Performance Prediction of Hulls with Transverse Steps

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Preface

This report is a Masters Thesis at the Royal Institute of Technology, KTH, Centre for Naval Architecture. For the most part it was conducted at the institute at KTH in Stockholm, Sweden, but the author did also spend some time at Lightcraft Design Group, LDG, in Öregrund. LDG are also the ones that initiated this project by contacting KTH. The masters thesis was performed between October 2008 and May 2009.

Abstract

In this Master Thesis a method to perform resistance predictions on planing hulls with transverse steps has been developed. For hulls without steps there are several methods available to predict their performances. One of the methods is called Savitsky's method and was published in 1964 and is one of the more famous. Where needed, this method has been modified and used together with other theories to become applicable for stepped hulls.

The stepped hull is viewed as two regular hulls following each other closely in the water. The first hull follows the same theory as a normal planing hull since this one meets a calm level water surface. The second hull does however not, as it travels in the wake behind the first hull. Because of this, the shape of the wake has been studied for different conditions like speeds and hull shapes.

There are no practical ways to give the shape of the wake as input to Savitsky's method, but it is possible to interpret the afterbody hull shape relative to the wake surface instead of relative to the level water surface as is usually done, and by doing so view the water as level.

Another issue is that it initially is unknown how the weight is distributed between the two hulls, since the two centre of pressures have different positions dependent on speed and positioning of the boat, while centre of gravity of course remains the same. As a solution the author decided to solve the weight distribution by iteration, where suitable start guess that 50-70% of the weight is carried by the forebody is made for most of the conventional hulls with transverse steps.

As platform to solve the problem above, MATLAB has been used. A program has been written where the dimensions, weight and speed of the craft are used as input and the new method predicts drag, required power, trim and draft of the stepped hull. The program is tested on a boat with detailed dimensions available. The required power predicted with the program corresponds very well with the available propeller effect at that speed after the air resistance has been added.

Contents

Nomenclature

e – distance below the transom/keel where the propeller shaft pass, [m]

ε – inclination of thrust line relative to keel line, [deg (if nothing else said)]

f – distance between T and centre of gravity (CG) measured normal to T, [m]

g – acceleration due to gravity, $= 9.81\ ^m/_{s^2}$

ν – kinematic viscosity of fluid, [$^{m^2}/_s$]

m – total mass of the boat, [kg]

ρ – density of water, [$^{kg}/_{m^3}$]

V – horizontal velocity of planing surface, [m/s]

T – propeller thrust, [N]

M_{tot} – total pitching moment, [Nm]

None stepped hulls only:

A_w – wet area, $\left[m^2 \right]$

a – distance between D_f and CG measured normal to D_f, [m]

b – beam of planing surface, [m]

β – angle of deadrise of planing surface, [deg (if nothing else said)]

C_V – speed coefficient

$C_{L\beta}$ – lift coefficient, deadrise surface

C_{L0} – lift coefficient, zero deadrise

C_f – frictional-drag coefficient

C_p – distance of centre of pressure measured along keel forward of transom

c – distance between N and CG, measured normal to N, [m]

D_f – frictional drag component along the bottom surface, [N]

D – total horizontal hydrodynamic drag component, [N]

d – vertical depth of trailing edge of boat, at keel, below level water surface, [m]

LCG – longitudinal distance of CG from the transom measured along the keel, [m]

L_1 – difference between wetted keel and wetted chine lengths, [m]

L_2 – difference between keel and chine lengths wetted by level water surface, [m]

L_C – wetted chine length, [m]

L_k – wetted keel length, [m]

l_p – dist. from transom along keel to where normal force, N, acts, [m]

λ – mean wetted length-beam ratio

N – hydrodynamic force normal to the bottom, [m]

Re   – Reynolds number

τ – trim angle of planing area, [deg (if nothing else said)]

V_m – mean velocity over bottom planing surface, [m/s]

VCG – distance of CG above the keel line, [m]

Hulls with one step only:

A_{w1} – wet area of forebody, $\left[m^2 \right]$

A_{w2} – wet area of afterbody, $\left[m^2 \right]$

a_1 – distance between D_{f1} and CG measured normal to D_{f1}, [m]

a_2 – distance between D_{f2} and CG measured normal to D_{f2}, [m]

b_1 – beam of planing surface of forebody, [m]

b_2 – beam of planing surface of afterbody, [m]

b_{2L} – local beam of planing surface of afterbody, [m]

β_1 – angle of deadrise of planing surface of forebody, [deg (if nothing else is said)]

β_2 – angle of deadrise of planing surface of afterbody, [deg (if nothing else is said)]

β_{2L} – local angle of deadrise of planing surface of afterbody, [deg (if nothing else is said)]

C_{f1} – frictional-drag coefficient, forebody

C_{f2} – frictional-drag coefficient, afterbody

C_{p1} – distance of centre of pressure measured along keel from step on forebody

C_{p2} – distance of centre of pressure measured along keel from transom on afterbody

C_{L0_1} – lift coefficient, zero deadrise, forebody

C_{L0_2} – lift coefficient, zero deadrise, afterbody

$C_{L\beta_1}$ – lift coefficient, deadrise surface, forebody

$C_{L\beta_2}$ – lift coefficient, deadrise surface, afterbody

C_{V1} – speed coefficient, forebody

C_{V2} – speed coefficient, afterbody

c_1 – distance between N_1 and CG, measured normal to N_1, [m]

c_2 – distance between N_2 and CG, measured normal to N_2, [m]

D_{f1} – frictional drag component along the bottom surface, forebody, [N]

D_{f2} – frictional drag component along the bottom surface, afterbody, [N]

D_1 – total horizontal hydrodynamic drag component of forebody, [N]

D_2 – total horizontal hydrodynamic drag component of afterbody, [N]

d_1 – vertical depth of step edge, below level water surface, [m]

d_2 – vertical depth of trailing edge of boat, at keel, below level water surface, [m]

F_{L1} – vertical component of N_1 [N]

F_{L2} – vertical component of N_2 [N]

F_{L2_L} – component of N_2 normal to local water level line, [N]

φ – angle between the keel in front, and keel behind the step, [deg (if nothing else is said)]

H_{CL} – height of wake profile above extended keel, [m]

$H_{1/4}$ – height of wake profile above extended ¼-beam buttock, [m]

LCG_1 – longitudinal distance of CG from the step measured along the keel, [m]

LCG_2 – longitudinal distance of CG from transom measured along the keel, [m]

LS – longitudinal distance of the step from transom measured along the keel, [m]

L_{C1} – wetted chine length, forebody, [m]

L_{C2} – wetted chine length, afterbody, [m]

L_{k1} – wetted keel length, forebody, [m]

L_{k2} – wetted keel length, afterbody, [m]

L_{1_1} – difference between wetted keel and wetted chine lengths on the forebody, [m]

L_{1_2} – difference between wetted keel and wetted chine lengths on the afterbody, [m]

L_{2_1} – diff. between keel and chine lengths wetted by level water surface on the forebody, [m]

L_{2_2} – diff. between keel and chine lengths wetted by level water surface on the afterbody, [m]

l_{p1} – dist. from step along keel to where normal force N_1 acts on forebody, [m]

l_{p2} – dist. from transom along keel to where normal force N_2 acts on afterbody, [m]

λ_1 – mean wetted length-beam ratio on the forebody

λ_2 – mean wetted length-beam ratio on the afterbody

λ_{2L} – local mean wetted length-beam ratio on the afterbody

N_1 – hydrodynamic force normal to the bottom of the forebody, [N]

N_2 – hydrodynamic force normal to the bottom of the afterbody, [N]

Ω – part of weight carried by the forebody hull

Re_1 – Reynolds number, forebody

Re_2 – Reynolds number, afterbody

τ_1 – trim angle of planing area, forebody, [deg (if nothing else is said)]

τ_2 – trim angle of planing area, afterbody, [deg (if nothing else is said)]

τ_{2L} – local trim angle of planing area, afterbody, [deg (if nothing else is said)]

VCG_1 – distance of CG above the forebody keel, [m]

VCG_2 – distance of CG above the afterbody keel, [m]

VS – height of step, [m]

V_{m1} – mean velocity over bottom planing surface of the forebody, [m/s]

V_{m2} – mean velocity over bottom planing surface of the afterbody, [m/s]

x_{CL} – distance behind step where centre line wake profile intersects with aft hull keel, [m]

$x_{1/4}$ – distance behind step where ¼-beam wake profile intersects with aft hull ¼-beam, [m]

1. Introduction

The intention with the introduction is to lay a foundation for the continued reading of this thesis and give the reader some basic information needed to understand the thesis as a whole. It starts with some fundamentals on planing hulls and continues with a description of stepped hulls and a little bit why boat constructers sometimes chose to use this design. The introduction section is then rounded off with a clear description of the task and boundaries of the thesis.

1.1. Planing Hulls

There are basically three types of hulls. Displacing hulls, semi-planing hulls and planing hulls. All hulls can however be considered displacing when travelling at low speeds or not moving. In this study all hulls are observed at their designed cruising speeds, which means speeds for fully developed planing. Planing hulls, unlike the other two, generate virtually all lift from hydrodynamic pressure, rather than hydrostatic pressure. The gain by designing a hull like this is that it partially rises out of the water, greatly reducing its wet area and thereby its friction drag making higher speeds possible.
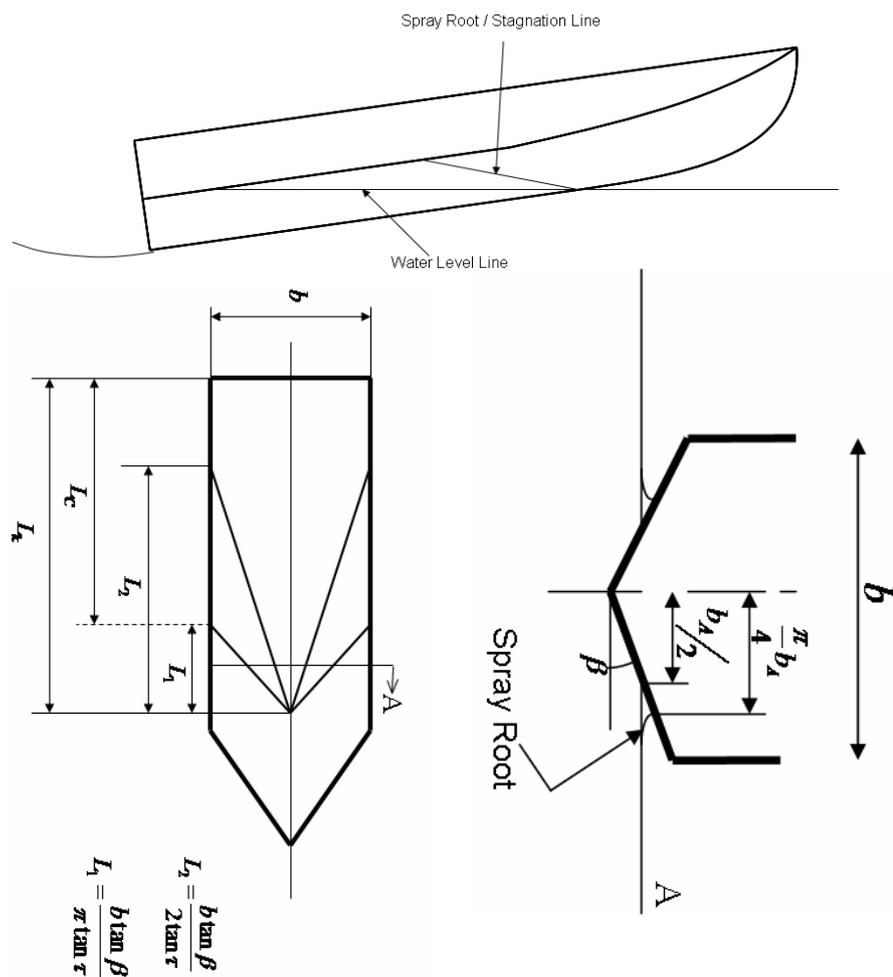

Fig. 1.1 Planing hull

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The horizontal component of the normal force, *N,* is called induced drag. This drag depends on the weight and the trim angle of the boat since the normal force is applied normal to the keel and not parallel to the weight, *mg*, which is illustrated in figure 1.2. Out of this perspective an as small trim as possible is wanted since this reduces the induced drag. However that would result in a larger wet area to compensate for the decreased lifting force one gets with lower trim.
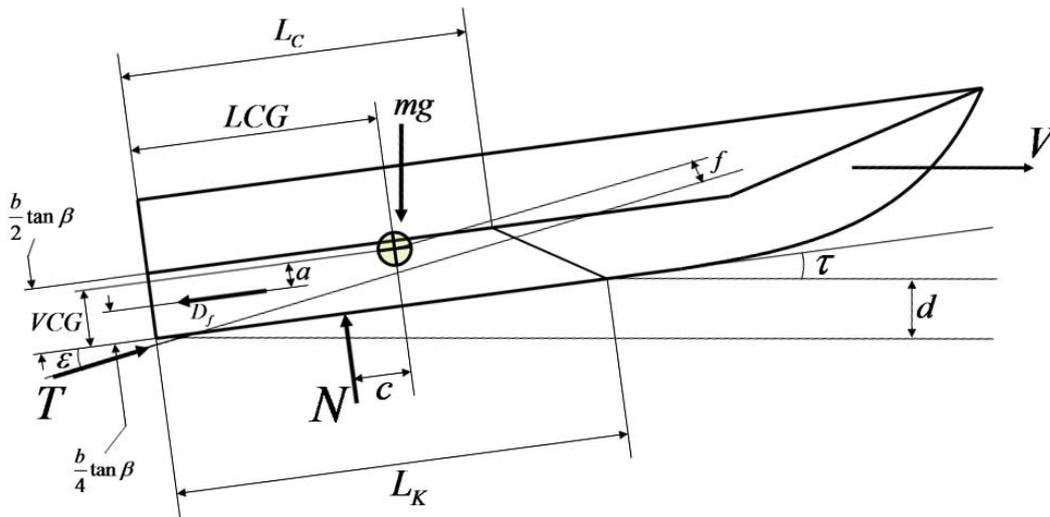

Fig. 1.2 Planing hull

Fig. 1.2 show a steady state condition and that means that forces and moment equilibrium is fulfilled giving:

$$\uparrow : \; N\cos\tau + T\sin(\tau+\varepsilon) - mg - D_f\sin\tau = 0 \qquad (1.1)$$

$$\rightarrow : \; T\cos(\tau+\varepsilon) - N\sin\tau - D_f\cos\tau = 0 \qquad (1.2)$$

$$\overset{\curvearrowleft}{CG} : \; N\cdot c + D_f a - T\cdot f = 0 \qquad (1.3)$$

As a rule of thumb boat constructers design these hulls so they trim about 4-5° at their cruising speeds and this is usually a good way to go. At this trim the Lift/Drag-ratio is as high as it gets for most conventional planing hull.

There are several methods available to predict the performance on a hull design. One of the more famous is Savitsky's method. With knowledge about the beam, deadrise angle, weight and centre of gravity, one can calculate the trim angle, draft and drag for a specific speed. This method will be explained in more detail later in the report.

The V-shape, or the deadrise, makes the hull less sensitive to waves, much like a suspension on a car deals with bumps in the road. But it also increases drag since it needs more wet area to generate enough lift. A fast boat that weighs about 4000-5000kg and measures 10-12meters does normally have a deadrise of 20-25°.

A problem with this hull is when designing faster and faster boats this also means that the centre of gravity must be moved further and further aft to avoid porpoising, which is an oscillating pitching motion. This makes these types of fast boats sensitive to changes in weight distribution, like a person walking aboard, while at high speeds. This is where transverse steps can be useful, which is further discussed in the following section.

1.2. Hulls with Transverse Steps

Hulls with transverse steps have been around over a century now. It is fast boats that can have use for steps. An example on stepped hull is shown in figure 1.3 and 1.4 below. At high speeds the water will separate completely from the step, creating a dry section on the hull from the step to a point somewhere between the step and the transom.

A normal misconception is that stepped hulls are faster or more efficient than regular hulls. It is not that simple. But one of the benefits of using steps is that the boat can be designed with the centre of gravity further from the transom and still keep most of its good performance at high speeds. But if the step was removed and the centre of gravity moved back towards the transom, it would actually need less power for same speeds. Important to know though, is that the step is well motivated in many cases. The step makes it possible to build boats where the designer wants to spread out the weight along the whole boat that still perform really well with reasonable engine requirements. Another benefit that comes with this system is the less obvious hump to overcome during the transitions speeds.

Fig. 1.3 Delta 29 SW with a Transverse Step

If the step is well ventilated the water will separate from it just like it does at the transom of a regular planing hull. "Well ventilated" means that air flows without restriction in behind the step. If it is not well ventilated the water will behave differently due to low pressure. In this thesis the step is assumed well ventilated.

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Steps are very common on modern fast boats, but there are still no real calculating methods to predict its performance. Eugene Clement [1] stated this fact as late as 2005 that after nearly 100 years of building these boats "calculation methods are nonexistent."

This issue is where this thesis origin. One of the questions asked is if it is possible to use and modify Savitsky's method [2] from 1964 to predict performance on boats with transverse steps. There are several reasons why one can not just use Savitsky's method on the two hulls separately. The first hull has pushed the water down thus changing the water level and its shape and Savitsky's method presupposes a calm water surface. A crucial question is the wake shape and its influence on the hull aft of the step.

Fig. 1.4 Delta 29 SW with a Transverse Step

1.3. Task

The aim is a computational method to predict the performance of stepped hulls. Especially in a simple and reliable way like Savitsky's method [2] that is used for regular hulls where the deadrise, beam and centre of gravity is used to find out the required power, draft and the trim at pitch equilibrium. With such a method it would be easier to determine what engine a specific stepped hull design would need to perform at desired speed, and also detect flaws in a design before it leaves the drawing board.

As long as possible known methods are used, and where needed these are modified and combined with new theories to fit the designs with transverse step.

This project was initiated by naval architects at Lightcraft Design Group, LDG in Öregrund, Sweden. This after coming to the conclusion that there are no public methods available to predict how a stepped hull will behave or how the performance is effected by the dimensioning and positioning of the step itself.

The master thesis focuses on performance. Trim, resistance, draft and required power depends on the position of the step, height of the step, the shape of the hull and the centre of gravity. Other aspects that could be interesting, like porpoising and bow steering is not considered in this thesis. Since the work concentrates on how the step is making a difference and that only, it is assumed that the surrounding water is calm and the boat is always travelling at steady state. The method developed in this thesis does not take dry chines into consideration, and neither does Savitsky's method, but it has become clear that dry chines are not uncommon when looking at stepped hulls, and it is important to note that the new method may not predict the performance as accurate for cases with dry chines.

2. The Mathematical Models

In this section the mathematical models for the stepped hull are showed and explained.

2.1. Equilibrium

As mentioned before, the craft is observed in steady state. This means that the boat is travelling at a constant speed and does not accelerate in any direction. Equilibrium showed here can be compared to the equilibrium of a regular hull shown in equations (1.1)-(1.3).

2.1.1. Vertical and Horizontal Equilibrium

The main forces in vertical equilibrium, as shown in figure 2.1, are the lifting forces and the weight of the boat, but since the boat has a trim it means that the friction drag actually adds to the weight, and the thrust adds to the lift.
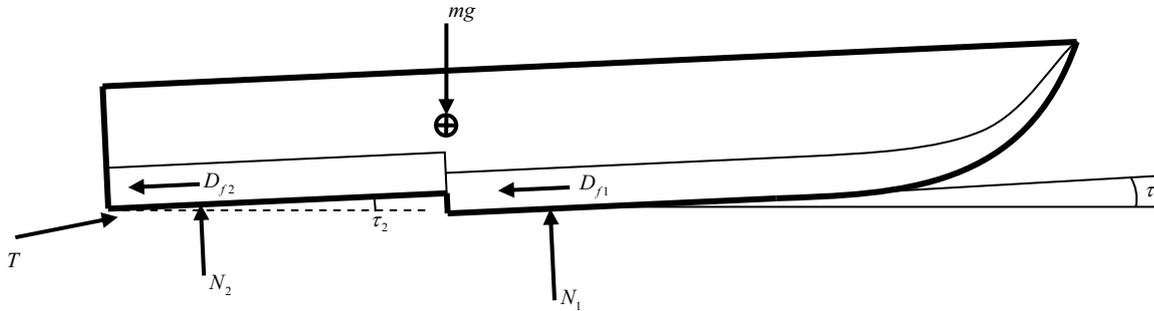
Fig. 2.1 Basic 2-D model of a planing Stepped Hull

$$\uparrow : \ N_1 \cos \tau_1 + N_2 \cos \tau_2 + T \sin(\tau_2 + \varepsilon) - mg - D_{f1} \sin \tau_1 - D_{f2} \sin \tau_2 = 0 \qquad (2.1)$$

Main horizontal forces are the drags and the thrust but since the boat has a trim the normal forces contribute too, just like in the vertical case.

$$\rightarrow : \ T \cos(\tau_2 + \varepsilon) - N_1 \sin \tau_1 - N_2 \sin \tau_2 - D_{f1} \cos \tau_1 - D_{f2} \cos \tau_2 = 0 \qquad (2.2)$$

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2.1.2. Pitching Moment Equilibrium

Steady state is assumed and so moment equilibrium is required. There are about twice as many measures to consider compared to the regular hull (compare figure 2.2 and 1.2).
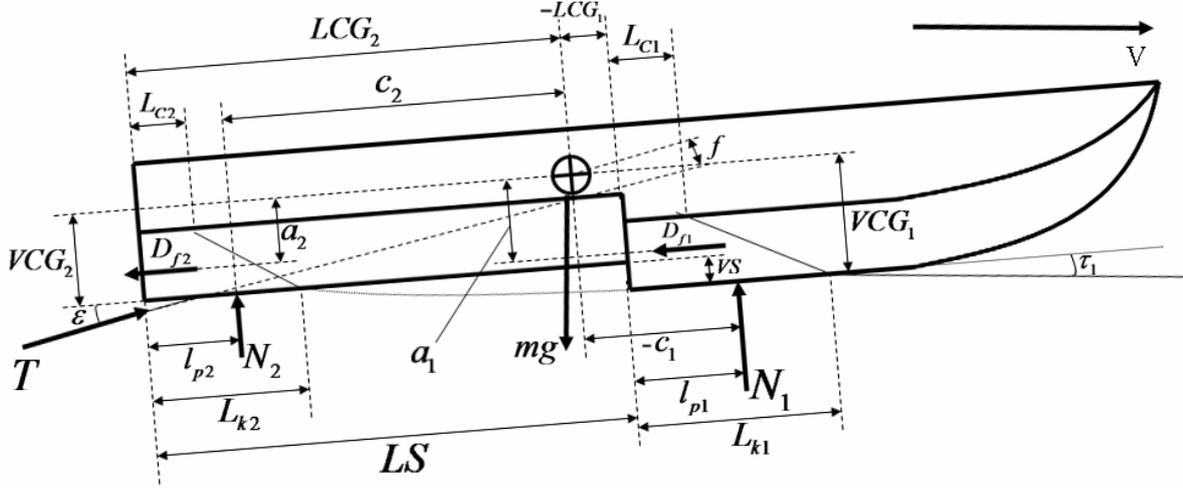
Fig. 2.2 Complete 2-D model of a planing Stepped Hull

$$\circlearrowright_{CG}: \quad N_1 c_1 + N_2 c_2 + D_{f1} a_1 + D_{f2} a_2 - T \cdot f = 0 \qquad (2.3)$$

where:

$$a_1 = VCG_1 - \frac{b_1}{4} \tan \beta_1 \qquad (2.4)$$

$$a_2 = VCG_2 - \frac{b_2}{4} \tan \beta_2 \qquad (2.5)$$

$$f = \left(VCG_2 + e \right) \cos \varepsilon - LCG_2 \sin \varepsilon \qquad (2.6)$$

Worth noticing is that c_1 has a negative value and could add confusion to an already complicated system of forces and distances. Due to this the first value in eq. (2.4), $N_1 c_1$ becomes negative. This is due to LCG_1 being measured the same way as LCG of a normal hull and the step would be representing the transom. In this case it would be interpreted as if the centre of gravity is located behind the transom, which is far from unusual. Keeping it this way should make it easier on anyone that wants to take this further adding more steps to the model.

2.2. Wake Theory

To model the aft part of the system knowledge of how the water behaves behind the step is required. How the water flows from the step edge to the point where it comes in contact with the aft hull is investigated. The step is viewed as a transom stern and the water lines behind it as a wake shape.

Some work was done in the 1950's at NACA on this matter when designing V-bottom shaped seaplanes. Even though the reports are there for anyone to read, it is hard to make out anything useful from them due to very different areas of interest. The seaplanes' bottom shapes and the steps just do not look very much like the boats with transverse steps that are of interest in this thesis. Other sources on the subject are rare. Most of the modern studies only apply on much lower speeds. Doctors and Robards [3] describe the flow line as a parabolic line behind a dry transom stern with the initial angle of the trim of the hull and intersects with the surface line. Faltinsen [4] describes the hollow line in a similar way.

Unfortunately there lack any published test results to verify the equations. Thus, this is not of much help besides giving an idea of the wake shape.

Luckily for this project there is a brand new report on this very subject written by Daniel Savitsky and his colleague Michael Morabito [5]. Savitsky and Morabito present detailed expressions of two wake lines profiles, one from the keel line (centre line) and the other one from ¼-beam line. The expressions are shown in equations (2.32)-(2.36) and are based on extensive towing tank results. According to this the wake shape depends on the wet keel length, trim, deadrise and boat speed. It also becomes clear that the water behaves differently depending on where it leaves the transom.
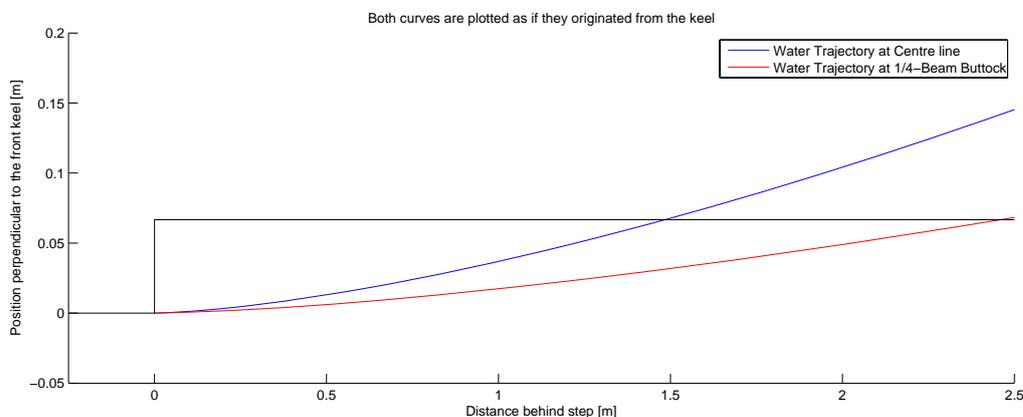
Fig. 2.3 An example of how the water line trajectories can look from the step to the aft hull

Figure 2.3 and the two curves show that the wake will flatten out rather quickly and that must be taken into consideration. Not only does this give two positions along the keel and ¼-beam buttock where the water intersects with the aft hull, it also reveals at what angle the water hits the hull surface. This can be interpreted as a local trim angle and is used for calculating the lift it will give on the aft hull.

2.3. Aft Hull

There is no handy way of giving the wake shape as input to Savitsky's method but it is possible to interpret the aft hull properties relative to the wake instead of relative to the level water surface as is usually done. In this way the water is viewed as level but the hull shape is modified. This modification is made by introducing local hull parameters.

With the wake theory in mind it is now possible to calculate the wetted area and forces acting on the aft hull, given that the properties of the first hull in front of the step is known. What wake theory really gives are two water curves and their angles relative to the keel at any given position behind the step. This, together with the corresponding line that represents the aft hull it is only a matter of solving the equations (2.32)-(2.35) to give the position where the water and hull intersect. A straight line can be drawn from the intersection point at the keel through the intersection point at ¼-beam to the chine, which is shown in figure 2.4. This line represents the water line that on the first hull is horizontal, but not on the aft hull, where it is called "local level water line". Besides this shape difference, it is assumed that the water behaves the same way meeting the aft hull as it does when hitting the first hull.
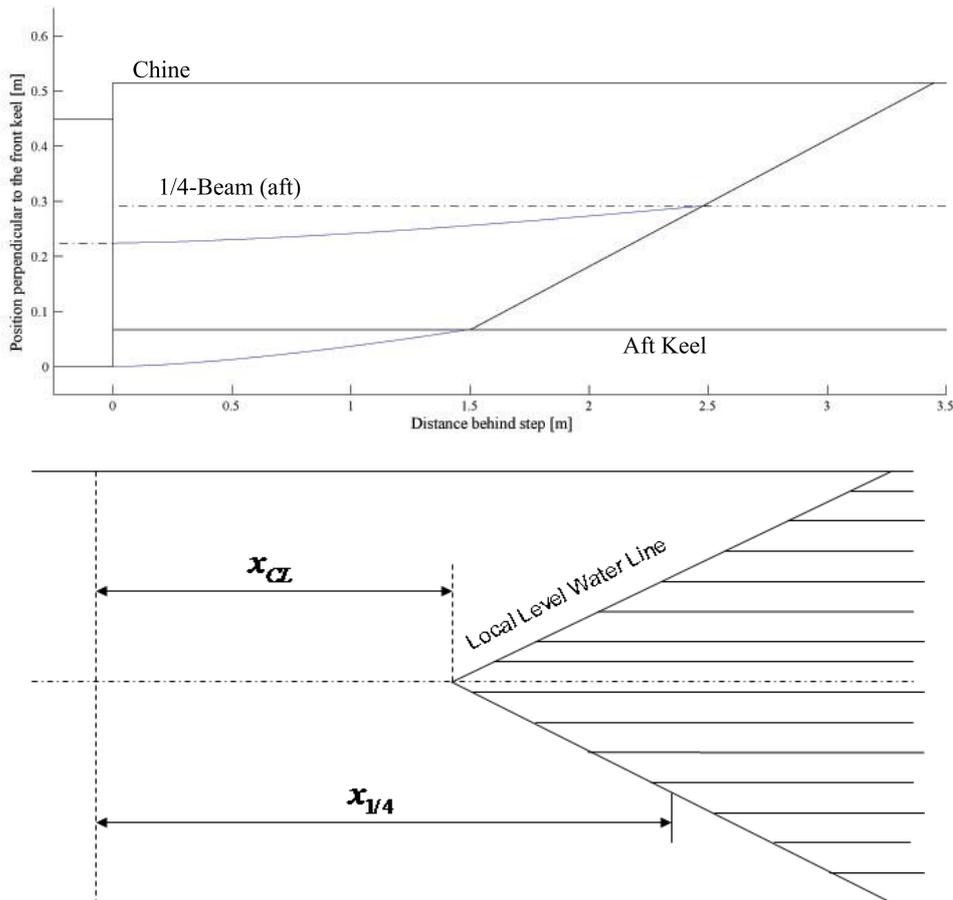
Fig. 2.4 An example of how the water line trajectories can look between the step and the aft hull with the correct relation to each others initial positions, and shown from above

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With this information a local deadrise can be determined. Basically it is just the difference between the hull deadrise and the angle the wake shape inclines towards the keel. This is illustrated in figures 2.5 and 2.6 below. Important to notice is that this local deadrise continuously increases the further behind the step one looks. The wake flattens. In the original Savitsky method only one value of the deadrise is used. So the wake shape that is found at the point behind the step where the ¼-beam water line intersects with the aft hull is used as a mean "local deadrise", β_{2L}. The same theory is used for deciding the local trim, τ_{2L} that varies along the level water line and the one found at ¼-beam line is the one used in the following calculations.
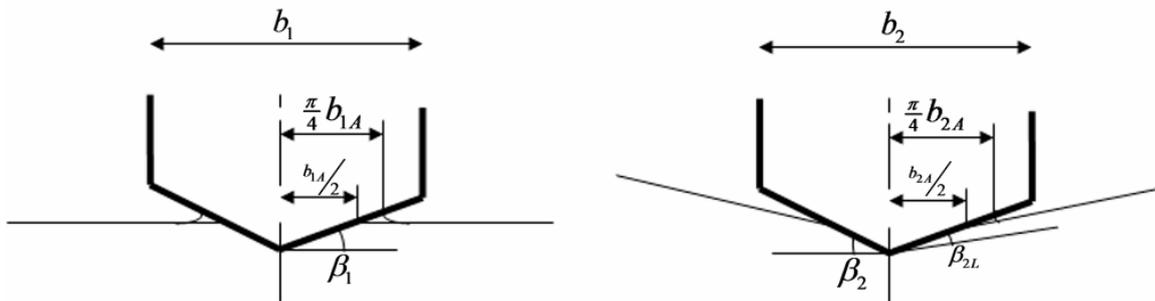
Fig. 2.5 Comparing the level water line and the local level water line at the aft hull
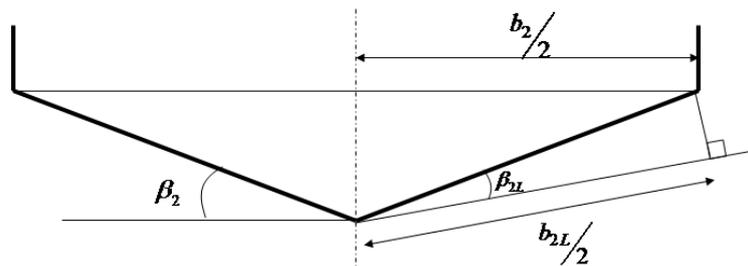
Fig. 2.6 Illustration of the local beam with respect to the local deadrise

In Savitsky's method the beam is a horizontal projection of the hull on the level water, but due to the wake shape the local beam, b_{2L} is projected on this inclining local level water line. This local beam, shown in figure 2.8, that is longer than the real beam, b_2, is used when calculating the spray-root.

2.3.1. Local Deadrise

The wake shape flattens the further behind the step the observation is made. That means that the local deadrise increases with the distance behind the step and a mean value of it is needed. It is decided that the ¼-beam will serve as a mean value. For any position x along the centre line and ¼-beam lines they will form a V-shaped wake, and the shape observed at x where the ¼-beam curve intersects with the hull will represent the approximated shape for the whole wake. This means that it is imagined that the centre line continues on its path as if the keel was not in its way. How the local deadrise, β_{2L}, is finally calculated is shown in equation (2.37) and illustrated in figure 2.7 below.

Fig.2.7 Illustration of factors in the wake profile giving the local deadrise, transverse section view

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2.3.2. Local Beam and Aspect Ratio

The beam is used when calculating the aspect ratio that in turn is used to determine the aft lift coefficient $C_{L\beta2}$. To compensate for local conditions in order to fit Savitsky's method a local beam is introduced. The local beam is the aft beam with respect to the shape of the wake, thus with respect to the local deadrise. Just as the normal beam is the hull's projection on the flat water surface, this local beam is its projection on the inclining water surface. The geometric connection between deadrise, trim and beam is given by L_{2_2} (in analogy with L_2, see figure 1.1). L_{2_2} originates from [2] where:

$$L_2 = \frac{b \tan \beta}{2 \tan \tau} \tag{2.7}$$

$$L_{2_2} = \frac{b_{2L} \tan \beta_{2L}}{2 \tan \tau_{2L}} \tag{2.8}$$

$$L_{1_2} = \frac{b_{2L} \tan \beta_{2L}}{\pi \tan \tau_{2L}} \tag{2.9}$$

(2.8) can be rewritten to,

$$b_{2L} = \frac{2 L_{2_2} \tan \tau_{2L}}{\tan \beta_{2L}} \tag{2.10}$$

with L_{2_2} calculated using (2.38). The aspect ratio and the local aspect ratio of the aftbody is given by,

$$\lambda_2 = \frac{L_{k2}}{b_2} - \frac{L_{1_2}}{2 b_2} \tag{2.11}$$

$$\lambda_{2L} = \frac{L_{k2}}{b_{2L}} - \frac{L_{1_2}}{2 b_{2L}} \tag{2.12}$$

With all geometrical parameters expressed the lift and drag forces can be determined.
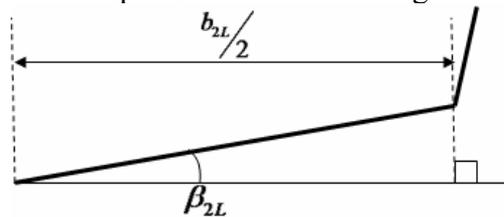
Fig. 2.8 Illustration of the local beam with respect to the local deadrise

2.3.3. Local Trim Angle

Another local condition that is easy to oversee is that the lift force that is calculated is perpendicular to the angle which the water meets the aft hull, and not vertical. This is why one has to recalculate the lift so the vertical component is correct.

$$F_{L2} = F_{L2_L} \cdot \cos\left(\tau_2 - \tau_{2L}\right) \qquad\qquad (2.13)$$

For the first hull the trim angle is the angle between level water surface and the keel, τ_1. But at the aft hull there is a local trim angle which is the angle between the aft keel and the angle which the water meets it, illustrated with exaggeration in figure 2.9. As a matter of fact there are two of these angles registered. One at the keel and one at ¼-beam buttock but only one angle is used in the calculations. As discussed earlier in this section the angle measured at ¼-beam is the one used as a mean local trim angle and it is extracted from known wake theory, equation (2.36).
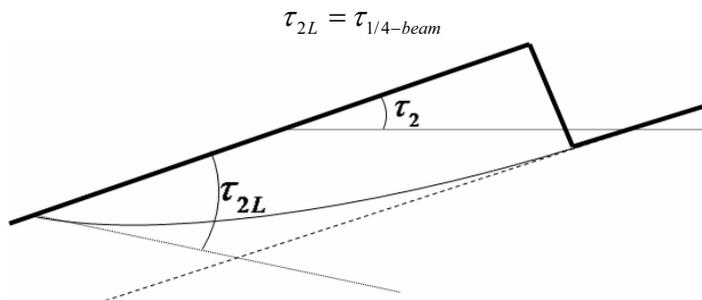
Fig. 2.9 Illustration of local trim angle behind the step

2.4. Savitsky's Method

In this section the method that Savitsky published in 1964 [2] is studied. The method is used to predict the performance for prismatic shaped planing hulls. It is important to investigate how the method works in order to use it to predict the performance of a stepped hull.

A specification of the craft is needed and also the speed one is interested in investigating the performance for. The required input are:

m – total mass of the boat, [kg]
b – beam, [m]
LCG – longitudinal distance of centre of gravity from transom, [m]
VCG – distance of centre of gravity above keel line, [m]
β – angle of deadrise of planing surface, [deg]
V – horizontal velocity, [m/s]
ε – inclination of thrust line relative to keel line, [deg]

Fig. 2.10 Regular Planing Hull

With Savitsky's method one basically investigates the pitching moment equilibrium for one trim angle at a time for the planing hull model seen in figure 2.10. The trim that gives moment equilibrium corresponds to the trim this boat would have for the given speed.

To understand the steps taken in Savitsky's method the different equations needs explanation. The original method consists of 30 steps for each investigated trim. These steps are shown in figure 2.11.

Two coefficients that does not change with the trim angle are speed coefficient, C_V and the lift coefficient, C_{L_β}. C_V is a nondimensional Froude number with respect to the beam:

$$C_V = \frac{V}{\sqrt{gb}}$$
(2.14)

C_{L_β} is the required lift coefficient to create enough lift to counter the weight of the boat.

$$C_{L_\beta} = \frac{mg}{\frac{1}{2}V^2 b^2 \rho}$$
(2.15)

Row	Quantity	Source	$\tau = 2°$	$\tau = 3°$	$\tau = 4°$
1	$\tau^{1.1}$	Figure 10	2.14	3.35	4.59
2	C_{L_0}	Figure 11	.085	.085	.085
3	$C_{L_0}/\tau^{1.1}$	(2)/(1)	.0397	.0254	.0185
4	λ	Figure 10	3.85	2.60	1.86
5	V_m	Figure 14	67.0	66.6	66.2
6	R_e	$V_m \lambda b / \nu$	3.61×10^8	2.42×10^8	1.73×10^8
7	C_f	Schoenherr	.00174	.00184	.00192
8	ΔC_f	ATTC Standard Roughness	.0004	.0004	.0004
9	$C_f + \Delta C_f$	(7) + (8)	.00214	.00224	.00232
10	D_f	$\dfrac{\rho V_m^2 \lambda b^2 (C_f + \Delta C_f)}{2 \cos\beta}$	7.340	5.160	3.760
11	$\tan\tau$		.0349	.0524	.0698
12	$\sin\tau$		.0349	.0524	.0698
13	$\cos\tau$		.9994	.9986	.9976
14	$\Delta \tan\tau$		2094	3144	4188
15	$D_f/\cos\tau$	(10)/$\cos\tau$	7340	5160	3760
16	D	(14) + (15)	9434	8304	7948
17	C_p	Figure 17	.59	.65	.70
18	$C_p \lambda b$		31.6	23.5	18.2
19	c	LCG - (18)	-2.6	5.5	10.8
20	$(b/4)\tan\beta$		.616	.616	.616
21	a	VCG - (20)	1.39	1.39	1.39
22	$\sin(\tau + \epsilon)$		.1045	.1219	.1392
23	$1 - \sin\tau \sin(\tau + \epsilon)$	1 - (12)(22)	.9964	.9964	.9903
24	$(23)(\frac{c}{\cos\tau})$		-2.59	5.46	10.70
25	$f \sin\tau$		.0174	.0262	.0349
26	(24) - (25)		-2.6	5.53	10.73
27	$\Delta (26)$		-156,500	332,000	645,000
28	$(a - f)$	(21) - f	.89	.89	.89
29	$D_f(a - f)$	(10)(28)	6540	4600	3350
30	(27) + (29)	Eq 35	-149,960	336,600	648,350

Fig. 2.11 Calculation chart from Savitsky's method published 1964
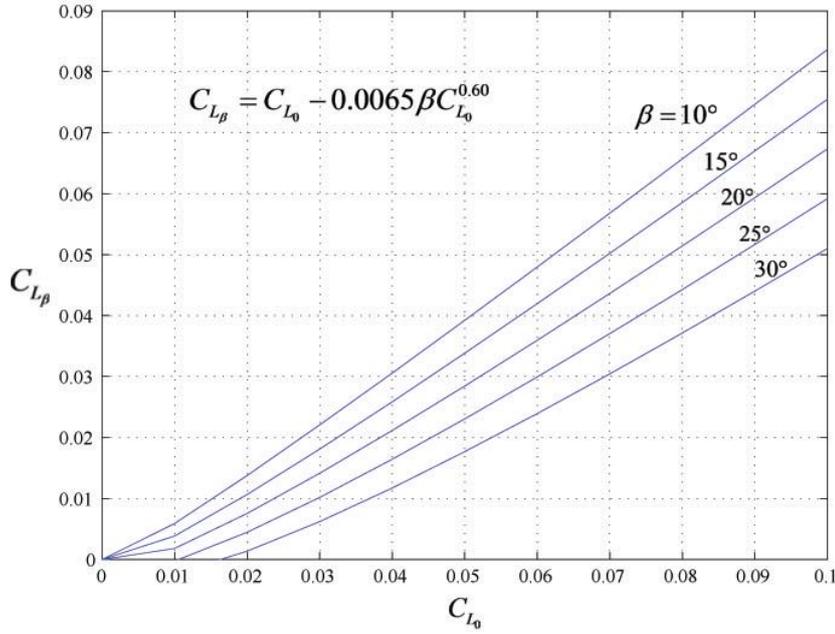
Fig. 2.12 Lift coefficient of a deadrise planing surface, recreated from Savitsky 1964

With these two and the input data, the first steps can be done. Steps 1-10 calculates the frictional drag, D_f, and can be summarized by following the equations (2.16)-(2.22) below that are also included in Savitsky's method:

$$C_{L_\beta} = C_{L_0} - 0.0065\beta C_{L_0}^{0.60} \qquad (2.16)$$

C_{L_β} is given from (2.15) and gives the value of C_{L_0}, either by solving (2.16) or using the plotted expression in figure 2.12. Required aspect ratio, λ, is then extracted in a similar fashion from,

$$C_{L_0} = \tau^{1.1}\left(0.0120\lambda^{\frac{1}{2}} + \frac{0.0055\lambda^{\frac{5}{2}}}{C_V^2}\right) \qquad (2.17)$$

Equations (2.16) and (2.17) are semi empirical expressions extracted from extensive towing tank testresults [2]. To perform plate friction calculations a mean flow velocity is needed. The following expression giving the mean velocity, V_m is based on Bernoulli's equation.

$$V_m = V\left[1 - \frac{0.0120\lambda^{\frac{1}{2}}\tau^{1.1} - 0.0065\beta\left(0.0120\lambda^{\frac{1}{2}}\tau^{1.1}\right)^{0.60}}{\lambda\cos\tau}\right]^{\frac{1}{2}} \qquad (2.18)$$

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And the plate friction coefficient is determined by (ITTC-57),

$$C_f = \frac{0.075}{\left(\log_{10}(R_e) - 2\right)^2}$$
(2.19)

Where Reynolds number is given from mean flow velocity and mean wetted length:

$$\mathrm{Re}_n = V_m \lambda \frac{b}{v}$$
(2.20)

with,

$$\Delta C_f = 0.0004 \text{, ATTC Standard Roughness}$$
(2.21)

The frictional drag is expressed by,

$$D_f = \frac{1}{2} \frac{\rho V_m^2 \lambda b^2}{\cos \beta} \left(C_f + \Delta C_f\right)$$
(2.22)

The wet area is included in equation (2.22). It can be extracted:

$$A_w = \frac{\lambda b^2}{\cos \beta}$$
(2.23)

To fulfill vertical equilibrium the vertical component of the normal force, N, must equal the weight of the boat, mg. Figure 2.10 gives a good overview of this, even if it has an exaggerated trim.

$$mg = N \cos \tau \quad \Rightarrow \quad N = \frac{mg}{\cos \tau}$$
(2.24)

Steps 11-13 in figure 2.11 provide values for $\tan \tau$, $\sin \tau$ and $\cos \tau$, and step 14 describes the induced drag. The induced drag is the horizontal component of the normal force, N and step 15 gives the horizontal component of the frictional drag, D_f. Finally these two are summed up in step 16 for the total drag, D.

$$D = mg \tan \tau + \frac{D_f}{\cos \tau}$$
(2.25)

Next in the list of steps in Savitsky's method is determining where the centre of pressure, C_p acts along the wet keel by using the expression,

$$C_p = 0.75 - \frac{1}{\dfrac{5.21 C_V^2}{\lambda^2} + 2.39}$$
(2.26)

For fully developed planing, thus for high C_V, centre of pressure will act at about 75% of the mean wet keel length from the transom, $C_p \approx 0.75$. Next is to determine the distance c, (step 18-19)

$$c = LCG - l_p = LCG - C_p \lambda b \qquad (2.27)$$

Moving on through the method, steps 20 and 21, where the distance, a, between D_f and CG (measured normal to D_f) is calculated,

$$a = VCG - \frac{b}{4} \tan \beta \qquad (2.28)$$

The remaining steps, 22-30, are a part of the final pitching moment calculation and sums up,

$$M_{tot} = mg \left[\frac{c}{\cos \tau} (1 - \sin \tau \sin(\tau + \varepsilon)) - f \cdot \sin \tau \right] + D_f (a - f) \qquad (2.29)$$

and can be related to equilibrium equation (1.3).

Equilibrium would mean that the moment equals zero, $M_{tot} = 0$ and this can only be true for one trim angle. If a negative value on the moment is found this means the investigated trim is too low and a higher trim is assumed and the method is restarted from step 1. This is repeated until a positive value of the moment is found which tells that the correct trim is somewhere between the last two investigated trims. A more exact trim can easily be found by linear interpolation between these two. The drag and aspect ratio are interpolated the same way since those are found for each investigated trim in step 16 and 4. With the interpolated value of the trim, τ_e and aspect ratio, λ_e one can finally calculate the wet keel length, L_K and the draft, d.

$$L_K = \lambda_e b + \frac{b \tan \beta}{2\pi \tan \tau_e} \qquad (2.30)$$

$$d = L_K \sin \tau_e \qquad (2.31)$$

2.5. Savitsky's Method used behind the Step

Supposing the running attitude of the hull in front of the step is known all local conditions can be determined thanks to the wake theory. The wake theory is used to determine where the water meets the aft hull. The function of the centre line profile depending on the deadrise is given in eq. (2.32)-(2.34), and the ¼-beam buttock by (2.35). These are taken straight out of Savitsky and Morabito's [5].

$$\beta_1 = 10°$$

$$VS + x_{CL} \tan\varphi = H_{CL}(x_{CL}) = b_1 \cdot 0.17\left(1.5 + 0.03\frac{L_{k1}}{b_1}\tau_1^{1.5}\right)\sin\left(\frac{\pi}{C_{V1}}\left(\frac{x_{CL}}{3b_1}\right)^{1.5}\right) \quad (2.32)$$

$$\beta_1 = 20°$$

$$VS + x_{CL} \tan\varphi = H_{CL}(x_{CL}) = b_1 \cdot 0.17\left(2.0 + 0.03\frac{L_{k1}}{b_1}\tau_1^{1.5}\right)\sin\left(\frac{\pi}{C_{V1}}\left(\frac{x_{CL}}{3b_1}\right)^{1.5}\right) \quad (2.33)$$

$$\beta_1 = 30°$$

$$VS + x_{CL} \tan\varphi = H_{CL}(x_{CL}) = b_1 \cdot 0.17\left(2.0 + 0.03\frac{L_{k1}}{b_1}\tau_1^{1.5}\right)\sin\left(\frac{\pi}{C_{V1}}\left(\frac{x_{CL}}{3b_1}\right)^{1.5}\right) \quad (2.34)$$

The left hand side of (2.32)-(2.34), $VS + x \cdot \tan\varphi$, describes the vertical position of the afterbody keel, where the x-axis follows the extension of the forebody keel aft of the step and origin at the step edge. φ is the angle difference the two keels have. A nonzero valued φ means the two keels are not parallel. The right hand side is the height of the wake profile relative the extended forebody keel line as a function of x (seen in the first part of fig. 2.7) and its function is shown,

$$H_{CL}(x) = b_1 \cdot 0.17\left(1.5 + 0.03\frac{L_{k1}}{b_1}\tau_1^{1.5}\right)\sin\left(\frac{\pi}{C_{V1}}\left(\frac{x}{3b_1}\right)^{1.5}\right), \text{ for } \beta_1 = 10°.$$

x_{CL} is the point on the x-axis where both left and right side of the equations are equal, meaning that is where the wake line and keel intersects, which is also illustrated in figure 2.4. The ¼-beam case follows the same pattern,

$$10° \geq \beta_1 \geq 30°$$

$$VS + 0.25 \cdot b_1(\tan\beta_2 - \tan\beta_1) + x_{1/4}\tan\varphi = H_{1/4}(x_{1/4}) = b_1 \cdot 0.17\left(0.75 + 0.03\frac{L_{k1}}{b_1}\tau_1^{1.5}\right)\sin\left(\frac{\pi}{C_{V1}}\left(\frac{x_{1/4}}{3b_1}\right)^{1.5}\right)$$

$$(2.35)$$

, but since deadrises β_1 and β_2 does not have to be equal for the forebody and afterbody the aft hull equation looks a little different in (2.35), unless $\beta_1 = \beta_2$, then that factor disappears.

In (2.36) the local trim angle is calculated. It is the differentiation with respect to the distance aft of the step, x, of the right part of (2.35) with the determined value of wake-hull intersection, $x_{1/4}$ as input. It is important to notice that to find the correct local trim, τ_{2L}, the difference in angle between the fore and aft keel lines must be subtracted from the derivate, as done in (2.36).

$$\tau_{2L} = 0.17 \left[0.75 + 0.03 \frac{L_{k1}}{b_1} \tau_1^{1.5} \right] \frac{1}{2} \frac{\pi}{C_{V1}} \left(\frac{x_{1/4}}{3b_1} \right)^{0.5} \cos\left(\frac{\pi}{C_{V1}} \left(\frac{x_{1/4}}{3b_1} \right)^{1.5} \right) - \varphi \qquad (2.36)$$

where,

VS	- height of step, [m]
x_{CL}	- dist. aft step where centre line wake profile intersects with aft hull keel, [m]
$x_{1/4}$	- dist. aft step where ¼-beam wake profile intersects with aft hull ¼-beam, [m]
H_{CL}	- height of wake profile above extended keel, [m]
$H_{1/4}$	- height of wake profile above extended ¼-beam buttock, [m]
φ	- angle difference between forebody and afterbody keels, [rad]
τ_1	- forebody trim angle, [deg]
τ_{2L}	- local aft trim angle, [rad]

Angles must be given in units according to this nomenclature even if it may seem strange to have them both in the same equation.

With H_{CL}, $H_{1/4}$ and τ_{2L} determined from (2.32-36) it is possible to determine the local deadrise, β_{2L} the way it is explained in 2.3 Aft Hull:

$$\beta_{2L} = \beta_2 - \arctan\left(\frac{H_{1/4}(x_{1/4}) + 0.25 \cdot b_2 \tan \beta_1 - H_{CL}(x_{1/4})}{0.25 \cdot b_2} \right) \qquad (2.37)$$

The length L_{2_2} (see figure 1.1) on the aftbody:

$$L_{2_2} = 2 \frac{x_{1/4} - x_{CL}}{\cos \varphi} \qquad (2.38)$$

The aftbody wet keel length is calculated:

$$L_{K2} = LS - \frac{x_{CL}}{\cos \varphi} \qquad (2.39)$$

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The velocity coefficient is given with respect to local conditions:

$$C_{V2} = \frac{V}{\sqrt{g \cdot b_{2L}}}$$

(2.40)

where b_{2L} is given by equation (2.10). Lift coefficients are then calculated using following equations. C_{L0_2} is the lift coefficient corresponding to a hull with zero local deadrise.

$$C_{L0_2} = \tau_{2L}^{1.1} \cdot \left(0.012 \cdot \lambda_{2L}^{0.5} + \frac{0.0055 \lambda_{2L}^{2.5}}{C_{V2}^2} \right)$$

(2.41)

The lift coefficient with respect to the local deadrise is given by,

$$C_{L\beta_2} = C_{L0_2} - 0.0065 \cdot \beta_{2L} \cdot C_{L0_2}^{0.6}$$

(2.42)

When using this lift coefficient the resulting lift will be normal to the water surface. To obtain the vertical component, F_{L2} two projections are made in the following equations (2.43), with respect to local deadrise, and (2.44) with respect to local trim angle.

$$F_{L2_L} = \frac{1}{2} C_{L\beta_2} \rho V^2 b_{2L}^2 \cdot \cos\left(\beta_2 - \beta_{2L} \right)$$

(2.43)

$$F_{L2} = F_{L2_L} \cos\left(\tau_2 - \tau_{2L} \right)$$

(2.44)

Plate friction is calculated similar to Savitsky's method but with local parameters:

$$C_{f2} = \frac{0.075}{(\log_{10} \text{Re}_2 - 2)^2}$$

(2.45)

$$\text{Re}_2 = \frac{V_{m2} \left(\lambda_2 b_2 + \lambda_1 b_1 \right)}{\nu}$$

(2.46)

$$V_{m2} = V \left[1 - \frac{0.0120 \lambda_{2L}^{0.5} \tau_{2L}^{1.1} - 0.0065 \beta_{2L} \left(0.0120 \lambda_{2L}^{0.5} \tau_{2L}^{1.1} \right)^{0.60}}{\lambda_2 \cos \tau_{2L}} \right]^{\frac{1}{2}}$$

(2.47)

Leading to the calculations of friction drag,

$$D_{f2} = \frac{1}{2} \frac{\rho V_{m2}^2 \lambda_2 b_2^2}{\cos \beta_2} \left(C_{f2} + \Delta C_f \right)$$

(2.48)

which is added to the induced drag,

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$$D_2 = (1-\Omega)mg \cdot \tan\tau_2 + \frac{D_{f2}}{\cos\tau_2} \qquad (2.49)$$

where Ω is the part of the weight that is not carried by the aft hull.

Equations (2.26) and (2.27) turn to (2.50) and (2.51) when used on the afterbody:

$$C_{p2} = 0.75 - \frac{1}{\dfrac{5.21C_{V2}^2}{\lambda_2^2} + 2.39} \qquad (2.50)$$

$$c_2 = LCG_2 - l_{p2} = LCG_2 - C_{p2}\lambda_2 b_2 \qquad (2.51)$$

All the pieces to express the system of equations have now been shown, and in the next section the whole equilibrium system is solved.

3. Solving the Stepped Hull Equilibrium

In this section the method to solve the equilibrium equations of stepped hulls is presented.

Fig. 3.1 Complete 2-D model of a planing Stepped Hull

The problem, as shown in figure 3.1, is interpreted as two hulls following each other very closely. The calculations for the first hull would then be exactly like in Savitsky's original method. But for the second hull several factors need to be taken into account. The water surface is not level like for the first hull. As discussed in section *2.2 Wake Theory* and *2.5 Savitsky's Method used behind the Step*, the wake flattens the further behind the step it is observed. Due to this the calculations of the wet chine length for the second hull requires the local deadrise, as defined in *2.3.1 Local Deadrise* and determined in equation (2.37).

Draft and trim of the forebody must be known to perform calculations on the aft hull. These two are determined by using the original method on the hull in front of the step, but there is a problem. The lift of the forebody must be known first. On a regular hull without steps the required lift is already known as there is only one lift force to counter the weight of the boat. In the current case there are two lift forces, and how the weight is distributed between those is unknown.

For each investigated trim the weight distribution can be solved through iteration until vertical equilibrium is fulfilled within acceptable tolerances. After this is done and both lift forces are known the total pitching moment of the whole system can be calculated.
The iteration of the weight distribution has been kept simple. To initiate, a start guess is needed for the amount of the weight that is carried by the first hull.

$$F_{L1} = \Omega \cdot mg, \quad \text{where } 0 \leq \Omega \leq 1 \qquad (3.1)$$

In a normal case 70% of the weight could be carried by the front hull, $\Omega = 0.70$ and this works well as a starting guess. The choice of start approximation is not so important and if one have no idea how a certain hull will behave, $\Omega = 0.60$ works fine as a start guess, although might result in one or two more iteration cycles if it turns out that $\Omega = 0.80$.

When the weight on the first hull is assumed by a start guess its wet keel, L_{k1}, is calculated through Savitsky's original method with equations (2.14)-(2.17) and (2.30) for this specific assumption, where mg is replaced by $\Omega \cdot mg$. When this is done all information is available to perform Savitsky's method behind the step with equations (2.32)-(2.44). Basically, if the forebody of the boat carries a certain amount of weight at given trim and speed, this says how much the afterbody will carry, F_{L2}. Here the main vertical forces are added to see if vertical equilibrium is fulfilled or not.

$$\uparrow: \quad F_{L1} + F_{L2} - mg = \gamma \tag{3.2}$$

The absolute value of γ must be less than a given tolerance to consider vertical equilibrium to be fulfilled. If $|\gamma| > tol$ a new value of the weight distribution, Ω is determined from,

$$\Omega_{n+1} = \frac{\dfrac{\left(mg - F_{L2}\right)}{mg} + \Omega_n}{2} = \frac{1}{2} + \frac{F_{L1} - F_{L2}}{2mg} \tag{3.3}$$

and is put into (3.1) to give a new F_{L1}.

When vertical equilibrium has been found for one trim it is time to calculate the total pitching moment of the system. For this all forces and their perpendicular positions must be calculated.

Starting with the forebody, drag force D_{f1} is found through equations (2.18)-(2.22) and then used to solve the total forebody vertical drag;

$$(2.25) \quad \Rightarrow \quad D_1 = \Omega \cdot mg \tan \tau_1 + \frac{D_{f1}}{\cos \tau_1} \tag{3.4}$$

The distance from the centre of gravity that D_{f1} acts, a_1, is found with eq. (2.4),

$$a_1 = VCG_1 - \frac{b_1}{4} \tan \beta_1 \tag{3.5}$$

Eq. (2.24) gives the normal force,

$$N_1 = \Omega \frac{mg}{\cos \tau_1} = \frac{F_{L1}}{\cos \tau_1} \qquad (3.6)$$

and its distance from *CG*, c_1, is calculated from (2.26) and (2.27).

Moving on to the afterbody (2.45)-(2.49) gives D_{f2} and D_2. The distance D_{f2} act from *CG*, a_2 is given in equation (2.5).

The second normal force:

$$N_2 = (1 - \Omega) \frac{mg}{\cos \tau_2} = \frac{F_{L2}}{\cos \tau_2} \qquad (3.7)$$

and c_2 that says how far N_2 acts from the centre of gravity is obtained by solving equations (2.50) and (2.51).

Finally the pitching moment for the whole system can be determined through equation (2.3). If the result is negative, it means that the trim angle is too low to fulfill equilibrium, and a higher trim needs to be investigated the same way. This continues until the total moment comes out positive.

To speed it up, one can assume that the Ω finally obtained from the investigation of for instance $\tau_1 = 3°$ is most likely about the same at $\tau_1 = 4°$. So it is a good idea to use that obtained weight distribution as a start guess for the next investigated trim. This means that after doing the weight distribution iteration for the first trim the work will precede much faster since every new trim already has a very good start guess and will converge in one or two loops.

In the example in *Appendix 1*, the moment shows negative at $\tau_1 = 4°$ but positive at $\tau_1 = 5°$, thus moment equilibrium is located somewhere between those two trims. A more exact trim is found through interpolation using the pitching moment results. This is not very complicated and can be found in the prediction example in *Appendix 1*, equations (A1:1)-(A1:6). Important to notice there is that the investigated trims are 1° apart and makes the interpolation very simple, but if one were to use smaller trim steps than 1° there has to be some minor changes to above mentioned equations.

In the example in Appendix 1, a made up boat and its dimensions are given and the whole procedure of solving a case like this is shown row by row continuously referring to the equations in this report. It is there to give a view of how the method works and in what order things are done. That example is a very simple one and it is not advised to solve real cases by hand like that, but instead put the algorithm into a program that can do the iterations, and also investigate trims at a much smaller sample, like 0.1° at the time. This is exactly what has been done for the next section where real boats are used as input to predict their performance at known top speeds. Figure 3.2 is a simplified illustration of how the method works.
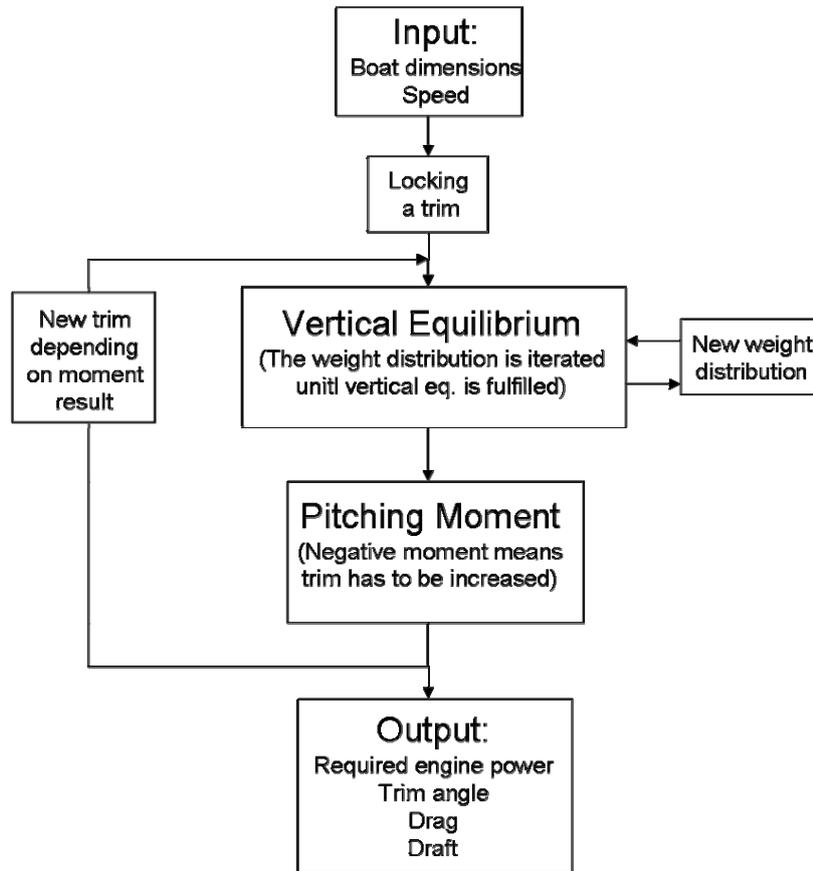
Fig. 3.2 Illustration of the prediction method

4. Benchmark Results

Detailed information [8] on dimensions, weights, engines and top speeds on three boats constructed by Delta Power Boats have been used to predict their required power. Even though there are no test results available, their actual engines give an approximation of how much propeller effect one can expect at the known top speeds of the crafts.

The three boats investigated are the Delta 29 SW, Delta 34 SW and Delta 40 WA, and below are the calculations for the Delta 29 SW presented. The results from all three boats are displayed in table 4.1.

Delta 29 SW:

Top speed, $V_{max} = 22.12$m/s $\quad\quad$ (43knots)

Engine effect, $E_E = 220.7$kW $\quad\quad$ (300 hp)

As a rule of thumb a good transmission system can deliver about 65% of the engine power to propeller effect. This would mean:

Available propeller effect, $E_P = 143.46$kW $\quad$ (195hp)

As mentioned before in this report added wave resistance has been neglected and the boat is modeled in calm water. But one added resistance that can not be neglected is the air resistance that has a significant impact on the final result.

$$R_A = \frac{1}{2} C_D \rho_{air} V^2 A_T \quad\quad\quad\quad (4.1)$$

Where:

R_A - air resistance, [N]

ρ_{air} - air density, $\left[{}^{kg}\!/_{m^3} \right]$

A_T - projected front area, $\left[m^2 \right]$

C_D - air drag coefficient

Drag coefficient is of course dependent on how the hull and structure above deck is shaped, and can vary between $0.5 < C_D < 1.2$ [7]. For this particular boat the projected area is approximated as a rectangle with the beam and approximated height measuring its sides.

$$A_T = 2.2 \cdot 2.7 = 5.94 m^2$$

$R_A = 1817 \cdot C_D$ [N] $\quad$ for $0.5 < C_D < 1.2 \Rightarrow 908 < R_A < 2180$ [N]

The added effect required to counter the air resistance can be solved:

$$E_{add} = R_A \cdot V_{max} \qquad \Rightarrow \qquad E_{add} = 34.2 \pm 14.1 kW$$

$$\text{or,} \qquad E_{add} = 46.4 \pm 19.1 \ hp$$

A program has been constructed in MATLAB [6] during the project with the method discussed. It is the same method used in the example in Appendix 1, but thanks to the computer this can be done much quicker and for smaller sample of trim. The dimensions of the Delta 29 SW were put in and here are the results:

$\tau_1 = 4.5°$ $\qquad\qquad \Omega = 0.682$

$P_{req} = 106.3kW$ $\qquad L_{k1} = 2.82m$

$D = 4806N$ $\qquad\qquad L_{k2} = 1.09m$

$d_1 = 0.219m$ $\qquad\quad L_{C1} = 0m$

$d_2 = 0.352m$ $\qquad\quad L_{C2} = 0.059m$

Trim, τ_1 and required power, P_{req} are the most interesting results here. Worth noticing is that the values of wet chine lengths, L_{C1} and L_{C2} says that the forebody chines are dry for this case which could result in some error to the rest of the results. The required power, P_{req} does not take air resistance into account, which must be added:

$$P_{req_{TOT}} = P_{req} + E_{add} \qquad \Rightarrow \qquad \boxed{\begin{array}{l} P_{req_{TOT}} = 140 \ \pm \ 14.1 \ kW \\ P_{req_{TOT}} = 190 \ \pm \ 19 \ hp \end{array}}$$

This should be compared with the approximated available propeller effect- $E_P = 195hp$. This makes $P_{req_{TOT}}$ look like a very good prediction in this particular case, even though the imprecise calculation of air resistance makes the prediction a bit uncertain. This and the result from the other two boats are displayed in table 4.1.

While the first two predictions are in very good agreement with the actual engines of those boats, the third prediction may seem a bit low. One reason to this difference in predicted required engine and the actual one may be because that particular boat has two engines and two rigs. This may cause the total transmission to deliver less than 65% of the engine power to propeller effect. Just to take this deviation into perspective, the predicted required propeller effect, on the Delta 40 WA with two engines, represents 60% of the totally available engine power, instead of 65%.

Boat model	Top speed [knots]	Predicted Required Propeller Effect [hp]	Predicted Required Engine Effect [hp]	Actual Engine Effect [hp]
Delta 29 SW	43	190 ± 19	292 ± 30	300
Delta 34 SW	38	254 ± 16	391 ± 25	370
Delta 40 WA	46	442 ± 43	680 ± 66	2 x 370 = 740

Table 4.1 Required Engine Predictions compared with engines already fitted

5. Summary and Conclusions

A method for performance prediction of stepped hulls has been developed based on the approach from "Savitsky's method" for planing crafts. The mayor contribution of the report is on calculating the lift and drag for the hull aft of the step. Crucial for the method is the description of the water surface shape aft of the step. Here the work of Savitsky and Morabito [5] has been used to interpret the hull water intersection geometry.

The comparison of calculations and full scale data show promising correlation and is an indication that this theory is on the right track. A limitation to the new method is that it does not take cases of dry chines into account.

6. Future Work

The next thing in line would be to make the method take dry chine cases into account. The problem would be to make the lift force continuous when going from a wet chine case to the dry chine case.

To better predict the required power a study of the air resistance for the investigated boat is of interest. That becomes evident from the benchmark results where 20-35% of the required power was there to counter the air resistance alone.

As always when making these kind of theoretical models real tests are pretty much the only way to finally verify them, and also a good way to discover factors one may have missed and find flaws in the model. Most interesting to measure at such tests would be drag and trim for different speeds. A way to calculate the real drag would be to measure the thrust that in turn can be given by the hydraulic pressure in the tilt circuit for instance. The trim is preferably measured onboard with an inclinometer. Observations that make it possible to determine wet chine lengths and wet keel lengths are also desired. Preferably these tests should be done on several different stepped hulls with different dimensions.

7. References

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5. Daniel Savitsky, Michael Morabito, "Surface Wave Contours Associated with the Forebody Wake of Stepped Planing Hulls", Davidson Laboratory Technical Memo #181, Stevens Institute of Technology, Hoboken, NJ USA, 2009

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Appendix 1

Example:

It is desired to know how a specific boat will behave at a certain speed, and what kind of engine is needed to power the boat at wanted speed. In this example the dimensions of a made up boat are given:

$VS = 0.05m$ $\quad\quad e = 0.15m$

$m = 4000kg$ $\quad\quad LS = 2.5m$

$\beta_1 = 11°$ $\quad\quad LCG_2 = 2.6m$

$\beta_2 = 10°$ $\quad\quad VCG_1 = 0.5m$ $\quad\quad V = 35knots$

$\varphi = 0°$ $\quad\quad b_1 = 2m$

$\varepsilon = 0°$ $\quad\quad b_2 = 2m$

Trims 3°, 4° and 5° are investigated in this rather simple example and at the first trim the weight distribution will start with the initiation guess of $\Omega = 0.60$.

Eqns:		$\tau_1 = 3°$	$\tau_1 = 3°$	$\tau_1 = 3°$	$\tau_1 = 4°$	$\tau_1 = 4°$	$\tau_1 = 5°$	$\tau_1 = 5°$
	Ω	0.60	0.6276	0.6328	0.6328	0.6416	0.6416	0.6507
(2.15)	$C_{L\beta 1}$	0.03632	0.03799	0.03831	0.03831	0.03884	0.03884	0.03939
Fig. 2.12 and (2.16)	C_{L01}	0.04787	0.04982	0.05018	0.05018	0.05080	0.05080	0.05145
(2.14)	C_{V1}	4.065	4.065	4.065	4.065	4.065	4.065	4.065
(2.17)	λ_1	1.296	1.386	1.403	0.7997	0.8182	0.5121	0.5248
(2.30) and (2.31)	L_{k1} [m]	3.772	3.952	3.986	2.484	2.521	1.731	1.757
(2.35)	$x_{1/4}$ [m]	1.692	1.677	1.674	1.687	1.682	1.696	1.691
(2.32)	x_{CL} [m]	1.346	1.339	1.337	1.344	1.341	1.347	1.345
(2.36)	τ_{2L} [rad]	0.03616	0.03648	0.03655	0.03625	0.03636	0.03607	0.03617
(2.35), (2.32) in (2.37)	β_{2L} [rad]	0.02216	0.02140	0.02126	0.02193	0.02169	0.02236	0.02213
(2.38)	L_{22} [m]	0.6920	0.6759	0.6729	0.6872	0.6821	0.6962	0.6912
(2.39)	L_{k2} [m]	1.154	1.161	1.163	1.157	1.159	1.153	1.155
(2.11)	b_{2L} [m]	2.259	2.306	2.314	2.272	2.287	2.247	2.261
(2.10)	L_{12} [m]	0.4405	0.4303	0.4284	0.4375	0.4342	0.4432	0.4401
(2.13)	λ_{2L}	0.4136	0.4105	0.4099	0.4127	0.4117	0.4144	0.4135

		$\tau_1 = 3°$	$\tau_1 = 3°$	$\tau_1 = 3°$	$\tau_1 = 4°$	$\tau_1 = 4°$	$\tau_1 = 5°$	$\tau_1 = 5°$
(2.14) $b = b_{2L}$	C_{V2}	3.825	3.786	3.779	3.814	3.801	3.835	3.823
(2.41)	$C_{L0_{2L}}$	0.01729	0.01739	0.01741	0.01732	0.01735	0.01726	0.01729
(2.42)	$C_{L\beta_2}$	0.01656	0.01669	0.01672	0.01660	0.01664	0.01653	0.01657
(2.43)	$F_{L_{2L}}$ [N]	13534	14209	14340	13728	13942	13364	13564
(2.44)	F_{L2} [N]	13532	14207	14338	13721	13934	13347	13546
(3.2)	$\uparrow_:$ [N]	-2164	-407.3	-72.56	-689.9	-131.8	-718.6	-160.4
(3.3)	Ω_{new}	0.6276	0.6328	-	0.6416	-	0.6507	-
(2.18)	V_{m1} [m/s]			17.77		17.58		17.32
(2.47)	V_{m2} [m/s]			17.64		17.64		17.64
(2.20)	Re_1			$3.8 \cdot 10^7$		$2.2 \cdot 10^7$		$1.39 \cdot 10^7$
(2.46)	Re_2			$5.06 \cdot 10^7$		$3.5 \cdot 10^7$		$2.68 \cdot 10^7$
(2.21)	ΔC_f			0.0004		0.0004		0.0004
(2.19)+ (2.21)	C_{ftot1}			0.002808		0.003028		0.003235
(2.45)+ (2.21)	C_{ftot2}			0.002705		0.002842		0.002946
(2.22)	D_{f1} [N]			2533		1559		1037
(2.48)	D_{f2} [N]			810.1		845.4		870
(3.4)	D_1 [N]			3839.8		3328.2		3282.2
(2.49)	D_2 [N]			1564.5		1826.4		2065.4
(3.4)+ (2.49)	D [N]			5404		5155		5348
(2.4)	a_1 [m]			0.4028		0.4028		0.4028
(2.5)	a_2 [m]			0.3618		0.3618		0.3618
(2.6)	f [m]			0.6		0.6		0.6
(2.26)	C_{p1}			0.7283		0.7424		0.7468
(2.50)	C_{p2}			0.747		0.7471		0.7472
(2.27)	c_1 [m]			-1.943		-1.115		-0.6838
(2.51)	c_2 [m]			1.891		1.897		1.902
(2.2)	T [N]			5403		5155		5354
(2.3)	M [Nm]			-23080		-3749		5949

This means that moment equilibrium (M) is fulfilled somewhere between $\tau_1 = 4°$ and $\tau_1 = 5°$. A more exact value of the trim, drag and aspect ratio is interpolated with the values of the moment:

$$\tau_{e1} = 4° + \frac{3748.8}{3748.8 + 5948.7} \approx 4.4° \tag{A1:1}$$

$$D_e = 5154.6 + \frac{3748.8}{3748.8 + 5948.7}(5347.7 - 5154.6) = 5229N \tag{A1:2}$$

$$\lambda_1 = 0.8182 + \frac{3748.8}{3748.8 + 5948.7}(0.5249 - 0.8182) = 0.7048 \tag{A1:3}$$

$$P_{req} = D_e \cdot V = 94.175kW \implies 128hp \tag{A1:4}$$

$$d_{e1} = \left(\lambda_{e1}b_1 + \frac{b_1 \tan \beta_1}{2\pi \tan \tau_{e1}}\right)\sin \tau_{e1} = 0.17m \tag{A1:5}$$

$$d_{e2} = LS \sin \tau_2 - VS \cos \tau_2 = 0.31m \tag{A1:6}$$

where, $\tau_2 = \tau_{e1}$ since, $\varphi = 0°$

Solution for V=35knots:
$\tau_{e1} = 4.4°$
$D_e = 5229N$
$P_{req} = 128hp$
$d_{e1} = 0.17m$
$d_{e2} = 0.31m$