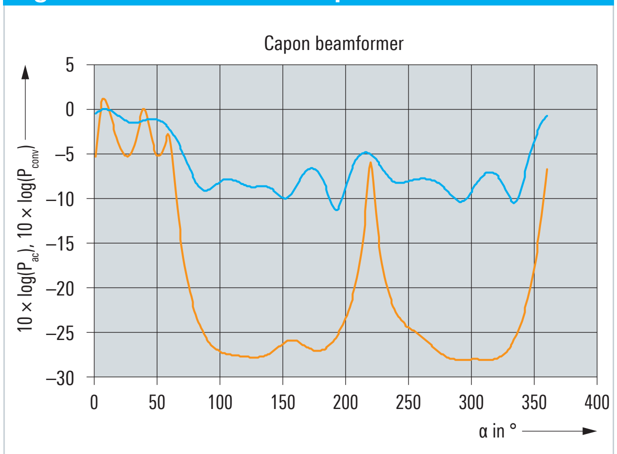


Fig. 26: DF function of Capon beamformer



Comparision with conventional beamformer (S/N = 100); wave angles: 5/15/40/60/220°.

Fig. 27: DF function of Capon beamformer



Comparision with conventional beamformer (S/N = 10).

Display of bearings

Operators of direction finders rely heavily on the display of DF results, which can generally be divided into two categories:

- Results from a direction finder operating at a single frequency channel
- Results from a multichannel direction finder

Single-channel direction finders

If a bearing is to be taken of a single frequency channel, the following parameters are usually displayed:

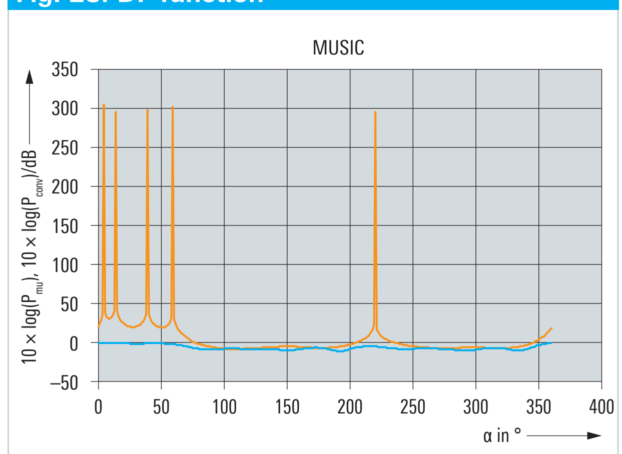
- Bearing as a numeric value
- Azimuth as a polar diagram
- Elevation as a bargraph or a polar diagram (combined with azimuth display)
- DF quality
- Level
- Bearing histogram
- Bearings versus time (waterfall)

Fig. 29 shows a choice of possible displays.

In addition to the usual receiver settings such as frequency and bandwidth, the following parameters are set and displayed on direction finders:

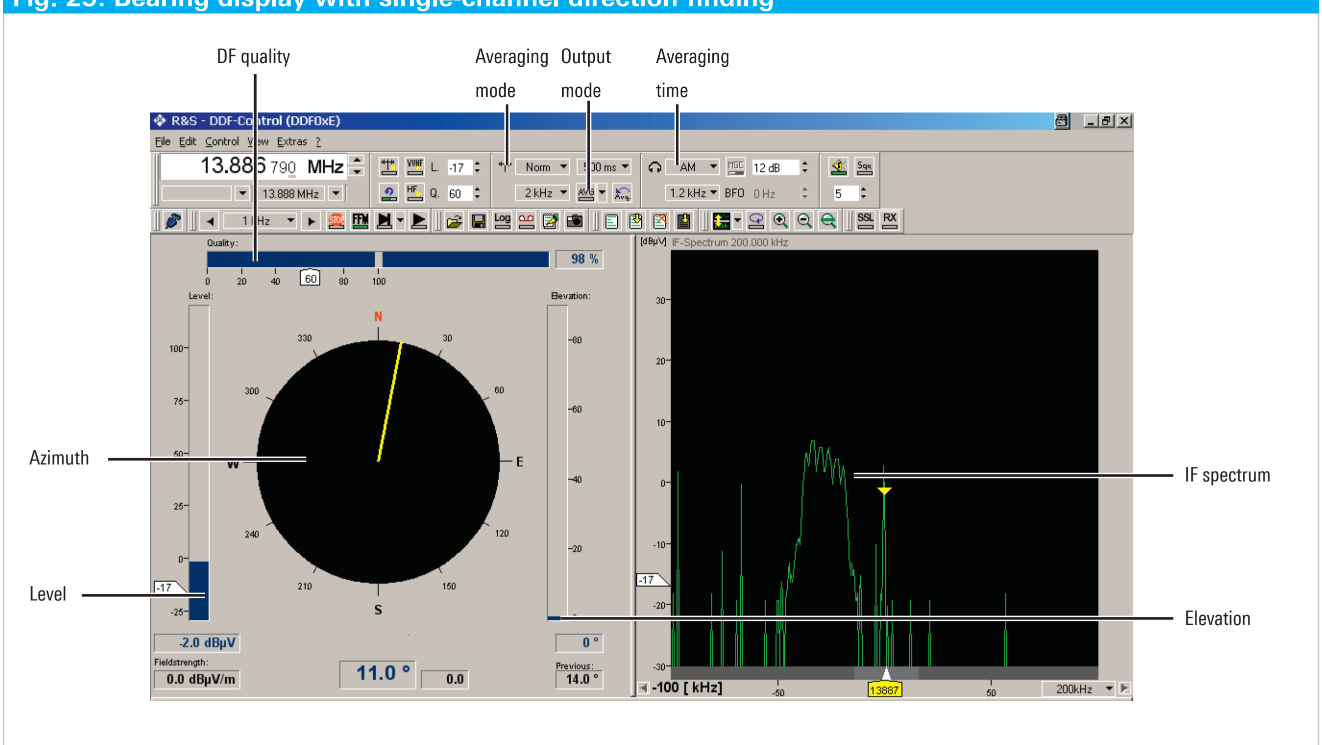
- Averaging mode (If the signal level drops below the preset threshold, averaging is stopped and either continued or restarted the next time the threshold is exceeded, depending on the averaging mode.)
- Averaging time
- Output mode
  - Refresh rate of display
  - Output of results as a function of the signal threshold being exceeded

Fig. 28: DF function



With MUSIC algorithm (S/N = 10).

Fig. 29: Bearing display with single-channel direction finding



Multichannel direction finders

Multichannel direction finders are implemented by means of digital filter banks (FFT and polyphase filters). Depending on the configuration level, this enables quasi-parallel direction finding in a frequency range from a few 100 kHz up to a few 10 MHz. Larger frequency ranges can be covered by direction finding in the scan mode (Fig. 30).

With multichannel direction finders, it is essential that the individual events can be quickly recognized and activities taking place on different channels correctly assigned. Therefore, the following display modes are usually provided:

- Bearings versus frequency
- Bearings versus frequency and time (e.g. by displaying the bearings in different colors)
- Level versus frequency (power spectrum)
- Level versus time and frequency (e.g. by displaying the level values in different colors)
- Histograms

Fig. 30: Multichannel direction finder

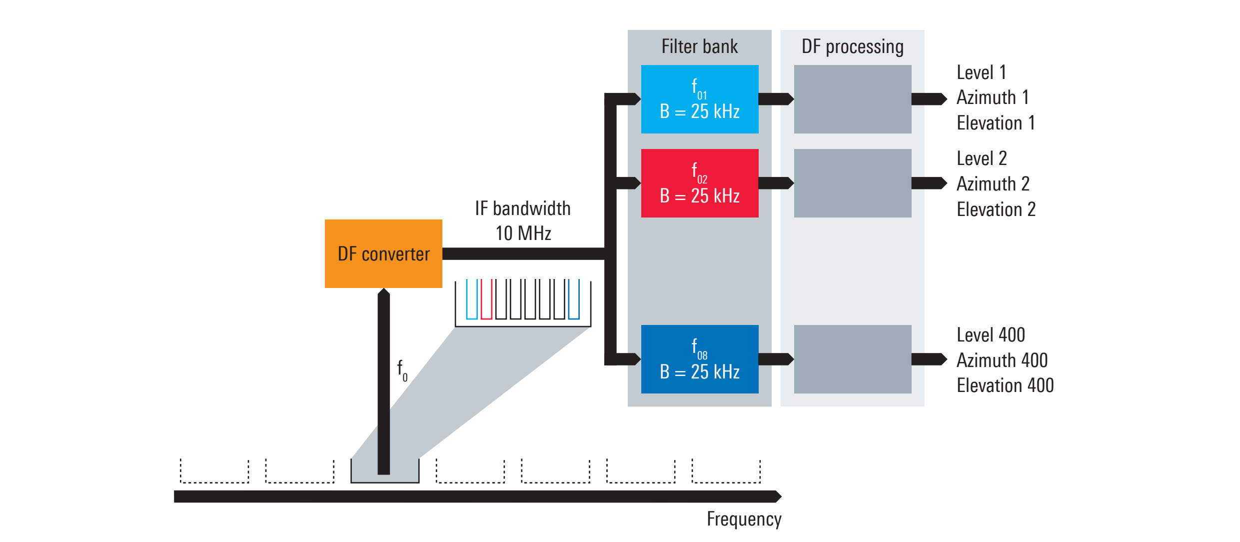
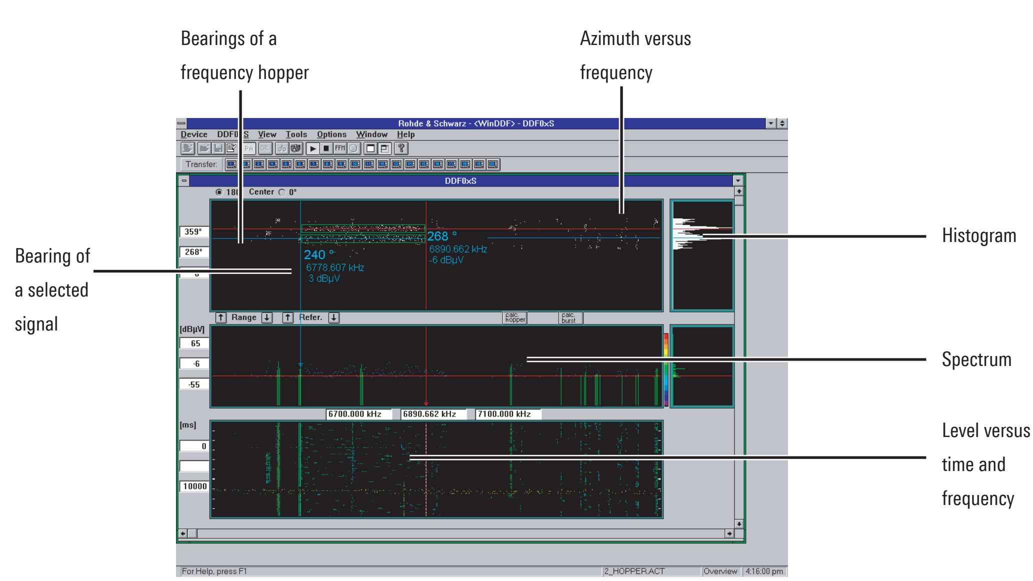


Fig. 31: Multichannel (wideband) display



Processing of bearings

Position finding

Bearings can be used in various ways to estimate the position of an emitter of interest, depending on the degree of sophistication of the DF system and the achievable accuracy. Maximum accuracy is attained if several direction finders are employed to determine the emitter position by way of triangulation [1]. If more than two bearings are used for position finding, an ambiguous result is usually obtained (Fig. 32).

The most probable position can be calculated in a variety of ways. For example, the position can be determined by minimizing the error squares or by maximum likelihood estimation.

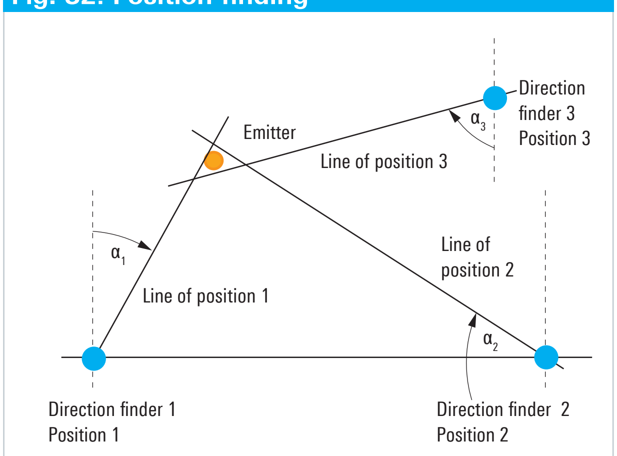
If a moving direction finder takes several bearings of an emitter, the emitter position can be determined by a running fix (Fig. 33). The emitter may move only slightly relative to the DF platform and must be active long enough to fix its position.

Emitters operating in the shortwave range can be located by means of a single direction finder under certain conditions. The direction finder must be able to measure azimuth and elevation, and the virtual height of the reflecting ionosphere layer must be known (Fig. 34).

Emitter preclassification

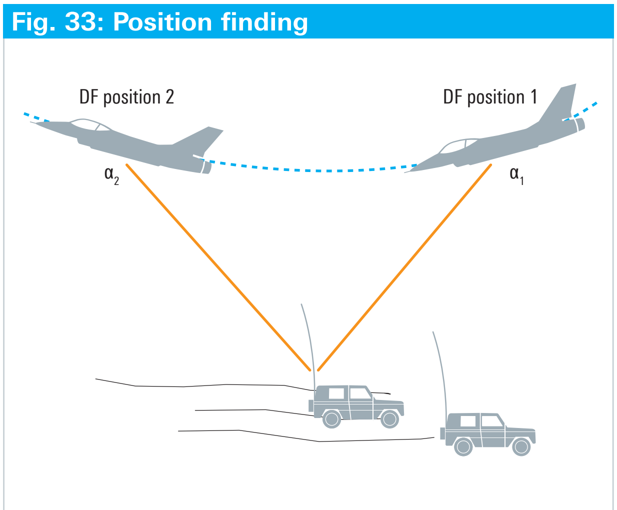
Bearings are vital characteristics of a detected signal. They facilitate quick segmentation, i.e. the assignment of subspectra to the overall spectrum of an emitter. This makes it possible to determine the center frequency and the bandwidth of an emission, which allows its automatic transfer to a hand-off receiver for analysis. Cluster analysis of bearings makes it possible to separate the signals of emitters operating in overlapping spectra, especially those of frequency hoppers.

Fig. 32: Position finding



Triangulation.

Fig. 33: Position finding



Running fix.

Fig. 34: Single station location

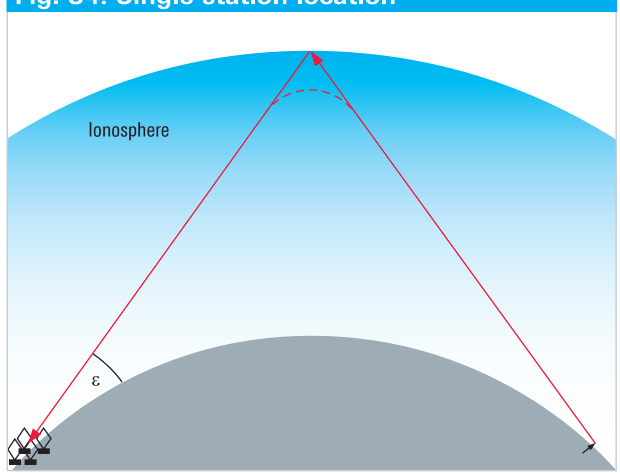
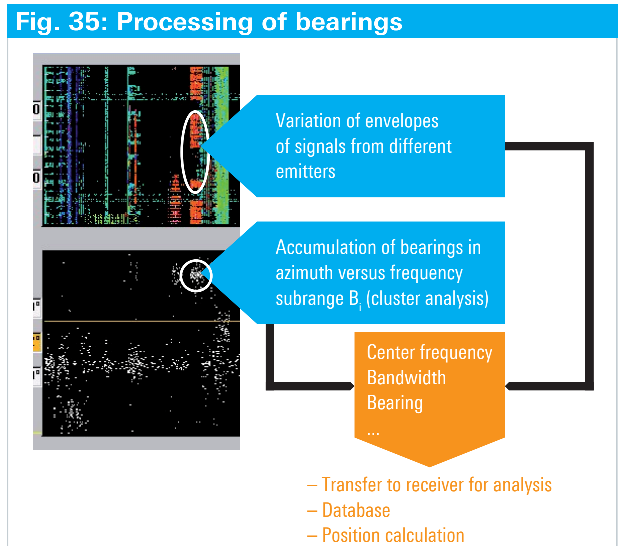


Fig. 35: Processing of bearings



DF accuracy

DF accuracy is affected by a number of factors:

- Wave propagation is usually disturbed by obstacles
- Signals radiated by emitters
  - are modulated
  - are limited in time
  - frequently operate at unknown carrier frequencies
- The following are superimposed on the reception field:
  - noise
  - co-channel interferers
- Noise and tolerances in the DF system

Noise

In addition to intermodulation distortion, interference caused by noise has a limiting effect on the sensitivity of a DF system.

Fig. 36: Model describing signal and noise

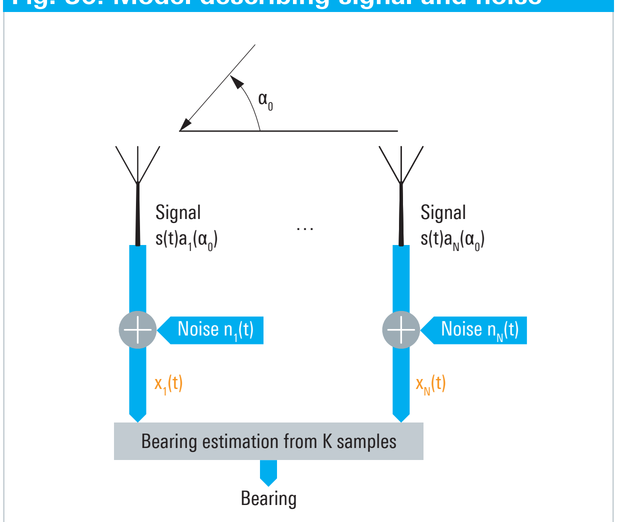
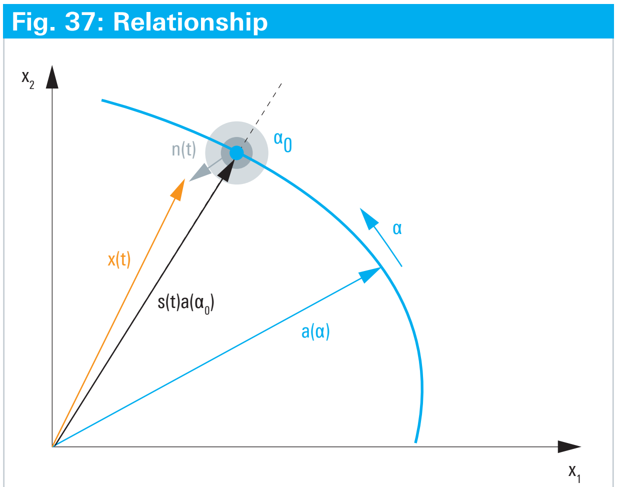


Fig. 37: Relationship



Relationship between bearing fluctuation and signal-to-noise ratio.

Sensitivity is understood to be the field strength at which the bearing fluctuation remains below a certain standard deviation.

Noise may occur in the following form:

- External noise (atmospheric, galactic, industrial noise)
- System-inherent noise (antenna amplifiers, DF converters, A/D converters)

The following basic considerations apply to system-inherent noise.

Uncorrelated noise in the receive sections (Fig. 36) causes statistically independent variations of the measured signals as a function of the S/N ratio. The variations become noticeable as bearing fluctuations.

The bearing fluctuation of general antenna arrays can be determined by comparing the variation of the measured vector **x** with the corresponding variation of α on the curve a(α). Fig. 37 shows this relationship for a two-element array. The "faster" the antenna vector "moves", the smaller the effect of the variation of the measured vector on the variation of α [17].

The relationship between the minimum bearing fluctuation and the signal-to-noise ratio for a given antenna geometry (expressed by a(α)) is determined by the Cramer-Rao bound (CRB) [13].

For a general antenna array at whose outputs K samples are taken, the variance σ^2 of the bearing fluctuation is therefore defined by the following equation:

$$\sigma^2 \geq \sigma^2_{CRB} = \frac{1}{2KSNR^2}\,\frac{1+2SNR\mathbf{a}^H\mathbf{a}}{\mathbf{a}^H\mathbf{a}}\left[\frac{\partial\mathbf{a}}{\partial\alpha}^H\left(1-\frac{\mathbf{a}\mathbf{a}^H}{\mathbf{a}^H\mathbf{a}}\right)\frac{\partial\mathbf{a}}{\partial\alpha}\right]^{-1}$$

The equation simplifies if omnidirectional receiving elements are used and the antenna array is of conjugate symmetrical design:

$$\sigma^2_{CRB} = \frac{1}{2KNSNR^2}\left(1+NSNR\right)\left\|\frac{\partial\mathbf{a}}{\partial\alpha}\right\|^{-2} .$$

For the important case of a circular array with diameter D and N omnidirectional receiving elements, the following applies at a constant elevation angle ε:

$$\sigma^2_{CRB\_UCA} = \frac{\lambda^2}{\pi^2KD^2\cos^2\varepsilon}\left(\frac{1}{NSNR}+\frac{1}{N^2SNR^2}\right) .$$

Fig. 38 shows the typical effect of the antenna diameter, the wavelength and the number of elements of a circular array on the S/N ratio that is required for a specific bearing fluctuation.

If direction finding is based solely on amplitude evaluation, as is the case with the Watson-Watt method, the Cramer-Rao bound is determined by:

$$\sigma^2_{CRB\_WW} = \frac{1}{2K}\left(\frac{1}{SNR} + \frac{1}{2SNR^2}\right)$$

The effect of the S/N ratio on the bearing fluctuation is particularly obvious in the case of a narrowband two-element interferometer. In narrowband systems, the noise voltage becomes approximately sine-shaped with slowly varying amplitude and phase so that, assuming large S/N ratios, the following applies to the phase variation [19]:

$$\sigma^2_{\varphi} = \frac{1}{2SNR}$$

Given a sufficiently long observation time, the phase variation can be reduced by way of averaging. If the data used for averaging is uncorrelated, averaging over K samples improves the phase variation as follows:

$$\sigma^2_{\varphi av} = \frac{\sigma^2_{\varphi}}{K}$$

By mapping the phase variation to virtual variations of the positions of the DF antennas (Fig. 39), the bearing standard deviation is as follows:

$$\sigma \cong \frac{\lambda}{\pi D\sqrt{2SNRK}}$$

For N = 2, this corresponds to the equation for $\sigma^2_{CRB\_UCA}$ mentioned above, if the term

$$\frac{1}{N^2 SNR^2}$$

which can be ignored for large S/N ratios, is removed and ε is set equal to 0.

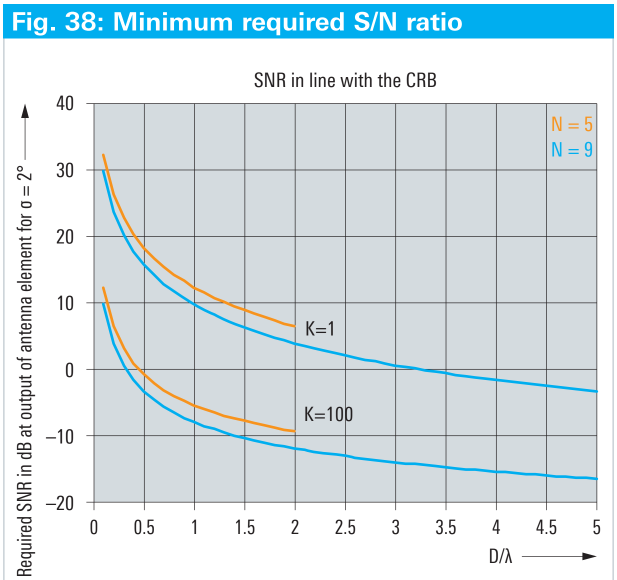
The above relationship clearly shows how important it is that the relative antenna basis D/λ is as large as possible.

Limitation of upper operating frequency because of ambiguity

The required S/N ratio decreases as the frequency increases, because the length of the array manifold **a**(α) also increases. With a constant number N of antenna elements (= dimension of the observation space), however, various sections of the curve **a**(α) approach each other (Fig. 40 on next page). At the same time, the probability of measured vectors being positioned on such curve sections increases. This leads to large bearing errors [17].

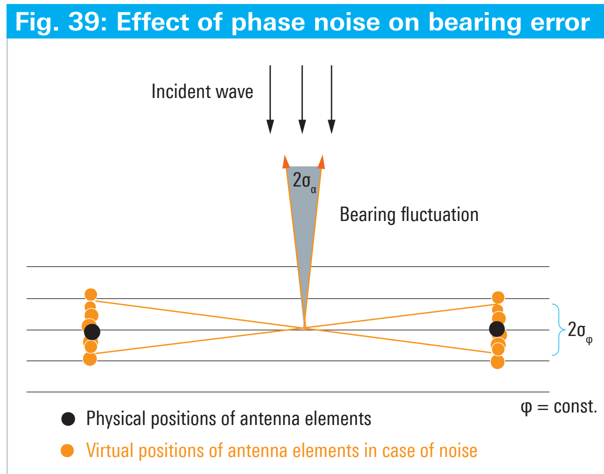
The problem can be solved by increasing the number of elements N: The increase in dimension of the observation space will accommodate the extended length of the array manifold.

Fig. 38: Minimum required S/N ratio



Minimum SNR for a bearing fluctuation (standard deviation) of 2° as a function of the antenna diameter as referenced to the wavelength and the number K of samples taken.

Fig. 39: Effect of phase noise on bearing error



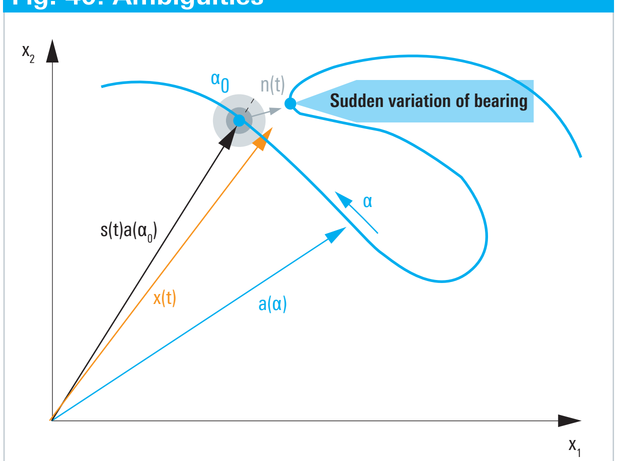
Measurement errors

Different gain and phase responses in the receive sections cause bearing errors that increase as the antenna aperture decreases relative to the wavelength. Fig. 41 shows this effect for a two-element interferometer.

Prior to the DF operation, the receive sections of most multipath direction finders are calibrated for synchronization by means of a test generator. The magnitude and phase responses are measured, and the level and phase differences are stored. In DF operation, the measured values are corrected by the stored difference values before the bearing is calculated.

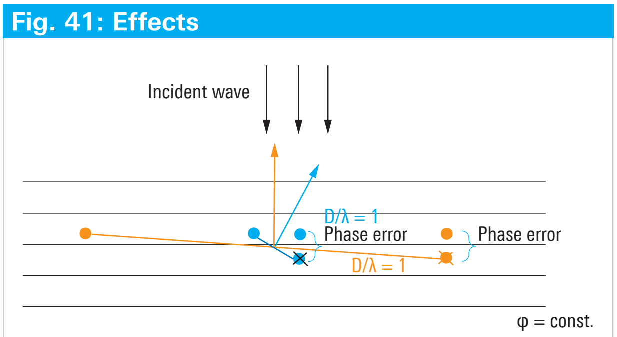
As regards the frequency response of the filters, synchronization must be ensured not only in the middle of the filter passband but also at the band limits. Digital filters have the decisive advantage that they can be implemented with absolutely identical transmission characteristics.

Fig. 40: Ambiguities



As D/λ increases, various sections of the curve a(α) approach each other. This causes large, sudden variations of the bearing if the S/N ratio and the number of elements remain constant.

Fig. 41: Effects



Impact of phase synchronization tolerances on bearing error for different antenna apertures.

Multiwave-related problems

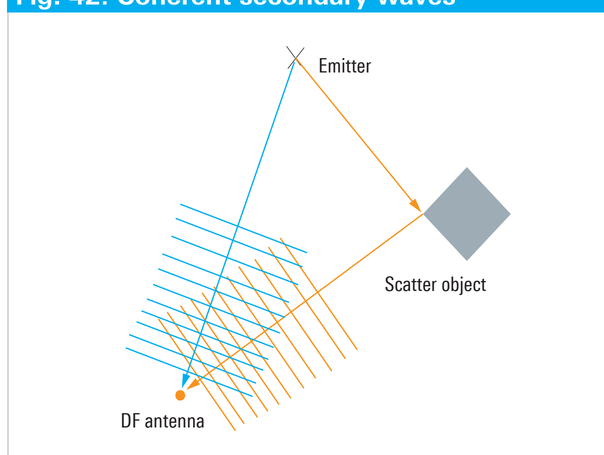
As already mentioned, the simple case of a plane wave hardly ever occurs in practice. In a real environment, other waves usually have to be taken into account:

- Waves from other emitters operating on the same frequency channel ▷ incoherent interference
- Secondary waves (caused by reflection, refraction, diffraction – see Fig. 42) ▷ coherent co-channel interference (prerequisite: detours are small relative to the coherence lengths determined by bandwidth B)

In practice, a large number of waves is involved [20]. Fig. 43 shows a typical azimuth distribution of waves generated by a mobile emitter in a built-up area. The direct wave component with amplitude 1 arrives at an angle of 90°.

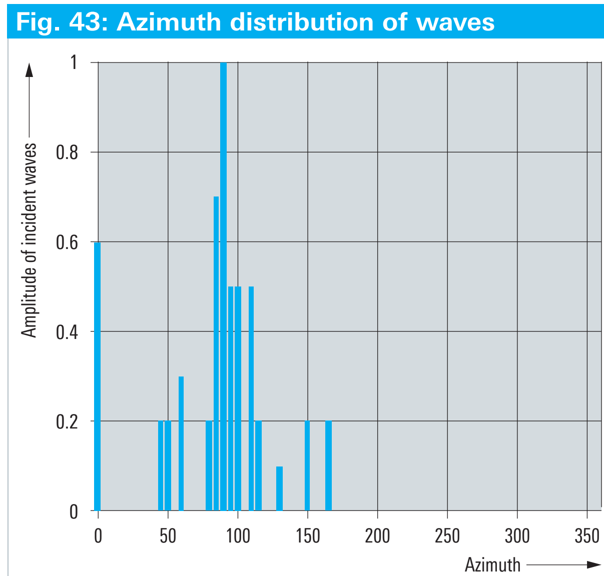
3

Fig. 42: Coherent secondary waves



Secondary waves caused by reflection.

Fig. 43: Azimuth distribution of waves



Waves radiated by an emitter in a built-up area.

Fig. 44 shows the resulting wavefront as a contour plot for the amplitude and phase [12].

If the majority of waves arrives from the direction of the emitter, the bearing error can be sufficiently reduced by increasing the aperture of the antenna system. This effect is shown by Fig. 45 for an interferometer direction finder. The pattern of the isophase curves in Fig. 44 shows that two measures are required in order to increase the DF accuracy:

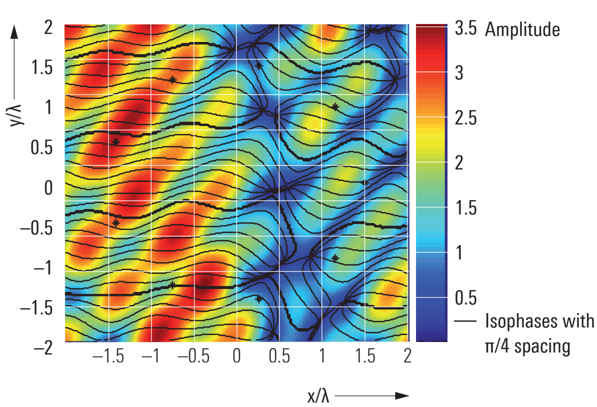
- The antenna aperture should be as wide as possible
- The number of antenna elements should be significantly higher than with interference-free reception

Fig. 46 shows the positive effect of a wide antenna aperture on the DF accuracy for a two-element interferometer. Arranging the antenna elements at a spacing that is large relative to the operating wavelength will prevent bearings from being obtained at points where the isophase curves strongly bend.

The effect of the number of antenna elements is obvious when looking at the density of the isophase curves. To avoid 180° ambiguities in areas of high density, the spacing between the antenna elements should be smaller in these areas than with interference-free reception. This requires a higher number of antenna elements.

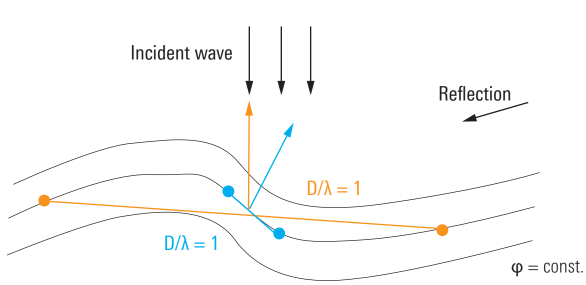
Fig. 46 shows the effect of the antenna size and the number of antenna elements on the bearing error for two circular antennas in a two-wave field with an amplitude ratio of 0.3 of the reflected wave to the direct wave. The nine-element antenna delivers increasingly more accurate bearings up to a diameter of five wavelengths, whereas the bearings obtained with the five-element antenna become instable already from a diameter of 1.6 wavelengths.

Fig. 44: Resulting field



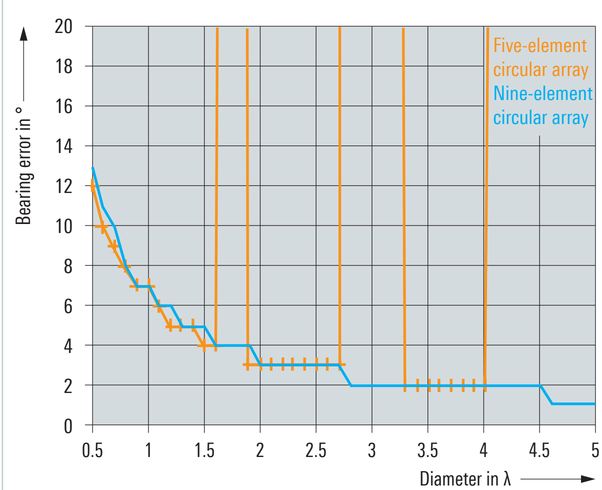
Field within a range of 4 × 4 wavelengths (the amplitudes are color-coded; the lines represent isophases with π/4 spacing).

Fig. 45: Reducing bearing error



Error of interferometer direction finder reduced by increasing antenna aperture.

Fig. 46: Bearing errors



Errors of a five- and a nine-element circular array antenna as a function of the antenna diameter expressed in operating wavelengths in a two-wave field.

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