Two-dimensional Semi-Lagrangian Vlasov Simulations of the Relativistic Modulational Instability

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Abstract

A semi-lagrangian two-dimensional full relativistic Vlasov code for multi-computer environment was developped to study the accelerated particle dynamics in phase space induced by the relativistic modulational instability in a strong nonlinear regime. Attention was focused on its accuracy, stability, and implementation facilities on parallel computers, efficiency properties of the model which allows a very good description of particle acceleration mechanism in phase space. Vlasov simulations show the occuring of coherent vortex structures due to the nonlinear saturation mechanism of the Relativistic Modulational Instability.

1 Introduction:

Newly developped high-intensity lasers have brought new attention of the field of electromagnetic wave propagation in plasmas in the nonlinear regime where the electron quiver velocity approaches the light velocity. As these high-intensity laser have very short pulses (less than the picosecond), the ion motion can be neglected, so that we have to consider electron parametric instability. At moderate intensities, these instabilities are clearly identified as the Stimulated Raman Scattering (SRS), the Relativistic Modulational Instability (RMI), the Relativistic Filamentation Instability (RFI) and in particular Two-plasmon Decay (TPD). This last instability normally occurs at the quarter critical density $n = n_c/4$. Recently a study of the fully relativistic 2D case has been investigated by Quesnel et al [1, 2] by solving the dispersion relation of electron parametric instability of a circularly polarized wave in a cold plasma at any laser intensity and plasma density. The authors are identified differents regimes and in particular the existence on unstable modes at high value of $k_{\perp}$ (the wave vector component perpendicular to the incident pump wave number $\vec{k_o}$). In the relativistic regime the unstable wavenumber area may extend to high values of $k_{\perp}$ with high growth rates. At ultra-high regimes, the TPD and RMI tend to merge. The central goal of this paper is to investigate the nonlinear regime of the RMI at ultra-high intensity using semi-lagrangian Vlasov simulations.

2 Basic equations and the numerical code.

Let us consider a circularly polarized electromagnetic wave propagating in the x-direction. The plasma is modeled by an infinite homogeneous system in both x and y directions. The elctron distribution function $F(x, y, \vec{p}, t)$ obeys the normalized relativistic Vlasov equation:

$$\frac{\partial F}{\partial t} + \frac{\vec{p}}{\gamma\prime}\frac{\partial F}{\partial \vec{r}} + \left(\vec{E} + \frac{\vec{p}}{\gamma\prime} \times \vec{B}\right)\frac{\partial F}{\partial \vec{p}} = 0 \tag{1}$$

where $\vec{r} = (x, y, 0)$ and $\vec{p} = (p_x, p_y, p_z)$ with the Lorentz factor given by $\gamma\prime^2 = 1 + p^2$. By considering the following class of exact solution of (1), Eq. (1) can be written in the form:

$$\frac{\partial f}{\partial t} + \frac{p_x}{\gamma}\frac{\partial f}{\partial x} + \frac{p_y}{\gamma}\frac{\partial f}{\partial y} + \left(E_x + \frac{p_y B_z - P_z B_y}{\gamma}\right)\frac{\partial f}{\partial p_x}$$

$$+ \left(E_y + \frac{P_z B_x - p_x B_z}{\gamma}\right)\frac{\partial f}{\partial p_y} = 0 \tag{2}$$

with a new expression of the Lorentz factor given by $\gamma^2 = 1 + p_x^2 + p_y^2 + P_z^2(x, y, t)$, and P_z verifies the "fluid" equation $\frac{\partial P_z}{\partial t} = E_z$. The electromagnetic field $\left(\vec{E}, \vec{B}\right)$ are computed using the Maxwell equations. The semi-lagrangian Vlasov code has been adapted to optimally use the particular parallel architecture of the Cray T3E. For example in the one-dimensional case described by the simplified Vlasov equation: $\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + G\frac{\partial f}{\partial p_x} = 0$ and where the function v and G are defined by $v(x, p_x, t) = p_x/\gamma$, $\gamma^2 = 1 + p_x^2 + a^2(x, t)$ where a denotes the normalized amplitude of the potential vector and $G(x, p_x, t) = E_x - \frac{1}{2\gamma}\frac{\partial}{\partial x}\left(a^2\right)$, the well-known fractional step or "splitting scheme" used to integrate the distribution function $f(x, p_x, t)$ is straightforward (in which we shift the distribution function alternatively in the x-direction leading to the mathematical expression:

$$f^*(x, p_x) = f(x - \alpha_x, p_x, t_n) \ \ with \ \alpha_x = \Delta t v\left(x - \alpha_x/2, p_x, t_{n+1/2}\right) \tag{3}$$

and then in the momentum p_x -space with the corresponding expression:

$$f(x, p_x, t_{n+1}) = f^*(x, p_x - \alpha_{p_x}) \ \ with \ \alpha_{p_x} = \Delta t G\left(x, p_x - \alpha_{p_x}/2, t_{n+1/2}\right) \tag{4}$$

Each shift is easily parallelized by just assigning a fraction of the distribution function to each node. Then a transposition of the distribution function is required to perform the parallelized shift in the second direction. The method can be directly generalized to higher dimensions of the distribution function.

3 Numericals Results.

We consider here a circularly polarized electromagnetic pump wave of frequency $\omega_o = \sqrt{2}\omega_p$ and of wavenumber $k_o c/\omega_p = 1.224$ (the corresponding value is 0.866 in $\omega_o c^{-1}$ units). The quiver momentum is $p_{osc}/mc = \sqrt{3}$, (with these definitions the value of $p_{osc} = \sqrt{2}$ corresponds to an intensity of $5 \times 10^{18} Wcm^{-2}$ for a 1.06 μ m wavelength). The normalized Lorentz factor is then given in the case of a circular polarization by $\gamma_o^2 = 1 + p_{osc}^2$. The ratio of the density of the critical density is then $n/n_{crit} = 0.5$, i.e. $n/\gamma_o n_{crit} = 0.25$. We choose a plasma box of length $L_x = L_x = 2\pi/k_o$, leading to values of the fundamental wave numbers of $c\Delta k_x/\omega_o = 0.866$ and $c\Delta k_y/\omega_o = 0.866$ for the transverse direction. For these parameters, the growth rate of the relativistic Modulational Instability reaches $\gamma_{th}/\omega_o \approx 0.36$ for $(k_x c/\omega_o, k_y c/\omega_o) = (1.730,\ 0.)$ for the pump longitudinal mode and $\gamma_{th}/\omega_o \approx 0.24$ for $(k_x c/\omega_o, k_y c/\omega_o)$ =(2.60, 0.866), the corresponding numerical values, obtained directly by the code are $\gamma_{num}/\omega_o \approx 0.35$ in the first case for $k_x c/\omega_o = 2\Delta k_x = 1.722$; $k_y c/\omega_o = 0$, and in the second case, $\gamma_{num}/\omega_o \approx 0.235$ for $k_x c/\omega_o = 3\Delta k_x = 2.598$ and $k_y c/\omega_o = \Delta k_y = 0.866$. The most striking advantage of the semi-lagrangian Vlasov code is the very fine resolution obtained in phase space, capable of resolving the finest mechanism of the particle acceleration. Fig. 1 exhibits clearly the modulation of the whole distribution function followed by trapping and formation of coherent Structures in the $x - p_x$ reduced phases space corresponding to the saturation of the RMI. The curves are shown at respective times $t\omega_p = 10.5,\ 12.5,\ 13.5,\ 15$.

4 Conclusion.

In order to investigate the transverse geometrical effects on the Relativistic Modulational Instability, numerical simulations using a semi-lagrangian Vlasov code have been carried out on a parallel computer in a ultra-intense electromagnetic regime. The numerical results show clearly the formation of coherent structures in phases space as a result of the saturation of the RMI instability.

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6 Reference:

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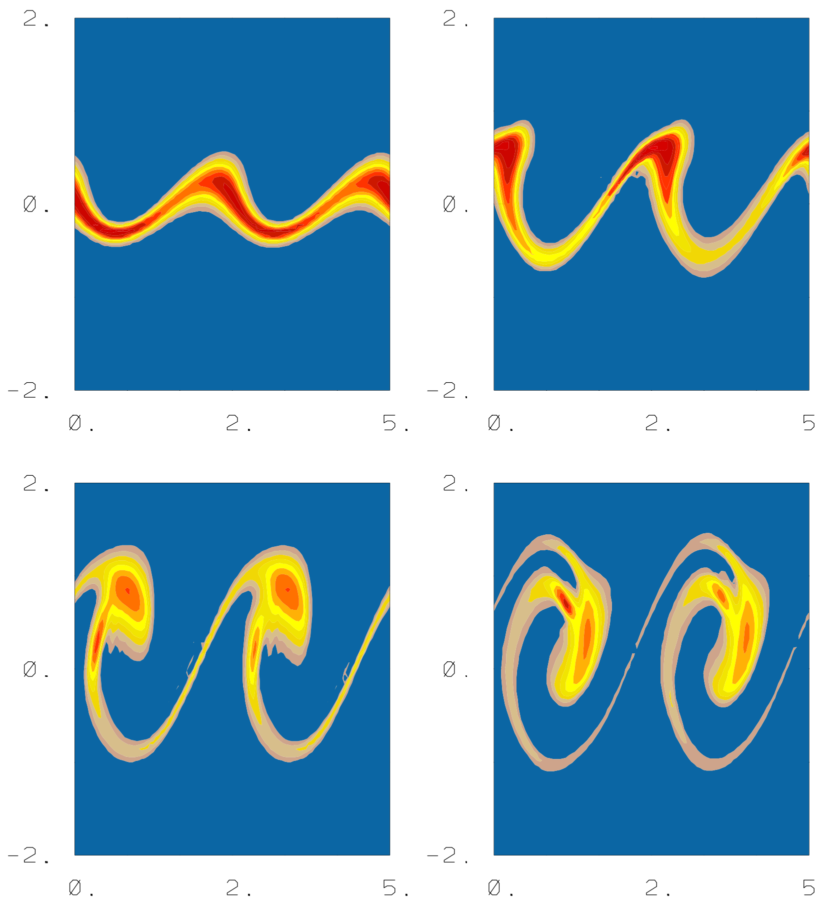

Fig.1 Phase space representation x-p_x of the distribution function.

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