

Analysis and Design of Prestressed Concrete Girder

Vishal U. Misal, N. G. Gore, P. J. Salunke

Abstract— In this present study, cost analysis and design of prestressed concrete girder is presented. The aim and objective can be summarized as to analyze and design the concrete girder under a IRC class 70 R loading. To formulate the entire problem for a couple of span under the loading mentioned above to obtain shear force and bending moment at regular intervals along the beam. To use the software STAAD PRO for the analysis and design of prestressed concrete girders. Before using the software for analysis it will be validated by comparing its results with the corresponding classical theory result. To carry out the parametric analysis for prestressed concrete I girder and box girder. To calculate the quantities of concrete and steel required as per the analysis and design carried out for the girders and to carry out the comparative study for the same.

Index Terms— Box girder, Deck slab, I girder, Prestressed concrete

I. INTRODUCTION

Prestressed concrete is basically concrete in which internal stresses of suitable magnitude and distribution are introduced so that the stresses resulting from external loads are counteracted to a desired degree. In reinforced concrete members, the prestress is commonly introduced by tensioning the steel reinforcement.

The development of early cracks in reinforced concrete due to incompatibility in the strains of steel and concrete was perhaps the starting point in the development of a new material like ‘prestressed concrete’. The application of permanent compressive stress to a material like concrete, which is strong in compression but weak in tension, increases the apparent tensile strength of that material, because the subsequent application of tensile stress must first nullify the compressive prestress. In 1904, Freyssinet attempted to introduce permanently acting forces in concrete to resist the elastic forces developed under the name of “prestressing”.

II. PRINCIPLE OF PRESTRESSING

The function of prestressing is to place the concrete structure under compression in those regions where load causes tensile stress. Tension caused by the load will first have to cancel the compression induced by the prestressing before it can crack the concrete. Figure 1 (a) shows a plainly reinforced concrete simple-span beam and fixed cantilever beam cracked under applied load. Figure 1(b) shows the same unloaded beams with prestressing forces applied by stressing high strength tendons. By placing the prestressing low in the simple-span beam and high in the cantilever beam, compression is induced in the tension zones; creating upward camber.

Manuscript received January 2014.

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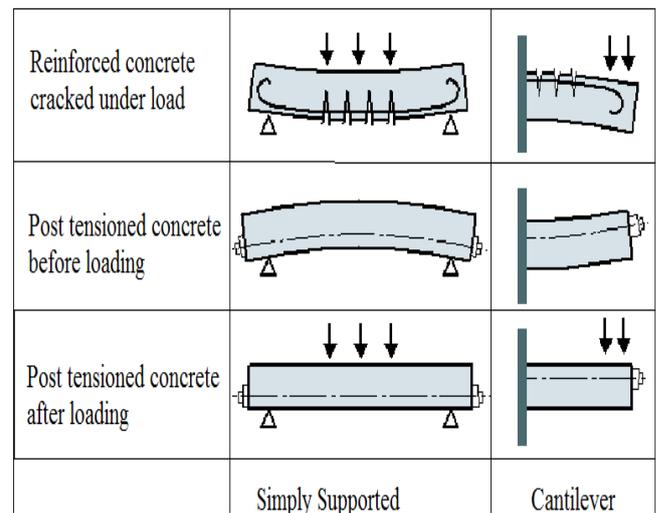


Fig. 1 Comparison of Reinforced and Prestressed Concrete Beams

Figure 1 (c) shows the two prestressed beams after loads have been applied. The loads cause both the simple-span beam and cantilever beam to deflect down, creating tensile stresses in the bottom of the simple-span beam and top of the cantilever beam. The Bridge Designer balances the effects of load and prestressing in such a way that tension from the loading is compensated by compression induced by the prestressing. Tension is eliminated under the combination of the two and tension cracks are prevented. Also, construction materials (concrete and steel) are used more efficiently; optimizing materials, construction effort and cost.

III. PROBLEM FORMULATION

Design a post-tensioned prestressed concrete I-beam slab bridge-deck.

1. Permissible stresses:

- For deck slab: As per IRC: 21-2000 for various grade of concrete and steel. Evaluate σ_{cb} , σ_{st} , m , Q and j .
- For prestressed concrete girder: As per IRC: 18-2000 for various grade of concrete. Evaluate f_{ck} , f_{ci} , $f_{ct} = 0.45 f_{ci}$, $f_{cw} = 0.33 f_{ck}$.

2. Design of interior slab panel:

a. Bending moments

- Dead weight of slab: $1 * 1 * \text{thickness of slab} * \text{density of concrete} \dots \dots \dots$ “(1)”
- Dead weight of wearing coat: $\text{thickness of wearing coat} * \text{density of wearing coat} \dots \dots \dots$ “(2)”
- Total dead load: eq “(1+2)”.
- Live load calculation: for IRC class AA tracked vehicle. One wheel is placed at the centre of panel

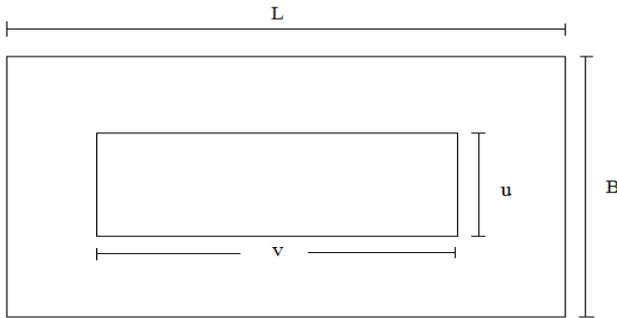


Fig. 2: Position of IRC Class AA Wheel Load for Maximum Bending Moment

Where,

- L= length of panel.
- B= Width of panel.
- u= 0.85+2*thickness of wearing coat.
- v= 3.60+2*thickness of wearing coat.
- Bending moment along short span (M_S): $W (m_1 + 0.15m_2)$.
- Bending moment along short span (M_L): $W (0.15m_1 + m_2)$.
- Where: m_1 and m_2 is computed by using Piegeauds curves & $W= 350$ KN.
- Design bending moment= $1.25 * 0.8 * \text{Bending moment}$.

b. Shear forces:

Dispersion in the direction of span= $[0.85 + 2 (\text{thickness of wearing coat} + \text{thickness of deck slab})]$

- For maximum shear, load is kept such that the whole dispersion is in the span. The load is kept at (dispersion in the direction of span/2) from the edge of beam (x).
- Effective width of the slab= $kx[1-(x/L)]+b_w$.
- Load per metre width= $350 / \text{Effective width of the slab}$.
- Shear force per metre width= $[\text{Load per metre width} * (\text{Clear width of panel} - x)] / \text{Clear width of panel}$.

c. Dead load Bending moments and Shear force:

- Total load on panel (W) = (length of panel * width of panel * total dead load).
- $M_S = W (m_1 + 0.15m_2)$.
- $M_L = W (m_2 + 0.15m_1)$.

d. Design moment: Including the continuity factor by multiplying 0.8 to Bending moments.

e. Design of slab section and reinforcement.

- Effective depth, $d = \sqrt{\frac{M}{Qb}}$.

- Area of steel: $A_{st} = \left(\frac{M}{\sigma_{st} jd} \right)$.

f. Check for shear: (As per IRC: 21-2000).

- $\tau_v = \left(\frac{V}{bd} \right)$.

3. Design of longitudinal girder:

a. Reaction factor

Using Courbon theory:

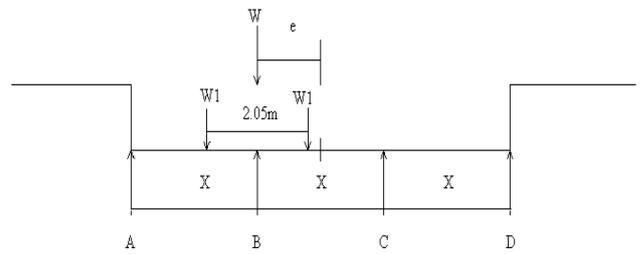


Fig. 3 Transverse Disposition of IRC Class AA Tracked vehicle

- Reaction factor of outer girder A is

$$R_A = \frac{2W_1}{4} * \left[\frac{4 * a * e}{2 * a^2 \quad 2 * b^2} \right]$$

- Reaction factor for inner girder B is

$$R_B = \frac{2W_1}{4} * \left[\frac{4 * b * e}{2 * a^2 \quad 2 * b^2} \right]$$

Where,

a= distance between c/l to girder A.

b= distance between c/l to girder B.

b. Dead load on slab per girder:

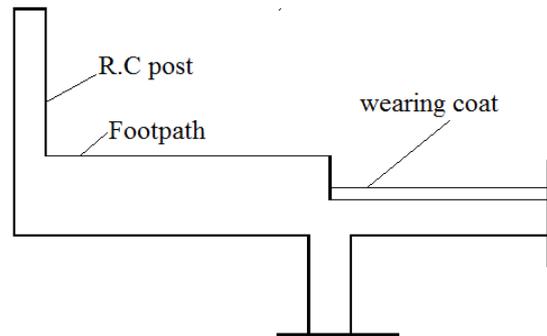


Fig. 4 Details of footpath, parapet and deck slab

Weight of

- Parapet railing = 0.92KN/m(assume)
- Footpath and kerb= depth * width * density.
- Deck slab= depth * width * density.

Total dead load of the deck= $[2 * \{ \text{weight of (parapet railing + footpath and kerb + deck slab)} \} + (\text{total dead load of interior slab panel} * \text{width of slab})]$.

It is assume that the deck load is shared equally by all four girders.

$$(\text{load on each girder}) = \frac{\text{Total dead load of the deck}}{\text{no of girder}}$$

c. Dead load on main girder: the overall depth of the girder is assumed to be 1800 mm at the rate of 60 mm of every meter of span.

Overall depth: $60 * \text{span of girder}$.

- Self-weight of girder= (dead weight of rib + dead weight of bottom flange)
- Weight of cross girder= (width of cross girder * depth of cross girder * density)
- d. Dead load moments and shears in the main girder:

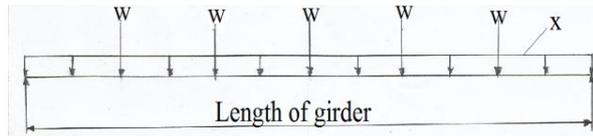


Fig. 5 Dead Load on Main Girder

Where,

X= total dead load on the girder.

W= reaction on main girder.

- Reaction from deck slab on each girder: load on each girder.
 - Reaction on main girder= Weight of cross girder * spacing of main girders.
 - Total dead load on the girder= (self-weight of the main girder + Reaction from deck slab on each girder)
 - Maximum bending moment: it will act at the centre.
 - Maximum shear force:
- e. Live load bending moments in the girder:

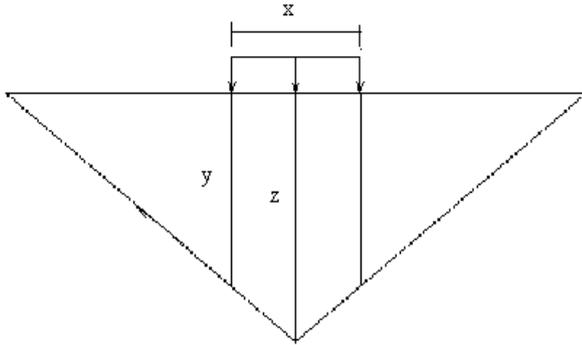


Fig. 6 Influence line for bending moment in girder

Where:

- $x = 3.6m$.
- $z = Wab/l$
- $L =$ length of girder.
- $a = b =$ length of girder/2.
- $W = 1$.
- B.M at centre of span = $0.5(y+z) * 700$.
- B.M, including the impact and reaction factors,
- For outer girder is = (reaction at A * 1.1)
- For inner girder = (reaction at B * 1.1)

f. Properties of main girder section:

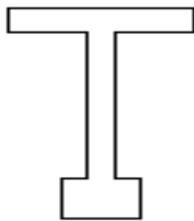


Fig. 7 Cross-section of main girder

Calculate A, Z_t , Z_b .

g. Check for minimum section modulus:

$$Z_b = \frac{M_q + (1 - \eta)M_d}{f_{br}}$$

$$f_{br} = (\eta f_{ct} - f_{tw})$$

h. Prestressing force:

$$P = \frac{A f_{inf} Z_b}{(Z_b + Ae)}$$

$$f_{inf} = \left(\frac{f_{tw}}{\eta} \right) + \left(\frac{M_D}{\eta Z_b} \right)$$

i. Permissible tendon zone:

$$e \leq \left(Z_b \frac{f_{ct}}{P} \right) - \left(\frac{Z_b}{A} \right)$$

$$e \geq \left(Z_b \frac{f_{tw}}{P} \right) - \left(\frac{Z_b}{A} \right)$$

4. Check for stresses:

A. At transfer stage

$$\sigma_t = \left[\left(\frac{P}{A} \right) - \left(\frac{Pe}{Z_t} \right) + \left(\frac{M_D}{Z_t} \right) \right]$$

$$\sigma_b = \left[\left(\frac{P}{A} \right) + \left(\frac{Pe}{Z_b} \right) - \left(\frac{M_D}{Z_b} \right) \right]$$

B. At working stage:

$$\sigma_t = \left[\eta \left(\frac{P}{A} \right) - \eta \left(\frac{Pe}{Z_t} \right) + \left(\frac{M_D}{Z_t} \right) + \left(\frac{M_L}{Z_t} \right) \right]$$

$$\sigma_b = \left[\eta \left(\frac{P}{A} \right) + \eta \left(\frac{Pe}{Z_b} \right) - \left(\frac{M_D}{Z_b} \right) - \left(\frac{M_L}{Z_b} \right) \right]$$

5. Check for ultimate flexural strength:

According to I.S: 1343-1980, the ultimate flexural strength of the centre of span section is computed as follow:

$$A_p = (A_{pw} + A_{pf})$$

Where,

$$A_{pf} = 0.45 f_{ck} (b - b_w) \left(\frac{D_f}{f_p} \right)$$

$$A_p = \frac{\prod}{4} d^2 \times \text{No of tendon in each cable} * \text{No of cable.}$$

$$A_{pw} = A_p - A_{pf}$$

$$\text{-Ratio: } \left(\frac{A_{pw} f_p}{b_w d f_{ck}} \right)$$

$$\frac{f_{pu}}{0.87 f_p} = x, \frac{x_u}{d} = y.$$

$$M_u = [f_{pu} A_{pw} d - 0.42 x_u + 0.45 f_{ck} b - b_w D_f d - 0.5 D_f]$$

$$M_u req = 1.5 M_D + 2.5 M_L$$

6. Check for ultimate shear strength:

Ultimate shear force: $V_u = (1.5 V_g + 2.5 V_q)$

According to IRC: 18-2000, the ultimate shear resistance of the support section uncracked in flexure is given by,

$$V_{cw} = 0.67 b_w h \sqrt{f_t^2 + 0.8 f_{cp} f_t} + \eta P \sin \theta$$

- $b_w =$ width of the web.
- $h =$ overall depth of girder.
- $f_t =$ maximum principal tensile stress at the centroidal axis.
- $f_t = 0.24 f_{ck}^{0.5}$.

- f_{cp} = compressive stress at the centroidal axis due to prestress = $\left(\frac{\eta P}{A}\right)$.
- Slope of cable= $(4e/L)$
Where,
- e = Eccentricity of cable at the centre of span - Eccentricity of cable at the support.
- A_{st} = provide 10 mm diameter two-legged stirrups of Fe 415 HYSD bars.
- Spacing= $S_v = \left(\frac{0.87 f_y A_{sv} d_t}{V}\right)$
- $V = V_u - V_{cw}$

Supplementary reinforcements: Longitudinal reinforcements of not less than 0.15 percent of the gross cross-sectional area are to be provided to safeguard against shrinkage cracking. Illustrative examples:

- For span of 16.30 m and 31.4 m we have design I girder and Box girder by using M 45 grade concrete and Fe 415 steel.

IV. RESULT

A. For 31.4 m span

	I girder	Box girder
Volume of steel	2.2313 m ³	2.652 m ³
Volume of concrete	181.81 m ³	92.3 m ³

Fig. 8 Total quantities of steel and concrete.

B. For 16.3 m span

	I girder	Box girder
Volume of steel	0.523 m ³	0.728 m ³
Volume of concrete	89.568 m ³	236.349 m ³

Fig. 9 Total quantities of steel and concrete.

V. CONCLUSION

- In view of achieving the aim and objectives of this study a detailed literature survey was being carried out. It gave us an idea regarding different methodologies adopted for analysis and design of prestressed concrete slabs and girders. It was decided to go for the use of the software STAAD PRO for the analysis and design of slab and girders. For validation purpose the analysis results of a problem of “I girder” using STAAD PRO were compared with the corresponding analytical results. It is observed that the result obtained by both method showing good agreement
- By using this software the analysis and design of I girder and Box girder was carried out. A comparative study was carried out between I girder and Box girder.
- By extracting result we have concluded that box girder is costlier than I girder.
- It has also seen that losses is more in I girder as compared to Box girder.

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