

# Mathematics

*Grade 7*



*Curriculum Guide*

*2013*

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## INTRODUCTION

### Background

*The curriculum guide communicates high expectations for students.*

The Mathematics curriculum guides for Newfoundland and Labrador have been derived from *The Common Curriculum Framework for K-9 Mathematics: Western and Northern Canadian Protocol*, May 2006. These guides incorporate the conceptual framework for Kindergarten to Grade 9 Mathematics and the general outcomes, specific outcomes and achievement indicators established in the common curriculum framework. They also include suggestions for teaching and learning, suggested assessment strategies, and an identification of the associated resource match between the curriculum and authorized, as well as recommended, resource materials.

This Grade 7 Mathematics course was originally implemented in 2008.

### Beliefs About Students and Mathematics

*Mathematical understanding is fostered when students build on their own experiences and prior knowledge.*

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in developing mathematical literacy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. Students at all levels benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students' experiences and ways of thinking, so that students feel comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must come to understand that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

## Affective Domain

*To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.*

A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting, assessing and revising personal goals.

## Goals For Students

*Mathematics education must prepare students to use mathematics confidently to solve problems.*

The main goals of mathematics education are to prepare students to:

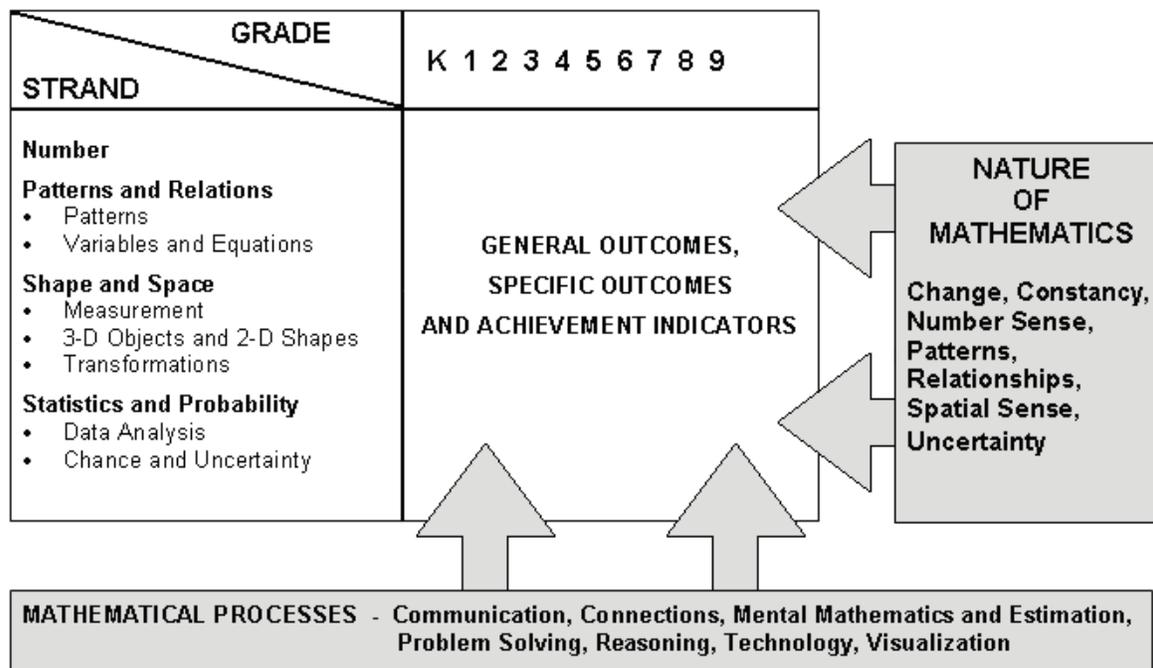
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

# CONCEPTUAL FRAMEWORK FOR K-9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



## Mathematical Processes

- *Communication [C]*
- *Connections [CN]*
- *Mental Mathematics and Estimation [ME]*
- *Problem Solving [PS]*
- *Reasoning [R]*
- *Technology [T]*
- *Visualization [V]*

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.

Students are expected to:

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and for solving problems
- develop visualization skills to assist in processing information, making connections and solving problems.

This curriculum guide incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

**Communication [C]**

*Students must be able to communicate mathematical ideas in a variety of ways and contexts.*

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

**Connections [CN]**

*Through connections, students begin to view mathematics as useful and relevant.*

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding ... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p.5).

## Mental Mathematics and Estimation [ME]

*Mental mathematics and estimation are fundamental components of number sense.*

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

“Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “... become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001, p. 442).

Mental mathematics “... provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers” (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

## Problem Solving [PS]

*Learning through problem solving should be the focus of mathematics at all grade levels.*

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you know?” or “How could you ...?”, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity requires students to determine a way to get from what is known to what is unknown. If students have already been given steps to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly seek and engage in a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

**Reasoning [R]**

*Mathematical reasoning helps students think logically and make sense of mathematics.*

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

**Technology [T]**

*Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.*

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- create geometric patterns
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

## Visualization [V]

*Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.*

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

## Nature of Mathematics

- *Change*
- *Constancy*
- *Number Sense*
- *Patterns*
- *Relationships*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this curriculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

## Change

*Change is an integral part of mathematics and the learning of mathematics.*

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

## Constancy

*Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.*

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS-Benchmarks, 1993, p.270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is  $180^\circ$ .
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

## Number Sense

*An intuition about number is the most important foundation of a numerate child.*

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p.146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by product of learning rather than through direct instruction. It can be developed by providing rich mathematical tasks that allow students to make connections to their own experiences and their previous learning.

## Patterns

*Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.*

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of mathematics.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics.

## Relationships

*Mathematics is used to describe and explain relationships.*

Mathematics is one way to describe interconnectedness in a holistic world view. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

## Spatial Sense

*Spatial sense offers a way to interpret and reflect on the physical environment.*

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

## Uncertainty

*Uncertainty is an inherent part of making predictions.*

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

## Essential Graduation Learnings

Essential graduation learnings are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression, citizenship, communication, personal development, problem solving, technological competence and spiritual and moral development.*

### Aesthetic Expression

*Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.*

### Citizenship

*Graduates will be able to assess social, cultural, economic and environmental interdependence in a local and global context.*

### Communication

*Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn and communicate effectively.*

### Personal Development

*Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.*

### Problem Solving

*Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts.*

### Technological Competence

*Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.*

## Spiritual and Moral Development

*Graduates will be able to demonstrate an understanding and appreciation for the place of belief systems in shaping the development of moral values and ethical conduct.*

See *Foundations for the Atlantic Canada Mathematics Curriculum*, pages 4-6.

The mathematics curriculum is designed to make a significant contribution towards students meeting each of the essential graduation learnings (EGLs), with the communication, problem-solving and technological competence EGLs relating particularly well to the mathematical processes.

## Strands

- *Number*
- *Patterns and Relations*
- *Shape and Space*
- *Statistics and Probability*

The learning outcomes in the program of studies are organized into four strands across the grades K–9. Some strands are subdivided into substrands. There is one general outcome per substrand across the grades K–9.

The strands and substrands, including the general outcome for each, follow.

### Number

#### *Number*

- Develop number sense.

### Patterns and Relations

#### *Patterns*

- Use patterns to describe the world and to solve problems.

#### *Variables and Equations*

- Represent algebraic expressions in multiple ways.

### Shape and Space

#### *Measurement*

- Use direct and indirect measurement to solve problems.

#### *3-D Objects and 2-D Shapes*

- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

#### *Transformations*

- Describe and analyze position and motion of objects and shapes.

### Statistics and Probability

#### *Data Analysis*

- Collect, display and analyze data to solve problems.

#### *Chance and Uncertainty*

- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

## Outcomes and Achievement Indicators

### *General Outcomes*

The curriculum is stated in terms of general outcomes, specific outcomes and achievement indicators.

**General outcomes** are overarching statements about what students are expected to learn in each course.

### *Specific Outcomes*

**Specific outcomes** are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given course.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

### *Achievement Indicators*

**Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

Specific curriculum outcomes represent the means by which students work toward accomplishing the general curriculum outcomes and ultimately, the essential graduation learnings.

## Summary

The conceptual framework for Kindergarten to Grade 9 Mathematics (p. 3) describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should result from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between topics.

# ASSESSMENT AND EVALUATION

## Purposes of Assessment

What learning is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others about what is really valued.

*Assessment* techniques are used to gather information for evaluation. Information gathered through assessment helps teachers determine students' strengths and needs in their achievement of mathematics and guides future instructional approaches.

Teachers are encouraged to be flexible in assessing the learning success of all students and to seek diverse ways in which students might demonstrate what they know and are able to do.

*Evaluation* involves the weighing of the assessment information against a standard in order to make an evaluation or judgment about student achievement.

Assessment has three interrelated purposes:

- assessment *for* learning to guide and inform instruction;
- assessment *as* learning to involve students in self-assessment and setting goals for their own learning; and
- assessment *of* learning to make judgements about student performance in relation to curriculum outcomes.

## Assessment *for* Learning

Assessment *for* learning involves frequent, interactive assessments designed to make student understanding visible. This enables teachers to identify learning needs and adjust teaching accordingly. It is an ongoing process of teaching and learning.

Assessment *for* learning:

- requires the collection of data from a range of assessments as investigative tools to find out as much as possible about what students know
- provides descriptive, specific and instructive feedback to students and parents regarding the next stage of learning
- actively engages students in their own learning as they assess themselves and understand how to improve performance.

**Assessment as Learning**

Assessment *as* learning actively involves students' reflection on their learning and monitoring of their own progress. It focuses on the role of the student as the critical connector between assessment and learning, thereby developing and supporting metacognition in students.

Assessment *as* learning:

- supports students in critically analysing their learning related to learning outcomes
- prompts students to consider how they can continue to improve their learning
- enables students to use information gathered to make adaptations to their learning processes and to develop new understandings.

**Assessment of Learning**

Assessment *of* learning involves strategies to confirm what students know, demonstrate whether or not they have met curriculum outcomes, or to certify proficiency and make decisions about students' future learning needs. Assessment *of* learning occurs at the end of a learning experience that contributes directly to reported results.

Traditionally, teachers relied on this type of assessment to make judgments about student performance by measuring learning after the fact and then reporting it to others. Used in conjunction with the other assessment processes previously outlined, however, assessment *of* learning is strengthened.

Assessment *of* learning:

- provides opportunities to report evidence to date of student achievement in relation to learning outcomes, to parents/guardians and other stakeholders
- confirms what students know and can do
- occurs at the end of a learning experience using a variety of tools.

Because the consequences of assessment *of* learning are often far-reaching, teachers have the responsibility of reporting student learning accurately and fairly, based on evidence obtained from a variety of contexts and applications.

## **Assessment Strategies**

Assessment techniques should match the style of learning and instruction employed. Several options are suggested in this curriculum guide from which teachers may choose, depending on the curriculum outcomes, the class and school/district policies.

### **Observation (formal or informal)**

This technique provides a way of gathering information fairly quickly while a lesson is in progress. When used formally, the student(s) would be aware of the observation and the criteria being assessed. Informally, it could be a frequent, but brief, check on a given criterion. Observation may offer information about the participation level of a student for a given task, use of a concrete model or application of a given process. The results may be recorded in the form of checklists, rating scales or brief written notes. It is important to plan in order that specific criteria are identified, suitable recording forms are ready, and all students are observed within a reasonable period of time.

### **Performance**

This curriculum encourages learning through active participation. Many of the curriculum outcomes promote skills and their applications. In order for students to appreciate the importance of skill development, it is important that assessment provide feedback on the various skills. These may be the correct manner in which to use a manipulative, the ability to interpret and follow instructions, or to research, organize and present information. Assessing performance is most often achieved through observing the process.

### **Paper and Pencil**

These techniques can be formative or summative. Whether as part of learning, or a final statement, students should know the expectations for the exercise and how it will be assessed. Written assignments and tests can be used to assess knowledge, understanding and application of concepts. They are less successful at assessing processes and attitudes. The purpose of the assessment should determine what form of paper and pencil exercise is used.

### **Journal**

Journals provide an opportunity for students to express thoughts and ideas in a reflective way. By recording feelings, perceptions of success, and responses to new concepts, a student may be helped to identify his or her most effective learning style. Knowing how to learn in an effective way is powerful information. Journal entries also give indicators of developing attitudes to mathematical concepts, processes and skills, and how these may be applied in the context of society. Self-assessment, through a journal, permits a student to consider strengths and weaknesses, attitudes, interests and new ideas. Developing patterns may help in career decisions and choices of further study.

**Interview**

This curriculum promotes understanding and applying mathematics concepts. Interviewing a student allows the teacher to confirm that learning has taken place beyond simple factual recall. Discussion allows a student to display an ability to use information and clarify understanding. Interviews may be a brief discussion between teacher and student or they may be more extensive. Such conferences allow students to be proactive in displaying understanding. It is helpful for students to know which criteria will be used to assess formal interviews. This assessment technique provides an opportunity to students whose verbal presentation skills are stronger than their written skills.

**Presentation**

The curriculum includes outcomes that require students to analyze and interpret information, to be able to work in teams, and to communicate information. These activities are best displayed and assessed through presentations. These can be given orally, in written/pictorial form, by project summary, or by using electronic systems such as video or computer software. Whatever the level of complexity, or format used, it is important to consider the curriculum outcomes as a guide to assessing the presentation. The outcomes indicate the process, concepts and context for which a presentation is made.

**Portfolio**

Portfolios offer another option for assessing student progress in meeting curriculum outcomes over a more extended period of time. This form of assessment allows the student to be central to the process. There are decisions about the portfolio, and its contents, which can be made by the student. What is placed in the portfolio, the criteria for selection, how the portfolio is used, how and where it is stored, and how it is evaluated are some of the questions to consider when planning to collect and display student work in this way. The portfolio should provide a long-term record of growth in learning and skills. This record of growth is important for individual reflection and self-assessment, but it is also important to share with others. For all students, it is exciting to review a portfolio and see the record of development over time.

# INSTRUCTIONAL FOCUS

## Planning for Instruction

Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There should be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

## Teaching Sequence

The curriculum guide for Grade 7 Mathematics is organized by units. This is only a suggested teaching order for the course. There are a number of combinations of sequences that would be appropriate.

Each two page spread lists the topic, general outcome, and specific outcome.

## Instructional Time Per Unit

The suggested number of weeks of instruction per unit is listed in the guide at the beginning of each unit. The number of suggested weeks includes time for completing assessment activities, reviewing and evaluating. The timelines at the beginning of each unit are provided to assist in planning. The use of these timelines is not mandatory. However, it is mandatory that all outcomes are taught during the school year, so a long term plan is advised. Teaching of the outcomes is ongoing, and may be revisited as necessary.

## Resources

The authorized resource for Newfoundland and Labrador students and teachers is *Math Makes Sense 7* (Pearson). Column four of the curriculum guide references *Math Makes Sense 7* for this reason. Teachers may use any other resource, or combination of resources, to meet the required specific outcomes.

## GENERAL AND SPECIFIC OUTCOMES

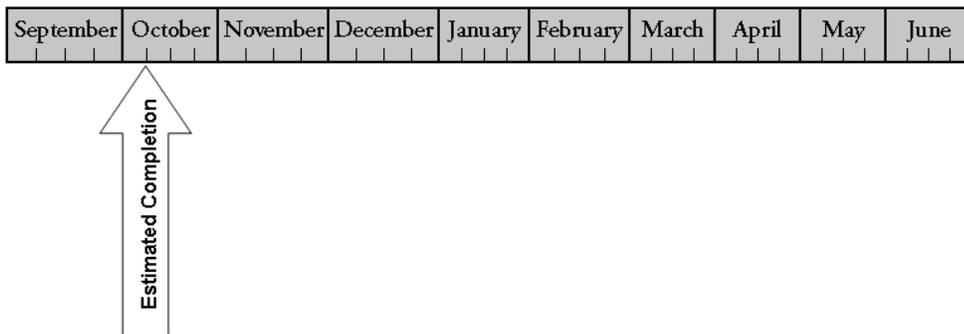
### GENERAL AND SPECIFIC OUTCOMES WITH ACHIEVEMENT INDICATORS (pages 19-170)

This section presents general and specific outcomes with corresponding achievement indicators and is organized by unit. The list of indicators contained in this section is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used to determine whether or not students have achieved a given specific outcome. Teachers may use any number of these indicators or choose to use other indicators as evidence that the desired learning has been achieved. Achievement indicators should also help teachers form a clear picture of the intent and scope of each specific outcome.

The curriculum is organized into eight units: *Patterns and Relations*, *Integers*, *Fractions*, *Decimals and Percents*, *Circles and Area*, *Operations with Fractions*, *Equations*, *Data Analysis* and *Geometry*.

# Patterns and Relations

Suggested Time: 4 Weeks



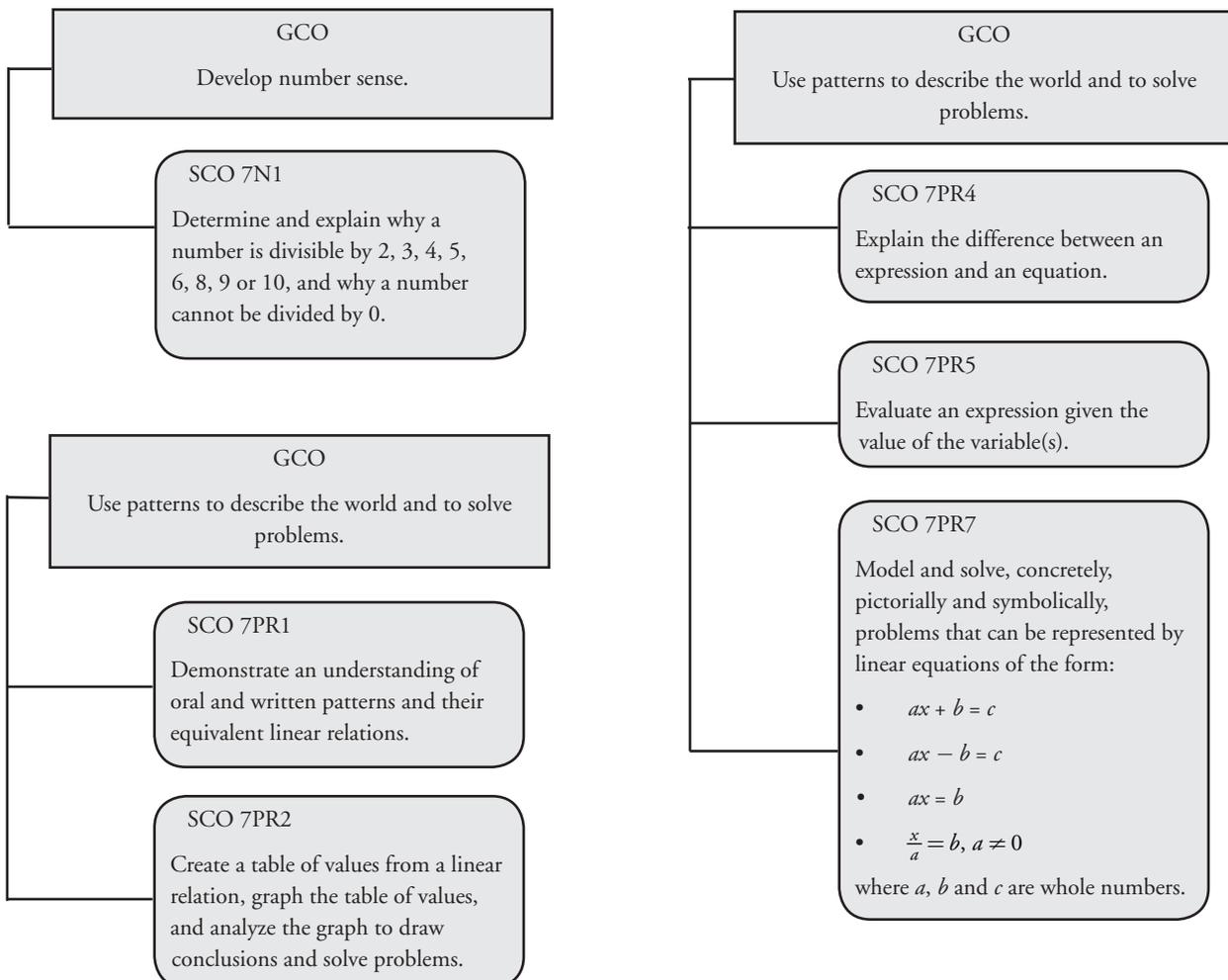
# Unit Overview

## Focus and Context

In this unit, students will explore a variety of situations involving patterns and change. They will continue to explore the rules of divisibility using number patterns.

Work will include both expressions and equations. Patterns will be represented by relations and these relations will be used to make predictions and solve problems. Relations will be represented symbolically, graphically, and in tabular form. Knowledge of expressions will then be extended to equations. Students will learn that the solution to an equation is the number that can be used to replace the variable and make the equation a true statement. To help find solutions to equations, students will model and solve equations using algebra tiles.

## Outcomes Framework



# Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

## SCO Continuum

Grade 6	Grade 7	Grade 8
<b>Number</b>		
<p>6N3. Demonstrate an understanding of factors and multiples by:</p> <ul style="list-style-type: none"> <li>determining multiples and factors of numbers less than 100</li> <li>identifying prime and composite numbers</li> <li>solving problems using multiples and factors.</li> </ul> <p>[CN, PS, R, V]</p>	<p>7N1. Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.</p> <p>[C, R]</p>	<p>not addressed</p>
<b>Patterns and Relations</b>		
<p>6PR1. Demonstrate an understanding of the relationships within the tables of values to solve problems.</p> <p>[C, CN, PS, R]</p> <p>6PR2. Represent and describe patterns and relationships, using graphs and tables.</p> <p>[C, CN, ME, PS, R, V]</p> <p>6PR3. Represent generalizations arising from number relationships, using equations with letter variables.</p> <p>[C, CN, PS, R, V]</p> <p>6PR5. Express a given problem as an equation in which a letter variable is used to represent an unknown number.</p> <p>[C, CN, PS, R]</p>	<p>7PR1. Demonstrate an understanding of oral and written patterns and their equivalent linear relations.</p> <p>[C, CN, R]</p> <p>7PR2. Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.</p> <p>[C, CN, PS, R, V]</p> <p>7PR4. Explain the difference between an expression and an equation.</p> <p>[C, CN]</p> <p>7PR5. Evaluate an expression, given the value of the variable(s). [CN, R]</p> <p>7PR7. Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form:</p> <ul style="list-style-type: none"> <li><math>ax + b = c</math></li> <li><math>ax - b = c</math></li> <li><math>ax = b</math></li> <li><math>\frac{x}{a} = b, a \neq 0</math></li> </ul> <p>where <math>a, b</math> and <math>c</math> are whole numbers.</p> <p>[CN, PS, R, V]</p>	<p>8PR1. Graph and analyze two-variable linear relations.</p> <p>[C, ME, PS, R, T, V]</p> <p>8PR2. Model and solve problems using linear equations of the form:</p> <ul style="list-style-type: none"> <li><math>ax = b</math></li> <li><math>\frac{x}{a} = b, a \neq 0</math></li> <li><math>ax + b = c</math></li> <li><math>\frac{x}{a} = b, a \neq 0</math></li> <li><math>a(x + b) = c</math></li> </ul> <p>concretely, pictorially and symbolically, where <math>a, b</math> and <math>c</math> are integers.</p> <p>[C, CN, PS, V]</p>

**Strand: Number****Outcomes**

*Students will be expected to*

**7N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0.**

[C, R]

**Achievement Indicator:**

7N1.1 *Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10 and explain why.*

**Elaborations—Strategies for Learning and Teaching**

In earlier grades, students built their understanding of sequences and number patterns. These patterns are used to develop division of larger numbers. It is assumed that students can:

- recognize number patterns in tables
- extend a table of values using a pattern
- describe the relationships among terms in a table

Exploration of the divisibility rules serves as an excellent opportunity to extend number sense.

When introducing divisibility, students may use number charts and/or a calculator to determine if the quotient is a whole number. Students should explore divisibility in such a manner that they arrive at the divisibility rules themselves. Knowledge of divisibility rules will be a very valuable tool for mental arithmetic as well as areas such as fractions and algebra. There are a number of tests for determining if a number is a multiple of certain factors. Most are straightforward. Divisibility rules for 2, 5 and 10 involve simple number patterns. Through patterning, students should quickly realize that an even number is divisible by 2, a number is divisible by 5 if the ones digit is 0 or 5, and a number is divisible by 10 if the ones digit is 0. They should also explore divisibility by 3 and 9. A number is divisible by 3 if the sum of the digits is divisible by 3. Similarly, if the sum of the digits of a number is divisible by 9, the number is also divisible by 9. Divisibility by 6 is possible if the number is divisible by 2 and by 3. A number is divisible by 4 if the number represented by the last 2 digits is divisible by 4 or divisible by 2 at least twice.

Students sometimes struggle with divisibility by 8. You may use rules such as the following for divisibility by 8.

- Ask students to check for divisibility by 4. Then divide the original number by 4. If the quotient is even, the number is also divisible by 8. If the quotient is odd, the number is not divisible by 8. For example,  $92 \div 4 = 23$ , so 92 is not divisible by 8;  $392 \div 4 = 98$ , so 392 is divisible by 8.
- Ask students to check for divisibility by 4, then find the quotient of the last two digits and 4. If the hundreds digit is even and the quotient of the last two digits and 4 is even, then the number is divisible by 8. If the hundreds value is odd and the quotient of the last two digits and 4 is odd, then the number is divisible by 8. Consider  $392 \div 4$ . The hundreds digit (3) is odd, and  $92 \div 4 = 23$  is odd. Therefore 392 is divisible by 8. On the other hand, when dividing 292 by 4, the hundreds digit (2) is even, but  $92 \div 4 = 23$  is odd. In this case, 292 is not divisible by 8.
- A number is divisible by 8 if it is divisible by 2 at least 3 times.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to complete the number by filling in each blank with a digit. Ask them to explain, using divisibility rules, how they know their answers are correct.
    - (i) 26\_ is divisible by 10
    - (ii) 154\_ is divisible by 2
    - (iii) \_6\_ is divisible by 6
    - (iv) 26\_ is divisible by 3
    - (v) 1\_2 is divisible by 9
    - (vi) 15\_ is divisible by 4
- (7N1.1)
- There will be 138 people at a party. Ask students to determine if the host can fill tables of 5. Tables of 6? They should support their answer by using divisibility rules.
- (7N1.1)

**Resources/Notes****Authorized Resource***Math Makes Sense 7*

Prep Talk Video: Patterns and Relations

**Lesson 1.1: Patterns in Division****Lesson 1.2: More Patterns in Division**

ProGuide: pp. 4-7, 8-11

Master 1.10, 1.12, 1.22, 1.13, 1.23

CD-ROM: Unit 1 Masters

Prep Talk Video: Patterns in Division

Student Book (SB): pp. 6-9, 10-13

Practice and HW Book: pp. 4-8

**Web Link**

Students can play a divisibility rule matching game. Refer to Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>).

**Strand: Number**

**Outcomes**

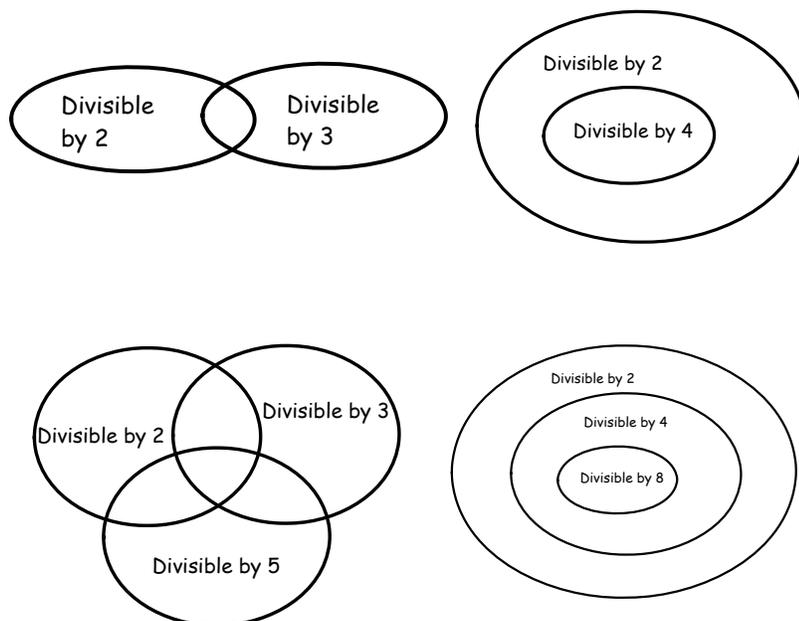
*Students will be expected to*  
7N1 Continued ...

**Achievement Indicators:**

7N1.2 *Sort a given set of numbers based upon their divisibility, using organizers such as Venn and Carroll diagrams.*

**Elaborations – Strategies for Learning and Teaching**

Venn diagrams are effective tools for sorting by multiple attributes because they make it easy to see when there are items that cross-classify. Students should be introduced to Venn diagrams with two loops before using a Venn diagram with three loops.



Carroll diagrams should only be used to compare numbers using two divisors. They work much like Venn diagrams. For example, ask students to sort the following numbers into the Carroll diagram given:

15, 82, 75, 23, 39, 90

	Divisible by 2	Not Divisible by 2
Divisible by 5	90	15, 75
Not Divisible by 5	82	23, 39

7N1.3 *Determine the factors of a given number, using the divisibility rules.*

Students should mentally determine whether 2, 3, 4, 5, 6, 8, 9 and 10 are factors of a given number. Then they can divide by those which are factors to determine other factors. There may also be other factors that students will identify which are not determined by the divisibility rules.

## General Outcome: Develop number sense.

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Students should choose three numbers that are divisible by both 6 and 9. Ask them to find the smallest number, other than 1, by which the chosen numbers are divisible. Ask students to share their answers with the class and discuss.

(7N1.1, 7N1.3)

- Each of Eli's four friends has a code number. Keile's number is divisible by 3, 5, and 8. Max's number is divisible by 2 and 3. Jennifer's number is divisible by 4 and 5, but not 3. Ben's number is divisible by 3 and 5, but not 8. Eli receives a message with the code number 5384 from one of his four friends. Ask students to determine who sent the message.

(7N1.1, 7N1.3)

- Ask students to create a Carroll Diagram or a Venn Diagram to sort the following numbers:
  - Divisibility rules for 3 and 5: 6, 8, 10, 15, 18, 25, 26, 36, 40, 45, 120
  - Divisibility rules for 6 and 9: 30, 79, 162, 3996; 23 517, 31 974

(7N1.2)

- Ask students to determine if each statement is true or false and identify an example that supports their answer.
  - All numbers divisible by 6 are divisible by 3.
  - Some, but not all, numbers divisible by 6 are divisible by 3.
  - No numbers divisible by 6 are divisible by 3.
  - All numbers divisible by 3 are divisible by 6.
  - Some, but not all, numbers divisible by 3 are divisible by 6.
  - No numbers divisible by 3 are divisible by 6.

(7N1.1, 7N1.2)

#### *Interview*

- The principal of Great School has to determine the number of classes of Grade 7 students in her school. Ask students to discuss the divisibility rules that can be used to determine the possible number of classes there could be if there are 240 Grade 7 students and all classes have an equal number of students.

(7N1.3)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 7*

##### Lesson 1.1: Patterns in Division

##### Lesson 1.2: More Patterns in Division

ProGuide: pp. 4-7, 8-11

Master 1.10, 1.12, 1.22, 1.13, 1.23

CD-ROM: Unit 1 Masters

Prep Talk Video: Patterns in Division

SB: pp. 6-9, 10-13

Practice and HW Book: pp. 4-8

**Strand: Number****Outcomes**

*Students will be expected to*

7N1 Continued ...

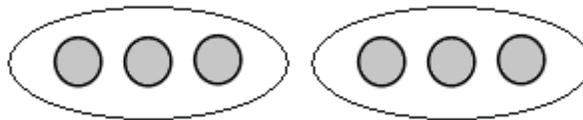
**Achievement Indicator:**

7N1.4 *Explain, using an example, why numbers cannot be divided by 0.*

**Elaborations—Strategies for Learning and Teaching**

Using the “repeated subtraction” meaning of division will help students understand why numbers cannot be divided by 0. Given  $20 \div 5$ , for example, students should understand that 5 can be subtracted from 20 four times ( $20 - 5 - 5 - 5 - 5 = 0$ ). Given  $6 \div 0$ , students should determine that no matter how many times they subtract 0, they will still have 6. Since there is no answer,  $6 \div 0$  is undefined.

Dividing by 0 can also be visualized using counters. For example, given  $6 \div 3$ , how many groups of 3 can be made from 6? Students separate 6 counters into 2 groups of 3.



Given  $6 \div 0$ , students will see that it is not possible to separate the 6 counters into groups of 0.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Interview*

- Ask students to explain why it is not possible to calculate  $12 \div 0$ .  
(7N1.4)
- Ask students to complete the following table and explain how the table shows division by 0 is not possible.

Division Statement	Related Multiplication Statement
$6 \div 2 = 3$	$3 \times 2 = 6$
$10 \div 5 = 2$	$2 \times 5 = \underline{\quad}$
$14 \div 2 = \underline{\quad}$	$2 \times 7 = 14$
$15 \div \underline{\quad} = 5$	$3 \times 5 = 15$
$\underline{\quad} \div 8 = 3$	$3 \times 8 = \underline{\quad}$
$12 \div 0 = \underline{\quad}$	$0 \times \underline{\quad} = 12$

(7N1.4)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 1.2: More Patterns in Division**

ProGuide: pp. 8-11

Master 1.13, 1.23

CD-ROM: Unit 1 Masters

SB: pp. 10-13

Practice and HW Book: pp. 6-8

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

*Students will be expected to*

**7PR4 Explain the difference between an expression and an equation.**

[C, CN]

#### Achievement Indicators:

7PR4.1 *Explain what a variable is and how it is used in a given expression.*

7PR4.2 *Identify and provide an example of a constant term, numerical coefficient and variable in an expression and an equation.*

7PR4.3 *Provide an example of an expression and an equation, and explain how they are similar and different.*

### Elaborations—Strategies for Learning and Teaching

Many students find the concept of a variable difficult to grasp. Using examples with real-life situations should help with this. For example, if Tyler earns \$15 for each soccer games he referees, a variable,  $g$ , can represent the number of soccer games he referees in a given time frame. Likewise,  $s$ ,  $n$ ,  $x$  or any other variable could be used. It is important that students understand what the variable represents. Variables are used to represent an unknown quantity (e.g.,  $x + 3 = 9$ , where  $x$  has a single value) or a quantity that can change (e.g.,  $4s$ , where  $s$  can be any value). Students often make inaccurate assumptions that different variables must have different numerical values. In  $7m + 2 = 23$  and  $7k + 2 = 23$ , both  $m$  and  $k$  have a value of 3.

When introducing the concepts of expressions and equations, real-life situations that are relevant to the students should be used. In the example *Mary earns \$8 each hour that she babysits*, the variable  $h$  can represent the number of hours Mary babysits.  $8h$  is an expression representing how much money she makes on any given babysitting job. An equation for this situation could be  $e = 8h$  ( $e$  = earnings,  $h$  = hours). In algebra, multiplication using a variable is often represented without a multiplication symbol. This is a new and sometimes confusing concept for students.

Discussion should lead to these definitions: An algebraic *expression* is a mathematical phrase that contains numbers and/or variables. An algebraic *equation* is a mathematical statement where two expressions are equal and contains at least one variable.

In the expression  $\frac{1}{2}k + 6$ ,  $k$  is the variable, 6 is the constant term, and  $\frac{1}{2}$  is the numerical coefficient. In the equation  $2a + 5 = 11$ , the variable is  $a$ , the numerical coefficient is 2, and the constant terms are 5 and 11.

Students should be exposed to expressions with numerical coefficients of 1, as well as with fractional coefficients. They sometimes have difficulty identifying the numerical coefficient in expressions such as  $x + 5$ . It may be beneficial for students to rewrite an equation like  $\frac{k}{2} + 6 = 10$  as  $\frac{1k}{2} + 6 = 10$  so that they clearly see the numerical coefficient is  $\frac{1}{2}$ .

Many students have a limited understanding of the meaning of the equal sign, and believe its purpose is to indicate an answer. They should understand that the equal sign is a symbol of equivalence and balance, and that equality is a relationship, not an operation (NCTM, 2000-2007).

Students will first work with expressions before moving to equations later in the unit.

**General Outcome: Represent algebraic expressions in multiple ways.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Ask students to write an algebraic expression that has a variable  $b$ , numerical coefficient 4 and constant term 11. (7PR4.2)
- Ask students to create a classroom chart with the following headings:

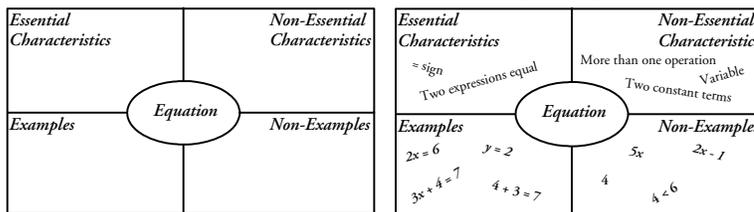
Algebraic Expression	Expression in Words	Variable	Numerical Coefficient	Constant
$3b + 1$	one more than 3 times a number	$b$	3	1
$y + 6$				

Equation	Equation in Words	Variable	Numerical Coefficient	Constant
$3b + 1 = 7$	one more than 3 times a number is 7	$b$	3	1 & 7

(7PR4.2, 7PR4.3)

- Ask students the following: Which are expressions? Which are equations? How are they similar? How are they different?
  - $2 - x$
  - $5v = 20$
  - $\frac{b}{3} = 4$
  - $w + 7$
- Ask students to complete the Frayer Model concept maps for expressions and equations. (7PR4.2, 7PR4.3)

*Sample Responses*



Once the diagram is complete, students could share their ideas with others, modifying their diagrams as necessary to incorporate new information.

(7PR4.3)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

Lesson 1.3: Algebraic Expressions

Lesson 1.7: Reading and Writing Equations

ProGuide: pp. 14-17, 33-35

Master 1.14, 1.24, 1.18, 1.28

CD-ROM: Unit 1 Masters

SB: pp. 16-19, 35-37

Practice and HW Book: pp. 9-11, 20-21

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

*Students will be expected to*

7PR4 Continued ...

#### Achievement Indicator:

7PR4.4 *Represent a given oral or written pattern using an algebraic expression.*

7PR5 Evaluate an expression, given the value of the variable(s).  
[CN, R]

#### Achievement Indicator:

7PR5.1 *Substitute a value for an unknown in a given expression, and evaluate the expression.*

### Elaborations—Strategies for Learning and Teaching

Using the concepts variable, constant term and numerical coefficient will help students determine the algebraic expression that represents a given situation. Students should be presented with situations and asked to represent the pattern with an expression. For example:

- Chris worked a certain number of hours yesterday and 8 more hours today. The unknown is the number of hours he worked yesterday, which will be replaced with a variable. The constant term is 8, the number of hours he worked today. The expression for this situation is  $h + 8$ .
- Loretta earned \$5 for each hour she worked. The variable represents the number of hours she worked. In this case the numerical coefficient is 5, given that you would multiply the number of hours by 5 in order to find out how much money she made. ( $5h$  is the expression.)
- Dennis has \$20 in his pocket. He earns \$6 for each hour he works. The variable represents the number of hours he works. The numerical coefficient is 6, and the constant term is 20. ( $6h + 20$  is the expression.)

Students applied the order of operations, excluding exponents and limited to whole numbers, in Grade 6 (6N9). A review is necessary prior to teaching this outcome.

To evaluate an algebraic expression, students substitute a number for the variable and carry out the computation. Using real-life situations relevant to students will help with this.

Draw students' attention to expressions such as  $4h + 8$ , where multiplication is used. If  $h = 5$ , for example, a common student error is writing  $45 + 8$  instead of  $4(5) + 8$  or  $4 \times 5 + 8$ .

Students should also be aware that division is often represented as a fraction, such as  $8 - \frac{m}{2}$  instead of  $8 - m \div 2$ .

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**General Outcome: Represent algebraic expressions in multiple ways.**


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**Suggested Assessment Strategies**
*Observation*

- Create cards with algebraic expressions and their equivalent word forms. Each student receives a card with either the expression or the word form. They find their partner in the class. Each group must then explain why their cards match.

(7PR4.4)

*Interview*

- Ask students to explain the steps used to evaluate the following expressions for the given value of the variable:

(i)  $3p + 5$ , for  $p = 1$

(ii)  $\frac{m}{2} - 3$ , for  $m = 6$

(7PR5.1)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*
**Lesson 1.3: Algebraic Expressions**

ProGuide: pp. 14-17

Master 1.14, 1.24

CD-ROM: Unit 1 Masters

SB: pp. 16-19

Practice and HW Book: pp. 9-11

**Lesson 1.3: Algebraic Expressions**
**Lesson 1.4: Relationships in Patterns**

ProGuide: pp. 14-17, 18-22

Master 1.14, 1.24, 1.15, 1.25

CD-ROM: Unit 1 Masters

SB: pp. 16-19, 20-24

Practice and HW Book: pp. 9-13

**Strand: Patterns and Relations (Patterns)**

**Outcomes**

*Students will be expected to*

**7PR1 Demonstrate an understanding of oral and written patterns and their equivalent linear relations.**

[C, CN, R]

**7PR2 Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.**

[C, CN, PS, R, V]

**Achievement Indicators:**

7PR1.1 *Formulate a linear relation to represent the relationship in a given oral or written pattern.*

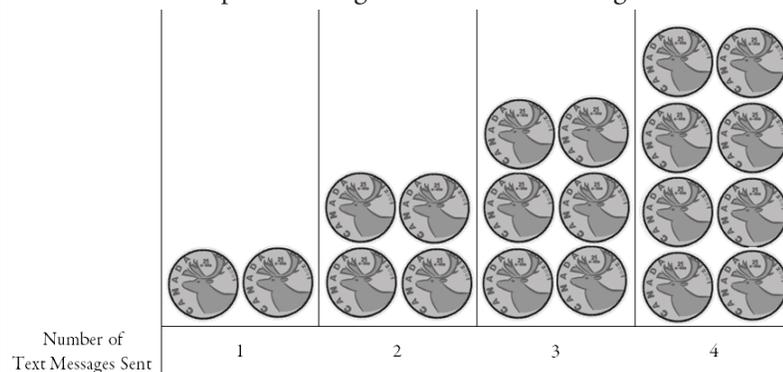
7PR2.1 *Create a table of values for a given linear relation by substituting values for the variable.*

7PR2.2 *Create a table of values, using a linear relation, and graph the table of values (limited to discrete elements).*

**Elaborations—Strategies for Learning and Teaching**

Student investigation of linear relations should start with concrete models, followed by oral and written descriptions. Where possible, the use of manipulatives will foster a better understanding of the concept. In Grade 6, students represented patterns and relationships using graphs and tables (6PR2). This continues here as they create a table of values from a linear relation and sketch the graph. The word linear may cause confusion but should become clear as students graph the table of values.

Consider the example involving the cost of text messages below:



The relationship between the number of text messages sent and the number of quarters can be represented in a table:

Number of text messages ( $t$ )	1	2	3	4	5
Number of quarters ( $q$ )	2	4	6	8	10

Students are encouraged to find a relationship between one quantity (variable) and the other. In this case, the number of quarters is equal to twice the number of text messages. They should then write the expression  $2t$  to represent this relation. A natural extension of this is expressing the relation as an equation,  $q = 2t$ . Students should substitute each of the values of  $t$  into the equation to verify that it represents the relationship in the table.

Students will often look at patterns such as the above and notice the amount of increase. For example, the number of quarters increases by 2 each time. This is true, but will not be very useful in finding the number of quarters for, say, the 52<sup>nd</sup> text message. The value 2 is important in the equation, but students must understand how it affects the equation.

**General Outcome: Use patterns to describe the world and to solve problems.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Ask students to continue the pattern and complete the chart to show pattern growth. Then ask them to describe the relationship between the variables, write a linear relation and graph the table of values.

(i) Price of Flowers in a Vase



Number of flowers ( $f$ )	1	2	3	4	5	6	7
Price in dollars ( $d$ )							

(ii) Number of tips on Moose Antlers



Age of moose in years ( $a$ )	1	2	3	4	5	6	7
Number of tips on antlers ( $t$ )							

(7PR1.1, 7PR2.2)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

Lesson 1.3: Algebraic Expressions

Lesson 1.4: Relationships in Patterns

Lesson 1.5: Patterns and Relationships in Tables

Lesson 1.6: Graphing Relations

ProGuide: pp. 14-17, 18-22, 23-26, 28-32

Master 1.14, 1.24, 1.15, 1.25, 1.11, 1.16, 1.26, 1.17, 1.27

CD-ROM: Unit 1 Masters

SB: pp. 16-19, 20-24, 25-28, 30-34

Practice and HW Book: pp. 9-19

**Strand: Patterns and Relations (Patterns)**

**Outcomes**

Students will be expected to  
7PR1, 7PR2 Continued ...

**Achievement Indicators:**

7PR1.1, 7PR2.1, 7PR2.2  
*Continued*

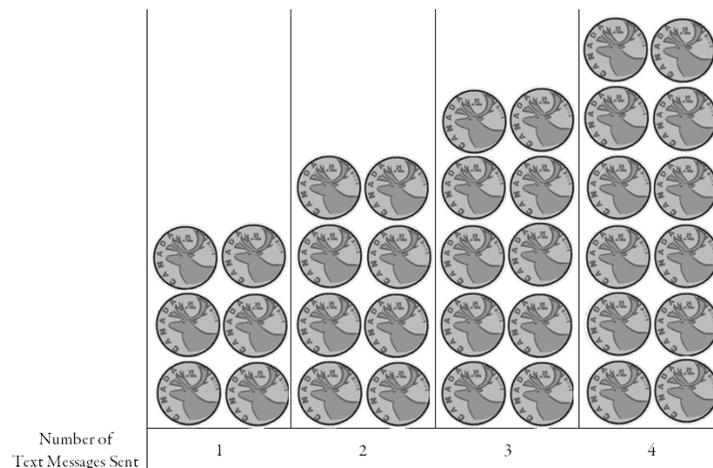
7PR2.3 *Sketch the graph from a table of values created for a given linear relation, and describe the patterns found in the graph to draw conclusions; e.g., graph the relationship between  $n$  and  $2n+3$ .*

7PR2.4 *Describe, using everyday language in spoken or written form, the relationship shown on a graph to solve problems.*

**Elaborations—Strategies for Learning and Teaching**

Students should then sketch the graph from a table of values using only the first quadrant of the Cartesian plane, label the axes and give the graph a title. They will tend to draw a line to connect the points. When using discrete data, however, it does not make sense to draw a line since you cannot deal with parts of one or both variables. For example, it is not possible to have 1.25 text messages. The graph can be used for interpolating and extrapolating data. This terminology is not the focus here. Students should be able to describe the general pattern of the graph (e.g., it goes upward to the right with the points in a straight line).

Students should investigate more complex patterns by examining modifications of simpler patterns. For example:



By adding four quarters to each diagram, they can create a new table.

Number of text messages ( $t$ )	1	2	3	4	5
Number of quarters ( $q$ )	6	8	10	12	14

Students should express the linear relationship in words (the number of quarters is equal to 2 times the number of text messages plus four) and as an expression ( $2t + 4$ ). It can also be expressed as an equation ( $q = 2t + 4$ ).

They should then move to creating a table of values for a given linear relation and then graph the table of values. Teachers may provide contextual examples, such as the amount of cutlery used for place settings is represented by  $c = 5p$  ( $c$  = amount of cutlery,  $p$  = number of place settings).

**General Outcome: Use patterns to describe the world and to solve problems.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Ask students to study the tables below, describe the relationship between the variables and write a linear relation and then graph the relation and describe the graph.

Cost of renting a scooter

Number of hours ( $h$ )	1	2	3	4	5
Cost ( $c$ )	50	70	90	110	130

Songs on an iPod

Number of Pop songs ( $p$ )	1	2	3	4	5
Number of Rock songs ( $r$ )	3	6	9	12	15

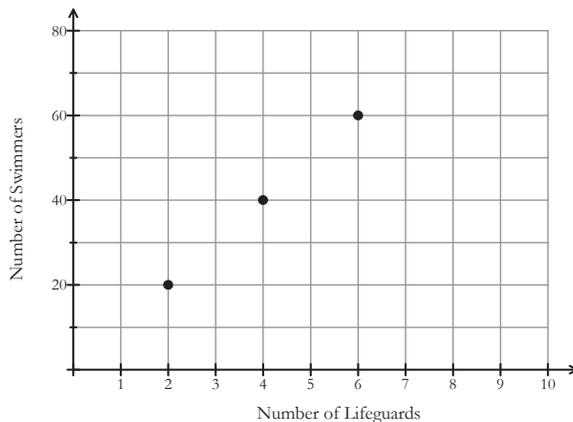
(7PR1.1, 7PR2.3, 7PR2.4)

- A taxi charges a base fare of \$4, plus \$1 for every kilometre traveled. This can be represented by the linear relation  $c = k + 4$  ( $c$  = cost,  $k$  = number of km). Ask students to make a table of values showing the total cost for the first 5 km. They can graph the table of values and describe the pattern. Ask: How much would a 10 km taxi ride cost?

(7PR2.1, 7PR2.2, 7PR2.3, 7PR2.4)

- Using the graph, students can answer questions such as the following:

Lifeguards needed for Swimmers



- How many swimmers would be allowed for 10 lifeguards?
- How many lifeguards would be needed for 50 swimmers?
- Describe the pattern in words.
- Write a relation for the number of swimmers for  $n$  lifeguards.

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

Lesson 1.3: Algebraic Expressions

Lesson 1.4: Relationships in Patterns

Lesson 1.5: Patterns and Relationships in Tables

Lesson 1.6: Graphing Relations

ProGuide: pp. 14-17, 18-22, 23-26, 28-32

Master 1.14, 1.24, 1.15, 1.25, 1.11, 1.16, 1.26, 1.17, 1.27

CD-ROM: Unit 1 Masters

SB: pp. 16-19, 20-24, 25-28, 30-34

Practice and HW Book: pp. 9-19

**Strand: Patterns and Relations (Patterns)**

**Outcomes**

*Students will be expected to*  
7PR1, 7PR2 Continued ...

**Achievement Indicators:**

7PR1.2 *Provide a context for a given linear relation that represents a pattern.*

7PR1.3 *Represent a pattern in the environment, using a linear relation.*

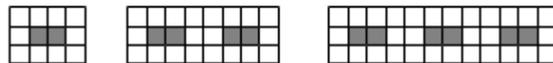
7PR2.5 *Match a given set of linear relations to a given set of graphs.*

7PR2.6 *Match a given set of graphs to a given set of linear relations.*

**Elaborations—Strategies for Learning and Teaching**

Prior to asking students to provide contexts to represent linear relations, teachers could give examples. The expression  $10b + 2$ , for example, could represent the amount of money a person makes if he/she is paid \$10 per hour plus a \$2 bonus.

Students could investigate a number of patterns that may be expressed using linear relations, such as the black-and-white tile pattern for kitchen flooring, as shown below. Students should be able to construct a table of values showing the number of black tiles and the number of white tiles in the first 5 designs, describe the pattern and write a linear relation.



Number of black tiles ( $b$ )	2	4	6	8	10
Number of white tiles ( $w$ )	10	20	30	40	50

The linear relation  $w = 5b$  describes the relationship between the two quantities.

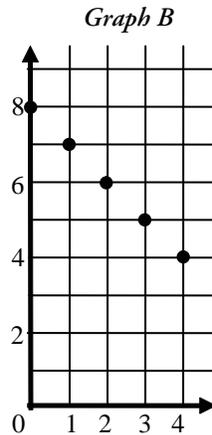
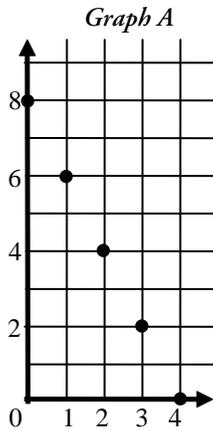
Students should be presented with graphs and relations (in word form and as algebraic equations). When matching a relation with its graph, students should be able to explain the reasoning for their choice.

**General Outcome: Use patterns to describe the world and to solve problems.**

**Suggested Assessment Strategies**

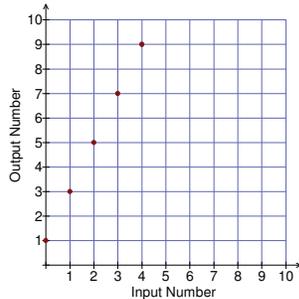
*Paper and Pencil*

- Faith says that Graph A shows  $y = 8 - 2x$  and Graph B shows  $y = 8 - x$ . Ask students to determine if she is correct. They should explain their reasoning.



(7PR2.5, 7PR2.6)

- Ask students to determine which relations can be matched with the graph. They should explain their reasoning.



- (i)  $y = 2x + 1$
- (ii)  $y = x + 2$
- (iii) The output number is equal to double the input number increased by 1.
- (iv) The output number is equal to double the input number decreased by 1.

(7PR2.5)

*Interview*

- Ask students to describe a real-life situation that could be represented by  $3p + 4$ .

(7PR1.2)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

Lesson 1.4: Relationships in Patterns

Lesson 1.6: Graphing Relations

ProGuide: pp. 18-22, 28-32

Master 1.15, 1.25, 1.17, 1.27

CD-ROM: Unit 1 Masters

SB: pp. 20-24, 30-34

Practice and HW Book: pp. 12-13, 17-19

**Strand: Patterns and Relations (Variables and Equations)****Outcomes**

*Students will be expected to*

7PR4 Continued ...

**Achievement Indicator:**

7PR4.5 *Represent a given oral or written pattern using an equation.*

**Elaborations—Strategies for Learning and Teaching**

Writing and solving algebraic equations is explored in great detail in the Equations unit, including the use of algebra tiles to represent positive and negative numbers. Teachers may decide to focus on Achievement Indicators 7PR4.5 and 7PR7.1 in conjunction with that unit.

Students translate verbal statements into equations as they have done with expressions throughout the work on linear relations. When writing the equation for *three more than a number is 8* students will first choose a variable (e.g.,  $n$ ) to represent a number. *Three more than a number* would be written as  $n + 3$ . “Is” represents the equality ( $=$ ). The equation, therefore, is  $n + 3 = 8$ .

It is unwise to only use a key word approach to writing equations. Students should be encouraged to read verbal statements for meaning. A common error occurs with statements such as *five less than a number is 12*, which is often mistakenly written as  $5 - n = 12$ , but is actually  $n - 5 = 12$ . To help students make sense of the statement, numerical examples, such as *5 less than 8 is 3* which is written as  $8 - 5 = 3$ , could be used.

**General Outcome: Represent algebraic expressions in multiple ways.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to complete a table such as the following:

Words	Unknown Quantity	Choose a Variable	Equation
Twice Jenny's age increased by 3 is 15.	Jenny's age	$j$	$2j + 3 = 15$
One more than 3 times a number is 10.	A number	$n$	
Six less than Mark's age is 10.			
			$4x - 1 = 19$

(7PR4.5)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 1.7: Reading and Writing Equations**

ProGuide: pp. 33-35

Master 1.18, 1.28

CD-ROM: Unit 1 Masters

SB: pp. 35-37

Practice and HW Book: pp. 20-21

**Strand: Patterns and Relations (Variables and Equations)**

**Outcomes**

*Students will be expected to*

**7PR7 Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form:**

- $ax + b = c$
- $ax - b = c$
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$

where  $a, b$  and  $c$  are whole numbers.

[CN, PS, R, V]

**Achievement Indicator:**

*7PR7.1 Model a given problem with a linear equation and solve the equation, using concrete models, e.g., counters, integer tiles.*

**Elaborations—Strategies for Learning and Teaching**

This unit focuses on solving equations concretely and pictorially, using whole numbers only. Solving linear equations symbolically will be addressed in the Equations unit. Equations of the form  $\frac{x}{a} = b, a \neq 0$  are not addressed in this unit, as the emphasis here is on the concrete and pictorial representation.

In Grade 5, students solved single-variable, one-step equations with whole number coefficients and whole number solutions (5PR2). This is now extended to include two-step equations.

Equations can be thought of as linear relations where one of the variables is known, and the goal is to find the numeric value of the unknown variable. Using a table of values to link the concepts will be useful. For example,

$n$	1	2	3	4	?
$2n + 1$	3	5	7	9	201

Students could perform a guess-and-check routine:

$2(10)+1 = 21$  (An input of 10 is not large enough.)

$2(50)+1 = 101$  (An input of 50 is not large enough.)

$2(100)+1 = 201$  (Therefore, 100 is the correct input value.)

This approach is valuable for students to explore, especially when verifying their solution to an equation. It is covered in greater detail in the Equations unit.

The use of concrete models is essential in developing students’ comprehension of solving equations. Students can draw pictures of their models and explain how they were used to solve the equation. This will help as they move from the concrete stage to the pictorial stage. It is also important that students verify the solutions to equations using their models.

Algebra tiles or other sets of counters are useful manipulatives for modeling algebraic equations. For this section, it is important to use tiles or counters of one colour, since only “positives” will be used. For the purposes of this curriculum guide, shaded tiles represent “positive” and unshaded tiles represent “negative”.

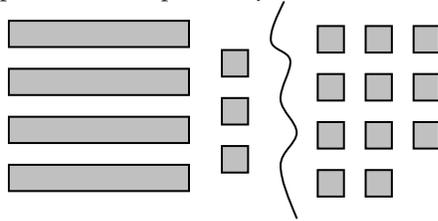
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**General Outcome: Represent algebraic expressions in multiple ways.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- The algebra tile diagram represents an equation. Ask students to identify the two expressions that make up the equation, then write the equation. Students should then solve the equation, drawing pictures to represent the steps as they work.



(7PR7.1)

- Ask students to use tiles to solve each equation and draw pictures to represent each step.

(i)  $7 + x = 10$

(ii)  $4x = 16$

(7PR7.1)

- Given the statement *three more than twice a number is 19*, ask students to write an equation that can be solved to find the number, use algebra tiles to solve, and verify the solution.

(7PR7.1)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*
**Lesson 1.8: Solving Equations Using Algebra Tiles**

ProGuide: pp. 36-40

Master 1.19, 1.29

PM 30

CD-ROM: Unit 1 Masters

Prep Talk Video: Solving Equations Using Algebra Tiles

Classroom Video: Solving Equations Using Algebra Tiles, Part 1-3

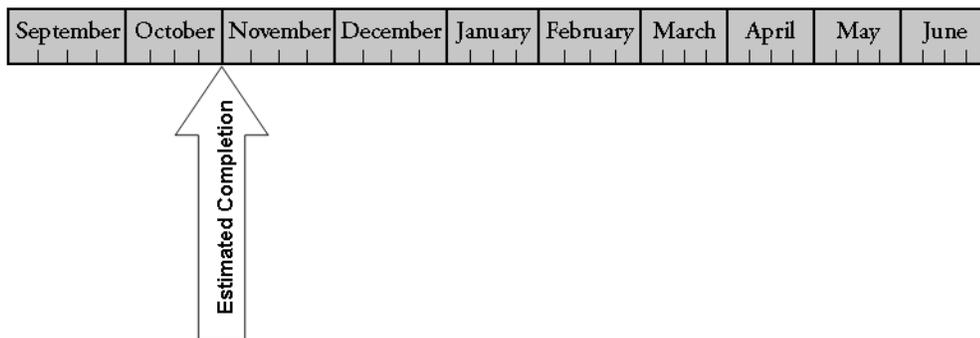
SB: pp. 38-42

Practice and HW Book: pp. 22-24



# Integers

Suggested Time: 3 Weeks



# Unit Overview

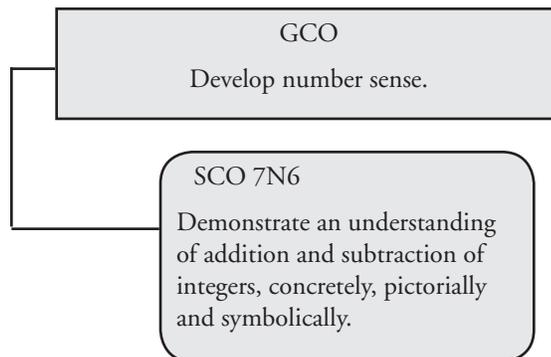
## Focus and Context

In this unit, students will add and subtract integers concretely, pictorially and symbolically. They will build upon prior experience with positive and negative numbers to represent real-life situations. Using models, such as counters and number lines, to add and subtract integers should enhance understanding of integers as they pertain to size and direction. Work with zero pairs will be important as students concretely add and subtract integers.

Students will progress to adding and subtracting integers symbolically. They will generalize and apply rules for these integer operations.

Proficiency with integers is crucial to future work with algebra. It is necessary when evaluating algebraic expressions and solving equations. It allows students to graph relations using all four quadrants. Work with integers will be applied to future study of rational expressions, and extended to irrational and real numbers. It continues to build number sense, preparing students for a wide range of problem solving activities.

## Outcomes Framework



Mathematical Processes

- [C] Communication
- [CN] Connections
- [ME] Mental Mathematics and Estimation
- [PS] Problem Solving
- [R] Reasoning
- [T] Technology
- [V] Visualization

SCO Continuum

Grade 6	Grade 7	Grade 8
Number		
<p>6N7. Demonstrate an understanding of integers, concretely, pictorially and symbolically.</p> <p>[C, CN, R, V]</p>	<p>7N6. Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.</p> <p>[C, CN, PS, R, V]</p>	<p>8N7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.</p> <p>[C, CN, PS, R, V]</p>

**Strand: Number**

**Outcomes**

*Students will be expected to*

**7N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically.**

[C, CN, PS, R, V]

**Achievement Indicators:**

7N6.1 *Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero.*

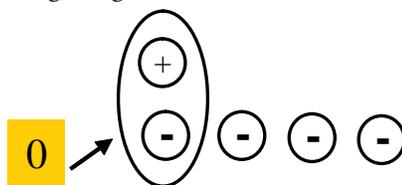
7N6.2 *Solve a given problem involving the addition and subtraction of integers.*

**Elaborations – Strategies for Learning and Teaching**

Students are introduced to addition and subtraction of integers in this unit. The set of integers includes zero and each natural number and its opposite. Operations with integers build on operations with whole numbers. It is assumed that students can add and subtract whole numbers. Students have also had previous experience with positive and negative numbers. In Grade 6, they placed and ordered integers using a number line, and described contexts in which integers are used (6N7). Multiplication and division of integers will be introduced in Grade 8 (8N7).

Mathematicians have defined  $(-1)$  as the number you add to  $(+1)$  to result in 0. The zero principle,  $(-1) + (+1) = 0$ , is the foundation for many computations involving integers.

A tile or counter representing  $(+1)$  and one representing  $(-1)$  form a zero pair. When combined, these tiles model the number zero. A given integer can be modelled in many different ways. For example, one way to represent  $-3$  using integer tiles is shown below.



Exploration of the zero principle using integer tiles or counters should lead students to conclude that adding a zero pair does not change the value of the integer being modelled.

Students should be given opportunities to make connections between integers and the world around them through exposure to contextualized problems (e.g., distance above or below sea level, temperature, bank deposits and withdrawals). Developing a good understanding of integers will enable students to represent real-life situations involving size and direction. Integers are required to describe rates of change, and are used in situations involving time, position, elevation, temperature, energy, and financial contexts such as net worth, balance sheets, and profit or loss.

This achievement indicator will be repeatedly addressed throughout this unit, and aspects of it will be developed under each of the other achievement indicators.

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**General Outcome: Develop number sense.**


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**Suggested Assessment Strategies**
*Observation*

- Create cards with opposite integers written on them. Distribute these cards to students. Ask them to locate their opposite and sit in zero pairs.

(7N6.1)

*Performance*

- Using integer tiles, students demonstrate three ways to model an integer such as  $-7$ ,  $8$ ,  $-2$ , etc. They should share their models with the class.

(7N6.1)

- Ask students to model as many integers as they can using exactly nine integer counters.

(7N6.1)

*Paper and Pencil*

- Present the following problem to students: You earn \$5 and then spend \$5. How much is your profit or loss? Draw diagrams with integer counters to represent this problem.

(7N6.1)

*Journal*

- Ask students if they can model  $+2$  with an odd number of counters. They should explain their reasoning.

(7N6.1)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*

Prep Talk Video: Integers

**Lesson 2.1: Representing Integers**

ProGuide: pp. 4-7

Master 2.10, 2.18

CD-ROM: Unit 2 Masters

Prep Talk Video: Representing Integers

Student Book (SB): pp. 52-55

Practice and HW Book: pp. 32-33

**Lessons 2.1-2.5**

**Strand: Number**

**Outcomes**

*Students will be expected to*  
**7N6 Continued ...**

**Achievement Indicators:**

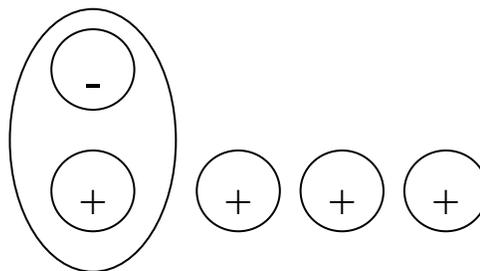
*7N6.3 Add two given integers, using concrete materials or pictorial representations, and record the process symbolically.*

*7N6.4 Illustrate, using a number line, the results of adding negative and positive integers.*

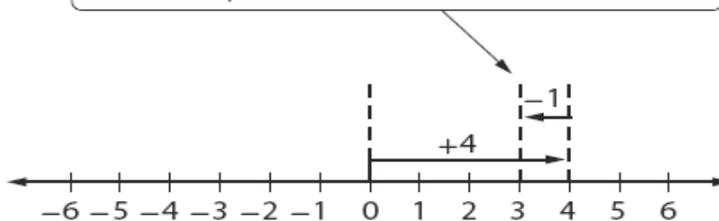
**Elaborations – Strategies for Learning and Teaching**

Two models commonly used for representing integers are coloured counters and number lines. Students should be exposed to both models. Parallel development, using both models at the same time, may be the most conceptual approach. Integers involve two concepts, “quantity” and “opposite”. Quantity is modeled by the number of counters or length of the arrows. Opposite is represented as different colours or different directions. An example of adding integers using each model is provided.

$$(+4) + (-1) = +3$$



Start at 0. Move 4 units to the right to show +4. From there, move 1 unit to the left to show -1.



The use of models should lead to a more conceptual understanding of the principles for adding integers.

1. The sum of two positives is positive.
2. The sum of two negatives is negative.
3. The sum of a negative and a positive can be negative or positive. The sum has the sign of the number that is further from zero.

Students should eventually move away from models as they practice integer arithmetic. However, it is important that they not view the procedural rules as arbitrary. While the correct answer is important, emphasis should be on the rationale and not on how quickly they can get answers.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Journal*

- Ask students if the sum of a negative number and a positive number is always negative. They should explain their reasoning.  
(7N6.3, 7N6.4)

*Performance*

- Arrange students in teams. Begin with all students standing. Write an addition expression on the board and have students write the answer. Students with correct answers remain standing. All others sit. After three questions, the team with the most people standing receives 10 points. One member of the team is selected to attempt a fun activity (e.g., shooting a foam basketball into a net) for an additional 5 points for the team. Then a new round of play begins. The team that reaches 100 points first wins.  
(7N6.3)

*Paper and Pencil*

- Ask students to write an addition equation to describe each situation, and explain its meaning.
  - Before you went to sleep last night the temperature was  $-3^{\circ}\text{C}$ . During the night the temperature dropped by  $5^{\circ}\text{C}$ . What was the temperature in the morning?
  - Mrs. Brown parked in the parking garage 10 m below street level. She then got on an elevator and went up 27 m to her office. How far above the street is her office?  
(7N6.2, 7N6.3)
- Ask students to find three pairs of integers with a sum of  $-29$ .  
(7N6.3)
- Ask students to find integer pairs with a sum of  $-16$  where:
  - one number is less than  $-16$ .
  - one number is greater than  $+16$ .
  - one number is greater than 0 and less than  $+5$ .  
(7N6.3)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 2.2: Adding Integers with Tiles****Lesson 2.3: Adding Integers on a Number Line**

ProGuide: pp. 8-11, 12-16

Master 2.11, 2.12, 2.15, 2.19, 2.20

CD-ROM: Unit 2 Masters

## Prep Talk Videos:

- Adding Integers with Tiles
- Adding Integers on a Number Line

SB: pp. 56-59, 60-64

Practice and HW Book: pp. 34-35, 36-38

**Web Links**

The following games are found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>):

- Integer War
- Connect Four: *Adding Small Integers*
- Connect Four: *Adding Big Integers*
- Magic Circles

**Strand: Number**

**Outcomes**

*Students will be expected to*  
**7N6 Continued ...**

**Achievement Indicators:**

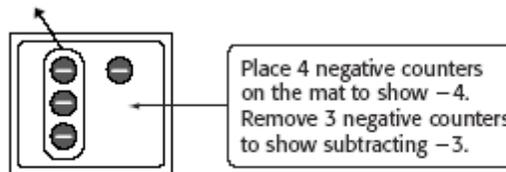
*7N6.5 Subtract two given integers, using concrete materials or pictorial representations, and record the process symbolically.*

*7N6.6 Illustrate, using a number line, the results of subtracting negative and positive integers.*

**Elaborations – Strategies for Learning and Teaching**

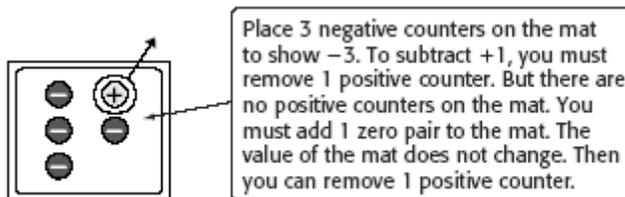
As with addition, it is important for students to have a conceptual understanding of integer subtraction. Students should model subtraction of integers using coloured counters and number lines.

One possibility when subtracting integers is to use a “take away” meaning. This is easily manageable when the integers have the same sign, and the subtrahend is closer to zero than the minuend, or starting amount. For example,  $(-4) - (-3)$  can be modelled with counters as follows.



So,  $-4 - (-3) = -1$ .

When taking away is not immediately possible, it is necessary to add zero pairs prior to removing tiles. A model of  $(-3) - (+1)$  is shown here.



When 1 is taken away from  $-3$ , the result is  $-4$ .

So,  $(-3) - (+1) = -4$ .

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to write a subtraction equation for each situation.
  - (i) A soccer ball was kicked 5 m in the forward direction on the first play of the game. The opposing team then kicked it back 6 m. What was the total change in distance?
  - (ii) If the average temperature in Gander is  $19^{\circ}\text{C}$  in July and  $-6^{\circ}\text{C}$  in January, what is the difference between the average highest and lowest temperatures?
  - (iii) Archie and Jughead were digging in the sand at the beach. Archie dug a hole that was 22 cm below the surface, and Jughead dug a hole that was 13 cm below the surface. What is the difference in the depths of the holes?  
(7N6.2, 7N6.5)
- Ask students to create and solve their own problems using real-life situations such as time zones, temperature, heights above and below sea level, profit/loss, etc.  
(7N6.2, 7N6.5)
- Students find charts in the daily newspaper showing the low and high temperatures for the day for cities around the world. They use information from the charts to make up two problems involving subtraction of positive and negative numbers. They find the solutions and then exchange problems with other students and solve.  
(7N6.2, 7N6.5)
- Ask students to answer the following:  
Tim has a debt of \$55. He earns \$30 on each of two days and spends \$39 on a pair of pants. How much money or debt does he now have? Include a diagram and a number sentence.  
(7N6.2, 7N6.5)

*Journal*

- Ask students to respond to the following:
  - (i) Is the difference between a negative number and a positive number always negative? Explain your reasoning.  
(7N6.5)
  - (ii) How do you subtract integers using tiles? Explain and give an example.  
(7N6.5)
  - (iii) Without actually calculating the difference between two integers, how do you know whether the answer will be positive, negative or zero? Explain with the aid of examples.  
(7N6.5)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 2.4: Subtracting Integers with Tiles****Lesson 2.5: Subtracting Integers on a Number Line**

ProGuide: pp. 18-22, 23-27

Master 2.13, 2.14, 2.15, 2.21, 2.22

CD-ROM: Unit 2 Masters

Prep Talk Video: Subtracting Integers with Tiles

Prep Talk Video: Subtracting Integers on a Number Line

SB: pp. 66-70, 71-75

Practice and HW Book: pp. 39-40, 41-43

**Note**

The questions in *Math Makes Sense 7* emphasize integers between -10 and 10. Teachers should take students beyond this range to explore their understanding.

**Strand: Number**

**Outcomes**

*Students will be expected to*

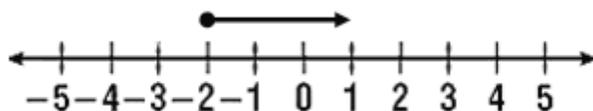
7N6 Continued ...

**Achievement Indicators:**

7N6.5, 7N6.6 *Continued*

**Elaborations—Strategies for Learning and Teaching**

To subtract integers, you can also use a “think addition” meaning. To determine  $(+1) - (-2)$ , ask “How much must be added to  $(-2)$  to get to  $(+1)$ ?” Using a number line, begin at  $(-2)$  and draw an arrow to  $(+1)$ . It has a length of 3 pointing right.  $\therefore (+1) - (-2) = +3$ .



Using the rule “To subtract an integer, add the opposite” allows students to reach the correct answer. However, the rule may not be understood by all students. Students should be led to this rule through the use of models. A good example would involve using the number line to subtract a negative from a positive. Such a situation should make it easier for students to see why you can add the opposite to subtract. From the previous example,  $(+1) - (-2)$  tells what to add to  $(-2)$  to get to  $(+1)$ . To go from  $(-2)$  to  $(+1)$ , move 2 to the right to get to 0, and then another 1 to the right to get to  $(+1)$ . The total amount to be added is  $(+2) + (+1)$  or, since addition is commutative,  $(+1) + (+2)$ . Students should now see that  $(+1) - (-2) = (+1) + (+2)$ .

Patterning can be used to develop this as well. Ask students to study a pattern such as the one given here.

$(+4) - (+2) = 2$	$(+4) + (-2) = 2$
$(+4) - (+1) = 3$	$(+4) + (-1) = 3$
$(+4) - (0) = 4$	$(+4) + (0) = 4$
$(+4) - (-1) = 5$	$(+4) + (+1) = 5$
$(+4) - (-2) = 6$	$(+4) + (+2) = 6$

As they compare each column, they should conclude that subtracting results in the same answer as adding the opposite.

Students should be aware that while addition of integers is commutative, subtraction is not. In fact, if the order of the integers in the subtraction statement changes, the differences are opposite integers.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Performance*

- Teachers can use masking tape (or other materials) to make a large number line on the floor. Students can discuss how a number line can be used for subtraction, as well as for addition of integers. Students can walk on the number line to show subtractions such as  $(+7) - (-3)$  or  $(-4) - (-2)$ . (7N6.6)

*Paper and Pencil*

- Ask students the following:  
When you add two negative integers, you always get a negative sum. When you subtract two negative integers, do you always get a negative difference? Explain with the aid of examples. (7N6.5)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 2.4: Subtracting Integers with Tiles****Lesson 2.5: Subtracting Integers on a Number Line**

ProGuide: pp. 18-22, 23-27

Master 2.13, 2.14, 2.15, 2.21, 2.22

CD-ROM: Unit 2 Masters

Prep Talk Videos:

- Subtracting Integers with Tiles
- Subtracting Integers on a Number Line

SB: pp. 66-70, 71-75

Practice and HW Book: pp. 39-40, 41-43

**Web Links**

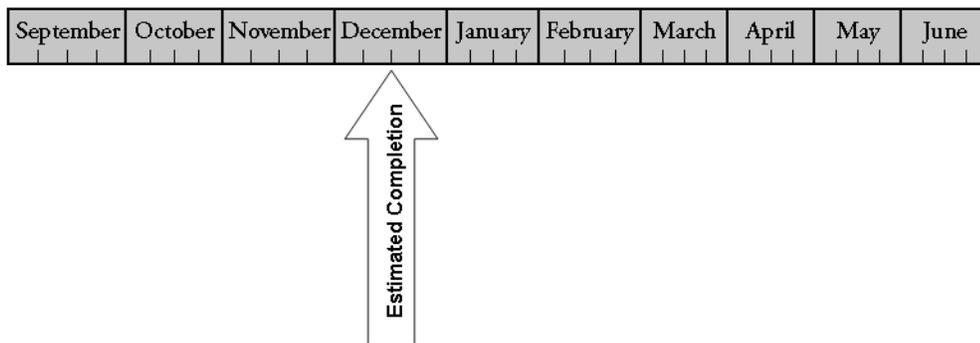
The following activities can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>):

- Line Jumper
- National Library of Virtual Manipulatives
- Exploring Integers and Temperature



# Fractions, Decimals, and Percents

Suggested Time: 5 Weeks



# Unit Overview

## Focus and Context

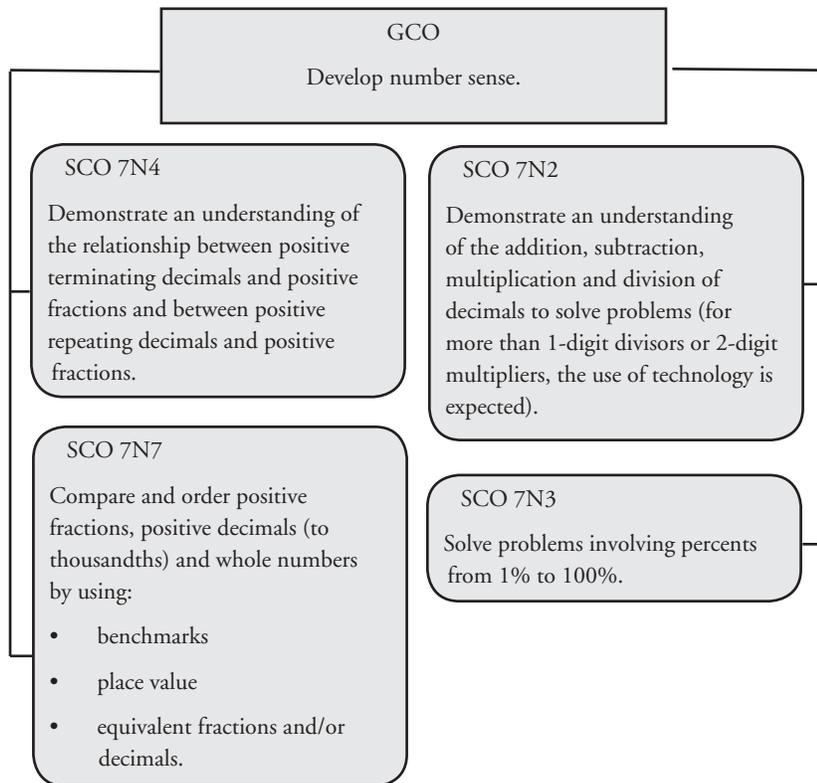
The focus of this unit is fractions, decimals and percents. The emphasis is on understanding the relationships between these three alternate forms.

Patterns will be useful once again in conversions between fractions and decimals. Tools used to compare and order fractions will include benchmarks, number lines, place values, equivalent fractions, and manipulatives such as fraction pieces and fraction strips.

When working with fractions, decimals and percents, the use of manipulatives, technology, and paper and pencil will be encouraged. Estimating continues to be important as students develop number sense. A “sense” of whether or not an answer is correct will be critical to problem solving. Operations with decimals will be subject to the order of operations. Exponents will not be introduced until later grades.

During this unit, students will examine the connections between fractions, decimals and percents and express them in all three forms. Percent problems will be limited to values between 1% and 100%.

## Outcomes Framework



## Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

## SCO Continuum

Grade 6	Grade 7	Grade 8
Number		
6N1. Demonstrate an understanding of place value, including numbers that are: <ul style="list-style-type: none"> <li>greater than one million</li> <li>less than one thousandth.</li> </ul> [C, CN, R, T]	7N4. Demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals and positive fractions.  7N7. Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: <ul style="list-style-type: none"> <li>benchmarks</li> <li>place value</li> <li>equivalent fractions and/or decimals.</li> </ul> [CN, R, V]	8N1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).  [C, CN, R, V]
6N2. Solve problems involving whole numbers and decimal numbers.  [ME, PS, T]	7N2. Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected).  [ME, PS, T]	8N2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).  [C, CN, ME, R, T]
6N5. Demonstrate an understanding of ratio, concretely, pictorially and symbolically.  [C, CN, PS, R, V]	7N3. Solve problems involving percents from 1% to 100%.  [C, CN, PS, R, T]	8N3. Demonstrate an understanding of percents greater than or equal to 0%.  [CN, PS, R, V]
6N6. Demonstrate an understanding of percent (limited to whole numbers), concretely, pictorially and symbolically.  [C, CN, PS, R, V]		8N4. Demonstrate an understanding of ratio and rate.  [C, CN, V]
6N8. Demonstrate an understanding of multiplication and division of decimals (1-digit whole number multipliers and 1-digit natural number divisors).  [C, CN, ME, PS, R, V]		8N5. Solve problems that involve rates, ratios and proportional reasoning.  [C, CN, PS, R]
6N9. Explain and apply the order of operations, excluding exponents, with and without technology (limited to whole numbers).  [C, CN, ME, PS, T]		

**Strand: Number**

**Outcomes**

*Students will be expected to*

**7N4 Demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals and positive fractions.**

[C, CN, R, T]

**Achievement Indicator:**

7N4.1 *Predict the decimal representation of a given fraction, using patterns; e.g.,  $\frac{1}{11} = 0.\overline{09}$ ,  $\frac{2}{11} = 0.18$ ,  $\frac{3}{11} = ?$ , ... (It is intended that repeating decimals be limited to decimals with 1 or 2 repeating digits).*

**Elaborations – Strategies for Learning and Teaching**

Decimals and proper fractions can both be represented using the part of a whole model. All fractions can be expressed as decimals and vice versa, including terminating (e.g.,  $\frac{1}{2} = 0.5$ ) or repeating decimals (e.g.,  $\frac{1}{3} = 0.\overline{3}$ ). Students may already know the decimal equivalents of some simple fractions (e.g.,  $\frac{1}{2} = 0.5$ ,  $\frac{1}{4} = 0.25$ ,  $\frac{1}{5} = 0.2$ ), as well as any fraction with a denominator of 10, 100, or 1000. Knowing common fraction-decimal relationships can help students interpret decimals meaningfully. For example, they see 0.23 and realize that it is almost  $\frac{1}{4}$ .

It is important that students become proficient at correctly reading a decimal number. If 0.37 is read as thirty-seven hundredths, the conversion to  $\frac{37}{100}$  is easily made. Students often read 0.37 as “decimal three seven” or “point three seven”, which does not provide context or frame of reference and should be avoided. Reinforce the importance of placing zero in front of the decimal to emphasize it is less than 1. Students should be introduced to the terminology *terminating*, *repeating*, and *period*, as well as the bar notation used to indicate repeating periods.

Students should be encouraged to use mental calculation and prior knowledge where possible. For example, the fraction  $\frac{4}{25}$  can easily be changed to a decimal by first finding the equivalent fraction with a denominator of 100. Using calculators is encouraged when necessary to find the decimal form for some fractions before predicting the decimal for other fractions. Students should investigate the difference in finding the decimal equivalents for sevenths and eighths:

On a calculator we find: $\frac{1}{7} = 0.\overline{142857}$ $\frac{2}{7} = 0.\overline{285714}$ $\frac{3}{7} = 0.\overline{428571}$ Although there is a pattern here, it is not easily observable.	Using a pattern we find: $\frac{1}{8} = 0.125$ $\frac{2}{8} = 0.250$ Therefore: $\frac{3}{8} = ?$ (0.375)
---	---

It is important to make students aware of the effect of calculator rounding caused by the limited number of digits which the calculator can display.

Students are expected to find the decimal representation of a set of fractions such as  $\frac{1}{9}$ ,  $\frac{2}{9}$ ,  $\frac{3}{9}$ , find a pattern and then use the pattern to predict the decimal for other fractions such as  $\frac{4}{9}$ ,  $\frac{5}{9}$ ,  $\frac{10}{9}$ . Students’ attention should be drawn to fractions in such patterns that will result in a whole number (e.g.,  $\frac{9}{9} = 1$ , not  $0.\overline{9}$ ). Decimal representations of sets of fractions such as  $\frac{1}{12}$  and  $\frac{1}{120}$  should also be explored. These patterns can be used to predict the decimal representation of other similar sets.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to determine the pattern for the following:

$$\frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = 0.\overline{27}, \frac{4}{11} = 0.\overline{36}$$

Ask them to:

- predict the decimals for  $\frac{5}{11}$  and  $\frac{9}{11}$ .
- predict the fraction which will have 0.636363... as a decimal.
- predict what the decimal for  $\frac{8}{11}$  would look like on a calculator display if the calculator is set to display 8 digits after the decimal.
- predict the fraction for which 0.909090... is the decimal form.

(7N4.1)

*Journal*

- Ask students to explain how knowing that  $\frac{1}{5} = 0.2$  helps in finding the decimal form of  $\frac{3}{5}$  and  $\frac{6}{5}$ .

(7N4.1)

**Resources/Notes****Authorized Resource***Math Makes Sense 7*

Prep Talk Video: Fractions, Decimals, and Percents

**Lesson 3.1: Fractions to Decimals**

ProGuide: pp. 4-8

Master 3.11, 3.21

CD-ROM: Unit 3 Masters

Student Book (SB): pp. 86-90

Practice and HW Book: pp. 50-51

**Note**

Some of the questions in *Math Makes Sense 7* (pp. 88-90) go beyond 2 repeating digits. This can be explored in class activities, but should not be formally assessed.

**Strand: Number****Outcomes**

*Students will be expected to*

7N4 Continued ...

**Achievement Indicators:**

7N4.2 *Match a given set of fractions to their decimal representations.*

7N4.3 *Sort a given set of fractions as repeating or terminating decimals.*

7N4.4 *Express a given fraction as a terminating or repeating decimal.*

7N4.5 *Express a given terminating decimal as a fraction.*

7N4.6 *Express a given repeating decimal as a fraction.*

7N4.7 *Provide an example where the decimal representation of a fraction is an approximation of its exact value.*

**Elaborations—Strategies for Learning and Teaching**

Students are already familiar with a variety of fractions and their decimal representations. Reviewing fractions such as  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{3}{4}$ ,  $\frac{1}{5}$ ,  $\frac{2}{5}$  and their decimal equivalents will provide a foundation for discussion for terminating decimals. Calculators may be used to explore both terminating and repeating decimals. When exploring repeating decimals such as  $\frac{1}{3} = 0.333333333\dots$  and  $\frac{2}{11} = 0.1818181818\dots$ , students should look for the period. This should lead into a discussion of how to write repeating decimals with the bar notation.

Students could then be provided with a set of fractions and asked to determine whether the decimal equivalents are terminating or repeating, and re-write repeating decimals using the bar notation. A graphic organizer, such as a T-Chart, may be useful in helping students sort fractions.

Terminating decimals can easily be expressed as decimal numbers by using equivalent fractions with denominators of 10, 100, 1000. Students are expected to reduce fractions to simplest form, which they have been exposed to in previous grades.

Expressing repeating decimals as fractions is more challenging since denominators of 10, 100, 1000 cannot be used. Repeating decimals can be changed into fractions using denominators of 9, 99, 999, etc., depending on the number of decimal places in the period. Student understanding of this should evolve through discussions of familiar examples, such as  $0.\overline{3}$ . Students know it is equivalent to  $\frac{3}{9}$ , not  $\frac{3}{10}$ . Ask students which denominator could be used for the numerator 3, since the 3 is in the decimal form. Students should easily identify  $\frac{3}{9}$ . In the example  $0.\overline{7}$ , the 7 is in the tenths place, but tenths cannot be used since it is not exactly seven tenths. In this case ninths would be used, giving the fraction  $\frac{7}{9}$ . In the example  $0.\overline{18}$ , hundredths cannot be used since it is not exactly 18 hundredths, so 99 is used as the denominator, resulting in the fraction  $\frac{18}{99}$ , which can be reduced to  $\frac{2}{11}$ .

Students should realize that fractions such as  $\frac{1}{6} = 0.\overline{16}$  are exact values whereas a calculator display that shows 0.16666667 is an approximation. When students round such values to 0.17 or 0.2, for example, it is important that they recognize that these are approximations, not exact values. Discussion may include real-life situations for which it might make sense to use approximations, such as the distance between towns, the amount of gas in a dirt-bike, mental calculation of discount amounts, etc.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Observation*

- Create cards with fractions and their decimal equivalents. Each student receives a card with either a decimal or a fraction. They circulate around the room to find the card which is equivalent to their own. Each group must explain why their cards belong together. (7N4.2)

*Paper and Pencil*

- Ask students to answer the following questions.
  - (i) Does the fraction  $\frac{7}{15}$  produce a repeating decimal? (7N4.3, 7N4.4)
  - (ii) Chris had a calculator which displayed 2.3737374. Chris concluded that it was not a repeating decimal. Explain why Chris made this conclusion and whether or not it is a correct conclusion. (7N4.4)
  - (iii) About 0.4 of a math class will be going on a field trip. Write the decimal in words, and as a fraction in simplest form. (7N4.5)
  - (iv) Of all life on Earth,  $0.\overline{72}$  live below the ocean's surface. Write this as a fraction in simplest form. (7N4.6)
- The following numbers appear on three calculator screens. Ask students to match the correct displays to the correct fractions. Encourage them to use their knowledge of repeating decimals and estimation.

0.5555556

$$\frac{2}{7}$$

0.28571429

$$\frac{5}{9}$$

0.30769231

$$\frac{4}{13}$$

(7N4.2)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.1: Fractions to Decimals**

ProGuide: pp. 4-8

Master 3.11, 3.21

CD-ROM: Unit 3 Masters

SB: pp. 86-90

Practice and HW Book: pp. 50-51

**Web Link**

Cards for matching fractions and their decimal equivalents can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>).

**Strand: Number****Outcomes**

*Students will be expected to*

7N7 Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using:

- benchmarks
- place value
- equivalent fractions and/or decimals.

[CN, R, V]

**Achievement Indicators:**

7N7.1 *Order the numbers of a given set that includes positive fractions, positive decimals and/or whole numbers in ascending or descending order, and verify the result using a variety of strategies.*

7N7.2 *Position fractions with like and unlike denominators from a given set on a number line, and explain strategies used to determine order.*

7N7.3 *Order the numbers of a given set by placing them on a number line that contains benchmarks, such as 0 and 1 or 0 and 5.*

7N7.4 *Position a given set of positive fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position.*

**Elaborations—Strategies for Learning and Teaching**

Students should develop a variety of strategies to compare fractions.

- Use benchmarks such as  $\frac{1}{2}$ ,  $\frac{1}{4}$  and  $\frac{3}{4}$ , and their decimal equivalents.
- Use common denominators. If both fractions have the same denominator, the larger numerator represents the larger fraction (e.g.,  $\frac{5}{8} > \frac{3}{8}$ ). If denominators differ, students will write equivalent fractions with like denominators and then compare the numerators.
- Use common numerators. If both fractions have the same numerator, the fraction with the smallest denominator is larger (e.g.,  $\frac{2}{7} > \frac{2}{9}$ ).
- Convert all fractions to decimals and then compare using place value.
- Model fractions and/or decimals using manipulatives such as base-10 blocks, fraction pieces, pattern blocks, etc.
- Place the fractions and/or decimals on a number line with benchmarks.

Students should practice the above strategies for use when ordering fractions and decimals. Students can change fractions greater than one, such as  $\frac{10}{8}$  or  $\frac{7}{5}$ , to mixed fractions if they choose. Repeated addition can be used as a strategy to write mixed numbers. Recognizing what makes a whole,  $\frac{10}{8}$  can be rewritten as  $\frac{8}{8} + \frac{2}{8}$ .

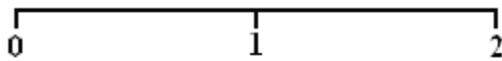
**General Outcome: Develop number sense.****Suggested Assessment Strategies***Observation*

- Create a set of cards with a variety of fractions and decimals. Each student gets five cards and must lay them out in the order they receive them. Students take turns trading one of the cards for a new one from the pack. They place the new card in whichever location best helps get the cards in order. The object is to be the first to get their cards in order. The pack should have sufficient cards to allow the game to run smoothly: for groups of 3, there should be at least 30 cards per group, for groups of 4, at least 40 cards per group. (7N7.1)
- Create a human number line. Each student is given a card with a fraction or decimal number. Have students order themselves into a line based on the relative size of their number. Ask students why they chose their position. (Alternate version: Use a skipping rope as the number line. Students attach their number to the line in an appropriate position.) (7N7.1, 7N7.2, 7N7.3, 7N7.4)

*Paper and Pencil*

- Give students 5 decimal numbers that have friendly fraction equivalents. The numbers should fall between two consecutive whole numbers. (Example: 3.5, 3.125, 3.4, 3.75, and 3.66 are between 3 and 4.) Provide a number line with the same two whole numbers, using subdivisions that are only thirds, fourths, or fifths without labelling them. Have students locate each decimal on the number line and provide the fraction equivalent for each (Van de Walle and Lovin, 2006, p.115). (7N7.1, 7N7.3)
- Ask students to place the numbers 2.3, 2.4, 2.32, 2.36, 2.327 on a number line. (7N7.1, 7N7.4)
- Ask students to arrange the numbers 0.96,  $0.\overline{9}$ , 0.9,  $0.\overline{96}$ , 0.09 from greatest to least. (7N7.1)
- Ask students to write each of the following numbers in an approximate location on the number line provided. They should explain the strategies they used to approximate each point on the line.

$$\frac{3}{7}, 1\frac{1}{3}, \frac{5}{9}, \frac{13}{12}, 1\frac{4}{9}, 0.45, 0.93$$



(7N7.3, 7N7.4)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.2: Comparing and Ordering Fractions and Decimals**

ProGuide: pp. 9-13

Master 3.12, 3.22

PM 23

CD-ROM: Unit 3 Masters

SB: pp.91-95

Practice and HW Book: pp. 52-54

**Suggested Resource**

Van de Walle and Lovin. *Teaching Student-Centered Mathematics*  
 Grades 5-8, p.115

**Strand: Number****Outcomes**

*Students will be expected to*

7N7 Continued ...

**Achievement Indicators:**

7N7.5 *Identify a number that would be between two given numbers in an ordered sequence or on a number line.*

7N7.6 *Identify incorrectly placed numbers in an ordered sequence or on a number line.*

**Elaborations—Strategies for Learning and Teaching**

Some students may have difficulty identifying a number that is between two given numbers, especially when the numbers given are fractions with the same denominator and are close in value (e.g.,  $\frac{3}{10} < ? < \frac{4}{10}$ ). Students have worked with equivalent fractions in previous grades.

This will be applied as they determine a fraction that is between two given fractions in an ordered sequence. The two fractions above could be changed to  $\frac{6}{20}$  and  $\frac{8}{20}$ . Students can now more easily identify that  $\frac{7}{20}$  is a possible answer. If the fractions are changed to  $\frac{12}{40}$  and  $\frac{16}{40}$ , students can see that there are more options.

When working with decimal numbers, students can use place value. With numbers such as 0.3 and 0.4, they can use the hundredths instead of tenths (e.g., 38 hundredths are between 3 tenths and 4 tenths).

Students should use similar strategies learned from placing fractions on a number line to identify a fraction between two decimals (e.g.,  $0.4 < \frac{?}{8} < 0.7$ ), a decimal between two fractions, or a number between a given decimal and a given fraction.

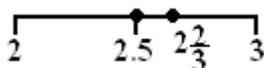
Students can use these same strategies (decimal or fraction equivalents, benchmarks, place value) to identify an incorrectly placed number in a given ordered sequence or on a given number line.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Journal*

- Ask students to respond to the following:  
Suzie and Polly both worked very hard and have nearly completed their math assignment. Suzie has completed  $\frac{5}{6}$  of the project and Polly has completed 0.8 of her project. Who was closer to completing the assignment? How do you know?  
(7N7.1)
- Ask students to choose three values that are not whole numbers and explain how to write them in order using benchmarks.  
(7N7.3)

*Paper and Pencil*

- Ask students to identify a number that would fit between the plotted points on these number lines, and have them explain their choice.



(7N7.5)

- Ask students to identify the number(s) which are not in the correct position in the sets of numbers below. They should record and justify their responses.

(i)  $0.75, 0.\overline{7}, \frac{8}{9}, \frac{3}{5}, \frac{10}{11}$

(ii)  $\frac{4}{5}, 0.8\overline{1}, \frac{9}{10}, \frac{13}{15}, 1.\overline{1}$

(7N7.6)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.2: Comparing and Ordering Fractions and Decimals**

ProGuide: pp. 9-13

Master 3.12, 3.22

PM 23

CD-ROM: Unit 3 Masters

SB: pp.91-95

Practice and HW Book: pp. 52-54

**Strand: Number****Outcomes**

*Students will be expected to*

**7N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected).**

[ME, PS, T]

**Achievement Indicators:**

7N2.1 *Solve a given problem involving the addition of two or more decimal numbers.*

7N2.2 *Solve a given problem involving the subtraction of decimal numbers.*

7N2.3 *Place the decimal in a sum or difference, using front-end estimation; e.g., for  $4.5 + 0.73 + 256.458$ , think  $4 + 256$ , so the sum is greater than 260.*

**Elaborations—Strategies for Learning and Teaching**

When working with operations involving whole numbers and/or decimals, students should use a mental procedure, an algorithm, or a calculator where appropriate. They need to understand the relationship between whole number and decimal number operations, including order of operations. Emphasis should be on place value and estimation. Instruction should not focus students on simply mastering procedural rules without a conceptual understanding. It is important that a problem solving context is used to help ensure the relevance of the operations.

To encourage alternative computational strategies which students have learned in previous grades, addition and subtraction questions should be presented horizontally, as well as vertically. Students should be able to choose algorithms when they calculate with paper and pencil methods. While it is important that the algorithms developed by students are respected, if they are inefficient, students should be guided toward more appropriate strategies.

Mental calculations should be encouraged whenever possible. In paper and pencil computations, we usually start at the right and work toward the left. To add mentally, students can start at the left.

To calculate  $1.7 + 3.6$ , think:

$$1 + 3 = 4$$

$$7 \text{ tenths} + 6 \text{ tenths} = 13 \text{ tenths or } 1 \text{ and } 3 \text{ tenths}$$

$$4 + 1 \text{ and } 3 \text{ tenths} = 5.3$$

When adding numbers such as 4.2 and 0.23, students should be encouraged to add corresponding place values. A common error students make is adding digit to digit starting at “the end” but ignoring place value. For example, using the above numbers, students could incorrectly arrive at an answer of 0.65.

Front-end estimation should be used to develop a sense of the size of an answer for any calculations involving decimals. In this simple strategy, students perform operations from left to right using only the whole number part of each value. When determining the sum  $9.2 + 3.5 + 12.72$ , students use  $9 + 3 + 12 = 24$  to estimate. Similarly, to find the difference  $14.31 - 5.2 - 3.6$ , students use  $14 - 5 - 3 = 6$  to estimate.

Once this estimation is complete, the calculation must be performed. Students can use the estimation to determine whether or not their computations make sense, considering the placement of the decimal.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to estimate the following sums or differences using front-end estimation. They should then compute the answers and compare them to the estimations.
  - (i)  $4.6 + 11.8 + 15.3$
  - (ii)  $19.6 - 15.9 - 1.7$

(7N2.1, 7N2.2, 7N2.3)
- Ask students to create three different addition/subtraction word problems with an answer of 4.2.
 

(7N2.1, 7N2.2)

*Interview*

- Ask the student to add or subtract the following mentally, and to explain the process being used.
  - (i)  $6.4 + 1.8$
  - (ii)  $4.75 - 1.32$

(7N2.1, 7N2.2)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.3: Adding and Subtracting Decimals**

ProGuide: pp. 14-17

Master 3.13, 3.23

CD-ROM: Unit 3 Masters

SB: pp.96-99

Practice and HW Book: pp. 55-56

**Strand: Number**

**Outcomes**

*Students will be expected to*

**7N2 Continued ...**

**Achievement Indicators:**

7N2.4 Solve a given problem involving the multiplication of decimal numbers with two digit multipliers (whole numbers or decimals) without the use of technology.

7N2.5 Place the decimal in a product, using front-end estimation; e.g., for  $\$12.33 \times 2.4$ , think  $\$12 \times 2$ , so the product is greater than  $\$24$ .

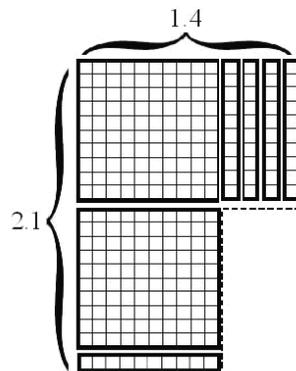
7N2.6 Solve a given problem involving the multiplication or division of decimal numbers with more than 2-digit multipliers or more than 1-digit divisors (whole numbers or decimals) with the use of technology.

**Elaborations – Strategies for Learning and Teaching**

Multiplication or division of two numbers will produce the same digits, regardless of the position of the decimal point. As a result, for most practical purposes, there is no reason to develop new rules for decimal multiplication and division. Rather, the computations can be performed as whole numbers with the decimal being placed by way of estimation (Van de Walle and Lovin, 2006, p.107).

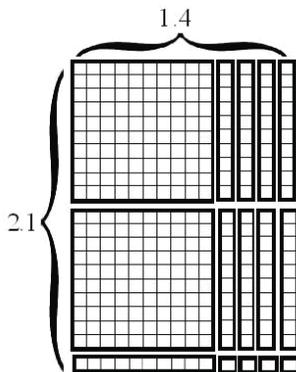
Students have been using base 10 blocks in previous grades to multiply a decimal number by a whole number.

The base 10 area model will now be extended to 2-digit multipliers. An example using the area model for 2-digit decimal numbers is shown below:



The dimensions of the rectangle are 1.4 and 2.1. The area of the rectangle represents the product of these numbers.

Fill the rectangle using the largest possible blocks first, progressing to the smallest (the unit block).



With the region completed, we determine the answer by counting 2 unit blocks, 9 tenths blocks, and 4 hundredths blocks. When combined, we get 2.94.

Students should use front-end estimation to check the reasonableness of their answer. In the above diagrams, students could arrive at 2.94, 29.4 or 294. Using front-end estimation ( $2 \times 1 = 2$ ) indicates that the answer is close to 2, making 29.4 and 294 unreasonable answers.

Students are expected to use manipulatives and algorithms when multiplying decimal numbers with two digits. When solving problems with more than 2-digit multipliers, technology can be used.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to answer the following question.  
*Mary said to Sharon, "I'm thinking of a number that when multiplied by 8.7 gives a product of about 7.2."*  
 Give five numbers that Sharon could have used to answer Mary's question. Show how Sharon's estimates are reasonable.  
 (7N2.4, 7N2.5)
- Ask students to use front-end estimation to determine the position of the decimal in the following products.
  - $7.8 \times 3.2 = 2496$
  - $28.39 \times 2.4 = 68136$
 (7N2.5)
- Ask students to solve the following problem using technology. Have them explain why they think the decimal is in the correct position.  
 Jolene bought 11.8 L of gas for her snowmobile. The gas cost \$1.10 per litre. How much did Jolene pay for her gas?  
 (7N2.5, 7N2.8)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.4: Multiplying Decimals**

ProGuide: pp. 18-21

Master 3.14, 3.24

PM 22

CD-ROM: Unit 3 Masters

SB: pp. 100-103

Practice and HW Book: pp. 57-59

**Strand: Number**

**Outcomes**

*Students will be expected to*

**7N2 Continued ...**

**Achievement Indicators:**

7N2.7 *Solve a given problem involving the division of decimal numbers for 1-digit divisors (whole numbers or decimals) without the use of technology.*

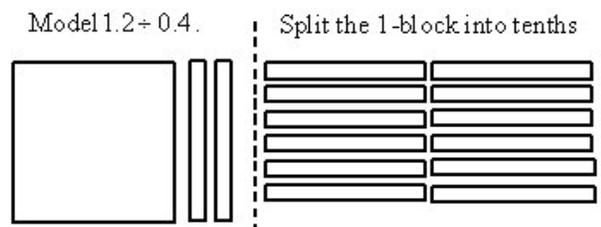
7N2.8 *Check the reasonableness of solutions using estimation.*

7N2.9 *Place the decimal in a quotient, using front-end estimation; e.g., for  $51.50 m \div 2.1$ , think  $50 m \div 2$ , so the quotient is approximately 25 m.*

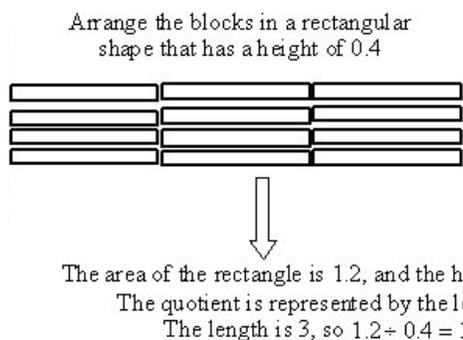
7N2.6 *Solve a given problem involving the multiplication or division of decimal numbers with more than 2-digit multipliers or more than 1-digit divisors (whole numbers or decimals) with the use of technology.*

**Elaborations – Strategies for Learning and Teaching**

In previous grades, students will have used base 10 blocks to divide a decimal number by a whole number. The base 10 area model will be extended to solve problems with 1-digit decimal divisors. The focus is on using division of decimals in a problem solving context.



Since a rectangle could not be created with one dimension 0.4 using 1 and 2 tenths, the 1-block was traded for 10 tenths.



Students should estimate before calculating using any method (base 10 blocks, algorithm or technology where appropriate.) This will continue to develop number sense. Front-end estimation will help students determine the correct position of the decimal, and some rounding may be necessary for a mental calculation. In the example,  $43.24 \div 4.7$ , front-end estimation would be  $43 \div 4$ . To make it easier for mental calculation, it can be considered that 43 is close to 44, and  $44 \div 4 = 11$ , so a reasonable answer is close to 11. When students calculate and find the digits 92, they should then determine that 9.2 makes more sense than 0.92 or 92, since 9.2 is closer to the estimate of 11.

Students may find estimation challenging when the divisor is less than 1. To estimate the value of  $4.2 \div 0.2$ , for example, students will often think  $4 \div 0$ , which is undefined. Using a real-life context can help students understand this better. As an example, ask them to think of a 1 m piece of board which must be sawed into 0.2 m pieces. Five equal pieces can be made. Using this idea, we know that a 4.2 m board is about 4 times larger than the 1 m board and will make 4 times as many pieces ( $5 \times 4 = 20$  pieces).

Technology can be used to solve division problems with more than 1-digit divisors.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to determine how many times a 0.3 L glass can be used to fill a 1.5 L bottle with water.

(7N2.6)

- Ask students to use front-end estimation to determine where the decimal should be placed in the following quotients.

(i)  $39.06 \div 4.2 = 93$

(ii)  $58.5 \div 3.9 = 15$

(7N2.9)

- Ask students to use technology to solve problems such as the following, and then have them explain how they know the decimal is in the correct place.

*John paid \$4.92 for a case of 32 bottles of water to take camping. How much did he pay per bottle?*

(7N2.9, 7N2.8)

*Journal*

- Ask students to answer the following question:  
Carole used her calculator to complete each of the following calculations. Should she accept her answer in each case? Why or why not?

(i)  $24.29 \times 3.8 = 923.02$

(ii)  $8.9 \times 0.4 = 3.56$

(iii)  $36.54 \div 2.9 = 12.6$

(iv)  $8.76 \div 0.4 = 21.9$

(7N2.5, 7N2.7, 7N2.9)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.5: Dividing Decimals**

ProGuide: pp. 22-25

Master 3.15, 3.25

PM 22

CD-ROM: Unit 3 Masters

SB: pp. 104-107

Practice and HW Book: pp. 60-63

**Strand: Number****Outcomes**

*Students will be expected to*

7N2 Continued ...

**Achievement Indicator:**

7N2.10 *Solve a given problem that involves operations on decimals (limited to thousandths), taking into consideration the order of operations.*

**Elaborations—Strategies for Learning and Teaching**

Students have already used the order of operations, excluding exponents, but limited to whole numbers. This will now be extended to calculations with decimal numbers. Remember that for more than 1-digit divisors or 2-digit multipliers, the use of technology is necessary. When students are not using technology, “friendly” numbers for which a calculator is not needed should be used.

The order of operations is necessary in order to maintain consistency of results. It is important to provide students with a variety of situations in which they can recognize the need for the order of operations, such as calculating the total cost for a family with two parents and three children for theatre tickets, where children’s tickets cost \$8.50 and adult tickets cost \$14.80. Students will write a number sentence such as  $C=3 \times \$8.50 + 2 \times \$14.80$ . Discuss with students how they would determine the total, and link the discussion to the order of operations. Students should indicate that it would be necessary to find the total for the adults and the total for the children and then add the totals together. It would not make sense, then, to calculate from left to right:

$$3 \times \$8.50 + 2 \times \$14.80$$

$$\$25.50 + 2 \times \$14.80$$

$$\$27.50 \times \$14.80$$

$$\$407.00$$

An example such as this emphasizes the need to follow the order of operations.

The order of operations for Grade 7 is as follows:

- Brackets
- Division/Multiplication (from left to right)
- Addition/Subtraction (from left to right).

Instruction should be given on calculator use with regard to the order of operations. Students should recognize the necessity of preparing problems for calculator entry. They should also be aware that different calculators process the order of operations in different ways. Some calculators are programmed to address the order of operations automatically, and others are not. Students could perform one calculation at a time and record the flow of their answers. In this way, if an error is made it is easier to identify where it occurred. They could also insert brackets as a reminder of the correct order to perform calculations.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Journal*

- Ask students to compare the solution of  $4 \times 7 - 3 \times 6$  with the solution of  $4 \times (7 - 3) \times 6$ . Are the solutions the same or different? They should explain their answer.

(7N2.10)

*Observation*

- Ask students to write an expression and then calculate the answer to following question: Chris found the attendance reports for hockey games at the stadium for the past nine days to be 2787, 2683, 3319, 4009, 2993, 3419, 4108, 3539, and 4602. If tickets were sold for \$12.75 each, and expenses amounted to \$258 712.00, what was the profit for the stadium?

(7N2.10)

*Paper and Pencil*

- Ask students to write an expression for each of the following and use the expression to answer the question.
  - Ms. Janes bought the following for her project: 5 sheets of pressboard at \$8.95 a sheet, 20 planks at \$2.95 each, and 2 liters of paint at \$9.95. What was the total cost?
  - Three times the sum of \$34.95 and \$48.95 represents the total amount of Jim's sales on April 29. When his expenses, which total \$75.00, were subtracted, what was his profit?
- Ask students to identify where the brackets should be placed in order for the answer to be correct. They should show calculations to demonstrate the correctness of their answer.

(i)  $4 + 6 \times 8 - 3 = 77$

(ii)  $26 - 4 \times 4 - 2 = 18$

(7N2.10)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.6: Order of Operations with Decimals**

ProGuide: pp. 26-27

Master 3.16, 3.26

CD-ROM: Unit 3 Masters

SB: pp. 108-109

Practice and HW Book: pp. 64-65

**Strand: Number****Outcomes**

*Students will be expected to*

**7N3 Solve problems involving percents from 1% to 100%.**

[C, CN, PS, R, T]

**Elaborations—Strategies for Learning and Teaching**

Percents, meaning *per hundred*, are simply hundredths and can be also written as fractions and decimals. Number sense for percent should be developed through the use of benchmarks:

- 100% is all
- 50% is one half
- 25% is one quarter
- 10% is one tenth
- 1% is one hundredth

Students should be able to easily shift between percent, fraction and decimal equivalents in problem solving situations. For example, when finding 25% of a number, it is often much easier to use  $\frac{1}{4}$  and then divide by 4 as a means of finding or estimating the percent.

Students should make immediate connections between other percentages, such as 50%, 75%,  $33\frac{1}{3}\%$  and 20%, 30%, 40%, etc., and their fraction equivalents. They should be encouraged to recognize that percents such as 51% and 12% are close to benchmarks, which could be used for estimation purposes. Students should be able to calculate 1%, 5% (one half of 10%), 10%, 15% and 50% mentally using their knowledge of benchmarks.

When exact answers are required, students should be able to employ a variety of strategies in calculating percent of a number.

7N3.1 *Express a given percent as a decimal or fraction.*

When students understand that percent means per hundred, they should be able to write the percent as a fraction with a denominator of 100 (which should be reduced if possible). Once students have a fraction with a denominator of 100, they can write the decimal since they already have experience with this. Students have also previously written a fraction as a decimal and a decimal as a fraction.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Interview*

- Ask students to change each of the following to a percent mentally and have them explain their thinking:
  - $\frac{2}{5}$
  - $\frac{4}{25}$
  - $\frac{6}{50}$
  - $\frac{7}{20}$

(7N3.1)
- Ask students to determine what percent of a book is left to read if the class read 60 out of 150 pages. They should explain their thinking.
 

(7N3.1)

*Observation*

- Create cards for *Mix Up Match Up with Percents, Decimals and Fractions*. Each card contains a fraction, a decimal, or a percent. Students circulate around the room to find the two other students whose cards correspond to theirs. Once students have found their partners, ask each group why they belong together.
 

(7N3.1)
- Create two sets of cards for *Percent War*: one set containing various percents and the other set containing numbers. Students draw a card from each pile and calculate the percent of the number using the two cards. The player with the largest answer collects all cards. Play continues until there aren't enough cards left for each player to select two. The player with the most cards wins.
 

(7N3.1)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.7: Relating Fractions, Decimals, and Percents**

ProGuide: pp. 29-31

Master 3.10, 3.17, 3.27

CD-ROM: Unit 3 Masters

SB: pp. 111-113

Practice and HW Book: pp. 66-69

**Web Links**

Refer to Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>) for the following activities:

- Mix Up Match Up with Percents, Decimals and Fractions
- Percent War

**Strand: Number****Outcomes**

*Students will be expected to*

**7N3 Continued ...**

**Achievement Indicators:**

*7N3.2 Solve a given problem that involves finding a percent.*

*7N3.3 Determine the answer to a given percent problem where the answer requires rounding, and explain why an approximate answer is needed; e.g., total cost including taxes.*

**Elaborations—Strategies for Learning and Teaching**

Students should solve problems that involve finding a percent in situations such as calculating sales tax, discounts, commissions, tips, etc. They can employ a variety of strategies when exact answers are required to calculate the percent of a number:

- changing percent to a decimal and multiplying  
 $12\% \text{ of } 80 = 0.12 \times 80 = 9.6$
- finding 1% and then multiplying.  
 $1\% \text{ of } 80 = 0.8$ , so  $12\% \text{ of } 80 = 0.8 \times 12 = 9.6$
- changing to a fraction and dividing  
 $25\% \text{ of } 60 = \frac{1}{4} \times 60 = 60 \div 4 = 15$

This method works best with percentages that are more common.

- finding equivalent fractions  
 $30\% \text{ of } 85$   
 $30\% \text{ is } \frac{30}{100} = \frac{3}{10}$   
 $\frac{3}{10} = \frac{?}{85}$   
 $? = 25.5$

It is not necessary that students become proficient in all four methods. The important thing is that they have a method which works well for them.

Students should realize when answers must be rounded in order to make sense in the context presented. For example, when calculating sales tax on a purchase, students may have an answer with as many as 4 decimal places. Students should understand that in real life, money is only presented as 2 decimal places, so the answer must be rounded to the nearest hundredth. Answers to problems which involve finding the number of people must be rounded to the nearest whole number, since it makes no sense to speak of a part of a person.

**General Outcome: Develop number sense.****Suggested Assessment Strategies***Paper and Pencil*

- Students could create problems that utilize percent. They can be given flyers from local supermarkets and/or department stores and use these to create problems which involve calculating the total savings when certain items are purchased at the sale price. (7N3.3)
- Ask students to answer the following question.  
Byron took \$85 to the mall to buy gifts. He wants to purchase a book for \$13, a video game for \$18 and a laptop bag for \$40. Sales tax is charged at 13%. Does he have enough money with him to make these purchases? If he does have enough money for all of his purchases, how much money will he have left after he finishes shopping? (7N3.3)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 3.8: Solving Percent Problems**

ProGuide: pp. 32-34

Master 3.18, 3.28

CD-ROM: Unit 3 Masters

SB: pp. 114-116

Practice and HW Book: pp. 70-72

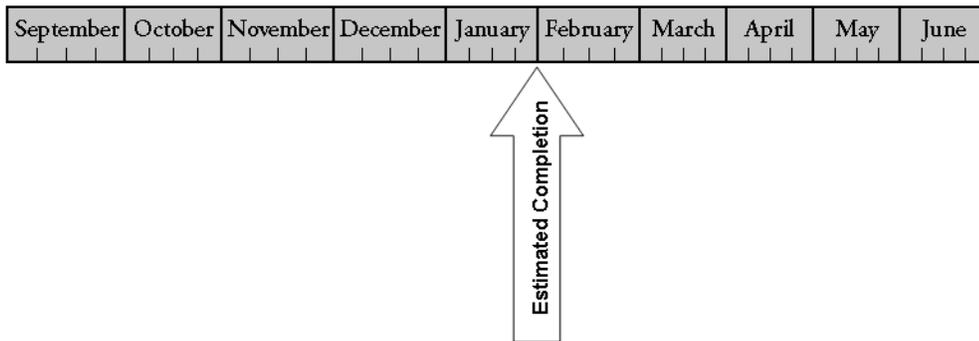
**Suggested Resource**

The resource *Mental Math in Junior High* includes good practice of these indicators in lessons 45-50



# Circles and Area

Suggested Time: 4 Weeks



# Unit Overview

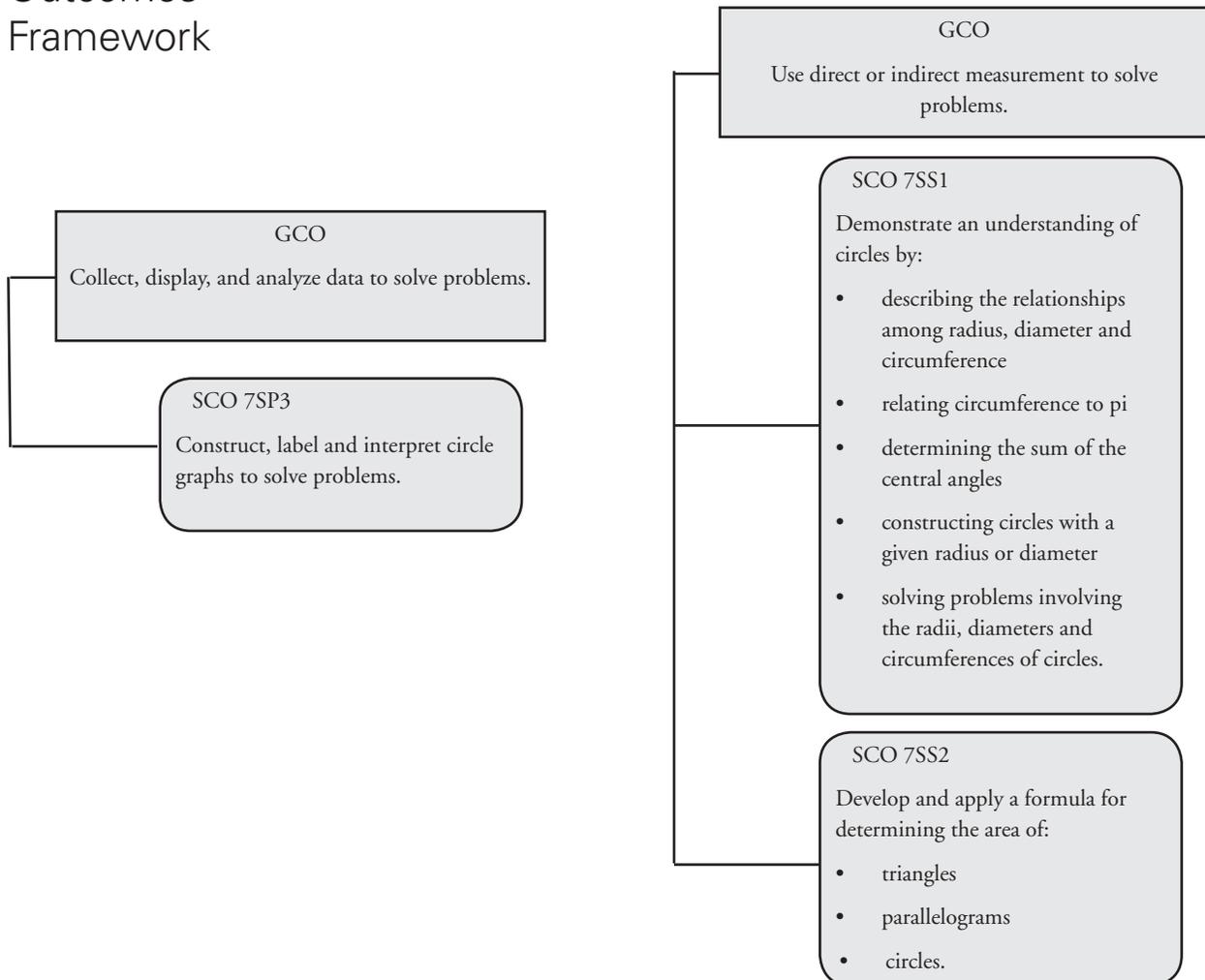
## Focus and Context

In this unit, direct or indirect measurement is used to solve problems. It also includes the construction of circles, the sum of the central angles of a circle, and specific reference to pi.

Students will build meaning for the area of a parallelogram based on their understanding of the area of a rectangle. They will then connect the area of a triangle to the area of a parallelogram. Finally, they will explore the area of a circle by rearranging it into a parallelogram or a rectangle.

Students will collect and organize data, and then use the data to create circle graphs and solve problems. A circle graph will be used to compare parts of a whole to the whole. Two circle graphs will be used to compare parts of two separate wholes to each other. Circle graphs are particularly useful for comparing the frequency of data in one category to the entire set of data, while still allowing for comparisons among categories.

## Outcomes Framework



## Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics  
 and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

## SCO Continuum

Grade 6	Grade 7	Grade 8
<b>Shape and Space (Measurement)</b>		
<p>6SS1. Demonstrate an understanding of angles by:</p> <ul style="list-style-type: none"> <li>identifying examples of angles in the environment</li> <li>classifying angles according to their measure</li> <li>estimating the measure of angles, using <math>45^\circ</math>, <math>90^\circ</math> and <math>180^\circ</math> as reference angles</li> <li>determining angle measures in degrees</li> <li>drawing and labelling angles when the measure is specified.</li> </ul> <p>[C, CN, ME, V]</p>	<p>7SS1. Demonstrate an understanding of circles by:</p> <ul style="list-style-type: none"> <li>describing the relationships among radius, diameter and circumference</li> <li>relating circumference to pi</li> <li>determining the sum of the central angles</li> <li>constructing circles with a given radius or diameter</li> <li>solving problems involving the radii, diameters and circumferences of circles.</li> </ul> <p>[C, CN, PS, R, V]</p>	<p>8SS1. Develop and apply the Pythagorean theorem to solve problems.</p> <p>[CN, PS, R, T, V]</p> <p>8SS2. Draw and construct nets for 3-D objects.</p> <p>[C, CN, PS, V]</p> <p>8SS3. Determine the surface area of:</p> <ul style="list-style-type: none"> <li>right rectangular prisms</li> <li>right triangular prisms</li> <li>right cylinders</li> </ul> <p>to solve problems.</p> <p>[C, CN, PS, R, V]</p> <p>8SS4. Develop and apply formulas for determining the volume of right prisms and right cylinders.</p> <p>[C, CN, PS, R, V]</p>
<p>6SS2. Demonstrate that the sum of interior angles is:</p> <ul style="list-style-type: none"> <li><math>180^\circ</math> in a triangle</li> <li><math>360^\circ</math> in a quadrilateral.</li> </ul> <p>[C, R]</p>	<p>7SS2. Develop and apply a formula for determining the area of:</p> <ul style="list-style-type: none"> <li>triangles</li> <li>parallelograms</li> <li>circles.</li> </ul> <p>[CN, PS, R, V]</p>	
<p>6SS3. Develop and apply a formula for determining the:</p> <ul style="list-style-type: none"> <li>perimeter of polygons</li> <li>area of rectangles</li> <li>volume of right rectangular prisms.</li> </ul> <p>[C, CN, PS, R, V]</p>		
<b>Statistics and Probability (Data Analysis)</b>		
<p>6SP1. Create, label and interpret line graphs to draw conclusions.</p> <p>[C, CN, PS, R, V]</p>	<p>7SP3. Construct, label and interpret circle graphs to solve problems.</p> <p>[C, CN, PS, R, T, V]</p>	Not addressed

## Strand: Shape and Space (Measurement)

### Outcomes

*Students will be expected to*

**7SS1 Demonstrate an understanding of circles by:**

- describing the relationships among radius, diameter and circumference
- relating circumference to pi
- determining the sum of the central angles
- constructing circles with a given radius or diameter
- solving problems involving the radii, diameters and circumferences of circles.

[C, CN, PS, R, V]

### Achievement Indicators:

7SS1.1 *Illustrate and explain that the diameter is twice the radius in a given circle.*

7SS1.2 *Draw a circle with a given radius or diameter, with and without a compass.*

### Elaborations—Strategies for Learning and Teaching

Students have been introduced to the concepts of circles, area, and perimeter in previous grades. From previous exposure, they should be able to:

- recognize circles, triangles, and parallelograms
- calculate the area of a rectangle
- measure perimeter in linear units, and measure area in square units

In using any type of measurement, the attribute to be measured must be identified, and the appropriate unit chosen. When measuring the circumference, radius and diameter of circles, length is being measured and appropriate units to measure length include millimeters, centimeters and metres. When finding the sum of the central angles of a circle, the attribute of angle measure is being used and the appropriate unit of measure is the degree.

Ask students to position themselves to make a human circle with one student as the centre of the circle. Ask how they can use the person at the centre to check that the circle made is really circular. This should lead to discussion of the circle characteristic that any point on the circle is the same distance from a point at its centre. Provide a student at the centre with a string about 3 metres long. Ask him or her to give the other end of the string to a student on the circle. The student must adjust his or her position so that the string is taut. Students continue the process until the string has been passed to all students on the circle.

Introduce the term *radius*, and ask students what represents the radius in their concrete circle. After they understand that the length of the string represents the radius, they can predict how many lengths of string are needed to extend from one student to another student on the opposite side of the circle, passing through the centre. They should conclude that double the radius equals the distance across the circle through the centre. Introduce the term *diameter* to represent this distance. It is important to explore the relation between the diameter and the radius in both directions (i.e.,  $d = 2r$  and  $\frac{d}{2} = r$ ).

In addition to the concrete circle described previously, students can also draw a circle without using a compass. One possibility is to trace a round object. A second method is to tie a piece of string near the bottom of a pencil. Hold the string the length of the radius away from the pencil with your finger. Hold the string down against the paper where you want the centre of the circle to be. Draw around the centre while keeping the string tight and the pencil upright. Students may also devise other ways to draw circles.

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**General Outcome: Use direct or indirect measurement to solve problems.**


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**Suggested Assessment Strategies**
*Journal*

- Students can make a list of sports in which circles play an important role and estimate the radius of each circle described. (7SS1.1)
- Ask students to consider the following statement:  
*If the radius of a circle is doubled to make a new circle, the diameter is also doubled.*  
Is this true? Use examples to support your answer. (7SS1.1)

*Paper and Pencil*

- Ask students to use words and diagrams to explain how to find the diameter of a circle if the radius is known. (7SS1.1)
- Students can practice drawing circles with a compass by drawing circles with a 10 cm radius, with a 5 cm radius, and with a 6 cm diameter. (7SS1.2)
- Ask students to write a set of instructions to describe how to draw a circle with a diameter of 8 cm, using a compass. Each student then gives his or her instructions to a classmate who will draw it. Students will then decide if the drawing of their circle is accurate. (7SS1.2)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*

Prep Talk Video: Circles and Area

**Lesson 4.1: Investigating Circles**

ProGuide: pp. 4-6

Master 4.10, 4.15, 4.24

PM19

CD-ROM: Unit 4 Masters

Prep Talk Video: Investigating Circles

Student Book (SB): pp. 130-132

Practice and HW Book: pp. 80-81

**Web Link**

 A link to the Circle Song can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>).

**Strand: Shape and Space (Measurement)****Outcomes**

*Students will be expected to*

7SS1 Continued ...

**Achievement Indicators:**

7SS1.3 *Illustrate and explain that the circumference is approximately three times the diameter in a given circle.*

7SS1.4 *Explain that, for all circles, pi is the ratio of the circumference to the diameter ( $\frac{C}{d}$ ) and its value is approximately 3.14.*

7SS1.5 *Solve a given contextual problem involving circles.*

**Elaborations—Strategies for Learning and Teaching**

The human circle activity referenced on page 82 can be used to develop the relationship between the diameter and the circumference of a circle. Introduce the term *circumference* as the distance around the circle and relate it to perimeter. You can connect the length of the diameter to the circumference of a circle by posing the following question:

About how many lengths the size of the radius would fit around the perimeter or circumference of the circle?

If done accurately, a little more than 6 radii are needed to measure the circumference.

Ask:

If about 6 lengths of string, each representing the radius of the circle, are needed to measure the circumference of the circle, how many lengths of string the size of the diameter would be needed?

This leads to the relationship  $C \approx 6r$  or  $C \approx 3d$ .

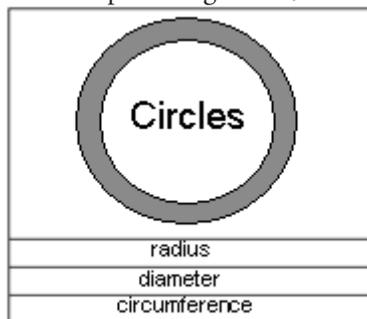
Pi is defined as the ratio of circumference to diameter. It is a non-repeating, non-terminating decimal that cannot be expressed as a fraction (i.e., irrational). Students should explore pi, and its relationship with the circumference and diameter should be discovered through investigation. Any exploration that is done should include collecting measures of circumference and diameter. Ratios for the circumference to the diameter should also be computed and the information can be recorded in a table similar to the one below.

Circular Object	Circumference	Radius	Diameter	$\frac{C}{d}$

$\pi$  is often approximated as 3.14. Most calculators have a button that can be used for calculations. For estimates, students may use 3 as an approximation of  $\pi$ .

**General Outcome: Use direct or indirect measurement to solve problems.****Suggested Assessment Strategies***Performance*

- Students may use a 3-tab foldable as an organizational tool when describing the relationships among radius, diameter and circumference.



(7SS1.1, 7SS1.3)

*Interview*

- Ask students if it is necessary to use hands-on measurement to determine the radius, diameter and circumference of a given circle. (7SS1.3)
- Ask students the following questions:
  - What is the best estimate for the circumference of a circle with a diameter of 12 cm? Justify your choice.
 

(a) 6 cm      (b) 18 cm      (c) 36 cm      (7SS1.3)
  - What is the best estimate for the circumference of a circle with a radius of 10 cm? Justify your choice.
 

(a) 30 cm      (b) 60 cm      (c) 90 cm      (7SS1.3)

*Paper and Pencil*

- Students can answer questions such as the following:
  - Jackie is constructing a round dining room table that will seat 12 people. She wants each person to have 60 cm of table space along the circumference. Determine the diameter of the dining room table.
  - A manufacturing company is producing dinner plates with a diameter of 30 cm. They plan to put a gold edge around each plate. Determine how much gold edging they need for an eight plate setting. If gold edging costs \$4 per cm, what would it cost to trim all of the plates?
  - A dog is tethered to a stake in a yard and can walk or run in a circle. The largest circumference of his runway is 56.52 m. What is the length of the dog's tether rope? Explain your thinking. (7SS1.5)

**Resources/Notes****Authorized Resource***Math Makes Sense 7*

Lesson 4.2: Circumference of a Circle

ProGuide: pp. 7-11

Master 4.16, 4.25

PM 20

CD-ROM: Unit 4 Masters

Prep Talk Video: Circumference of a Circle

SB: pp. 133-137

Practice and HW Book: pp. 82-83

**Web Link**

Instructions on how to create a 3-tab foldable can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>).

**Suggested Resource***Playing around with "Mono-pi-ly"**Mathematics Teaching in the Middle School*

Vol. 11, No. 6, February 2006

pp. 294-297

## Strand: Shape and Space (Measurement)

### Outcomes

*Students will be expected to*

**7SS2 Develop and apply a formula for determining the area of:**

- triangles
- parallelograms
- circles.

[CN, PS, R, V]

### Achievement Indicators:

7SS2.1 *Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram.*

7SS2.2 *Generalize a rule to create a formula for determining the area of parallelograms.*

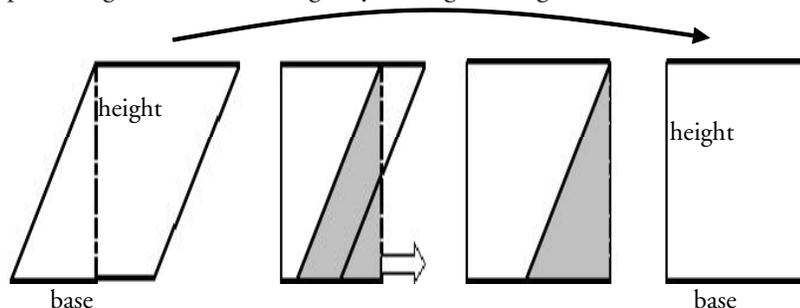
7SS2.3 *Solve a given problem involving the area of triangles, parallelograms and/or circles.*

### Elaborations—Strategies for Learning and Teaching

Area can be defined as a measure of the space inside a region or how many square “units” it takes to cover a region. The areas of rectangles, parallelograms, triangles and circles are related, with the area of rectangles forming the foundation for the areas of the other 2-D shapes. In Grade 6, students developed and applied a formula for determining the area of rectangles (6SS3). In this unit, this prior knowledge will be used to develop a formula for determining the area of triangles, parallelograms and circles. In Grade 8, students will determine the surface area of right rectangular and triangular prisms, and right cylinders to solve problems (8SS3).

An understanding of conservation of area is critical. That is, students should come to realize that an object retains its size when the orientation is changed or when it is broken into smaller parts and the parts are rearranged. When measuring area, some appropriate units include  $\text{cm}^2$  and  $\text{m}^2$ .

Students should be introduced to the area of a parallelogram by building on their prior knowledge of area. One way to do this is to convert a parallelogram into a rectangle by sliding a triangle.



Students should recognize that the area of a parallelogram is the same as the area of a related rectangle with the same base and height. An activity such as this one develops the formula  $A_{\text{parallelogram}} = (\text{base})(\text{height})$ , and builds awareness of conservation of area.

Students should be given the opportunity to explore a variety of parallelograms in various orientations. They can work in groups or independently to find the area of the parallelograms and generalize a formula. They can cut out the parallelograms and rearrange them if desired. They should generalize that any parallelogram can be rearranged to form a rectangle. The area can be found, therefore, by multiplying the base of the parallelogram by the height of the parallelogram, using the formula for the area of a rectangle. Emphasize that the height of the parallelogram is always the perpendicular height because a rectangle always has a base and height perpendicular to each other.

**General Outcome: Use direct or indirect measurement to solve problems.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to draw a parallelogram, on grid paper, with an area of  $24 \text{ cm}^2$ . Then ask them to create three other parallelograms with a different base length but the same area.

(7SS2.2, 7SS2.3)

*Journal*

- Ask students to explain why the area of a rectangle and the area of a parallelogram made from the rectangle are the same. They should include diagrams with their responses.

(7SS2.1)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 4.3: Area of a Parallelogram**

ProGuide: pp. 13-16

Master 4.17, 4.26

PM 23

CD-ROM: Unit 4 Masters

Prep Talk Video: Area of a Parallelogram

SB: pp. 139-142

Practice and HW Book: pp. 84-86

**Strand: Shape and Space (Measurement)****Outcomes**

*Students will be expected to*

7SS2 Continued ...

**Achievement Indicators:**

7SS2.1, 7SS2.2 and 7SS2.3  
*Continued*

7SS2.4 *Illustrate and explain how the area of a rectangle or a parallelogram can be used to determine the area of a triangle.*

7SS2.5 *Generalize a rule to create a formula for determining the area of triangles.*

7SS2.3 *Solve a given problem involving the area of triangles, parallelograms and/or circles.*

**Elaborations—Strategies for Learning and Teaching**

Ask students to measure the base and height of the parallelograms and apply the formula  $A = bh$  to calculate the area. Students could also place centimetre grid transparencies over the parallelogram and count the number of square centimetres inside the parallelogram. They can then compare the calculated area to the estimated area of the parallelogram.

Students should be able to determine the base or height, given the area and the other dimension, and recognize that a variety of parallelograms can have the same area.

The area of triangles is related to the area of rectangles and parallelograms. The following activity is a good example of leading students to the area of a triangle by exploring its relationship with the area of a rectangle. Teachers can ask students to follow the procedure:

- On grid paper, draw a rectangle that has a base of 8 units and a height of 5 units.
- Cut out the rectangle.
- Record the number of squares in the rectangle as the area of the rectangle. (This reinforces the idea of square units to measure area.)
- Draw a diagonal line from one corner of the rectangle to the opposite corner. Cut along the diagonal to separate the rectangle into two sections. What shapes have been created?
- Place these two shapes on top of each other. How do they compare?
- How does the area of one triangle compare to the area of the original rectangle?
- Suggest a formula for the area of a triangle recalling that the area of a rectangle is  $A = bh$ .

Students can report their formulas to the class, leading to a discussion of any similarities and/or differences in their formulas.

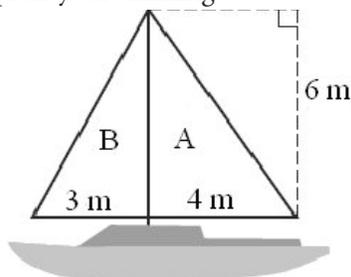
Alternately, the area of triangles can be determined using parallelograms. Following the same procedure as outlined previously, students should discover that the triangle has the same base and perpendicular height as the related parallelogram but has only half the area of the parallelogram. Therefore,  $A_{\text{triangle}} = \frac{bh}{2}$ . Students may wish to use  $A_{\text{triangle}} = \frac{1}{2}bh$  particularly if numbers are large.

**General Outcome: Use direct or indirect measurement to solve problems.**

### Suggested Assessment Strategies

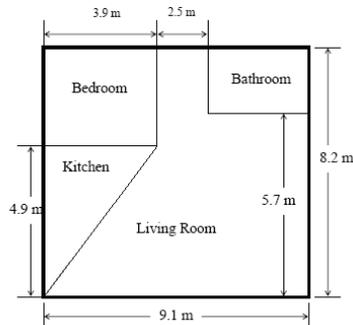
#### *Paper and Pencil*

- Students can answer questions such as the following:
  - Daniel just bought a used sailboat with two sails that need replacing. How much sail fabric will Daniel need if he replaces sail A? Explain your thinking.



- How much sail fabric will Daniel need if he replaces sail B?  
(7SS2.5, 7SS2.3)

- Ask students to respond to the following:  
Daisy wants new flooring and carpeting for her rectangular apartment. A floor plan of her apartment is shown below.



- If bathroom flooring costs \$12.95 per square meter, how much will it cost Daisy to put new flooring on her bathroom?
- If Daisy has \$700 to spend on carpet in her living room and bedroom and carpet costs \$9.98 a square meter, does she have enough money to carpet the two rooms?  
(7SS2.4)

#### *Journal*

- Ask students to respond to the following:
  - A triangle and a parallelogram have the same base and the same height. Explain how their areas compare. Include diagrams in your explanation.  
(7SS2.4)
  - Explain how the formulas for the area of rectangles, parallelograms and triangles are the same. Explain how they are different.  
(7SS2.4)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 7*

#### Lesson 4.4: Area of a Triangle

ProGuide: pp.17-21

Master 4.18, 4.27

PM 23, 25

CD-ROM: Unit 4 Masters

Prep Talk Video: Area of a Triangle

SB: pp. 143-147

Practice and HW Book: pp. 87-89

**Strand: Shape and Space (Measurement)****Outcomes**

*Students will be expected to*

7SS2 Continued ...

**Achievement Indicators:**

7SS2.6 *Illustrate and explain how to estimate the area of a circle without the use of a formula.*

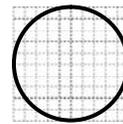
7SS2.7 *Apply a formula for determining the area of a given circle.*

**Elaborations—Strategies for Learning and Teaching**

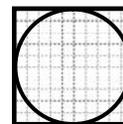
Students often memorize mathematical formulas without much understanding. Engaging students in activities involving estimating the areas of circles provides a foundation for developing the formula for the area of a circle.

The following activity appears in the student book as an Assessment Focus and occurs after the formula is developed. It would be appropriate, however, to address it prior to introducing the formula for the area of a circle.

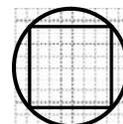
- Using a compass, draw a circle on 1-cm grid paper.
- Count squares inside the circle and estimate the area.



- Draw a square outside the circle and calculate the area of the square.

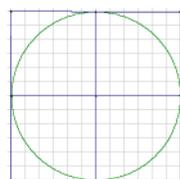


- Draw a square inside the circle and calculate the area of the square.



- Estimate the area of the circle by relating it to areas of the outer and inner squares (average the areas of the two squares).
- Discuss the advantages and disadvantages of the above method for measuring the area of a circle.

An alternate activity for estimating the area of a circle is described here. It provides an effective transition to the development of the formula.



Students cover as much of the circle as possible with beans. Because of their curved shape, the beans should fill more space inside the circle than the square.



Transfer the beans used to cover the circle onto the squares. These squares are the four squares from the original diagram.

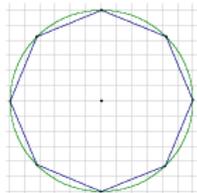
They can be called  $r$ -squares since their sides are the radius of the circle. Ask students to count the number of smaller squares covered by the beans to get an estimate of the circle's area. They should have covered a little more than 3 of the  $r$ -squares, resulting in an estimate that is approximately 3  $r$ -squares. This leads to the formula for the area of a circle.

**General Outcome: Use direct or indirect measurement to solve problems.**

### Suggested Assessment Strategies

#### *Paper and Pencil*

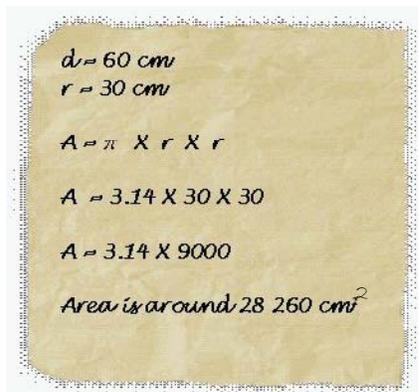
- Ask students to estimate the area of the circle using the octagon as a benchmark. (Notice that the octagon fills more of the circle than a square would.)



(7SS2.6)

#### *Journal*

- Jackie's mom was decorating Jackie's bedroom and placed a round mat on the floor near the bed. Jackie had just learned about circles in math class and wondered about the area of the mat. The tag on the mat said that it was 60 cm wide. She performed the following calculations:



Are Jackie's calculations reasonable? Explain.

(7SS2.7)

#### *Performance*

- Students can use a tri-fold foldable to keep notes about the three shapes discussed in this unit: parallelograms, triangles, and circles. Topics such as estimation, definitions and area formulas could be included for fast and easy access.

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 7*

Lesson 4.5: Area of a Circle

ProGuide: pp. 22-26

Master 4.19, 4.28

PM 22

CD-ROM: Unit 4 Masters

Prep Talk Video: Area of a Circle

Classroom Videos: Area of a Circle, Parts 1, 2, 3

See It Video: Game - Packing Circles

SB: pp.148-152

Practice and HW Book: pp. 90-92

#### Web Link

Instructions on how to create a tri-fold foldable can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>).

**Strand: Shape and Space (Measurement)****Outcomes**

*Students will be expected to*

7SS2 Continued ...

**Achievement Indicators:**

7SS2.6, 7SS2.7 *Continued*

7SS2.3 *Solve a given problem involving the area of triangles, parallelograms and/or circles.*

**Elaborations—Strategies for Learning and Teaching**

The area of a circle can also be developed using a parallelogram. The circle is cut into many equal sectors and rearranged into the approximate shape of a parallelogram. The estimate becomes more accurate as more sectors are used because more and more space inside the parallelogram gets “filled up”. To effectively develop the area of a circle with this activity, the measures of the circle (radius and half the circumference) should be transferred to the measures of the parallelogram. The base of the parallelogram can be represented by  $\pi r$  and the height of the parallelogram is  $r$ . Then the formula for the area of a parallelogram can be applied to create the formula for the area of a circle.

Students have not been exposed to powers or exponents. When developing the formula  $A_{\text{circle}} = \pi r^2$ , it can be introduced as  $A = \pi \times r \times r$ . Students have only seen the “square” notation when working with area units.

Students can solve problems involving everyday contexts, using only the area of a circle first. After students are exposed to simpler problems involving a single formula, present them with more complex problems in which they apply one or more of the formulas created. When solving problems, students should be encouraged to focus on the needed information, draw and label diagrams if necessary, estimate the answer, write the appropriate formula and finally substitute the numbers into the formula to solve the problem. A complete answer includes both a numerical value and the correct measurement unit.

**General Outcome: Use direct or indirect measurement to solve problems.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to answer questions such as the following:
  - Mr. McGowan made an apple pie with diameter of 25 cm. He cut the pie into 6 equal slices. Find the approximate area of each slice.

(7SS2.7, 7SS2.3)

- The outer section on the Canadian Toonie has an outside radius of 14 mm, and an inside radius of 8 mm. What is the area of the outer section?



(7SS2.7, 7SS2.3)

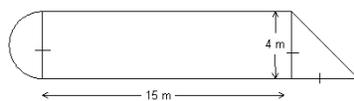
- A garden plot was made in the following shape:



- Will the total area of the garden plot be greater than  $40 \text{ m}^2$ ? Explain your thinking.
- Calculate the total area of the garden plot.

(7SS2.2, 7SS2.3, 7SS2.7)

- A garden plot was made in the following shape:



- Estimate the area of the garden plot. Explain your thinking.
- Find the area of the plot.
- If the width (4 m) of the plot is doubled, is the area of the plot doubled? Explain.

(7SS2.3, 7SS2.5, 7SS2.7)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 4.5: Area of a Circle**

ProGuide: pp. 22-26

Master 4.19, 4.28

PM 22

CD-ROM: Unit 4 Masters

Prep Talk Video: Area of a Circle

Classroom Videos: Area of a Circle, Parts 1, 2, 3

See It Video: Game - Packing Circles

SB: pp.148-152

Practice and HW Book: pp. 90-92

## Strand: Statistics and Probability (Data Analysis)

### Outcomes

*Students will be expected to*

**7SP3 Construct, label and interpret circle graphs to solve problems.**

[C, CN, PS, R, T, V]

**7SS1 Continued ...**

### Achievement Indicators:

*7SP3.1 Find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines and the Internet.*

*7SP3.2 Identify common attributes of circle graphs, such as:*

- *title, label or legend*
- *the sum of the central angles is  $360^\circ$*
- *the data is reported as a percent of the total, and the sum of the percents is equal to 100%.*

*7SS1.6 Explain, using an illustration, that the sum of the central angles of a circle is  $360^\circ$ .*

### Elaborations—Strategies for Learning and Teaching

Comparisons can be made between circle graphs and the more familiar bar graphs. Students began working with bar graphs in Grade 3 (3SP2), and were introduced to double bar graphs in Grade 5 (5SP2). The two are similar in that they provide information arranged in categories.

On a circle graph, the categories are represented by sectors, while bars represent the categories on a bar graph.

Circle graphs are particularly useful for comparing the frequency of data in one category to the entire set of data, while still allowing for comparisons among categories. For example, the percentage of people in one age group for a city might be compared to another age group, and it may also be compared to the city. Since circle graphs display ratios rather than quantities, the small set of data can be compared to the large set of data. That could not be done with bar graphs (Van de Walle and Lovin, 2006, p. 234).

When students interpret graphs constructed by others, they learn to appreciate the features that can help them make sense of a visual display of data. The title, legend and labels are crucial to interpreting circle graphs. The graphs can be labeled with the actual data and/or percents. Each category sector must be labelled. Use real data if at all possible when having students interpret or draw circle graphs. Data will typically be given as percentages or as raw data to be converted to percents.

A variety of strategies are available to illustrate that the sum of the central angles is always equal to  $360^\circ$ . Students first need to be aware that a central angle is one with its vertex at the centre of the circle and its arms intersecting the circumference. Build on students' understanding of  $180^\circ$  being a straight line and, therefore, a semi-circle, so the complete circle will be  $360^\circ$ . Similarly, students should understand that a right angle is  $90^\circ$  and there are four  $90^\circ$  central angles in a circle. Another possibility is to build on the knowledge that the sum of the angles of any quadrilateral is  $360^\circ$ . Given any quadrilateral, students can "tear off" the vertices and arrange them so that they all meet in the centre of a circle.

**General Outcome: Collect, display, and analyze data to solve problems.****Suggested Assessment Strategies***Journal*

- Ask students how a circle graph can provide information about how parts of a whole are related. (7SP3.2)
- Students could search newspapers, magazines, and the Internet for information that has been represented as a circle graph. Ask them to print or cut out a graph and glue or tape it into their notebook. Students should analyze the graph according to criteria such as the following:
  - (i) Is a title given? Does the title say what the graph is about?
  - (ii) Are sectors labelled or is a legend or key provided?
  - (iii) Do the percents add up to 100%?
  - (iv) Does the graph effectively get the reader's attention?
 (7SP3.1, 7SP3.2)

*Paper and Pencil*

- Ask students to complete *Parts of a Circle Graph*.

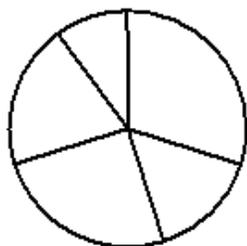
Mike is a student in grade 7 and is learning about circle graphs. Mike has to study regularly in order to keep his grades up. He decided how he should use his study time and recorded it in the table below.

Using the data in the table, label the circle graph correctly. Match the correct percentages with the correct sectors. Create an appropriate title for the graph. Complete the legend and shade the circle graph to match.

Math	30%
Social Studies	15%
Language Arts	25%
Science	20%
French	10%

*Parts of a Circle Graph*

Title



Legend


(7SP3.2)

**Resources/Notes****Authorized Resource***Math Makes Sense 7***Lesson 4.6: Interpreting Circle Graphs**

ProGuide: pp. 30-34

Master 4.20, 4.29

CD-ROM: Unit 4 Masters

SB: pp. 156-160

Practice and HW Book: pp. 93-95

**Lesson 4.7: Drawing Circle Graphs**

ProGuide: pp. 35-38

Master 4.12, 4.21, 4.30

CD-ROM: Unit 4 Masters

SB: pp.161-164

Practice and HW Book: pp. 96-99

**Strand: Statistics and Probability (Data Analysis)****Outcomes**

*Students will be expected to*

7SP3 Continued ...

**Achievement Indicators:**

7SP3.3 *Translate percentages displayed in a circle graph into quantities to solve a given problem.*

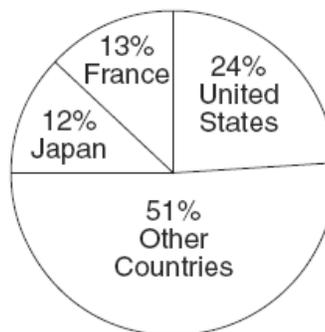
7SP3.4 *Interpret a given circle graph to answer questions.*

**Elaborations—Strategies for Learning and Teaching**

Circle graphs show each measure in ratio to the sum of the measures (i.e., a percentage of total measures). The primary function of a circle graph is to show relationships among the parts of a whole and, at the same time, show relationships between the part and the whole. In the Fractions, Decimals, and Percents unit studied earlier in Grade 7, students calculated the percent of a number (7N3). When interpreting a circle graph, they can use a percentage to determine what portion of the total graph corresponds to that percentage.

For example, the following circle graph can be interpreted to determine a quantity for each sector:

**Locations of Nuclear Reactors  
in Operation 2007**



Students should answer questions such as:

If there are 435 nuclear reactors in operation, how many are in the United States? Who has more reactors: France or Japan?

To translate the percentage displayed in the circle graph into a quantity, students must calculate 24% of 435. There are approximately 104 nuclear reactors in the United States.

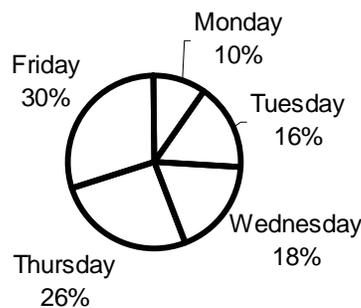
**General Outcome: Collect, display, and analyze data to solve problems.**

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to answer the following:

#### **Amount of Chocolate Milk Sold in a Week**



Jan wants to show that the sales of chocolate milk are higher at the end of the week, so that more chocolate milk can be ordered for that time. She creates the circle graph above. Analyze the graph and answer these questions.

- What percentage of the milk is sold on Wednesday?
- Identify a group of days that accounts for about one half of the total sales. (There is more than one possible answer.)
- If Friday is a holiday, discuss how that would affect ordering chocolate milk for that week.
- In a regular week 500 cartons of chocolate milk are sold. How many cartons should be ordered if Friday was a holiday?
- If weekly sales of chocolate milk are \$200, how much money is made on Monday?
- Why do you think chocolate milk sales increased steadily as the week progressed?

(7SP3.3, 7SP3.4)

#### *Journal*

- Ask students to respond to the following:  
When you are studying a circle graph, what kind of questions should you ask yourself about the information it shows?

(7SP3.2, 7SP3.4)

### Resources/Notes

#### **Authorized Resource**

*Math Makes Sense 7*

Lesson 4.6: Interpreting Circle Graphs

Lesson 4.7: Drawing Circle Graphs

ProGuide: pp. 30-34, 35-38

Master 4.12, 4.20, 4.21, 4.29, 4.30

CD-ROM: Unit 4 Masters

Prep Talk Video: Drawing Circle Graphs

SB: pp. 156-160, 161-164

Practice and HW Book: pp. 93-95, 96-99

**Strand: Statistics and Probability (Data Analysis)****Outcomes**

*Students will be expected to*

7SP3 Continued ...

**Achievement Indicator:**

7SP3.5 *Create and label a circle graph, with and without technology, to display a given set of data.*

**Elaborations—Strategies for Learning and Teaching**

Prior to constructing circle graphs, you may wish to use an informal activity such as the “Human Circle Graph” described in *Teaching Student-Centered Mathematics* (Van de Walle and Lovin, 2006, p. 324). Choose a topic, such as having students select their favourite hockey team in the Stanley Cup semi-final round, and line them up so that students favouring the same team are together. Other suggestions include having strips of paper or wearing t-shirts corresponding to eye color. Form the entire group into a circle. You can tape the ends of four long strings in the centre of the circle, and extend them to the circle at each point where the team changes. This results in a pie graph with no measuring and no percentages. If students experience an activity such as the human pie graph, using their own calculations to make circle graphs should have more meaning.

The ability to find the percent of a number and the ability to use a protractor are necessary skills when constructing circle graphs from raw data. The construction using paper and pencil can be time consuming and should be done with the aid of a calculator when the numbers used are not “friendly”. When rounding percents, the numbers may need to be adjusted slightly to ensure a total of exactly 100%.

Once students have been engaged in generating circle graphs by hand, the focus should be on when a circle graph is the most appropriate form of data display and how to use technology to construct them. Technology options include, but are not limited to, Microsoft Excel, websites and graphing calculators. Whenever students construct data displays, these displays should be used for interpretation. The ability to organize and display data provides quick visual representations of the data, and the ability to predict future related events based on the data.

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**General Outcome: Collect, display, and analyze data to solve problems.**


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**Suggested Assessment Strategies**
*Performance*

- Conduct surveys with your class and ask students to use the results to create circle graphs. Possible survey ideas include:
  - (i) How many children are in your family?
  - (ii) What kind of pet do you have?
  - (iii) In what month were you born?
  - (iv) What color are your eyes?
  - (v) What is your favourite hockey team?

(7SP3.5)

- Ask students to make bar graphs. When completed, they should cut out the bars and tape them end to end. The two ends taped together form a circle. Ask students to estimate the centre of the circle, draw lines to the points where the different bars meet, and trace around the full loop. They can then estimate the percentages. (Van de Walle and Lovin, 2006, p.324)

(7SP3.5)

*Presentation*

- Students use the Internet to find the most recent population figures for Newfoundland and Labrador, Nova Scotia, New Brunswick, and Prince Edward Island. Ask them to record the population figures from approximately 20 years ago.

Based on the data, they can answer the following questions:

- (i) Make two circle graphs:
  - Graph A: using the most recent figures
  - Graph B: using the older set of data
- (ii) How can you tell from the circle graphs which provinces showed the greatest change in total population?
- (iii) Write two questions that could be answered using the circle graphs you drew.

(7SP3.4, 7SP3.5)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*
**Lesson 4.7: Drawing Circle Graphs**

ProGuide: pp. 35-38

Master 4.12, 4.21, 4.30

CD-ROM: Unit 4 Masters

Prep Talk Video: Drawing Circle Graphs

SB: pp. 161-164

Practice and HW Book: pp. 96-99

**Web Links**

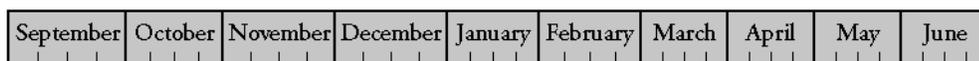
Links to the following sites can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>).

- The Stats Canada website is useful for finding statistics.
- A circle graph maker can be found at the National Library of Virtual Manipulatives.



# Operations with Fractions

Suggested Time: 4 Weeks



# Unit Overview

## Focus and Context

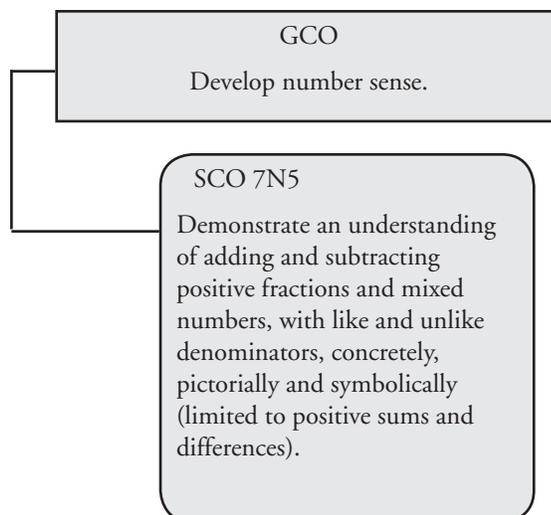
In this unit, students will perform addition and subtraction of fractions. They will use manipulatives such as fraction strips and fraction circles, number lines, and pattern blocks to model these fraction operations. This provides a concrete representation for a traditionally difficult concept.

The use of these concrete materials will lead students to the need for common denominators when adding, subtracting, comparing and ordering fractions. They will then be introduced to the common denominator algorithm. Finally, work with proper fractions will be extended to include addition and subtraction of mixed numbers. Throughout the unit, estimation using benchmarks will play an important role in helping students to decide if their answers are reasonable.

Developing a good understanding of adding and subtracting fractions will enable students to understand real-life situations that require fractions such as using measurements to follow recipes, budgeting money, and understanding timing in music. As well, numerous occupations require a solid understanding of fractions, including skilled trades such as plumbing and carpentry. Chefs, pharmacists and engineers all use fractions on a daily basis. Offering students the opportunity to work with fractions in a real-life context can be very beneficial.

Developing a solid foundation with fractions will prepare students for future study of algebra, rational expressions and equations, proportional reasoning and trigonometry.

## Outcomes Framework



## Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics  
 and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

## SCO Continuum

Grade 6	Grade 7	Grade 8
Number		
6N4. Relate improper fractions to mixed numbers. [CN, ME, R, V]	7N5. Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]	8N6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]

**Strand: Number****Outcomes**

*Students will be expected to*

**7N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences).**

[C, CN, ME, PS, R, V]

**Achievement Indicators:**

*7N5.1 Model addition of positive fractions, using concrete representations, and record symbolically.*

*7N5.2 Determine the sum of two given positive fractions with like denominators.*

**Elaborations—Strategies for Learning and Teaching**

Students are introduced to addition and subtraction of fractions in this unit. Throughout primary and elementary mathematics, students developed conceptual and procedural understandings of operations with whole numbers and decimals. This understanding of operations should be used to give meaning to fraction computations. Students have had previous experience relating improper fractions with mixed numbers in Grade 6 (6N4). Earlier in Grade 7, students compared and ordered positive fractions by using benchmarks and equivalent fractions (7N7). Multiplication and division of fractions will be introduced in Grade 8 (8N6).

This outcome limits questions to those resulting in positive sums and differences. This foundation will be further developed in Grade 9 with the study of rational numbers (9N3).

NCTM (2000) recommends that students be encouraged to use a variety of representations and be able to translate from one representation to another to enhance their understanding. Manipulatives allow students to visualize fractions concretely. As their conceptual understanding develops, students connect their drawings and symbolic representations of these abstract concepts.

When teaching students addition of fractions, an appropriate sequence begins with fractions with like denominators, then to fractions with unlike denominators, and finally to improper fractions and mixed numbers.

Pattern blocks and fraction circles can be used to model addition of fractions with like denominators. Pattern blocks are a good model for addition when the fractions have denominators 2, 3 or 6. Fraction circles lend themselves to more denominators, including 2, 3, 4, 5, 6, 8, 10, or 12.

Students should be encouraged to use drawings to help visualize their thinking. However, it is possible for students to make incorrect conclusions based on inaccurate drawings. Fractional parts of regions, especially circles, are often difficult to draw. The whole can be defined using other shapes, such as rectangles. Students should be encouraged to use and reflect on their drawings. Evaluating the strengths and weaknesses of various representations for a particular problem enhances students' understanding.

Once students have modelled addition of fractions with like denominators they should have little difficulty determining their sum.

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### *Observation*

- Create cards with addition expressions and their equivalent manipulative representations. Each student receives a card with either the addition expression, or the representation. Ask them to find their partner in the class. Each group must then explain why their cards match.

(7N5.1)

#### *Journal*

- Ask students to determine, with the aid of drawings, if the following is correct:  $\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$ . Ask them to explain their reasoning.

(7N5.1, 7N5.2)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 7*

Prep Talk Video: Operations with Fractions

#### Lesson 5.1: Using Models to Add Fractions

ProGuide: pp. 4-6 Master 5.13, 5.18, 5.27

CD-ROM: Unit 5 Masters

Classroom Videos: Using Models to Add Fractions, Parts 1, 2, 3

Student Book (SB): pp. 178-180

Practice and HW Book: pp. 106-108

#### Web Links

Links for the following activities can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>):

- The National Library of Virtual Manipulatives
- No Matter What Shape Your Fractions Are In

**Strand: Number**

**Outcomes**

*Students will be expected to*  
**7N5 Continued ...**

**Achievement Indicators:**

7N5.2 *Continued*

7N5.3 *Simplify a given positive fraction by identifying the common factor between the numerator and denominator.*

7N5.1 *Continued*

7N5.4 *Determine a common denominator for a given set of positive fractions.*

7N5.5 *Determine the sum of two given positive fractions with unlike denominators.*

**Elaborations – Strategies for Learning and Teaching**

Building from whole number addition, students can generalize addition of parts of a whole. For example, they can think of  $\frac{2}{9} + \frac{5}{9}$  as adding 2 ninths and 5 ninths to total 7 ninths, or  $\frac{7}{9}$ .

Throughout the unit, students should be encouraged to simplify fractions. They have previously worked with factors of whole numbers (6N3). The use of models can facilitate an understanding of fractional equivalents and expressing fractions in simplest form.

Once students use models to add fractions with like denominators, they can begin using pattern blocks, fraction circles, fraction strips and number lines to add fractions with related denominators (e.g., thirds and sixths, or fifths and tenths), and then to move to fractions with unrelated denominators (e.g., thirds and fourths).

Most of the tasks should involve fractions with “friendly” denominators no greater than 12. At this stage, avoid adding numbers that cannot be easily represented with any model or drawing.

Using models to add fractions with unrelated denominators leads students to discover the need for common denominators. For example, when students model  $\frac{1}{3} + \frac{1}{2}$ , they should quickly realize that the manipulative being used should be divided into sixths to find the sum. Students look for fraction pieces that can exactly cover one third and one half, in this case sixths. They may try several possibilities from their fraction circles first before arriving at this conclusion. Giving them the opportunity to come to this conclusion allows them to develop fraction number sense prior to being introduced to common denominators and other rules of computation.

Ideally, the least common multiple of the unlike denominators should be the common denominator used.

Progression can then be made to the symbolic level.

$$\frac{1}{3} + \frac{1}{2}$$

and

$$\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$$

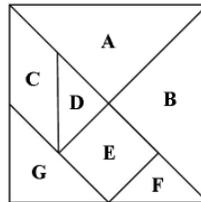
**General Outcome: Develop Number Sense.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Explain to students that a tangram is a square puzzle that is divided into seven shapes. Based on the tangram below, students can answer the following questions.

- (i) Given that piece A is  $\frac{1}{4}$  of the whole square, what are the values of pieces B, C, D, E, F and G?
- (ii) What is the sum of A and B? B and G? E and F?
- (iii) Which two tangram pieces add up to the value of B? C?
- (iv) Invent a problem on your own and solve it.



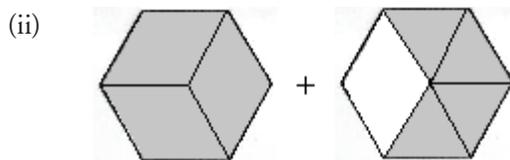
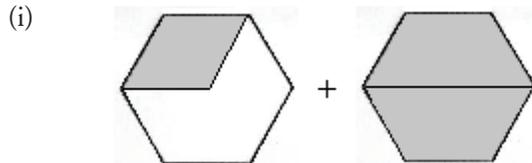
(7N5.1, 7N5.2, 7N5.5)

- Ask students to create three addition expressions with unlike denominators that are equivalent to  $\frac{6}{12} + \frac{3}{12}$ . (7N5.5)
- Provide students with the magic square. The sum of each row, column and diagonal in this magic square must equal 1. Ask them to find the missing values.

		$\frac{5}{12}$
$\frac{7}{12}$	$\frac{1}{3}$	
$\frac{1}{4}$		

(7N5.5)

- Ask students to write an addition sentence to represent the total fraction of each hexagon that is shaded and use the addition sentence to find the total value of the shaded hexagons in each case.



(7N5.1, 7N5.5)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

**Lesson 5.1: Using Models to Add Fractions**

**Lesson 5.2: Using Other Models to Add Fractions**

ProGuide: pp. 4-6, 7-11

Master 5.13, 5.18, 5.27

Master 5.10, 5.11, 5.14, 5.15, 5.16, 5.17, 5.19, 5.28

CD-ROM: Unit 5 Masters

Prep Talk Video: Using Other Models to Add Fractions

SB: pp. 178-180, 181-185

Practice and HW Book: pp. 106-108, 109-111

**Lesson 5.3: Using Symbols to Add Fractions**

ProGuide: pp. 12-15

Master 5.14, 5.15, 5.16, 5.17, 5.20, 5.29

CD-ROM: Unit 5 Masters

SB: pp. 186-189

Practice and HW Book: pp. 112-114

**Strand: Number****Outcomes***Students will be expected to*

7N5 Continued ...

**Achievement Indicator:**7N5.5 *Continued***Elaborations—Strategies for Learning and Teaching**

It is important for students to keep focus on the meanings of the numbers and the operations. Estimation should play a role in the development of strategies for working with fractions. Through the use of benchmarks (close to 0,  $\frac{1}{2}$  or 1) developed earlier (7N7), students should be encouraged to estimate the solution and use their estimate to verify the reasonableness of the answer obtained using the algorithm.

When calculating  $\frac{1}{3} + \frac{5}{8}$ , students should reason that  $\frac{1}{3}$  is a little bit less than  $\frac{1}{2}$  and  $\frac{5}{8}$  is more than  $\frac{1}{2}$ , so the answer should be close to 1. Then they can use the common denominator algorithm to determine the sum.

$$\begin{aligned} & \frac{1}{3} + \frac{5}{8} \\ & \frac{1}{3} \times \frac{8}{8} + \frac{5}{8} \times \frac{8}{8} \\ & \frac{8}{24} + \frac{15}{24} \\ & \frac{23}{24} \end{aligned}$$

Finally, they should check the reasonableness of their calculation by comparing it to their initial estimate.

It is important for students to work with problems such as  $\frac{1}{4} + \frac{1}{6} = ?$ . Many students quickly conclude that the lowest common denominator is the product of the given denominators because in so many questions this turns out to be correct. They need to see that the lowest common denominator is often smaller than the product of the two given denominators. Exposing students to an extreme case often makes this point well. For example, when adding  $\frac{1}{12}$  and  $\frac{1}{18}$ , the product of the denominators is 216, whereas the LCD is 36. The teacher can ask students which would be easier to calculate:  $\frac{18}{216} + \frac{12}{216} = ?$  or  $\frac{3}{36} + \frac{2}{36} = ?$

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### Performance

- Students could use pattern blocks to create a design on triangular grid paper and then use fraction addition to name the design. It is possible to use several different addition sentences to name the same design.

(7N5.1, 7N5.5)

- Connect Three - Addition of Fractions

This two-player game provides opportunity for students to practice fraction addition.

**Materials:** game board, two coloured counters, paper clips

#### How to Play:

- First player chooses two numbers on the bottom strip and places a paper clip on each. Player then adds those two numbers, and places a counter on the answer on the game board.
- Second player moves ONLY ONE of the paper clips on the bottom strip to make a second operation. Player then places a counter on the answer.
- Play continues until a player connects three answers in a horizontal, vertical, or diagonal row.

This game can be modified for subtraction of fractions.

(7N5.5)

#### Interview

- Ask students if:
  - adding fourths and thirds results in sixths
  - adding fourths and thirds results in sevenths
 They should justify their answers.

(7N5.1, 7N5.4, 7N5.5)

#### Journal

- Ask students to respond to the following:

Your friend missed yesterday's lesson. When solving a problem today, he suggested that  $\frac{5}{6} + \frac{5}{8} = \frac{10}{14}$ . How can you convince him that this is not a reasonable solution?

(7N5.1, 7N5.5)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 7*

Lesson 5.3: Using Symbols to Add Fractions

ProGuide: pp. 12-15

Master 5.14-5.17, 5.20, 5.29

CD-ROM: Master

SB: pp. 186-189

Practice and HW Book: pp. 112-114

#### Web Links

Links for the following activities can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>):

- Creating a Design With Fraction Addition
- Connect Three - Addition of Fractions

**Strand: Number**

**Outcomes**

*Students will be expected to*  
**7N5 Continued ...**

**Achievement Indicators:**

7N5.6 *Model subtraction of positive fractions, using concrete representations, and record symbolically.*

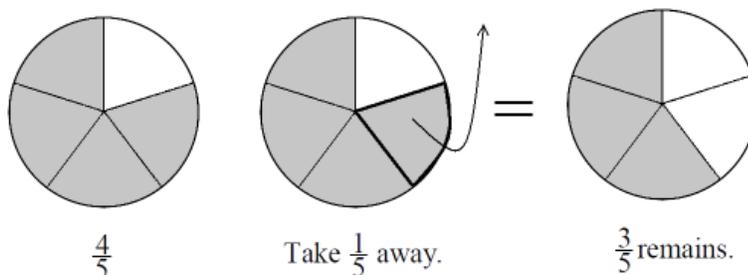
7N5.7 *Determine the difference of two given positive fractions.*

**Elaborations – Strategies for Learning and Teaching**

Subtraction of fractions, as with whole numbers, is the inverse operation of addition.

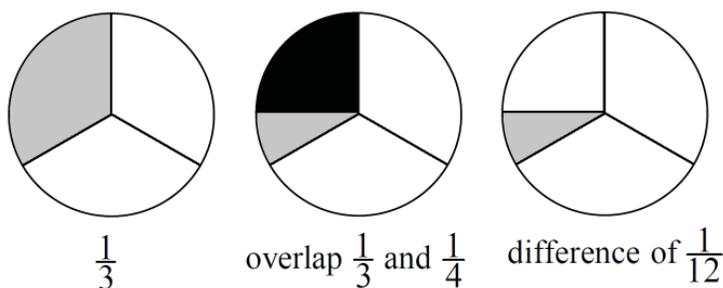
Subtraction should be visualized through the use of various models, including pattern blocks, fraction circles, fraction strips and number lines. Modelling subtraction of fractions with like denominators, students can physically remove a fraction piece to determine the difference.

For example:  $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$



Modelling subtraction of fractions with unlike denominators, students can concretely overlap the minuend and subtrahend to determine the difference.

For example,  $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$   
 (minuend) (subtrahend) (difference)



Ensure all subtractions have the minuend larger than the subtrahend to result in a positive difference.

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### Observation

- Ask students to use concrete materials or diagrams to show why the following is an incorrect procedure:  

$$\frac{3}{8} - \frac{1}{4} = \frac{3-1}{8-4} = \frac{2}{4} = \frac{1}{2}$$

(7N5.6, 7N5.7)
- Create cards with subtraction expressions and their equivalent manipulative representations. Each student would receive a card with either the subtraction expression or the representation. They find their partner in the class. Each group must then explain why their cards match.  

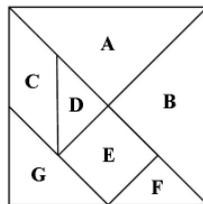
(7N5.6)

#### Paper and Pencil

- When one fraction is subtracted from another fraction, the difference is zero. The fractions have different denominators. Ask students to determine what the fractions could be. They should give two possible answers.  

(7N5.7)
- Ask students to use concrete materials of their choice to make up two subtraction questions. Students draw diagrams to show their questions. They can challenge a classmate to answer their questions using the materials they chose.  

(7N5.6)
- The tangram piece labelled “A” is removed from a finished tangram. Ask students to write a subtraction sentence to show the fraction of the completed tangram that remains.



(7N5.6, 7N5.7)

- Using the tangram above, ask students to write and answer subtraction questions for:
  - $A - D$
  - $B - E$

(7N5.6, 7N5.7)

### Resources/Notes

#### Authorized Resource

#### *Math Makes Sense 7*

#### Lesson 5.4: Using Models to Subtract Fractions

#### Lesson 5.5: Using Symbols to Subtract Fractions

ProGuide: pp.17-20, 21-24

Master 5.12, 5.14-5.17, 5.21, 5.22, 5.30, 5.31

CD-ROM: Master

Prep Talk Video: Using Models to Subtract Fractions

SB: pp. 191-194, 195-198

Practice and HW Book: pp. 115-117, 118-120

**Strand: Number****Outcomes**

*Students will be expected to*

7N5 Continued ...

**Achievement Indicators:**

7N5.8 *Model addition and subtraction of mixed numbers, using concrete representations, and record symbolically.*

7N5.9 *Determine the sum or difference of two mixed numbers.*

7N5.10 *Simplify the solution to a given problem involving the sum or difference of two positive fractions or mixed numbers.*

7N5.11 *Solve a given problem involving the addition or subtraction of positive fractions or mixed numbers and determine if the solution is reasonable.*

**Elaborations – Strategies for Learning and Teaching**

Addition and subtraction of mixed numbers is developed from the models and algorithms for addition and subtraction of positive fractions.

Various models, including Cuisenaire rods and fraction strips, can help students visualize working with mixed numbers. A link for using Cuisenaire rods is referenced in the resource section on page 113.

There are two numerical approaches for adding and subtracting mixed numbers. For addition and subtraction, students may start by converting mixed numbers to improper fractions. Alternately, for addition they may add the whole number portion separately from the fraction portion. In this case, they may have to simplify a mixed number containing an improper fraction to a mixed number with a proper fraction (e.g.,  $2\frac{19}{18} = 3\frac{1}{18}$ ). For subtraction, students may subtract the fraction portions separately if the fractional portion of the minuend is larger than the fractional portion of the subtrahend (e.g.,  $3\frac{6}{7} - 2\frac{1}{3}$ , note  $\frac{6}{7} > \frac{1}{3}$ ). Otherwise, students will have to regroup from the whole number portion (e.g.,  $2\frac{4}{7} - 1\frac{2}{3} = 1\frac{33}{21} - 1\frac{14}{21} = \frac{19}{21}$ ).

All sums and differences should be simplified to lowest terms as a proper fraction or mixed number.

Connections to real world applications should be used throughout the development of adding and subtracting positive fractions and mixed numbers. Various examples include recipes involving cups, timed tasks involving hours, or capacity involving portions.

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### Interview

- Give students a variety of addition and subtraction expressions including positive fractions and mixed numbers. Ask them to explain how to determine the sum or difference using concrete materials, drawings, or descriptions.

(7N5.8, 7N5.9)

#### Journal

- Give students solutions to a variety of addition or subtraction expressions, some of which have errors in them. For example,

$$12\frac{1}{4} - 9\frac{2}{3} = 3\frac{5}{12}$$

$$\frac{7}{8} + \frac{1}{3} = \frac{8}{11}$$

$$2\frac{1}{5} - 1\frac{3}{5} = \frac{8}{5}$$

Ask them to find the errors and explain how they would correct them. (7N5.9)

- Ask students if it is possible to find two mixed numbers which add together to form a whole number. They should explain their answer and, if possible, give an example. (7N5.9)

#### Paper and Pencil

- Students could answer questions such as the following:
  - Andrew plays guitar in a rock band. For a song that is 36 measures long he plays for  $4\frac{1}{2}$  measures, rests for  $8\frac{3}{8}$  measures, plays for another 16 measures, rests for  $2\frac{1}{4}$  measures and plays for the last section. How many measures are in the last section?
  - This week, Mark practised piano for  $3\frac{1}{2}$  hours, played soccer for  $6\frac{1}{4}$  hours, and talked on the phone for  $4\frac{1}{3}$  hours.
 

How many hours did Mark spend practising piano and playing soccer?

Hour many more hours did Mark spend playing soccer than talking on the phone?

(7N5.9, 7N5.10, 7N5.11)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 7*

##### Lesson 5.6: Adding with Mixed Numbers

##### Lesson 5.7: Subtracting with Mixed Numbers

ProGuide: pp. 25-29, 30-34  
 Master 5.13, 5.14, 5.15, 5.16,  
 5.17, 5.23, 5.24, 5.32, 5.33

CD-ROM: Master

Prep Talk Video: Subtracting with Mixed Numbers

SB: pp. 199-203, 204-208

Practice and HW Book: pp. 121-122, 123-124

#### Web Link

A link for an introduction to Cuisenaire rods and their use in the study of fractions can be found in Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>).



# Equations

Suggested Time: 3 Weeks

September	October	November	December	January	February	March	April	May	June
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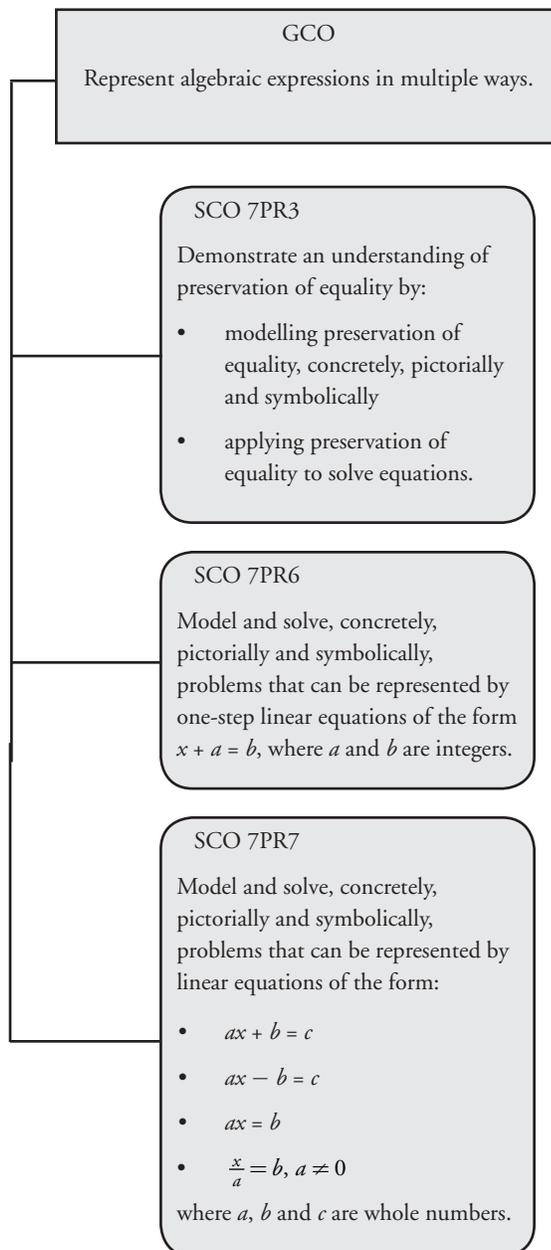


# Unit Overview

## Focus and Context

In this unit, the focus is on understanding the preservation of equality and solving equations concretely, pictorially and symbolically. Students will begin to solve equations using systematic trial and inspection. They will sometimes recognize the solution to an equation instantly. They will, however, be expected to explain their reasoning before they move on to solving equations with two-pan balance models and algebra tiles. Students will solve equations limited to a maximum of two steps. Ultimately, students will apply algebraic techniques, requiring the use of preservation of equality, to solve equations.

## Outcomes Framework



## Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics  
 and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

## SCO Continuum

Grade 6	Grade 7	Grade 8
Patterns and Relations (Variables and Equations)		
<p>6PR4. Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.            [C, CN, PS, R, V]</p>	<p>7PR3. Demonstrate an understanding of preservation of equality by:</p> <ul style="list-style-type: none"> <li>• modelling preservation of equality, concretely, pictorially and symbolically</li> <li>• applying preservation of equality to solve equations.</li> </ul> <p>[C, CN, PS, R, V]</p> <p>7PR6. Model and solve, concretely, pictorially and symbolically, problems that can be represented by one-step linear equations of the form <math>x + a = b</math>, where <math>a</math> and <math>b</math> are integers.            [CN, PS, R, V]</p> <p>7PR7. Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form:</p> <ul style="list-style-type: none"> <li>• <math>ax + b = c</math></li> <li>• <math>ax - b = c</math></li> <li>• <math>ax = b</math></li> <li>• <math>\frac{x}{a} = b, a \neq 0</math></li> </ul> <p>where <math>a, b</math> and <math>c</math> are whole numbers.            [CN, PS, R, V]</p>	<p>8PR2. Model and solve problems using linear equations of the form:</p> <ul style="list-style-type: none"> <li>• <math>ax = b</math></li> <li>• <math>\frac{x}{a} = b, a \neq 0</math></li> <li>• <math>ax + b = c</math></li> <li>• <math>\frac{x}{a} = b, a \neq 0</math></li> <li>• <math>a(x + b) = c</math></li> </ul> <p>concretely, pictorially and symbolically, where <math>a, b</math> and <math>c</math> are integers.            [C, CN, PS, V]</p>

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

*Students will be expected to*

**7PR7 Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form:**

- $ax + b = c$
- $ax - b = c$
- $ax = b$
- $\frac{x}{a} = b, a \neq 0$

where  $a$ ,  $b$  and  $c$  are whole numbers.

[CN, PS, R, V]

### Achievement Indicator:

*7PR7.2 Solve a given linear equation by inspection and by systematic trial.*

### Elaborations—Strategies for Learning and Teaching

Students have had exposure to equations in earlier grades. In Grade 5, they worked with single variable, one-step equations with whole number coefficients and solutions (5PR2). In Grade 6, they studied the meaning of preservation of equality (6PR4). Earlier in Grade 7, students began developing the foundation necessary for work with linear equations. In the Patterns and Relations unit, they distinguished between linear expressions and equations and began using algebra tiles to solve equations. This concept will be further explored in this unit with the inclusion of tiles to represent negative integers (7PR6). When choosing examples for this outcome, be certain to only use whole numbers for  $a$ ,  $b$  and  $c$ . In the Integers unit, work was limited to the addition and subtraction operations. Since multiplication and division will not be introduced until Grade 8, students are not expected to multiply or divide integers when solving equations in this unit. Therefore, when selecting equations of the form  $ax + b = c$ , ensure that  $b < c$ . In the equation  $3x + 9 = 6$ , although the  $a$ ,  $b$  and  $c$  values are whole numbers, the value of  $3x$  is  $-3$ . Since  $3x$  is a multiplication statement, students are not expected to be able to determine that  $3(-1) = -3$  and, therefore,  $x = -3$ .

While some students may immediately arrive at the value of the unknown (the variable), it is important to work through the various strategies presented in this unit since work with algebraic equations will become increasingly complex throughout later grades.

When using systematic trial, students are expected to choose a reasonable value to substitute for the variable, and then evaluate using the order of operations to determine if the chosen value for the variable maintains the equality of the two expressions. If the chosen value does not work, students should question whether it is too small or too large, and then choose another value. They continue in this way until the correct value is found. At first, students might begin by using the guess-and-check strategy. By observing patterns in their results, they should become more systematic in the guesses they make. An example was provided in the Patterns and Relations unit.

Inspection differs from systematic trial. It is not a guess-and-check approach. To solve  $3x + 7 = 19$ , students will replace the  $3x$  with the value needed to add to 7 to get 19. They will then determine that the value, 12, is equal to  $3(4)$ . Therefore  $x = 4$ . This can also be thought of as the “cover-up” method. Using the same equation, cover up the  $3x$  and ask “What added to 7 makes 19?” Next, cover up the  $x$  and ask “What multiplied by 3 makes 12?”

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**General Outcome: Represent algebraic expressions in multiple ways.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Ask students to answer the questions below based on the following situation:  
A hockey school charges \$80 per day to use the facility plus \$20 per player per day for food, equipment and lessons. A team raised \$320 for a one-day practice.
  - (i) Write an equation to represent this situation.
  - (ii) Solve the equation, first using systematic trial and then by inspection, to determine how many players are on the team. Which method do you prefer? Why?

(7PR7.2)

*Journal*

- Ask students to respond to the following:
  - (i) When solving  $4d + 24 = 36$ , Sarah chose 3 for her first value for  $d$  and Billy chose 6. Which number is the better choice? Explain how you made your decision.
  - (ii) Ryan was asked to solve the equation  $5d + 7 = 22$  for  $d$ . Using inspection, he found that  $d = 15$ . He was told that his answer was incorrect. Explain Ryan's mistake and how he should solve the equation correctly.

(7PR7.2)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*

Prep Talk Video: Equations

**Lesson 6.1: Solving Equations**

ProGuide: pp. 4-9

Master 6.9, 6.18

CD-ROM: Unit 6 Masters

Student Book (SB): pp. 220-225

Practice and HW Book: pp. 132-134

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

Students will be expected to

**7PR3 Demonstrate an understanding of preservation of equality by:**

- modelling preservation of equality, concretely, pictorially and symbolically
- applying preservation of equality to solve equations.

[C, CN, PS, R, V]

7PR7 Continued ...

#### Achievement Indicators:

7PR3.1 Model the preservation of equality for each of the four operations using concrete materials or pictorial representations, explain the process orally and record it symbolically.

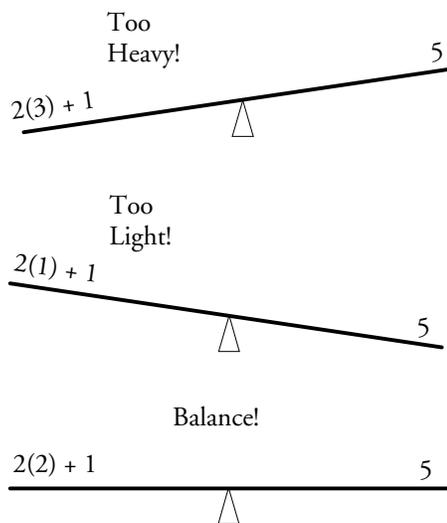
7PR3.2 Write equivalent forms of a given equation by applying the preservation of equality, and verify, using concrete materials, e.g.,  $3b = 12$  is the same as  $3b + 5 = 12 + 5$  or  $2r = 7$  is the same as  $3(2r) = 3(7)$ .

7PR7.1 Model a given problem with a linear equation and solve the equation using concrete models, e.g., counters, integer tiles.

### Elaborations—Strategies for Learning and Teaching

The National Council of Teachers of Mathematics states that “In understanding equality, one of the first things students must realize is that equality is a relationship, not an operation” (2000-2007). Students often think of the equal sign as a symbol that tells them to do something or find the answer. “They should come to view the equals sign as a symbol of equivalence and balance” (NCTM 2000, p. 39).

Students can use the two-pan balance scale approach in solving equations using systematic trial or inspection. Consider the example  $2x + 1 = 5$ .



You may find examples such as the following useful.



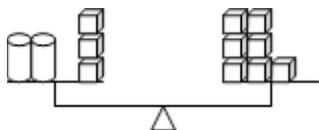
Ask students to consider what would happen if 5 is added to the left pan, why, and what would be needed to balance the pans. Teachers should provide ample visual examples using each of the four operations so that students realize that in order to preserve equality, what is done to one side must also be done to the other. This understanding of preserving equality between the two sides (expressions) of the equation is crucial to work with equations, especially solving equations symbolically.

## General Outcome: Represent algebraic expressions in multiple ways.

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Students should write the equation represented by the balance scale below. Ask them to solve the equation pictorially and explain their solution.

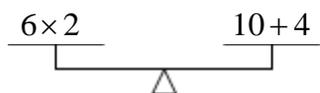


(7PR3.1)

- Ask students to write two equations that are equivalent to  $3n + 1 = 5$  and verify using a model.

(7PR3.2)

- Ask students if the following diagrams are correct. Ask them to explain their reasoning.



(7PR3.2)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 7*

Lesson 6.2: Using a Model to Solve Equations

Lesson 6.3: Solving Equations Involving Integers

Lesson 6.4: Solving Equations Using Algebra

Lesson 6.5: Using Different Methods to Solve Equations

ProGuide: pp. 10-14, 15-19, 21-23, 24-28

Master 6.10-6.13, 6.19-6.22

PM 30

CD-ROM: Unit 6 Masters

SB: pp. 226-230, 231-235, 237-239, 240-244

Practice and HW Book: pp. 135-137, 138-140, 141-144, 145-147

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

Students will be expected to

7PR3 and 7PR7 Continued ...

### Achievement Indicators:

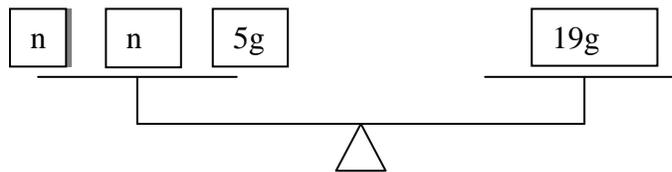
7PR3.1, 7PR3.2, 7PR7.1  
Continued

7PR7.3 Draw a visual representation of the steps used to solve a given linear equation.

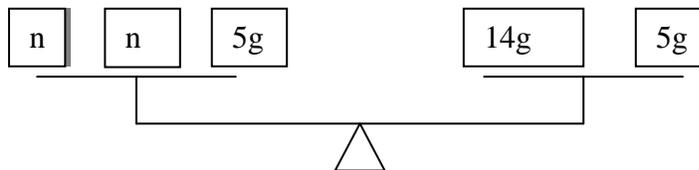
7PR7.4 Solve a given problem, using a linear equation, and record the process.

### Elaborations—Strategies for Learning and Teaching

The two-pan balance scale can then be used to help students move from the pictorial to the symbolic representation, as in the example below.



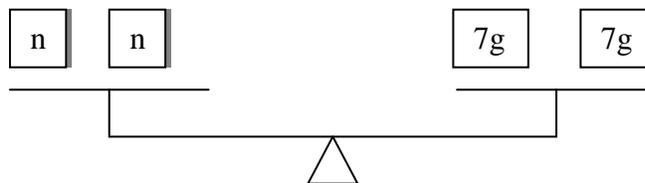
$$2n + 5 = 19$$



To isolate  $2n$ , subtract 5 from each side.

$$2n + 5 - 5 = 19 - 5$$

$$2n = 14$$



Divide each side by 2.

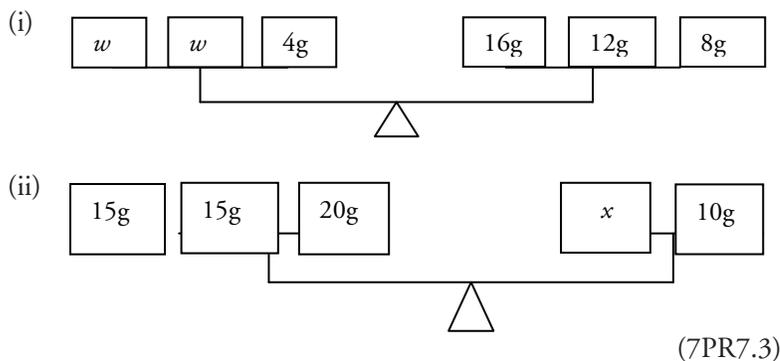
$$\frac{2n}{2} = \frac{14}{2}$$

$$n = 7$$

This approach is very effective in modeling the preservation of equality when writing equivalent forms of equations. Explore changes to each pan of the balance scale when a change is made to one of the pans.

**General Outcome: Represent algebraic expressions in multiple ways.**
**Suggested Assessment Strategies**
*Paper and Pencil*

- Ask students to find the values of the unknown mass on each balance scale and to sketch the steps used.


**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*
**Lesson 6.2: Using a Model to Solve Equations**
**Lesson 6.3: Solving Equations Involving Integers**
**Lesson 6.4: Solving Equations Using Algebra**
**Lesson 6.5: Using Different Methods to Solve Equations**

ProGuide: pp. 10-14, 15-19, 21-23, 24-28

Master 6.10-6.13, 6.19-6.22

PM 30

CD-ROM: Unit 6 Masters

SB: pp. 226-230, 231-235, 237-239, 240-244

Practice and HW Book: pp. 135-137, 138-140, 141-144, 145-147

**Strand: Patterns and Relations (Variables and Equations)**

**Outcomes**

*Students will be expected to*  
7PR3 and 7PR7 Continued ...

**Achievement Indicators:**

7PR3.1, 7PR3.2, 7PR7.3 and  
7PR7.4 *Continued*

**Elaborations—Strategies for Learning and Teaching**

Algebra tiles or a similar manipulative as used in the Patterns and Relations unit (7PR7) should also be incorporated. In this unit, students should now be able to record the steps symbolically as well as pictorially using either the two-pan balance scale or algebra tiles. Students should practice recording the steps symbolically using equations with one operation before recording the steps symbolically using two operations as used in the example  $2x + 1 = 5$ .

Concrete Representation	Symbolic Representation
	$2x + 1 = 5$
	Remove a unit tile from each side: $2x + 1 - 1 = 5 - 1$  Simplify: $2x = 4$
	Since we have two $x$ tiles, we separate both sides into two equal groups.
	Each $x$ -tile is paired with 2 unit tiles. Therefore, the solution is: $x = 2$

7PR7.5 *Verify the solution to a given linear equation, using concrete materials and diagrams.*

7PR7.6 *Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.*

Once students solve a linear equation, it is important that the solution is verified. This can be done by redrawing the diagram if the two-pan balance scale was used. When algebra tiles are used, students should replace each variable tile with the value in the solution to determine if both sides of the equation remain equal.

Students can then symbolically verify a solution by substituting the variable with the value identified, similar to the method used in systematic trial. Students can also use substitution to verify a solution when encountering a problem where a suggested solution is given.

## General Outcome: Represent algebraic expressions in multiple ways.

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to sketch balance scales to represent each equation, solve and verify the solution.
  - $2y = 18$
  - $3n + 2 = 17$

(7PR7.3, 7PR7.5)

- Ask students to show whether or not  $x = 7$  is the solution to each equation.
  - $6x = 48$
  - $3x + 2 = 20$
  - $\frac{x}{7} = 1$

(7PR7.6)

- Students can verify the following solution to the equation  $\frac{f}{8} = 10$ . If they determine that the solution is incorrect, ask them to draw a model with the correct solution.

Solution:

$$\frac{f}{8} = 10$$

$$\frac{f}{8} - 8 = 10 - 8$$

$$f = 2$$

(7PR7.5)

#### *Performance*

- Students can work in pairs for the activity *Pass the Problem*. Each pair gets a problem that can be modelled with a linear equation. Ask one student to write the first line of the solution and then pass it to the second student. The second student verifies the workings and checks for errors. If there is an error, students should discuss what the error is and why it occurred. The student then writes the second line of the solution and passes it to their partner. This process continues until the solution is complete. Students should then verify the solution.

#### Sample Problem:

Jacob paid \$19 for two shirts and a pair of sunglasses. The sunglasses cost \$5. How much did each shirt cost?

(7PR7.4, 7PR7.5)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 7*

Lesson 6.2: Using a Model to Solve Equations

Lesson 6.3: Solving Equations Involving Integers

Lesson 6.4: Solving Equations Using Algebra

Lesson 6.5: Using Different Methods to Solve Equations

ProGuide: pp. 10-14, 15-19, 21-23, 24-28

Master 6.10-6.13, 6.19-6.22

PM 30

CD-ROM: Unit 6 Masters

SB: pp. 226-230, 231-235, 237-239, 240-244

Practice and HW Book: pp. 135-137, 138-140, 141-144, 145-147

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

Students will be expected to

7PR6 Model and solve, concretely, pictorially and symbolically, problems that can be represented by one-step linear equations of the form  $x + a = b$ , where  $a$  and  $b$  are integers.

[CN, PS, R, V]

### Achievement Indicators:

7PR6.1 Represent a given problem with a linear equation and solve the equation using concrete models, e.g., counters, integer tiles.

7PR6.2 Draw a visual representation of the steps required to solve a given linear equation.

7PR6.3 Solve a given problem using a linear equation.

7PR6.4 Verify the solution to a given linear equation using concrete materials and diagrams.

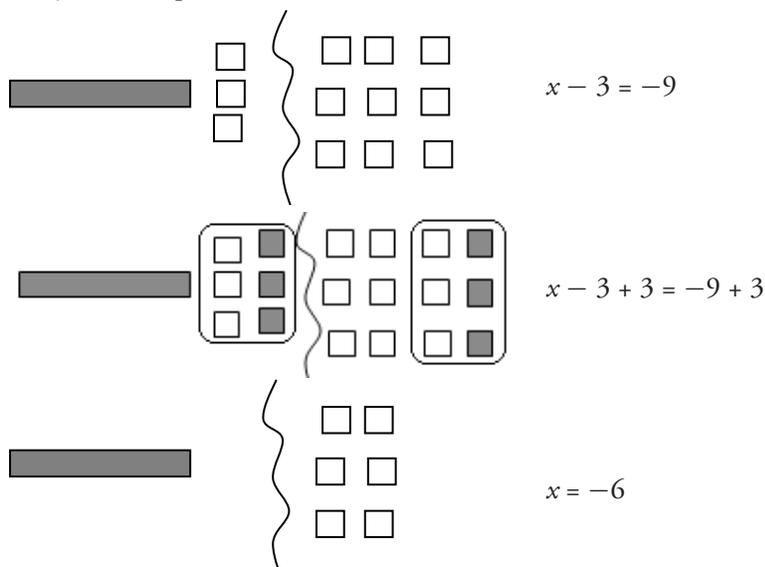
7PR6.5 Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.

### Elaborations—Strategies for Learning and Teaching

Work with equations from outcome 7PR7 will now be extended. Students will apply their knowledge of integer addition and subtraction to solve one-step equations of the form  $x + a = b$ .

Students will now begin using two different coloured algebra tiles. Regardless of the colour tiles you have available, decide which colour will represent positive and which will represent negative. Throughout this curriculum guide, shaded tiles represent positive values and white tiles represent negative values.

Students have used algebra tiles or a similar manipulative to solve linear equations involving whole numbers, and will extend this knowledge to include all integers. This outcome, however, does not include equations which involve multiplication and division. Students will be required to draw upon outcome 7N6 from the Integers unit. To model an equation that uses subtraction such as  $x - 3 = -9$  students must recall that subtracting 3 is equivalent to adding negative 3, represented by 3 unit tiles in a colour different than the positive tiles. To isolate the variable, zero pairs are made by adding 3 positive tiles to each side. Once the zero pairs are removed, the tiles show that  $x = -6$ . As equations are modelled, it is a good idea for students to record the process symbolically. This will help with the transition from the concrete and pictorial representations to the symbolic representation.



Students should then verify the solution by replacing the variable tile in the original equation with the appropriate number of unit tiles. In the above example, 6 negative tiles would be used. They should consider in advance what might be a reasonable solution, and be aware that once they acquire a solution, it can be checked for accuracy by substitution into the original equation.

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**General Outcome: Represent algebraic expressions in multiple ways.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Students should sketch the steps used to solve each equation, and then verify the solution.
  - (i)  $n - 3 = 4$
  - (ii)  $h + 1 = -2$
  - (iii)  $2 = y - 6$
  - (iv)  $w - 4 = 1$

(7PR6.2, 7PR6.4)
  
- Ask students to write an equation for each problem and then use algebra tiles to solve and verify.
  - (i) The temperature dropped  $5^{\circ}\text{C}$  to  $-2^{\circ}\text{C}$ . What was the original temperature?
  - (ii) Frank is 9 years old. He is 4 years older than Joe. How old is Joe?
  - (iii) Susan borrowed books from the library. She then returned 4 books. If she still has 3 books at home, how many did she borrow?

(7PR6.1, 7PR6.2, 7PR6.3, 7PR6.4)
  
- Ask students to identify which of the equations have the solution  $x = -2$ .
  - (i)  $x + 3 = -5$
  - (ii)  $x - 3 = -5$
  - (iii)  $x - 7 = -5$
  - (iv)  $x + 3 = 1$

(7PR6.5)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*
**Lesson 6.3: Solving Equations Involving Integers**
**Lesson 6.4: Solving Equations Using Algebra**
**Lesson 6.5: Using Different Methods to Solve Equations**

ProGuide: pp. 15-19, 21-23, 24-28

Master 6.11-6.13, 6.20-6.22

CD-ROM: Unit 6 Masters

Prep Talk Video: Solving Equations Involving Integers

SB: pp. 231-235, 237-239, 240-244

Practice and HW Book: pp. 138-140, 141-144, 145-147

**Strand: Patterns and Relations (Variables and Equations)****Outcomes**

*Students will be expected to*

7PR3 Continued ...

**Achievement Indicator:**

7PR3.3 *Solve a given problem by applying preservation of equality.*

**Elaborations—Strategies for Learning and Teaching**

Having modeled the solutions to numerous linear equations, students should begin solving linear equations symbolically (algebraically). They should be encouraged to continue to visualize models as needed while they solve the equations symbolically, using algebra and preserving equality as they work through the solutions.

Be sure to expose students to examples of each of the following types of linear equations:

- $x + a = b$ , where  $a$  and  $b$  are integers
- one and two-step equations where  $a$ ,  $b$  and  $c$  are whole numbers:
  - (i)  $ax + b = c$
  - (ii)  $ax - b = c$
  - (iii)  $ax = b$
  - (iv)  $\frac{x}{a} = b$ ,  $a \neq 0$

It can be challenging to create problem situations which require algebra to solve them. Many problems can be solved using methods such as guess-and-check and systematic trial. It may be necessary, therefore, to specify the strategy in some instances to ensure that problem solving using algebra is done. When the numbers used are large, it can be more easily illustrated that algebra is a tool which can be used to readily solve problems that might otherwise be very tedious to solve using methods such as guess-and-check.

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**General Outcome: Represent algebraic expressions in multiple ways.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Students can solve equations such as the following algebraically.
  - (i)  $3x = 24$
  - (ii)  $\frac{x}{7} = 7$
  - (iii)  $6x + 5 = 29$
  - (iv)  $x - 8 = 19$
  - (v)  $x + 7 = -3$

(7PR3.3)
  
- Ask students to write an equation for sentences such as the following:
  - (i) The cost shared by 5 people amounts to \$35 each.
  - (ii) There are 38 boys. This is 6 more than double the number of girls.
  - (iii) 60 centimetres is one half of Bob's height.

Students should solve the equations algebraically and verify the solution.

(7PR3.3)
  
- Ask students to explain whether or not the following equations were solved correctly.
  - (i)  $f - 3 = -2$   
 $f - 3 - 3 = -2 - 3$   
 $f = -5$
  
  - (ii)  $2w + 4 = 12$   
 $2w + 4 - 4 = 12 - 4$   
 $2w = 8$   
 $w = 8 \times 2$   
 $w = 16$

(7PR3.3)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*
**Lesson 6.4: Solving Equations Using Algebra**
**Lesson 6.5: Using Different Methods to Solve Equations**

ProGuide: pp. 21-23, 24-28

Master 6.12, 6.13, 6.21, 6..22

CD-ROM: Unit 6 Masters

See It Video: Equation Baseball

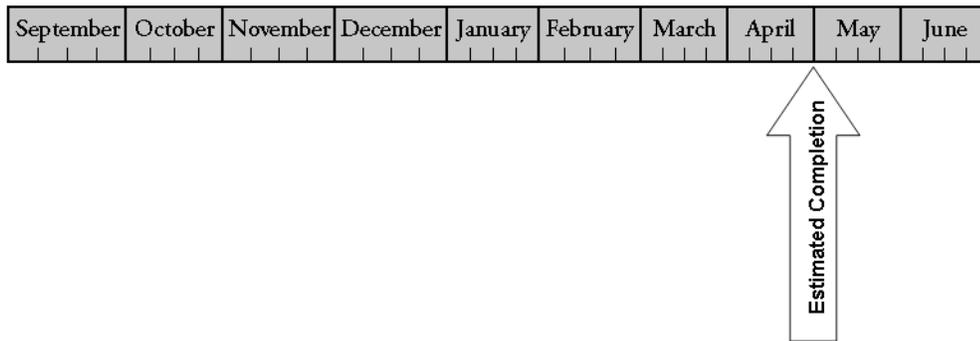
SB: pp. 237-239, 240-244

Practice and HW Book: pp. 141-144, 145-147



# Data Analysis

Suggested Time: 3 Weeks



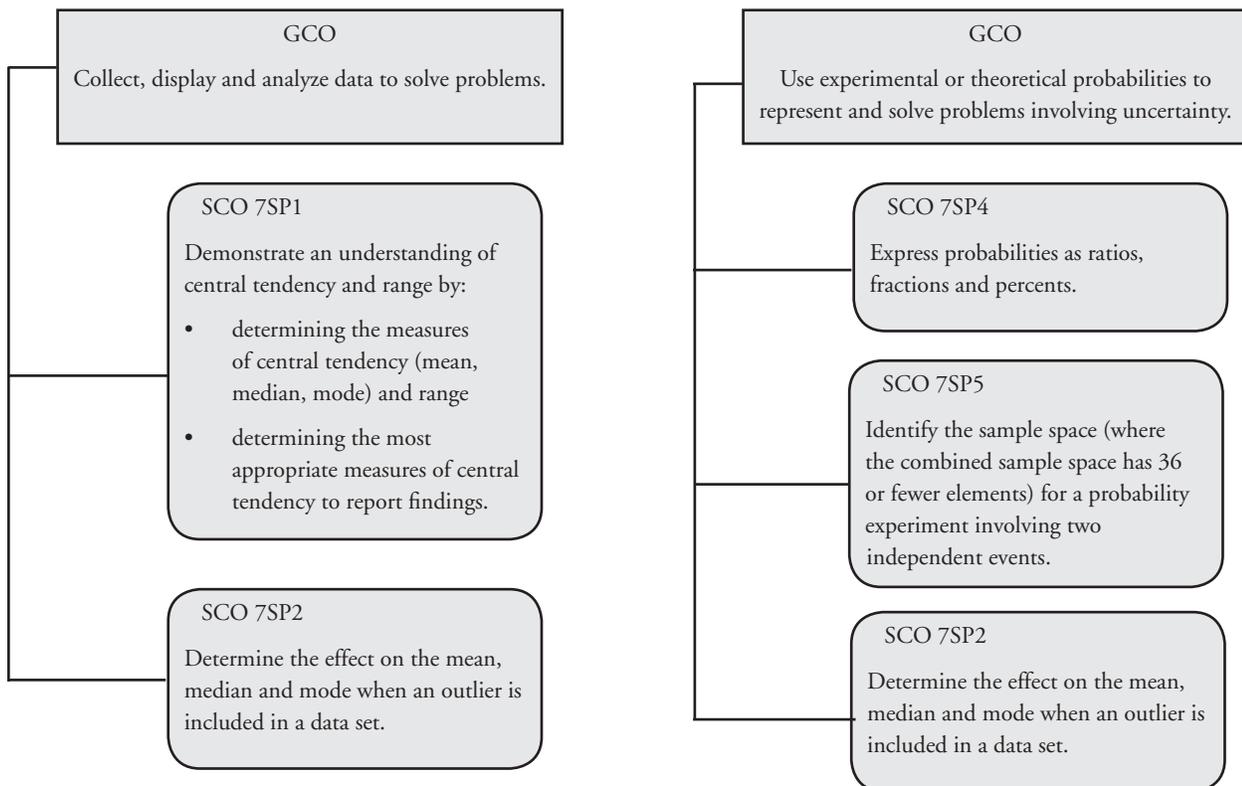
# Unit Overview

## Focus and Context

The three measures of central tendency are mean, median and mode. In this unit, students will examine sets of data to determine the measures of central tendency. Each one of these is a way to describe a set of data with a single meaningful number. In some situations, only one of the averages makes sense, but in other situations, two or three averages are meaningful, even if they are different. Students will determine which measure is the best representative of a given data set. The presence and effects of outliers will be considered.

Following data analysis, students will work with probability. These topics are examined together because it is through collecting, organizing, representing, and analyzing data that students are able to draw conclusions about probability. Using tree diagrams and tables, students will develop sample spaces for events, and then determine the probability of two independent events. They will compare theoretical probabilities with experimentally determined probabilities, and see that as the number of trials in an experiment increases the experimental probability of an event occurring gets closer to the theoretical probability of that event occurring.

## Outcomes Framework



## Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics  
 and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

## SCO Continuum

Grade 6	Grade 7	Grade 8
Statistics and Probability		
<p>6SP1. Create, label and interpret line graphs to draw conclusions.            [C, CN, PS, R, V]</p> <p>6SP2. Select, justify and use appropriate methods of collecting data, including:</p> <ul style="list-style-type: none"> <li>questionnaires</li> <li>experiments</li> <li>databases</li> <li>electronic media.</li> </ul> <p>[C, CN, PS, R, T]</p> <p>6SP3. Graph collected data, and analyze the graph to solve problems.</p> <p>6SP4. Demonstrate an understanding of probability by:</p> <ul style="list-style-type: none"> <li>identifying all possible outcomes of a probability experiment</li> <li>differentiating between experimental and theoretical probability</li> <li>determining the theoretical probability of outcomes in a probability experiment</li> <li>determining the experimental probability of outcomes in a probability experiment</li> <li>comparing experimental results with the theoretical probability for an experiment.</li> </ul> <p>[C, ME, PS, T]</p>	<p>7SP1. Demonstrate an understanding of central tendency and range by:</p> <ul style="list-style-type: none"> <li>determining the measures of central tendency (mean, median and mode) and range</li> <li>determining the most appropriate measures of central tendency to report findings.</li> </ul> <p>[C, PS, R, T]</p> <p>7SP2. Determine the effect on the mean, median and mode when an outlier is included in a data set.            [C, CN, PS, R]</p> <p>7SP4. Express probabilities as ratios, fractions and percents.            [C, CN, R, T, V]</p> <p>7SP5. Identify sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.            [C, ME, PS]</p> <p>7SP6. Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events.            [C, PS, R, T]</p>	<p>8SP1. Critique ways in which data is presented.            [C, R, T, V]</p> <p>8SP2. Solve problems involving the probability of independent events.            [C, CN, PS, T]</p>

## Strand: Statistics and Probability (Data Analysis)

### Outcomes

*Students will be expected to*

**7SP1 Demonstrate an understanding of central tendency and range by:**

- determining the measures of central tendency (mean, median, mode) and range
- determining the most appropriate measures of central tendency to report findings.

[C, PS, R, T]

### Achievement Indicator:

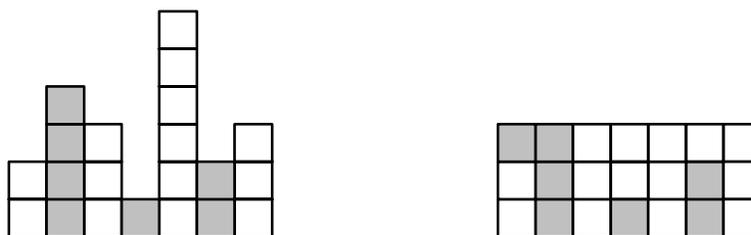
7SP1.1 *Determine mean, median and mode for a given set of data, and explain why these values may be the same or different.*

### Elaborations—Strategies for Learning and Teaching

Students are introduced to statistical measures of central tendency and range in this unit. The centre may be viewed in five ways:

- Point of balance - arithmetic mean
- Algorithm - arithmetic mean
- Midpoint - median
- Most often - mode
- Reasonable - represents a situation

The mean (arithmetic mean) can be computed as a point of balance or by using an algorithm. As a point of balance, students can find the arithmetic mean of a data set using blocks. The blocks can be rearranged so each column has equal height. When using this strategy, ensure the data sets used have a small number of elements with low numbers, and that they balance to a whole number. The data set 2, 4, 3, 1, 6, 2, 3, for example, has a mean of 3, as can be seen from the diagram below.



Alternatively, students could arrange themselves into two groups of 2, one group of 4, two groups of 3, one group of 1, and one group of 6, and then redistribute themselves into seven equal groups to determine how many students would be in each new group.

Using an algorithm, students can compute the mean by adding all the numbers in a data set and dividing the sum by the number of elements added. The mean of the data set 40, 51, 65, 75, 75, 90 is calculated below.

$$\text{Mean} = \frac{(40 + 51 + 65 + 75 + 75 + 90)}{6} = 66$$

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**General Outcome: Collect, display and analyze data to solve problems.**


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**Suggested Assessment Strategies**
*Performance*

- Students can create a tri-fold foldable to define and create examples of each of the measures of central tendency. On each of the outside panels, they can name and define mean, median or mode. On the corresponding inside panel, they can create and solve an example of a problem using the measure of central tendency on the front.

(7SP1.1)

*Journal*

- Students can create a set of data for each of the following. Each set must have at least 6 pieces of data.
  - Situation 1: The mean, median and mode are the same.
  - Situation 2: The mean, median and mode are different.
 Ask them to write about which situation they found more challenging.

(7SP1.1)

*Paper and Pencil*

- Ask students to create a set of five numbers where the median and mode are the same. They should explain why they chose those numbers.

(7SP1.1)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*

Prep Talk Video: Data Analysis

**Lesson 7.1: Mean and Mode**
**Lesson 7.2: Median and Range**

ProGuide: pp. 4-7, 8-12

Master 7.11, 7.12, 7.19, 7.20

CD-ROM: Unit 7 Masters

Student Book (SB): pp. 258-261, 262-266

Practice and HW Book: pp. 154-155, 156-157

**Web Link**

A template for a tri-fold foldable can be found in Grade 7 Mathematics Curriculum Resources:

(<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>)

**Strand: Statistics and Probability (Data Analysis)****Outcomes***Students will be expected to*

7SP1 Continued ...

**Achievement Indicators:**7SP1.1 *Continued***Elaborations—Strategies for Learning and Teaching**

The median is a measure of the “middle” of a data set. For an odd number of data values arranged in ascending or descending order, the median is the middle data value. The data set 35, 45, 60, 70, 75, 80, 80 has a median of 70 (the fourth of the sorted seven values). For an even number of data values arranged in ascending or descending order, it is the value halfway between the two middle data values. The median of 35, 45, 60, 70, 70, 80 is 65 (midway between the third and fourth of the sorted six values).

Rolls of tickets may be used to introduce the concept of median. On a strip of tickets, students write a data value on each ticket in ascending order. If there is an odd number of tickets, the strip is folded at the median.



The following set with an odd number of values has a median of 56.

12	16	31	42	48	56	63	64	78	83	91
----	----	----	----	----	----	----	----	----	----	----

The data set, containing an even number of values, has a median of 50.

12	16	27	31	42	46	54	56	63	64	78	82
----	----	----	----	----	----	----	----	----	----	----	----

Students sometimes forget to arrange the numbers in order when calculating the median. Another common error occurs with data sets that have an even number of values. Students often use one or the other of the middle values, rather than their mean, as the median. Remind them that there should be the same number of data values above the median as below it. If this does not occur, the median was incorrectly identified. This provides a quick check that can identify errors in the calculation of the median.

The mode is the data value that occurs most often in a data set. A data set may have no mode or several modes. The mode is arguably the least useful as a descriptor of a data set as a whole. It is a statistic that does not always exist, does not necessarily reflect the centre of the data, and can be highly unstable, changeable with very small changes in the data.

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**General Outcome: Collect, display and analyze data to solve problems.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Ask students to answer the following questions:
  - (i) Between January and March one year school was cancelled in Snowytown seven times due to blizzards. The following data gives the number of days each blizzard lasted.

1 day	6 days
4 days	2 days
2 days	3 days
3 days	

Find the mean, median and mode for this data.

(7SP1.1)

- (ii) The mean of a set of data is much lower than the median. What do you know about the data?

(7SP1.1)

**Resources/Notes**
**Authorized Resource**

*Math Makes Sense 7*

Lesson 7.1: Mean and Mode

Lesson 7.2: Median and Range

ProGuide: pp. 4-7, 8-12

Master 7.11, 7.12, 7.19, 7.20

CD-ROM: Unit 7 Masters

SB: pp. 258-261, 262-266

Practice and HW Book: pp. 154-155, 156-157

## Strand: Statistics and Probability (Data Analysis)

### Outcomes

*Students will be expected to*

7SP1 Continued ...

#### Achievement Indicators:

7SP1.2 *Determine the range for a given set of data.*

7SP1.3 *Provide a context in which the mean, median or mode is the most appropriate measure of central tendency to use when reporting findings.*

7SP1.4 *Solve a given problem involving the measures of central tendency.*

### Elaborations—Strategies for Learning and Teaching

Although the main focus of this outcome is on the centre of data sets, students can get a better representation of these sets by exploring how the data is spread out. The simplest measure of this dispersion is range. The range is the difference between the largest and smallest data values.

Students sometimes incorrectly describe the range using the minimum and maximum data values. Remind them that, similar to the mean and median, the range is a single value.

When asked to calculate the average, students often choose the arithmetic mean, saying it is “more mathematical.” In fact, all three measures of central tendency can represent the average or the centre of the data. Students should consider the appropriateness of each measure depending upon the situation presented.

Students worked with discrete and continuous data in Grade 6 (6SP1). When data are discrete, there are values between those given that are not meaningful in the context of the problem. Continuous data have an infinite number of values between data points which all make sense in the context of the problem.

When asked to identify a typical value, students may choose the mode, as they have previous exposure to bar graphs (3SP2, 4SP2, 5SP2). When discrete data is displayed as a bar graph, students often see the tallest bar standing out in the display. There are certain real-life situations where the mode is the appropriate measure. A shoe store, for example, might order new stock based on the shoe sizes it sells most often. In this case, a mean or median shoe size may not be useful. If these are values such as 6.2, they do not make sense. Other examples of discrete data sets may include sizes of cereal boxes or dresses, as one is unable to sell a partial size of these products.

The mean and median statistics can be chosen for continuous data including monetary values, temperatures, or test scores. The mean is affected by extreme values, whereas the median is not. The effects of these extreme measures will be elaborated on in the next outcome.

## General Outcome: Collect, display and analyze data to solve problems.

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students how they can determine the largest value in a data set if they are given the range and the smallest value. They should use an example to explain their answers.

(7SP1.2)

#### *Journal*

- The following data was collected to represent the progress of two students in science class. Each of the students would have the same mark based on the calculation of the mean. Ask students to find the range of the data for each student and explain how the range can add valuable information to the representation of the progress of each student.

(i) Student 1: 76%, 78%, 80%, 82%, 84%

(ii) Student 2: 60%, 70%, 80%, 90%, 100%

(7SP1.2)

- Darryl, Gordon and Joan are captains of the school math teams. Their contest results are recorded in the table below.

	Darryl	Gordon	Joan
Contest 1	82	84	85
Contest 2	82	84	85
Contest 3	88	90	85
Contest 4	100	71	81
Contest 5	77	78	81
Contest 6	81	87	85
Contest 7	87	89	82
Contest 8	83	88	85
Contest 9	83	86	83

Ask students to respond to the following:

Which measure would you choose to determine whose team is the best? Why? Why might someone disagree with you?

(7SP1.1, 7SP1.3)

#### *Interview*

- Present students with the following situations. Ask them to determine whether mean, median, or mode would be most helpful and have them justify their choice.
  - You are ordering bowling shoes to rent at a bowling alley.
  - You want to know if you read more or fewer books per month than most people in your class.
  - You want to know the “average” amount spent per week on junk food in your class.

(7SP1.3)

### Resources/Notes

#### Authorized Resource

#### *Math Makes Sense 7*

Lesson 7.1: Mean and Mode

Lesson 7.2: Median and Range

Lesson 7.4: Applications of Averages

ProGuide: pp. 4-7, 8-12, 17-21

Master 7.11, 7.12, 7.14, 7.19, 7.20, 7.22

CD-ROM: Unit 7 Masters

SB: pp. 258-261, 262-266, 271-275

Practice and HW Book: pp. 154-155, 156-157, 161-163

## Strand: Statistics and Probability (Data Analysis)

### Outcomes

*Students will be expected to*

**7SP2 Determine the effect on the mean, median and mode when an outlier is included in a data set.**

[C, CN, PS, R]

### Achievement Indicators:

7SP2.1 *Analyze a given set of data to identify any outliers.*

7SP2.2 *Explain the effect of outliers on the measures of central tendency for a given data set.*

7SP2.3 *Identify outliers in a given set of data, and justify whether or not they are to be included in reporting the measures of central tendency.*

7SP2.4 *Provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency.*

### Elaborations—Strategies for Learning and Teaching

In statistics, extreme values in data sets, known as outliers, are numerically distant from the rest of the data values. When students compare the measures of central tendency for a situation, they must consider the impact of outliers. This may affect which statistic to choose.

In some cases, the presence of outliers may not affect the measures of central tendency. Ask students to explore the effect of 38 and 98 on the measures of central tendency for the data set: 38, 64, 68, 71, 72, 75, 98. In this case, they should conclude that the values on opposite extremes of the data set will have virtually no effect on the average score.

Students should also analyze cases where there is only one outlier or multiple outliers on the same extreme. Sometimes the median can be affected, as it would be in data sets such as 1, 2, 4, 6, 63 and 3, 5, 26, 33, 37, 42.

When the data displays outliers, the median may be a better representation than the mean. If one is studying the average temperature of objects in a kitchen, for example, most would be at room temperature, between 20°C and 25°C. If one includes a warm oven at 300°C, the median would be close to room temperature, but the mean temperature would be much higher. For this situation, the median would be the better choice.

Outliers may occur in data sets due to a human error (e.g., incorrect measurements or recordings). In these cases, outliers should be ignored when computing statistics. If no error has occurred, the extreme values should be included. At times, the occurrence of outliers may not be obvious. Their identification is then a matter of choice.

Ask students to discuss with a partner the following situation to determine if an outlier exists:

Drag racing is usually done on a  $\frac{1}{4}$  mile track and cars are timed over that distance. The data collected for a challenger SRTB was:

9.11 s, 9.10 s, 9.54 s, 8.01 s, 9.76 s, 9.32 s

Follow this up with a class discussion to determine if there is agreement about the identification of an outlier in this data, and if so, whether the outlier should be excluded before calculating measures of central tendency.

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**General Outcome: Collect, display and analyze data to solve problems.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Ask students to define the term outlier. Ask them to provide an example of a situation in which an outlier must be excluded from the data before calculating the measures of central tendency, and explain why it would be excluded.

(7SP2.4)

- Students could answer questions such as the following:  
Tanya received the following scores on her first five math quizzes: 75%, 75%, 80%, 77%, 82%

- What is the mean, median and mode?
- On her next quiz, Tanya only achieved a mark of 25%. What effect, if any, did this mark have on the measures of central tendency calculated in (i)?

(7SP1.1, 7SP2.2)

- Ask students to answer the following:  
Players on the grade 7 basketball team were asked to record their height in cm on a chart. The data obtained were used to represent the height of the team.

155 cm	153 cm	150 cm	167 cm
164 cm	182 cm	170 cm	159 cm
185 cm	19 cm	182 cm	174 cm

- What is the outlier in this data set?
- Suggest a reason for this outlier. Should it be included in the calculations for the measures of central tendency? Why or why not?
- Calculate the mean, median and mode for these heights.
- Which measure(s) of central tendency would you use to represent the height of the team? Why?

(7SP1.1, 7SP1.4, 7SP2.3)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*
**Lesson 7.3: The Effects of Outliers on Average**

ProGuide: pp. 13-16

Master 7.13, 7.21

CD-ROM: Unit 7 Masters

Prep Talk Video: The Effects of Outliers on Average

SB: pp. 267-270

Practice and HW Book: pp. 158-160

## Strand: Statistics and Probability (Chance and Uncertainty)

### Outcomes

*Students will be expected to*

**7SP4 Express probabilities as ratios, fractions and percents.**

[C, CN, R, T, V]

### Achievement Indicators:

7SP4.1 *Determine the probability of a given outcome occurring for a given probability experiment, and express it as a ratio, fraction and percent.*

7SP4.2 *Provide an example of an event with a probability of 0 or 0% (impossible) and an example of an event with a probability of 1 or 100% (certain).*

### Elaborations—Strategies for Learning and Teaching

Students are to express probabilities in a variety of ways, including ratios, fractions, and percents. In Grade 6, students calculated the theoretical and experimental probability of a single event (6SP4). Earlier in Grade 7, students expressed percents as decimals and fractions (7N3).

The probability is a number between 0 and 1 that measures the likelihood of an event. The probability of a single event occurring is the ratio of the number of favourable outcomes to the number of possible outcomes. For example, the probability of rolling a prime number on a regular decadic (10 sides) numbered 1 to 10 can be expressed in multiple ways.

Ratio:  $P(\text{prime}) = 4:10 = 2:5$

Fraction:  $P(\text{prime}) = \frac{4}{10} = \frac{2}{5}$

Percent:  $P(\text{prime}) = \frac{4}{10} = \frac{40}{100} = 40\%$

In Grade 5, students determined the likelihood of an event as impossible, possible, or certain (5SP3). Students can now express the probabilities of impossible events as 0 or 0% and of certain events as 1 or 100%.

Using a scale with benchmarks 0 (0%),  $\frac{1}{4}$  (25%),  $\frac{1}{2}$  (50%),  $\frac{3}{4}$  (75%), and 1(100%), have students assess the reasonable probability of the events such as:

- The next baby born in your town will be a boy.
- It will snow at least once in the month of June.
- A person will live 6 months without water.
- The sun will set tomorrow.

They should explain their choices.

**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Ask students to answer the following questions.  
Chris noted the results of spinning a spinner in the table below.

2	4	1	1	2
1	1	4	1	3
3	5	2	2	3
2	4	1	2	1
3	2	5	3	1

Find each of the following probabilities. Express your answer as a ratio, as a fraction and as a percent each time.

- P(spin of 2)
- P(spin of 5)
- P(spin of an even number)
- P(spin of 7)
- P(spin of 1, 2, 3, 4 or 5)

(7SP4.1, 7SP4.2)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

**Lesson 7.5: Different Ways to Express Probability**

ProGuide: pp. 25-29

Master 7.10, 7.15, 7.23

CD-ROM: Unit 7 Masters

SB: pp. 279-283

Practice and HW Book: pp. 164-166

**Strand: Statistics and Probability (Chance and Uncertainty)**

**Outcomes**

*Students will be expected to*

**7SP5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events.**

[C, ME, PS]

**Achievement Indicators:**

7SP5.1 *Provide an example of two independent events, such as:*

- *spinning a four section spinner and rolling an eight-sided die*
- *tossing a coin and rolling a twelve-sided die*
- *tossing two coins*
- *rolling two dice*

*and explain why they are independent.*

7SP5.2 *Identify the sample space (all possible outcomes) for each of two independent events, using a tree diagram, table or other graphic organizer.*

**Elaborations—Strategies for Learning and Teaching**

In Grade 6, students identified all the possible outcomes of a probability experiment of a single event (6SP4). In Grade 7, the study of sample space is limited to independent events.

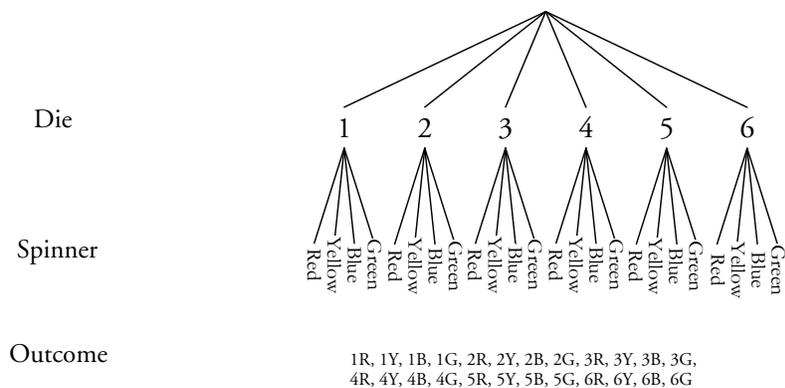
Two events are independent if they do not influence each other. The occurrence of one event does not influence the chance of the other event occurring. Students should understand that spinning a four section spinner, for example, does not in any way affect the number an eight-sided die will land on when tossed.

A common error for both tossing two coins or rolling two dice is a failure to distinguish between the two events, especially when the outcomes are combined. When determining the probability of tossing two coins and getting heads and tails, some students might treat HT and TH as the same outcome. Identifying the sample space before calculating probability should help students avoid this error.

A sample space is a specification of all the events that might happen in a given random experiment. The sample space may be represented as a tree diagram or in a table.

Students can display sample space for a regular 6-sided die, numbered 1 to 6, and a 4-coloured spinner, coloured red, yellow, blue, and green in either a vertical or horizontal tree diagram, a table or a fishbone organizer.

*Vertical Tree Diagram*



**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Students can answer questions such as the following:  
 Bob is a creative boy who loves interesting colour combinations. In his closet, he has a variety of shirts and pants to choose from. He has shirts that are blue, green, yellow, red, orange and pink. As pants, he chooses between shorts, jeans, dress pants and casual pants.
  - (i) What are the two independent events?
  - (ii) Explain why these events are independent.
  - (iii) Using an appropriate method, identify the sample space which describes all possible combinations of shirts and pants Bob can create.
  - (iv) Bob's mom buys him a new purple shirt. How many different outfits can he now create?

(7SP5.1, 7SP5.2)
- Ask students to decide whether each pair of events are independent or not, and to explain their reasoning.
  - (i) Roll a die and then roll a different die.
  - (ii) Roll a die and then roll the same die again.
  - (iii) Choose a name from a hat and then choose a second name from the hat without replacing the first name.
  - (iv) Choose a student from grade 7 and choose a student from grade 8.

(7SP5.1)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

**Lesson 7.6: Tree Diagrams**

ProGuide: pp. 30-34

Master 7.10, 7.16, 7.24

CD-ROM: Unit 7 Masters

SB: pp. 284-288

Practice and HW Book: pp. 167-170

**Strand: Statistics and Probability (Chance and Uncertainty)**

**Outcomes**

*Students will be expected to*

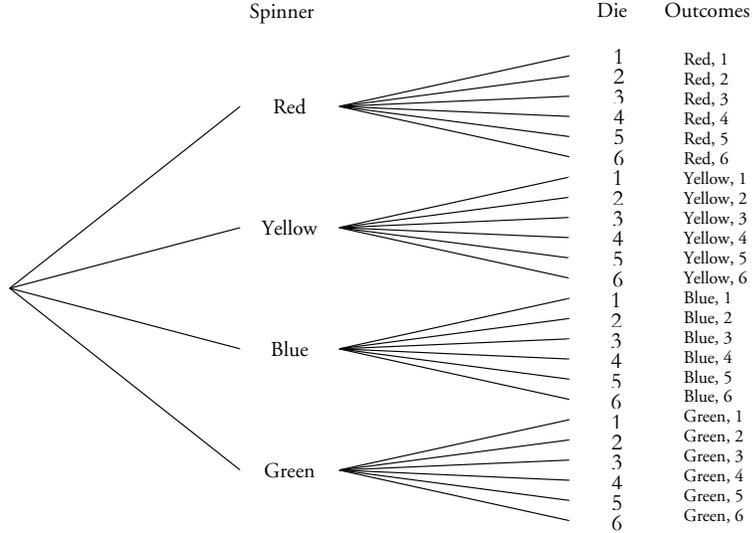
7SP5 Continued ...

**Achievement Indicator:**

7SP5.2 *Continued*

**Elaborations—Strategies for Learning and Teaching**

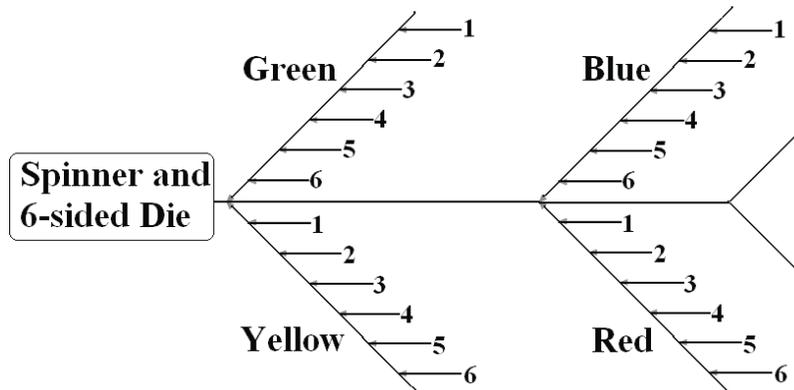
*Horizontal Tree Diagram*



*Table*

		Die					
		1	2	3	4	5	6
Spinner	Red	R, 1	R, 2	R, 3	R, 4	R, 5	R, 6
	Yellow	Y, 1	Y, 2	Y, 3	Y, 4	Y, 5	Y, 6
	Blue	B, 1	B, 2	B, 3	B, 4	B, 5	B, 6
	Green	G, 1	G, 2	G, 3	G, 4	G, 5	G, 6

*Fishbone Organizer*



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**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

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**Suggested Assessment Strategies**

*Performance*

- Students should work in pairs for this activity.  
One student teaches the other how to use a tree diagram to organize the outcomes and identify the sample space for rolling a six-sided die and flipping a coin.

(7SP5.2)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

**Lesson 7.6: Tree Diagrams**

ProGuide: pp. 30-34

Master 7.10, 7.16, 7.24

CD-ROM: Unit 7 Masters

SB: pp. 284-288

Practice and HW Book: pp. 167-170

**Strand: Statistics and Probability (Chance and Uncertainty)**

**Outcomes**

*Students will be expected to*

**7SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events.**

[C, PS, R, T]

**Achievement Indicators:**

7SP6.1 *Determine the theoretical probability of a given outcome involving two independent events.*

7SP6.2 *Conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability with the theoretical probability.*

7SP6.3 *Solve a given probability problem involving two independent events.*

**Elaborations – Strategies for Learning and Teaching**

Students will compare the theoretical probability of two independent events to the experimental probability by using a graphic organizer.

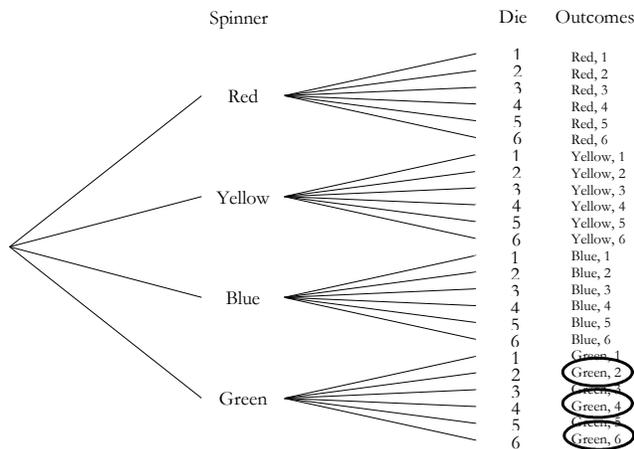
In Grade 6, students determined both the theoretical and experimental probability of a single event (6SP4). In Grade 8, students will determine the probability of independent events as the product of the probabilities of each event occurring separately (8SP2), after they study multiplication of fractions (8N6).

The theoretical probability of an event is the ratio of the number of favourable outcomes in an event to the total number of possible outcomes, when all possible outcomes are equally likely. Students use the sample space to determine the number of favourable outcomes and the number of possible outcomes. They then display this proportion as a ratio, percent, or fraction.

Using the die-spinner example from the previous outcome, if students were asked the theoretical probability of rolling an even number and spinning the colour green, they would indicate or circle all of the possible outcomes of the sample space using any graphic organizer.

$$P(\text{even, green}) = \frac{3}{24} = \frac{1}{8} = 1:8 = 0.125 = 12.5\%$$

*Using Tree Diagram*



*Using Table*

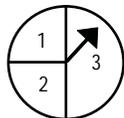
		Die					
		1	2	3	4	5	6
Spinner	Red	R, 1	R, 2	R, 3	R, 4	R, 5	R, 6
	Yellow	Y, 1	Y, 2	Y, 3	Y, 4	Y, 5	Y, 6
	Blue	B, 1	B, 2	B, 3	B, 4	B, 5	B, 6
	Green	G, 1	G, 2	G, 3	G, 4	G, 5	G, 6

## General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

### Suggested Assessment Strategies

#### Observation

- Students can conduct an experiment by spinning the spinner twice, and finding the sum of the numbers from the two spins. Ask them to predict which sum will appear most often and explain their thinking.



Teachers could ask students to work in pairs to carry out the experiment. Collate the results to obtain at least 100 trials. Students can compare the experimental results to their prediction and explain why there may be differences. (7SP6.2)

#### Paper and Pencil

- Students can answer questions such as the following:
  - Matthew's brand new iPod™ has only 5 songs in memory. They are all different. He hits the shuffle button to randomly select a song. Matthew's favourite song begins to play. At the end of that song, he hits the shuffle button again to get another random selection.
    - Organize the sample space (possible outcomes) for choosing two songs at random.
    - What is the probability that Matthew will hear his favourite song two times in a row? Show clearly how you obtained your answer. (7SP5.2, 7SP6.1)
  - At a carnival, you will receive a prize if you toss two dice and the sum is a prime number. Which of the following options provides the greatest likelihood of winning a prize? Explain your thinking.
    - tossing two six-sided dice
    - tossing one six-sided die and one four-sided die (7SP6.1, 7SP6.3)
  - A probability experiment consists of tossing two six-sided fair dice.
    - Does this experiment describe two independent events? Explain.
    - Draw a tree diagram or create a table to show all possible outcomes for this experiment.
    - Find the theoretical probability of obtaining a sum of 5 on the two dice in this experiment. Show all your work.
    - Describe how you could conduct this experiment by using two spinners instead of two six-sided dice. (7SP6.1, 7SP6.2)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 7*

#### Lesson 7.6: Tree Diagrams

ProGuide: pp. 30-34

Master 7.10, 7.16, 7.24

CD-ROM: Unit 7 Masters

See It Video: Game - All the Sticks

SB: pp. 284-288

Practice and HW Book: pp. 167-170

#### Web Link

Refer to Grade 7 Mathematics Curriculum Resources (<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>) for a link to a free download of Winstats software. It simulates experiments such as dealing cards, rolling dice and tossing coins.

**Strand: Statistics and Probability (Chance and Uncertainty)****Outcomes**

*Students will be expected to*

7SP6 Continued ...

**Achievement Indicators:**

7SP6.1, 7SP6.2, 7SP6.3

*Continued*

**Elaborations—Strategies for Learning and Teaching**

Students should realize that the probability in many situations cannot be characterized as equally likely. The theoretical probability of a thumb tack landing with the point up or down, for example, is more difficult to determine. In such cases, experiments or simulations may be conducted to determine the experimental probability.

Before conducting experiments, students should predict the probability whenever possible, and use the experiment to verify or refute the prediction. Materials such as spinners, dice, coins or coloured marbles may be used to conduct experiments. Graphing calculator simulations or computer software could also be used.

Theoretical and experimental probabilities can be compared in situations such as the following:

An experiment was conducted where two fair coins were tossed. Ask students to consider how many times in 64 trials they would expect to get heads on both coins. They should explain their thinking.

Students can then work in pairs to carry out the experiment, with each group doing 10 or 20 trials. Collate the results to obtain 64 trials and then add more trials as needed to show that experimental probability approaches theoretical probability as the number of trials increases. They should then calculate the theoretical probability of getting two heads when two coins are tossed and compare the experimental probability to the theoretical probability.

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**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

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**Suggested Assessment Strategies**

*Performance*

- Three students play a coin toss game in which points are awarded depending on the following rules:

Player A gets a point if two tosses result in two heads.

Player B gets a point if two tosses result in two tails.

Player C gets a point if two tosses result in one head and one tail.

Students play the game for twenty times. The player who has the most points wins.

Discussion around this activity should focus on questions such as:

- Is there a favoured player? How do you know? Why is this player favoured? Is this player likely to win the next game? Is this player guaranteed to win the next game?
- How many ways can two heads occur? Two tails? One head and one tail?
- Is this game fair? It will be useful to consider both the theoretical and experimental probabilities.

(adapted from Van de Walle, p. 335)

(7SP6.1, 7SP6.2, 7SP6.3)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

**Lesson 7.6: Tree Diagrams**

ProGuide: pp. 30-34

Master 7.10, 7.16, 7.24

CD-ROM: Unit 7 Masters

See It Video: Game - All the Sticks

SB: pp. 284-288

Practice and HW Book: pp. 167-170

**Suggested Resource**

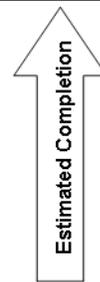
Van de Walle et al. *Teaching Student-Centered Mathematics Grades 5-8*. Boston, MA: Pearson Education, 2006.



# Geometry

Suggested Time: 4 Weeks

September	October	November	December	January	February	March	April	May	June
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# Unit Overview

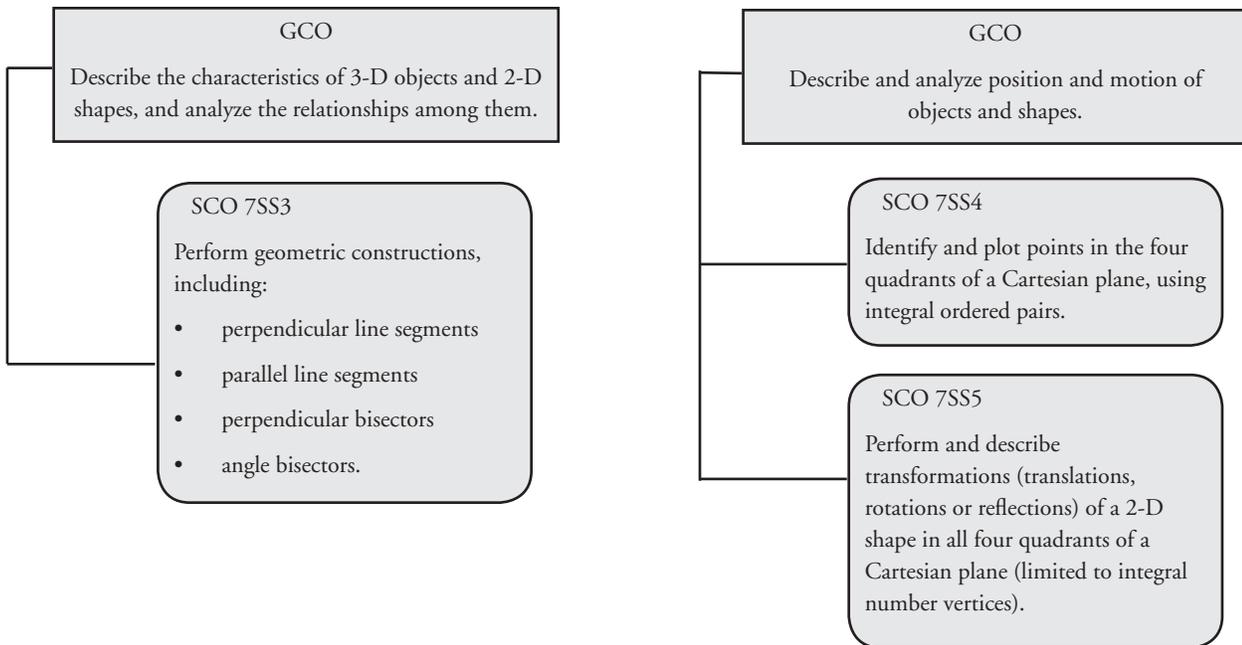
## Focus and Context

In this unit, the focus is on simple constructions, the Cartesian plane, and transformations. Students will begin by identifying parallel and perpendicular line segments found in the environment. They will explore a variety of methods to construct their own parallel and perpendicular segments, while developing a vocabulary of geometric terms. The concept of bisector, both line and angle, will be introduced and the construction of bisectors will be developed.

The Cartesian plane will be presented and points on the plane will be identified using ordered pairs.

Congruence and orientation will be addressed for each transformation. Prime notation will be used and students will explore the properties of combined transformations.

## Outcomes Framework



Process Standards

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics  
 and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

SCO Continuum

Grade 6	Grade 7	Grade 8
<b>Shape and Space (3-D Objects and 2-D Shapes)</b>		
<p>6SS4. Construct and compare triangles, including:</p> <ul style="list-style-type: none"> <li>• scalene</li> <li>• isosceles</li> <li>• equilateral</li> <li>• right</li> <li>• obtuse</li> <li>• acute</li> </ul> <p>in different orientations.                      [C, PS, R, V]</p> <p>6SS5. Describe and compare the sides and angles of regular and irregular polygons.                      [C, PS, R, V]</p>	<p>7SS3. Perform geometric constructions including:</p> <ul style="list-style-type: none"> <li>• perpendicular line segments</li> <li>• parallel line segments</li> <li>• perpendicular bisectors</li> <li>• angle bisectors.</li> </ul> <p>[CN, R, V]</p>	<p>8SS5. Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.                      [C, CN, R, T, V]</p>
<b>Shape and Space (Transformations)</b>		
<p>6SS6. Perform a combination of translations, rotations and/or reflections on a single 2-D shape, with and without technology, and draw and describe the image.                      [C, CN, PS, T, V]</p> <p>6SS7. Perform a combination of successive transformations of 2-D shapes to create a design, and identify and describe the transformations.                      [C, CN, T, V]</p> <p>6SS8. Identify and plot points in the first quadrant of a Cartesian plane, using whole number ordered pairs.                      [C, CN, V]</p> <p>6SS9. Perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).                      [C, CN, PS, T, V]</p>	<p>7SS4. Identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs.                      [C, CN, V]</p> <p>7SS5. Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).                      [C, CN, PS, T, V]</p>	<p>8SS6. Demonstrate an understanding of tessellation by:</p> <ul style="list-style-type: none"> <li>• explaining the properties of shapes that make tessellating possible</li> <li>• creating tessellations</li> <li>• identifying tessellations in the environment.</li> </ul> <p>[C, CN, PS, T, V]</p>

## Strand: Shape and Space (3-D Objects and 2-D Shapes)

### Outcomes

*Students will be expected to*

**7SS3 Perform geometric constructions, including:**

- perpendicular line segments
- parallel line segments
- perpendicular bisectors
- angle bisectors.

[CN, R, V]

### Achievement Indicators:

7SS3.1 *Identify line segments on a given diagram that are either parallel or perpendicular.*

7SS3.2 *Describe examples of parallel line segments in the environment.*

7SS3.3 *Draw a line segment parallel to another line segment, and explain why they are parallel.*

### Elaborations—Strategies for Learning and Teaching

In Grade 5 (5SS5, 5SS6), students were introduced to:

- lines (or line segments) that are parallel or that are perpendicular in familiar shapes and in the real world
- identifying the parallel sides of squares, rectangles, hexagons, trapezoids, and parallelograms
- identifying pairs of adjacent sides that are perpendicular.

Since some of the constructions for parallel line segments involve perpendiculars, an option is to introduce perpendicular line segments and bisectors first (7SS3.4, 7SS3.5).

Teachers should challenge students to give examples of parallel lines from their everyday life. Students might suggest examples such as:

- Opposite sides of picture frames
- Railroad/Roller Coaster tracks
- Lines on loose-leaf paper
- Rows of siding on a house
- Lines of latitude
- Guitar strings

Ask students why it is important for these things to be parallel. Ask for their ideas on how companies and engineers ensure that they are parallel.

Various methods can be used to construct parallel line segments. Two such possibilities are referred to here.

Constructing a rhombus:

- Place two points A and B on a line.
- With centre A and radius AB make an arc above AB.
- With centre B and the same radius make an arc intersecting the previous arc and extend it to the right. Mark the intersection point D.
- With centre D and the same radius make an arc to intersect the extension of the previous arc. Mark this point C.
- Join ABCD to make a rhombus.

Ask students: How do you know it is a rhombus? What do you know about the sides of a rhombus?

Answers will vary, but be sure to bring out that opposite sides are parallel. Therefore,  $CD \parallel AB$ .

**General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.**

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to make a list of as many pairs of parallel lines as they can find in the classroom in two minutes. After the time is up, ask students to pass their list to another student. Then ask them, one at a time, to read an entry from the list in front of them. Everybody who has that entry on their list will cross it off. At the end, the list with the most remaining entries will be the winner. (Students can be paired up for this activity.)

(7SS3.2)

- Students could draw a line that is neither vertical nor horizontal. Then, using a method of their choice, they draw a second line that is parallel to the first.

(7SS3.3)

- Present a diagram such as the one below to students. They should consider where they might see this pattern and what the purpose of these parallel lines is.



(7SS3.2)

#### *Journal*

- Ask students to think of 2-D shapes (excluding quadrilaterals) that have parallel sides. Ask them to include diagrams to illustrate their thinking.

(7SS3.2)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 7*

Prep Talk Video: Geometry

#### Lesson 8.1: Parallel Lines

ProGuide: pp. 4-6

Master 8.8, 8.24, 8.15

CD-ROM: Unit 8 Masters

Prep Talk Video: Parallel Lines

Student Book (SB): pp. 300-302

Practice and HW Book: pp. 178-181

## Strand: Shape and Space (3-D Objects and 2-D Shapes)

### Outcomes

*Students will be expected to*

7SS3 Continued ...

#### Achievement Indicators:

7SS3.3 *Continued*

7SS3.4 *Describe examples of perpendicular line segments in the environment.*

7SS3.5 *Draw a line segment perpendicular to another line segment, and explain why they are perpendicular.*

### Elaborations—Strategies for Learning and Teaching

Constructing two right angles:

- Place point A on a line.
- With centre A and any radius make an arc which intersects the line twice. Label these intersection points P and Q.
- With centres P and Q and any radius make two arcs above the line. Label their intersection point B. Join AB and extend.
- With centre B and any radius make an arc to intersect AB and two points. Label the points R and S.
- With centres R and S and any radius, make two arcs to the right of AB. Label their intersection point C. Join BC.

Ask students: How do you know that BC is parallel to the original line?

Examples of perpendicular lines in the environment include:

- Crosses
- Railway tracks and railway ties
- Fence posts and fence rails
- Four way stops
- Lines of latitude and longitude
- A wall and a shelf

Perpendicular line segments can be constructed using paper folding, a transparent mirror, a protractor and straight edge, or a compass and straightedge. Students should be exposed to a variety of construction methods.

Quite often students don't distinguish between perpendicular line segments and perpendicular bisectors. Reinforce that line segments are perpendicular if they meet at a right angle.

**General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.**

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to make a list of as many pairs of perpendicular lines as they can find in the classroom in two minutes. After the time is up, students can pass their list to another student. Then ask them, one at a time, to read an entry from the list in front of them. Everybody who has that entry on their list will cross it off. At the end, the list with the most remaining entries will be the winner. (Students can be paired up for this activity)

(7SS3.4)

- Ask students to draw a line that is neither vertical nor horizontal. Then, using a method of their choice, draw a second line that is perpendicular to the first.

(7SS3.5)

#### *Journal*

- Ask students to respond to the following:
  - (i) Can two lines be both parallel and perpendicular?  
(7SS3.1, 7SS3.2, 7SS3.3)
  - (ii) Can a line have more than one line that is perpendicular to it? Explain your reasoning. Can you think of an example?  
(7SS3.1, 7SS3.2, 7SS3.3)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 7*

#### Lesson 8.1: Parallel Lines

ProGuide: pp. 4-6

Master 8.8, 8.24, 8.15

CD-ROM: Unit 8 Masters

SB: pp. 300-302

Practice and HW Book: pp. 178-181

#### Lesson 8.2: Perpendicular Lines

ProGuide: pp. 7-9

Master 8.9, 8.25, 8.16

CD-ROM: Unit 8 Masters

Prep Talk Video: Perpendicular Lines

SB: pp. 303-305

Practice and HW Book: pp. 182-185

**Strand: Shape and Space (3-D Objects and 2-D Shapes)**

**Outcomes**

*Students will be expected to*  
7SS3 Continued ...

**Achievement Indicators:**

7SS3.6 *Describe examples of perpendicular bisectors in the environment.*

7SS3.7 *Draw the perpendicular bisector of a line segment, using more than one method, and verify the construction.*

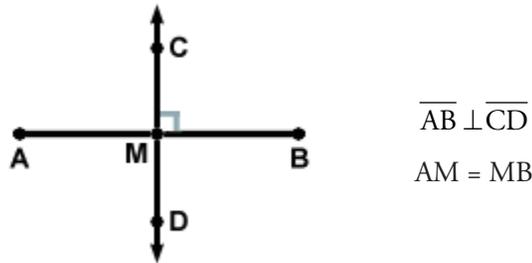
7SS3.8 *Describe examples of angle bisectors in the environment.*

7SS3.9 *Draw the bisector of a given angle, using more than one method, and verify that the resulting angles are equal.*

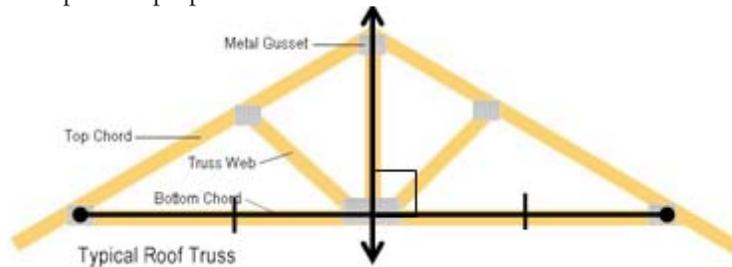
**Elaborations—Strategies for Learning and Teaching**

Introduction to angle and perpendicular bisectors can be supported through paper folding, transparent mirrors, tracing paper, compass and straight edge, or appropriate computer software. Teachers can set up stations, with a different method of bisecting at each station. The focus for students should be on accomplishing constructions in a variety of ways, as well as communicating, informally, how the construction was completed. Ask students which construction method they prefer.

A perpendicular bisector is a line or segment that intersects another segment at a right angle and also divides it into two equal parts.

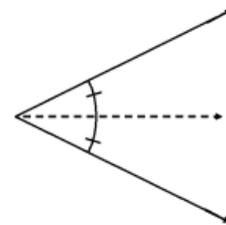


An example of a perpendicular bisector in the real world is a roof truss.



This design makes the roof very strong.

For every angle, there exists a line that divides the angle into two equal parts. This line is known as the angle bisector.



Students should discuss examples of angle bisectors in the environment. One situation occurs when a carpenter is installing mouldings in a corner (not always restricted to a right angle). The mouldings must be cut at an angle so that the two pieces fit together tightly. The carpenter is creating an angle bisector of the corner angle. This is very challenging and requires great skill.

**General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.**

### Suggested Assessment Strategies

#### *Performance*

- Mix-up Match-up: Create a set of cards. On one half write the terms in the list below; on the second half write definitions that match the terms on the first half. Distribute these cards to the students and ask them to circulate the room trying to match up the correct cards. Instruct them to sit together when they find their match.

parallel lines	right angle
perpendicular lines	straight angle
angle bisector	reflex angle
perpendicular bisector	vertex
line segment	radius
obtuse angle	diameter
acute angle	circumference

- Students could make a graffiti wall. On a sticky note, each student writes an example in the environment of either a pair of parallel lines, a pair of perpendicular lines, a perpendicular bisector, or an angle bisector. Ask them to post their notes on the wall. Students then choose a sticky other than their own and determine which category it falls under. They then place it in the appropriate section under parallel lines, perpendicular lines, perpendicular bisector, or angle bisector.

(7SS3.2, 7SS3.4, 7SS3.6, 7SS3.8)

#### *Journal*

- Ask students to choose the method for constructing an angle bisector and a perpendicular bisector that they like best. They should write directions for using their method of choice.

(7SS3.7, 7SS3.9)

#### *Paper and Pencil*

- Ask students to answer the following:  
A student in art class wants to draw a picture of the sun with eight light rays coming out from it. The rays must be spaced apart equally. Show how this can be done using perpendicular bisectors and angle bisectors. The teacher has asked that the radius of the sun be 3 cm. Construct a circle using your compass.

(7SS3.7, 7SS3.9)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 7*

#### Lesson 8.3: Constructing Perpendicular Bisectors

ProGuide: pp. 10-13

Master 8.10, 8.26

CD-ROM: Unit 8 Masters

Prep Talk Video: Constructing Perpendicular Bisectors

SB: pp. 306-309

Practice and HW Book: pp. 186-189

#### Lesson 8.4: Constructing Angle Bisectors

ProGuide: pp. 14-17

Master 8.11, 8.27, 8.17, 8.18a,b

CD-ROM: Unit 8 Masters

Prep Talk Video: Constructing Angle Bisectors

SB: pp. 310-313

Practice and HW Book: pp. 190-192

**Strand: Shape and Space (Transformations)**

**Outcomes**

*Students will be expected to*

**7SS4 Identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs.**

[C, CN, V]

**Achievement Indicators:**

7SS4.1 *Label the axes of a four quadrant coordinate plane (or Cartesian plane), and identify the origin.*

7SS4.2 *Identify the location of a given point in any quadrant of a Cartesian plane, using an integral ordered pair.*

7SS4.3 *Plot the point corresponding to a given integral ordered pair on a Cartesian plane with units of 1, 2, 5 or 10 on its axes.*

**Elaborations—Strategies for Learning and Teaching**

In Grade 6 (6SS8), students were introduced to the Cartesian plane. They identified and plotted whole number ordered pairs, limited to the first quadrant. This prior knowledge will now be extended to include integral ordered pairs in all four quadrants. Students should already be familiar with key terms such as coordinate plane, ordered pairs, origin,  $x$ -axis,  $y$ -axis,  $x$ -coordinates and  $y$ -coordinates. Continued use of appropriate terminology is important.

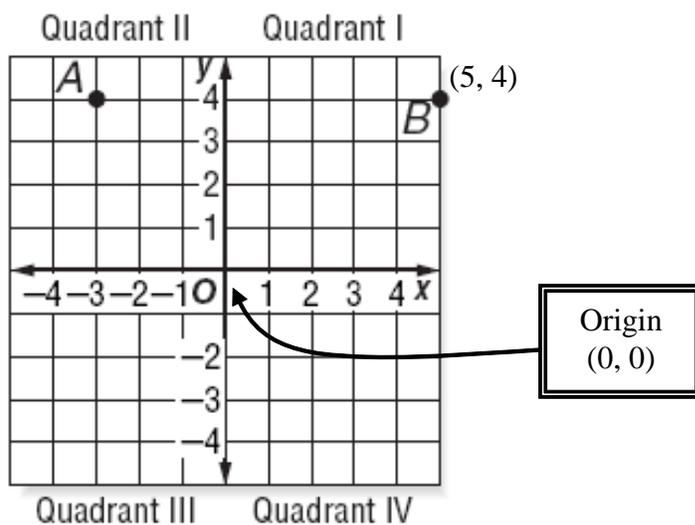
Each of the achievement indicators associated with this outcome have previously been addressed in Grade 6, restricted to the first quadrant.

Identify the ordered pair that names point A.

Step 1: Move left on the  $x$ -axis to find the  $x$ -coordinate of point A, which is  $-3$ .

Step 2: Move up the  $y$ -axis to find the  $y$ -coordinate, which is 4.

Point A is named by  $(-3, 4)$ .



Model the correct labelling of the coordinate plane, with numbers placed to identify locations where two grid lines intersect rather than in the interior of the grid square. Remind students of the similarity to a number line, with positive values to the right of zero and negative values to the left of zero.

A common error when identifying and plotting points is to reverse the order of the  $x$ -coordinate and the  $y$ -coordinate. To avoid making this mistake, students should label the  $x$  and  $y$  axes of a Cartesian plane.

**General Outcome: Describe and analyze position and motion of objects and shapes.**

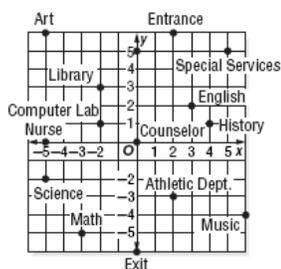
**Suggested Assessment Strategies**

*Performance*

- Create a game similar to Battleship™ using the coordinate plane. Creating a game would be beneficial because you can utilize all four quadrants. Note that many online battleship games use the interior of the block instead of grid points. In using battleships in created games, be careful to use grid points as opposed to interiors as is the case for map coordinates. (7SS4.2, 7SS4.3)
- A variation of Battleship™ is a game called Buried Treasure. Each student has two grids. On the first grid they bury 4 treasure chests of lengths 1, 2, 3 and 4 coordinate points. They use the second to keep track of their guesses. Students take turns guessing coordinates to try to find each others treasure. Hint: use a smaller grid size to make the game shorter such as having each axis extend only from -4 to +4. A game on a larger grid can result in using a significant amount of class time. (7SS4.2, 7SS4.3)

*Paper and Pencil*

- Students should research why coordinate planes are often called Cartesian planes. Ask them to write a brief paragraph explaining their findings. (7SS4)
- Ask students to use the coordinate plane below to answer the questions that follow. It shows a map of the rooms in a junior high school.



- Jessica is in the room located at (5, 5). What room is she in? Describe in words how to get to the nurse from this point.
- Jessica's next class is 8 units to the right and 2 units up on the map from the nurse. In what room is Jessica's next class? Find the ordered pair that represents the location of that room.
- Lucas is in the Music room, but his next class is in the Library. Give Lucas directions on how to get to the Library.

(7SS4.2, 7SS4.3)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

**Lesson 8.5: Graphing on a Coordinate Grid**

ProGuide: pp. 19-23

Master 8.20, 8.12, 8.28

PM 22

CD-ROM: Unit 8 Masters

SB: pp. 315-319

Practice and HW Book: pp. 193-195

**Web Link**

Refer to Grade 7 Mathematics Curriculum Resources

(<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>) for the following:

- A four-door foldable to organize students' notes on a Cartesian plane

**Strand: Shape and Space (Transformations)**

**Outcomes**

*Students will be expected to*

7SS4 Continued ...

**Achievement Indicators:**

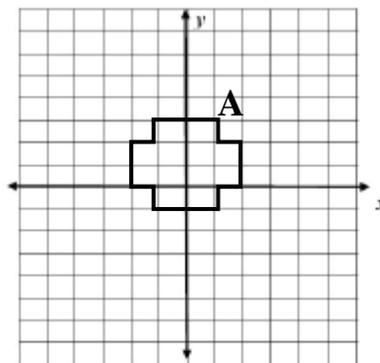
7SS4.4 *Draw shapes and designs in a Cartesian plane, using integral ordered pairs.*

7SS4.5 *Create shapes and designs, and identify the points used to produce the shapes and designs, in any quadrant of a Cartesian plane.*

**Elaborations – Strategies for Learning and Teaching**

Students should answer questions similar to the following:

1. Given sets of points as ordered pairs, such as A(1, 3), B(-1, 3), C(-1, 2), D(-2, 2), E(-2, -1), F(2, -1), G(2, 2) and H(1, 2), plot them on a coordinate plane and join those points to create a shape.
2. Using shapes drawn on a coordinate plane, identify the locations of the vertices.



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**General Outcome: Describe and analyze position and motion of objects and shapes.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Present the following to students:

Natasha is creating an X-pattern for her needlepoint project in home economics. She has plotted the X on a coordinate plane using these ordered pairs.

A(3, 0)            B(2, -1)            C(1, -2)

D(-3, -2)        E(-1, -4)        F(-1, 0)

G(0, -1)        H(2, -3)        I(3, -4)

Ask students to determine if Natasha will make an X. If not, what ordered pair will she need to change to fix it?

(7SS4.4)

- Ask students to write the coordinates to draw a simple picture. Trade coordinates to see if their partner can follow their instructions to produce the picture they have in mind. (7SS4.4, 7SS4.5)

**Resources/Notes**
**Authorized Resource**

*Math Makes Sense 7*

**Lesson 8.5: Graphing on a Coordinate Grid**

ProGuide: pp. 19-23

Master 8.20, 8.12, 8.28

PM 22

CD-ROM: Unit 8 Masters

SB: pp. 315-319

Practice and HW Book: pp. 193-195

## Strand: Shape and Space (Transformations)

### Outcomes

*Students will be expected to*

**7SS5 Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).**

[C, CN, PS, T, V]

### Achievement Indicators:

*7SS5.1 Identify the coordinates of the vertices of a given 2-D shape on a Cartesian plane.*

*7SS5.2 Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.*

*7SS5.3 Determine the distance between points along horizontal and vertical lines in a Cartesian plane.*

### Elaborations – Strategies for Learning and Teaching

Students have been exposed to transformational geometry in previous grades. The three transformations that change the location of an object in space, or the direction in which it faces, but not its size or shape, are translations, reflections and rotations. These transformations result in images that are congruent to the original object.

In primary grades, students used the more informal terms slides, flips and turns. They were later introduced to the formal mathematical language. When students first began studying these transformations, they worked with concrete shapes on a flat surface. Later, they worked with simple grids, providing them with an opportunity to develop and apply positional and movement language.

In Grade 5, students worked with a single transformation (5SS7, 5SS8). In Grade 6, this was extended to a combination of transformations (6SS6, 6SS7). They also worked with a single transformation in the first quadrant of a Cartesian plane (6SS9).

In this unit, students will work with transformations and combinations of transformations in all four quadrants of the Cartesian plane. It is intended that the original shape and its image have vertices with integral coefficients.

When describing transformations, students should be able to recognize a given transformation as a reflection, a translation, a rotation, or some combination of these. In addition, when given an object and its image, students should be able to describe:

- a translation, using words and notation describing the translation (e.g.,  $\Delta A'B'C'$  is the translation image of  $\Delta ABC$ ). Given two shapes students should be able to say:  $\Delta ABC$  has been translated 2 units to the right and 3 units up to produce its image  $\Delta A'B'C'$ . Continue to remind students that they must describe the horizontal change first and the vertical change second.
- a reflection, by determining the location of the line of reflection. Reflections should be limited to use of the  $x$  or  $y$ -axis as reflection lines.
- a rotation, using degree or fraction-of-turn measures, both clockwise and counterclockwise, and identify the location of the centre of a rotation. A centre of rotation may be located on the shape (such as a vertex of the original image) or off the shape.

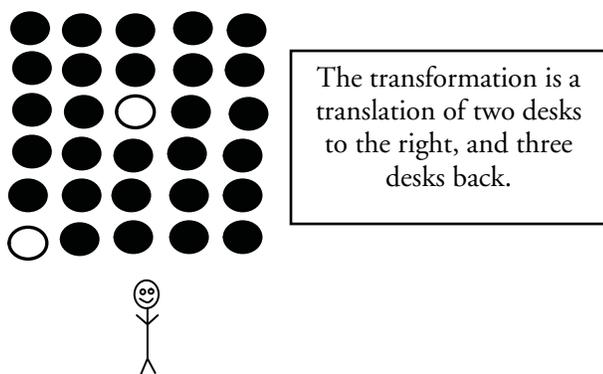
**General Outcome: Describe and analyze position and motion of objects and shapes.**

**Suggested Assessment Strategies**

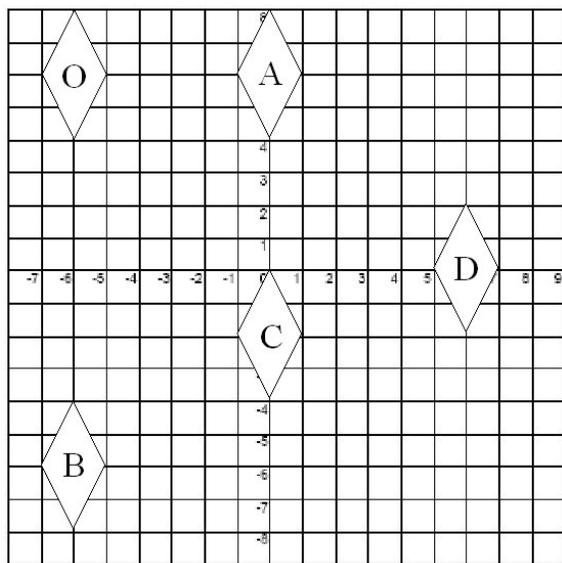
*Paper and Pencil*

- Ask students to identify identical objects in the classroom (desks, books, posters, etc.). Discuss how these objects could be regarded as objects and images, and describe the transformations that relate them to each other.

E.g., Desks are arranged in 5 rows with 6 desks in each row. What transformation could be applied to relate the first desk in the first row to the fourth desk in the third row? (7SS5.2,7SS5.4)



- If O is the original object, and A, B, C, and D are images of O, ask students to answer the following questions.
  - Identify the coordinate pairs of the vertices for object O and its images.
  - Describe the movement required to move from any point on O to each of its image points.



(7SS5.1, 7SS5.2, 7SS5.6)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 7*

**Lesson 8.6: Graphing Translations and Reflections**

**Lesson 8.7: Graphing Rotations**

ProGuide: pp. 24-28, 29-33

Master 8.20, 8.13, 8.14, 8.29, 8.30

PM 22

CD-ROM: Unit 8 Masters

Prep Talk Videos:

- Graphing Translations and Reflections
- Graphing Rotations

SB: pp. 320-324, 325-329

Practice and HW Book: pp. 196-198, 199-201

## Strand: Shape and Space (Transformations)

### Outcomes

*Students will be expected to*

7SS5 Continued ...

#### Achievement Indicators:

7SS5.4 *Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation, or successive transformations, on a Cartesian plane.*

7SS5.5 *Perform a transformation or consecutive transformations on a given 2-D shape, and identify coordinates of the vertices of the image.*

7SS5.6 *Describe the image resulting from the transformations of a given 2-D shape on a Cartesian plane by identifying the coordinates of the vertices of the image.*

### Elaborations—Strategies for Learning and Teaching

When investigating properties of transformations, students should consider the concepts of congruence, which were developed informally in previous grades. In discussing the properties of transformations, students should consider if the transformation of the image:

- has side lengths and angle measures the same as the given image
- is both similar to and congruent to the given image
- has the same orientation as the given image
- appears to have remained stationary with respect to the given image

Transformational geometry is another way to investigate and interpret geometric figures by moving every point in a plane figure to a new location. To help students form images of shapes through different transformations, students can use concrete objects such as cardboard cut-outs or geometry sets, figures drawn on graph paper, mirrors or other reflective surfaces, or appropriate technology. Students should be able to recognize a given transformation as a reflection, a rotation, a translation, or some combination of these.

Successive transformations are defined as more than one transformation being applied to an object in succession. One transformation is applied to point  $A$  to create  $A'$ , and then a second transformation is applied to  $A'$  creating  $A''$ . The double prime notation is used to label the point that matches point  $A$  after a second transformation. If  $A(2, -2)$  is reflected in the  $x$ -axis, for example, the resulting image is  $A'(2, 2)$ . If it is then translated 4 units left and 2 units up, this results in  $A''(-2, 4)$ . Students should then be able to describe the positional change of  $A$  to  $A''$ . It has moved 4 units horizontally left and 6 units vertically up.

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**General Outcome: Describe and analyze position and motion of objects and shapes.**


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**Suggested Assessment Strategies**
*Journal*

- Ask students to respond to the following:  
If a shape undergoes two transformations, one after the other, does it matter in what order they are applied? Will you get the same final image either way?  
(7SS5.4)

*Performance*

- Create a grid on the floor with masking tape. Use rope or coloured tape to place the axes. One student chooses a spot. Another student directs him/her to translate the position. This activity could progress to more than 3 students on the grid holding elastic to form a two dimensional shape. A different student directs them (as vertices) to “walk” through various transformations.  
(7SS5.4, 7SS5.5)

*Paper and Pencil*

- Using grid paper, ask students to answer the following:  
You are employed by a graphic design firm which creates graphic designs for companies who manufacture wallpaper, wrapping paper, tile, and fabric. Your supervisor has assigned you to develop a new design, using the following design elements:
  - it must use at least two types of transformations
  - it must use at least two colours
  - it must be extended to cover at least 75% of the grid
 Create a design and write an explanation of how you created your design.  
(7SS5.5)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 7*
**Lesson 8.6: Graphing  
Translations and Reflections**
**Lesson 8.7: Graphing Rotations**

ProGuide: pp. 24-28, 29-33

 Master 8.20, 8.13, 8.14, 8.29,  
8.30

PM 22

CD-ROM: Unit 8 Masters

SB: pp. 320-324, 325-329

 Practice and HW Book: pp. 196-  
198, 199-201

**Web Link**

 Refer to Grade 7 Mathematics  
Curriculum Resources  
(<https://www.k12pl.nl.ca/curriculumresources/7-9/mathematics/grade7.html>) for the following:

- Transformation Golf:  
students can reflect, rotate and translate the ball around the course.



# **Appendix:**

## **Outcomes with Achievement Indicators Organized by Strand (With Curriculum Guide Page References)**

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Number	General Outcome: Develop number sense.	
Specific Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	Page Reference
7N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and why a number cannot be divided by 0. [C, R]	7N1.1 Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10, and explain why.	p. 22
	7N1.2 Sort a given set of numbers based upon their divisibility, using organizers such as Venn and Carroll diagrams.	p. 24
	7N1.3 Determine the factors of a given number, using the divisibility rules.	p. 24
	7N1.4 Explain, using an example, why numbers cannot be divided by 0.	p. 26
7N2 Demonstrate an understanding of the addition, subtraction, multiplication and division of decimals to solve problems (for more than 1-digit divisors or 2-digit multipliers, the use of technology is expected). [ME, PS, T]	7N2.1 Solve a given problem involving the addition of two or more decimal numbers.	p. 66
	7N2.2 Solve a given problem involving the subtraction of decimal numbers.	p. 66
	7N2.3 Place the decimal in a sum or difference, using front-end estimation; e.g., for $4.5 + 0.73 + 256.458$ , think $4 + 256$ , so the sum is greater than 260.	p. 66
	7N2.4 Solve a given problem involving the multiplication of decimal numbers with two digit multipliers (whole numbers or decimals) without the use of technology.	p. 68
	7N2.5 Place the decimal in a product, using front-end estimation; e.g., for $\$12.33 \times 2.4$ , think $\$12 \times 2$ , so the product is greater than \$24.	p. 68
	7N2.6 Solve a given problem involving the multiplication or division of decimal numbers with more than 2-digit multipliers or 1-digit divisors (whole numbers or decimals) with the use of technology.	pp. 68, 70
	7N2.7 Solve a given problem involving the division of decimal numbers for 1-digit divisors (whole numbers or decimals) without the use of technology.	p. 70
	7N2.8 Check the reasonableness of solutions, using estimation.	p. 70
	7N2.9 Place the decimal in a quotient, using front-end estimation; e.g., for $51.50 \text{ m} \div 2.1$ , think $50 \text{ m} \div 2$ , so the quotient is approximately 25 m.	p. 70
	7N2.10 Solve a given problem that involves operations on decimals (limited to thousandths), taking into consideration the order of operations.	p. 72

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Number	General Outcome: Develop number sense.	
Specific Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	Page Reference
7N3 Solve problems involving percents from 1% to 100%. [C, CN, PS, R, T]	7N3.1 Express a given percent as a decimal or fraction.	p. 74
	7N3.2 Solve a given problem that involves finding a percent.	p. 76
	7N3.3 Determine the answer to a given percent problem where the answer requires rounding, and explain why an approximate answer is needed; e.g., total cost including taxes.	p. 76
7N4 Demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals and positive fractions. [C, CN, R, T]	(It is intended that repeating decimals be limited to decimals with 1 or 2 repeating digits.)	
	7N4.1 Predict the decimal representation of a given fraction, using patterns; e.g., $\frac{1}{11} = 0.\overline{09}$ , $\frac{2}{11} = 0.\overline{18}$ , $\frac{3}{11} = ? \dots$	p. 58
	7N4.2 Match a given set of fractions to their decimal representations.	p. 60
	7N4.3 Sort a given set of fractions as repeating or terminating decimals.	p. 60
	7N4.4 Express a given fraction as a terminating or repeating decimal.	p. 60
	7N4.5 Express a given terminating decimal as a fraction.	p. 60
	7N4.6 Express a given repeating decimal as a fraction.	p. 60
7N4.7 Provide an example where the decimal representation of a fraction is an approximation of its exact value.	p. 60	

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Number		General Outcome: Develop number sense.		
Specific Outcomes		Achievement Indicators		
<i>It is expected that students will:</i>		<i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>		
		Page Reference		
7N5	Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]	7N5.1	Model addition of positive fractions, using concrete representations, and record symbolically.	pp. 104, 106
		7N5.2	Determine the sum of two given positive fractions with like denominators.	pp. 104, 106
		7N5.3	Simplify a given positive fraction by identifying the common factor between the numerator and denominator.	p. 106
		7N5.4	Determine a common denominator for a given set of positive fractions.	p. 106
		7N5.5	Determine the sum of two given positive fractions with unlike denominators.	pp. 106, 108
		7N5.6	Model subtraction of positive fractions, using concrete representations, and record symbolically.	p. 110
		7N5.7	Determine the difference of two given positive fractions.	p. 110
		7N5.8	Model addition and subtraction of mixed numbers, using concrete representations, and record symbolically.	p. 112
		7N5.9	Determine the sum or difference of two mixed numbers.	p. 112
		7N5.10	Simplify the solution to a given problem involving the sum or difference of two positive fractions or mixed numbers.	p. 112
		7N5.11	Solve a given problem involving the addition or subtraction of positive fractions or mixed numbers, and determine if the solution is reasonable.	p. 112

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Number	General Outcome: Develop number sense.	
Specific Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	Page Reference
<b>7N6</b> Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]	7N6.1 Explain, using concrete materials such as integer tiles and diagrams, that the sum of opposite integers is zero.	p. 46
	7N6.2 Solve a given problem involving the addition and subtraction of integers.	p. 46
	7N6.3 Add two given integers, using concrete materials or pictorial representations, and record the process symbolically.	p. 48
	7N6.4 Illustrate, using a number line, the results of adding negative and positive integers.	p. 48
	7N6.5 Subtract two given integers, using concrete materials or pictorial representations, and record the process symbolically.	pp. 50, 52
	7N6.6 Illustrate, using a number line, the results of subtracting negative and positive integers.	pp. 50, 52
<b>7N7</b> Compare and order positive fractions, positive decimals (to thousandths) and whole numbers by using: <ul style="list-style-type: none"> <li>• benchmarks</li> <li>• place value</li> <li>• equivalent fractions and/or decimals.</li> </ul> [CN, R, V]	7N7.1 Order the numbers of a given set that includes positive fractions, positive decimals and/or whole numbers in ascending or descending order, and verify the result using a variety of strategies.	p. 62
	7N7.2 Position fractions with like and unlike denominators from a given set on a number line, and explain strategies used to determine order.	p. 62
	7N7.3 Order the numbers of a given set by placing them on a number line that contains benchmarks, such as 0 and 1 or 0 and 5.	p. 62
	7N7.4 Position a given set of positive fractions, including mixed numbers and improper fractions, on a number line and explain strategies used to determine position.	p. 62
	7N7.5 Identify a number that would be between two given numbers in an ordered sequence or on a number line.	p. 64
	7N7.6 Identify incorrectly placed numbers in an ordered sequence or on a number line.	p. 64

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Patterns and Relations (Patterns)	General Outcome: Use patterns to describe the world and to solve problems.	
Specific Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	Page Reference
7PR1 Demonstrate an understanding of oral and written patterns and their equivalent linear relations. [C, CN, R]	7PR1.1 Formulate a linear relation to represent the relationship in a given oral or written pattern.	pp. 32, 34
	7PR1.2 Provide a context for a given linear relation that represents a pattern.	p. 36
	7PR1.3 Represent a pattern in the environment, using a linear relation.	p. 36
7PR2 Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, PS, R, V]	7PR2.1 Create a table of values for a given linear relation by substituting values for the variable.	pp. 32, 34
	7PR2.2 Create a table of values, using a linear relation, and graph the table of values (limited to discrete elements).	pp. 32, 34
	7PR2.3 Sketch the graph from a table of values created for a given linear relation, and describe the patterns found in the graph to draw conclusions; e.g., graph the relationship between $n$ and $2n + 3$ .	p. 34
	7PR2.4 Describe, using everyday language in spoken or written form, the relationship shown on a graph to solve problems.	p. 34
	7PR2.5 Match a given set of linear relations to a given set of graphs.	p. 36
	7PR2.6 Match a given set of graphs to a given set of linear relations.	p. 36

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

<b>Strand:</b> Patterns and Relations (Variables and Equations)	<b>General Outcome:</b> Represent algebraic expressions in multiple ways.	
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	<b>Page Reference</b>
<p><b>7PR3</b> Demonstrate an understanding of preservation of equality by:</p> <ul style="list-style-type: none"> <li>• modelling preservation of equality, concretely, pictorially and symbolically</li> <li>• applying preservation of equality to solve equations. [C, CN, PS, R, V]</li> </ul>	<p>7PR3.1 Model the preservation of equality for each of the four operations, using concrete materials or pictorial representations, explain the process orally and record it symbolically.</p> <p>7PR3.2 Write equivalent forms of a given equation by applying the preservation of equality, and verify, using concrete materials; e.g., <math>3b = 12</math> is the same as <math>3b + 5 = 12 + 5</math> or <math>2r = 7</math> is the same as <math>3(2r) = 3(7)</math>.</p> <p>7PR3.3 Solve a given problem by applying preservation of equality.</p>	<p>pp. 120, 122, 124</p> <p>pp. 120, 122, 124</p> <p>p. 128</p>
<p><b>7PR4</b> Explain the difference between an expression and an equation. [C, CN]</p>	<p>7PR4.1 Explain what a variable is and how it is used in a given expression.</p> <p>7PR4.2 Identify and provide an example of a constant term, numerical coefficient and variable in an expression and an equation.</p> <p>7PR4.3 Provide an example of an expression and an equation, and explain how they are similar and different.</p> <p>7PR4.4 Represent a given oral or written pattern using an algebraic expression.</p> <p>7PR4.5 Represent a given oral or written pattern using an equation.</p>	<p>p. 28</p> <p>p. 28</p> <p>p. 28</p> <p>p. 30</p> <p>p. 38</p>
<p><b>7PR5</b> Evaluate an expression, given the value of the variable(s). [CN, R]</p>	<p>7PR5.1 Substitute a value for an unknown in a given expression, and evaluate the expression.</p>	<p>p. 30</p>

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

<b>Strand:</b> Patterns and Relations (Variables and Equations) (continued)	<b>General Outcome:</b> Represent algebraic expressions in multiple ways.	
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	<b>Page Reference</b>
<b>7PR6</b> Model and solve, concretely, pictorially and symbolically, problems that can be represented by one-step linear equations of the form $x + a = b$ , where $a$ and $b$ are integers. [CN, PS, R, V]	7PR6.1 Represent a given problem with a linear equation and solve the equation using concrete models, e.g., counters, integer tiles.  7PR6.2 Draw a visual representation of the steps required to solve a given linear equation.  7PR6.3 Solve a given problem using a linear equation.  7PR6.4 Verify the solution to a given linear equation using concrete materials and diagrams.  7PR6.5 Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.	p. 126          p. 126          p. 126          p. 126          p. 126
<b>7PR7</b> Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form: <ul style="list-style-type: none"> <li>• <math>ax + b = c</math></li> <li>• <math>ax - b = c</math></li> <li>• <math>ax = b</math></li> <li>• <math>\frac{x}{a} = b, a \neq 0</math></li> </ul> where $a$ , $b$ and $c$ are whole numbers. [CN, PS, R, V]	7PR7.1 Model a given problem with a linear equation and solve the equation, using concrete models, e.g., counters, integer tiles.  7PR7.2 Solve a given linear equation by inspection and by systematic trial.  7PR7.3 Draw a visual representation of the steps used to solve a given linear equation.  7PR7.4 Solve a given problem, using a linear equation, and record the process.  7PR7.5 Verify the solution to a given linear equation, using concrete materials and diagrams.  7PR7.6 Substitute a possible solution for the variable in a given linear equation into the original linear equation to verify the equality.	pp. 40, 120, 122          p. 118          pp. 122, 124          pp. 122, 124          p. 124          p. 124

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Shape and Space (Measurement)		General Outcome: Use direct and indirect measurement to solve problems.	
Specific Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	Page Reference	
<b>7SS1</b> Demonstrate an understanding of circles by: <ul style="list-style-type: none"> <li>describing the relationships among radius, diameter and circumference</li> <li>relating circumference to pi</li> <li>determining the sum of the central angles</li> <li>constructing circles with a given radius or diameter</li> <li>solving problems involving the radii, diameters and circumferences of circles.</li> </ul> [C, CN, PS, R, V]	7SS1.1 Illustrate and explain that the diameter is twice the radius in a given circle.	p. 82	
	7SS1.2 Draw a circle with a given radius or diameter, with and without a compass.	p. 82	
	7SS1.3 Illustrate and explain that the circumference is approximately three times the diameter in a given circle.	p. 84	
	7SS1.4 Explain that, for all circles, pi is the ratio of the circumference to the diameter $\left(\frac{C}{d}\right)$ and its value is approximately 3.14.	p. 84	
	7SS1.5 Solve a given contextual problem involving circles.	p. 84	
	7SS1.6 Explain, using an illustration, that the sum of the central angles of a circle is 360°.	p. 94	
<b>7SS2</b> Develop and apply a formula for determining the area of: <ul style="list-style-type: none"> <li>triangles</li> <li>parallelograms</li> <li>circles.</li> </ul> [CN, PS, R, V]	7SS2.1 Illustrate and explain how the area of a rectangle can be used to determine the area of a parallelogram.	pp. 86, 88	
	7SS2.2 Generalize a rule to create a formula for determining the area of parallelograms.	pp. 86, 88	
	7SS2.3 Solve a given problem involving the area of triangles, parallelograms and/or circles.	pp. 86, 88, 92	
	7SS2.4 Illustrate and explain how the area of a rectangle or a parallelogram can be used to determine the area of a triangle.	p. 88	
	7SS2.5 Generalize a rule to create a formula for determining the area of triangles.	p. 88	
	7SS2.6 Illustrate and explain how to estimate the area of a circle without the use of a formula.	pp. 90, 92	
	7SS2.7 Apply a formula for determining the area of a given circle.	pp. 90, 92	

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

<b>Strand:</b> Shape and Space (3-D Objects and 2-D Shapes)	<b>General Outcome:</b> Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	<b>Page Reference</b>
7SS3 Perform geometric constructions, including: <ul style="list-style-type: none"> <li>• perpendicular line segments</li> <li>• parallel line segments</li> <li>• perpendicular bisectors</li> <li>• angle bisectors.</li> </ul> [CN, R, V]	7SS3.1 Identify line segments on a given diagram that are parallel or perpendicular.	p. 156
	7SS3.2 Describe examples of parallel line segments in the environment.	p. 156
	7SS3.3 Draw a line segment parallel to another line segment, and explain why they are parallel.	pp. 156, 158
	7SS3.4 Describe examples of perpendicular line segments in the environment.	p. 158
	7SS3.5 Draw a line segment perpendicular to another line segment, and explain why they are perpendicular.	p. 158
	7SS3.6 Describe examples of perpendicular bisectors in the environment.	p. 160
	7SS3.7 Draw the perpendicular bisector of a line segment, using more than one method, and verify the construction.	p. 160
	7SS3.8 Describe examples of angle bisectors in the environment.	p. 160
	7SS3.9 Draw the bisector of a given angle, using more than one method, and verify that the resulting angles are equal.	p. 160

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

<b>Strand:</b> Shape and Space (Transformations)	<b>General Outcome:</b> Describe and analyze position and motion of objects and shapes.	
<b>Specific Outcomes</b> <i>It is expected that students will:</i>	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	<b>Page Reference</b>
<b>7SS4</b> Identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs. [C, CN, V]	7SS4.1 Label the axes of a four quadrant coordinate plane (or Cartesian plane), and identify the origin. 7SS4.2 Identify the location of a given point in any quadrant of a Cartesian plane, using an integral ordered pair. 7SS4.3 Plot the point corresponding to a given integral ordered pair on a Cartesian plane with units of 1, 2, 5 or 10 on its axes. 7SS4.4 Draw shapes and designs in a Cartesian plane, using integral ordered pairs. 7SS4.5 Create shapes and designs, and identify the points used to produce the shapes and designs, in any quadrant of a Cartesian plane.	p. 162 p. 162 p. 162 p. 164 p. 164
<b>7SS5</b> Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). [C, CN, PS, T, V]	(It is intended that the original shape and its image have vertices with integral coordinates.) 7SS5.1 Identify the coordinates of the vertices of a given 2-D shape on a Cartesian plane. 7SS5.2 Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane. 7SS5.3 Determine the distance between points along horizontal and vertical lines in a Cartesian plane. 7SS5.4 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation, or successive transformations, on a Cartesian plane. 7SS5.5 Perform a transformation or consecutive transformations on a given 2-D shape, and identify coordinates of the vertices of the image. 7SS5.6 Describe the image resulting from the transformation of a given 2-D shape on a Cartesian plane by identifying the coordinates of the vertices of the image.	p. 166 p. 166 p. 166 p. 168 p. 168 p. 168

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Statistics and Probability (Data Analysis)		General Outcome: Collect, display and analyze data to solve problems.		
Specific Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>		Page Reference
7SP1	Demonstrate an understanding of central tendency and range by: <ul style="list-style-type: none"> <li>determining the measures of central tendency (mean, median, mode) and range</li> <li>determining the most appropriate measures of central tendency to report findings.</li> </ul> [C, PS, R, T]	7SP1.1	Determine mean, median and mode for a given set of data, and explain why these values may be the same or different.	pp. 134, 136
		7SP1.2	Determine the range for a given set of data.	p. 138
		7SP1.3	Provide a context in which the mean, median or mode is the most appropriate measure of central tendency to use when reporting findings.	p. 138
		7SP1.4	Solve a given problem involving the measures of central tendency.	p. 138
7SP2	Determine the effect on the mean, median and mode when an outlier is included in a data set. [C, CN, PS, R]	7SP2.1	Analyze a given set of data to identify any outliers.	p. 140
		7SP2.2	Explain the effect of outliers on the measures of central tendency for a given data set.	p. 140
		7SP2.3	Identify outliers in a given set of data, and justify whether or not they are to be included in reporting the measures of central tendency.	p. 140
		7SP2.4	Provide examples of situations in which outliers would and would not be used in reporting the measures of central tendency.	p. 140
7SP3	Construct, label and interpret circle graphs to solve problems. [C, CN, PS, R, T, V]	7SP3.1	Find and compare circle graphs in a variety of print and electronic media, such as newspapers, magazines and the Internet.	p. 94
		7SP3.2	Identify common attributes of circle graphs, such as: <ul style="list-style-type: none"> <li>title, label or legend</li> <li>the sum of the central angles is <math>360^\circ</math></li> <li>the data is reported as a percent of the total, and the sum of the percents is equal to 100%.</li> </ul>	p. 94
		7SP3.3	Translate percentages displayed in a circle graph into quantities to solve a given problem.	p. 96
		7SP3.4	Interpret a given circle graph to answer questions.	p. 96
		7SP3.5	Create and label a circle graph, with and without technology, to display a given set of data.	p. 98

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

<b>Strand:</b> Statistics and Probability (Chance and Uncertainty)		<b>General Outcome:</b> Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	
<b>Specific Outcomes</b> <i>It is expected that students will:</i>		<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome.</i>	<b>Page Reference</b>
7SP4	Express probabilities as ratios, fractions and percents. [C, CN, R, T, V]	7SP4.1 Determine the probability of a given outcome occurring for a given probability experiment, and express it as a ratio, fraction and percent.	p. 142
		7SP4.2 Provide an example of an event with a probability of 0 or 0% (impossible) and an example of an event with a probability of 1 or 100% (certain).	p. 142
7SP5	Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events. [C, ME, PS]	7SP5.1 Provide an example of two independent events, such as: <ul style="list-style-type: none"> <li>• spinning a four section spinner and rolling an eight-sided die</li> <li>• tossing a coin and rolling a twelve-sided die</li> <li>• tossing two coins</li> <li>• rolling two dice</li> </ul> and explain why they are independent.	p. 144
		7SP5.2 Identify the sample space (all possible outcomes) for each of two independent events, using a tree diagram, table or other graphic organizer.	pp. 144, 146
7SP6	Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events. [C, PS, R, T]	7SP6.1 Determine the theoretical probability of a given outcome involving two independent events.	pp. 148, 150
		7SP6.2 Conduct a probability experiment for an outcome involving two independent events, with and without technology, to compare the experimental probability with the theoretical probability.	pp. 148, 150
		7SP6.3 Solve a given probability problem involving two independent events.	pp. 148, 150



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