Ontario

Measurement, Grades 4 to 6

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

2008

Measurement, Grades 4 to 6

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

CONTENTS

Introduction 5

The Pleasure of Mathematical Surprise and Insight 5
Working Towards Equitable Outcomes for Diverse Students 6
Accommodations and Modifications 7
The Mathematical Processes 9
Addressing the Needs of Junior Learners 11
Learning About Measurement in the Junior Grades 13

The "Big Ideas" of Measurement 15

About Big Ideas 15
Big Ideas and Tiered Instruction 17
The Big Ideas of Measurement in Grades 4 to 6 18

Attributes, Units, and Measurement Sense 21

Overview 21
Example: A Cubic Metre 22
Method 1: Understanding the Cubic Metre 23
Method 2: Extending the Problem 27

Measurement Relationships 31

Overview 31
Example: Developing Formulas From Student Experiences 31
Method 1: Understanding Measurement Formulas 32
Method 2: Exploring Measurement Relationships 36

References 40

Learning Activities 41

Introduction to the Learning Activities 43

Grade 4 Learning Activities 45

The First Decade of My Life
(Attributes, Units, and Measurement Sense) 45
Designing a Kindergarten Play Enclosure
(Measurement Relationships) 55

Grade 5 Learning Activities 67

Weather or Not...
(Attributes, Units, and Measurement Sense) ... 67

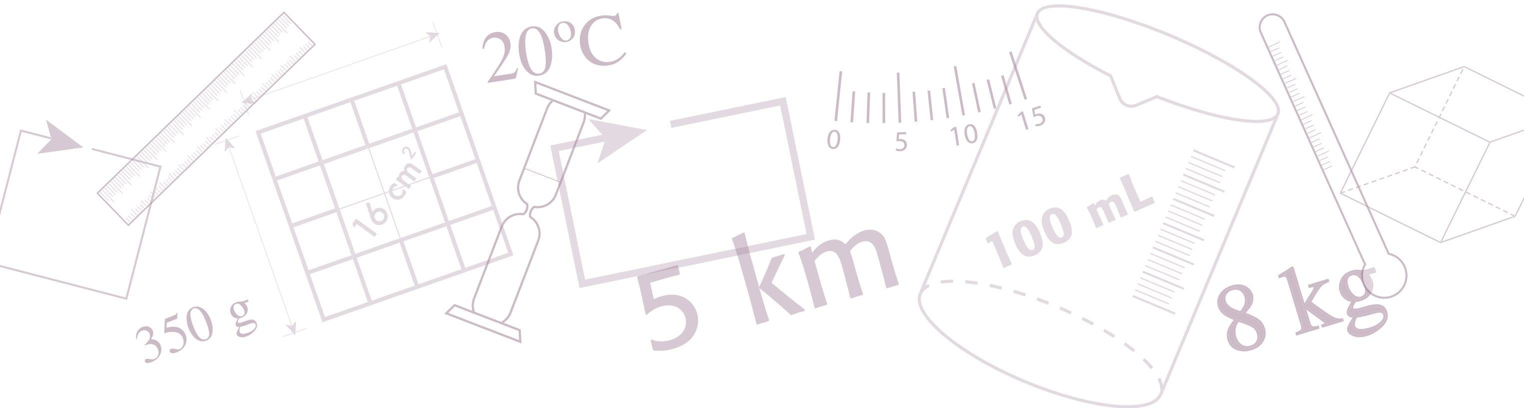
Hiking the Bruce Trail
(Measurement Relationships) ... 89

Grade 6 Learning Activities 99

Measurement at the Track Meet
(Attributes, Units, and Measurement Sense) ... 99

Packaging the Chocolongo Bar
(Measurement Relationships) ... 109

Glossary 121



INTRODUCTION

Measurement, Grades 4 to 6 is a practical guide that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Measurement strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories that are elaborated in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The first part of the guide provides a detailed discussion of the two “big ideas”, or major mathematical themes, in Measurement, and provides a discussion of mathematical models and instructional strategies that have proved effective in helping students understand the mathematical concepts related to each big idea. The guide emphasizes the importance of focusing on the big ideas in mathematical instruction to achieve the goal of *helping students gain a deeper understanding of mathematical concepts*. At the end of the first part of the guide is a list of references cited.

The second part of the guide provides sample learning activities, for Grades 4, 5, and 6, that illustrate how a learning activity can be designed to:

- focus on an important curriculum topic;
- involve students in applying the seven mathematical processes described in the mathematics curriculum document;
- develop understanding of the big ideas in Measurement.

At the end of the second part of the guide is a glossary that includes mathematical and other terms that are used in the guide.

The Pleasure of Mathematical Surprise and Insight

Young children enter school mathematically curious, imaginative, and capable. They have to learn to be otherwise (Papert, 1980). The aim of this resource is to help consolidate and extend junior students’ mathematical capacity and their potential for mathematical growth by providing ideas and classroom activities that draw their attention to relationships embedded in the “big ideas” of the Measurement strand in the Ontario mathematics curriculum and that offer them opportunities to experience the pleasure of mathematical surprise and insight (Gadanidis, 2004).

The teaching of mathematics around big ideas offers students opportunities to develop a sophisticated understanding of mathematics concepts and processes, and helps them to maintain their interest in and excitement about doing and learning mathematics.

The activities in this resource incorporate the ideas and practice of classroom teachers. The activities have been field-tested in Ontario classrooms, and feedback from practising teachers has been used to create the final versions.

Working Towards Equitable Outcomes for Diverse Students

All students, whatever their socio-economic, ethnocultural, or linguistic background, must have opportunities to learn and to grow, both cognitively and socially. When students can make personal connections to their learning, and when they feel secure in their learning environment, their true capacity will be realized in their achievement. A commitment to equity and inclusive instruction in Ontario classrooms is therefore critical to enabling all students to succeed in school and, consequently, to become productive and contributing members of society.

To create effective conditions for learning, teachers must take care to avoid all forms of bias and stereotyping in resources and learning activities, which can quickly alienate students and limit their learning. Teachers should be aware of the need to provide a variety of experiences and to encourage multiple perspectives, so that the diversity of the class is recognized and all students feel respected and valued. Learning activities and resources for teaching mathematics should be inclusive, providing examples and illustrations and using approaches that recognize the range of experiences of students with diverse backgrounds, knowledge, skills, interests, and learning styles.

The following are some strategies for creating a learning environment that acknowledges and values the diversity of students and enables them to participate fully in the learning experience:

- providing mathematics problems with situations and contexts that are meaningful to all students (e.g., problems that reflect students' interests, home-life experiences, and cultural backgrounds and that arouse their curiosity and spirit of enquiry);
- using mathematics examples drawn from diverse cultures, including those of Aboriginal peoples;
- using children's literature that reflects various cultures and customs as a source of mathematical examples and situations;
- understanding and acknowledging customs and adjusting teaching strategies as necessary. For example, a student may come from a culture in which it is considered inappropriate for a child to ask for help, express opinions openly, or make direct eye contact with an adult;
- considering the appropriateness of references to holidays, celebrations, and traditions;
- providing clarification if the context of a learning activity is unfamiliar to students (e.g., describing or showing a food item that may be new to some students);

- evaluating the content of mathematics textbooks, children's literature, and supplementary materials for cultural or gender bias;
- designing learning and assessment activities that allow students with various learning styles (e.g., auditory, visual, tactile/kinaesthetic) to participate meaningfully;
- providing opportunities for students to work both independently and interdependently with others;
- providing opportunities for students to communicate orally and in writing in their home language (e.g., pairing English language learners with a first-language peer who also speaks English);
- using diagrams, pictures, manipulatives, sounds, and gestures to clarify mathematical vocabulary that may be new to English language learners.

For a full discussion of equity and diversity in the classroom, as well as a detailed checklist for providing inclusive mathematics instruction, see pages 34–40 in Volume 1 of *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

Accommodations and Modifications

The learning activities in this document have been designed for students with a range of learning needs. Instructional and assessment tasks are open-ended, allowing most students to participate fully in learning experiences. In some cases, individual students may require *accommodations* and/or *modifications*, in accordance with their Individual Education Plan (IEP), to support their participation in learning activities.

The term accommodations *is used to refer to the special teaching and assessment strategies, human supports, and/or individualized equipment required to enable a student to learn and to demonstrate learning. Accommodations do not alter the provincial curriculum expectations for the grade.*

Modifications *are changes made in the age-appropriate grade-level expectations for a subject . . . in order to meet a student's learning needs. These changes may involve developing expectations that reflect knowledge and skills required in the curriculum for a different grade level and/or increasing or decreasing the number and/or complexity of the regular grade-level curriculum expectations.*

(Ontario Ministry of Education, 2004, pp. 25–26)

PROVIDING ACCOMMODATIONS

Students may require accommodations, including special strategies, support, and/or equipment to allow them to participate in learning activities. There are three types of accommodations:

- *Instructional accommodations* are adjustments in teaching strategies, including styles of presentation, methods of organization, or the use of technology or multimedia.
- *Environmental accommodations* are supports or changes that the student may require in the physical environment of the classroom and/or the school, such as preferential seating or special lighting.
- *Assessment accommodations* are adjustments in assessment activities and methods that enable the student to demonstrate learning, such as allowing additional time to complete tasks or permitting oral responses to test questions.

Some of the ways in which teachers can provide accommodations with respect to mathematics learning activities are listed in the chart on page 8.

Instructional Accommodations

- Vary instructional strategies, using different manipulatives, examples, and visuals (e.g., concrete materials, pictures, diagrams) as necessary to aid understanding.
- Rephrase information and instructions to make them simpler and clearer.
- Use non-verbal signals and gesture cues to convey information.
- Teach mathematical vocabulary explicitly.
- Have students work with a peer.
- Structure activities by breaking them into smaller steps.
- Model concepts using concrete materials and computer software, and encourage students to use them when learning concepts or working on problems.
- Have students use calculators and/or addition and multiplication grids for computations.
- Format worksheets so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Encourage students to use graphic organizers and graph paper to organize ideas and written work.
- Provide augmentative and alternative communications systems.
- Provide assistive technology, such as text-to-speech software.
- Provide time-management aids (e.g., checklists).
- Encourage students to verbalize as they work on mathematics problems.
- Provide access to computers.
- Reduce the number of tasks to be completed.
- Provide extra time to complete tasks.

Environmental Accommodations

- Provide an alternative work space.
- Seat students strategically (e.g., near the front of the room; close to the teacher in group settings; with a classmate who can help them).
- Reduce visual distractions.
- Minimize background noise.
- Provide a quiet setting.
- Provide headphones to reduce audio distractions.
- Provide special lighting.
- Provide assistive devices or adaptive equipment.

Assessment Accommodations

- Have students demonstrate understanding using concrete materials, computer software, or orally rather than in written form.
- Have students record oral responses on audiotape.
- Have students' responses on written tasks recorded by a scribe.
- Provide assistive technology, such as speech-to-text software.
- Provide an alternative setting.
- Provide assistive devices or adaptive equipment.
- Provide augmentative and alternative communications systems.
- Format tests so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Provide access to computers.
- Provide access to calculators and/or addition and multiplication grids.
- Provide visual cues (e.g., posters).
- Provide extra time to complete problems or tasks or answer questions.
- Reduce the number of tasks used to assess a concept or skill.

MODIFYING CURRICULUM EXPECTATIONS

Students who have an IEP may require modified expectations, which differ from the regular grade-level curriculum expectations. When developing modified expectations, teachers make important decisions regarding the concepts and skills that students need to learn.

Most of the learning activities in this document can be adapted for students who require modified expectations. The following chart provides examples of how a teacher could deliver learning activities that incorporate individual students' modified expectations.

Modified Program	What It Means	Example
Modified learning expectations, *same* activity, *same* materials	The student with modified expectations works on the same or a similar activity, using the same materials.	The learning activity involves measuring the dimensions of a rectangular object to the nearest tenth of a centimetre and calculating the area. Students with modified expectations measure the dimensions to the nearest centimetre.
Modified learning expectations, *same* activity, *different* materials	The student with modified expectations engages in the same activity, but uses different materials that enable him/her to remain an equal participant in the activity.	The learning activity involves determining the surface area of a rectangular prism using diagrams and paper-and-pencil computations. Students with modified expectations may also use a calculator.
Modified learning expectations, *different* activity, *different* materials	Students with modified expectations participate in different activities.	Students with modified expectations work on measurement activities that reflect their learning expectations, using a variety of concrete materials and measurement tools.

(Adapted from *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6, 2005,* p. 119.)

It is important to note that some students may require both accommodations and modified expectations.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or an inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this document provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions that teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this document provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and to represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections between concrete, pictorial, and symbolic mathematical representations. Advice on helping students develop conceptual understanding is also provided. The problem-solving experience in many of the learning activities allows students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas by using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The following chart outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.

Characteristics of Junior Learners and Implications for Instruction

Area of Development	Characteristics of Junior Learners	Implications for Teaching Mathematics
Intellectual development	Generally, students in the junior grades: • prefer active learning experiences that allow them to interact with their peers; • are curious about the world around them; • are at a concrete, operational stage of development, and are often not ready to think abstractly; • enjoy and understand the subtleties of humour.	The mathematics program should provide: • learning experiences that allow students to actively explore and construct mathematical ideas; • learning situations that involve the use of concrete materials; • opportunities for students to see that mathematics is practical and important in their daily lives; • enjoyable activities that stimulate curiosity and interest; • tasks that challenge students to reason and think deeply about mathematical ideas. *(continued)*

Area of Development	Characteristics of Junior Learners	Implications for Teaching Mathematics
Physical development	Generally, students in the junior grades: • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); • are concerned about body image; • are active and energetic; • display wide variations in physical development and maturity.	The mathematics program should provide: • opportunities for physical movement and hands-on learning; • a classroom that is safe and physically appealing.
Psychological development	Generally, students in the junior grades: • are less reliant on praise, but still respond well to positive feedback; • accept greater responsibility for their actions and work; • are influenced by their peer groups.	The mathematics program should provide: • ongoing feedback on students' learning and progress; • an environment in which students can take risks without fear of ridicule; • opportunities for students to accept responsibilities for their work; • a classroom climate that supports diversity and encourages all members to work cooperatively.
Social development	Generally, students in the junior grades: • are less egocentric, yet require individual attention; • can be volatile and changeable in regard to friendship, yet want to be part of a social group; • can be talkative; • are more tentative and unsure of themselves; • mature socially at different rates.	The mathematics program should provide: • opportunities to work with others in a variety of groupings (pairs, small groups, large group); • opportunities to discuss mathematical ideas; • clear expectations of what is acceptable social behaviour; • learning activities that involve all students regardless of ability.
Moral and ethical development	Generally, students in the junior grades: • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask "why" questions; • begin to consider others' points of view.	The mathematics program should provide: • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas and evaluate the ideas of others.

(Adapted, with permission, from *Making Math Happen in the Junior Grades.* Elementary Teachers' Federation of Ontario, 2004.)

Learning About Measurement in the Junior Grades

The development of understanding of measurement concepts and relationships is a gradual one – moving from experiential and physical learning to theoretical and inferential learning. Measurement thinking in the junior years begins to bridge the two.

PRIOR LEARNING

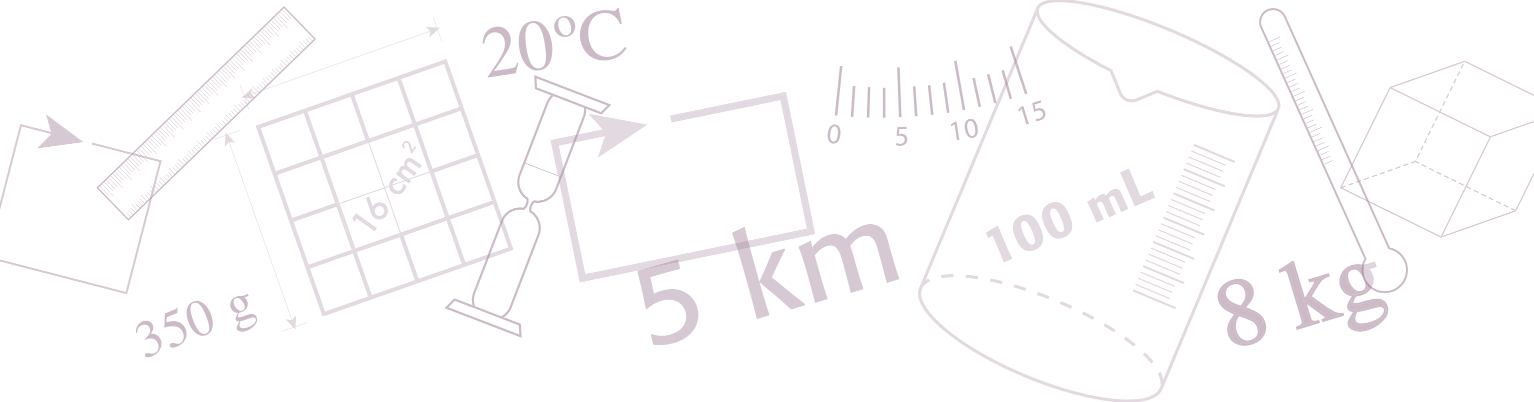
In the primary grades, students learn to estimate, measure, and record length, height, distance, area, capacity, and mass, using non-standard and standard units. They learn to compare, order, and describe objects, using attributes measured in non-standard and standard units. They learn to tell and write time and to measure temperature. Learning about measurement allows students to develop the concepts and language they need for describing objects and events in the world around them.

Experiences in the primary classroom include constructing measurement tools, selecting and justifying the choice of units, investigating relationships between the size of a unit and the number of units needed to measure an object, determining relationships between units, such as days and weeks and centimetres and metres, and solving measurement problems.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

In the junior grades, students continue to estimate, measure, and record attributes of objects and to compare, order, and describe objects, using measured attributes. They learn about volume and surface area. They extend their knowledge of time and temperature measurement. Junior students become more precise when estimating, measuring, and recording attributes of objects. They develop and use formulas for perimeter, area, volume, and surface area.

Junior students extend their understanding of measurement relationships through investigation. They compare shapes that have the same area or the same perimeter and make and test conjectures about problems they investigate. For example, what are the possible dimensions of a rectangular pen with a perimeter of 24 m? Or, what is the smallest length of fence needed for a rectangular pen with an area of 36 cm^2? Such problems offer junior students opportunities to integrate their knowledge of measurement and algebra. Measurement problems are often situated in real-life settings. Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to measurement.



THE "BIG IDEAS" OF MEASUREMENT

> *All learning, especially new learning, should be embedded in well-chosen contexts for learning – that is, contexts that are broad enough to allow students to investigate initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. Such rich contexts for learning open the door for students to see the "big ideas", or key principles, of mathematics, such as pattern or relationship.*
>
> (Ontario Ministry of Education, 2005, p. 25)

About Big Ideas

Ginsburg, who has extensively studied young children doing mathematics, suggests that, although "mathematics is big", children's minds are bigger (2002, p. 13). He argues that "children possess greater competence and interest in mathematics than we ordinarily recognize", and we should aim to develop a curriculum for them in which they are challenged to understand big mathematical ideas and have opportunities to "achieve the fulfillment and enjoyment of their intellectual interest" (p. 7).

In developing a mathematics program, it is important to concentrate on major mathematical themes, or "big ideas", and the important knowledge and skills that relate to those big ideas. Programs that are organized around big ideas and focus on problem solving provide cohesive learning opportunities that allow students to explore mathematical concepts in depth. An emphasis on big ideas contributes to the main goal of mathematics instruction – to help students gain a deeper understanding of mathematical concepts.

Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004, states that "when students construct a big idea, it is big because they make connections that allow them to use mathematics more effectively and powerfully (Fosnot and Dolk, 2001b). The big ideas are also critical leaps for students who are developing mathematical concepts and abilities" (p. 19).

Students are better able to see the connections in mathematics, and thus to *learn* mathematics, when it is organized in big, coherent "chunks". In organizing a mathematics program, teachers should concentrate on the big ideas in mathematics and view the expectations in the curriculum policy documents for Grades 4 to 6 as being clustered around those big ideas.

The clustering of expectations around big ideas provides a focus for student learning and for teacher professional development in mathematics. Teachers will find that investigating and discussing effective teaching strategies for a big idea is much more valuable than trying to determine specific strategies and approaches to help students achieve individual expectations. In fact, using big ideas as a focus helps teachers to see that the concepts presented in the curriculum expectations should not be taught as isolated bits of information but rather as a network of interrelated concepts.

In building a program, teachers need a sound understanding of the key mathematical concepts for their students' grade level, as well as an understanding of how those concepts connect with students' prior and future learning (Ma, 1999). Such knowledge includes an understanding of the "conceptual structure and basic attitudes of mathematics inherent in the elementary curriculum" (p. xxiv), as well as an understanding of how best to teach the concepts to students. Concentrating on developing this knowledge will enhance teaching and provide teachers with the tools to differentiate instruction.

Focusing on the big ideas provides teachers with a global view of the concepts represented in the strand. The big ideas also act as a "lens" for:

- making instructional decisions (e.g., choosing an emphasis for a lesson or set of lessons);
- identifying prior learning;
- looking at students' thinking and understanding in relation to the mathematical concepts addressed in the curriculum (e.g., making note of the ways in which a student solves a measurement problem);
- collecting observations and making anecdotal records;
- providing feedback to students;
- determining next steps;
- communicating concepts and providing feedback on students' achievement to parents[1] (e.g., in report card comments).

1. In this document, *parent(s)* refers to parent(s) and guardian(s).

Focusing on the big ideas also means that teachers use strategies for advancing all students' mathematical thinking (Fraivillig, 2001) by:

- eliciting from students a variety of solution methods through appropriate prompts, collaborative learning, and a positive, supportive classroom environment;
- helping students develop conceptual understanding by attending to relationships among concepts; and
- extending students' mathematical thinking by (a) encouraging students to try alternative ways of finding solutions and to generalize, and (b) setting high standards of mathematical performance for all students.

Big Ideas and Tiered Instruction[2]

How students experience a "big idea", and how "big" it becomes, depends greatly on how it is developed pedagogically in the classroom. It is not enough to label a mathematical concept as a "big idea". Big ideas must be coupled with a pedagogy that offers students opportunities to attend deeply to mathematical concepts and relationships and to experience the pleasure of mathematical insight (Gadanidis, 2004).

Big ideas, and a pedagogy that supports student learning of big ideas, naturally provide opportunities for meeting the needs of students who are working at different levels of mathematical performance. The reason for this is that teaching around big ideas means teaching around ideas that incorporate a variety of levels of mathematical sophistication. For example, consider the problem of *finding all the different rectangular areas that can be enclosed by a fence of* 24 m, and the tiers at which the problem can be approached or extended:

Tier 1: Using square tiles or grid paper, students construct various areas whose perimeter is 24. Students record length, width, area, and perimeter in a table of values and look for patterns.

Tier 2: Students also use the table of length and area values to create ordered pairs. They plot these ordered pairs on a graph and investigate relationships. For example, they might investigate which dimensions result in the greatest area, and consider whether their solution can be generalized.

Tier 3: Students might also work on extensions of the problem. They could be asked, for example: "What if we wanted to create a rectangular pen whose area is 36 m^2? How many different rectangular shapes are possible? Is there a pattern? Which dimensions result in the least perimeter? Can we generalize the solution?"

---

2. A tiered approach to instruction is suggested in *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6, 2005*, pp. 60, 120, 123.

Finding all the different rectangular areas that can be enclosed by a fence of 24 m is a variation of a sample problem listed on page 70 of The Ontario Curriculum, Grades 1–8: Mathematics, 2005 for meeting Grade 4 expectations in the Measurement strand. It is a problem that can also be used to address Algebra expectations in the junior grades (patterning and generalization). It addresses Data Management expectations (collecting and recording data that arise from the various rectangles drawn). It also uses a context from the Geometry and Spatial Sense strand (quadrilaterals and their properties).

It should also be noted that the problem of *finding all the different rectangular areas that can be enclosed by a fence of* 24 m is used at the Grade 10 level in the study of quadratic functions and at the Grade 12 level in the study of calculus concepts.

Big ideas, big problems, and a pedagogy that supports them at the classroom level provide opportunities for students to engage with *the same mathematical situation* at different levels of sophistication.

The Big Ideas of Measurement in Grades 4 to 6

The goal of teaching and learning mathematics through big ideas is an integral component of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. In each of the strands and in each of the grades, the specific expectations have been organized around big ideas in mathematics. The subheadings in the strands reflect the big ideas.

The big ideas in Measurement are:

- attributes, units, and measurement sense
- measurement relationships

The following tables show how the expectations for each of these big ideas progress through the junior grades.

The sections that follow offer teachers strategies and content knowledge to address these expectations in the junior grades while helping students develop an understanding of measurement. Teachers can facilitate this understanding by helping students to:

- investigate measurement problems in real-life settings;
- extend their knowledge of measurement units and their relationships;
- investigate the relationships between, and develop formulas for, area and perimeter and surface area and volume;
- extend their knowledge of time and temperature measurement.

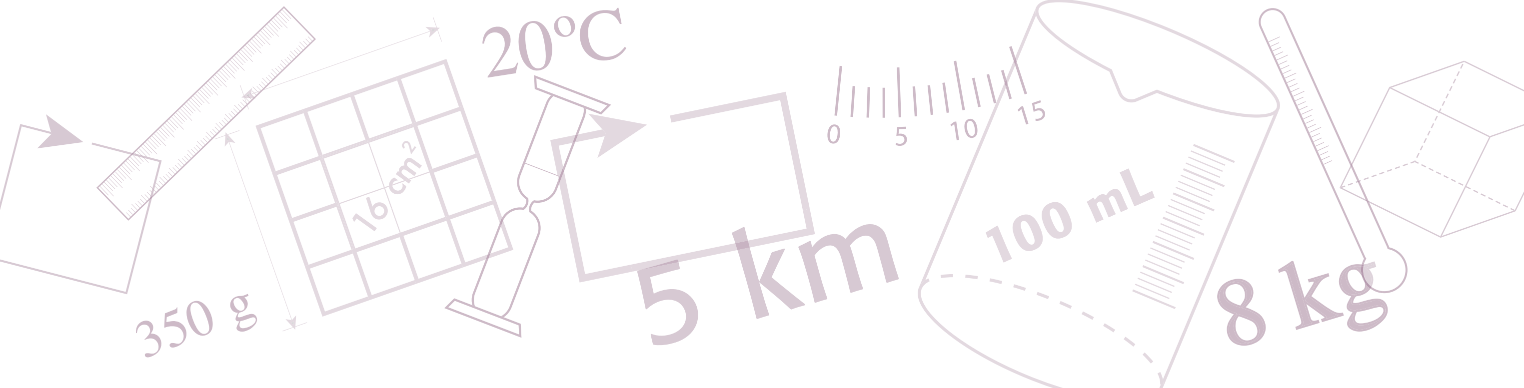
Curriculum Expectations Related to Attributes, Units, and Measurement Sense, Grades 4, 5, and 6		
By the end of Grade 4, students will:	**By the end of Grade 5, students will:**	**By the end of Grade 6, students will:**
Overall Expectation • estimate, measure, and record length, perimeter, area, mass, capacity, volume, and elapsed time, using a variety of strategies. ***Specific Expectations*** • estimate, measure, and record length, height, and distance, using standard units (i.e., millimetre, centimetre, metre, kilometre); • draw items using a ruler, given specific lengths in millimetres or centimetres; • estimate, measure (i.e., using an analogue clock), and represent time intervals to the nearest minute; • estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in five-minute intervals, hours, days, weeks, months, or years; • estimate, measure using a variety of tools and strategies, and record the perimeter and area of polygons; • estimate, measure, and record the mass of objects, using the standard units of the kilogram and the gram; • estimate, measure, and record the capacity of containers, using the standard units of the litre and the millilitre; • estimate, measure using concrete materials, and record volume, and relate volume to the space taken up by an object.	***Overall Expectation*** • estimate, measure, and record perimeter, area, temperature change, and elapsed time, using a variety of strategies. ***Specific Expectations*** • estimate, measure (i.e., using an analogue clock), and represent time intervals to the nearest second; • estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in minutes, hours, days, weeks, months, or years; • measure and record temperatures to determine and represent temperature changes over time; • estimate and measure the perimeter and area of regular and irregular polygons, using a variety of tools and strategies.	***Overall Expectation*** • estimate, measure, and record quantities, using the metric measurement system. ***Specific Expectations*** • demonstrate an understanding of the relationship between estimated and precise measurements, and determine and justify when each kind is appropriate; • estimate, measure, and record length, area, mass, capacity, and volume, using the metric measurement system.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)

Curriculum Expectations Related to Measurement Relationships, Grades 4, 5 and 6

By the end of Grade 4, students will:	By the end of Grade 5, students will:	By the end of Grade 6, students will:
Overall Expectation • determine the relationships among units and measurable attributes, including the area and perimeter of rectangles. ***Specific Expectations*** • describe, through investigation, the relationship between various units of length (i.e., millimetre, centimetre, decimetre, metre, kilometre); • select and justify the most appropriate standard unit (i.e., millimetre, centimetre, decimetre, metre, kilometre) to measure the side lengths and perimeters of various polygons; • determine, through investigation, the relationship between the side lengths of a rectangle and its perimeter and area; • pose and solve meaningful problems that require the ability to distinguish perimeter and area; • compare and order a collection of objects, using standard units of mass (i.e., gram, kilogram) and/or capacity (i.e., millilitre, litre); • determine, through investigation, the relationship between grams and kilograms; • determine, through investigation, the relationship between millilitres and litres; • select and justify the most appropriate standard unit to measure mass (i.e., milligram, gram, kilogram) and the most appropriate standard unit to measure the capacity of a container (i.e., millilitre, litre); • solve problems involving the relationship between years and decades, and between decades and centuries; • compare, using a variety of tools, two-dimensional shapes that have the same perimeter or the same area.	***Overall Expectation*** • determine the relationships among units and measurable attributes, including the area of a rectangle and the volume of a rectangular prism. ***Specific Expectations*** • select and justify the most appropriate standard unit (i.e., millimetre, centimetre, decimetre, metre, kilometre) to measure length, height, width, and distance, and to measure the perimeter of various polygons; • solve problems requiring conversion from metres to centimetres and from kilometres to metres; • solve problems involving the relationship between a 12-hour clock and a 24-hour clock; • create, through investigation using a variety of tools and strategies, two-dimensional shapes with the same perimeter or the same area; • determine, through investigation using a variety of tools and strategies, the relationships between the length and width of a rectangle and its area and perimeter, and generalize to develop the formulas [i.e., $\textit{Area} = \textit{length} \times \textit{width}$; $\textit{Perimeter} = (2 \times \textit{length}) + (2 \times \textit{width})$]; • solve problems requiring the estimation and calculation of perimeters and areas of rectangles; • determine, through investigation, the relationship between capacity (i.e., the amount a container can hold) and volume (i.e., the amount of space taken up by an object), by comparing the volume of an object with the amount of liquid it can contain or displace; • determine, through investigation using stacked congruent rectangular layers of concrete materials, the relationship between the height, the area of the base, and the volume of a rectangular prism, and generalize to develop the formula (i.e., $\textit{Volume} = \textit{area of base} \times \textit{height}$); • select and justify the most appropriate standard unit to measure mass (i.e., milligram, gram, kilogram, tonne).	***Overall Expectation*** • determine the relationships among units and measurable attributes, including the area of a parallelogram, the area of a triangle, and the volume of a triangular prism. ***Specific Expectations*** • select and justify the appropriate metric unit (i.e., millimetre, centimetre, decimetre, metre, decametre, kilometre) to measure length or distance in a given real-life situation; • solve problems requiring conversion from larger to smaller metric units; • construct a rectangle, a square, a triangle, and a parallelogram, using a variety of tools, given the area and/or perimeter; • determine, through investigation using a variety of tools and strategies, the relationship between the area of a rectangle and the areas of parallelograms and triangles, by decomposing and composing; • develop the formulas for the area of a parallelogram (i.e., $\textit{Area of parallelogram} = \textit{base} \times \textit{height}$) and the area of a triangle [i.e., $\textit{Area of triangle} = (\textit{base} \times \textit{height}) \div 2$], using the area relationships among rectangles, parallelograms, and triangles; • solve problems involving the estimation and calculation of the areas of triangles and the areas of parallelograms; • determine, using concrete materials, the relationship between units used to measure area (i.e., square centimetre, square metre), and apply the relationship to solve problems that involve conversions from square metres to square centimetres; • determine, through investigation using a variety of tools and strategies, the relationship between the height, the area of the base, and the volume of a triangular prism, and generalize to develop the formula (i.e., $\textit{Volume} = \textit{area of base} \times \textit{height}$); • determine, through investigation using a variety of tools and strategies, the surface area of rectangular and triangular prisms; • solve problems involving the estimation and calculation of the surface area and volume of triangular and rectangular prisms.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)



ATTRIBUTES, UNITS, AND MEASUREMENT SENSE

Overview

Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004 (p. 1) states:

> *The junior years are an important time of transition and growth in students' mathematical thinking. […] Junior students investigate increasingly complex ideas, building on their capacity to deal with more formal concepts.*

Students enter the junior grades with knowledge of and experience in estimating, measuring, and recording length, perimeter, area, mass, capacity, time, and temperature, using standard and non-standard units in various contexts. As they move through the junior grades, they consolidate these estimation and measurement skills, anchoring their knowledge to meaningful measurement benchmarks (e.g., the freezing point of water is 0ºC, a standard paper clip weighs about 1 g, the width of a forefinger is about 1 cm).

> *They identify benchmarks to help them recognize the magnitude of units such as the kilogram, the litre, and the metre.*
>
> (Ontario Ministry of Education, 2005, p. 8)

Students extend their understanding of what it means to measure an object, and they measure more attributes of objects (such as area, volume, and angle), choosing appropriate units.

> *Concrete experience in solving measurement problems gives students the foundation necessary for using measurement tools and applying their understanding of measurement relationships.*
>
> (Ontario Ministry of Education, 2005, p. 9)

Students come to understand that measurements are approximations. They develop this understanding by discussing factors that result in measurement discrepancies. These factors could be human error, differences in the instruments used, the perceived need for accuracy, and the precision with which measurement scales are read. Students also explore measurement in open-ended situations, such as Fermi questions (e.g., “How many basketballs would fit on the floor of our classroom?”, where estimation is necessary in order to solve the problem. *The Ontario Curriculum, Grades 1–8: Mathematics, 2005* (p. 9) states:

> *Estimation activities help students to gain an awareness of the size of different units and to become familiar with the process of measuring. As students’ skills in numeration develop, they can be challenged to undertake increasingly complex measurement problems, thereby strengthening their facility in both areas of mathematics.*

Example: A Cubic Metre

Let’s see how the “big idea” of “attributes, units, and measurement sense” might be developed through a student exploration of a cubic metre.

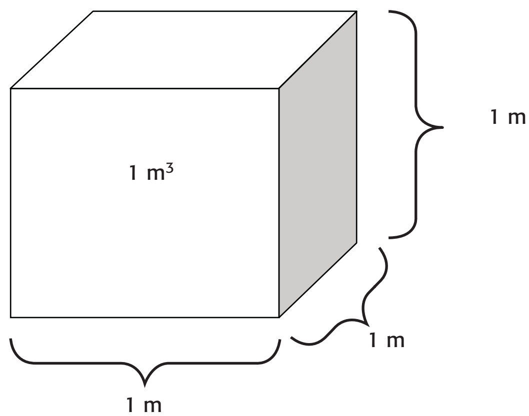


Figure 1. One cubic metre

The teacher may present to students the cubic metre in figure 1 by constructing a model, using concrete materials such as the plastic rods and connectors found in many junior classrooms. If such resources are not available, a model can be built with metre sticks or rolled newspaper poles attached with masking tape to a corner of the classroom.

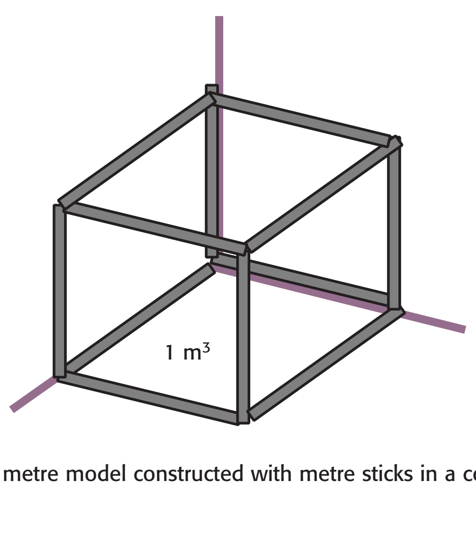


Figure 2. A cubic metre model constructed with metre sticks in a corner of the classroom

The model of the cubic metre may then be investigated so that students develop an understanding of this unit of measurement and its relationship with other units and other common physical objects in the everyday world (Method 1 below). The model of the cubic metre may also serve as a starting point for posing and solving measurement problems that engage students in estimation and in higher-level thinking (see Method 2, pp. 27–29).

Method 1: Understanding the Cubic Metre

HOW MANY STUDENTS WILL FIT INSIDE A CUBIC METRE?

Experiencing measurement units with their bodies is an excellent way for students to anchor and make meaningful what might otherwise be abstract measurement concepts. For example, having a physical model of a cube constructed in the classroom (as in figures 1 and 2 on p. 22) would allow students to explore the question "How many students would fit inside a cubic metre?"

- Students could step inside the model and crouch down so that their bodies would be contained within the model.

Suppose that four students could fit comfortably inside the cubic metre model.

- Students could then try to visualize how many more students might possibly fit in the gaps between the bodies of the four students.
- They could also estimate the volume of their own bodies as a fraction of a cubic metre. For example, if four students fit inside the model, then each student's volume is about 1/4 of a cubic metre (or 0.25 m^3). Ask: "Is this an accurate estimate? How might it be improved? How many cubic metres would be needed to contain all the students in the class?"

HOW MANY LARGE BASE TEN CUBES (DECIMETRE CUBES) WILL FIT INSIDE A CUBIC METRE?

Base ten blocks, which students have been familiar with since the primary grades, are excellent models for cubic metre measurement. The smallest base ten cubes are centimetre cubes (1 cm × 1 cm × 1 cm, or 1 cm^3). The larger base ten cubes are decimetre cubes (10 cm × 10 cm × 10 cm, or 1000 cm^3). Students can verify these facts by using a ruler or metre stick to measure the dimensions of the cubes.

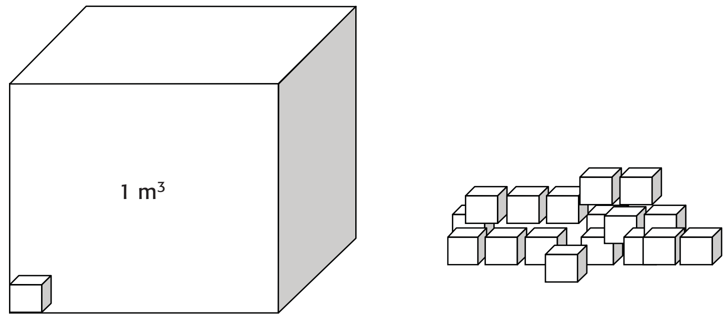


Figure 3. How many decimetre cubes will fit inside?

Figure 3 shows how the cubic metre model constructed earlier could be used to pose the following problem: “How many large base ten cubes (decimetre cubes) fit inside one cubic metre?”

- Students could explore how many base ten cubes would be needed to cover the base of the cubic metre model. Chances are that a classroom would not have the 100 large base ten cubes needed to cover the base of the cubic metre model. If students created one row of large base ten cubes along one side, they could then consider how many rows they would need to cover the base. This task would involve some measurement, some physical modelling, and some visualization.
- Students would then calculate how many layers would be needed to fill the cubic metre model.

Realizing that the cubic metre model would hold 1000 large base ten blocks is typically a surprise to students. Surprises are good. They focus student attention and create excellent opportunities for experiencing the pleasure of mathematical insight (Gadanidis, 2004).

Large numbers tend to be abstract concepts in students’ minds. By physically experiencing that 1000 large base ten cubes fit inside a cubic metre, students are better able to meaningfully visualize numbers in that range.

The experience can be further enriched by using other anchors from the everyday world. For example, using a container with a volume of one cubic decimetre and a litre of milk, students can verify that a litre of milk fits exactly inside a container that is 10 cm × 10 cm × 10 cm. Therefore, 1000 L of milk fit inside a container whose volume is one cubic metre or 1 m^3. This context may stimulate students’ mathematical imaginations and raise more interesting questions to be explored – questions that engage students in using measurement units to do meaningful problem solving (see Method 2 on pp. 27–29). For example, you might ask students to determine how many days it would take to drink the amount of milk that would fit inside a 1 m^3 container. This context also makes an important link between the volume magnitude of one cubic decimetre (1000 cm^3) and the capacity magnitude of one litre, and provides an ideal setting for incidentally discussing the capacity units of litre and millilitre and their relationships to 1000 cm^3 and 1 cm^3 respectively.

HOW MANY SMALL BASE TEN CUBES (CENTIMETRE CUBES) WILL FIT INSIDE A CUBIC METRE?

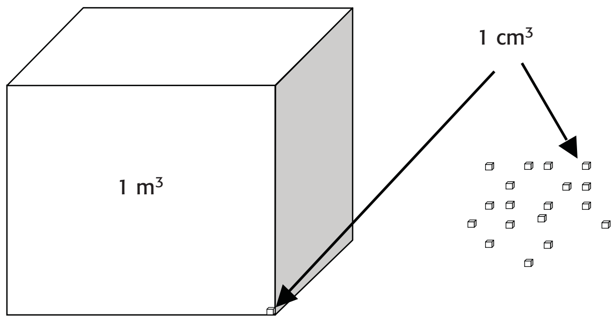


Figure 4. How many centimetre cubes will fit inside?

Figure 4 shows how the cubic metre model constructed earlier can be used to pose the following problem: How many small base ten cubes (centimetre cubes) will fit inside one cubic metre? From the previous activity, students know that 1000 large base ten cubes fit inside one cubic metre. As a first step in solving the present problem, they could consider how many small base ten cubes fit inside a large base ten cube. This question could be explored in at least two physical ways:

- The students could use a container with a volume of one cubic decimetre (which is the same size as a large base ten cube) and fill the base layer with small base ten cubes. Then they could consider how many of these layers would be needed to fill the container.
- They could also use a large base ten block that has centimetre grid etchings on each of its six faces. Or they could use a ruler to measure the base dimensions and then use that measurement to determine how many small base ten cubes would fit in the base layer. After this, they could consider how many of these layers would be needed to fill the large base ten cube.

There are 1000 cm^3 in one cubic decimetre or 1 dm^3, and there are 1000 dm^3 in 1 m^3. Therefore, there are 1 000 000 cm^3 in 1 m^3.

Another way to approach this problem is by laying metre sticks along the base edges of the cubic metre model to help students see that there are 100 cm along each edge.

$$\textit{Area of Base} = \textit{length} \times \textit{width} = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2.$$

$$\textit{Volume} = \textit{area of base} \times \textit{height} = 10\,000 \text{ cm}^2 \times 100 \text{ cm} = 1\,000\,000 \text{ cm}^3.$$

Or:

$$\textit{Volume} = \textit{length} \times \textit{width} \times \textit{height} = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} = 1\,000\,000 \text{ cm}^3.$$

Students could also use centimetre grid paper cut into 10 cm × 10 cm sheets to help visualize how many square centimetres fit into the base. Ten 10 cm × 10 cm sheets could be used to create a row, which is then slid to determine the number of rows needed.

Realizing that 1 000 000 small base ten blocks would fit inside the cubic metre model provides students with a physical representation of the number 1 000 000. By physically experiencing that 1 000 000 small base ten cubes fit inside one cubic metre, students are better able to meaningfully visualize numbers in the range of 1 000 000.

WHY CUBIC CENTIMETRES (cm^3)?

The above experiences also provide a context within which students can experience and discuss the differences between the units the centimetre (cm), the square centimetre (cm^2), and the cubic centimetre (cm^3) and their relationships to linear, area, and volume measurement respectively. All three units are useful for describing various attributes of the cube, as shown in figure 5:

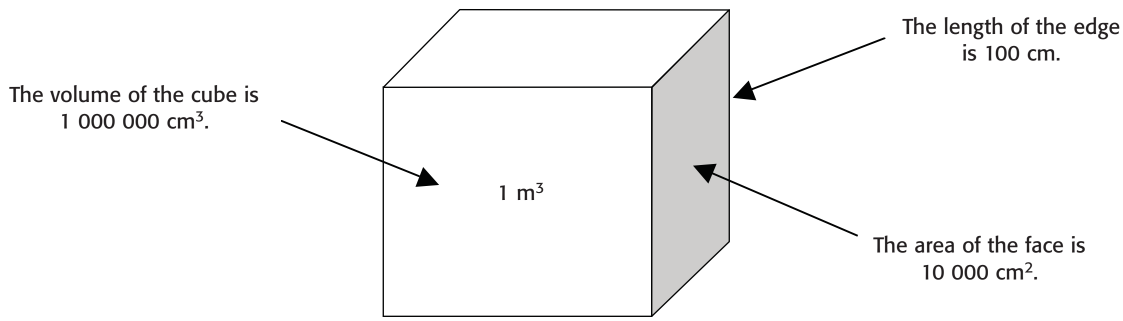


Figure 5. The centimetre (cm), square centimetre (cm^2), and cubic centimetre (cm^3)

MEASUREMENT BENCHMARKS

Measurement sense develops as students anchor the meaning of measurement units to measurement benchmarks in their everyday world. For example:

- a teaspoon is about five millilitres, or 5 mL;
- a large thumbtack has an approximate mass of one gram, or 1 g;
- a litre of milk or a litre of water has a mass of about one kilogram, or 1 kg.

Students need opportunities to measure and discover such benchmarks, and to discuss them in classroom settings. Ask, for instance:

- “What benchmark would you use for 1 cm? Why?”
- “What benchmark would you use for 1 Gm (a metric unit of distance equal to one million kilometres)? Why?”
- “What benchmark would you use for 1 L? Why?”

CHARACTERISTICS OF STUDENT LEARNING AND INSTRUCTIONAL STRATEGIES

Junior students who possess measurement sense are able to do the following:

- choose units appropriately (by type and magnitude) to measure attributes of objects;
- use measurement instruments effectively;
- use meaningful measurement benchmarks to make sense of and visualize the magnitude of measurement units; and
- make reasonable measurement estimates and justify their reasoning.

For students to develop such an understanding, the teacher needs to use instructional strategies that help students become aware of the following:

- the differences between units, in terms of the attributes and the magnitude of the object they measure;
- the various measurement instruments and methods and their appropriate use;
- benchmarks in the everyday world that help anchor the meaning of measurement units; and
- estimation strategies and the degree of accuracy required in various contexts.

Method 2: Extending the Problem

It is the questions that drive mathematics. Solving problems and making up new ones is the essence of mathematical activity.

(Hersh, 1997, p. 18)

Classroom experiences such as those described in "Method 1: Understanding the Cubic Metre" help students develop a strong basic sense of measurement units. However, students also need to experience measurement in a problem-solving setting, in which math investigations are extended by the use of a new context or a "what if" question. This goal is reflected in the *The Ontario Curriculum, Grades 1–8: Mathematics, 2005* (p. 11): "Problem solving is central to learning mathematics. By learning to solve problems and by learning *through* problem solving, students are given numerous opportunities to connect mathematical ideas and to develop conceptual understanding". Students are expected to "pose and solve problems and conduct investigations, to help deepen their mathematical understanding" (p. 77). *Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004* (p. 7) states that "a variety of problem-solving experiences and a balanced array of pedagogical approaches" are necessary for effective mathematics instruction in the junior grades. An essential aspect of an effective mathematics program is balance (Kilpatrick, Swafford, & Findell, 2001).

The extension examples that follow aim to give teachers a sense of the richness – the "bigness" – of mathematical ideas. Knowing the connections discussed below will help teachers plan investigations that engage students with big mathematical ideas and will also help them be flexible and responsive to mathematical questions and directions initiated by students.

EXTENSION: HOW MUCH MILK?

Earlier, we discussed an investigation that related capacity (litres) to volume (cubic metres) and engaged students' mathematical imaginations by posing problems such as this one: How many days would pass before you drank the amount of milk that would fit inside a 1 m^3 container?

This problem can be further extended as follows: If it were possible for us to take all the milk consumed in one year by the students in this school and to pour it into the classrooms (with doors and windows shut tight), how many classrooms would it fill?

How do we solve such a problem? Below is one possible sequence of solution steps.

1. We might start by determining the number of students in the school. We might know the answer to this or we might have to estimate, taking into account the number of grades and the number of classes of each grade in the school.
2. We would estimate how many litres of milk the average student drinks in a year.
3. We would have to consider such issues as: Do students drink more or less milk on weekends? Do students drink more or less milk in the summer? How does milk consumption change with age? What is the average glass or cup size, and how many glasses or cups make up a litre? Should we include milk used on cereal? Do all students drink milk? Making a reasonable guess as to the amount of milk typically consumed by a student in a day or in a week could involve some data management skills (conducting student surveys, for example).
4. Then we would have to calculate the number of litres of milk consumed by all students. We could use a calculator or we could round numbers and use mental computation.
5. We would also need to estimate how many litres of milk might fit inside a typical classroom. We already know that there are 1000 L in 1 m^3. We would then need to estimate the volume of the classroom in cubic metres.

Problems or questions such as these are called Fermi questions, named after the physicist/mathematician Enrico Fermi (1901–1954).

> *Throughout his work, Fermi was legendary for being able to figure out things in his head, using information that initially seems too meagre for a quantitative result. He used a process of "zeroing in" on problems by saying that the value in question was certainly larger than one number and less than some other amount. He would proceed through a problem in that fashion and, in the end, have a quantified answer within identified limits.*
>
> (Talamo, 1996)

A quick Internet search will yield numerous creative Fermi questions, such as the following:

- What fraction of your town is covered by roads?
- How many hairs are on your head?
- How many blinks are there in a lifetime?

Fermi questions offer students the opportunity to engage with measurement in an imaginative and creative manner. In solving Fermi questions, students get practice in using measurement units and formulas. They also get practice in thinking creatively in a mathematical setting. Fermi questions can be solved in a variety of ways and require the exploration of creative ways of using measurement skills.

EXTENSION: MATHEMATICAL LITERATURE

Whitin and Wilde (1992) identify the many benefits of using children's literature in mathematics teaching. Children's literature:

- provides a meaningful context for mathematics;
- celebrates mathematics as language;
- demonstrates that mathematics develops out of human experience;
- addresses humanistic, affective elements of mathematics;
- integrates mathematics into other curriculum areas;
- restores an aesthetic dimension to mathematical learning;
- provides a meaningful context for posing problems.

Mathematical literature can be a starting point for engaging and extending students' measurement thinking. Books such as *Counting on Frank* (Clement, 1990) give students opportunities to explore Fermi questions in the context of an enjoyable story.

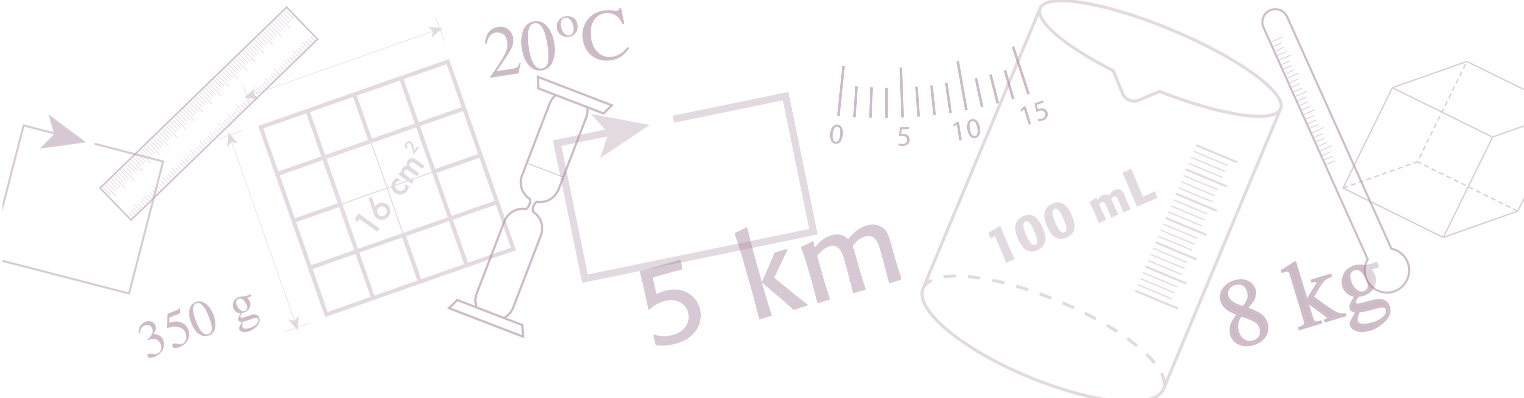
CHARACTERISTICS OF STUDENT LEARNING AND INSTRUCTIONAL STRATEGIES

Extending a problem in mathematically more sophisticated ways requires more in-depth mathematical knowledge from the teacher, who must be flexible and responsive to (possibly unanticipated) directions that mathematical exploration takes in a classroom setting. In addition, the teacher will need to use more elaborate instructional strategies in which students:

- are encouraged to pose and explore "what if" questions;
- are encouraged to model the problem situations, using a variety of representations: numbers, diagrams, concrete materials, computer models, and real-life applications;
- are encouraged to make connections with measurement concepts explored previously and with concepts learned in other strands;
- have opportunities to attend to deep mathematical ideas and to experience the pleasure of mathematical insight.

Students who develop problem-solving skills that enable them to explore extensions to measurement problems have the following learning characteristics:

- They have a rich understanding of measurement units and estimation strategies.
- They can pose “what if” questions to extend problems in new mathematical directions.
- They are willing to persevere in their mathematical thinking and solve mathematical problems.
- They can work cooperatively and constructively with others.



MEASUREMENT RELATIONSHIPS

Overview

In addition to learning about the attributes of objects to be measured and the appropriate units to use in each case, students in the junior grades explore measurement relationships (such as those of area to perimeter and volume to surface), as well as relationships that help add meaning to measurement formulas (such as relationships between the area formulas for rectangles, parallelograms, and triangles). In the Measurement strand, learning investigations often use real-life, concrete settings and contexts.

For students in the middle grades, measurement acts as a context for connecting ideas in mathematics with those in other disciplines, since many disciplines make some use of measurement concepts. *The Ontario Curriculum: Mathematics, Grades 1–8, 2005* (p. 24) states:

> *Measurement concepts and skills are directly applicable to the world in which students live. Many of these concepts are also developed in other subject areas, such as science, social studies, and physical education.*

Measurement also serves as a context for helping students understand concepts from other strands – for example, fractions and decimals, multiplication using an area model, geometric shapes and their properties, the nature of variables in formulas, and data collection.

Example: Developing Formulas From Student Experiences

Measurement formulas can be developed from student experiences with measurement. For example, by using a centimetre grid copied onto a transparency, students can determine how many square centimetres (cm^2) make up the area of each rectangle shown in figure 6 on page 32. This experience helps students identify patterns that can be generalized to develop the area formula for a rectangle (as in Method 1, pp. 32–33).

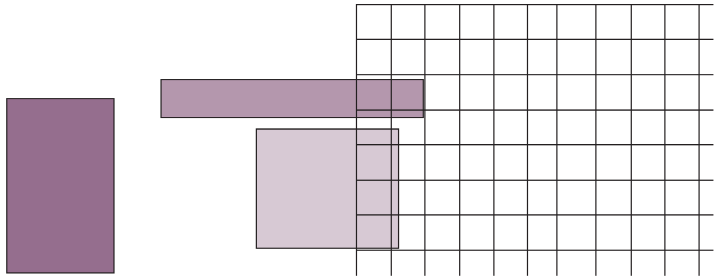


Figure 6. Using a centimetre grid transparency to find the areas of rectangles

Similarly, by exploring and observing the growing volume and surface area patterns in a tower of cubes (figure 7), students can generalize to develop the surface area and volume formulas for a rectangle prism (see Method 2, p. 35).

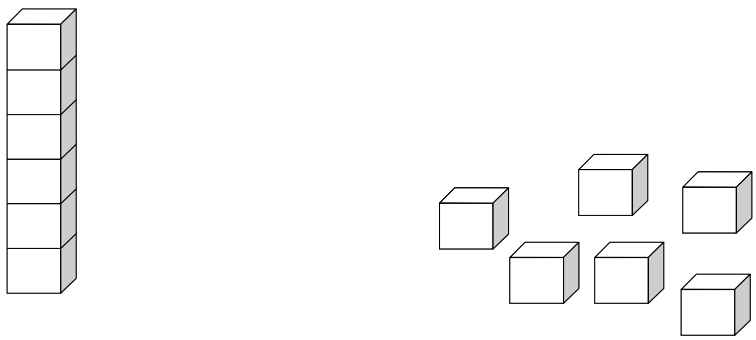


Figure 7. Connecting cubes to make a tower

Such starting points help students develop an understanding of measurement formulas. These activities may also be extended to help students explore relationships among measurements and formulas in growth/change patterns. For example, students can explore what areas are possible and what might be the greatest area for a rectangle with a perimeter of 32 m (see Method 2, p. 38).

Method 1: Understanding Measurement Formulas

DEVELOPING THE AREA AND PERIMETER FORMULAS FOR RECTANGLES

Using a centimetre grid copied onto a transparency, students can measure and record the following attributes of rectangles: length, width, perimeter, and area. These measurements can be recorded in a table:

Rectangle	Length (ℓ)	Width (w)	Fencing (Perimeter)	Area
#1	12 cm	3 cm	30 cm	36 cm^2
#2	8 cm	10 cm	36 cm	80 cm^2
#3	15 cm	11 cm	52 cm	165 cm^2
#4	4 cm	17 cm	42 cm	68 cm^2
#5				
#6				
#7				
#8				

Table 1. Measuring attributes of rectangles and noticing patterns

By observing the relationships between the dimensions and the perimeter, and between the dimensions and the area, students can find patterns and express them as generalized statements. For example, for calculating perimeter, students might suggest the following:

- add the measurements of the four sides, or $P = \ell + w + \ell + w$;
- add the two dimensions and multiply by 2, or $P = (\ell + w) \times 2$;
- multiply each dimension by 2 and then add the results, or $P = 2 \times \ell + 2 \times w$.

Investigating this type of problem deepens students' understanding of the relationships expressed by measurement formulas. It also helps students realize that formulas can be expressed in more than one way. Noting patterns and generalizing also overlap with the Patterning and Algebra expectations for the junior grades, creating opportunities to integrate instruction and assessment for both strands.

DEVELOPING THE AREA FORMULAS FOR PARALLELOGRAMS AND TRIANGLES

Once students understand the area formula for rectangles, they are able to follow the development of the area formulas for parallelograms and triangles.

For example, using paper and scissors, students can cut a corner off a rectangle and slide it to the opposite side, as shown in figure 8. The rectangle and the parallelogram have the same area, since moving a part of the rectangle to an opposite side does not add to or take away from the original area.

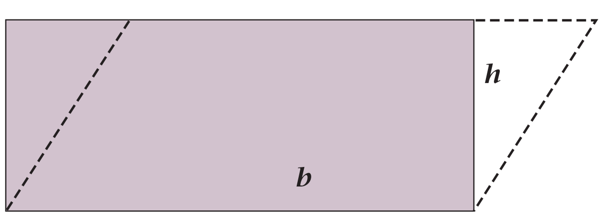


Figure 8. Developing the area formula for parallelograms

> ***Note:*** It is appropriate to use *l* and *w* as labels for rectangles, but not for parallelograms, since *ℓ* and *w* apply to sides that meet at 90º. We need to adjust the rectangular area formula, $A = \ell \times w$, and write it as $A = b \times h$, where b = base and h = height. Notice that the new formula, $A = b \times h$, applies for both rectangles and parallelograms.

Then, by considering a parallelogram that is cut into two congruent pieces along one of its diagonals, we can see that the area of a triangle is one-half the area of a parallelogram.

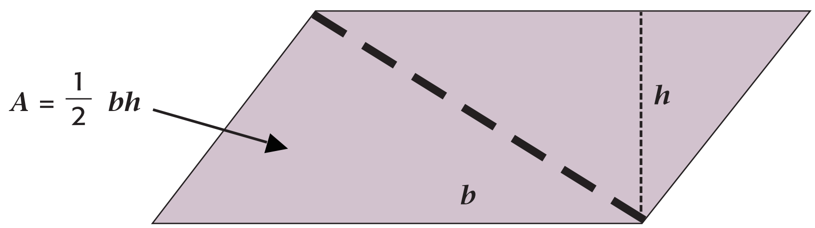


Figure 9. Developing the area formula for triangles

The relationship between base (*b*) and height (*h*) in a parallelogram can be illustrated and explored using dynamic geometry software, such as The Geometer's Sketchpad, which is licensed to all Ontario schools. For example, by drawing a parallelogram between two parallel lines and then sliding the parallelogram along one of the parallel lines, students can see that as long as the base and height remain the same, the area does not change.

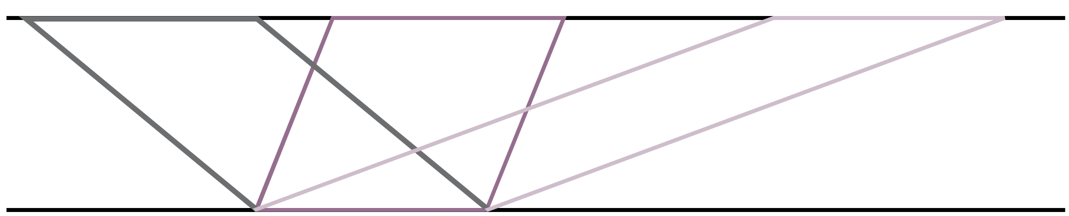


Figure 10. Three parallelograms with the same area

A similar relationship can be illustrated and explored for triangles:

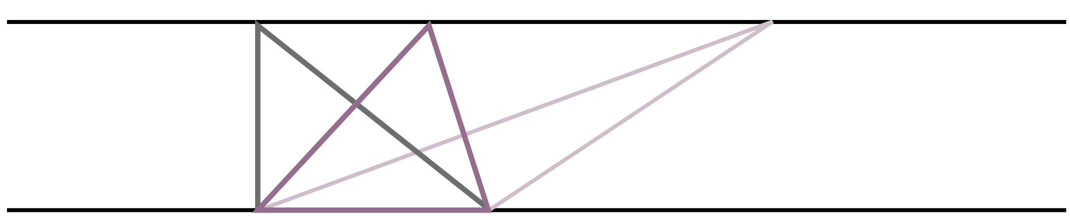


Figure 11. Three triangles with the same area

DEVELOPING THE SURFACE AREA AND VOLUME FORMULAS FOR RECTANGULAR PRISMS

Using the growing tower of cubes shown in figure 7 (p. 32), students can record the measurements as the tower grows (table 2). Assume that the tower is being built with 2 cm × 2 cm × 2 cm cubes (e.g., interlocking cubes).

Number of Cubes	Length (ℓ)	Width (w)	Height (h)	Surface Area	Volume
1	2 cm	2 cm	2 cm	24 cm^2	8 cm^3
2	2 cm	2 cm	4 cm	40 cm^2	16 cm^3
3	2 cm	2 cm	6 cm	56 cm^2	24 cm^3
4	2 cm	2 cm	8 cm	72 cm^2	32 cm^3
5	2 cm	2 cm	10 cm	88 cm^2	40 cm^3
6	2 cm	2 cm	12 cm	104 cm^2	48 cm^3
7	…	…	…	…	…

Table 2. Exploring the surface area and volume of a growing rectangular tower (prism)

As students create and use a table such this, they can see patterns in the Volume column and in the Surface Area column. For example, they might notice and explore the following:

- As we move down the volume column, we see that the volume increases by 8 cm^3, which is the volume of the 2 cm × 2 cm × 2 cm cube we add to the tower each time.
- The formula $\ell \times w \times h$ gives us the volume.
- The area of the base ($\ell \times w$) times the height (h) also gives us the volume.
- As we move down the surface area column, we see that the surface area increases by 16 m^2. How does this increase relate to the surface area of the new cube added?
- Each cube has a surface area of 24 cm^2; for 5 cubes, the surface area would be 5 × 24 cm^2 = 120 cm^2. But our volume in the 5th row is 88 cm^2. Why is this the case? What happened to the missing 32 cm^2? How can we explain the "missing" surface area, and how can we use the explanation to develop a formula for the surface area of the tower?

CHARACTERISTICS OF STUDENT LEARNING AND INSTRUCTIONAL STRATEGIES

Junior students who understand measurement formulas:

- know and apply measurement formulas;
- can generalize from investigations in order to develop measurement formulas;
- can demonstrate relationships among measurement formulas (e.g., the area formulas for squares, rectangles, parallelograms, and triangles);
- recognize the role of variables in measurement formulas (as in $A = \ell \times w$ and $P = 2 \times \ell + 2 \times w$); and
- recognize that formulas can be expressed in more than one way (e.g., $P = 2 \times \ell + 2 \times w$ or $P = \ell + \ell + w + w$ or $P = 2 \times (\ell + w)$.

To help students develop an understanding of measurement formulas, teachers should provide them with opportunities to observe the patterns and relationships in measurement formulas and their algebraic representations.

Method 2: Exploring Measurement Relationships

To help students develop an understanding of measurement relationships, teachers should provide them with opportunities to investigate such relationships in real-life problem-solving settings to which they can relate. Three examples are provided below.

PACKAGING: WHAT THREE-DIMENSIONAL SHAPE REDUCES PACKAGING WASTE?

Packaging constitutes a significant percentage of the trash produced in Canada. The packaging of cereals, cookies, laundry detergents, and other such products comes in various three-dimensional shapes. Is there an optimum shape that would reduce the amount of packaging required for products we see on grocery store shelves?

By bringing various common packages to the classroom, the teacher can set the stage for a discussion of the social and environmental issues related to the waste produced by Canadians, and for instruction on the use of measurement concepts to better understand the issues.

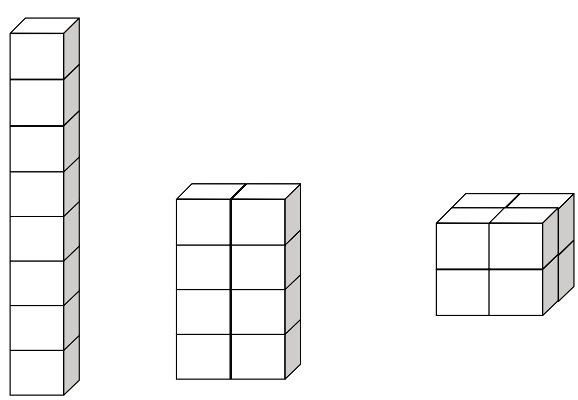


Figure 12. Different rectangular prisms made up of 8 interlocking cubes

Model the problem by giving pairs of students centimetre cubes and asking them to construct as many different rectangular prisms as they can, where each rectangular prism is made up of 8 centimetre cubes (see figure 12 on the opposite page). Ask students to record the findings in a table (table 3), and to calculate the surface area for each prism.

Volume	Length (ℓ)	Width (w)	Height (h)	Surface Area
8 cm^2	1 cm	1 cm	8 cm	34 cm^2
8 cm^2	1 cm	2 cm	4 cm	28 cm^2
8 cm^2	2 cm	2 cm	2 cm	24 cm^2

Table 3. Measurements of different rectangular prisms made up of 8 centimetre cubes.

Ask the students to consider the following:

Suppose we have a package whose volume is 1000 cm^3. What different dimensions are possible? Which dimensions result in the least amount of surface area? Is there a pattern? This problem offers an excellent opportunity to incorporate the use of a spreadsheet. A spreadsheet would allow students to use the formulas for volume and surface area in a meaningful way; that is, to "tell" the spreadsheet how to calculate the Volume and Surface Area columns. A spreadsheet like the one in table 4 can help students focus on relationships and prevent them from getting bogged down by calculations.

Volume	Length (ℓ)	Width (w)	Height (h)	Surface Area
1000 cm^3	5 cm	5 cm	40 cm	850 cm^2
1000 cm^3	5 cm	10 cm	20 cm	700 cm^2
1000 cm^3	8 cm	25 cm	5 cm	730 cm^2
1000 cm^3	10 cm	10 cm	10 cm	600 cm^2
1000 cm^3	4 cm	25 cm	10 cm	780 cm^2
1000 cm^3	2 cm	5 cm	100 cm	1420 cm^2
1000 cm^3	…	…	…	…

Table 4. Different dimensions for a volume of 1000 cm^3

The results of this exploration provide opportunities for a broader discussion. The cube is the most efficient rectangular prism in terms of the packaging needed to construct it. Why then do companies use less efficient packaging shapes? One reason is that they want to have a box front with a large area on which to advertise the product inside. Another is a matter of aesthetics. A cube-shaped package on a grocery store shelf presents a square face to the customer. The ancient Greeks would have argued (and the argument still holds true today) that the most pleasing shape is not a square but a rectangle, with sides in the approximate ratio of 1 to 1.6 (a shape known as the golden mean or golden ratio). Ask students if they have ever noticed that most paintings are rectangular. It is rare to see a square painting. There is a conflict here between efficiency, beauty, and marketing.

AREA AND PERIMETER: HOW DO WE FENCE IN THE LARGEST POSSIBLE AREA?

Farmers often use fencing to create pens for animals and, as a way of keeping costs down, often try to make the most efficient use of fencing material they have on hand. If a farmer has 32 m of fencing material with which to create a rectangular pen for a goat, what dimensions would produce the pen with the greatest area? Can a pattern be found in the various options?

Fencing (Perimeter)	Length (ℓ)	Width (w)	Area
32 m	1 m	15 m	15 m^2
32 m	2 m	14 m	28 m^2
32 m	3 m	13 m	39 m^2
32 m	4 m	12 m	48 m^2
32 m	5 m	11 m	55 m^2
32 m	6 m	10 m	60 m^2
…	…	…	…

Table 5. What is the largest area?

In a related problem, we want to enclose a rectangular area of 64 m^2. What would be the least amount of fencing that we would need?

Area	Length (ℓ)	Width (w)	Fencing (Perimeter)
64 m^2	1 m	64 m	130 m
64 m^2	2 m	32 m	68 m
64 m^2	4 m	16 m	40 m
64 m^2	8 m	8 m	32 m
64 m^2	10 m	6.4 m	32.8 m
64 m^2	16 m	4 m	40 m
…	…	…	…

Table 6. What is the least amount of fencing needed?

These area and perimeter activities can also serve as settings in which students can graph ordered pairs and see them come to life as graphs (by plotting length versus area for table 5, and length versus perimeter for table 6). In a study of Grade 5 students exploring such problems (Gadanidis, 2001) and using technology to represent the ordered pairs as graphs, one of the teachers expressed surprise that students were able to understand such relationships: “All of them got it.” The teacher went on to say that some students asked, “Is the biggest area always a square? … wouldn’t a circle be bigger? … they were really thinking” (p. 228).

Investigating such problems deepens students’ understanding of measurement relationships as well as their algebraic understanding. Such problems also provide a contextualized opportunity for practising the use of the area and perimeter formulas for rectangles, and for finding missing numbers in equations.

REFERENCES

Clement, R. (1990). *Counting on Frank*. Sydney, Australia: William Collins Pty Ltd.

Elementary Teachers' Federation of Toronto. (2004). *Making math happen in the junior grades*. Toronto: Author.

Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs. (2005). *Education for all: The report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6*. Toronto: Ontario Ministry of Education.

Expert Panel on Mathematics in Grades 4 to 6 in Ontario. (2004). *Teaching and learning mathematics: The report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario*. Toronto: Ontario Ministry of Education.

Fraivillig, J. (2001). Strategies for advancing students' mathematical thinking. *Teaching Children Mathematics*, *7*(8), 454–459.

Gadanidis, G. (2001). Web-based multimedia activities as pedagogical models. *Proceedings of the Asian Technology Conference in Mathematics*, RMIT University, Melbourne, Australia, 223–232.

Gadanidis, G. (2004). The pleasure of attention and insight. *Mathematics Teaching, 186*(1), 10–13.

Ginsburg, H.G. (2002). Little children, big mathematics: Learning and teaching in the pre-school. *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*, University of East Anglia , Volume 1, 3–14.

Hersh, R. (1997). *What is mathematics really?* London: Oxford University Press.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates.

Ontario Ministry of Education. (2005). *The Ontario curriculum, Grades 1–8: Mathematics*. Toronto: Author.

Ontario Ministry of Education. (2006). *A guide to effective instruction in mathematics, Kindergarten to Grade 6*. Toronto: Author.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York: Basic Books.

Talamo, S. (1996). What is a Fermi question? Retrieved May 10, 2007, from http://mathforum.org/workshops/sum96/interdisc/sheila1.html

Whitin, D.J., & Wilde, S. (1992). *Read any good math lately? Children's books for mathematical learning, K–6*. Portsmouth, NH: Heinemann.

Learning Activities

Introduction to the Learning Activities

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to measurement. The learning activities also support students in developing their understanding of the big ideas outlined in the first part of this guide.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, and instructional groupings for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or task.

WORKING ON IT: In this part, students work on the mathematical task, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

TIERED INSTRUCTION: These are suggestions for ways to meet the needs of all learners in the classroom.

HOME CONNECTION: This section is addressed to parents or guardians, and includes a task for students to do at home that is connected to the mathematical focus of the learning activity.

ASSESSMENT: This section provides guidance for teachers on assessing students' understanding of mathematical concepts. A rubric is included.

BLACKLINE MASTERS: These pages are referred to and used throughout the activities.

Grade 4 Learning Activity
The First Decade of My Life

OVERVIEW

In this learning activity, students create a time line of significant events that occurred during the first decade of their life. Significant events may include important world events, important innovations, and personal or family milestones. Time lines such as these provide a rich context for posing and responding to more complex mathematical problems, and for working with a variety of Fermi questions (see pp. 28–29 in this volume). This learning activity can be effectively linked with the Grade 4 topic "Medieval Times" in the Heritage and Citizenship strand of the curriculum in social studies.

The learning task builds upon students' prior learning of relationships between years and decades, and extends their understanding of the relationships between years and decades, and between decades and centuries. Before starting this learning activity, students will need opportunities to read and interpret sample time lines. They will also need to have an understanding of how to solve problems involving relationships between minutes and hours, hours and days, days and weeks, and weeks and years, using a variety of tools.

BIG IDEA

Attributes, units, and measurement sense

CURRICULUM EXPECTATIONS

ATTRIBUTES, UNITS, AND MEASUREMENT SENSE

This learning activity addresses the following **specific expectation**.

Students will:

- estimate and determine elapsed time, with and without using a time line, given the duration of events expressed in five-minute intervals, hours, days, weeks, months, or years.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- estimate, measure, and record length, perimeter, area, mass, capacity, volume, and elapsed time, using a variety of strategies.

MEASUREMENT RELATIONSHIPS

This learning activity addresses the following **specific expectation**.

Students will:

- solve problems involving the relationship between years and decades, and between decades and centuries.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- determine the relationship among units and measurable attributes, including the area and perimeter of rectangles.

ABOUT THE LEARNING ACTIVITY

TIME:
4 hours

MATERIALS

- sheets of paper (e.g., Bristol board, chart paper, or butcher paper) (1 per student)
- sets of markers (1 per group of students)
- glue (1 container per group of students)
- family pictures (optional)
- math journals (optional)
- informational texts and/or Internet access
- toothbrush and toothpaste (optional)
- clocks or stopwatches (1 per group of students)
- calculators (1 per group of students)
- **M.BLM4a.1: Personal Time Line** (1 per student)

MATH LANGUAGE

- units of measurement for time (e.g., *century*, *day*, *decade*, *hour*, *minute*, *year*, *month*)
- *elapsed*
- *duration*
- *time line*
- *frequency*
- *innovation*
- *time anchor*

INSTRUCTIONAL GROUPING:
individuals and partners

ABOUT THE MATH

FERMI QUESTIONS

Sheila Talamo, in “Fermi Questions” (1996) (Retrieved May 10, 2007, from http://mathforum.org/workshops/sum96/interdisc/sheila1.html) explains the significance of Fermi questions in mathematical learning:

> *Fermi questions emphasize estimation, numerical reasoning, communicating in mathematics, and questioning skills. Students often believe that “word problems” have one*

exact answer and that the answer is derived in a unique manner. Fermi questions encourage multiple approaches, emphasize process rather than "the answer", and promote non-traditional problem solving strategies.

GETTING STARTED

INTRODUCING THE PROBLEM

Describe the following scenario to the class:

> "A time line is an effective organizer for recording important events and innovations. Over the next several days you will be creating your own personal time line, detailing the first decade of your life. Your time line will include significant personal and family events, world events, and important innovations. In order to complete this time line, you will be required to gather information from your family as well as from secondary resources, such as informational texts or the Internet. Your time line will be shared with your classmates and used to investigate interesting facts from the first decade of your life."

> ***Note:*** Having students create and share personal time lines will provide an authentic opportunity to celebrate diversity in your classroom.

> ***Note:*** When discussing elapsed time on the sample time lines, direct conversation to the appropriateness of the unit used to describe the duration, frequency, and time between events. Select a variety of events or innovations in order to ensure that various units are used to describe elapsed time. Model the language of approximation when describing elapsed time.

WORKING ON IT

STAGE ONE: CREATING A PERSONAL TIME LINE

Before beginning this task, give students time to reflect and connect by brainstorming what they know about time lines. Working with the students as a group, review sample time lines to examine and note organizational structures and features. Discuss the supplies that are available – for example, Bristol board, chart paper, or butcher paper; markers; glue. Have the students do research on resources (informational texts or the Internet). Provide direction on the number of events and innovations that should be included per year on each time line. Encourage students to strive for a balance between personal events, world events, and important innovations.

> ***Note:*** You may decide to share a historical time line linked to the Grade 4 topic "Medieval Times" in the Heritage and Citizenship strand of the social studies curriculum. A historical time line will provide opportunities to discuss elapsed time and to investigate the relationships between years and decades, and between decades and centuries.

To facilitate comparison through shared discussion, you may decide to select specific events that must be represented on all time lines. Examples of questions related to personal events might include:

- "When did you learn to talk?"
- "When did you learn to walk?"
- "When did you start school?"
- "When did you get your first tooth?"

Note: When discussing key events on a sample time line, it is best to focus on elapsed time, given the time and duration of specific events. In discussions and shared investigations, elapsed time can be expressed in intervals of five minutes or in hours, days, weeks, months, or years.

A key feature of each time line will be notations indicating the amount of elapsed time within and between events. Specify a reasonable number of notations per time line. It is important for students to recognize that certain notations indicating elapsed time will require a greater degree of precision than others.

Note: This component of the learning task offers an excellent opportunity for a home connection. Interviewing a parent about important personal milestones, family events, and significant world events will enrich this task for students. With parental permission, family photographs could be used to visually represent key events on the time line. (See **M.BLM4a.1: Personal Time Line**).

As students work on their personal time lines, circulate and conduct individual conferences. During this phase, you will be able to assess students' understanding of elapsed time by discussing their notations of the duration of specific events, milestones, or innovations. Focus on whether students have selected an appropriate unit of measurement, and also on their recognition of the degree of precision required. Students can then share their completed time lines in a Gallery Walk (where students display their work for others to view) or in Sharing Circles (where students share their work in small or large group settings). At this point in the activity you may decide to have students complete a math journal entry focused on describing elapsed time and on noting an appropriate degree of precision.

Note: Using the language of approximation, relate events that occur naturally throughout the school day to various units of time. Experiences that allow students to estimate, measure, and record time intervals to the nearest minute will provide foundational knowledge for this learning task. Everyday references and experiences will help students to develop benchmarks for time, thus providing an anchor for reasonable estimation.

Note: See the music video of the song "Help me Fermi", by George Gadinidis, at http://publish.edu.uwo.ca/george.gadanidis/fermi. The Fermi questions asked in this song may motivate students to ask and explore their own questions.

Scaffolding Suggestion: You might ask students to sketch a time line with notches to indicate units of elapsed time. A student who learned to walk at 18 months could be asked to locate this point on his or her time line. By counting forward six months and eight years, the student would be able to determine the amount of elapsed time between the time when he or she learned to walk and the present.

STAGE TWO: FERMI QUESTIONS

Students focus on the elapsed time related to a specific event in order to solve rich problems involving the relationship between years and decades, and between decades and centuries.

ESTIMATING TOOTHBRUSHING TIME

Say to the students:

"You have been sharing personal events, world events, and important innovations, using your personal time line of the first decade of your life.

"Every day, we spend considerable time completing daily routines. Toothbrushing is one of those routines. In the next part of the time line activity you will be adding time anchors related to toothbrushing. You will be working with a partner to consider these questions:

- Approximately how much time might a person spend brushing his or her teeth in one year?
- Approximately how much time might that person spend brushing his or her teeth in one decade?
- Approximately how much time could that person spend brushing his or her teeth in half a century?"

Note: Discuss student estimations, connecting them to benchmark references noted in daily classroom routines. You may decide to chart these estimates on a class tally.

"Let's begin by estimating, to the nearest minute, how long it takes to brush your teeth:

- Approximately how much time do you think it takes you to brush your teeth?
- How many times per day do you brush your teeth?
- Approximately how much time do you think you spend on toothbrushing each day?"

There are several ways to proceed with the task at this point. For instance:

- You could consider a home-school connection. Have students ask a parent to measure the time they (the students) take to brush their teeth at home. The data will be shared at school the next day.

- If a classroom sink is available, you could brush your teeth (or a student volunteer could do so) while the rest of the class uses an appropriate tool (e.g., a clock or a stopwatch) to measure the duration of the event to the nearest minute.
- You could discuss the fact that toothbrush timers usually run for two minutes. Generally speaking, two minutes is the recommended minimum time for a toothbrushing session.

SOLVING THE PROBLEM

If your students have collected data at home, provide the following instructions:

> "The amount of time per toothbrushing session will vary, as will the number of times per day that you brush your teeth. You and your partner will have to use the data you have collected to decide on a reasonable estimate of the number of times you brush your teeth each day and a reasonable estimate of how long each toothbrushing session lasts."

Alternatively, the class can decide on a specific toothbrushing time per session and a frequency of toothbrushing sessions per day that all students will use while working on this task. This specific length of time and frequency could be based on the toothbrushing demonstration or on the two-minute recommended guidelines. Now is an ideal time to clarify students' understanding of the task. Ask them:

- "What is this problem asking you to determine?"
- "What strategies could you use to begin solving this problem?"
- "What materials and tools could you use to solve this problem?"
- "How might you organize your thinking effectively so that you can share your solution with your classmates?"

> ***Note:*** It might be useful to create an anchor chart with your students to display the relationships between minutes and hours, hours and days, days and weeks, weeks and years, years and decades, and decades and centuries.

Working in pairs, students record their thinking on chart paper. As they investigate the relationships between years and decades, and between decades and centuries, they will be engaging in computations with increasingly large values. Calculators will allow them to focus on mathematical reasoning and communication during this task. The task will culminate in a whole-group sharing session, after which students will indicate on their personal time lines time anchors drawn from the calculations.

As the students work on this task, observe how effectively they use the relationships between minutes and hours, hours and days, days and weeks, weeks and years, years and decades, and decades and centuries.

> ***Note:*** Rich assessment data can be gathered while you observe the degree to which students work flexibly with units of time. The solution to the problem could be presented in minutes, but a student who works flexibly will be able to recognize and use larger units of time.

REFLECTING AND CONNECTING

Skilfully led discussions provide opportunities for students to ask questions of one another, to share ideas, and to justify their reasoning. As students reflect and connect through shared discussion, they deepen their understanding of attributes, units, measurement sense, and measurement relationships. Draw students' attention to the different formats used to create their personal time lines. Discuss the toothbrushing problem, focusing on process. In sharing sessions, such as a Gallery Walk or Sharing Circles, students can compare approaches, self-assess, and set goals as they continue to work on the problem.

Draw a horizontal bar on the board, placing a 0 at the start of the bar and a 10 at its end. Explain to students that this bar represents their first decade. Ask them to consider what portion of this decade was spent on toothbrushing and whether it is possible to represent this portion visually on the bar by shading the portion of the bar that represents the total toothbrushing time. Students should realize that it would be difficult to do this, because toothbrushing is a very short activity. Ask them to brainstorm daily activities that take longer than toothbrushing – for example, sleeping, walking, or talking. Ask them to estimate the portion of the decade spent on each of these activities by indicating the portion of the decade bar that might be shaded. For example, if someone sleeps an average of 8 hours per night, then one-third of the decade bar will be shaded.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- This learning activity provides excellent opportunities for differentiated instruction; it requires students to make choices and offers multiple entry points. The open-ended nature of the Fermi question allows students to use varying levels of sophistication to interpret information and select units.
- Ongoing assessment will allow you to provide feedback and to scaffold instruction. For example, you might simplify the time line task by having students use more approximate, larger units of time to calculate elapsed time.
- Some students may require individual assistance to organize their information. Anchor charts, particularly those created by the class, as well as sample time lines, will be critical reference tools for some students.

EXTENSIONS

Wasting Water. According to Environment Canada, the average Canadian uses 335 L of water per day. Daily water use in Canada is higher per person than in most other countries. Conservationists are urging Canadians to protect our fresh water supplies and not waste them. Every time someone leaves the tap running while brushing his or her teeth, 10 L to 20 L of water are wasted. Challenge students to determine the answer to the following Fermi question: If you were to leave your tap running every time you brushed your teeth, how much water would you use in one year, one decade, and one century? This extension connects to the Grade 4 topic "Habitats and Communities" in the Life Systems strand of the curriculum in science and technology.

Happiness Scale. A second possible extension involves the measurement of happiness in relation to events on the time lines. As students reflect on significant events, some events may evoke a stronger emotional response than others. Ask students to assign happiness values to a selection of events, using a scale of 0–10. The results may then be represented by a broken-line graph, where the horizontal axis is the time line and the vertical axis is the happiness scale.

Accounting For Our Time. Have each student create a personal time line to track, record, and account for a 24-hour period of his or her life. The data will be used to determine elapsed time and to analyse how time is being spent. Students will represent elapsed time using a variety of units, which can then be converted to fractions. For example, "I sleep for about 8 hours each day; therefore, I spend approximately one-third of my day sleeping."

Exploring Additional Fermi Questions. Many other Fermi questions would provide rich learning connections for this learning activity. For example: How much time might a person spend sleeping in one year? In one decade? In a lifetime?

HOME CONNECTION

See **M.BLM4a.1: Personal Time Line**.

ASSESSMENT

Ongoing assessment opportunities are embedded throughout this activity as suggested prompts and questions. Some additional assessment questions are:

- "How did you decide what degree of precision was required when calculating elapsed time?"
- "How did you use benchmarks to estimate time?"
- "How did you use relationships between units to solve problems?"

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
The student:	☐ limited	☐ some	☐ considerable	☐ thorough
– estimates and determines elapsed time	☐ limited	☐ some	☐ considerable	☐ thorough
– uses and understands the relationships among minutes, hours, days, weeks, months, years, decades, and centuries	☐ limited	☐ some	☐ considerable	☐ thorough
– identifies linear patterns and non-linear patterns	☐ limited	☐ some	☐ considerable	☐ thorough
– constructs tables, graphs, and diagrams				
Thinking				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– creates a plan of action for exploring Fermi questions	☐ limited	☐ some	☐ considerable	☐ high degree
– identifies and uses patterns in problem solving	☐ limited	☐ some	☐ considerable	☐ high degree
– makes predictions for pattern growth in time lines and Fermi questions	☐ limited	☐ some	☐ considerable	☐ high degree
– explores alternative solutions				
Communication				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– explains mathematical thinking	☐ limited	☐ some	☐ considerable	☐ high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	☐ limited	☐ some	☐ considerable	☐ high degree
– uses appropriate vocabulary and terminology				
Application				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– applies measurement skills in familiar contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– transfers knowledge and skills to new contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– makes connections among concepts				

Personal Time Line

Dear Parent/Guardian:

As part of our measurement unit, your child will be creating a personal time line at school. Students will be using these time lines to calculate elapsed time. They will be determining the duration of events as well as the amount of time between events. To assist your child in this activity, and to ensure that the time lines are personally relevant, please answer the following questions related to your child's first decade:

- When did your child learn to walk?
- When did your child learn to talk?
- When did your child get his or her first tooth?
- What were three important family events, and when did they take place?
- What were three significant world events that occurred during your child's first decade, and when did they take place?

You and your child may wish to select family photographs or artefacts to provide visual representations for the time line.

> You may decide to extend the time line by going back in time to the birth date of other family members.

Thank you for assisting your child by providing details regarding the timing of important milestones and events in your child's first decade. Please take time to celebrate the completion of this task by having your child explain the mathematics involved.

Grade 4 Learning Activity Designing a Kindergarten Play Enclosure

OVERVIEW

In this learning activity, students investigate the relationship between perimeter and area in the context of designing a Kindergarten play enclosure. Students use a variety of tools to measure and record, to the nearest metre, the perimeter of the existing play enclosure or the perimeter of a space delineated by the teacher. Working with this defined perimeter, students explore the areas of possible rectangular enclosures, using a variety of manipulatives to model their findings. Students are asked to determine the most effective use of the space, while taking into account the way in which it is to be used and structural features of the school.

Students will need to bring to this task an understanding of the attributes of perimeter and area, as well as experiences in using concrete materials to measure lengths and cover classroom surfaces. In addition, they must be able to recognize a number of familiar benchmarks for a metre.

BIG IDEA

Measurement relationships

CURRICULUM EXPECTATIONS

MEASUREMENT RELATIONSHIPS

This learning activity addresses the following **specific expectations**.

Students will:

- determine, through investigation, the relationship between the side lengths of a rectangle and its perimeter and area;
- pose and solve meaningful problems that require the ability to distinguish perimeter and area (e.g., "I need to know about area when I cover a bulletin board with construction paper. I need to know about perimeter when I make the border.");
- compare, using a variety of tools (e.g., geoboard, patterns blocks, dot paper), two-dimensional shapes that have the same perimeter or the same area.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- determine the relationships among units and measurable attributes, including the area and perimeter of rectangles.

ATTRIBUTES, UNITS, AND MEASUREMENT SENSE

This learning activity addresses the following **specific expectation**.

Students will:

- estimate, measure, and record length, height, and distance, using standard units (i.e., millimetre, centimetre, metre, kilometre).

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- estimate, measure, and record length, perimeter, area, mass, capacity, volume, and elapsed time, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

TIME:
2 hours

MATERIALS

- measurement tools (e.g., string cut into metre lengths, metre sticks, trundle wheel) (1 per group of students)
- math journals (optional)
- manipulatives for modelling area (e.g., dot paper, grid paper, geoboards and geobands, colour tiles, interlocking cubes) (1 set per group of students)
- overhead transparencies and overhead projector
- sets of markers (1 per group of students)
- **M.BLM4b.1: Recording Chart** (1 per group of students)
- **M.BLM4b.2: Measuring Tables at Home** (1 per student)

MATH LANGUAGE

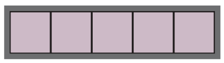
- *area*
- *length*
- *metre*
- *square metre* (m^2)
- *pattern*
- *perimeter*
- *rectangle*
- *relationship*

INSTRUCTIONAL GROUPING:
pairs and/or small groups

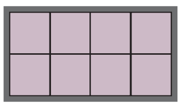
ABOUT THE MATH

AREA-PERIMETER RELATIONSHIPS

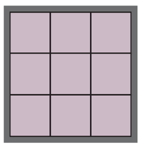
A key learning of this activity is that two rectangles with the same perimeter will not always have the same area. When comparing rectangles with the same perimeter, students discover that the rectangle with the largest area will be a square. For example, for a perimeter of 12 units, we can use square tiles to show a variety of configurations. Notice that the area changes, and that the largest area is a square.



Perimeter = 12 units
Area = 5 square units



Perimeter = 12 units
Area = 8 square units

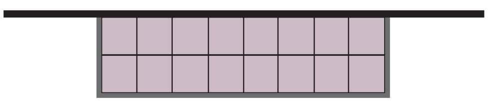


Perimeter = 12 units
Area = 9 square units

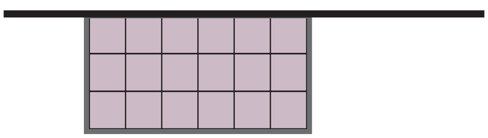
FENCING A RECTANGULAR AREA AGAINST AN EXISTING WALL

When we fence an area against an existing structure (such as a school), we only need to fence three sides. In this case, notice that the greatest area is not a square; it is a rectangle whose width is twice its length. In an extension of this learning task, students consider the effect on the perimeter and the area, given a fixed amount of fencing and the option to use any length of the school wall.

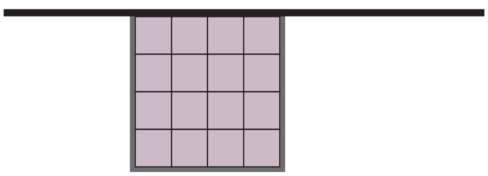
Fence = 12 m
Area = 10 m^2



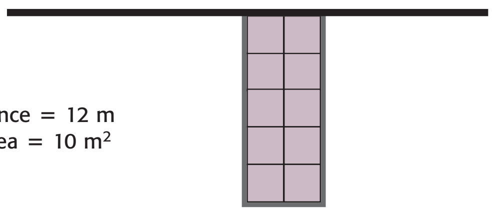
Fence = 12 m
Area = 16 m^2



Fence = 12 m
Area = 18 m^2



Fence = 12 m
Area = 16 m^2



Fence = 12 m
Area = 10 m^2

GETTING STARTED – MEASURING AND RECORDING

INTRODUCING THE PROBLEM – INSTRUCTIONS TO STUDENTS

For schools with a Kindergarten play enclosure:

Describe the following scenario to the class:

> "From time to time the pavement in school play areas needs to be resurfaced. Because this involves removing the fence around the Kindergarten play enclosure, we have the opportunity to decide if this defined space has been designed in the most effective manner. The perimeter of the space cannot be changed, and the shape of the space must be rectangular. However, the dimensions of the rectangular space can change. In order to make an informed decision, we will first need to estimate and measure the dimensions and the perimeter of the existing play space."

For schools without a Kindergarten play enclosure:

Describe the following scenario to the class:

> "Some parents and teachers have expressed a concern regarding the safety of Kindergarten students on the playground. Some adults feel that it would be safer if there were an enclosed space in which Kindergarten students could play. I have marked off a rectangular space outside that I feel would be an appropriate size to allow the Kindergarten students to enjoy their favourite activities without interfering with the play space of other students. The perimeter of the space cannot be changed, and the shape of the space must be rectangular. You will have the opportunity to decide if the space I have created has been effectively planned. In order to make this decision, you will first need to estimate and measure the dimensions of the space I have created."

Note: Before you introduce this activity, mark a rectangular space outside, using pylons.

MEASURING THE KINDERGARTEN PLAY AREA

Both scenarios:

> "Consider the following questions:
>
> - How will we estimate and measure the dimensions of this play enclosure?
> - Which of our measuring tools will be most efficient in measuring the dimensions?
> - How will the dimensions help us to determine the perimeter of this play space?
>
> "When you visit the Kindergarten play enclosure, you will be creating a visual representation of this space. You will need to measure and record the lengths of each side of the enclosure and calculate the perimeter. In your visual representation, include structural features (e.g., doors, windows) that could have an impact on possible play area designs. Remember that hedges, the proximity of parking lots, and other factors might affect your final design choice. The space has to be organized in a way that will allow Kindergarten students to engage in their favourite activities."

> ***Note:*** You may wish to have students observe the Kindergarten students at play or reflect on their own experiences as Kindergarten students. This information could be recorded in a visual format such as a mind map or other brainstorming web.

Have the students visit the Kindergarten play enclosure. Have them use benchmarks to estimate the perimeter of the space, and have them work in pairs to note and record structural features. Select class representatives to measure the length of the sides of the enclosure to the nearest metre, using a variety of measurement tools (e.g., string cut into metre lengths, metre sticks, trundle wheel).

> ***Note:*** Where students are measuring a Kindergarten enclosure that is not rectangular, it will be important to consider how best to proceed with the problem. You might choose to have students work with only a rectangular portion of the yard or to subdivide the current space into manageable sections.

MODELLING THE PROBLEM

After students have returned to the classroom, have them compare the measurements recorded by the class representatives. Comparing these measurements will allow the class to generate a set of data that will be used to measure the effectiveness of benchmark estimates and to complete the rest of the task. Students must be able to recognize the standard units that apply to this task. At this point, each student might use a math journal entry to reflect on the accuracy of his or her benchmark estimate as it compares with actual measurements of the space.

Ask students to use a range of appropriate manipulatives (e.g., dot paper, grid paper, geoboards and geobands, colour tiles, interlocking cubes) representing the standard unit to:

- model the perimeter of the existing play space and determine the area;
- model alternative rectangular play areas with the same perimeter;
- record, for each design, the length of each side and the area;
- look for patterns in their data;
- select a design to best meet the needs of the Kindergarten students, while taking into account structural features of the building and the surrounding space, and justify their reasoning.

> ***Note:*** You may wish to generate specific criteria with your students – for example:
>
> - create a space with the largest area;
> - address how the students are going to use the space; for example, a space that is one metre wide and very long will not allow the Kindergarten students to safely ride their tricycles.

The goal is for the students to use their knowledge and understanding of measurement and shape to determine the most effective use of space, while taking into account both the way the space is to be used and the impact of the structural features of the school.

WORKING ON IT

DESIGNING THE NEW KINDERGARTEN PLAY AREA

Asking the following key questions will allow you to check for understanding:

- "What data have we already gathered and recorded?"
- "As we explore this challenge, what data must remain constant?"
- "How would you describe the challenge in your own words?"

Explain to students that they will be required to work with partners or in small groups and represent their work on a sheet of chart paper that will be displayed on the walls of the classroom. Ask each student pair to select one rectangular area as its new design for the Kindergarten enclosure. Have student pairs share their selections with the whole class, justifying their choices.

> ***Note:*** Students should share their solutions in a personally relevant manner. This may include highlighting their preferred rectangular designs by using overhead transparencies or computer software or by referring to diagrams or manipulative representations they have created.

Circulate to observe and interact with students. Focus on the types of manipulatives students are choosing and how efficiently students are organizing data. Prompt them to look for patterns in their data. Ask:

- "What do you notice about the area of your rectangle as the length of the sides changes?"

> ***Note:*** A square is a special type of rectangle.

CHECKING STUDENT PROGRESS

> ***Note:*** On the basis of the observations you are making as you circulate and interact with your students, you may determine that this is an ideal time to bring students together to reflect on work in progress. Students will be at various points in determining the relationship that exists between area and perimeter. Engaging in math talk at this time will allow students to clarify their thinking and to reflect on the work of others.

If groups have difficulty organizing their data and seeing relationships between perimeter and area, you may facilitate their learning by using strategies that promote talk and the sharing of ideas. Whole-group strategies for sharing might include a Gallery Walk.

> ***Prompt:*** "That's an interesting observation. I wonder if that relationship is present in anyone else's data?"

Alternatively, you may choose to invite particular students to visit another group. Some students might benefit from guided instruction and support.

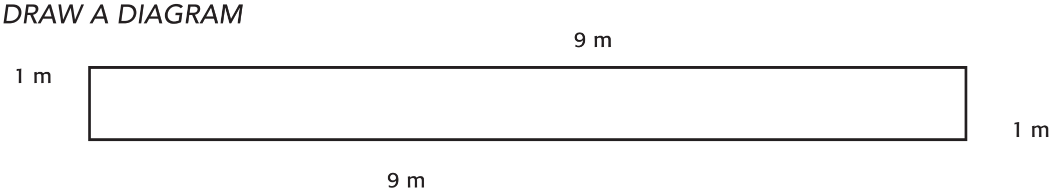
If groups continue to have difficulty organizing their data, provide them with **M.BLM4b.1: Recording Chart**, which will allow students to transfer the information they have generated to an organized format in order to focus on patterns and relationships.

STRATEGIES STUDENTS MIGHT USE

CREATE A TABLE OF VALUES

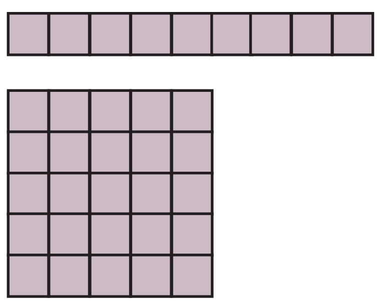
Length of Side One	Length of Side Two	Length of Side Three	Length of Side Four	Perimeter	Area
4 m	6 m	6 m	4 m	20 m	24 m^2
9 m	1 m	1 m	9 m	20 m	9 m^2
5 m	5 m	5 m	5 m	20 m	25 m^2

DRAW A DIAGRAM



MODEL WITH CONCRETE MATERIALS

Have a variety of commercial and non-commercial manipulatives available for students to choose from. Manipulatives may include, but need not be restricted to, the following: square sticky notes, colour tiles, geoboards, grid paper, dot paper, interlocking cubes.



USE NUMERICAL REPRESENTATION

4 m + 6 m + 4 m + 6 m = 20 m

5 m + 5 m + 5 m + 5 m = 20 m

6 m^2 + 6 m^2+ 6 m^2 + 6 m^2 = 24 m^2

5 m^2 + 5 m^2 + 5 m^2 + 5 m^2 + 5 m^2 = 25 m^2

REFLECTING AND CONNECTING

Have pairs or small groups of students share their choices and justify their solution by discussing how the Kindergarten play enclosure they have designed provides the most effective space and meets the needs of Kindergarten students.

Ask students to clarify their understanding of the relationship between perimeter and area by responding to questions such as the following:

- "How did the relationship between perimeter and area affect your decision?"
- "What strategies did you use to help develop your understanding of this relationship?"
- "What strategies did you see or hear others using? Which strategies seemed to be most effective?"

Draw students' attention to the different formats used to record solutions. Ask questions such as the following:

- "In what different ways did pairs or groups record their strategies and solutions?"
- "Which forms are easy to understand?"

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- Some students may benefit from prompts that encourage them to think about which manipulatives and tools will help them to arrive at a solution and organize their work in a manner that can be clearly communicated.
- For students who experience difficulty, simplify the problem by providing an organized list - great value in having students record their learning in personally relevant ways, **M.BLM4b.1: Recording Chart** may be made available to those students requiring further support in organizing their data.

EXTENSIONS

Tangram Teasers. Provide each student with a set of tangrams and several sheets of grid paper. Challenge students to find the configuration of the 7 tangram pieces having the shortest perimeter and the longest perimeter. Using a cooperative learning strategy such as think-pair-share or partner to partner, direct students to justify their thinking by discussing the following question: "Is it possible to order our drawings by size of area?"

Literature Link: *Grandfather Tang's Story*, by Ann Tompert (New York: Dragonfly Books, 1997).

> ***Note:*** In this task, area will remain constant as perimeter changes. Students may not yet realize this fact. Challenge them to verify their conjectures by measuring.

Perplexing Pentomino Perimeter. Pentominoes are made by joining five squares so that each square shares at least one edge with another. Challenge students to find and create on grid paper all possible pentominoes. Students can then use these templates to create a personal set of pentomino manipulatives. Have them record the perimeter for each pentomino piece. Ask: "Which pentominoes have a greater area?" (All have the same area.) "Which pentominoes have the greatest/least perimeter?" "What is the smallest/greatest perimeter possible when two pentominoes are joined?"

HOME CONNECTION

See **M.BLM4b.2: Measuring Tables at Home**.

ASSESSMENT

Ongoing assessment opportunities are embedded throughout this activity. Use curriculum expectations to focus your observations and assess how effectively students:

- choose and apply personal benchmarks for one metre;
- reflect on estimates and measurement strategies;
- select and use measurement tools;
- communicate and justify their findings regarding the best use of space;
- express their understanding of the relationship between perimeter and area.

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
The student:				
– distinguishes between area and perimeter	☐ limited	☐ some	☐ considerable	☐ thorough
– identifies relationships among units and measurable attributes	☐ limited	☐ some	☐ considerable	☐ thorough
– compares shapes that have the same perimeter or same area	☐ limited	☐ some	☐ considerable	☐ thorough
– describes the relationship between perimeter and rectangular area	☐ limited	☐ some	☐ considerable	☐ thorough
– constructs tables, graphs, and diagrams to represent data	☐ limited	☐ some	☐ considerable	☐ thorough
Thinking				
The student:				
– creates a plan of action for exploring measurement relationships	☐ limited	☐ some	☐ considerable	☐ high degree
– identifies and uses patterns in problem solving	☐ limited	☐ some	☐ considerable	☐ high degree
– makes predictions for pattern growth in area and perimeter	☐ limited	☐ some	☐ considerable	☐ high degree
– explores alternative solutions	☐ limited	☐ some	☐ considerable	☐ high degree
Communication				
The student:				
– explains mathematical thinking	☐ limited	☐ some	☐ considerable	☐ high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	☐ limited	☐ some	☐ considerable	☐ high degree
– uses appropriate vocabulary and terminology	☐ limited	☐ some	☐ considerable	☐ high degree
Application				
The student:				
– applies measurement skills in familiar contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– transfers knowledge and skills to new contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– makes connections among concepts	☐ limited	☐ some	☐ considerable	☐ high degree

M.BLM4b.1

Recording Chart

Length of Side One	Length of Side Two	Length of Side Three	Length of Side Four	Perimeter	Area

Measuring Tables at Home

Dear Parent/Guardian:

In our homes we use table surfaces for various purposes. We use rectangular countertops in the kitchen for food preparation; we have smaller rectangular tables near chairs in our living rooms. The perimeter and area of table surfaces is often determined by function and by the available space in our homes.

Start this activity by asking your child to predict which table in your home has the greatest surface area and which table has the smallest perimeter. Your child should use the first two rows of the table below to record his or her estimates of the length and width of each table, and to calculate the estimated area and perimeter for each. Take the time to discuss with your child how he or she arrived at the estimates. Then, ask your child to measure the actual length and width of each of these tables, record these measurements in rows 3 and 4 of the chart below, and calculate the area and perimeter. Your child should then measure the length and width for at least four other tables, and calculate the area and perimeter, to test their prediction of which table has the greatest area and which table has the smallest perimeter.

Table	Length	Width	Perimeter	Area

Grade 5 Learning Activity
Weather or Not …

OVERVIEW

In this learning activity, students work through a series of "weather centres" to measure and record temperatures, research and record changes in the length of days, and measure and record precipitation. They will use this information to identify local climate patterns and to formulate their own weather predictions. Weather data are collected and recorded in an almanac format. Students make decisions about how to record their information in order to see patterns and draw conclusions from their data. During the final week of this study, students synthesize their data to generate a week of predictions that will allow them to determine whether outdoor electives should be scheduled or postponed for a given week. They will also make recommendations as to appropriate attire for the week. This measurement study has strong cross-curricular and multi-strand connections. The measurement tasks also support and align with the Earth and Space Systems strand related to weather in the Grade 5 science and technology curriculum and the Data Management strand of the mathematics curriculum.

Students need to bring to this task an understanding of how to read a standard thermometer, how to determine whether temperature is rising or falling, and how to determine benchmarks for freezing, cold, cool, and warm temperatures.

BIG IDEA

Attributes, units, and measurement sense

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in minutes, hours, days, weeks, months, or years;
- measure and record temperatures to determine and represent temperature changes over time.

These expectations contribute to the development of the following **overall expectation**.

Students will:

- estimate, measure, and record perimeter, area, temperature change, and elapsed time, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

TIME:
6 hours over 4 weeks

MATERIALS

- newspaper and/or Internet access
- plain and grid chart paper, and grid paper
- markers (1 per group of students)
- overhead projector (optional)
- thermometers (1 per group of students)
- rain gauges (classes may choose to make their own) (1 per group of students)
- prepared "almanacs" – one exercise book or journal per student, divided into sections with the following subtitles:
  - Student Almanac: Final Week (*data synthesis and prediction section*)
  - Temperature Measurement and Analysis (*Centre 1 data and analysis*)
  - Sunrise/Sunset Measurement and Analysis (*Centre 2 data and analysis*)
  - Precipitation Measurement and Analysis (*Centre 3 data and analysis*)
- **M.BLM5a.1: Student Instruction Sheet for Centre 1** (1 per student)
- **M.BLM5a.2a–b: Student Instruction Sheet for Centre 2** (1 per student)
- **M.BLM5a.3: Sunrise/Sunset Recording Sheet** (1 per group of students)
- **M.BLM5a.4: Sunrise/Sunset Data for August 3, 2006** (1 per group of students)
- **M.BLM5a.5a–b: Student Instruction Sheet for Centre 3** (1 per student)
- **M.BLM5a.6: Measuring Temperatures at Home** (1 per student)

Note: To facilitate the organization and presentation of the measurement data used in this study, you might provide students with notebooks containing grid paper.

MATH LANGUAGE

- units of temperature (e.g., *degree Celsius*)
- temperature tools (e.g., *thermometer*)
- *mean*
- graphing terms (e.g., *double bar graph, broken line graph, pictograph*)
- metric units (e.g., *millimetre, centimetre*)
- units of time (e.g., *minute, hour, day, week, year*)
- *annual*
- *probability*

Note: You may choose to divide your class into 6 groups. If you do this, a second set of materials will be required for each centre.

Students also work in expert groups:

INSTRUCTIONAL GROUPING: small groups working at three math centres

Students in Expert Group 1 compare their primary data with data from a location in the Pacific Coast climatic region. Students in Expert Group 2 compare their primary data with data from a location in the Arctic tundra climatic region. Students in Expert Group 3 compare their data with data from a location in the Atlantic Canada climatic region.

ABOUT THE MATH

TEMPERATURE

Although the use of negative integers is not introduced as a formal expectation until Grade 7, students have already encountered below-zero temperature readings in their everyday lives.

ELAPSED TIME

Working with authentic data for sunrise and sunset times, students calculate changes in elapsed "daylight" time. They identify patterns or trends in the measurement data and predict measures of daylight for an upcoming week or for specific dates in an upcoming month.

PRECIPITATION

Students measure precipitation, analyse precipitation statistics, and explore the relationship between the specific attributes being measured and the measurement tools and units being used (e.g., the rain gauge is used as an indicator of the amount, in millimetres, of rain that has fallen in a given location).

GETTING STARTED – INTRODUCING THE WEATHER CENTRES

Describe the following scenario to the class:

> "This study is organized as a series of weekly 'weather centres' that groups of students will visit on a rotating basis. There will be an introductory lesson related to each of the centres. To begin each math session, you will work as a whole group to complete two tasks: add the daily forecast for your area to a class T-chart, then compare the forecast for the previous day with the actual weather measured by your class. To determine the accuracy of the forecasts, use information from each centre group: consider actual precipitation, temperature highs and lows, and other factors."

Note: November or April may provide the greatest variety of weather conditions and patterns.

Note: Some 24-hour weather data for Ontario locations can be found on the Environment Canada website: http://www.weatheroffice.ec.gc.ca/forecast/canada/index_e.html?id=ON.

A Sunrise/Sunset Calculator can be found on the National Research Council of Canada website: http://www.hia-iha.nrc-cnrc.gc.ca/sunrise_e.html.

During the final week of this measurement study, students will synthesize the data they have collected and generate a 5-day prediction to be recorded in the first section of their almanacs.

Introduce centres through whole-class guided and shared lessons. In addition, involve students in collecting and recording primary data to be used at the centres.

DAY ONE: INTRODUCING CENTRE 1 – MEASURING TEMPERATURE AND RECORDING TEMPERATURE CHANGE

In this minilesson, review how to read a standard thermometer and conduct a discussion on possible ways to record temperature data. Model the creation of a temperature graph, using a 24-hour data sample.

> ***Note:*** Students working at independent centres will be responsible for measuring hourly outdoor temperatures. You may wish to establish classroom routines for taking these measurements.

Have students work in groups to explore how weather data are presented on websites and/or in newspapers. Create a Weather Data chart, displaying the measurements taken, the units used, and the formats in which this information is displayed. Ask:

- "What information was available from your source?"
- "Was it available in a variety of formats? What formats were presented?"
- "Why is it important to present weather information in a variety of formats?"

> ***Note:*** Radio and television reports, websites, and newspapers provide a variety of formats for the presentation of weather data. Ensure that students have access to multiple representations of data as they make their own choices for data recording throughout this unit. A rich Weather Data anchor chart will include pictographs, tables, graphs, charts, and a variety of weather maps.

Select samples that will be used to add visuals to the Weather Data anchor chart.

Using a shared lesson format, explore particular data representations in greater depth. For example, a line graph outlining weather trends may be displayed on the overhead projector. Using the think-aloud strategy, model how the data are read and used to make informed decisions.

Explicitly communicate expectations for recording data during the introductory input lesson so as to ensure consistency. Students will then be able to focus on patterns in their data and to use information effectively to make predictions.

For the first three days of introductory lessons, have students collect hourly temperature data during the school day. To enrich the data set, you might also ask students to collect daily measurements for late afternoon and early evening (see the Home Connection **M.BLM5a.6: Measuring Temperature at Home**). The data will be used in the Working on It section to identify high and low temperatures, variations in temperature throughout the day, and weekly temperature trends.

DAY TWO: INTRODUCING CENTRE 2 – HOURS OF DAYLIGHT

Model how to access the present day's sunrise and sunset data, and challenge students to determine from these data the elapsed time between sunrise and sunset.

> ***Note:*** An anchor chart highlighting various approaches enables students to reference personally meaningful strategies in their independent work.

Ask students to work in pairs and use their own strategies to calculate the elapsed time between sunrise and sunset. Next, ask them to share their solutions, strategies, and thinking with the rest of the class. Identify efficient strategies for determining elapsed time, and prepare a class anchor chart to be used as a reference for Centre 2.

Invite students to use what they know about the amount of elapsed time between sunrise and sunset to determine the hours of darkness in one day from midnight to midnight. Ask:

- "What strategy did you use to calculate the hours of darkness between midnight and sunrise, and between sunset and midnight the following night?"
- "How could you incorporate into your calculations information about the length of a day?"
- "How are the two lengths of elapsed time (daylight/darkness) related?"
- "Are there times in the year when the days seem longer or shorter? Explain."
- "What evidence could you use to support your theory?"

Refer students to the prepared classroom graph (Recording Sunrise and Sunset Times on p. 72), and model how to plot the sunrise and sunset times. Add daily information throughout the week.

> ***Note:*** The length of a day changes by about 2.4 minutes each day, getting longer or shorter depending on the season. Therefore, both the sunrise and sunset times change by about 1.2 minutes daily. The dimensions of the grid needed for a 30-day period would be 30 days across and 40 minutes vertically.

RECORDING SUNRISE AND SUNSET TIMES

Day

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Sunrise Times																														
Sunset Times																														

DAY THREE: INTRODUCING CENTRE 3 – MEASURING PRECIPITATION AND ANALYSING PRECIPITATION STATISTICS

This centre involves accessing measurement information and using it in a variety of contexts. To prepare students, model the skills and strategies they will use in their independent work. Because students will be responsible for daily precipitation measurements, use a gauge to model the measurement of precipitation, and record the measurements on a graph. This centre requires students to access and use a variety of data. Decisions need to be made as to how information will be accessed by students. You might consider providing statistics in a variety of print formats or identifying websites for student use. Regardless of the format chosen, there is a need to model how to access, read, and use the information to create graphs and tables.

WORKING ON IT

CENTRE 1: MEASURING TEMPERATURE AND RECORDING TEMPERATURE CHANGE

Provide each student with a copy of the instructions for Centre 1 (**M.BLM5a.1: Student Instruction Sheet for Centre 1**). Go over these instructions with the students as a group, reading them aloud and explaining them as follows:

"Each day, measure and record temperatures every hour on the hour throughout the school day and at home and record these in your almanac. When you are unable to measure the temperature personally, use secondary data. Use data from the previous day for your area, as well as for the climatic region assigned to your group, in order to graph hourly temperatures and identify:

- daily high and low temperatures from data collected
- the range of temperatures on a given day
- the mean daily high and low temperatures for the week
- the weekly temperature trends"

> ***Note:*** Data will need to be gathered over the weekend for use on Monday. Because students will be plotting 24-hour data, they will need to measure and record each hourly temperature and gather information from a secondary source for the hours during which they were asleep.

"Compare the daily high and low temperatures of your area with those of the climatic region of Canada designated for your group. You may also choose to compare your area with another world location (e.g., near the equator, near one of the poles, in another hemisphere). Once you have determined the weekly temperature trends, display your information in a format that can be posted. Include explanatory notes that interpret the information. You may also wish to include information detailing what you learned about daily high and low temperatures, daily ranges of temperatures, and daily mean temperatures."

CENTRE 2: GRAPHING SUNRISE AND SUNSET TIMES

Provide each student with a copy of the instructions for Centre 2 (**M.BLM5a.2a–b: Student Instruction Sheet for Centre 2**). Go over these instructions with the students as a group, reading them aloud and explaining them as follows:

"There are two components to this centre. Each day, you must check the sunrise and sunset times and plot this information on a class graph. Calculate the elapsed time for daylight as well as darkness, and record your calculation in the Sunrise/Sunset Measurement and Analysis section of your almanacs. In addition to this assignment, you must complete a related daily task.

"**Day One:** Check the sunrise and sunset times for your area and for the climatic region assigned to your expert group. Plot this information on the class graph. Calculate the elapsed daylight hours and hours of darkness, and record these measurements in your almanac.

"**Day Two:** Check the sunrise and sunset times for your area and for the climatic region assigned to your expert group. Plot this information on the class graph. Calculate the elapsed daylight hours and hours of darkness, and record these measurements in your almanac. Using the sunrise and sunset information for both regions, write a prediction for the sunrise and sunset times for your fifth day at this centre. Justify your prediction, using specific measurement vocabulary and visual supports, and explain your calculations.

"**Day Three:** Check the sunrise and sunset times for your area and for the climatic region assigned to your expert group. Plot this information on the class graph. Calculate the elapsed daylight hours and hours of darkness, and record these measurements in your almanac.

"The sunrise and sunset times, as recorded by Environment Canada on August 3, 2006, for Iqaluit, Nunavut, and for Toronto, Ontario, are shown in the table below:

August 3, 2006	Sunrise	Sunset
Toronto, Ontario	**06:08**	**20:39**
Iqaluit, Nunavut	**03:50**	**21:30**

(*Source*: Environment Canada)

"Using the sunrise and sunset data, discuss the information contained in the table."

Note: The following key questions and prompts might be helpful to guide discussion:

1. "Determine the elapsed time between sunrise and sunset in Toronto on August 3, 2006."
2. "Determine the elapsed time between sunrise and sunset in Iqaluit on August 3, 2006."
3. "How many more hours of daylight were there in one location than the other?"
4. "How might the difference in daylight hours affect day-to-day living in these locations?"

> ***Note:*** This activity may be extended by having students track sunrise and sunset times for these two locations over a number of days.

"**Day Four:** Check the sunrise and sunset times for your area and for the climatic region assigned to your expert group. Plot this information on the class graph. Calculate the elapsed daylight hours and hours of darkness, and record these measurements in your almanac.

"For today's date, locate the sunrise and sunset times from the following Canadian weather stations: Gjoa Haven, NU; Iqaluit, NU; Arviat, NU; Sanikiluaq, NU; Moosonee, ON; Thunder Bay, ON; Toronto, ON; and your local area weather station. Record your data on **M.BLM5a.3: Sunrise/Sunset Recording Sheet**.

"Consider: How does latitude affect the amount of elapsed time between sunrise and sunset?

"Record your work in your almanac."

> ***Note:*** Where access to the current daily information about these locations is limited, you may wish to have students use **M.BLM5a.4: Sunrise/Sunset Data for August 3, 2006**, which contains data for August 3, 2006, for these locations.

"**Day Five:** Check the sunrise and sunset times for your area and for the climatic region assigned to your expert group. Plot this information on the class graph. Calculate the elapsed daylight hours and hours of darkness, and record these measurements in your almanac. Using your graphed data, look for trends and patterns, and record your findings in your almanac.

"You may also choose to compare your area with another world location (e.g., near the equator, near one of the poles, in another hemisphere).

"Gather information on daylight saving time, identify important ideas in the information, and discuss your new learning. Create a mind map to record this information in your almanac."

CENTRE 3: PRECIPITATION STATISTICS

Provide each student with a copy of the instructions for Centre 3 (**M.BLM5a.5a–b: Student Instruction Sheet for Centre 3**). Go over these instructions with the students as a group, reading them aloud and explaining them as follows:

"There are two components to this centre. You will use your personally created rain gauges to collect and record daily precipitation statistics, and you will also complete a specific daily assignment."

> ***Note:*** Because students will be measuring and recording precipitation for the previous 24-hour period, they will need to take their measurement tools home with them the Friday before their week at this centre.

"**Day One:** Observe your rain gauge. Measure and record the precipitation that has accumulated over the past 24 hours. Record your measurement on the class graph. Empty your rain gauge to ensure that tomorrow's reading will be accurate.

"Check annual precipitation statistics for your area and use this information to create a double bar graph detailing the mean amount of rain and snow that falls in your area each month.

- What trends do you see?
- Why do you think that two units of measure are used for recording precipitation instead of one standard unit?

"Record your findings in the Precipitation Measurement and Analysis section of your almanac."

Note: Rainfall is normally recorded in millimetres, and snowfall is generally recorded in centimetres. It will be important for students to know this as they transfer statistical data into a double bar graph format. As they work with statistics in millimetres and centimetres, they will need to make decisions about the most appropriate units and scale to use when creating their graph.

"**Day Two:** Observe your rain gauge. Measure and record the precipitation that has accumulated over the past 24 hours. Record your measurement on the class graph. Empty your rain gauge to ensure that tomorrow's reading will be accurate.

"Check annual precipitation statistics for the climatic region assigned to your expert group and use this information to create a double line graph detailing the mean amount of rain and snow that falls in that region each month. What trends do you see?

"Use the graphs of average precipitation for your area and for your designated climatic region to compare and contrast the two regions. Summarize your findings, identifying mathematical relationships you observe in the data. Record your findings in the Precipitation Measurement and Analysis section of your almanac. You may also choose to compare your area with another world location (e.g., near the equator, near one of the poles, in another hemisphere).

"**Day Three:** Observe your rain gauge. Measure and record the precipitation that has accumulated over the past 24 hours. Record your measurement on the class graph. Empty your rain gauge to ensure that tomorrow's reading will be accurate.

"In the past, people developed and relied on weather sayings, such as 'April showers bring May flowers', to help them predict the weather. Many of these sayings exist, but how valid are they? Using a variety of sources, identify three weather sayings. Use statistical data to comment on the reliability of the sayings. Do the measurements related to weather support these folkloric sayings? Choose one saying for which you have strong supporting or contradictory data. Choose a visual format for presenting the saying, and comment on the reliability of the saying.

"**Day Four:** Observe your rain gauge. Measure and record the precipitation that has accumulated over the past 24 hours. Record your measurement on the class graph. Empty your rain gauge to ensure that tomorrow's reading will be accurate.

"Check the weather reports for the last 6 days. Focus on the 'probability of precipitation' section. Create a T-chart with the headers Probability of Precipitation (POP) and Actual Precipitation. Compare the forecast probability of precipitation data with the precipitation that actually occurred in your area. Use this information to make judgements about the accuracy of precipitation forecast for your 6-day sample. Record your ideas in the Precipitation Measurement and Analysis section of your almanac.

"Use a variety of sources to answer the following key questions as a group:

- What does the term *probability of precipitation* mean?
- How is the probability of precipitation determined?
- How do people use probability of precipitation measures in their daily lives?

"Identify three new learnings or key mathematical ideas to record and explain in the Precipitation Measurement and Analysis section of your almanac.

"**Day Five:** Observe your rain gauge. Measure and record the precipitation that has accumulated over the past 24 hours. Record your measurement on the class graph. Access a meteorological website to check the precipitation statistics for these dates.

- How do your measurements compare?
- If there are differences in the data, what factors might account for them?"

Note: Precipitation measurements taken by students reflect precipitation at the school location. Where the location of students' gauges is geographically distant from the locations used for published reports, there may be significant variation between the two sets of data. Such a situation provides students with an opportunity to see an authentic application of data sets, where a set of data is a sample of a larger population or, in this case, of a larger geographic area. In addition, student measurement instruments may not afford the same level of precision as the instruments used in published reports.

"Create a table comparing the published daily precipitation measurements for the past 5 days with your daily measurements for the same dates. What trends do you see? Use the Precipitation Measurement and Analysis section of your almanac to record your work."

REFLECTING AND CONNECTING

Ongoing reflection has been embedded in each centre of this measurement study.

Divide your class into three groups to participate in a Gallery Walk. Have students visit each set of posted data to look for trends and patterns. Ask them to discuss and analyse the measurement information presented at each data centre and make jot notes in the appropriate section of their almanac.

Reconvene the class to discuss findings. Advise students that they may add to their jot notes during the discussion. As a conclusion for each section of their almanacs, have students write a brief summary of key learnings related to the measurement of temperature, daylight hours, and precipitation.

Make sure that the expert groups have opportunities to share their comparative data and analysis related to climatic regions. Rich discussions may be facilitated by having expert groups detail the trends they notice in each region and apply the language of measurement when comparing regions.

To connect learning with a context that is relevant in the world beyond the classroom, introduce students to the *Farmer's Almanac*.

Challenge students to predict the weather for the coming week, using the measurement information they have gathered. Have them use their predictions to decide whether outdoor electives should be scheduled or postponed for the next week. Ask them to use their almanacs to make recommendations for suitable attire for participants in these electives.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

This measurement study provides many opportunities for differentiated instruction; it requires students to make choices and it promotes cooperative learning. Model measurement procedures, data recording, and analysis, using think-alouds. As students progress through centre-based learning tasks, you will have the opportunity to circulate and assess student needs, provide feedback, and scaffold instruction. Encourage journal writing by providing sentence starters, checklists, and prompts or picture cues. Simplify the investigations by selecting key activities at each centre and allowing additional time for their completion.

EXTENSIONS

Daylight Saving Time. Have students complete a research project on daylight saving time. They may choose any format for a presentation focusing on how people have used mathematics to make informed decisions regarding the adoption of daylight saving time.

A Meteorological Measurement Guide. Have students choose from the following topics to write procedural texts:

- how to calculate the number of daylight hours that will elapse in a day
- how to build and use a rain gauge to measure precipitation
- how to determine the daily high and low temperatures

Students are more likely to engage in this writing task if an authentic purpose and audience are provided. For instance, they could share their texts with meteorologists at a local weather station or news affiliate.

Meteorological Newsletter. Engage students in writing articles for a meteorological newsletter. Topics could be linked to the measurements students have collected.

HOME CONNECTION

See **M.BLM5a.6: Measuring Temperature at Home**.

ASSESSMENT

Ongoing assessment opportunities are embedded throughout this measurement study.

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
The student:	☐ limited	☐ some	☐ considerable	☐ thorough
– estimates, collects, measures, and records weather data	☐ limited	☐ some	☐ considerable	☐ thorough
– estimates and determines elapsed time	☐ limited	☐ some	☐ considerable	☐ thorough
– determines weather data change over time	☐ limited	☐ some	☐ considerable	☐ thorough
– organizes and displays measurement data	☐ limited	☐ some	☐ considerable	☐ thorough
– reads, interprets, and draws conclusions from data				
Thinking				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– creates a plan of action for exploring weather over time	☐ limited	☐ some	☐ considerable	☐ high degree
– identifies and uses patterns in measurement data	☐ limited	☐ some	☐ considerable	☐ high degree
– makes predictions for weather patterns over time	☐ limited	☐ some	☐ considerable	☐ high degree
– explores alternative solutions				
Communication				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– explains mathematical thinking	☐ limited	☐ some	☐ considerable	☐ high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	☐ limited	☐ some	☐ considerable	☐ high degree
– uses appropriate vocabulary and terminology				
Application				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– applies measurement skills in familiar contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– transfers knowledge and skills to new contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– makes connections among concepts				

Student Instruction Sheet for Centre 1

CENTRE 1 – MEASURING TEMPERATURE AND RECORDING TEMPERATURE CHANGE

Record all of your daily data in the Temperature Measurement and Analysis section of your almanac.

DAYS ONE TO FOUR

- Measure and record temperatures every hour on the hour throughout the school day (and at home, when possible).
- Graph the temperatures from the previous day, using a line graph. (The first day you complete this activity you will need to use a secondary source, such as the Internet, to collect your data.)
- Record the range of temperatures, noting the times of both the high and the low.

Repeat steps 2 and 3 for the climatic region assigned to your group.

DAY FIVE

Complete steps 1–3, and then proceed to the following tasks:

- Calculate the mean daily high and low temperatures for the week.
- Using a double line graph, record your findings in a format that can be posted. Include explanatory notes that interpret your information. You may wish to include information detailing what you learned about daily high and low temperatures, ranges of temperatures, and mean.

Repeat steps 1 and 2 for the climatic region assigned to your group.

Student Instruction Sheet for Centre 2

CENTRE 2 – HOURS OF DAYLIGHT

Record all of your daily data in the Temperature Measurement and Analysis section of your almanac.

EVERY DAY

1. Check **today's** sunrise and sunset times for your area and for the climatic region assigned to your group.
2. Plot this information on the class graph.
3. Calculate the number of elapsed daylight hours and hours of darkness, and record this information in the Sunrise/Sunset Measurement and Analysis section of your almanac.

DAY ONE

Complete the daily activity.

DAY TWO

1. Write a prediction of the sunrise and sunset times for your area and climatic region for your fifth day at this centre.
2. Justify your prediction, using specific measurement vocabulary and visual supports, and explain any calculations you may have used.

DAY THREE

The sunrise and sunset times on August 3, 2006, for Iqaluit, Nunavut, and Toronto, Ontario, are shown in the table below:

August 3, 2006	Sunrise	Sunset
Toronto, Ontario	06:08	20:39
Iqaluit, Nunavut	03:50	21:30

(*Source*: Environment Canada)

Using the sunrise and sunset data in the table, determine:

- the elapsed time between sunrise and sunset in Toronto on August 3, 2006;
- the elapsed time between sunrise and sunset in Iqaluit on August 3, 2006;
- how many more hours of daylight there were in one location than in the other;
- how the difference in daylight hours might affect day-to-day living in these locations.

DAY FOUR

Locate today's sunrise and sunset times for the following Canadian weather stations:

1. Gjoa Haven, NU	**2. Iqaluit, NU**
3. Arviat, NU	**4. Sanikiluaq, NU**
5. Moosonee, ON	**6. Thunder Bay, ON**
7. Toronto, ON	**8. Your area weather station**

Record your data on **M.BLM5a.3: Sunrise/Sunset Recording Sheet**.

Consider:

- How does moving farther north affect the amount of elapsed time between sunrise and sunset?

Record your work in your almanac.

DAY FIVE

1. Using your graphed data, look for patterns and trends and record these findings in your almanac, in the Final Week section.
2. Investigate daylight saving time using print sources and Web-based resources such as http://www.google.ca and http://www.wikipedia.org.
3. Identify important ideas and new learning through discussion.
4. Create a mind map to record this information in your almanac.

Sunrise/Sunset Recording Sheet

Location	Latitude	Sunrise	Sunset
Gjoa Haven, Nunavut	68 N		
Iqaluit, Nunavut	63 N		
Arviat, Nunavut	61 N		
Sanikiluaq, Nunavut	56 N		
Moosonee, Ontario	51 N		
Thunder Bay, Ontario	48 N		
Toronto, Ontario	43 N		

Sunrise/Sunset Data for August 3, 2006

The following table details the sunrise and sunset times for August 3, 2006, in a number of Canadian locations, as reported by Environment Canada.

Location	Latitude	Sunrise	Sunset
Gjoa Haven, Nunavut	68 N	02:37	22:22
Iqaluit, Nunavut	63 N	03:50	21:30
Arviat, Nunavut	61 N	04:54	21:51
Sanikiluaq, Nunavut	56 N	05:21	21:25
Moosonee, Ontario	51 N	05:50	21:08
Thunder Bay, Ontario	48 N	06:34	21:32
Toronto, Ontario	43 N	6:08	20:39

Student Instruction Sheet for Centre 3

CENTRE 3 – MEASURING PRECIPITATION AND ANALYSING PRECIPITATION STATISTICS

Record all of your daily data in the Temperature Measurement and Analysis section of your almanac.

EVERY DAY

1. Observe your rain gauge.
2. Measure and record the precipitation that has accumulated over the past 24 hours.
3. Empty your rain gauge to ensure that tomorrow's reading will be accurate.
4. Add your measurement to the class graph.

DAY ONE

1. Check annual precipitation statistics for your area.
2. Create a double bar graph detailing the mean amount of rain and snow that falls in your area each month.
3. In the Precipitation Measurement and Analysis section of your almanac, record any trends you see.
4. Why do you think snow accumulation is measured in centimetres and rainfall accumulation is measured in millimetres? Record your thoughts.

DAY TWO

1. Check annual precipitation statistics for the climatic region assigned to your group.
2. Create a double bar graph detailing the mean amount of rain and snow that falls in that region each month.
3. In the Precipitation Measurement and Analysis section of your almanac, record any trends you see.
4. Compare and contrast precipitation in your local area with precipitation in this climatic region.

DAY THREE

In the past, people developed and relied on weather sayings to help them predict the weather. Many such sayings exist, but are they valid?

1. Using a variety of sources, identify three weather sayings.

2. Use statistical data to comment on the reliability of the sayings.
3. Choose one saying for which you have strong supporting or contradictory data.
4. Create a poster to illustrate the saying, and comment on the reliability of the saying.

DAY FOUR

1. Check the weather reports for the last 6 days, focusing on the "probability of precipitation" section.
2. Create a T-chart with the headers: Probability of Precipitation (POP) and Actual Precipitation.
3. Compare the forecast data on the probability of precipitation with the precipitation that actually occurred in your area.
4. Use this information to make judgements about the accuracy of precipitation forecasting in weather reports for your 6-day sample.
5. Answer the following questions:
   - What does the term probability of precipitation mean?
   - How is the probability of precipitation determined?
   - How do people use probability of precipitation measures in their daily lives?
6. Identify three new learnings or key mathematical ideas based on your findings and discussion. Record these in your almanac.

DAY FIVE

1. Access a meteorological website to check your area's precipitation statistics for the past week.
2. Create a T-chart to compare your findings with the secondary precipitation statistics.
3. How do your findings compare with the findings from the secondary source?
4. Record in the Precipitation Measurement and Analysis section of your almanac any trends you see.

Measuring Temperature at Home

Dear Parent/Guardian:

At school we have been investigating temperature measurement and temperature change. We have been measuring temperatures in degrees Celsius at one-hour intervals on the hour. As we now wish to expand our data set, we are encouraging students to measure and record outdoor temperatures at home as their schedules permit.

Ask your child to tell you about our study and to describe and explain patterns that we have been noticing in temperature, rainfall, and daylight hours.

Grade 5 Learning Activity
Hiking the Bruce Trail

OVERVIEW

Canada is world renowned for its natural beauty and vast expanses of wilderness. These areas afford Canadians opportunities to enjoy a wide variety of outdoor activities. In Ontario, the Bruce Trail, extending from Niagara to Tobermory, provides an ideal setting for hiking adventures. In this learning activity, students explore measurement problems and relationships in the context of a Bruce Trail hike. Working with conversions, students will be required to recognize and apply the relationship between kilometres and metres. They will use their understanding of elapsed time in dynamic ways to determine distances covered over time.

Students should bring to this task an understanding of the relationship between various units of length. They should be familiar with selecting the most appropriate standard units in problem-solving contexts, and with justifying their choices. Students also need previous experience in displaying information on graphs. In particular, familiarity using broken-line graphs will allow students to focus on interpreting, drawing conclusions, and comparing their data with related sets.

BIG IDEA

Measurement relationships

CURRICULUM EXPECTATIONS

MEASUREMENT RELATIONSHIPS

This learning activity addresses the following **specific expectation**.

Students will:

- solve problems requiring conversion from metres to centimetres and from kilometres to metres.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- determine the relationships among units and measurable attributes, including the area of a rectangle and the volume of a rectangular prism.

TIME:
3 hours

INSTRUCTIONAL GROUPING:
whole group and triads

ATTRIBUTES, UNITS AND MEASUREMENT SENSE

This learning activity addresses the following **specific expectation**.

Students will:

- estimate and determine elapsed time, with and without using a time line, given the durations of events expressed in minutes, hours, days, weeks, months, or years.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- estimate, measure, and record perimeter, area, temperature change, and elapsed time, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

MATERIALS

- overhead projector and overhead transparency (of a math problem) or LCD projector, and highlighter (optional)
- sheets of chart paper (2 per group of students)
- sheets of grid paper (3 per group of students)
- markers (1 set per group of students)
- rulers (1 per group of students)
- **M.BLM5b.1: Clocking Kilometres** (1 per student)

MATHEMATICS LANGUAGE

- *broken-line graph*
- *conversion*
- *data*
- *distance*
- *elapsed time*
- *rate*
- units of distance (*kilometre, metre*)
- time intervals (*minute, hour*)

ABOUT THE MATH

RATES OF TRAVEL

If a hiker travels 9 km in 3 hours, the rate of travel is 3 km/h. This is equivalent to 3000 m in 1 hour, or 750 m every 15 minutes, or 500 m every 10 minutes, or 50 m every minute.

GETTING STARTED – PROBLEM 1

The Bruce Trail, extending from Niagara to Tobermory, provides an ideal setting for hiking adventures. At nearly 800 km, it is Ontario's longest trail.

INTRODUCING THE PROBLEM

Describe the following scenario to the class.

> "An Ontario hiker has just completed a three-hour hike along this trail. At a number of picturesque locations, the hiker stopped to take photographs. The digital camera recorded the time each photograph was taken. The hiker wants to use these recorded times to pinpoint the locations on a map for a hiking club's website. Over the three-hour period the hiker travelled nine kilometres and took 5 photographs. The hiker left the trailhead at 9:00 a.m. and took photographs at the following times:
>
> - Photograph 1 9:15 a.m.
> - Photograph 2 9:20 a.m.
> - Photograph 3 10:25 a.m.
> - Photograph 4 11:20 a.m.
> - Photograph 5 11:56 a.m.
>
> "How far is each photograph location from the trailhead, assuming the hiker walked at a steady pace?"

> ***Note:*** Students may seek clarification regarding the amount of time that elapsed as the photographer paused to take each photograph. For the purposes of this task, assume that the photographer took each picture quickly and then resumed the hike.

SHARED READING OF THE PROBLEM

Shared reading is an effective instructional approach that can be applied in this context as the students read and interpret the problem together as a class. They will benefit from explicit reading instruction regarding the format and features of mathematics text. A shared-reading approach will support students as they learn to isolate the key pieces of information they need to solve this problem. Through a skilfully led discussion, you can prompt students to justify the reasoning behind their choices.

> ***Note:*** Showing text on an overhead transparency or using an LCD projector is an effective shared-reading approach. The format of a math problem frequently presents the key question at or near the end of the text. Modelling the "skimming and scanning" strategy during the initial read will help students to recognize an effective reading approach for such math problem formats. On subsequent readings, shift the focus to locating pertinent facts. You may wish to use a highlighter to facilitate this process. The shared-reading format also provides opportunities for students to engage in mathematical talk and to clarify their understanding of the task.

> ***Student Thinking:*** Because I know that the key question is often presented near the end of a math problem, scanning the text will allow me to become familiar with the context and to identify the key question. Identifying the key question will help me to read for the specific information I need to solve this problem.

WORKING ON IT – PROBLEM 1

CREATING A VISUAL REPRESENTATION

At this point in the learning task, it may be helpful if each triad of students used chart paper to create a visual representation (such as a time line or a hiking route) of the key information identified during the shared reading of the math problem. The reading comprehension strategy of visualization encourages students to represent key information to synthesize what they know. This visual representation also provides a personally relevant referent from which each student can work.

CALCULATING THE DISTANCE FOR EACH PHOTOGRAPH

Have the triads determine the distance from the trailhead of each photograph location. While students are engaged in this task, circulate and encourage mathematical talk. Ask:

- "How are you using the information in the problem to determine the hiker's rate of travel?"
- "How could the information on the time each photograph was taken help you to determine the distance from the trailhead of each photograph location?"
- "How will you decide which units of measure to use in your calculations?"

Note: This task provides a rich opportunity for students to reason mathematically as they determine relationships. If the rate of travel is 3 km/h, students may use this information to determine distances travelled over time. For example, the knowledge that a hiker travels 3 km/h allows students to determine that the hiker will travel 1.5 km per half hour and 0.75 km or 750 m in 15 minutes. This line of reasoning will help them identify distances travelled over smaller increments of time. If 750 m can be travelled in 15 minutes, then 250 m can be travelled in 5 minutes, and 50 m can be travelled in 1 minute.

Note: Ensure that students are able to recognize the relationships between rate, time, and distance. An integral part of these calculations will involve working flexibly with conversions from kilometres to metres. It may be necessary to engage in small- or whole-group mathematics instruction. Alternatively, you may invite students to share and discuss their strategies for determining rate and distance.

SHARING IDEAS

Once triads have determined the distance from the trailhead of each photograph location, reconvene the class and have students communicate their findings. As they share their work, record (or have a student record) accurate distances related to time on a class T-chart.

Ask students to summarize the information on the T-chart, using a broken-line graph that displays time on the horizontal axis and distance on the vertical axis. The graph will be a straight line, since the distance/time rate is constant (3 km/h).

While students are engaged in the reading, interpretation, and analysis of their data, ask:

- "What conclusions can you draw about the relationship between time and distance by examining your graph?"

GETTING STARTED – PROBLEM 2

Another hiker travels the 9 km trail at the rate of 2.4 km/h. This hiker leaves at 9:00 a.m. and stops at all the same locations to take photographs. Determine the time at which each photograph will be taken. Ask:

- "What impact will this different rate of travel have on the time at which each photograph will be taken?"

WORKING ON IT – PROBLEM 2

Note: In the previous problem, students developed and shared efficient strategies to identify and apply the relationship between time and distance. Problem 2 will allow students to work from what they already know to be true, rethink the problem within a new context, and search for related information that may be helpful.

Have students work in triads to determine the time each photograph was taken. While they are engaged in this task, circulate and encourage them to reflect on strategies shared in stage one of the task. Ask:

- "Which strategies do you think will be most useful in approaching this new challenge? Explain your thinking."

SHARING IDEAS

There are many formats for communicating observations on the comparison of the two sets of data. You may wish to:

- have a class discussion;
- have each triad share graphs and observations;
- have students reflect individually in a math journal.

Once triads have determined the time at which the second hiker took each photograph, reconvene the class and have students communicate their findings. During sharing, record (or have a student record), on a second class T-chart, calculations related to this problem.

After students have completed this stage of the task, ask them to summarize the information on the same graph used in Problem 1 (that is, two broken-line graphs will be drawn on the same grid or set of axes, with time on the horizontal axis). Before they plot the new values, have students anticipate how this data set will compare with the representation of values from Problem 1. The graphs will be straight lines, since the distance/time rates are constant (3 km/h and 2.4 km/h). However, the steepness (or slope) of the graph lines will differ: the faster the rate, the steeper will be the graph line. Ask:

- "How do you think the graphed data for the second hiker will compare with the plotted values for the first hiker?"

Completed graphs provide a rich opportunity for comparing two related sets of data. Ask:

- "What can be determined by comparing the two related sets of data?"

REFLECTING AND CONNECTING

At key points in the process of solving the problem, there may be a need for shared discussion or guided math instruction. Questions and prompts that encourage student reflection have been embedded in each stage of this learning task. Draw students' attention to different visual formats that could be used to represent and solve the problem. Sharing opportunities that demonstrate and model a wide range of efficiency in strategy use allow students to focus on process. Students benefit from comparing approaches. Such comparisons help them to self-assess and to set goals as they continue to work on the problem. Provide opportunities for students to ask questions of one another, share ideas, and justify their reasoning.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

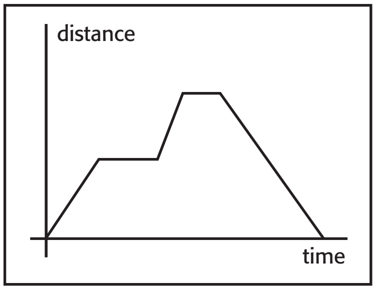
SUPPORTS FOR STUDENT LEARNING

- This measurement task provides many opportunities for differentiated instruction; it requires students to make choices and it promotes cooperative learning.
- As students progress through this task, you have the opportunity to circulate and assess student needs, provide feedback, and scaffold instruction.
- The task allows for multiple entry points. You have the opportunity to control the number of variables students are using, and can thus make the calculations more manageable. For example, you might decide to provide the rate of travel for the first problem, enabling students to focus solely on the relationship between time and distance.
- Some students may require help to organize their data and calculations. Differentiate instruction for these students by assisting them in the creation of a table or graphic organizer.

EXTENSIONS

Staggered Starting Times. Create a scenario in which the hikers' starting times are staggered and students are challenged to predict outcomes. For example, if one hiker leaves the trail-head at 9:00 a.m., travelling at a rate of 2.4 km/h, and another hiker leaves the trailhead at 9:30 a.m., travelling at a rate of 3 km/h: Will the hikers meet on the trail? Who will be the first to take photograph number 4? Who will be first to complete the hike?

Graphical Stories. Provide small groups of students with a variety of broken-line graphs (like the one shown on the right) representing distance and time. Ask the groups to select a particular graph and create a related math story.



Hiking Story. Ask students to create a story involving distance travelled over time. For example, the following story could match the graph shown above: "I started at 8 a.m. I walked at a rate of 4 km/h for 15 minutes. I stopped for 10 minutes to talk to a friend. We walked together for 5 minutes, covering a distance of 400 m. We stopped for a break. Then I walked home at a rate of 5 km/h. The whole trip took 60 minutes." Notice that a steeper slope indicates a faster rate. Advise students that they may choose to write math stories with realistic contexts related to their own experiences or they may create and use imaginative, fictional scenarios.

HOME CONNECTION

See **M.BLM5b.1: Clocking Kilometres**.

ASSESSMENT

Ongoing assessment opportunities are embedded throughout this learning activity. Suggested prompts and questions have been provided in the Getting Started and Working on It sections. Focus your observations in order to assess how well students:

- express their understanding of measurement relationships (time, distance, rate);
- work flexibly with conversions (kilometres to metres);
- select and compare units of measure and justify their reasoning;
- draw upon their understanding of quantity and fractional relationships with respect to time;
- apply reasoning and logical thinking;
- communicate and justify their solutions.

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
The student:				
– estimates and determines elapsed time	☐ limited	☐ some	☐ considerable	☐ thorough
– works flexibly with measurement unit conversions	☐ limited	☐ some	☐ considerable	☐ thorough
– identifies relationships among units and measurable attributes	☐ limited	☐ some	☐ considerable	☐ thorough
– constructs tables, graphs, and diagrams to represent data	☐ limited	☐ some	☐ considerable	☐ thorough
Thinking				
The student:				
– creates a plan of action for analysing measurement data	☐ limited	☐ some	☐ considerable	☐ high degree
– identifies and uses patterns in problem solving	☐ limited	☐ some	☐ considerable	☐ high degree
– explores alternative solutions	☐ limited	☐ some	☐ considerable	☐ high degree
Communication				
The student:				
– explains mathematical thinking	☐ limited	☐ some	☐ considerable	☐ high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	☐ limited	☐ some	☐ considerable	☐ high degree
– uses appropriate vocabulary and terminology	☐ limited	☐ some	☐ considerable	☐ high degree
Application				
The student:				
– applies measurement skills in familiar contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– transfers knowledge and skills to new contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– makes connections among concepts	☐ limited	☐ some	☐ considerable	☐ high degree

Clocking Kilometres

Dear Parent/Guardian:

Your child has been asked to select and complete one of the tasks below and to share with you his or her work and thought processes. Please take a few minutes to discuss this activity with your child.

TASK 1

Travelling to school each day, we generally follow a consistent route. Determine the distance from your front door to the entrance of the school. One way to obtain this information is to use a scale map of your neighbourhood. Alternatively, a Web-based tool such as MapQuest or Google Maps will provide an exact distance between two addresses, as will the odometer of a car. Determine the time it takes you to travel to school, noting your time of departure and time of arrival. Use this information to determine your average hourly rate of travel. Depending upon the measured values with which you are working, it may be necessary to use a calculator as a computational tool.

TASK 2

Travelling to school each day, we generally follow a consistent route. Determine the distance from your front door to the entrance of the school. One way to obtain this information is to use a scale map of your neighbourhood. Alternatively, a Web-based tool such as MapQuest.com or Google Maps will provide an exact distance between two addresses, as would the odometer of a car. Determine the time it takes you to travel to school, noting your time of departure and time of arrival. In addition, note the time and duration of any stops along your route. Create a graph to represent your data (time on the horizontal axis, and distance on the vertical axis). How does your graph tell the math story of your trip to school?

Grade 6 Learning Activity
Measurement at the Track Meet

OVERVIEW

In this learning activity, students explore estimated and precise measurements in the context of a school track meet. Students participate in a "mini track meet" as athletes and officials and plan two events: the 100 m dash and the long jump. They rotate through each of the two events. They record time results to the nearest hundredth of a second, which will require prior experience in working with decimal numbers. Students also need to be familiar with a variety of measurement tools, including stopwatches, trundle wheels, and measuring tapes, and need to recognize the units related to these measurement tools.

BIG IDEA

Attributes, units, and measurement sense

CURRICULUM EXPECTATIONS

ATTRIBUTES, UNITS, AND MEASUREMENT SENSE

This learning activity addresses the following **specific expectations**.

Students will:

- demonstrate an understanding of the relationship between estimated and precise measurements, and determine and justify when each kind is appropriate;
- estimate, measure, and record length, area, mass, capacity, and volume, using the metric measurement system.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- estimate, measure, and record quantities, using the metric measurement system.

MEASUREMENT RELATIONSHIPS

This learning activity addresses the following **specific expectation**.

Students will:

- select and justify the appropriate metric unit to measure length or distance in a given real-life situation.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- determine the relationships among units and measurable attributes, including the area of a parallelogram, the area of a triangle, and the volume of a triangular prism.

TIME:
2 hours

INSTRUCTIONAL GROUPING:
small groups of 4 to 6

ABOUT THE LEARNING ACTIVITY

This learning activity is divided into three stages:

- Planning the mini track meet – 40 minutes
- Collecting track meet data – 40 minutes
- Analysing and reflecting on data to make recommendations – 40 minutes

MATERIALS

- **M.BLM6a.1a–c: Recording Sheet** (1 per student)
- measurement tools (e.g., metre sticks, measuring tapes, trundle wheel, stopwatch) (2 sets per group of students)
- track meet equipment (e.g., pylons, whistles or coloured paper or flags to mark the start of races, finish line tape/string, rake, clipboards) (2 sets per group of students)
- sheets of chart paper (2 per group of students)
- set of markers (1 per group of students)
- **M.BLM6a.2: Measurement in Sports** (1 per student)

MATHEMATICS LANGUAGE

- *elapsed time*
- measures of length (*metre, centimetre*)
- measures of time (*minute, second, fraction of second*)
- measurement tools (*stopwatch, trundle wheel, measuring tapes*)

ABOUT THE MATH

DEGREE OF ACCURACY OF TRACK AND FIELD EVENTS

The 100 m dash is measured in seconds and hundredths of a second. As of 2006:

- The 100 m record for men is 9.77 seconds and is held by Asafa Powell, of Jamaica (2006).
- The 100 m record for women is 10.49 seconds and is held by Florence Griffith Joyner, USA (1988).

The long jump is measured in metres and centimetres. As of 2006:

- The long jump record for men is 8.95 m and is held by Mike Powell, USA (1991).
- The world record for women is 7.52 m and is held by Galina Chistyakova, of the former Soviet Union (1988).

GETTING STARTED

INTRODUCING THE PROBLEM

Describe the following scenario to the class:

"Our class has been asked to assume a leadership role in the Junior Division Track and Field Meet this year. We will be "measurement mentors", helping to plan and execute this year's meet. We will have to advise the teachers who are working on the schedule on how much time to allow for the entire track meet, as well as on how much time will be required for each of the individual events. We will also need to determine when estimated or precise measurements should be used.

"To gather our information, we will plan a two-event track and field meet for our class. We will communicate our findings, and provide a map suggesting locations for each event, to the teachers planning the larger school meet.

"First, we will identify all the measurable attributes of the track meet events. Using a brainstorming web, we will record our ideas. The web will be available for your reference as you work through the task. We will have to determine the following:

- When will precise measurements be needed and when will estimates be appropriate?
- How can we justify our decisions?
- How will estimating some of the measurements make our planning more efficient?"

Note: While creating the brainstorming web, draw students' attention to the wide range of measurement opportunities at a track meet (pre-event measuring of distances and designation of event space, measuring of the transition time between events, measuring of the time required to complete an event, measuring a jump or the time of a run).

WORKING ON IT

COLLECTING DATA

Divide the class into four groups. Explain that each group is responsible for planning one event (e.g., long jump) and participating in another (e.g., 100 m run). Each event will be run twice. Each athlete group will be paired with a track-official group for the first event. At the conclusion of the event, the groups will switch roles and rotate to the second event.

	First Event	Second Event
Group A	Long Jump – Recorders	100 m – Athletes
Group B	Long Jump – Athletes	100 m – Recorders
Group C	100 m – Recorders	Long Jump – Athletes
Group D	100 m – Athletes	Long Jump – Recorders

Provide students with **M.BLM6a.1a–c: Recording and Reflection Sheet**. Allow the groups time to collect the measurement tools (e.g., metre sticks, measuring tapes, trundle wheel, stopwatch) and track meet equipment (e.g., pylons, whistles or coloured paper or flags to mark the start of races, finish line tape/string, rake, clipboards) that will be needed for each event.

ANALYSING DATA

Explain that once data collection has been completed, students will form two event expert groups to analyse the data collected for each event. The following key questions (from **M.BLM6a.1a–c: Recording and Reflection Sheet**) will assist students in focusing their discussions and recording their data analysis:

- What similarities and differences are evident in the data collected by each group? How can we account for the differences?
- What is the average amount of time required for one person to complete your event? Why would this information be important for planning the junior division track meet?
- How might we expect our data to change with greater numbers of students or with younger or less-experienced students participating in the event?

REFLECTING AND CONNECTING

Following expert group discussion and analysis, reconvene the class to share findings and determine final recommendations. In preparation for this discussion, divide a piece of chart paper in half, one half per event. The chart will be used to collate the class data and recommendations. Within each half, insert subtitles such as “Measurement tools required”, “Total time required for the event”, “Number of event officials required to run the event”, and add other considerations generated by your class.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

- Thoughtful attention to student partnering will embed support within the task.
- Provide recording sheets to help students organize data.
- While students progress through the analysis stage of the task, circulate and assess student needs, give feedback, and scaffold instruction.

EXTENSIONS

Precision Over Time. There are certain sports events in which the recording of world record times has become increasingly precise. Have students select a specific sports event and research the historical world records related to that event. Ask them to investigate how the precision of timing has changed and how that precision has affected the number of people holding the world record in the chosen event.

Record Book. Following the completion of the Junior Division Track and Field Meet, have students create a Junior Division Record Book based on data collected at each event.

HOME CONNECTION

See **M.BLM6a.2: Measurement in Sports**.

ASSESSMENT

Opportunities for assessment are embedded in this learning activity, as are many occasions for observing students as they measure and record lengths and discuss the relationship between estimated and precise measurement. Ask: “In our 100 m dash we used precision to hundredths of a second. Why is this precision necessary?”

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
The student:				
– estimates, collects, measures, and records data	☐ limited	☐ some	☐ considerable	☐ thorough
– estimates and determines elapsed time	☐ limited	☐ some	☐ considerable	☐ thorough
– organizes and displays measurement data	☐ limited	☐ some	☐ considerable	☐ thorough
– reads, interprets, and draws conclusions from data	☐ limited	☐ some	☐ considerable	☐ thorough
Thinking				
The student:				
– creates a plan of action for collecting data	☐ limited	☐ some	☐ considerable	☐ high degree
– identifies and uses patterns in measurement data	☐ limited	☐ some	☐ considerable	☐ high degree
– makes predictions for patterns in measurement data	☐ limited	☐ some	☐ considerable	☐ high degree
– explores alternative solutions	☐ limited	☐ some	☐ considerable	☐ high degree
Communication				
The student:				
– explains mathematical thinking	☐ limited	☐ some	☐ considerable	☐ high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	☐ limited	☐ some	☐ considerable	☐ high degree
– uses appropriate vocabulary and terminology	☐ limited	☐ some	☐ considerable	☐ high degree
Application				
The student:				
– applies measurement skills in familiar contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– transfers knowledge and skills to new contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– makes connections among concepts	☐ limited	☐ some	☐ considerable	☐ high degree

Recording and Reflection Sheet

NAME OF EVENT: LONG JUMP (2 JUMPS PER JUMPER)

What needs to be measured for this event? ______________________

Attribute to be measured: ______________________

Unit of measurement to be used: ______________________

Rationale: ______________________

Tools selected: ______________________

Estimated time to complete your event: ______________________

Justify your estimate.

____

____

Actual start time for your event: ______________________

Actual finish time for your event: ______________________

Total elapsed time for your event: ______________________

How close was your estimate to the total elapsed time?

____

EXPERT GROUP DISCUSSION QUESTIONS

1. What similarities and differences are evident in the data collected by each group? How can we account for the differences?
2. What is the average amount of time required for one person to complete your event? Why would this information be important for planning the junior division track meet?
3. How might we expect our data to change with greater numbers of students or with younger or less-experienced students participating in the event?
4. What is our final recommendation for the junior division track meet?

____

____

____

____

____

NAME OF EVENT: 100 m DASH (2–3 HEATS [2 RUNNERS PER HEAT] AND 1 FINAL)

What needs to be measured for this event?

____

____

Attribute to be measured: ______________________________

Unit of measurement to be used: __________________________

Rationale: ____

Tools selected: _______________________________________

Estimated time to complete your event: ____________________

Justify your estimate.

____

____

Actual total times for individual heats: ___________________

While the race itself may be very short, the time involved in setting up the runners, beginning the race, and recording the times will all need to be considered in the length of time needed to run this event. Use this chart to record the actual timing of this event.

Heat	Start Time (watch)	Finish Time (watch)	Elapsed Time
1			
2			
3			
Final			

Actual start time for your event: _________________________

Actual finish time for your event: ________________________

Total elapsed time for your event: ________________________

1. How close was your estimate to the total elapsed time?

____

2. What was the average heat time (not including final)?

____

EXPERT GROUP DISCUSSION QUESTIONS

1. What similarities and differences are evident in the data collected by each group? How can we account for the differences?
2. What is the average amount of time required for one person to complete your event? Why would this information be important for planning the junior division track meet?
3. How might we expect our data to change with greater numbers of students or with younger or less-experienced students participating in the event?
4. What is our final recommendation for the junior division track meet?

____

____

____

____

____

Measurement in Sports

Dear Parent/Guardian:

Our class has been investigating measurement in sport. Please take the time to do the following activities with your child.

HOME CONNECTION 1

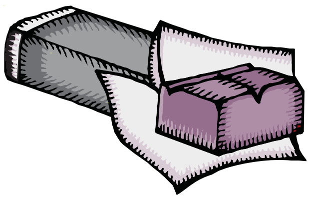
Our class has been exploring the relationships between estimated and precise measurements. Sporting events provide one context for examining the need for precision. Help your child to locate sports statistics involving length or time, using a newspaper, sports magazine, or other resource. Ask your child to discuss the degree of precision used in these measurements.

HOME CONNECTION 2

Our class has just held a mini track and field meet for the purpose of investigating measurement concepts. Part of the task involved the analysis of data in order to make recommendations to the staff for the Junior Division Track and Field Meet. Please ask your child to share what he or she learned about the relationship between estimated and precise measurement.

Grade 6 Learning Activity
Packaging the Chocolongo Bar

OVERVIEW



In this learning activity students explore measurement relationships between the dimensions and the surface area of rectangular prisms. The purpose of the activity is to identify the chocolate bar format that requires the least amount of packaging. Working with a defined volume, students determine possible chocolate bar dimensions. They use manipulatives to represent possible solutions and communicate their findings. Once they have identified the ideal chocolate bar format, they begin to explore possible packaging formats for Chocolongo bars, using specific dimensions of store shelving displays.

Students need to have an understanding of the relationships between the length and width of a rectangle and its area and perimeter.

BIG IDEA

Measurement relationships

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- determine, through investigation using a variety of tools and strategies, the surface area of rectangular and triangular prisms;
- solve problems involving the estimation and calculation of the surface area and volume of triangular and rectangular prisms.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- determine the relationships among units and measurable attributes, including the area of a parallelogram, the area of a triangle, and the volume of a triangular prism.

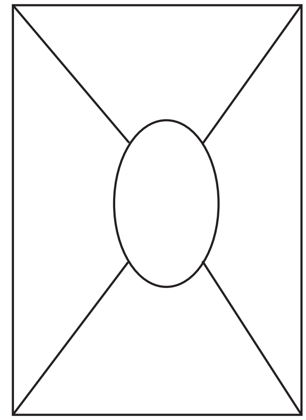
ABOUT THE LEARNING ACTIVITY

TIME:
Stage One - 80 minutes

Stage Two - 40 minutes

MATERIALS

- 36 interlocking cubes per student
- markers (1 per student)
- place mat recording charts (image shown on the right) (1 per group). Have students reproduce the place mat on chart paper.
- spreadsheet software (optional)
- **M.BLM6b.1: Investor Fact Sheet** (1 per student)
- **M.BLM6b.2: Exploring Volume and Surface Area** (1 per student)



MATHEMATICS LANGUAGE

- *area of base*
- *height*
- *surface area*
- *volume*

INSTRUCTIONAL GROUPING: groups of 4

ABOUT THE MATH

See "Packaging: What Three-Dimensional Shape Reduces Packaging Waste?" on pp. 36–38 of this volume.

GETTING STARTED: STAGE ONE – PACKAGING THE CHOCOLONGO BAR

INTRODUCING THE PROBLEM

Describe the following scenario to the class:

> "Since the public is becoming increasingly concerned about the impact that food packaging is having on the environment, the president of Chocolate Company X has called a crisis management meeting of her strategy team. Sales of the company's signature chocolate bar, the Chocolongo, have plummeted since a recent article named the company as the largest producer of packaging waste in the chocolate bar industry. This is very distressing news for the company. Before the appearance of the article, the Chocolongo bar had always met with rave reviews because of its unique long, thin shape."

* * * * * * * * c * h * o * c * o * l * o * n * g * o * * * * * * * * *

> "The president wants to continue providing her loyal customers with the same volume of chocolate while reducing the amount of packaging used. Therefore, her strategy team must determine a different format for the bar. To preserve some similarity between the original Chocolongo bar and the new one, the team

Prompts: How many possible formats do you think will be found for the Chocolongo chocolate bar? What makes you think that?

leader requires that the new bar have only a single wrapping. No additional sleeve is to be used. The team must provide proof that the selected format will result in the least amount of packaging. A member of the strategy team has asked our class for assistance with this challenge.

"You have 36 interlocking cubes, which represent the total volume of a Chocolongo bar. Your task is to work with the 36 interlocking cubes to find all other possible formats for the new and improved bar. For shipping and storage purposes, the final product must be in the form of a rectangular prism."

PLACE MAT

"You will be working in teams of four. As a first step, you will each work independently to complete a section of a place mat, noting every possible format for the new Chocolongo bar and identifying the total surface area. During sharing, you will compare your possibilities with those of your team members and determine which format best meets requirements. Record this solution in the centre of your place mat. You will use your place mat as a reference as you present your work to the other strategy teams and justify your selection."

> ***Note:*** The place mat organizer provides students with a structure for recording individual thinking, and for group consensus built through sharing. If students are unfamiliar with this organizer, model it before introducing the task.

WORKING ON IT: STAGE ONE – PACKAGING THE CHOCOLONGO BAR

While students work with their interlocking cubes and record their individual findings, circulate and encourage mathematical talk. The following questions may be helpful when assessing and promoting purposeful talk:

- "How has your group decided to measure the surface area?"
- "Are there other methods?"
- "How will your group organize the data?"

> ***Note:*** As students create models and work to complete individual sections of the place mat, encourage them to engage in purposeful talk. Such talk is essential, as it allows students to express, clarify, and expand on their ideas while they work to solve problems.

As groups work through the problem, encourage generalizations related to determining surface area for rectangular prisms by asking the following questions or using the prompts:

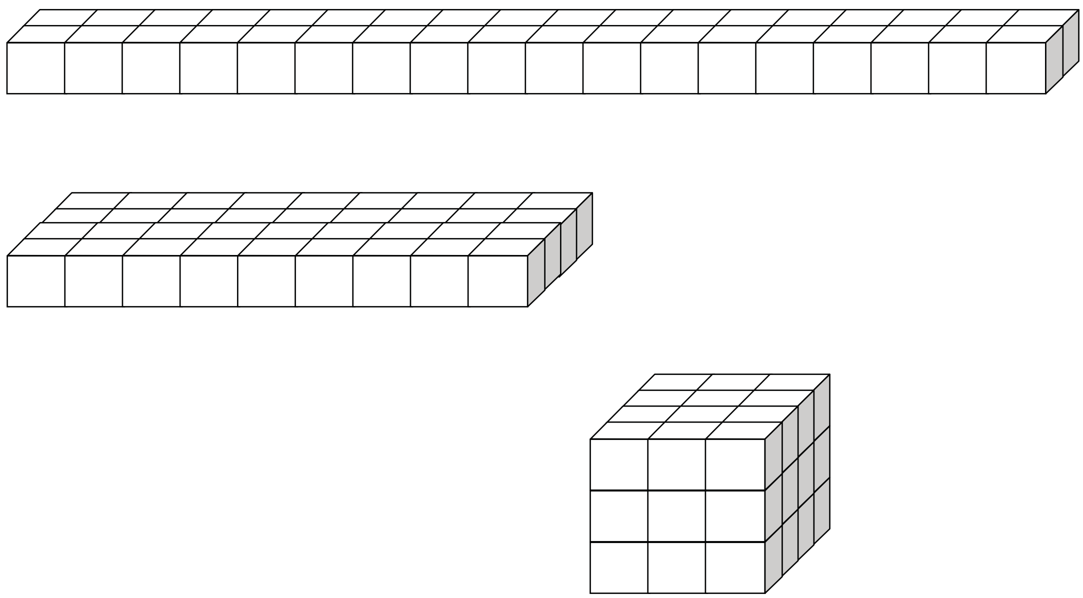
- "How did your group decide which rectangular prism used the least packaging?"
- "Describe the steps your group used to determine how much packaging each rectangular prism needed."
- "How could you prove that your strategy would apply to any rectangular prism?"
- "If you were given the specific dimensions of a very large rectangular prism, such as a rectangular-shaped building, how would you determine the surface area of that rectangular prism?"

- "How many sides of your rectangular prisms had the same area? Explain any patterns you noticed in your data."
- "Is it possible, using your 36 cubes, to find a rectangular prism whose sides all have the same area? Explain your thinking."
- "Given a rectangular prism in which the area of all six sides is equal, how would you calculate total surface area?"

STRATEGIES STUDENTS MIGHT USE

MAKE A MODEL WITH CONCRETE MATERIALS

Students use 36 interlocking cubes to concretely represent the Chocolongo bar.



DRAW A DIAGRAM

They could draw diagrams like the one below and label the dimensions.

USE THE GUESS-AND-CHECK METHOD

Once students have generated two or three possible formats, they might guess and check to direct their thinking about possible dimensions.

MAKE A TABLE

Students could create a table to record dimensions of possible chocolate bars and the area of each face of the rectangular prism.

	Length	Width	Height	Area of face 1	Area of face 2	Area of face 3	Area of face 4	Area of face 5	Area of face 6	Total Surface Area
Bar 1	4 units	9 units	1 unit	36 $units^2$	4 $units^2$	36 $units^2$	4 $units^2$	9 $units^2$	9 $units^2$	98 $units^2$
Bar 2										
Bar 3										

Note: As students experiment with three-dimensional figures, they notice that the more cube-like the package (with the same volume), the more efficient the packaging. This realization should lead them to conjecture that although they cannot construct a cube with the materials provided, a cube would be the most efficient shape. It is beneficial to encourage this type of thinking. One way to proceed is to use technology. A spreadsheet can be prepared on which students can enter different dimensions, after which the volume and surface area are calculated automatically. Another way would be to try a problem with a volume that is a perfect cube (e.g., 64 cm^3), making it possible for students to create a cube without using decimals ($4 \times 4 \times 4 = 64$).

Note: Prompt students to consider whether the name Chocolongo is still appropriate for the new design of the chocolate bar.

REFLECTING AND CONNECTING: STAGE ONE – PACKAGING THE CHOCOLONGO BAR

Following group-work time, reconvene the class for a whole-group discussion. Have students post their place mats around the classroom. Invite them to participate in a Two Stay Two Stray Gallery Walk, in which teams designate two people to stay with the place mat to explain their solution and strategy to other teams while the other two members view other place mats and listen to the solutions and strategies explained by other teams. Allow time for home-group discussion following the Gallery Walk. Team members who “strayed” are responsible for explaining other group strategies to those who “stayed”. Teams may decide to amend their solution on the basis of additional information gained from the “straying”. Invite the class to consider all the solutions to the problem and identify which solution best meets the criteria. Discuss what can be generalized regarding the relationship between surface area and volume. You might ask students to reflect on their learning by writing in their mathematics journal. Possible journal prompts include the following:

- When volume remains constant, what is the impact on the surface area of a rectangular prism of a change in the dimensions?
- Why would it be important for companies that sell packaged products to know about these relationships?

GETTING STARTED: STAGE TWO – USING SHELF SPACE

Explain the following scenario to the class:

"Boxes of Chocolongo bars are located on shelves in grocery stores, corner stores, gas stations, and big chain stores all over Ontario. Chocolongo bar displays are allotted space measuring 10 units deep × 5 units high × 36 units across.

"The original Chocolongo bars were sent in boxes of 50. There were 10 Chocolongo bars in each layer and 5 layers in each box. This format made restocking shelves a manageable task. The sales associates are asking for an estimate of how many Chocolongo bars will now be in one box. Estimate how many new Chocolongo bars can be displayed using the same volume of space.

- Do you think it will be possible to use all the space with the new Chocolongo format? Justify your thinking.

"The president will need exact information for her report to investors. Record your estimate for the president on the formal fact sheet **M.BLM6b.1: Investor Fact Sheet**."

WORKING ON IT: STAGE TWO – USING SHELF SPACE

Have students use a model of their solution as a reference for determining how Chocolongo bars will be displayed in stores. The challenge is to find a way to display the greatest number of Chocolongo bars within the constraints of the existing display space (10 units deep × 5 units high × 36 units across). Ask:

- "How will you represent the existing shelf space?"
- "How does the placement of the Chocolongo bars on the shelf affect how much of the space is used?"
- "How many possible placements would there be for the chocolate bar format our class has selected?"

Note: Students can draw a rectangle to represent the width and length of the shelf, and can use their blocks to explore how the chocolate bars might fit on the shelf.

Note: Students may attempt to answer this question by determining the total volume available for display (10 units × 5 units × 36 units) and dividing that by the volume of the Chocolongo (36 units^3). Ask prompting questions that will guide them to recognize that while there might be available volume, it may not be usable. The Chocolongo bar is a solid and has to remain intact.

STRATEGIES STUDENTS MIGHT USE

MAKE A MODEL WITH CONCRETE MATERIALS
In order to visualize this problem, some students may need to delineate a space equal to that of the shelving volume available and use concrete materials to fill that space. Students choosing this strategy will require additional manipulatives.

DRAW A DIAGRAM
Students may choose to draw the shelf and layers of chocolate bars. Prompt them to consider all the possible ways of placing the Chocolongo bars in the space.

Note: The base the student chooses, and its position on the shelf, will influence how many Chocolongo bars will fit on the shelf and how much space will be unusable.

REFLECTING AND CONNECTING: STAGE TWO – USING SHELF SPACE

Observe students as they work. Identify groups you will ask to share their solutions and strategies with the class. Include a variety of solutions and formats. Sharing opportunities that reflect a wide range of efficiency, strategy use, and solutions allow students to focus on process. Reflecting on less-efficient strategies or partial solutions allows students to identify gaps in reasoning, thereby gaining a deeper understanding of the problem.

TIERED INSTRUCTION

Supports and extensions can be beneficial for all students. For any given activity, there will always be some students who require more or less support, or for whom extensions will increase interest and deepen understanding.

SUPPORTS FOR STUDENT LEARNING

This learning task provides many opportunities for differentiated instruction; it requires students to make choices and it promotes cooperative learning. Students are able to select personally meaningful strategies and materials and to represent their ideas in a variety of formats. Instructional groupings promote purposeful mathematics talk as students share their problem-solving approaches and solutions and justify their reasoning. As students progress through this task, you have opportunities to circulate and assess student needs, provide feedback, and scaffold instruction. Encourage journal writing by providing sentence starters, checklists, and prompts or picture cues. You may also scaffold this task by giving careful consideration to the creation of student groupings. Consider "chunking" this task, providing time accommodations, and giving guiding instruction as needed.

EXTENSIONS

Shipping Container. Extend this problem by having students consider what size container would be required to ship a large quantity of boxes. For example, ask:

- "What size container would be required to ship 64 boxes of the best design of Chocolongo bars?"
- "Working with the same best design, can you think of other possible dimensions for a container of this size?"

Ratio of Dimensions. Bring various packages to the classroom, and challenge students to determine the ratio between the height and width of each package. Direct students to look for patterns in the data.

HOME CONNECTION

See **M.BLM6b.2: Exploring Volume and Surface Area**.

ASSESSMENT

While students are working on this task, you can effectively observe and assess math talk and strategy use. Ensure that students are given the opportunity to reason and to record their work in personally meaningful ways.

Focus your observations in order to assess how effectively students:

- determine the surface area of rectangular prisms;
- estimate and calculate surface area and volume of rectangular prisms.

Using probing questions, assess the depth and breadth of understanding that students bring to the task, and invite students to explain and justify their thinking. Ask:

- "Do you have a strategy for identifying all the possible chocolate bar formats?"
- "How will you know when you have found all the possibilities?"
- "How are you planning to record your work?"
- "How will you know that you have included the area of all surfaces in your calculation of total surface area?"
- "What strategy are you using to ensure that your measurements of surface area are accurate?"
- "What patterns are evident in your data?"
- "On the basis of the formats that you have created and the surface area measurements that you have recorded, can you predict the shape of chocolate bar that the president will likely want to use? What is leading you to this prediction?"

Once students have completed their work, use the following assessment prompts:

- We solved the problem by …
- The steps we followed were …
- We've shown our thinking by …
- Our strategy was successful because …
- The most important thing we learned about the relationship between volume and surface area is …

RUBRIC

Assessment Category	Level 1	Level 2	Level 3	Level 4
Knowledge and Understanding				
The student:	☐ limited	☐ some	☐ considerable	☐ thorough
– makes use of relationships among units	☐ limited	☐ some	☐ considerable	☐ thorough
– calculates surface area and volume of rectangular prisms	☐ limited	☐ some	☐ considerable	☐ thorough
– constructs tables, graphs, and diagrams to represent measurement data				
Thinking				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– creates a plan of action for analysing measurement data	☐ limited	☐ some	☐ considerable	☐ high degree
– identifies and uses patterns in problem solving	☐ limited	☐ some	☐ considerable	☐ high degree
– explores alternative solutions				
Communication				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– explains mathematical thinking	☐ limited	☐ some	☐ considerable	☐ high degree
– communicates using a variety of modes (short answers, lengthy explanations, verbal and written reports)	☐ limited	☐ some	☐ considerable	☐ high degree
– uses appropriate vocabulary and terminology				
Application				
The student:	☐ limited	☐ some	☐ considerable	☐ high degree
– applies measurement skills in familiar contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– transfers knowledge and skills to new contexts	☐ limited	☐ some	☐ considerable	☐ high degree
– makes connections among concepts				

Investor Fact Sheet

Important Considerations	Original Chocolongo Bar	New and Improved Chocolongo Bar
Dimensions		
Surface area		
Volume		
Number per shelving display		

On the basis of the charted data, how much will the packaging be reduced with the new and improved Chocolongo Bar?

Qualities of our new Chocolongo Bar that we intend to highlight in our advertising campaign:

*

*

*

Exploring Volume and Surface Area

Dear Parent/Guardian:

Our class has been learning about the relationship between surface area and volume. Please take the time to do the following activities with your child.

HOME CONNECTION 1

Have your child demonstrate how to calculate the volume and surface area of a rectangular package in your home. Work with your child to list the different dimensions possible for a package of this volume.

Ask your child:

- "Which dimensions result in the least amount of surface area?"
- "Is there a pattern?"

HOME CONNECTION 2

Using the list of possible dimensions from Home Connection 1, help your child investigate other packages in your home.

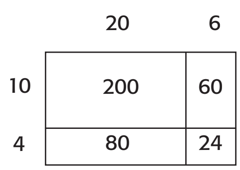
Ask your child:

- "Do companies generally use efficient packaging shapes?"
- "If a cube is the most efficient rectangular prism, why do you think companies sell their products (e.g., cereals or pancake mix) in other shaped boxes?"

Glossary

almanac. A calendar for a given year with information such as sunrise times, sunset times, and weather predictions.

area model. 1. A diagrammatic representation that uses area to demonstrate other mathematical concepts. In an area model for multiplication, for example, the length and width of a rectangle represent the factors, and the area of the rectangle represents the product. The diagram shows the use of an area model to represent a 26 × 14 array.
2. A rectangular arrangement of objects into rows and columns, used to represent multiplication (e.g., 5 × 3 can be represented by 15 objects arranged into 5 columns and 3 rows).



$26 \times 14 = 200 + 60 + 80 + 24$
$= 364$

attribute. A quantitative or qualitative characteristic of a shape, an object, or an occurrence; for example, colour, size, thickness, or number of sides. An attribute may or may not be a property. *See also* **property (geometric)**.

bar graph. *See under* **graph**.

base ten materials. Learning tools that help students learn a wide variety of concepts in number sense, including place value; the operations (addition, subtraction, multiplication, and division); and fractions and decimals. Sets of base ten materials typically include ones (small cubes called "units"), tens ("rods" or "longs"), hundreds ("flats"), and thousands (large cubes).

benchmark. A number or measurement that is internalized and used as a reference to help judge other numbers or measurements. For example, the width of the tip of the little finger is a common benchmark for one centimetre. Also called *referent*.

big ideas. In mathematics, the important concepts or major underlying principles.

broken-line graph. *See under* **graph**.

calculus. The algebraic study of rates of change.

capacity. The greatest amount that a container can hold; usually measured in litres or millilitres.

cardinal directions. The four main points of the compass: north, east, south, and west.

Cartesian coordinate grid. *See* **coordinate plane**.

Cartesian plane. *See* **coordinate plane**.

circle graph. *See under* **graph**.

conceptual understanding. The connection of mathematical ideas with one another that provides a deep understanding of mathematics. Students develop their understanding of mathematical concepts through rich problem-solving experiences.

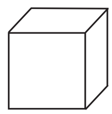
concrete materials. Objects that students handle and use in constructing or demonstrating their understanding of mathematical concepts and skills. Some examples of concrete materials are base ten blocks, interlocking cubes, construction kits, number cubes, games, geoboards, geometric solids, measuring tapes, Miras, pattern blocks, spinners, and tiles. Also called *manipulatives*.

connecting cubes. *See* **interlocking cubes**.

coordinate graph. *See under* **graph**.

coordinate plane. A plane that contains an *x*-axis (horizontal) and a *y*-axis (vertical), which are used to describe the location of a point. Also called *Cartesian coordinate grid* or *Cartesian plane*.

cube. A right rectangular prism with six congruent square faces. A cube is one of the Platonic solids. Also called a *hexahedron*.



data. Facts or information.

degree of precision. The degree of accuracy of a measurement. For example, the 100 m dash is typically measured to one one-hundredth of a second.

double line graph. *See under* **graph**.

dynamic geometry software. Computer software that allows the user to explore and analyse geometric properties and relationships through dynamic dragging and animations. Uses of the software include plotting points and making graphs on a coordinate system; measuring line segments and angles; constructing and transforming two-dimensional shapes; and creating two-dimensional representations of three-dimensional objects. An example of the software is The Geometer's Sketchpad.

equation. A mathematical statement that has equivalent expressions on either side of an equal sign.

estimation strategies. Strategies used to obtain an approximate answer. Students estimate when an exact answer is not required, and to check the reasonableness of their mathematics work.

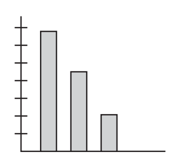
Fermi. Enrico Fermi (1901–1954) won the 1938 Nobel Prize in Physics. Fermi enjoyed posing and solving problems that involve a quantity that is difficult or impossible to measure directly; for example, "How many breaths have you breathed?" Such problems have come to be known as "Fermi questions".

geoboard. A commercially produced learning tool that helps students learn about perimeter, area, fractions, transformations, and so on. A geoboard is a square piece of plastic or wood with pins arranged in a grid or in a circle. Elastics are used to connect the pins to make different shapes.

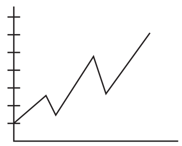
golden ratio. Approximately 1.62:1. The ancient Greeks believed that a rectangle with dimensions in this proportion was the most pleasing. This ratio was used by many artists of the Renaissance. Also called *golden mean*.

graph. A visual representation of data. Some types of graphs are:

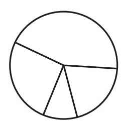
– **bar graph**. A graph consisting of horizontal or vertical bars that represent the frequency of an event or outcome. There are gaps between the bars to reflect the categorical or discrete nature of the data.



– **broken-line graph**. A graph formed by line segments that join points representing the data. The horizontal axis represents discrete quantities such as months or years, whereas the vertical axis can represent continuous quantities.

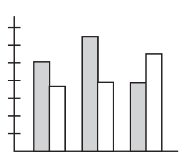


– **circle graph**. A graph in which a circle is used to display categorical data, through the division of the circle proportionally to represent each category.



– **coordinate graph**. A graph that has data points represented as ordered pairs on a grid; for example, (4, 3). *See also* **ordered pair**.

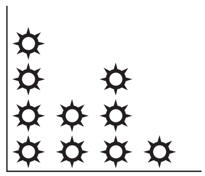
– **double bar graph**. A graph that combines two bar graphs to compare two aspects of the data in related contexts; for example, comparing the populations of males and females in a school in different years. Also called comparative bar graph.



– **double line graph**. A graph that combines two line graphs to compare two aspects of the data in related contexts; for example, comparing the distance travelled by two cars moving at different speeds.

– **line graph**. A graph formed by a straight line.

– **pictograph**. A graph that uses pictures or symbols to compare frequencies.



grid. A network of regularly spaced lines that cross one another at right angles to form squares or rectangles.

integer. Any one of the numbers … , –4, –3, –2, –1, 0, 1, 2, 3, 4, …

interlocking cubes. Commercially produced learning tools that help students learn about spatial sense, volume, surface area, patterning, and so on. Some interlocking cubes attach on only one face, while others attach on any face.

interval. The set of points or the set of numbers that exist between two given endpoints. The endpoints may or may not be included in the interval. For example, test score data can be organized into intervals such as 65–69, 70–74, 75–79, and so on.

line graph. *See under* **graph**.

linear pattern. A numeric pattern in which numbers grow at a constant rate. For example, 3, 5, 7, 9, 11, …

magnitude. An attribute relating to size or quantity.

manipulatives. *See* **concrete materials**.

mass. The amount of matter in an object; usually measured in grams or kilograms.

mathematical communication. The process through which mathematical thinking is shared. Students communicate by talking, drawing pictures, drawing diagrams, writing journals, charting, dramatizing, building with concrete materials, and using symbolic language (e.g., 2, =).

mathematical language. The conventions, vocabulary, and terminology of mathematics. Mathematical language may be used in oral, visual, or written forms. Some types of mathematical language are:

– terminology (e.g., factor, pictograph, tetrahedron);

– visual representations (e.g., 2 × 3 array, parallelogram, tree diagram);

– symbols, including numbers (e.g., 2, 1/4), operations [e.g., 3 × 8 = (3 × 4) + (3 × 4)], and signs (e.g., =).

mean. One measure of central tendency. The mean of a set of numbers is found by dividing the sum of the numbers by the number of numbers in the set. For example, the mean of 10, 20, and 60 is (10 + 20 + 60) ÷ 3 = 30. A change in the data produces a change in the mean, similar to the way in which changing the load on a lever affects the position of the fulcrum if balance is maintained.

meteorologist. A scientist who studies weather (and other atmospheric conditions).

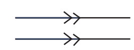
modelling. The process of describing a relationship using mathematical or physical representations.

non-standard units. Common objects used as measurement units; for example, paper clips, cubes, and hand spans. Non-standard units are used in the early development of measurement concepts.

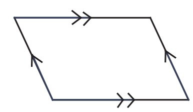
odometer. A device for measuring the distance travelled. Cars have odometers.

ordered pair. Two numbers, in order, that are used to describe the location of a point on a plane, relative to a point of origin (0, 0); for example, (2, 6). On a coordinate plane, the first number is the horizontal coordinate of a point, and the second is the vertical coordinate of the point.

parallel lines. Lines in the same plane that do not intersect.

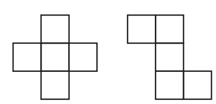


parallelogram. A quadrilateral whose opposite sides are parallel.



pattern blocks. Commercially produced learning tools that help students learn about shapes, patterning, fractions, angles, and so on. Standard sets include: green triangles; orange squares; tan rhombuses and larger blue rhombuses; red trapezoids; yellow hexagons.

pentomino. A shape made of five identical squares attached edge to edge. There are 12 possible pentominoes.



perimeter. The length of the boundary of a shape, or the distance around a shape. For example, the perimeter of a rectangle is the sum of its side lengths; the perimeter of a circle is its circumference.

pictograph. *See under* **graph**.

population. The total number of individuals or objects that fit a particular description; for example, salmon in Lake Ontario.

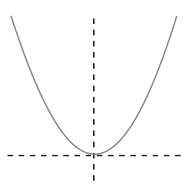
primary data. Information that is collected directly or first-hand; for example, observations and measurements collected directly by students through surveys and experiments. Also called *first-hand data* or *primary-source data*. *See also* **secondary data**.

prism. A three-dimensional figure with two parallel and congruent bases. A prism is named by the shape of its bases; for example, rectangular prism, triangular prism.

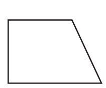
probability. A number from 0 to 1 that shows how likely it is that an event will happen.

property (geometric). An attribute that remains the same for a class of objects or shapes. A property of any parallelogram, for example, is that its opposite sides are congruent. *See also* **attribute**.

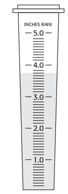
quadratic function. A function of the form $f(x) = ax^2 + bx + c$. The shape of its graph is a parabola (bell-shaped). The graph of the distance travelled by a falling object is (approximately) parabolic.



quadrilateral. A polygon with four sides.



rain gauge. A device used to capture and measure rainfall.



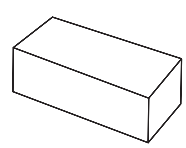
range. The difference between the highest and lowest numbers in a group of numbers or set of data. For example, in the data set 8, 32, 15, 10, the range is 24, that is, 32 – 8.

rate. A comparison, or a type of ratio, of two measurements with different units, such as distance and time; for example, 100 km/h, 10 kg/m^3, 20 L/100 km.

rate of change. A change in one quantity relative to the change in another quantity. For example, for a 10 km walk completed in 2 h at a steady pace, the rate of change is 10 km/2 h or 5 km/h.

rectangle. A quadrilateral in which opposite sides are equal, and all interior angles are right angles.

rectangular prism. A prism with opposite congruent rectangular faces.



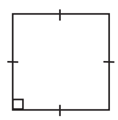
sample. A representative group chosen from a population and examined in order to make predictions about the population.

scale (on a graph). A sequence of numbers associated with marks that subdivide an axis. An appropriate scale is chosen to ensure that all data are represented on the graph.

secondary data. Information that is not collected first-hand; for example, data from a magazine, a newspaper, a government document, or a database. Also called *second-hand data* or *secondary-source data*. *See also* **primary data**.

spreadsheet. A tool that helps to organize information using rows and columns.

square. A rectangle with four equal sides and four right angles.



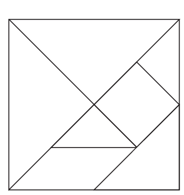
surface area. The total area of the surface of a three-dimensional object.

survey. A record of observations gathered from a sample of a population. For example, observations may be gathered and recorded by asking people questions or interviewing them.

table. An orderly arrangement of facts set out for easy reference; for example, an arrangement of numerical values in vertical columns and horizontal rows.

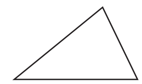
tally chart. A chart that uses tally marks to count data and record frequencies.

tangram. A Chinese puzzle made from a square cut into seven pieces: two large triangles, one medium-sized triangle, two small triangles, one square, and one parallelogram.

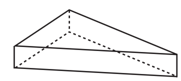


time line. A number line on which the numbers represent time values, such as numbers of days, months, or years.

triangle. A polygon with three sides.



triangular prism. A prism with opposite congruent triangular faces.



trundle wheel. A device for measuring distances that are too long for a measuring tape or that are not straight; made of a wheel that is rolled on the ground.

variable. A letter or symbol used to represent an unknown quantity, a changing value, or an unspecified number (e.g., $a \times b = b \times a$).

volume. The amount of space occupied by an object; measured in cubic units, such as cubic centimetres.

***x*-axis**. The horizontal number line on a coordinate plane.

***y*-axis**. The vertical number line on a coordinate plane.

Ministry of Education
Printed on recycled paper
ISBN 978-1-4249-5489-6
07-053