Conceptual design of long-span cantilever constructed concrete bridges

(Konceptuell utformning av konsolutbyggda betongbroar med långa spann)

by

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Abstract

Bridge design is a very delicate matter. One may argue that being a masterpiece, the beauty of a bridge can only be seen and felt from individual to individual and not accepted by the whole community. There was always the curiosity to know if this assumption was true and, in that case, the reason why.

There will be a brief introduction both to the cantilever method and the evolution of this method itself through time and a closer look and the world leading long-span bridges of today.

As this thesis is a conceptual study of bridge design for cantilever constructed concrete bridges, we aim to get good design notions, that is, the guidelines we need to follow in order to project a pleasant looking bridge, and then evaluate this type of bridges throughout the world to see if what we have learned is what it is being made. And if not, the reason behind it.

The second part of the thesis is more objective. Using case studies we will see the difference, in terms of material usage and consequent cost, between bridges built with the main purpose of good design and bridges built with the main purpose of being economic. From there we will learn the consequences of our choice basing ourselves on the terms of comparison between the two solutions.

By the end of our work, we will have developed a critical analysis towards a bridge, in terms of achieved design, and also distinguish the case were we should privilege economy over design, and vice-versa. With this thesis we hope we could enlighten a bit more the subject of bridge design for cantilever constructed prestressed concrete bridges.

Preface

From the very first day I began my academic studies I had the dream to go and study abroad. For that, I thank my home university IST and KTH for giving me that opportunity and let me live this indescribable experience.

I begin to thank my Professor and Mentor Håkan Sundquist who was always available to help and motivate me with great passion for the theme and work itself.

I would also like to thank to my amazing group of friends both in Portugal and the new I met during my stay in Stockholm, with a special regard to both my Tyresö friends and the Hammarby Rugby team for their great family spirit.

Finally a special “thank you” to my family and girlfriend for supporting me everyday.

Notation

Latin characters	**Unit**	
dg_{inf}	m	Distance between the bottom flange and the center of gravity
dg_{sup}	m	Distance between the top flange and the center of gravity
$e_{bottom\ flange}$	m	Thickness of the bottom flange
$e_{top\ flange}$	m	Thickness of the top flange
e_{web}	m	Thickness of the web
f_{cd}	MPa	Design compressive strength of concrete
f_t	MPa	Tension of the prestress tendons
h	m	Cross section height in the pier section
l	m	Half of the length of span (L/2)
t	m	Cross section height in the middle of the span section
y_i	m	Ordinate of the center of gravity of the element i
y_g	m	Ordinate of the center of gravity of the cross section

Capital Latin characters

$A_{bottom\ flange}$	m^2	Area of the bottom flange
A_t	m^2	Area of the prestress tendons
A_{webs}	m^2	Area of the webs
$F_{prestress}$	KN	Prestressing force
I	m^4	Moment of Inertia in relation to a neutral axis
L	m	Length of the span
M	KNm	Moment
$M_{bottom\ flange}$	KNm	Moment of the bottom flange
M_t	Kg	Mass of prestress tendons
M_{webs}	KNm	Moment of the webs
P	KN/m	Deadweight

$P_{top\ flange}$	KN/m	Load caused by the top flange
P_{webs}	KN/m	Load caused by the webs
V	KN	Shear force
$V_{top\ flange}$	m^3	Volume of the top flange
$V_{bottom\ flange}$	m^3	Volume of the bottom flange
V_t	m^3	Volume of the prestress tendons
$V_{TOT}a$	m^3	Total volume of the superstructure when the ratio h/t=2.7
$V_{TOT}b$	m^3	Total volume of the superstructure when the ratio h/t=4
V_{webs}	m^3	Volume of the webs
W_{inf}	m^3	Flexion module of the bottom part of the cross section
W_{max}	m^3	Flexion module of the upper part of the cross section at the pier
W_{min}	m^3	Flexion module of the upper part of the cross section in the middle of span
W_{sup}	m^3	Flexion module of the upper part of the cross section

Greek characters

γ	KN/m^3	Volumetric weight
σ_c	MPa	Compression tension
σ_t	MPa	Traction tension

Capital Greek characters

ΔA	m^2	Difference between two areas

Contents

1. Introduction

"When the history of our time is written, posterity will know us not by a cathedral or temple, but by a bridge"

- Montgomery Shuyler, 1877, writing about John Roebling's Brooklyn Bridge

1.1. Objective

Long span concrete box girder bridges allowed Man to build longer and better bridges. Due to its size and importance these types of structures are sure to create an impact. Consequently, there is, or should be, an effort made in order to make the bridge not only a structure but a piece of art as well.

Throughout this work we are going to study the aesthetic guidelines for good design and build our case studies based on these same guidelines. Then, a conceptual study will be made and the case studies will be evaluated and compared according to the volume of material (concrete and steel) used by each and, therefore, its final cost. By the end of our studies our objective is to get a notion of the values implicit when referring to design, dimensions, material and cost of the superstructure of a long span concrete box girder bridge.

1.2. Bridge Design

Ever since the ancient times, when it comes to large scale constructions, there is the general need to make a good impact among the beholders, whether for the greatness, for its simple beauty or even both.

Bridges are structures that, due to its connecting function, tend to be more isolated from other constructions thus, creating a bigger impact. So, Humankind has always tried to find new ways of improving the aesthetics and the design of bridges. Due to these constant advances, the length of the bridges started to get bigger and bigger along with the impact that they caused. After the basic functions of the bridge were fulfilled (security and safety), there was the need to make to turn a structure into a monument, a symbol of the place where it was built. Bearing this in mind, engineers and bridge designers tried to cope size with beauty. With this, bridges were no longer seen as just a way to connect two places, but as monument or construction which represented the city. The structural elements of the bridge were now carefully aimed to be organized in a way that produced a pleasantly looking outcome.

However, good design has a cost, a price. Sometimes the cost for a better looking solution doesn't justify its improvement. Other times, the importance of the construction itself can justify the extra amount of money.

All in all a bridge with a good design surpasses the mere concept of a linking construction and becomes a mark for all the years yet to come.

1.3. Case Studies

One of our objectives is to find the difference of material usage and respective cost for long span concrete box girder bridges; therefore, we will study bridges with different lengths of span (**Figure 1** - *L*), ranging from 100 to 300m, and each example is spaced by 50m from the next – 100, 150, 200, 250 and 300m.

Our case studies will have a varying height of the cross section, as we see in **Figure 1**. And, as we will further see, the ratio *h/t* plays an important part in both bridge aesthetics and cost. So, for each span length we will study two superstructures: One with a ratio *h/t* of 2.7 and the other with a ratio *h/t* of 4.

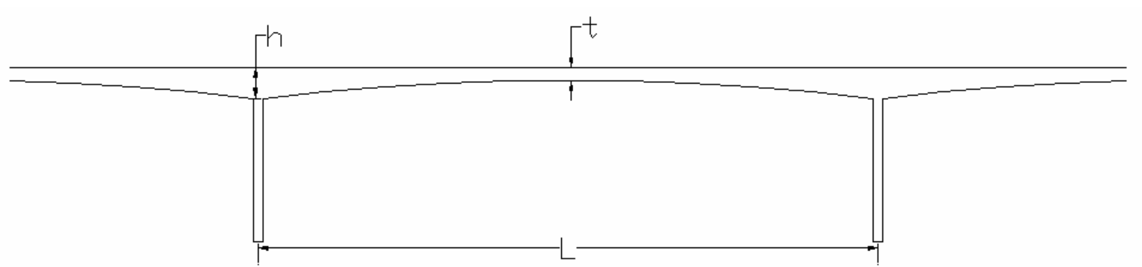


Figure 1 – Generic model of our case studies

As for the cross section used, we know that for this type of bridges the only compatible cross section, due to the properties that will later be listed, is the box girder – **Figure 2.a)**

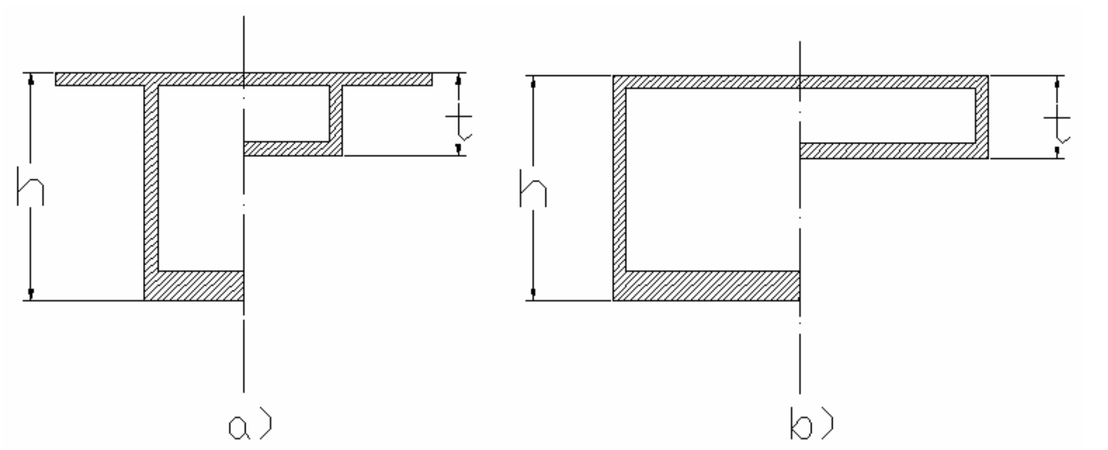


Figure 2 – Box girder section at the pier section and at mid span, respectively.

However, for our project we will simplify the box girder into **Figure 2.b)**. As this is a study made especially for comparing solutions we know that our interest is not the final result of one solution alone but its comparison with another one. For that reason we chose to simplify our cross section.

2. Cantilever Method

The Cantilever Method consists in building the bridge from a supporting end, such as a pier, using segments which range form 3 to 6m. This method can be executed:

- Symmetrically, for each side of the pier;
- Asymmetrically, from one end.

In the case of presstressed concrete bridges, each segment is presstressed as it is built – **Figure 3**

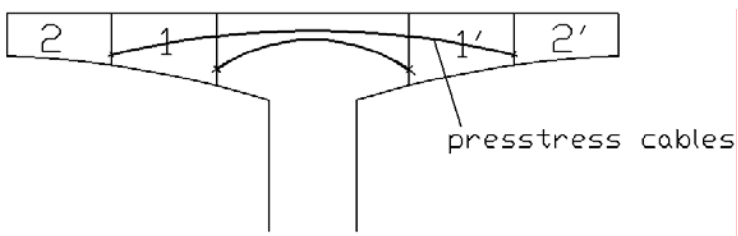


Figure 3 – Scheme of the cantilever method starting from a pier

Both the deadweights of each segment and the equipment are supported by the parts of the structure which are already built and presstressed. The connection of the deck is then made trough a "closing segment" with a length from 2 to 3m. In the following figure we can see the final stage of the cantilever method in the building of the Norwegian bridge – *Raftsundet*:

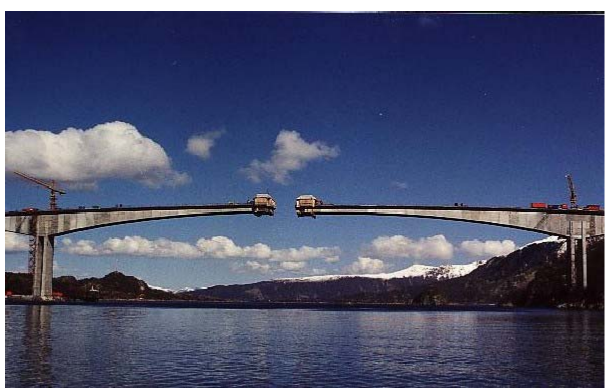


Figure 4 – *Raftsundet* Bridge, Norway

This method has the advantage of not needing any kind of structure supported in the terrain in order to hold the superstructure. Therefore it is extremely useful to build over difficult or inaccessible terrains, such as water and incoherent soil as we will see further on in a brief historical overview.

Due to its high cost, the cantilever method is, when possible, used with other construction methods:

- Scaffolding towers– **Figure 5**;
- Counterweight in one end of the cantilever – **Figure 6**.

The choice between the first and the second auxiliary methods relies on the accessibility of the terrain below the bridge. That is, in situations such as deep valleys or traffic roads that cannot be obstructed.

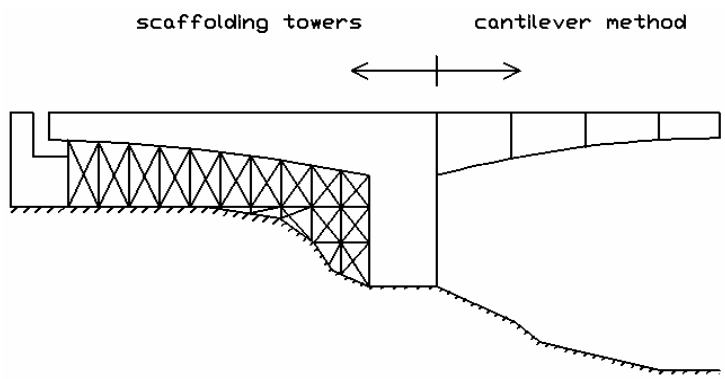


Figure 5 – Construction of a deck using the cantilever method and scaffolding towers

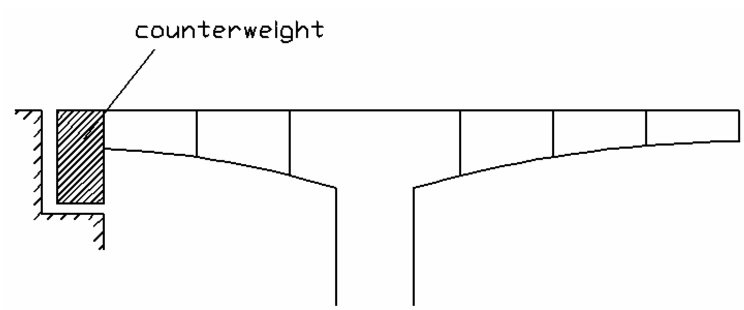


Figure 6 – Construction of a deck using the cantilever method and a counterweight on the opposite side

The closure of cantilevers (**Figure 4**) has also changed from the first solutions to the up up-to-date ones. The first bridges used a hinged positioned in the middle of span (where the cantilevers from each side met) – **Figure 7**

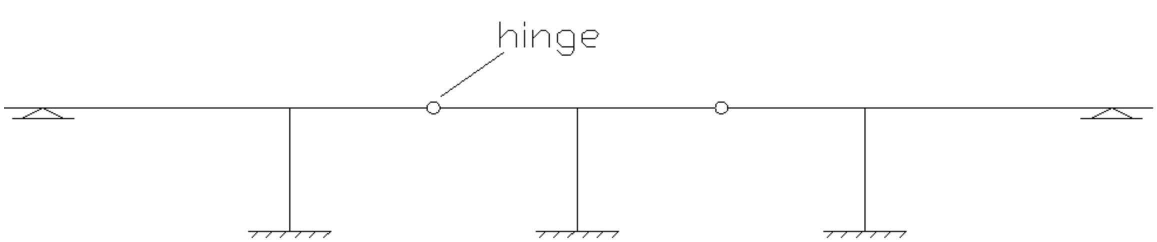


Figure 7 – Bridge built with the cantilever Method using a central hinge

The hinges allowed axial displacements and rotations. Like this, the effects caused by variations of temperature, creep and retraction of the concrete were eliminated. However, this solution had the following inconvenient:

- The need of a joint in the middle of each span;

- A possible problem with the span's articulations throughout the years.

Nowadays, it is preferable to use continuous systems – **Figure 8**

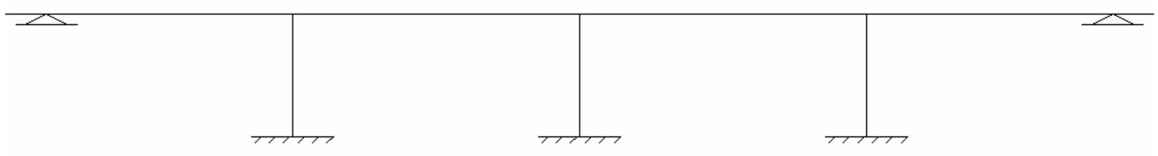


Figure 8 - Bridge built with the cantilever Method

No only do they avoid using joints, the weak points of a bridge, but they also have a good capacity of stress redistribution which allows the structure to absorb the effects caused by creep, retraction of the concrete, variations of temperature and settlement of supports. Nevertheless, when projecting a bridge of thus kind, one must bear in mind these effects.

3. Historical Overview

It is in the Human Nature to try to reach the unreachable, to keep on pursuing more goals and to acquire more knowledge in every field of interest.

Bridge construction has been suffering many changes throughout the years, whether for the type of material or the construction technique used. The basic function of a bridge is to serve mans need to surpass geographical obstacles, and as these obstacles kept getting bigger, man had do find new ways of reaching the other side.
The box girder section was the last solution found, for prestressed concrete bridges, to built greater spans due to its characteristics:

- A bottom flange which allows the cross section to be more resistant to compression forces, thus, less deformations caused by creep actions;

- Increased resistance to torsion making this cross section ideal for bridges with a horizontal radius;

- Increased slenderness and, therefore, a superstructure with less height, making the bridge more transparent;

As a matter of fact, these properties allowed these bridges to be built using the Cantilever Method. This method is used for long span bridges and every time the terrains bellow the bridge deck are inaccessible.

Historically, the Cantilever Method began to be used with wooden bridges, but became more commonly used with steel bridges. In 1930, in Brazil, the first concrete bridge was built using this method. The *Bridge over Rio do Peixe* (**Figure 9**), with a main span of 68.5m, had to be built out of both piers, as we can see, in order to eliminate the flood risk which could raise the water level up to 10 m in just a few hours.

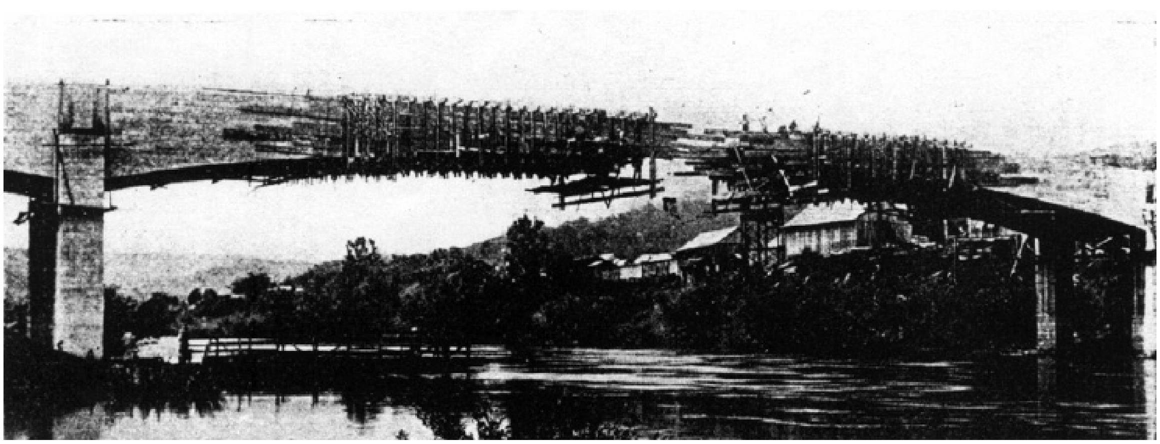


Figure 9 – *Bridge over Rio do Peixe*, Brazil

Although this was this achievement was a turning point for concrete bridge building, it was not recognized at that time.
A great pioneer of concrete bridge building and designing was Freyssinet (1879 – 1962) with the creation of prestress. Although the initial purpose of using prestress was to eliminate cracks and possible deformations through the creation of a beneficial state of stress, the increase of load capacity gained from the use of high-strength reinforcement was an important side effect. Among his projects one can highlight the *Luzancy Bridge* (**Figure 10**), in France, with a main span of 55 m, where simplicity and beauty is well achieved through the use of prestressed concrete.

Figure 10 – *Luzancy Bridge*, France

Freysinnet considered that prestressed concreted was a completely new material and would only accept the use of *full prestressing*, that is, the complete elimination of tensile stresses in the concrete, under the action of service loads. His ideas were kept for many years.

After World War II, there was a boom in bridge construction. The first years that followed the war were very important for the development of prestressed concrete bridges where several new construction techniques and new design were tested and approved. From this period, the major contributions were given by, the German, Fritz Leonhardt (1909-1999) and his book - *Prestressed Concrete – Design and Construction*.

It was in the beginning of the 1950s that the cantilever method was fully recognized to be extremely useful to prestressed concrete bridge building by, the German, Ulrich Finsterwalder (1897 – 1988). His first construction was the *Lahn Bridge*, 1951; with a span of 62 m, but his knowledge in this particular subject lead him to the construction of *Nibelungen Bridge* (**Figure 11**). This structure, with considerably bigger spans – 101.65m, 114,2m and 104.2m – managed to capture worldwide attention and became a mark for long span bridges, in prestressed concrete.

Figure 11 – *Nibelungen Bridge*, Germany

So, for spans, the cantilever method was the only one perfectly viable. With this method, Finsterwalder, surpassed himself and built the *Bendorf Bridge over the Rhine* with a, remarkable, 202 m span. With this achievement he managed to prove that prestressed concrete could compete with steel both in costs and deck height reduction.

Nowadays, the longest span belongs to *Shibanpe Bridge* (**Figure 12**), built in 2005, with a main span of 330 m. However, it is the only one to use steel girder in its main span and, therefore, its achievement is not fully acknowledged by most of the structural engineers.

Figure 12 – *Shibanpe Bridge*, China

When building the *Shibanpe Bridge*, in order to eliminate one central pier as well as maintaining the desired span-length while minimizing the effects caused by shear and bending, a 100 m long steel box section was placed middle of span between the prestressed concrete box girders.

In spite of having a span 29 m shorter than the *Shibanpe Bridge*, the *Stolmasundet Bridge* (**Figure 13**) is, actually, considered to hold the present world record span for free-cantilevering concrete bridge due to the fact the superstructure materials consist purely in concrete and prestressed concrete.

Figure 13 – *Stolmasundet Bridge*, Norway

In the *Stolma Bridge*, parts of the main span were built using a mix of high strength and lightweight concrete. The design and construction were carried out on the basis of the high experience Norwegian have with this type of bridges. In fact, as we can see in **Table 1**, four, out of the leading bridges in the world, are in Norway.

Nº	Bridge	Span	Location	Year
1	Stolmasundet	301 m	Austevoll, Norway	1998
2	Raftsundet	298 m	Lofoten, Norway	1998
3	Sundoy	298 m	Mosjöen, Norway	2003
4	Humen-2	270 m	Guangdong, China	1997
5	Gateway	260 m	Brisbane, Australia	1986
6	Varodd	260 m	Kristiansand, Norway	1994
7	Luzhou-2	252 m	Sichuan, China	2000
8	Schottwien	250 m	Semmering, Austria	1989
9	Ponte S.Joao	250 m	Oporto, Portugal	1991
10	Skye	250 m	Skye Island, Scotland	1995
11	Confederation	43 x 250 m	Northumberland, Canada	1997
12	Huanghuayuan	3 x 250 m	Chongqing, China	1999

Table 1 – The leading long-span prestressed concrete girder box Bridges in the World.

4. Aesthetics

"Successful design of a perfect structure can never be performed only on the basis of general rules concerning structural system, dimensions and proportions alone, as long as the design lacks in originality and individuality."

- Christian Menn

In the second half of the 20th century there was the general worldwide concept of building economical solutions. Nowadays, the concern in building esthetically pleasing bridges is growing again. In fact, when bridges started to be built by the ancient civilizations, such as the Romans and the Greek, they were created in order to fulfill two purposes: Functionality and Beauty. For that reason, we still admire and look upon most of the bridges and monuments made by them.

The bridge concept goes far beyond being a mere construction, it is a link between to place, two communities. It is a way of reaching new places. It is in the human nature to be proud of what one owns or of the place ones lives in. With bridges is not different, they cause so much impact within the society that, in the good cases, they become a symbol of that region, for the beauty they have or the status and prosperity they represent. Like the *Crni Kal Viaduct* (**Figure 14**), in Slovenia, that, besides belonging to the motorway, also serves as a stage for the well known cycling sports event – Giro d'Italia.

Figure 14 – *Crni Kal Viaduct,* Slovenia

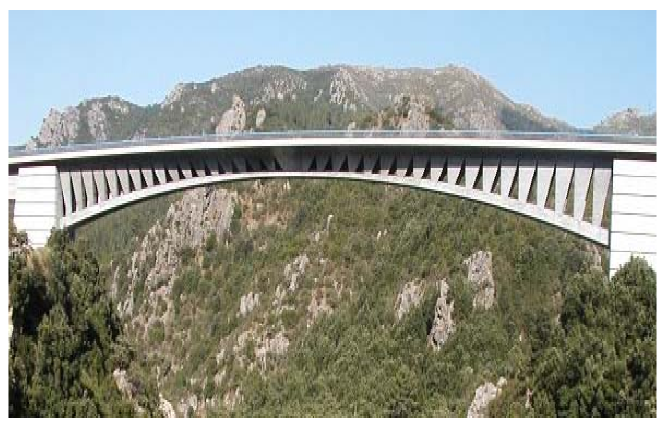


Figure 15 – *Vecchio Bridge,* France

One can evaluate a bridge through two different aspects: as an independent identity or as an element of a larger landscape. These two can, either, cope or be independent, however there can be structures that are aesthetically appealing as an independent element but do not integrate in their surroundings and vice-versa.

Therefore, when dealing with a bridge project, the designer must not neglect either aspect, as it can be seen in Michel Placidi's *Vecchio Bridge* – **Figure 15**. This bridge was completed in 1999, in Corsica. This structure is part of the highway that makes the connection between Bastia and Ajaccio; this highway not only crosses beautiful valleys but is also implanted in difficult terrains. The bridge has a total length of 222 m and a main span of 137.5 m. The structural combination of open truss form webs represented a world premier.

Bridge aesthetics can be evaluated in terms of harmony and efficiency. The first one concerns mainly the bridge integration with its surroundings, and the last one focuses more on the bridge itself.

Harmony

A bridge always creates an impact and, therefore, it is absolutely necessary that there is total harmony between the bridge and its landscape surroundings. This does not mean that the bridge has to blend in and try to hide itself, but it means that the structure has to be compatible with the environment. So, the bridge building process has to be dealt with great care. We know that the bridge is there, we can see it. It is the way that the bridge correlates with its surroundings that either creates a positive or negative impact.

So, the bridge should follow the topographical features of the terrain. For instance, a viaduct that goes along a mountain slope should be curved in order to follow the contours of it – **Figure 16.**

Figure 16 – *Chillon Viaduct,* Switzerland

Figure 17 – *Kubholmleia Bridge,* Norway

Or a bridge connecting islands can also adjust the arch of the span according to the island's topography – **Figure 17**.

Knowing that the beauty is in the eye of the beholder, is it then possible to establish certain standard ideas about better-looking bridges? Yes. Our perception of harmony comes to us from natural forms in which exist using minimum energy and materials.

Sometimes these forms are not invented by man but they are a part of Nature which man adapts to an urban construction, like Christian Menn's *Ganter Bridge* (**Figure 18**), where the triangular walls above the roadway show a great resemblance to a mountain's peak partially covered in snow, like the ones we can see in the back of the picture. Other times, these forms derive from the purpose of the bridge itself as we can see in the *Sunshine Skyway Bridge* (**Figure 19**), by Jean Muller, the golden cables appear to be the sail of a boat. Just from observing the bridges' surroundings and purposes both designers were capable of making worldwide masterpieces because, like so, they had the ability to make the structure cope with its environment.

Figure 18 – *Ganter Bridge*, Switzerland

Figure 19 – *Sunshine Skyway Bridge*, USA

The bridge components, themselves, must be all charmingly integrated into one coherent structural entity. Both visual and technical means contributed to this.
The visual means can be summarized into simplicity, symmetry, order and regularity, while the technical means refer to the structural function of each component.

<u>Simplicity</u> – The goal is to have as much as individual elements which are similar in function, size and shape as possible. The use of continuous lines also gives the observer a sense of continuity throughout the entire bridge;

<u>Symmetry</u> – Since the Roman times, the majority of bridges that have been acclaimed for their elegance have a symmetric structural form. This form creates an impression of balance and stability which corresponds to a state of a stable equilibrium and, also, low stress.

<u>Order</u> – This aspect concerns the orientation and arrangement of the bridge components. For instance, within similar structural members, and unless the members form a soft gently curve, there should be as minimum different inclination angles as possible.
If there are twin bridges parallel in plan, the roadways should be arranged in order to make only one of the two visible from the most important view point, or a combination of both that appear as only one when looked at from the same location (**Figure 20**).

Figure 20 – *Crni Kal Viaduct,* Slovenia

Regularity – There has to be certain regularity between the span length and the bridge height. For long bridges, if the bridge is of constant height, the uses of constant length spans tend to be more visual appealing – **Figure 21**.

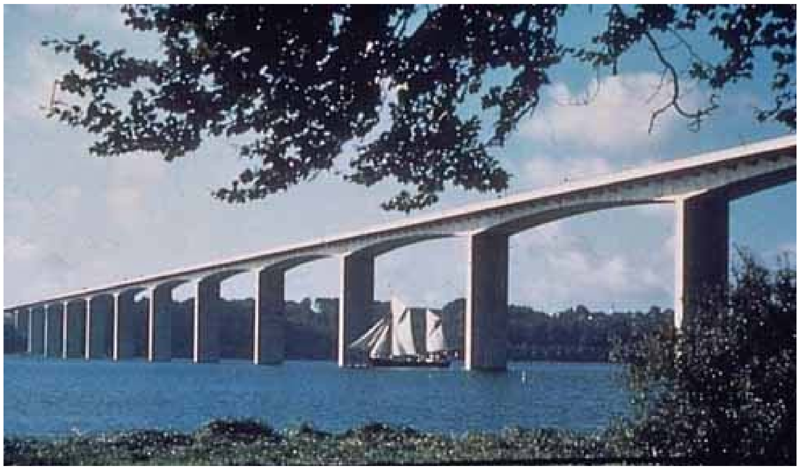


Figure 21 – *Veijle Fjord Bridge*, Denmark

When the height of the bridge varies so should the span length. The rule is that the ratio between the bridge height and the each span length remain the same. This will cause in a more balanced appearance – **Figure 22** and **Figure 23**.

Figure 22 – *Tanus Bridge*, France

Figure 23 –*Tanus Bridge*, France

Structural function of each component – Although constant cross section heights can be normally appealing due to its regularity. The shape of the structural member should be made according to forces that occur on them: the higher the force, the thicker the structural part. Therefore, in concrete box girder bridges, the varying height of the cross sections (greater deck depth in the piers than in middle of span) is a better-looking solution because it gives the observer a feeling of slimness and transparency.

Efficiency

This concerns, basically, the visual perception we get from observing a bridge and thus consider whether it is elegant or not. Efficiency is expressed through the transparency and slenderness of a bridge.

Transparency – It is evaluated regarding two different aspects: girder depth and number and width of columns. The first one concerns only the low bridges due to its proximity to the ground line. However, for most bridges, the achievement of transparency relies on the number and width of columns they have. In order to get maximum transparency, one column per axis is recommended. Two columns should only be used in low bridges, all the twin bridges and can be used in bridges with few, but long, spans. For wide, or twin, bridges, where there is the need to have two supports, it is visually better if the columns join each other and become one, if the height allows so (**Figure 14**). The width of the piers is the more important of these two aspects due to the fact that the transparency effect it may, or may not, cause is directly observed. In long bridges with many short spans, for instance, *Sandnessund Bridge* (**Figure 24**), the transparency is achieved through the use of suitably narrow piers.

Figure 24 - *Sandnessund Bridge*, Norway

This bridge is 1248 m long, a main span of 150 m and a total of 36 spans and yet managed to successfully achieve the transparency concept.

Slenderness – It is basically a function of the superstructure arrangement. The use of a box girder section is an advantage. The cantilevers of the box girder produces shadow effects which will create contrasting dark and light parallel strips, this will cause the bridge to appear longer and, therefore, making the superstructure thinner. If we choose to haunch the beams, varying the cross section, it will also improve the visual slenderness of the bridge. In fact, this is very important because not only will make the bridge appear more slender but, as we have seen before, it will create a greater sense of harmony in the observer. When looking at the picture of *Skye Bridge* (**Figure 25**) we get the notion that in the middle of span the bridge is much thinner than in reality. This effect was achieved both by the shadow effects and by varying the cross section height.

Figure 25 - *Skye Bridge*, United Kingdom

But how much should the cross section vary? What should be the ration between the box girder heights in the pier and in the middle of span? These questions can be answered both separately in terms of design and economy. The main goal, today, is to achieve the perfect relationship between these two different aspects.

Due to constructive and maintenance reasons the box girder must be at least 2.20 m – 2.30 m. This gives an effective height of 1.70 m – 1.80 m, inside de box girder, in order to give the workers the minimum comfort necessary to their well-being and productive work.

So what should be the ratio in order to achieve transparency and slenderness when projecting a bridge?

5. Evaluation of built bridges

According to the aesthetic ideas we have seen so far, the cross section should vary smoothly. To create a state of continuity the girder depth ratio should be around 2.5, and 3 in the limit cases. This is a good solution when transparency and slenderness are at stake. Such examples are the pictures **Figure 21** – **Figure 23**. The *Veijle Fjord Bridge* (**Figure 21**) has a main span of 110 m. The deck depths are 6.0 m and 2.50 m for the pier and middle of the span sections, respectively. Thus, the ratio between the two is 2.40. Knowing that this bridge's span is 110 m long, the depth of the deck is *L*/18, in the pier, and *L*/44 in the middle of the span (*L*=span length). With a main span of 190 m, the bridge in **Figure 22-23** – *Tanus Bridge* - has, for the same sections, the depths of 12.00 m (*L*/16) and 4.50 m (*L*/42), and a ratio of 2.67.
However, in some places over the world, namely in Australia, Norway and Sweden, the ratio is considerably higher – 4 to 5.

Figure 26 – *Raftsundet Bridge*, Norway

In **Figure 26**, we can see the second biggest span in the world – 298 m – and also the evidential difference of heights of the superstructure. Although the bridge appears relatively thick and heavy in the pier section, in the middle span it looks remarkably thin. In fact depth of the deck varies from 3.5 (*L*/21) to 14.5 m (*L*/85).

Figure 27 – *Sundoya Bridge*, Norway

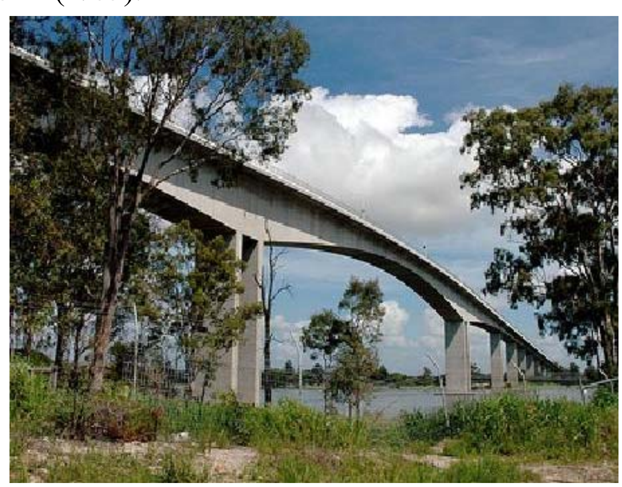


Figure 28 – *Gateway Bridge*, Australia

The *Sundoya Bridge* (**Figure 27**) has main span of 298 m and a ratio of 4.14 (14.5 / 3.5 m). Both the *Raftsundet* and the *Sundoya* bridges use prestressed lightweight concrete for the deck in order to reduce the dead weight and dynamic mass achieving, therefore, a greater span length.

The *Gateway Bridge* (**Figure 28**) is the 6^{th} long span leading bridge of the world (for concrete box girder bridges) with a main span of 260 m. In 1980, when the bridge began to be built, it was said that for spans of this range, a cable stayed model with be both more economical and elegant. However, the bridge had a height restriction caused by air traffic – 80 m and the shipping needed a navigational clearance of 55 m. So the solution found was the one we see in the picture. This bridge, with deck depths of 15.0 (L/17) m in the pier and 2.90 (L/90) m in middle of span, has an amazing ratio of 5.17.

However these last bridges, in spite of having a bigger ratio also have longer spans. So we may want to assume that in order to achieve distance we also have to increase the ratio between the girder depths and, consequently, tend to disregard the aesthetic part when it comes to the referenced ratio values – 2.5. A quick scan through the "Leading Bridges of the World" (**Table 1**) can help us to reach some conclusion.

The next bridges all have a main span of 250 m: *S.João Bridge* (**Figure 29**), *Schottwien Bridge* (**Figure 30**), *Skye Bridge* (**Figure 25**), *Confederation Bridge* (**Figure 31**) and *Huanghuayuan Bridge* (**Figure 32**). The first three are constituted by 3 beams whilst the *Confederation Bridge* is one of the longest bridges in the world and the *Huanghuayuan Bridge* is also considered to be a long bridge with 3 main spans of 250 m.

Figure 29 – *S.João Bridge*, Portugal

Figure 30 – *Schottwien Bridge*, Austria

The π-shaped *S.João Bridge* (**Figure 29**) was built in 1991. We can see that the cross section height varies smoothly and the ratio is, in fact, 2.7 which is the value wanted for aesthetic purposes. The varying shape of the piers also helps the structure to appear more slender.

We can easily see that the cross section height in *Schottwien Bridge* (**Figure 30**) is more variable. In spite of having a ratio value near to the previous bridge – 3 – we get the feeling that the ratio is bigger due to the linear variation of the cross section which creates a noticeable angle in middle of span.

The *Skye Bridge* (**Figure 25**) was built in order to connect Scotland to the Skye Island in 1995. The ratio is 2.7, the cross section varies in a parabolic curve and this bridge is considered to be elegant. The girder depth ranges from 12.5 m (*L*/18) to 4.7 m (*L*/53).

Figure 31 – Confederation Bridge, Canada

This remarkable, 12.9 Km long, bridge was built in 1997 with 43 spans of 250 m each. Due to its length, this multi-span concrete box girder structure was designed, with slight curves in order to ensure that drivers remain focused on the road. The cross section height ranges from 14.0 m (L/18) to 4.5 (L/56) making a ratio of 3.11.

Figure 32 – *Huanghuayuan Bridge*, China

Using some of the data we have used until now, mainly about the long-span leading bridges of the world, and adding some more useful data, we are able to build a table (**Table 2**) which may help us to reach some conclusions:

Bridge	Location	Year	L	L/h	L/t	Ratio	Fig.
Arrêt-Darré	France	1987	100	14	31	2.2	46
Avignon	France	1999	100	12	20	1.7	38
Indalsälven	Sweden	1967	105	18.3	68.9	3.8	
Österdalälven	Sweden	1955	107	19.0	35.0	1.9	
Källösund	Sweden	1959	107	14.3	62.9	4.4	
Ottmarsheim	France	1979	110	16	44	2.8	43
Veijle Ford	Denmark	1980	110	18.3	44.0	2.4	21
Felsenau	Switzerland	1975	114	14.3	38.0	2.7	47
Kubholmleia	Norway		115	18.3	67.7	3.7	17
Beaumont-sur-Oise	France	1997	120	18	40	2.2	48
Magnan	France	1975	120	16.0	42.9	2.8	37
Seskarö	Sweden	1978	120	17.1	60.0	3.5	
Hammarsund	Sweden	1994	120	18.5	48.0	2.6	49
Tan De	Vietnam	2002	120	18.5	40.0	2.2	39
Stocksund (west)	Sweden	1992	122	18.8	61.0	3.3	
Stocksund (east)	Sweden	1992	122	18.8	61.0	3.3	
Vignasses	France	1978	122	16	41	2.5	35
Strängnä	Sweden	1981	124	15.5	41.3	2.7	
Nantua	France	1988	124	15	41	2.8	50
Angered	Sweden	1979	129	16.1	51.6	3.2	
Öland	Sweden	1972	130	14.4	43.3	3.0	
Bellegarde-sur-Valserine	France	1982	130	20.0	40.6	2.0	51
Otira	New Zealand	2000	134	17.3	48.7	2.8	52
Alnö	Sweden	1964	134	17.9	89.3	5.0	
Aakviksundet	Norway	1999	135	19	64	3.3	34
Vecchio	France	1999	137.5	12.5	39.3	3.1	15
Torsö	Sweden	1994	140	18.7	70.0	3.8	
Tricastin	France		142.5	19.0	57.0	3.0	53
Gimsøystraumen	Norway	1980	148	20	74	3.7	44
Instö	Sweden	1991	150	16.7	50.0	3.0	
Sandnessund	Norway	1973	150	17.9	75.0	4.2	24
Calix	France	1974	156	16	82	5.1	54
Saltstraumen	Norway	1978	160	18.5	72.1	3.9	55
over Vätösundet	Sweden	1993	165	17.4	55.0	3.2	
Grand canal d'Alsace	France	1979	172	19	57	3.0	42
Bolte	Australia	1999	173	13.6	65.3	4.8	40
Deutzer	Germany	1980	184.45	23.7	57.6	2.4	41
Tanus	France	1998	190	15.8	42.2	2.7	22;23
Farstasund	Sweden	1985	200	16.7	66.7	4.0	56

Bridge	Location	Year	L	L/h	L/t	Ratio	Fig.
Boknasund	Norway	1991	224	15	41	2.6	36
Confederation	Canada	1997	250	18.0	56.0	3.1	31
Skye	Scotland	1995	250	20.0	53.2	2.7	25
S.Joao	Portugal	1991	250	18.0	50.0	2.7	29
Gateway	Australia	1986	260	17.3	90.0	5.2	28
Sundoy	Norway	2003	298	20.6	85.1	4.1	27
Raftsundet	Norway	1998	298	20.6	85.1	4.1	26
Stolmasundet	Norway	1998	301	20.1	86.0	4.3	13

Table 2 – Concrete Long Span Bridge Data

The values of *L*, *h*, *t* and all the combinations used with these variables are obtained from the measures of the different bridges (**Figure 33**). The meaning of both the variables and the ratios showed are also explained in **Table 3**.

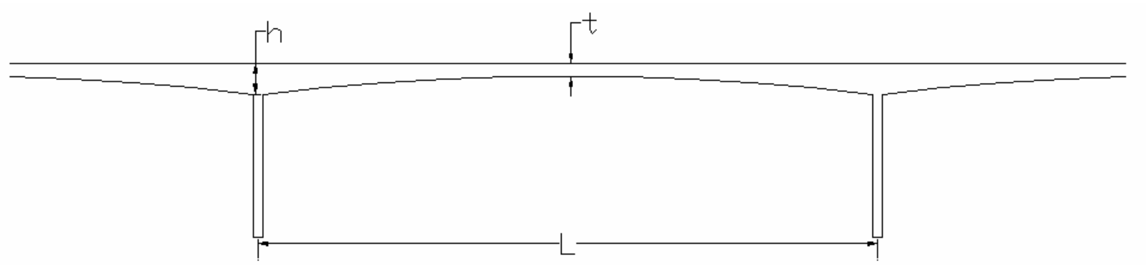


Figure 33 – General dimensions

L	Span length
h	Girder depth in the pier section
t	Grider depth in the middle of span
L/h	Ratio between the span length and the girder depth - pier
L/t	Ratio between the span length and the girder depth - middle of span
h/t	Ratio between the girder depths

Table 3 – Variable list

A quick scan through **Table 2** gives us the idea that the ratio *h/t* increases with the span's length. In fact, if we build a chart with these values we obtain the following:

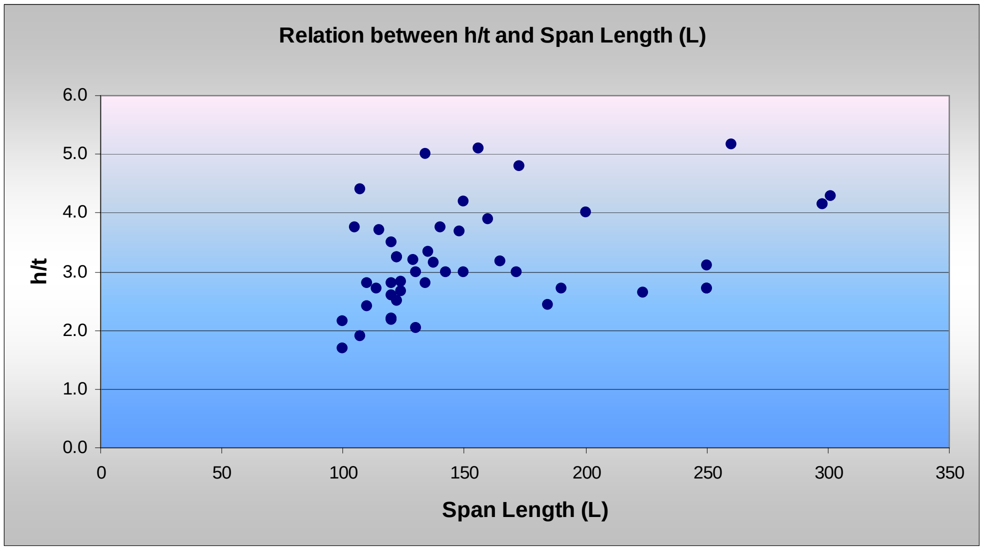


Chart 1 – Relation between h/t and the span's length

There are, however, some exceptions. Whether for having a noticeable bigger ratio - signalized in red - or the opposite situation - green.

Beginning with the "red" bridges, we can see that they were built in Norway and Australia. These are countries that use a big ratio for this type of bridges, as we have seen before.

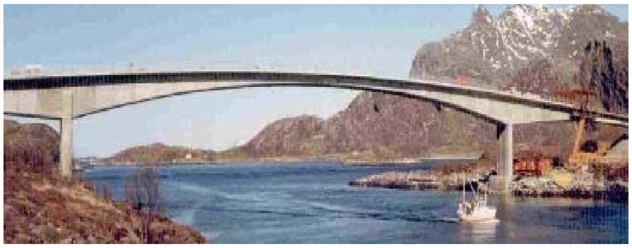


Figure 34 – *Aakviksundet Bridge*, Norway

Figure 35 –*Vignasses Viaduct*, France

Figure 36 - *Boknasund Bridge*, Norway

Figure 37 – *Magnan Viaduct*, France

In order to get a more clear observation of this relation, the odd values ("red" and "green") will be removed and a linear trendline will be traced for an easier perception of the chart – **Chart 2**.

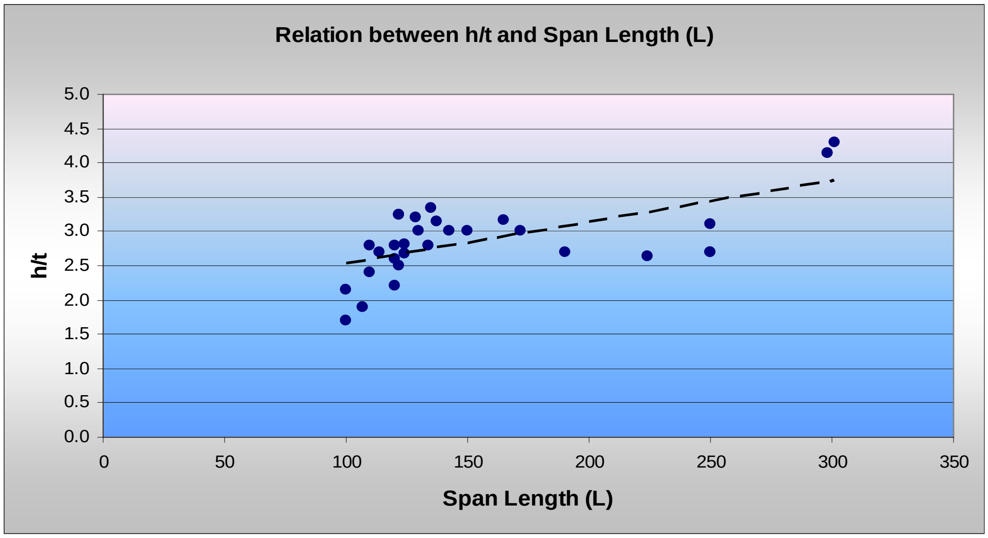


Chart 2 – Relation between h/t and the span's length

Now we can easily see a linear relation between the ratio *h/t* and the length of span. We can also see that the *h/t* values remain relatively close as the span increases until it reaches 250 m, then, in the following 50 m there is a big gap of ratio values passing from, an average, 3 to 4.2. The lowest ratio value is 1.7 – *Avignon Bridge*, **Figure 38** – and the highest is 5.2 – *Gateway Bridge*, **Figure 28**.

Figure 38 –*Avignon Bridge*, France

However, we can also see that the bridge pictured above is not so aesthetically pleasing. It looks quite massive and the variation on the cross section height is almost unnoticeable. In fact, it appears to have constant height except from the pier section. From this we see that a low ratio *h/t* is also not recommended when aesthetics and good design are at stake. Another example of this is the *Tan de Bridge* in Vietnam – **Figure 39**.

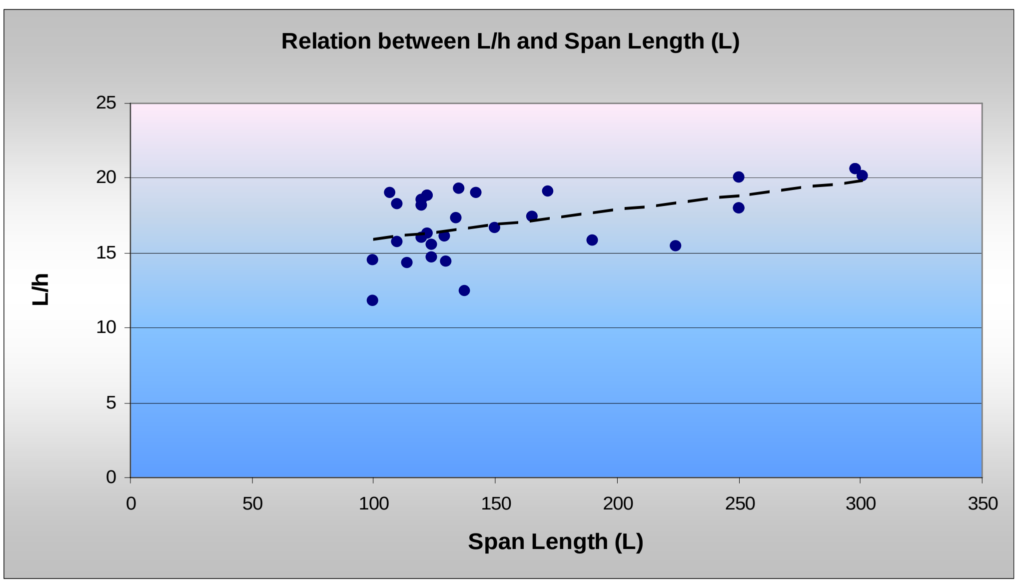
Figure 39 – *Tan De Bridge*, Vietnam

Figure 40 – *Bolte Bridge*, Australia

In the *Bolte Bridge* (**Figure 40**) we see that the cross section height varies considerably trough a linear line originating a noticeable angle in the middle of the span. This means that the bridge designers did no follow all the recommended guidelines and the sense of harmony was not fully achieved.

Still observing **Table 2** and with the help of **Chart 3** we see that the *L/h* values remain relatively close, apart some odd cases, and also that the *L/h* values also increase with the span's length.

Chart 3 – Relation between L/h and the span length



A closer look at **Chart 3** allows us to know that almost all the L/h values are between 15 and 20. From the trendline traced we can also see that the longer the span gets, the closer L/h is to 20. The highest L/h value is 23.7 – *Deutzer Bridge*, **Figure 41** – and the lowest is 12 – *Avignon Bridge*, **Figure 38**.

Figure 41 – *Deutzer Bridge*, Germany

The greatest variations of values occur in the *L/t* column. Once again, the longer the span, the higher the *L/t* value, as seen on **Chart 4**.

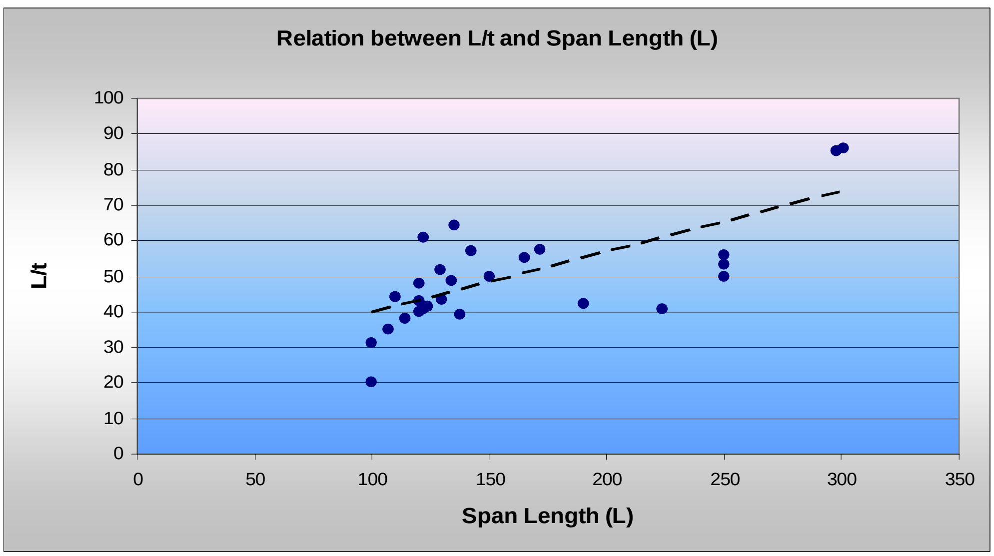


Chart 4 – Relation between L/t and the span length

From this chart we can take say that for a span between 100 m and 150 m, the preferred *L/t* value is around 45 and this value keeps on growing until 85 when it reaches the greatest span dimension ever – 301 m.

When comparing **Chart 4** with, the previous, **Chart 3** we can see that the gap of values in the latest is quite bigger, ranging from 20 - *Avignon Bridge*, **Figure 38** - to 90 - *Gateway Bridge*, **Figure 28**. Both the lowest and highest *L/t* values are from the same bridges that, also, have the lowest and highest ratio *h/t*, respectively, which allows us to think that *L/t* and the *h/t* values are strongly connected. That is, only by analyzing the *L/t* value we can have an immediate approximate value of the ratio of the deck depth.

However, due to the considerable oscillations of these values, we will build a chart in order to study, more carefully, the relation between *L/t* and *h/t* to see if we can jump to any further conclusions.

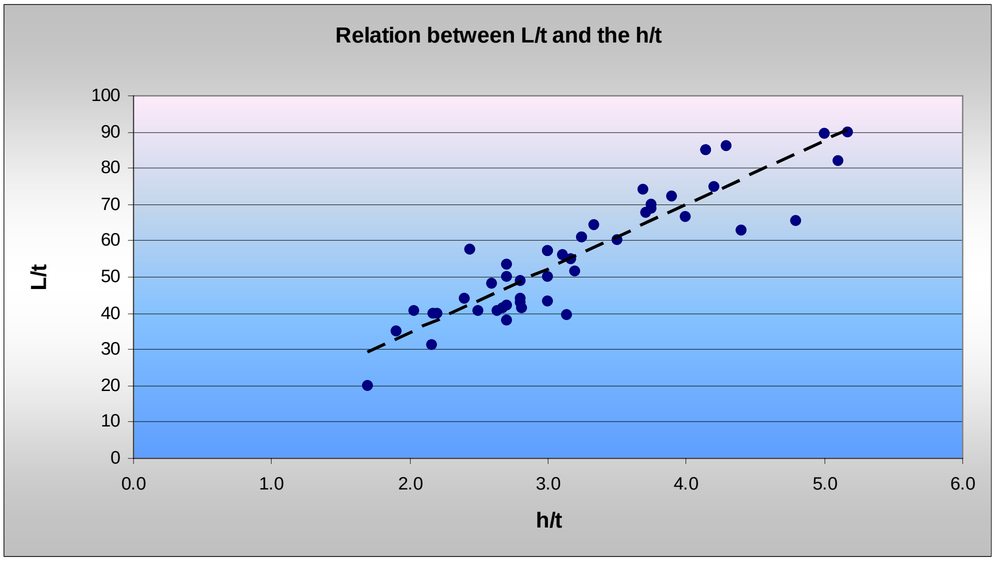


Chart 5 – Relation between L/t and h/t

When analyzing **Chart 5**, our initial suspicions are confirmed. There is almost a perfect linear relation between *L/t* and *h/t*.
For *h/t* values around 4, *L/t* is about 75. When the ratio is between 2.5 and 3, the aesthetically recommended, *L/t* has an average value of 45.

Figure 42 – *Grand Canal d'Alsace Bridge*, France

Figure 43 – *Ottmarsheim Bridge*, France

Figure 44 – *Gimsøystraumen Bridge*, Norway

Comparing, now, the relation between *L/h* and the ratio *h/t* we have:

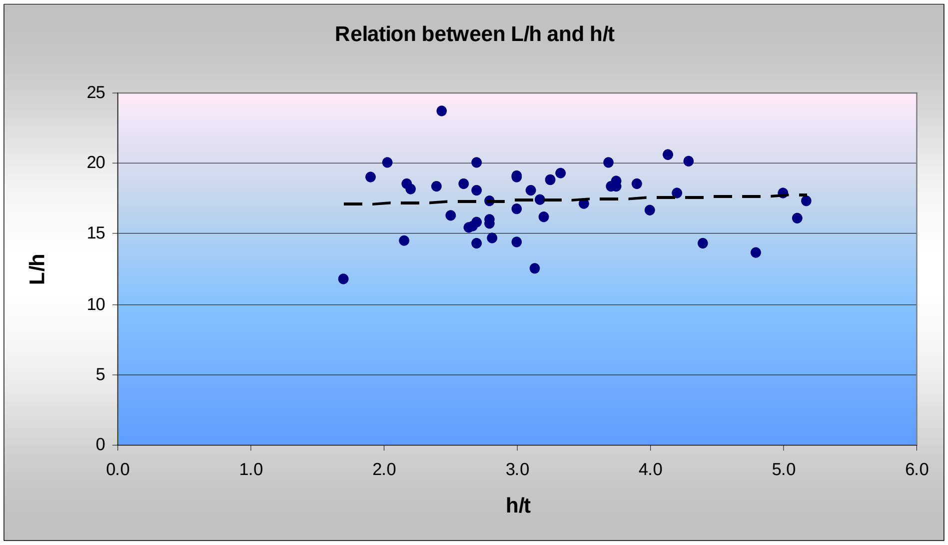


Chart 6 – Relation between L/h and the h/t

There is also a linear relation though not so perfect as the one seen on **Chart 5**. For *h/t* values around 2.7 *L/h* is about 16-17 and for an *h/t* value close to 4 L/h varies between 20 and 21.

From the charts shown above we can conclude that the variation on the ratio *h/t* is noticed mostly in the middle of span. In the figure below it is possible to see the consequences of different ratio *h/t* values. Although the red line is remarkably thin in the middle of span, in the pier section the bridge is quite massive. The green line is more continuous enhancing the sense of inner harmony. However, we can only compare these two for spans up to 250 m, using real examples.

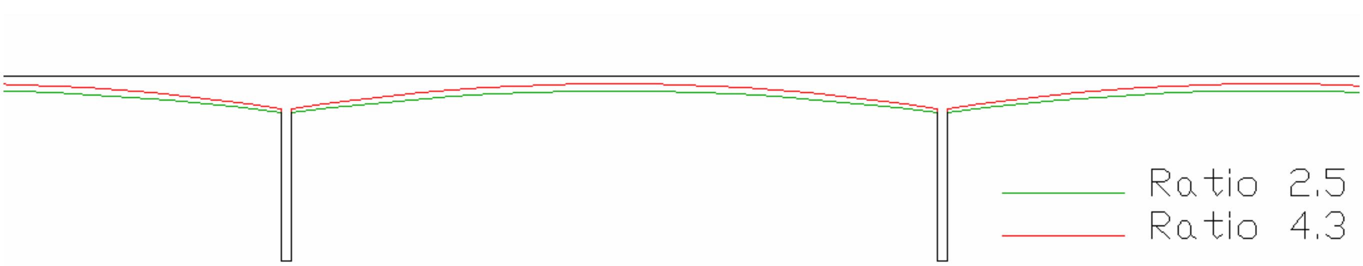


Figure 45 – Ratio differences

In the following figures we can see all these differences applied in real bridges with the necessary support data displayed in **Table 2**. The following bridges are organized by the length of span.

Figure 46 - *Arrêt-Darré Viaduct*, France

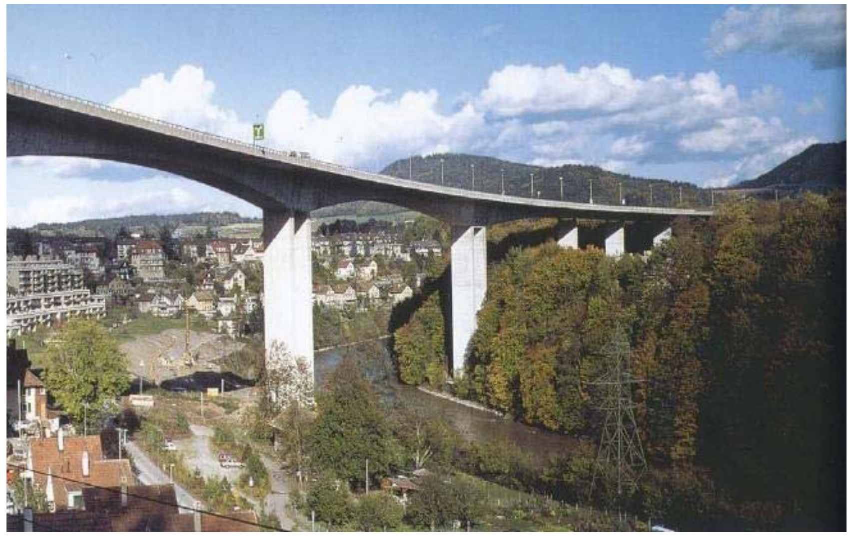


Figure 47 – *Felsenau Bridge*, Switzerland

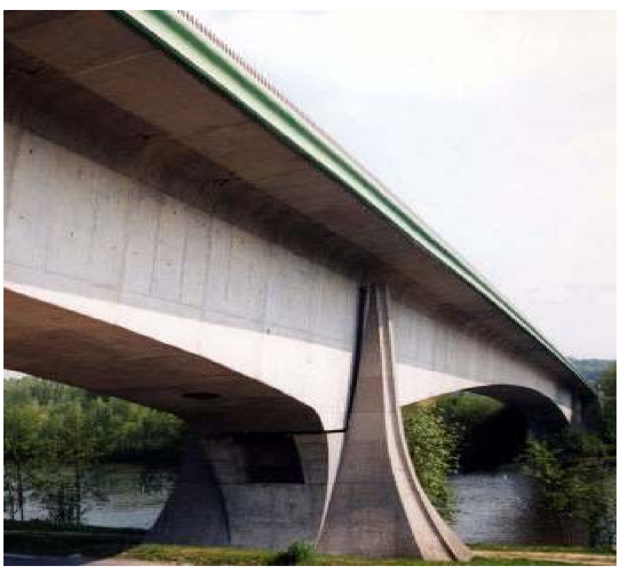


Figure 48 - *Beaumont-sur-Oise Bridge*, France

Figure 49 – *Hammarsund*, Sweden

Figure 50 –*Nantua Viaduct*, France

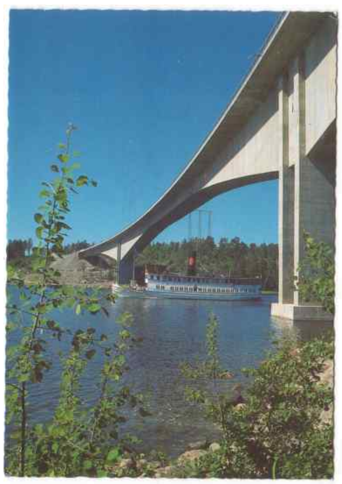
Figure 51 –*Bellegarde-sur-Valserine Bridge*, – Fance

Figure 52 *Otira Viaduct*, New Zealand

Figure 53 – *Tricastin Bridge*, France

Figure 54 – *Calix Viaduct*, France

Figure 55 – *Saltstraumen Bridge*, Norway



The *Farstasund Bridge* (on the left) has the longest span in Sweden, for concrete box girder bridges, with 200m. We can see that the cross section height varies but not always according to the same standards. That is, from the piers its variation follows a parabolic curve but as it starts approaching mid span, the deck height becomes constant and remains the same. Like the *Bolte Bridge* (**Figure 40**) this bridge failed to capture the harmony concept by creating angles and changing variation lines in the span.

Figure 56 – *Farstasund Bridge*, Sweden

The former pictures range from a span length of 100 to 200m. We could see that as the length of the span increased so did the variation of the deck height become more noticeable, as we knew it would happen from the chart observations.

So the only way to build a span 300 m long is by using a big ratio value?

We could also see that, for bridges with a span around 250 m, it is possible to achieve the desired ratio. So, if it is possible to build elegant long-span bridges why isn't that the most common choice of both engineers and designers?

These two questions are the goal of this thesis and in order to be fully answered we will need to study both the bridge's structure and cost.

6. Optimum measures

6.1. Economy

"It pays to spend some money for better esthetic quality. It pays no only because of the favorable social and health effects of beauty for the well-being of our fellow citizens but also for the reputation of our profession."

- Fritz Leonhardt

It is commonly believed that a better-looking solution leads to greater costs. However, it is very important to improve this aspect along with the other two – harmony and efficiency - in order to achieve the perfect solution. Therefore, every person who intervenes in the project knows what to do and how to do from the start.

There is a relation between aesthetics and the efficient use of material, as we have seen before; the design wanted is guided by simplicity, slimness, slenderness which requires less material. Therefore, sometimes a tight budget can act as a spur to creativity. But this does not mean that economy guarantees beauty.

In a raw point of view, the cost of a bridge will be the sum of the costs of man-work, mechanical equipment and quantity of material used. Nonetheless, the total cost will always depend on the complexity of the project, whether it is to make the bridge better-looking or even just due to the difficult landscape the bridge will be built in. Therefore an economical solution may vary from one place to another.

So how much cost is reasonable? What percentage of the total cost of the bridge should be taken into consideration?

There are several beliefs. Some claim that aesthetics are inherent to a good structural design and, therefore, "it costs nothing". Others defend an idea which consists in saving money on the less important bridges in order to spend on the most important ones.
I believe that in every project a careful study should be made in order to evaluate both the aesthetics and the cost of the solution. Sometimes, the difference a between a normal and a better-looking solution can be insignificant. Other times, the huge cost gap between the two solutions can justify a less beautiful solution.

For long span concrete box girder bridges, economy and aesthetics can always successfully cope. Knowing that the higher forces concentrate in the piers (support) it is possible to vary the cross section height making it bigger in the pier section and smaller in the middle of the span. Like this, we are respecting the harmony aspect of aesthetics and walking towards a more economical solution since less material is required. Examples of this are the several pictures showed up to know where we can see that the deck depth varies.

The materials used in this type of bridge are concrete and prestressed steel. Therefore, if we use more of one material we will, automatically, use less of the other. The key, is to use the perfect combination of these two materials that allow us to build both the most aesthetic and economical solution.

By now, we know that the aesthetically recommended ratio between the deck depth is a value around 2.7-3 but, as we have also seen in Chapter 4, there are some bridges that use a bigger ratio value – 4-5. So, in this Chapter we will aim to find out why does this happen, especially when the span surpasses 150 m. In order to do this, we will calculate the volume of concrete and steel needed for the bridges with different lengths of span (100, 150, 200, 250 and 300m) and different ratios of the deck depths as well (2.7 and 4).

6.2. Dimensioning

Before we calculate the volume of material used in each case study we will have to do the dimensioning of the cross section in order to get the values of *h* and *t* as closest as possible to reality. This is possible by giving concrete values to *h* and *t* and then see if with these the structural and safety conditions are satisfied. As exemplified on **Figure 57**.

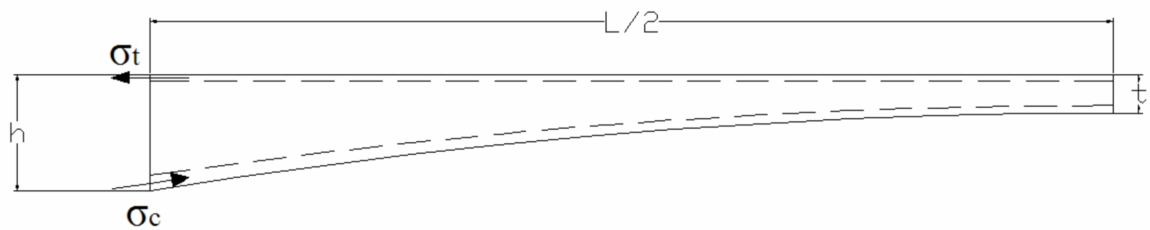


Figure 57 – Dimensioning basis

The values of *h* and *t* are obtained from the analysis of the Charts 5 and 6 from Chapter 4. In these charts we obtain the average values of *L/h* and *L/t* according to the desired value of the ratio *h/t* (**Table 4**). Then the values of *h* and *t* are obtained in function of the length of span – *L*.

Ratio (h/t)	***L/h***	***L/t***
2.7	16.5	45
4	17.5	70

Table 4 – Values of *L/h* and *L/t* according to the ratio *h/t*

Then, we proceed to some calculations so that we can verify the following conditions:

$$\begin{cases} \sigma_t > 0 \\ \sigma_c < f_{cd} \end{cases}$$

Where, f_{cd} – Design compressive strength of concrete

The calculation of the Traction and Compression Tensions, σ_t and σ_c respectively, is obtained trough the next formulas:

$$\sigma_t = \frac{M}{W_{sup}} \quad [MPa]$$

$$\sigma_c = \frac{M}{W_{inf}} \quad [MPa]$$

Where, M – Moment generated due to the dead weight P (**Figure 58**)
W_{sup} - Flexion module of the upper part of the cross section
W_{inf} - Flexion module of the bottom part of the cross section

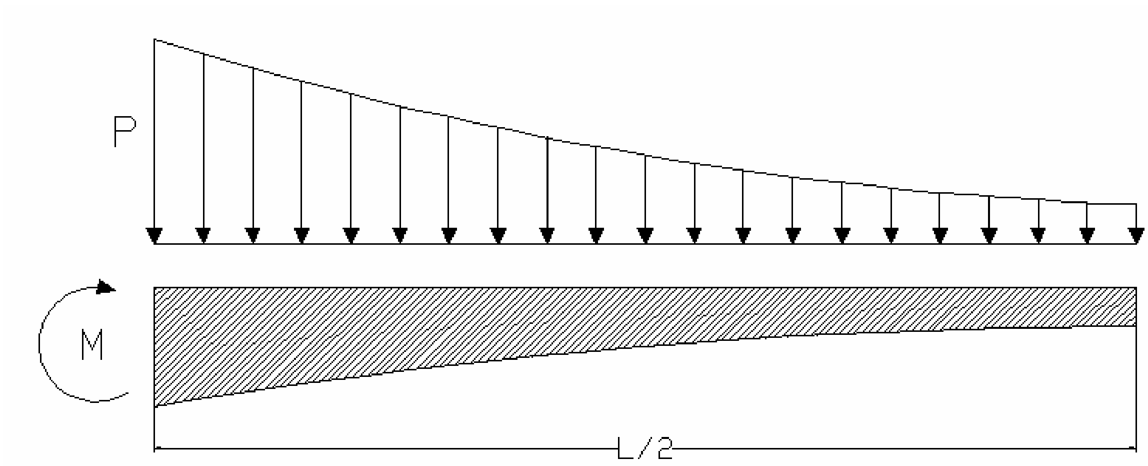


Figure 58 – Deadweight (*P*) and Moment (*M*)

Due to the parabolic variation of the deck depth, its deadweight will thus vary in the same way.

To start, our cross section used will have the top flange and the webs with constant thickness – 0.35m.

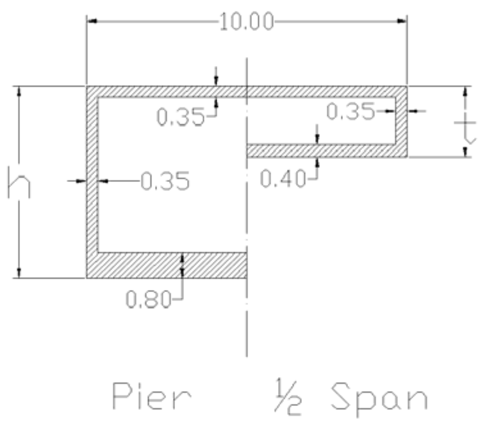


Figure 59 – Cross in the Pier and ½ Span sections [m]

The bottom flange will have a thickness of 0.8m in the pier section and 0.4m in the middle of span, and will vary in a parabolic curve, as we can see in **Figure 57** where the dashed line signifies the thickness of the webs.

Like so, our working model will be as represented in **Figure 60**. The bridge considered will have 10m of width having, therefore, two lane designed for traffic purposes.

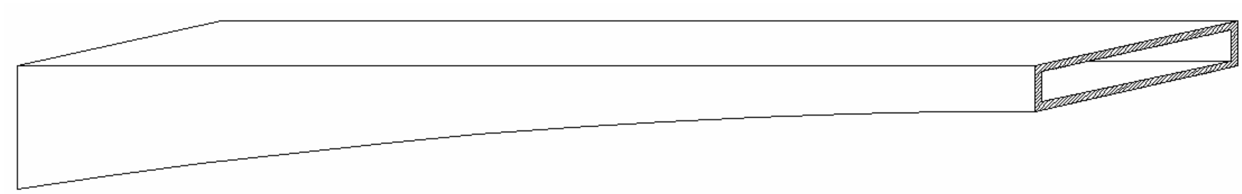


Figure 56 – Working model in 3D

In order to calculate the moment (*M*) generated by the dead weight (*P*) the working model will be divided in two separate parts: One concerning the top flange and the webs until the height of $t\text{-}e_{bottom\ flange}$ – A1; Other with the bottom flange and the remaining part of the webs – A2 (**Figure 61**).

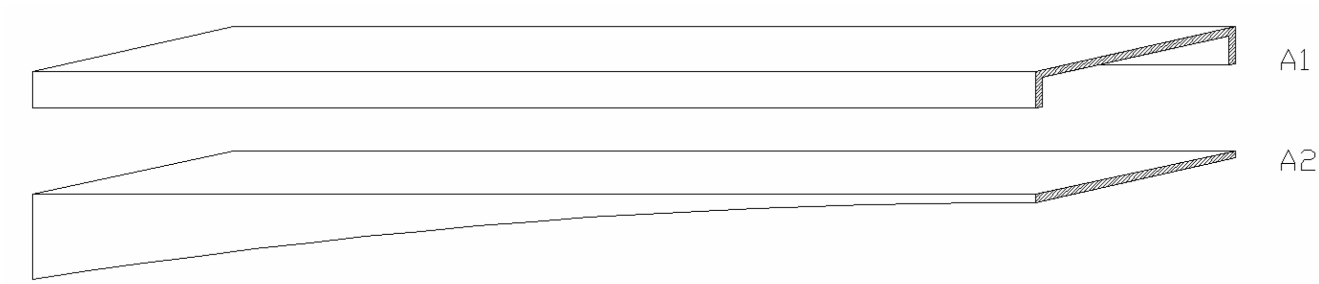


Figure 61 – Deadweight division areas

This allows us to calculate two values of a moment, one being generated by a rectangular load of the deadweight (A1) and the other by a parabolic load (A2). The model used to calculate both moments will be with an encastre in one end while the other end is free of restrictions. Applying this model to A1 and A2 we get:

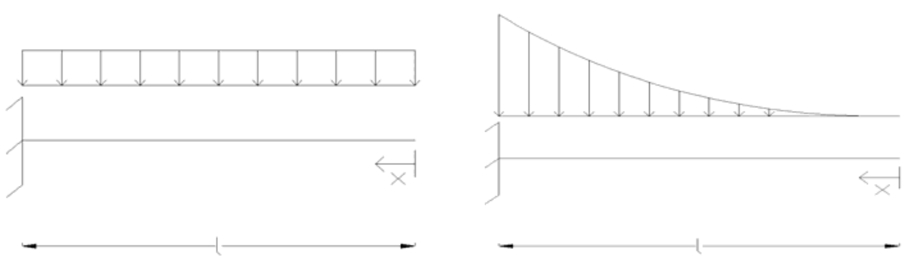


Figure 62 – Calculus Models

A1

The load caused in A1 is a combination of both the top flange and the webs (**Figure 63**); therefore we will calculate separately the load of these two, sum them and then calculate the moment.

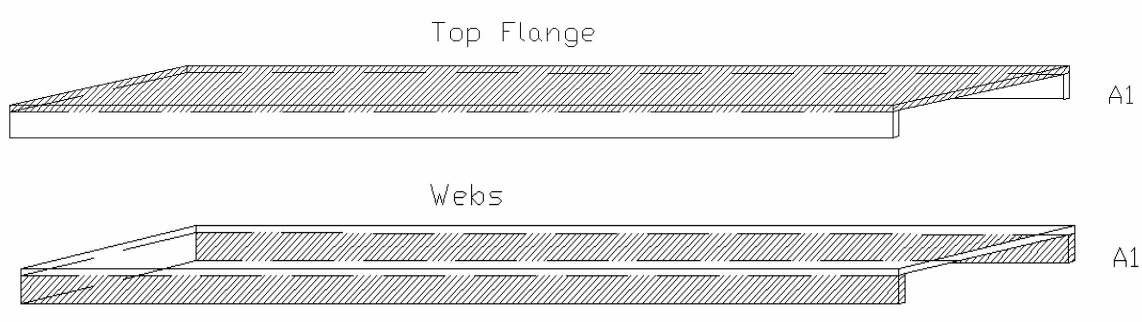


Figure 63 – Components of A1

The load caused by the top flange is:

$$P_{top\,flange} = width \times \gamma \times e_{top\,flange} \quad \text{[KN/m]}$$

Where, $width$ = 10m
γ = 25 KN/m^3
$e_{top\,flange}$ = thickness of the top flange (**Figure 59**)

This load is equal for all the studied cases and is 87.5 KN/m.

The load concerning the webs is given by:

$$P_{webs} = \gamma \times (t - e_{topflange} - e_{bottomflange}) \times (2 \times e_{web}) \quad \text{[KN/m]}$$

Where, t – height of the deck in the middle of the span (**Figure 59**)
e_i – thickness of the flange (**Figure 59**)

The total load and moment of A1 can now be calculated with reference to the calculus model displayed in **Figure 62**.

$$P = P_{top\,flange} + P_{webs} \quad \text{[KN/m]}$$

$$M_{A1} = P \times \frac{l^2}{2} \quad \text{[KNm]}$$

And, $l = L/2$ = (length of span)/2

A2

This part, like the former, is a combination of two webs and one flange.

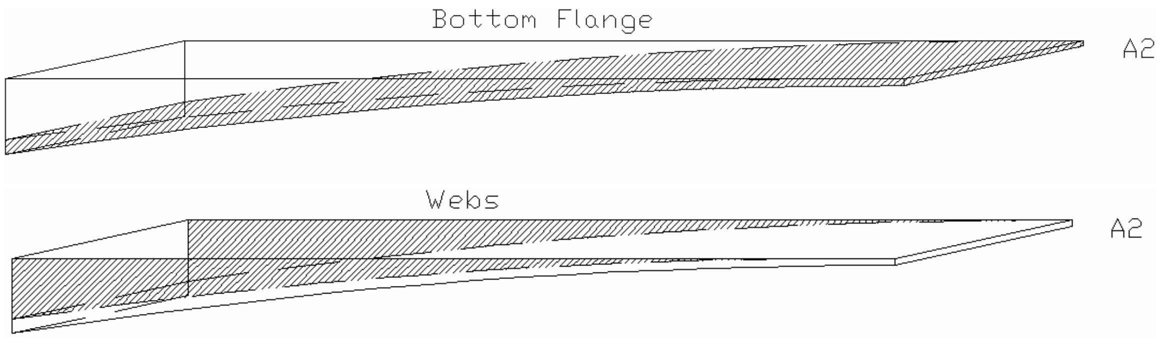


Figure 64 – Components of A2

The moment can be calculated trough the formula:

$$P = -\frac{\partial^2 M}{\partial x}$$

Where, P – load give by the expression of the parabolic curve

Due to the fact that, to get the value of the moment, each expression of the parabolic curve will have to be integrated twice, there will also appear two constants of integration in the final expression of the moment. Therefore, we will have to use the frontier conditions to determine them. So, we will say that in the free end of the cantilever the moment (*M*) and the shear force (*V*) have to be equal to zero.

The Shear force (*V*) is obtained integrating the parabolic expression only once:

$$P = \frac{\partial V}{\partial x}$$

In our case, and according to **Figure 62**, verifying the frontier conditions leads us to:

$$x = 0 \Rightarrow \begin{cases} M = 0 \\ V = 0 \end{cases} \Rightarrow \begin{cases} C1 = 0 \\ C2 = 0 \end{cases}$$

This means that in all our case studies we will not have any integration constants.

The thickness of the bottom flange is given by two parabolic curves. One which varies from the height measures *h* to *t*, and the other from *h-0.8* to *0.4m*.

The moment caused by the flange's deadweight is obtained by calculating the values of the moment for each of the two parabolic curves and then subtract one from the other. In **Figure 65** this happens by subtracting the blue are from the red one.

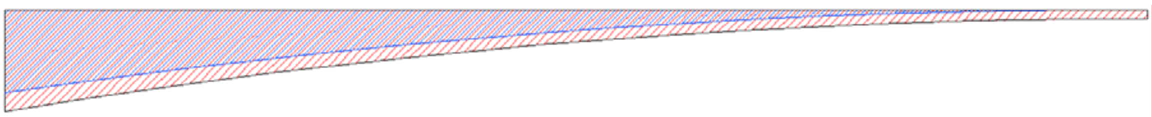


Figure 65 – Calculus model for the moment caused by the bottom flange

The value of the moment will then be:

$$M_{bottom\ flange} = \gamma \times width \times M \quad \text{[KNm]}$$

Where M is the value obtained from the difference of the integrals.

The moment concerning the webs uses only the value of one of the parabolic curves, the blue area in **Figure 65**, and is given by:

$$M_{webs} = 2 \times \gamma \times e_{web} \times M \quad \text{[KNm]}$$

And the moment for A2:

$$M_{A2} = M_{bottom\ flange} + M_{webs} \quad \text{[KNm]}$$

<u>Total Moment</u> (M_{TOT})

The total moment is the sum between M_{A1} and M_{A2}:

$$M_{TOT} = M_{A1} + M_{A2} \quad \text{[KNm]}$$

To continue the calculations to verify the safety conditions we now have to obtain the values of W_{sup} and W_{inf}.

$$W_{\sup} = \frac{I}{d_{g_{\sup}}} \quad [\text{m}^3]$$

$$W_{\inf} = \frac{I}{d_{g_{\inf}}} \quad [\text{m}^3]$$

The values of I and d_g are obtained trough the usage of the measures of the cross section shown in **Figure 59**.

To calculate the Moment of Inertia (I) of our cross section, we proceed as shown below.

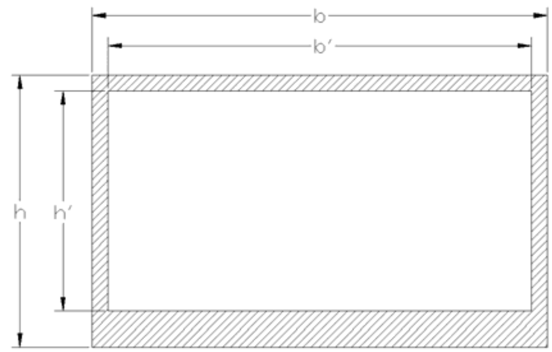


Figure 66 – Calculus model for the Moment of Inertia (I)

$$I = \frac{b \times h^3}{12} - \frac{b' \times h'^3}{12} \quad [\mathrm{m}^4]$$

The distance d_g comes from the formula:

$$y_g = \frac{\sum A_i y_i}{\sum A_i} \quad [\mathrm{m}]$$

A_i – area of the element i
y_i – position of the center of gravity of the element i

The model used to calculate the y_g of the box:

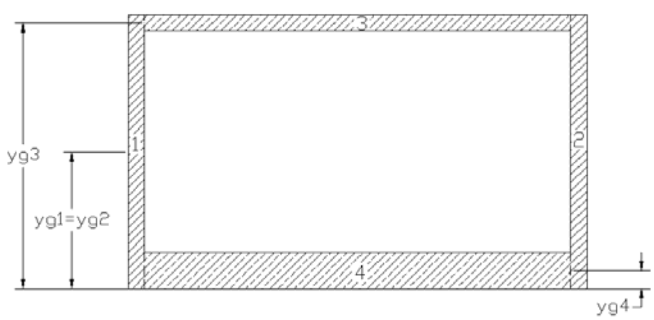


Figure 67 – Calculus model for y_g

Having reached the value of y_g, d_{gsup} and d_{ginf} come as indicated below:

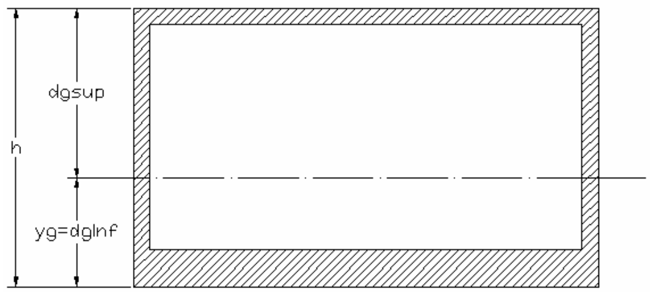


Figure 68 - d_{gsup} and d_{ginf}

We have now all the necessary values to engage in the dimensioning process throughout the case studies shown in the next table:

Case	L[m]	Ratio h/t
i.a)	**100**	2.7
i.b)		4
ii.a)	**150**	2.7
ii.b)		4
iii.a)	**200**	2.7
iii.b)		4
iv.a)	**250**	2.7
iv.b)		4
v.a)	**300**	2.7
v.b)		4

Table 6 – Case Studies

Should any of the case studies fail to verify the safety conditions, then the cross section will have to be changed by modifying the thickness of the bottom flange since is the part of the box that has most influence in the out coming result.

Results

L [m]	100.00		150.00		200.00		250.00		300.00	
Ratio *h/t*	2.70	4.00	2.70	4.00	2.70	4.00	2.70	4.00	2.70	4.00
h	6.06	8.80	9.09	8.80	12.12	11.43	15.15	14.29	18.18	17.14
t	2.22	2.20	3.33	2.20	4.44	2.86	5.56	3.57	6.67	4.29

Table 7 – Cross section dimensions

In the table above we have the dimensions of the cross section resulting from the conditions referred in **Table 4**. We can see that for the ratio *h/t* = 2.7 both the values of *h* and *t* are higher apart from the two first span lengths. This is due to constructive restrictions. That is, if we were to follow the calculations of **Table 4** we would obtain:

$$L/t = 70 \rightarrow \begin{cases} L = 100 \Rightarrow t = 1.43 \\ L = 150 \Rightarrow t = 2.14 \end{cases} \text{ [m]}$$

We have seen in Chapter 4 that the minimum height of the cross section should be around 2.20-2.30m for serving constructing and maintenance purposes. Therefore, as in this two cases the minimum height, in the middle of the span, was not verified they automatically resize in order to verify the necessary conditions. The height at the pier section is then obtained by multiplying 2.20m by the ratio *h/t* which is 4.

If these measures verify the dimensioning conditions we can expect that, apart from the first case – L=100m – the quantity of material used will be greater for the span using the aesthetically recommended ratio *h/t* - 2.7.

***L* [m]**	**100.00**		**150.00**		**200.00**		**250.00**		**300.00**	
Ratio *h/t*	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**
A1										
$P_{top\ flange}$	87.50									
P_{web}	25.76	25.38	45.21	25.38	64.65	36.88	84.10	49.38	103.54	61.88
P	113.26	112.88	132.71	112.88	152.15	124.38	171.60	136.88	191.04	149.38
M_{A1}	141580	141094	373242	317461	760764	621875	1340603	1069336	2149219	1680469
A2										
$M_{bottom\ lange}$	20313	20313	46907	48513	82500	83500	131226	134277	196172	227813
M_{web}	12578	22641	43965	50757	106225	119140	209351	234985	364563	405759
M_{A2}	32891	42953	90872	99270	188725	202640	340576	369263	560735	633572
M_{TOT}	**174470**	**184047**	**464114**	**416731**	**949489**	**824515**	**1681179**	**1438599**	**2709954**	**2314041**
I	93.74	220.93	238.02	220.93	460.63	402.34	771.30	672.98	1179.77	1028.10
dg_{inf}	2.34	3.45	3.57	3.45	4.86	4.56	6.20	5.81	7.57	7.10
dg_{sup}	3.72	5.35	9.09	5.35	12.12	6.86	8.95	8.47	10.61	10.05
W_{sup}	25.21	64.03	26.18	41.30	38.00	58.61	86.17	79.45	111.17	102.34
W_{inf}	40.02	41.30	66.65	64.03	94.71	88.16	124.40	115.74	155.86	144.87
σ_t	**6.92**	**2.72**	**17.73**	**10.09**	**24.99**	**14.07**	**19.51**	**18.11**	**24.38**	**22.61**
σ_c	**4.36**	**4.22**	**6.96**	**6.51**	**10.03**	**9.35**	**13.51**	**12.43**	**17.39**	**15.97**

Table 8 – Dimensioning results

When we look at the table above, the first thing to do is to verify is the safety conditions have been respected. As we can see on the two last lines of **Table 8**, σ_t>0 and σ_c<20 in all cases, which means that our initial cross section does not need to be modified.

Another scan through the tension values allows us to see that the traction tension (σ_t) is always the bigger one, when comparing between the ratios *h/t* for the same length of span. In the compression tension (σ_c) case, for a span length up to 200m the tension values are quite close but for the remaining examples the tension of the smaller *h/t* ratio is always the bigger one.

As we have seen before, a tension results from the ratio between the moment (M_{TOT}) and the flexion module (*W*). Looking at the values of the variables mentioned we can see that M_{TOT} is the most responsible for the values obtained for both σ_t and σ_c.
We know that M_{TOT} is the sum of the moments from *A1* and *A2*, respectively. Scanning the lines of M_{A1} and M_{A2} we verify that it is in the first that the variation of values is more noticeable and this is due to the value of the deadweight (*P*).

Apart from the two first cases, where *L* = 100m, the value of *P* is always higher when ratio *h/t* = 2.7. This can be explained by the dimensions of the cross section, that is, for a span length of 150, 200, 250 and 300m, the height of the box is always higher for the ratio *h/t* = 2.7. In order to explicate this we must recall that we assumed the values for both *h* and *t* in the beginning of the dimensioning process (**Table 4**) based on the dimensions of the bridges studied worldwide.

Having defined the dimensions of the cross section used for each case, we are now in condition to calculate the volume of material needed – Concrete and Steel.

Also, from observing the tension values, we can already expect that the higher tensions require a greater amount of concrete, so we can make a pre-assumption that the bridges using a ratio *h/t* of 2.7 will use more material and that they will also need more presstressed tendons due to the higher values of M_{TOT} and σ_t.

6.3. Quantity of material

6.3.1. Concrete

In the previous topic we confirmed that our section is secured in each pier. Therefore, the calculations made in this part concern the total length of the span. Since the main span is symmetric we will do the calculations for half of the span and, in the end, multiply the out coming result by two.

We saw in Chapter 1 that the length of each segment is normally comprehended between 3 and 6m, so we will use segment of 5m each along our calculus model shown in the next figure.

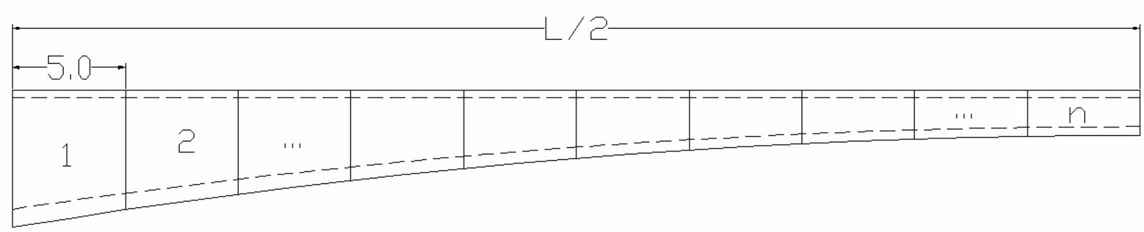


Figure 69 – Calculus model for the quantity of material needed

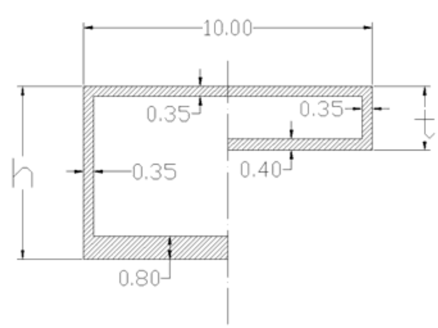


Figure 70 – Cross in the Pier and ½ Span sections [m]

The total volume of concrete needed is the sum of the volume of the top and bottom flanges and of the webs. For that we need both the longitudinal scheme of the bridge (**Figure 69**) and the cross section used (**Figure 70**).

Top Flange

It is the most simplest of the three intervenient to calculate since it uses constant measures:

$$V_{top\,flange} = e_{top\,flange} \times width \times L \quad [\mathrm{m}^3]$$

Bottom Flange

To calculate the volume of material used in the bottom flange we will run a process which consists in:

i) Calculating the longitudinal area defined by the lower parabolic curve – **Figure 71**;

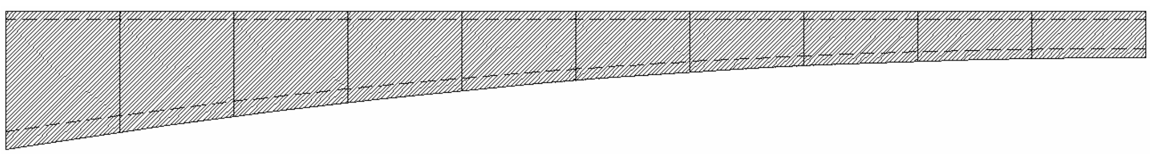


Figure 71 – Longitudinal area defined by the lower parabolic curve

ii) Calculating the longitudinal area defined by the upper parabolic curve – **Figure 72**;

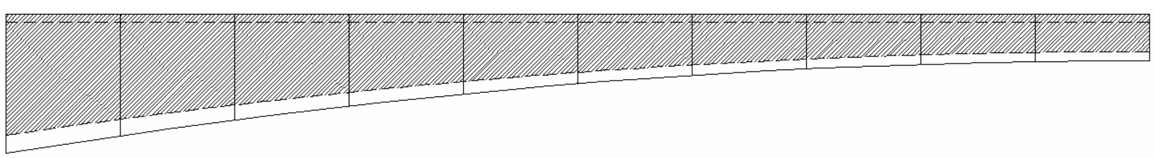


Figure 72 – Longitudinal area defined by the upper parabolic curve

iii) Subtract topic ii) from i) in order to have the longitudinal area of the bottom flange – **Figure 73**;

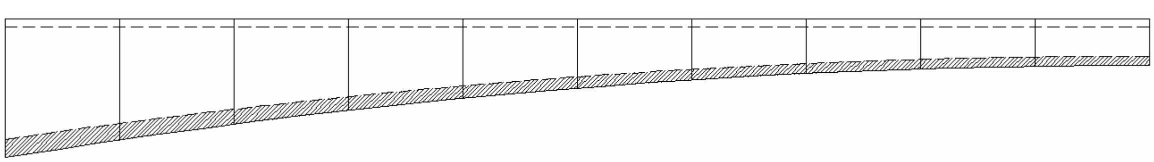


Figure 73 – Area of the bottom flange

iv) Obtain the final volume by multiplying the area of the flange by its width.

For the topic i)

We will use the expression of the parabolic curve that defines the bottom flange in order to obtain the heights y_i of the necessary defining points of each segment – **Figure 74**

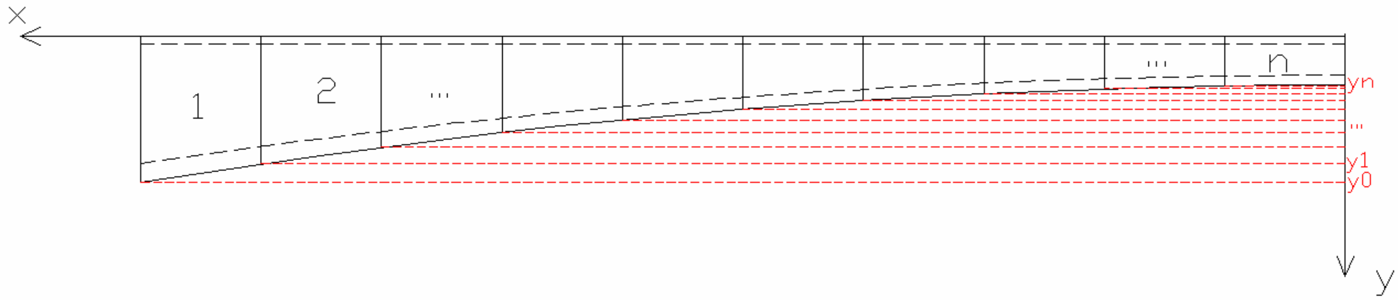


Figure 74 – y_i's for the bottom curve

Then, we will simplify each segment into a linear figure – the trapeze – **Figure 75**.

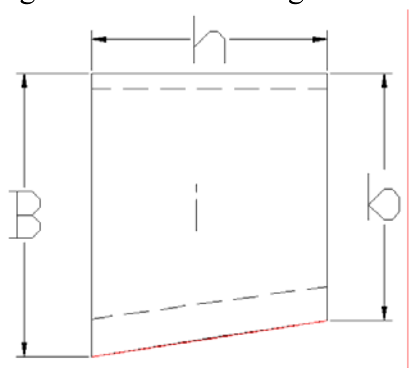


Figure 75 – General model of a simplified segment

As we can see, after simplifying the segment into a trapeze, with red line, the difference is irrelevant when compared to the original surface. The area of each one is calculated by the following formula:

$$A_i = \frac{B+b}{2} \times h \quad [\text{m}^2]$$

The total area, of the whole span, can then be obtained from the expression:

$$A = 2 \times \sum_{i=1}^{n} A_i \quad [\text{m}^2]$$

The way of thinking for topic ii) is analog to the prior topic. However the reference line is now the upper parabolic curve that defines the bottom flange – **Figure 76** and **77**

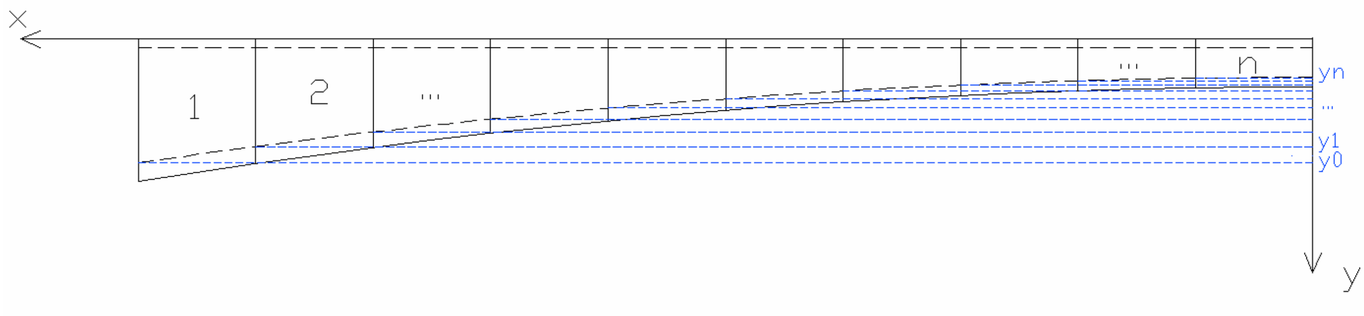


Figure 76 – y_i's for the upper curve

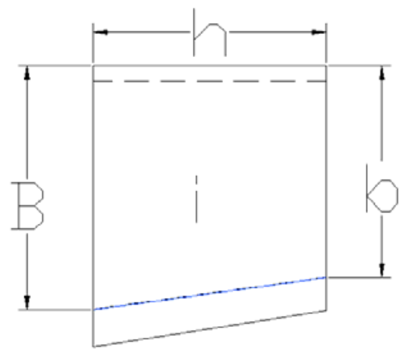


Figure 77 – General model of a simplified segment

As said before, the longitudinal area of the bottom flange comes from the subtraction between the two values of *A* obtained from each parabolic curve.

$$A_{bottom\ flange} = \Delta A \quad [\mathrm{m}^2]$$

Finally, the total volume of the bottom flange comes from the formula:

$$V_{bottom\ flange} = A_{bottom\ flange} \times (width - 2 \times e_{web}) \quad [\mathrm{m}^3]$$

Webs

The webs are defined between the top and bottom flange – **Figure 78**.

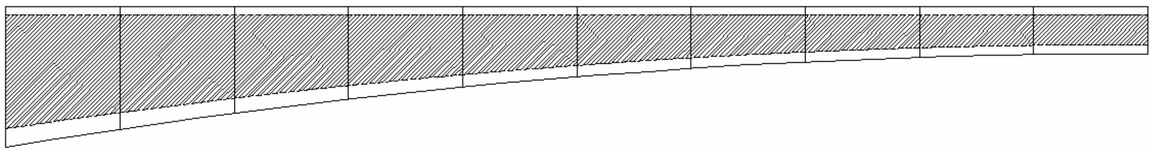


Figure 78 – Longitudinal area of the Webs

Consequently, in the calculations of the volume of concrete needed most of the necessary values have already been determined. That is, in order to get the longitudinal area of the webs we just have to go to **Figure 72** and subtract the longitudinal area of the top flange from it. So, from the results of topic ii) we can, either calculate the area of the web one segment at a time;

$$A_{web} = A_i - (5 \times e_{top\,flange}) \quad [\mathrm{m}^2]$$

Or basing ourselves on the total longitudinal area:

$$A_{webs} = A - (L \times e_{top\,flange}) \quad [\mathrm{m}^2]$$

The total volume of concrete needed for the webs is calculated with the next formula:

$$V_{webs} = A_{webs} \times 2 \times e_{web} \quad [\mathrm{m}^3]$$

Total Volume

The total volume of concrete needed for the span is a sum of the volumes of the different parts of the cross section:

$$V_{TOT} = V_{top\,flange} + V_{bottom\,flange} + V_{webs} \quad [\mathrm{m}^3]$$

After dimensioning and do the respective calculations concerning the quantity of concrete needed, we can now summarize all the results and expect to reach a valid and objective conclusion.

Results

L [m]	100.00		150.00		200.00		250.00		300.00	
Ratio h/t	2.70	4.00	2.70	4.00	2.70	4.00	2.70	4.00	2.70	4.00
$V_{top\ flange}$	350.00		525.00		700.00		875.00		1050.00	
$V_{bottom\ flange}$	496.25	496.62	744.38	749.36	991.94	992.31	1242.60	1245.10	1504.58	1102.20
V_{webs}	184	247	459	369	857	677	1378	1096	2021	1615
V_{TOT}	1029.82	1093.51	1728.30	1643.66	2548.70	2369.45	3495.71	3215.64	4575.95	3767.06

Table 9 – Volume of concrete [m^3]

As we already expected with the measures of **Table 7** having verified the safety conditions. The volume of concrete used by each span is always greater for the smaller *h/t* ratio value apart from the first length of span.

The volume of the top flange is the same for the two cases of each span, so the bottom flange and the webs are the responsible for the results.

Analyzing the $V_{bottom\ flange}$ line we see that the values, of each span, are relatively close until we reach the last span of 300m were we have a difference of, approximately, 400 m^3.

The webs are directly related to the height of the cross section. Therefore, it is without surprise that the bigger the box girder, the higher is the volume of concrete needed for the webs.

Taking our analysis one step further, we can make a more objective conclusion if we relate the two volumes obtained for each different span length. We will use the following ratio (multiplied by 100 in order to have the result in percentage):

$$\frac{V_{TOT}a}{V_{TOT}b}\times 100$$

$V_{TOT}a$ – V_{TOT} for ratio $h/t = 2.7$,

$V_{TOT}b$ - V_{TOT} for ratio $h/t = 4$

Applying to our case studies we have:

L **[m]**	**100.00**	**150.00**	**200.00**	**250.00**	**300.00**
$V_{TOT}a$	1029.82	1728.30	2548.70	3495.71	4575.95
$V_{TOT}b$	1093.51	1643.66	2369.45	3215.64	3767.06
$V_{TOT}a/V_{TOT}b$	94%	105%	108%	109%	121%

Table 10 – Relation between the concrete volumes of different ratio *h/t*

From the values displayed we can see that the volume of concrete used by the spans which have the aesthetically recommended measures is bigger for long span bridges. For bridges with a main span between 150 and 250m there may exist some discussion as to the measures the cross section should have, that is, as to which ratio *h/t* should be used. In these cases it may appear be acceptable to pay the extra 7% (average value) for the design touch. However, for a 300m span the price for a more visually appealing bridge is about 20%, which means 1/5 more concrete.

6.3.2. Steel

The quantity of prestress tendons needed for each of the case study is directly connected to the values of the total moment (M_{TOT}) and the traction tension (σ_t). That is, we want to pass from our current situation (**Figure 79**) to a situation where the traction values are all supported by the prestress tendons (**Figure 80**).

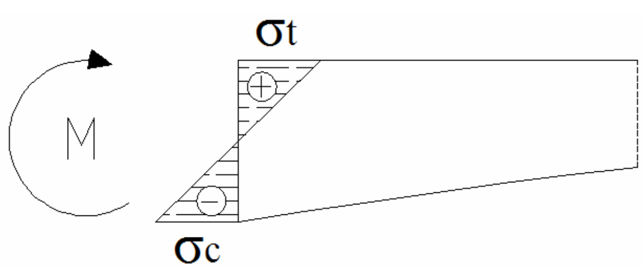


Figure 79 – Diagram of Tensions in the pier section

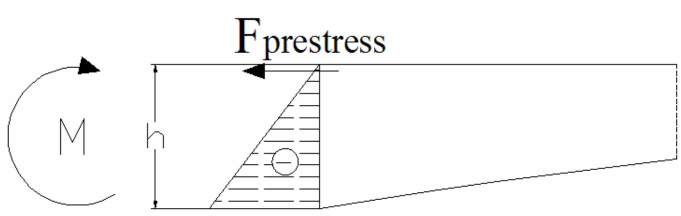


Figure 80 – Calculus model for the prestressing force – Diagram of Tensions desired

In order to obtain the prestressing force ($F_{prestress}$) we will use the calculus model as shown on **Figure 80**.

$$F_{prestress} = \frac{M_{TOT}}{h} \quad [\text{KN}]$$

Then, the area of tendons (A_t) will be given by the expression:

$$A_t = \frac{F_{prestress}}{f_t} \quad [\text{m}^2]$$

Where, f_t – tension stress = 1000 MPa

Finally, the total volume of prestress tendons (V_t):

$$V_t = \int_0^l A_t \times \frac{M_x}{M_{TOT}} \partial x \quad [\text{m}^3]$$

Due to the fact that, in our model, we assumed that the Moment in the free end of the cantilever with be equal to zero and that we have a deck depth varying in a defined parabolic curve, we can make the approximation:

$$\frac{M_x}{M_{TOT}} \approx \frac{W_x}{W_{\max}}$$

Where,

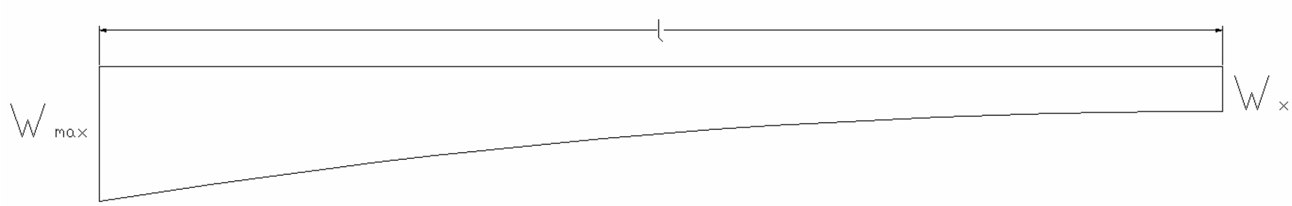


Figure 81 – Variables for the calculation of the volume of prestress tendons (V_t)

l – Half of the length of span
W_{max} - Flexion module of the upper part of the cross section at the pier
W_{min} - Flexion module of the upper part of the cross section in the middle of span

Therefore we have the final formula to obtain the volume of tendons, for half of the span:

$$V_t = \int_0^l A_t \times \frac{W_x}{W_{max}} \partial x \quad [m^3]$$

The mass of tendons (M_t), needed for the bridge, will be:

$$M_t = 2 \times (V_t \times 7800) \quad [Kg]$$

Results

***L* [m]**	**100.00**		**150.00**		**200.00**		**250.00**		**300.00**	
Ratio *h/t*	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**
M_{TOT}	174470	184047	464114	416731	949489	824515	1681179	1438599	2709954	2314041
$F_{prestress}$	28787	20914	51052	47355	78332	72145	110957	100701	149047	134985
f_t	1000									
A_t	0.03	0.02	0.05	0.05	0.08	0.07	0.11	0.10	0.15	0.13
W_{min}	5.80	5.71	10.11	5.71	14.77	8.22	19.75	11.08	25.03	14.08
W_{max}	25.21	64.03	26.18	41.30	38.00	58.61	86.17	79.45	111.17	102.34
V_t	0.33	0.09	1.48	0.49	3.04	1.01	3.18	1.76	5.03	2.79
M_t	**5148**	**1451**	**23088**	**7644**	**47424**	**15756**	**49608**	**27456**	**78468**	**43524**

Table 11 – Results of the calculations for the quantity of steel

The values shown above were somewhat expected after the tension values obtained as well as the results for the quantity of material needed. However, in this case, the bridge decks with a ratio *h/t* of 2.7 have always to use a greater amount of prestress no matter the length of the span.

A closer look to the values obtained allows concluding that it is the variation of the depth of the deck that influences, the most, the quantity of prestress needed for the bridge.

***L* [m]**	**100.00**		**150.00**		**200.00**		**250.00**		**300.00**	
Ratio *h/t*	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**	**2.70**	**4.00**
W_{min}	5.80	5.71	10.11	5.71	14.77	8.22	19.75	11.08	25.03	14.08
W_{max}	25.21	64.03	26.18	41.30	38.00	58.61	86.17	79.45	111.17	102.34
Ratio	**0.23**	**0.09**	**0.39**	**0.14**	**0.39**	**0.14**	**0.23**	**0.14**	**0.23**	**0.14**

Table 12 – Flection Module Results

That is, we can easily see that the values, of the two cases for each span, in the A_t line are quite close. However, both values of the Flexion Module (*W*), (**Table 12**) are very different for each ratio *h/t*. This is explained by the fact that the higher the *h/t* ratio, the more the cross section height varies and, therefore the smaller is the ratio W_{min}/W_{max} which will make the amount of total prestress, needed for the bridge, also smaller.

Making the same analysis as we made when studying the quantity of concrete needed we can compare the amount of steel needed by each type of bridge in **Table 13**.

***L* [m]**	**100.00**	**150.00**	**200.00**	**250.00**	**300.00**
M_ta	5148	23088	47424	49608	78468
M_tb	1451	7644	15756	27456	43524
M_ta/M_tb	355%	302%	301%	181%	180%

Table 13 – Relation between the steel volumes of different ratio *h/t*

Where, M_ta – Total volume of steel for ratio *h/t*=2.7
M_tb – Total volume of steel for ratio *h/t*=4

From the table above we see that, as the span length increases, the values of M_ta/M_tb decrease. However, we must also take into account that the values of M_ta and M_tb for *L* =100m are both about 15 and 30 times smaller than for *L* = 300m.

Nonetheless we still see that, even in the best of possibilities (*L* = 300m), the bridge deck built with a ratio h/t = 2.7 uses almost two times as much steel as a bridge built with a deck ratio *h/t* = 4.

6.4. Cost analysis

Previously, we saw how the quantity of material used changed according to the ratio *h/t* used in the bridge deck. In this part of the project we will translate that into money so that we can also have a financial idea of the differences.

6.4.1. Concrete

Assuming that, in Sweden, the price of 1 m^3 of concrete (with transport, pumping and formwork included) is around 800 € we have:

Sweden					
L [m]	**100**	**150**	**200**	**250**	**300**
$€_c a$	823,853	1,382,643	2,038,957	2,796,570	3,660,757
$€_c b$	874,808	1,314,927	1,895,563	2,572,512	3,013,650
$€_c a$ - $€_c b$	-50,955	67,716	143,394	224,059	647,107

Table 14 – Cost of concrete in Sweden [€]

Where, $€_c a$ - Cost of concrete for ratio h/t = 2.7
$€_c b$ - Cost of concrete for ratio h/t = 4

Although we already had a good idea of the consequences of choosing design over economy, now we see that, in the extreme case where L = 300m, the difference between the two options is about 650,000 €, just for the superstructure.

6.4.2. Steel

For this part, we make the assumption that the price of steel, in Sweden, is 5€/Kg. Using the values of M_t from **Tables 11** and **13** we have:

Sweden					
***L* [m]**	**100.00**	**150.00**	**200.00**	**250.00**	**300.00**
$€_t a$	25,740	115,440	237,120	248,040	392,340
$€_t b$	7,254	38,220	78,780	137,280	217,620
$€_t a$-$€_t b$	18,486	77,220	158,340	110,760	174,720

Table 15 – Cost of steel in Sweden [€]

Where, $€_s a$ - Cost of steel for ratio h/t = 2.7
$€_s b$ - Cost of steel for ratio h/t = 4

Here, we can see that the choice for, what we can now call as, the economical solution (ratio *h/t* = 4) is always the preferred solution when it comes to saving money.

6.4.3. Deck

Combining the values of both **Tables 14** and **15**:

$€_t i = €_c i + €_s i$

Sweden					
***L* [m]**	**100.00**	**150.00**	**200.00**	**250.00**	**300.00**
$€_t$a	849,593	1,498,083	2,276,077	3,044,610	4,053,097
$€_t$b	882,062	1,353,147	1,974,343	2,709,792	3,231,270
$€_t$a-$€_t$b	-32,469	144,936	301,734	334,819	821,827

Table 16 – Cost of the deck in Sweden [€]

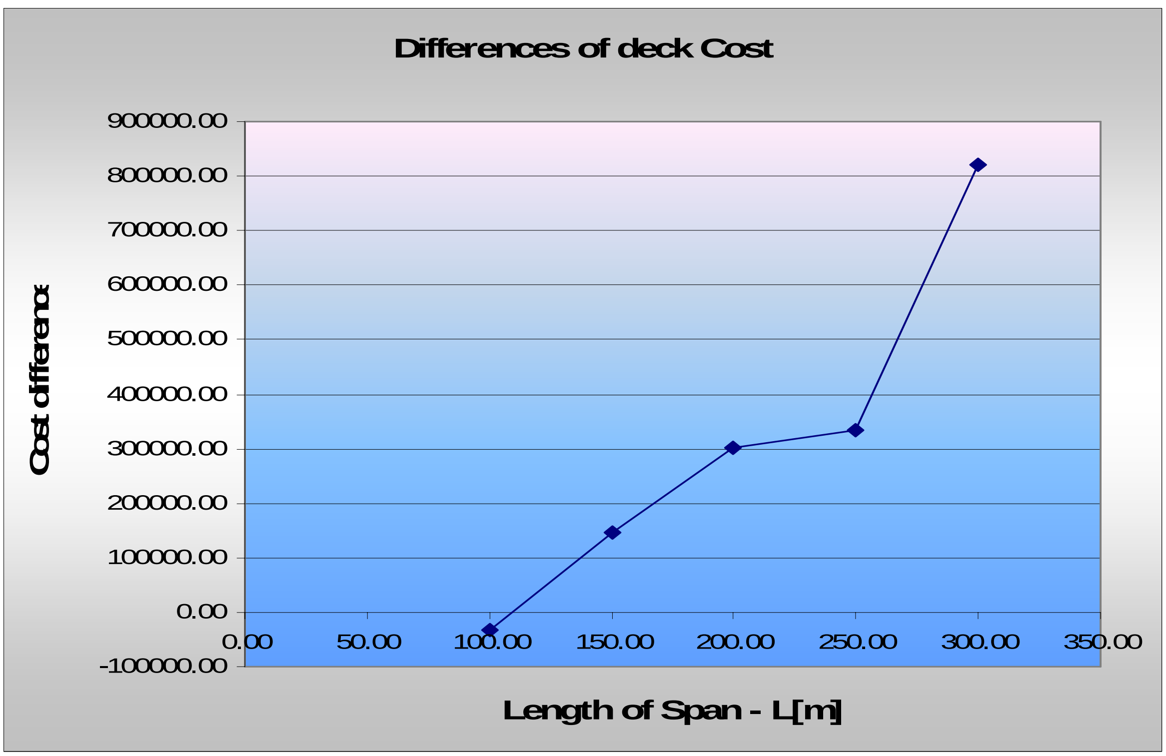


Chart 7 – Variation of the deck cost differences according to the length of span

The chart above was created from the values of the differences of the deck cost between the beautiful and the economical solutions.

From this global analysis we conclude that the beautiful solution (ratio *h/t* = 2.7) is only viable until a length of span of approximately 110m. Another curious observation is that for a span between 200 and 250m the price difference between the two solutions seems to stagnate, and when reaches the 250m span mark the graph turns into an almost vertical straight line, showing what we have seen before: For a 300m span, the choice of beauty over economy is too expensive.

7. Conclusion

Generally speaking, when a project comes to an end we feel that what we learned important things from the theme we were studying and with that we can make a difference in the world and help the improvement in our field of studies. With this project it was not different.

Although it was a conceptual study for cantilever constructed concrete bridges, we learned the good design guidelines and tips when projecting a bridge. We also learned the consequences of our possible choices, whether speaking in terms of aesthetics or economy. We were not only able to see and analyze the final results, but also understand how we got there and which factors influenced them the most. Like so, we were able to see that it was the moment caused by the top flange's deadweight which influenced most the total moment caused by the deadweight of the span in the pier section; that, when calculating the total volume of concrete, the volume of the webs was the one that varied the most between the two *h/t* ratios due to the considerable cross section height differences. A fact which was also responsible for the results obtained for the amount of steel.

The main conclusion that we get after finishing this work is that a bridge using a deck in which the cross section height ratio *h/t* is the aesthetically recommended starts to be more expensive as soon as the span surpasses the length of 110m. Moreover, if we choose to build a bridge with a 300m main span, the extra price to make it with the acknowledged design can be too much to justify it, according to some.

When a bridge building decision is being made, engineers follow this basic hierarchy:

- Performance: structural capacity, safety, durability and maintainability;
- Cost: construction and maintenance;
- Appearance

Looking at it we can get the wrong idea that it is not possible to make the best out of every topic without sacrificing any of them. The ideal solution is achieved when all of these topics ate being worked on and improved at the same time. However, we all agree that structural safety is the most important and must never me compromised.

We must not stick to the basic assumptions that limit creativity, such as "The client will never consider a different idea" or "We have always done it this way". The permanent advances in bridge appearance are due to innovations made by engineers who are permanently seeking and trying new materials, new construction techniques and new methods of analysis. Some examples of this are the bridges designed by the Swiss's Robert Maillard (1872-1940) and Christian Menn, the Portuguese Edgar Cardoso (1913 – 2000), the French Jean Muller (1925-2005), and, in our days, Santiago Calatrava, from Spain.

Confederation Bridge, Canada - Designed by Jean Muller

"The future, what does it hold? Nobody knows. The work initiated through the genius of the great constructor Eugène Freyssinet has been continued by his disciples, following on in the footsteps of their master. Many things remain to be done; one in particular, which is continuing in passing the knowledge on to the next generations. However, in order to perpetuate the work achieved, we must keep our technological lead and, wherever possible, increase it".

- Jean Muller

References

- Gottemoeller, Frederick, *bridgescape*, John Wiley & Sons, Inc., 1998.

- Menn, Christian, *Prestressed Concrete Bridges*, Birkhäuser, 1986.

- Menn, Christian, *Brückenbauer*, Birkhäuser, 1997.

- Leonhardt, Fritz, *Bridges*, Deutsche Verlags – Anstalt, 1982.

- Reis, A.J. – Departamento de Engenharia Civil e Arquitectura, Instituto Superior Técnico, 2002, Lisboa.

- Watson, Stewart C., Hurd, M.K., *Esthetics in concrete bridge design*, Aci, 1990.

- *Bridge aesthetics around the world* / Committee on General Structures, Subcommittee on Bridge Aesthetics, 1991.

- *Guidance for good bridge design*, federation international du béton (*fib*), 2000.

- http://www.structurae.net

- http://www.aaj.no/english/index.html (Aas-Jakobsen)

- http://www.tkk.fi/Units/Bridge/longspan.html (World's Longest Bridge Spans)