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Chaos synchronization of nonlinear Bloch equations

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Abstract

In this paper, the problem of chaos synchronization of Bloch equations is considered. A novel nonlinear controller is designed based on the Lyapunov stability theory. The proposed controller ensures that the states of the controlled chaotic slave system asymptotically synchronizes the states of the master system. A numerical example is given to illuminate the design procedure and advantage of the result derived.

1. Introduction

Chaos is very interesting nonlinear phenomenon and has been intensively studied in the last three decades [1–11]. It is found to be useful or has great potential in many disciplines [1]. Especially, the subject of chaotic synchronization has received considerable attentions since 1990. In the literature, various synchronization schemes, such as variable structure control, OGY method, parameters adaptive control, observer-based control, active control, time-delay feedback approach, backstepping design technique, and so on, have been successfully applied to the chaos synchronization. Using these methods, numerous works for the synchronization problem of well-known chaotic systems such as Lorenz, Chen, Lü, and Rossler systems have been done by many scientist.

On the other hand, the dynamics of an ensemble of spins which do not exhibit mutual coupling, except for some interactions leading to relaxation, is well described by the simple Bloch equations. Recently, Abergel [12] examined the linear set of equations originally proposed by Bloch to describe the dynamics of an ensemble of spins with minimal coupling, and incorporated certain nonlinear effects that were caused by a radiation damping based feedback field. Ucar et al. [13] extend the calculation of Abergel [12] and demonstrate that is is possible to synchronize two of these nonlinear Bloch equations. However, these works are based on the exactly knowing of the system parameters. But in real situation, some or all of the parameters are unknown.

In this paper, the chaotic synchronization of nonlinear Bloch equations with uncertain parameters is investigated. A class of novel nonlinear control scheme for the synchronization is proposed, and the synchronization is achieved by the Lyapunov stability theory.

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The organization of this paper is as follows. In Section 2, the problem statement and master–slave synchronization scheme are presented for the chaotic system. In Section 3, we provide an numerical example to demonstrate the effectiveness of the proposed method. Finally concluding remark is given.

2. Chaos synchronization

In dimensionless units, the dynamic model of nonlinear modified Bloch equations with feedback field [12] is given by

$$\begin{cases} \dot{x} = \delta y + \lambda z(x\sin\psi - y\cos\psi) - \frac{x}{\tau_2}, \\ \dot{y} = -\delta x - z + \lambda z(x\cos\psi + y\sin\psi) - \frac{y}{\tau_2}, \\ \dot{z} = y - \lambda\sin\psi(x^2+y^2) - \frac{z-1}{\tau_1}, \end{cases} \tag{1}$$

where δ, λ and ψ are the system parameters and τ_1 and τ_2 are the longitudinal time and transverse relaxation times, respectively. In the work of Abergel [12], the dynamic behavior of the system has been extensively investigated for a fixed subset of the system parameters $(\delta, \lambda, \tau_1, \tau_2)$ and for a space area range of the radiation damping feedback ψ. Especially, the regions of the parameter ψ that would admit chaotic behavior were derived. For instance, when the parameters are $\delta = -0.4\pi$, $\lambda = 30, \psi = 0.173$, $\tau_1 = 5$, $\tau_2 = 2.5$, the system is actually chaotic [13]. For details of other dynamic properties of the system, see the paper [12,13].

In order to observe the synchronization behavior in Bloch equations, when some parameters of the drive system are fully unknown and different with those of the response system, we assume that we have two Bloch equations where the master system with the subscript m drives the slave system having identical equations denoted by the subscript s. For the systems (1), the master (or drive) and slave (or response) systems are defined below, respectively,

$$\begin{cases} \dot{x}_m = \delta y_m + \lambda z_m(x_m\sin\psi - y_m\cos\psi) - \frac{x_m}{\tau_2}, \\ \dot{y}_m = -\delta x_m - z_m + \lambda z_m(x_m\cos\psi + y_m\sin\psi) - \frac{y_m}{\tau_2}, \\ \dot{z}_m = y_m - \lambda\sin\psi(x_m^2+y_m^2) - \frac{z_m-1}{\tau_1}, \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{x}_s = \delta_1 y_s + \lambda_1 z_s(x_s\sin\psi - y_s\cos\psi) - \frac{x_s}{\tau_2} + u_1, \\ \dot{y}_s = -\delta_1 x_s - z_s + \lambda_1 z_s(x_s\cos\psi + y_s\sin\psi) - \frac{y_s}{\tau_2} + u_2, \\ \dot{z}_s = y_s - \lambda_1\sin\psi(x_s^2+y_s^2) - \frac{z_s-1}{\tau_1} + u_3, \end{cases} \tag{3}$$

where δ_1 and λ_1 are parameters of the slave system which needs to be estimated, and u_1, u_2 and u_3 are the nonlinear controller such that two chaotic systems can be synchronized.

Define the error signal as

$$\begin{cases} e_1(t) = x_s(t) - x_m(t), \\ e_2(t) = y_s(t) - y_m(t), \\ e_3(t) = z_s(t) - z_m(t). \end{cases} \tag{4}$$

By differentiating Eq. (4), we have the following error dynamics:

$$\begin{aligned} \dot{e}_1(t) &= \delta_1 y_s - \delta y_m + \lambda_1 z_s(x_s\sin\psi - y_s\cos\psi) - \lambda z_m(x_m\sin\psi - y_m\cos\psi) - \frac{1}{\tau_2}e_1 + u_1, \\ \dot{e}_2(t) &= -\delta_1 x_s + \delta x_m - e_3 + \lambda_1 z_s(x_s\cos\psi + y_s\sin\psi) - \lambda z_m(x_m\cos\psi + y_m\sin\psi) - \frac{1}{\tau_2}e_2 + u_2, \\ \dot{e}_3(t) &= e_2 - \lambda_1\sin\psi(x_s^2+y_s^2) + \lambda\sin\psi(x_m^2+y_m^2) - \frac{1}{\tau_1}e_3 + u_3. \end{aligned} \tag{5}$$

Here, our goal is to make synchronization between two Bloch equations by using adaptive control scheme u_i, $i = 1,2,3$ when some parameters of the drive system are unknown and different with those of the response system, i.e.,

$$\lim_{t\to\infty} \|e(t)\| = 0,$$

where $e = [e_1 \ e_2 \ e_3]^{\mathrm{T}}$.

For two identical chaotic systems without control ($u_i = 0$), if the initial condition $(x_m(0), y_m(0), z_m(0)) \neq (x_s(0), y_s(0), z_s(0))$, the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled chaotic systems, the two systems will approach synchronization for any initial condition by appropriate controller scheme. For this end, we propose the following control law for the system (3):

$$\begin{aligned} u_1 &= -k_1e_1 - \lambda_1 \sin\psi(z_sx_s - z_mx_m) + \lambda_1 \cos\psi(z_sy_s - z_my_m) \\ u_2 &= -k_2e_2 - \lambda_1 \cos\psi(z_sx_s - z_mx_m) - \lambda_1 \sin\psi(z_sy_s - z_my_m), \\ u_3 &= -k_3e_3 + \lambda_1 \sin\psi[e_1(x_m + x_s) + e_2(y_m + y_s)], \end{aligned} \tag{6}$$

and the update rule for two unknown parameters δ and λ

$$\begin{aligned} \dot{\delta}_1 &= -y_me_1 + x_me_2, \\ \dot{\lambda}_1 &= -z_m(x_m \sin\psi - y_m \cos\psi)e_1 - z_m(x_m \cos\psi + y_m \sin\psi)e_2 + \sin\psi(x_m^2 + y_m^2)e_3, \end{aligned} \tag{7}$$

where k_1, k_2 and k_3 are positive scalars.

Then, we have the following theorem.

Theorem 1. *The two Bloch systems (2), (3) are synchronized for any initial conditions* $(x_m(0), y_m(0), z_m(0))$ *and* $(x_s(0), y_s(0), z_s(0))$ *by the control law (6) and update law (7).*

Proof. Choose the following Lyapunov candidate:

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2) \tag{8}$$

where $e_a = \delta_1 - \delta$ and $e_b = \lambda_1 - \lambda$.

The differential of the Lyapunov function along the trajectory of error system (5) is

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} &= \dot{e}_1e_1 + \dot{e}_2e_2 + \dot{e}_3e_3 + \dot{e}_ae_a + \dot{e}_be_b \\ &= e_1\left[\delta_1y_s - \delta y_m + \lambda_1z_s(x_s \sin\psi - y_s \cos\psi) - \lambda z_m(x_m \sin\psi - y_m \cos\psi) - \frac{1}{\tau_2}e_1 + u_1\right] \\ &\quad + e_2\left[-\delta_1x_s + \delta x_m - e_3 + \lambda_1z_s(x_s \cos\psi + y_s \sin\psi) - \lambda z_m(x_m \cos\psi + y_m \sin\psi) - \frac{1}{\tau_2}e_2 + u_2\right] \\ &\quad + e_3\left[e_2 - \lambda_1 \sin\psi(x_s^2 + y_s^2) + \lambda \sin\psi(x_m^2 + y_m^2) - \frac{1}{\tau_1}e_3 + u_3\right] + \dot{\delta}_1(\delta_1 - \delta) + \dot{\lambda}_1(\lambda_1 - \lambda). \end{aligned} \tag{9}$$

Substituting Eq. (7) into Eq. (9) gives that

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} &= [\lambda_1 \sin\psi(z_sx_s - z_mx_m) - \lambda_1 \cos\psi(z_sy_s - z_my_m)]e_1 - \frac{1}{\tau_2}e_1^2 \\ &\quad + e_1u_1 + [\lambda_1 \cos\psi(z_sx_s - z_mx_m) + \lambda_1 \sin\psi(z_sy_s - z_my_m)]e_2 - \frac{1}{\tau_2}e_2^2 + e_2u_2 \\ &\quad - \lambda_1 \sin\psi[e_1e_3(x_m + x_s) + e_2e_3(y_m + y_s)] - \frac{1}{\tau_1}e_3^2 + e_3u_3. \end{aligned} \tag{10}$$

Again, substituting Eq. (6) into Eq. (10) gives that

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -(k_1 + 1/\tau_2)e_1^2 - (k_2 + 1/\tau_2)e_2^2 - (k_3 + 1/\tau_1)e_3^2 = -e^{\mathrm{T}}Pe \tag{11}$$

where

$$P=\begin{bmatrix} k_1+(1/\tau_2) & 0 & 0 \\ 0 & k_2+(1/\tau_2) & 0 \\ 0 & 0 & k_3+(1/\tau_1) \end{bmatrix}.$$

Since $\dot{V}$ is negative semidefinite, we cannot immediately obtain that the origin of error system (5) is asymptotically stable. In fact, as $\dot{V}\leqslant 0$, then $e_1,e_2,e_3,e_a,e_b\in\mathscr{L}_\infty$. From the error system (5), we have $\dot{e}_1,\dot{e}_2,\dot{e}_3\in\mathscr{L}_\infty$. Since $\dot{V}=-e^{\mathrm{T}}Pe$ and P is a positive-definite matrix, then we have

$$\int_0^t \lambda_{\min}(P)\|e\|^2\mathrm{d}t\leqslant\int_0^t e^{\mathrm{T}}Pe\ \mathrm{d}t\leqslant\int_0^t -\dot{V}\mathrm{d}t=V(0)-V(t)\leqslant V(0),$$

where $\lambda_{\min}(P)$ is the minimum eigenvalue of positive-definite matrix P. Thus $e_1,e_2,e_3\in\mathscr{L}_2$. According to the Barbalat's lemma, we have $e_1(t)$, $e_2(t)$, $e_3(t)\to 0$ as $t\ \to\infty$. Therefore, the response system (3) synchronize the drive system (2) by the controller (6). This completes the proof. □

Remark 1. The rate of convergence of error signals can be controlled by the adjusting the values of the parameters k_1, k_2, and k_3.

3. Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for Bloch equations. In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems with time step size 0.001.

For this numerical simulation, we assume that the initial condition, $(x_m(0),y_m(0),z_m(0))=(0.5,-0.5,0)$, and $(x_s(0),y_s(0),z_s(0))=(-0.5,0.5,0.3)$ is employed. Hence the error system has the initial values $e_1(0)=-1$, $e_2(0)=1$ and $e_3(0)=0.3$. In simulation, the radiation damping feedback ψ, τ_1, and τ_2 are fixed as 0.173, 5 and 2.5, respectively, and the two unknown parameters are chosen as $\delta=-0.4\pi$ and $\lambda=35$ so that the Bloch equations exhibits a chaotic behavior. Synchronization of the systems (2) and (3) via adaptive control law (6) with $k_i=5$, $i=1$, 2, 3 and (7) with the initial estimated parameters $\delta_1(0)=0$ and $\lambda_1(0)=25$ are shown in Fig. 1. The figure display the state responses of systems (2), (3).

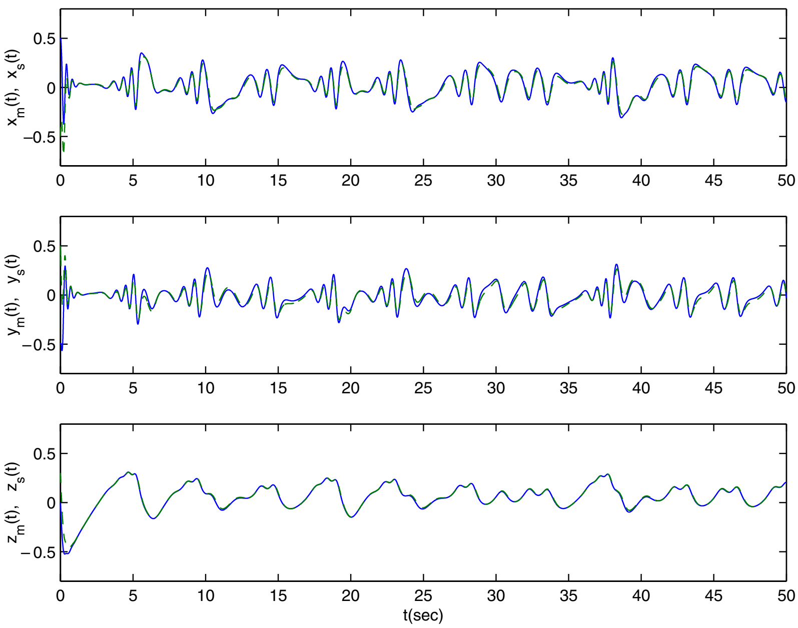

Fig. 1. Responses of two Bloch systems.

4. Concluding remark

In this paper, we investigate the synchronization of controlled nonlinear Bloch equations. We have proposed a novel nonlinear control scheme for asymptotic chaos synchronization using the Lyapunov method. Finally, a numerical simulation is provided to show the effectiveness of our method.

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