Planck Fluctuations, Measurement Uncertainties and the Holographic Principle

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Abstract

Starting from a critical analysis of recently reported surprisingly large uncertainties in length and position measurements deduced within the framework of quantum gravity, we embark on an investigation both of the correlation structure of Planck scale fluctuations and the role the holographic hypothesis is possibly playing in this context. While we prove the logical independence of the fluctuation results and the holographic hypothesis (in contrast to some recent statements in that direction) we show that by combining these two topics one can draw quite strong and interesting conclusions about the details of the fluctuation structure and the microscopic dynamics on the Planck scale. We further argue that these findings point to a possibly new and generalized form of quantum statistical mechanics of strongly (anti)correlated systems of degrees of freedom in this fundamental regime.

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1 Introduction

In recent years it has been argued that the at first glance quite remote Planck scale might perhaps become observationally accessible by devising certain ingeneous (thought) experiments. More specifically, arguments were given that the quantum fluctuations of the space-time metric or of distance measurements may come within the reach of observability using already existing equipment like, for example, the large and extremely sensitive interferometers, designed to detect gravitational waves. This argument was in particular advanced by Amelino-Camelia (see [1] or [2]) and supported by various thought experiments and qualitative calculations given by Jack Ng et al (see e.g. [3] or [4]). Interesting arguments concerning the interface of general relativity and quantum physics are also advanced in [5].

These arguments are provisional as a generally accepted theory of quantum gravity does not yet exist and are of a character similar to the quantum mechanical calculations before the advent of true quantum mechanics in 1926. Nevertheless it is believed that they will hold in a qualitative sense in any future theory of quantum gravity. In the following we will also stick to this provisional reasoning.

The quantum gravity literature of the past decades abounds with such heuristic arguments concerning the quantum behavior on the Planck scale (see in particular the numerous remarks in [6] or [7] or the paper by Padmanabhan, [8]) with the expected result that the relevant fluctuation effects are essentially of Planck scale character. We note also the discussion in e.g. [9] where a possible minimal length is related to large extra dimensions of space-time. One should however note that the reasoning is not always complete as it is of course important to take for example also the uncertainty in position of the (relevant parts of the) measuring devices into account. We took some pains to discuss this particular point in more detail in the following. In contrast to these findings there are more recent arguments claiming that some of these fluctuations (induced by quantum gravity) can already be seen on a (in general) much larger scale, depending in a somewhat surprising way also on the size of the quantities or objects being measured. For example in [3] the uncertainty, δl, of a length or distance measurement is claimed to go as

$$\delta l \gtrsim l_p (l/l_p)^{1/3} \tag{1}$$

with l the length being measured and l_p the Planck length.

The other interesting step consists of amalgamating this reasoning with a version of the hypothetical *holographic principle*, stating that on the Planck scale the number of degrees of freedom or the information capacity of a spatial volume, V, go with the surface area of V and not! with the volume itself as in ordinary (statistical) physics. A nice recent review is for example [10] (we note that we plan to give a more complete list of references elsewhere). It is claimed that the holographic principle, stated in this particular form, supports the above findings ([3]). In the following we want to scrutinize both lines of reasoning and show that we come to different results. In this connection we note that there are arguments that this particular form of the holographic principle cannot hold in all possible situation. In the following we mainly deal with weak gravitational fields and relatively small but macroscopic subvolumes of practically infinite space where these arguments do not apply. Anyway, we think that the last word is not yet said on this subject matter as there exist possible modifications of this relatively simple variant of the principle also in more general situations. We will come back to this question at the end of the paper.

We then proceed to show that both the fluctuation results and the holographic hypothesis imply particular (anti)correlation constraints of their own. By putting these two observations together we are able to derive strong constraints on the correlations and dynamics of degrees of freedom on the Planck scale. These findings seem to support the view that in this fundamental regime a new or extended form of statistical mechanics of strongly coupled or entangled degrees of freedom and open systems may become necessary. We make some remarks in this direction in the last section.

Concluding this brief résumé we want to mention two recent papers which discuss the interesting point of a possible change of particle dispersion relations near the Planck scale and its consequences for area laws and entropy bounds ([11] and [12]). We plan to come back to possible relations to our work elsewhere.

2 A Discussion of Some Recent Results

The first line of reasoning we mentioned above starts from an earlier finding of Wigner et al ([13]), dealing with quantum effects concerning clocks and mirrors treated as test particles in a gravitational field, $g_{ik}(x)$, and the setting-up of a coordinate system on the space-time manifold. This line of thought is supplemented by e.g. Ng et al by the wellknown argument concerning the emergence of a black hole if too much mass is concentrated in a very small region of space, the critical parameter being the *Schwarzschild* *radius*

$$r_s = 2GM/c^2 \tag{2}$$

Abbreviating the more detailed calculations in [13], one can argue as follows. If one insists to measure distances in a gravitational field by exchanging light signals between freely moving small and sufficiently localized clocks and mirrors (tacitly assuming that they do not disturb too much the given field $g_{ik}(x)$), the following conclusion seems to be inevitable. We assume the clock (and the mirror) initially to be localized with uncertainty δl. Standard quantum mechanics leads to a momentum uncertainty

$$\delta p \gtrsim \hbar/\delta l \tag{3}$$

The average time, τ, it takes for a light pulse to reach the mirror is

$$\tau = l/c \tag{4}$$

with l the average distance between clock and mirror. In the time interval τ the initial relative position uncertainty of clock and mirror increases roughly as

$$\delta l + \delta v \cdot \tau = \delta l + \hbar/m\delta l \cdot l/c = \delta l + \hbar l/mc\delta l \tag{5}$$

with the minimum δl being

$$\delta l_{min} = (\hbar l/mc)^{1/2} \tag{6}$$

(see [3]).

Quantum mechanics alone suggests to make the mass, m, of the clock (and mirror) large in order to reduce the uncertainty in distance measurement. Here now general relativity comes into the play. Realizing for example the clock as a spherical cavity of diameter d, surrounded by a mirrored wall of mass m, in which a light signal bounces back and forth, the clock must tick off time at a rate so that

$$d/c \lesssim \delta l/c \tag{7}$$

in order that the uncertainty in distance measurement is not greater than δl. On the other hand d must be larger than the Schwarzschild radius of the clock, r_s, so that signals can be exchanged at all. This implies

$$\delta l \gtrsim Gm/c^2 \tag{8}$$

Ng et al now combine these two estimates to get

$$\delta l^3 \gtrsim Gm/c^2 \cdot \hbar l/mc \tag{9}$$

or

$$\delta l \gtrsim l_p(l/l_p)^{1/3} = (l l_p^2)^{1/3} \tag{10}$$

with $l_p = (\hbar G/c^3)^{1/2}$ the Planck length. Correspondingly we get

$$\delta\tau \gtrsim (\tau t_p^2)^{1/3} \tag{11}$$

with $t_p = l_p/c$ the Planck time.

We briefly want to recapitulate how the *true* spatial distance is measured in a gravitational field in general relativity. This is particularly clearly discussed in [14], see also [15]. It comes out that for "infinitesimal" distances (which can in fact be macroscopic in a sufficiently weak field for practical purposes) we have

$$dl^2 = (g_{\alpha\beta} - g_{0\alpha}g_{0\beta}/g_{00})dx^\alpha dx^\beta = \gamma_{\alpha\beta}dx^\alpha dx^\beta > 0 \tag{12}$$

with greek indices running from 1 to 3 and $\gamma_{\alpha\beta}$ being the spatial metric (the sign convention being $-+++$). We note that in a physically realisable reference system we have $g_{00} < 0$ (and corresponding constraints for the $\gamma_{\alpha\beta}$). Note that in general gravitational fields the notion of true distance (measured for example with little measuring rods or light signals) has only an absolute meaning in the small. This is only different in particular cases like a static field (g_{ik} independent of the time coordinate).

The crucial point in the analysis of Wigner et al was that clocks and mirrors are treated as strongly localized freely moving test particles tracing out their individual world lines (or geodesics) in a given gravitational field (similar discussions can e.g. be found in [16] or [17]). This is reasonable in a certain context as e.g. in discussions of introducing appropriate coordinate grids or material reference systems which do not distort too much the given space-time. The situation however changes if questions of principle are adressed in certain thought experiments concerning fundamental limitations of e.g. length measurements in quantum gravity. We argue in the following section that in that case some of the above constraints can be avoided or at least relaxed so that the lower limit provided by e.g. Ng et al can be considerably improved upon.

3 An Alternative Thought Experiment

We now describe a different set up which is not designed to create for example a full coordinate grid or minimally disturb a given gravitational field. We rather concentrate on the important question of the existence of a priori limitations of measuring distances in the very small with uncertainties much larger than l_p.

We note that a severe restriction in the approach of Wigner et al or Ng et al derives from the assumption that clocks and mirrors are freely moving particles of mass m. This resulted in the additional (l-dependent) position uncertainty $\hbar l/mc\delta l$. We speculate what an experimenter in the laboratory would do. He would fix clock and mirror on an optical bench, which we, for calculational convenience, realize as a three-dimensional harmonic (macroscopic) quantum crystal in a way described below. The unavoidable *zero point motion* of the atoms of the solid is of the order

$$\Delta q \sim (\hbar/M_0)^{1/2} \tag{13}$$

with M_0 the mass of the atoms and Δq their position uncertainty (see e.g. [18]).

There exist various possibilities to implement the coupling between clock, mirror and solid quantum mechanically. One possibility is to confine both clock and mirror, as it is done with ordinary quantum objects, in macroscopic (ionic or optical) traps which, on their part, are attached to the solid. One may assume that these devices are attached to a (macroscopic) part of the solid and not to a single atom. This will yield an extra uncertainty of the order $(\hbar/M)^{1/2}$ instead of $(\hbar/M_0)^{1/2}$ with M the mass of the respective part of the solid.

We approximate possible experimental set-ups by assuming the clock (and mirror) to be bounded in the ground state of a harmonic oscillator potential. This yields

$$\Delta q^2 =< x^2 >= (m\omega/\hbar)^{1/2} \cdot \int x^2 \exp(-m\omega x^2/\hbar)dx \sim (\hbar/m\omega) \tag{14}$$

The momentum uncertainty

$$\Delta p \gtrsim \hbar/\Delta q \tag{15}$$

does now no longer matter as the particle cannot drift away during the measurement process.

Remark: The solid of course generates a gravitational field of its own but we think, this does not represent a real problem in this context as we are only interested in questions of principle. Assume for example that the original metric was the Minkowski metric which is now sligthly disturbed by the field of the solid.

We now observe that by increasing m and/or ω, we still have to obey the Schwarzschild constraint, but as the clock can not wander away we get a bound like

$$\delta l \gtrsim (Gm/c^2 \cdot \hbar/m\omega)^{1/3} + (\hbar/M\omega)^{1/2} = (G\hbar/c^2\omega)^{1/3} + (\hbar/M\omega)^{1/2} \\ = (l_p^2 c/\omega)^{1/3} + (\hbar/M\omega)^{1/2} \quad (16)$$

which does no longer contain an explicit l-dependence. M is possibly limited by practical or experimental constraints. But as the respective region of the lattice need not be an infinitesimal one the Scharzschild-constraint is at least not openly manifest.

At this place it is perhaps helpful to add a remark concerning another deep question of principle which tacitly underlies all the discussions of the kind presented above and similar ones but which, on the other hand, is seldomly openly addressed. In quantum mechanics proper it turned out that *uncertainty in measurement* (Heisenberg) is essentially the same as *uncertainty in definability* (Bohr) which mirrors sort of a preexisting harmony and is by no means a trivial property from an epistomological point of view. As to this important point cf. the discussion in [19] about the seemingly different viepoints of Heisenberg and Bohr.

To put it briefly and relating it to our present problem concerning the much more remote Planck scale: in our view it is not always entirely obvious that every seeming limitation concerning the measurement of a certain quantity like e.g. a distance by using a particular measuring device really corresponds to a truely fundamental limitation of definability of the quantity under discussion, that is, as having its roots in for example irreducible primordial fluctuations of space-time as such. To really decide this may be a touchy business given the great recent advances in measurement techniques.

4 Anticorrelated Space-Time Fluctuations

In [3] it is argued that the l-dependent fluctuation formula for length measurements, derived there, is further corroborated by an application of the so-called *holographic principle*. We want to show in this and the following section that the holographic principle is not really a cause for the given fluctuation formula but has rather a logical status which is independent of that result.

We start with a simple thought experiment concerning the nature of Planck fluctuations which we presented already quite some time ago ([20], we however presume that many other researchers in the field are aware of this phenomenon). We assume that the quantum vacuum on Planck scale is a fluctuating system behaving similar to systems in quantum statistical mechanics with the characteristic correlation parameters

$$l_p = (\hbar G/c^3)^{1/2}\ ,\ t_p = l_p/c\ ,\ E_p = \hbar\nu_p = \hbar t_p^{-1} \tag{17}$$

We make, to begin with, the simplest possible but naive assumption, assuming that in each Planck cell of volume l_p^3 we have essentially independent energy fluctuations of size E_p which implies that the characteristic correlation length is assumed to be l_p.

Picking now a macroscopic spatial volume (compared to the Planck scale!), V, we have $N = V/l_p^3 \gg 1$ of such cells labelled by $1 \leq i \leq N$. Defining the stochastic variable $E_V := \sum_1^N E_i$, the *central limit theorem* tells us that the expected fluctuation of E_V is

$$\Delta E_V := < E_V \cdot E_V >^{1/2} \sim E_p \cdot N^{1/2} \tag{18}$$

where we assumed $< E_i >= 0$ (a point we comment upon later). ΔE_V would still be very large as both $N^{1/2}$ and E_p are large because N itself is typically gigantic for macroscopic V. The question is now, why are these volume-dependent large fluctuations not observed?

Remark: In ordinary statistical mechanics extensive quantities like e.g. E go with the volume, V, or N. In that case it is frequently reasonable to neglect fluctuations, being of order $N^{1/2}$, as the scale used in our measurement devices is typically adjusted to the occurring values of the extensive variables. Regarding the Planck scale, we may however take the average of the vacuum energy to be zero, or put differently, we do not measure it with our local devices, but the fluctuations are expected to be large locally and should be detectable in principle (see also the remarks in [8]).

We note that a similar reasoning yields for the momentum fluctuations

$$\Delta p_V \sim p_{pl} \cdot N^{1/2} \tag{19}$$

Conclusion 4.1 *On the Planck scale the hypothetical individual fluctuations must be strongly negatively or anti-correlated so that the integrated fluctuations in the volume V are almost zero.*

We infer that what is called for are effective microscopic screening mechanisms!

The above picture is of course quite crude but the same result would essentially follow under much weaker and more realistic assumptions as they are, for example, frequently made in (quantum) statistical mechanics (cf. [21]). To be specific, let $q(x)\,,\, x\in\mathbb{R}^d$ be a certain (quantum) observable density like e.g. some charge, current or particle density. We normalize, for calculational convenience, its expectation value, $\langle q(x)\rangle$, to zero. We assume the system to be translation invariant and the *correlation function*

$$F(x-y) := \langle q(x)\cdot q(y)\rangle \tag{20}$$

to be integrable, i.e.

$$F(s)\in L^1(\mathbb{R}^d) \tag{21}$$

With $Q_V := \int_V q(x)d^dx$ the integral over a certain volume, V, we get

$$\begin{aligned} 0\leq\, \langle Q_V Q_V\rangle &= \int_V dx \int_V dy F(x-y) = \int_V dx \left(\int_{x-V} ds F(s)\right) \\ &\leq \int_V dx\, sup|(\ldots)| \leq \int_V dx\cdot\int_{\mathbb{R}^d} ds\,|F(s)| = V\cdot const \end{aligned} \tag{22}$$

as $\int_{\mathbb{R}^d} ds\,|F(s)|$ is finite by assumption. We can infer the following:

Conclusion 4.2 *With* $\langle q(x)q(y)\rangle \in L^1(\mathbb{R}^d)$ *and*

$$\lim_{V\to\mathbb{R}^d}\int_V F(s)\neq 0 \tag{23}$$

we get

$$\langle Q_V Q_V\rangle^{1/2}\sim V^{1/2} \tag{24}$$

as in the case of complete independence of random fluctuations.

Remark: Note that $N\sim V$ and summation is replaced by integration over $q(x)$. The same reasoning holds of course for discrete degrees of freedom.

This Gaussian type of fluctuation can only be avoided if the correlation function $F(s)$ displays a peculiar fine tuned (oscillatory) behavior, more precisely, it must hold that

$$\lim_{V\to\mathbb{R}^d}\int ds\, F(s) = 0 \tag{25}$$

Lemma 4.3 *The rate of the vanishing of the above integral is encoded in the rate of vanishing of the Fourier transform, $\hat{F}(k)$, near $k = 0$.*

(see [21] and [22] and further references given there). We hence can conclude that the behavior near $k = 0$ of $\hat{F}(k)$ is relevant for the degree of fluctuation of Q_V.

To establish this relation rigorously and also for other reasons ($q(x)$ is frequently not an operator function of x but more singular, i.e. only an operator valued distribution) it is customary in quantum statistical mechanics and field theory to replace the sharp volume integration over V by a smooth scaling function. One may take for example

$$f_R(x) := f(|x|/R) \tag{26}$$

with $f \geq 0$ $f = 1$ for $|x| \leq 1$ and being of compact support. Instead of Q_V we use in the following

$$Q_R := \int q(x) f_R(x) d^d x \tag{27}$$

Remark: We note, without going into any details, that one can give numerical estimates of the difference in behavior of the two quantities. A certain disadvantage of a sharp volume cut off is that it introduces an artificial *non-integrability* in Fourier space as the F.Tr. of a discontinuous function cannot be L^1! One can show that in our case it is only in L^2.

We get after Fourier transformation:

$$\langle Q_R \cdot Q_R \rangle = R^{2d} \cdot \int d^d k \, \hat{F}(k) |\hat{f}(Rk)|^2 \tag{28}$$

Making a variable transform we get

$$\langle Q_R \cdot Q_R \rangle = R^d \int d^d k \, \hat{F}(k/R) |\hat{f}(k)|^2 \sim const \cdot R^d \cdot R^{-\alpha} \cdot \int k^\alpha \cdot |\hat{f}(k)|^2 d^d k \tag{29}$$

asymptotically for $R \to \infty$ if

$$\hat{F}(k) \sim |k|^\alpha \quad \text{near} \quad k = 0 \tag{30}$$

and vice versa. To show this we simply express $\hat{F}(k)$ as $k^\alpha \cdot G(k)$ with $G(k)$ finite and nonvanishing at $k = 0$.

Observation 4.4 *Small or almost vanishing fluctuations in macroscopic volumes (compared to the Planck scale) can be achieved by certain covariance properties of the quantities under discussion, that is, a certain degree of vanishing of the F.Tr. of* $\langle q(x)q(y)\rangle$ *at* $k = 0$*. For example, if* $q(x)$ *is the zero (charge) component of a conserved 4-current, we have typically a behavior* $\sim |k|^2$ *near* $k = 0$ *for space dimension* $d = 3$*. If* $q(x)$ *is the* 00*-component of a conserved 2-tensor current, like e.g. the energy-momentum tensor, we have in general a behavior* $\sim |k|^4$*.*

(See [23] and [22]).

From the preceding discussion we hence can conclude that small-fluctuations in x-space can obviously be achieved by fine-tuned anticorrelations in $\langle q(x)q(y)\rangle$, which, nevertheless, can be of short range, i.e. integrable.

Corollary 4.5 *Small fluctuations as such in* V *imply fine-tuned anticorrelations but not necessarily correlations having a long range.*

This is remarkable as we will show in the following that the *holographic principle* is intimately connected with long-range correlations of a peculiar type. This implies that it is not a necessary prerequisite for establishing small Planck fluctuations.

Remark: Recently Brustein et al (see for example [24] and [25]) used field theoretic fluctuation results similar to our results derived in e.g. [21] and [22] to argue that such area-like scaling of fluctuations (occurring however in only very particular situations) may be related to the area laws of the holographic principle. We have to refrain in this letter-size format to go into more details but plan to discuss this subtle point elsewhere.

We conclude this section with providing a, as we think, instructive example taken from ordinary physics which shows how easily these strongly anticorrelated fluctuations appear even in non-relativistic physics. We take again the 3-dimensional harmonic crystal mentioned already above. We assume it to be fixed macroscopically in a definite position, so that in the language of statistical mechanics its state represent a *pure phase*, in other words we assume a (spontaneous) breaking of translation invariance. We concentrate in the following for convenience on the atoms lying on the x-axis, their equilibrium positions being the coordinates $\{j \cdot a\}$, $j \in \mathbb{Z}$, a the lattice constant. The momentary position of the j-th particle is x_j. We know from statistical mechanics (cf. e.g. [18]) that the fluctuations of the microscopic particle positions are finite (in three space dimensions!), i.e.

$$\delta x_j^2 = \langle (x_j - j \cdot a)^2 \rangle < \infty \tag{31}$$

for all j. On the other hand we have

$$x_j - j \cdot a = \sum_{k=1}^{j} (x_k - x_{k-1}) - ja + x_0 \tag{32}$$

with $< x_0 >= 0$.

The stochastic variables $(x_k - x_{k-1}) =: u_k$ with $< u_k >= a$ play here the role of the individual length fluctuations, δl, in the corresponding Planck scale model. The crystal condition we assume to be implemented by $\delta x_j^2 \leq a^2$ (or a slightly weaker condition) which is independent of j due to the assumed translation invariance. We note that there exist various possibilities to formulate such a condition but this is of no relevance for our present discussion.

We have

$$\delta x_j^2 = \langle \left(\sum_{k=1}^{j} (u_k - a) + x_0 \right)^2 \rangle =$$
$$\langle \sum_{k=1}^{j} (u_k - a)^2 \rangle + \langle \sum_{k \neq k'=1}^{j} (u_k - a)(u_{k'} - a) \rangle + 2\langle x_0 \cdot \sum_{1}^{j} (u_k - a) \rangle + \langle x_0^2 \rangle \tag{33}$$

With both δx_k^2 and δu_k^2 independent of k and roughly of the same order, i.e. being $\lesssim (2a)^2$ we see that, while the lhs of the equation is of order $(2a)^2$, the first sum on the rhs is of order $j \cdot (2a)^2$. The third term on the rhs can be calculated as follows.

$$\langle x_0 \cdot \sum_{1}^{j} (u_k - a) \rangle = \langle x_0 \cdot (x_j - x_0 - ja) \rangle = \langle x_0 \cdot (x_j - x_0) \rangle =$$
$$\langle (x_0 - \langle x_0 \rangle) \cdot (x_j - \langle x_j \rangle) \rangle - \langle x_0^2 \rangle \tag{34}$$

as $\langle x_0 \rangle = 0$ by assumption. In a pure phase we have clustering of correlation functions, hence the first term on the rhs of the last equation goes to zero for j large. The third term of equation (33) is therefore of order $\sim \langle x_0^2 \rangle$ for j large and is hence compensated by the fourth term.

We hence arrive at the important constraint equation

$$\langle \sum_{k \neq k'=1}^{j} (u_k - a)(u_{k'} - a) \rangle \approx -\langle \sum_{k=1}^{j} (u_k - a)^2) \rangle \lesssim -j \cdot (2a)^2 \tag{35}$$

Conclusion 4.6 *As δx_j^2 is globally bounded, $\delta x_j^2 \lesssim a^2$, we find that the contributions in $\langle \sum_{k \neq k'=1}^{j} (u_k - a)(u_{k'} - a) \rangle$ are to a large part strongly negatively correlated in order to compensate the linear positive increase in j of the first term on the rhs of equation (33) (remember that $< u_k >= a$).*

Tranferring these observations to our Planck scale model, we can associate x_j or $j \cdot a$ with the momentary and averaged macroscopical length we are going to measure, δx_j with its fluctuation and the u_k with the individual but strongly anticorrelated length fluctuations of the respective pieces of roughly Planck size. We see that an organized anticorrelation over large length scales is obviously not entirely unnatural. In the example we just studied it is connected with the occurrence of spontaneous symmetry breaking (of translation invariance), that is, a phase transition. We hence conclude that it may happen under certain circumstances that the fluctuation of a macroscopic length is of the same order as the fluctuations of its much smaller parts in contrast to what may be infered from a naive application of the central limit theorem.

5 Implications of the Holographic Principle

In the preceding section we dealt with the possibility of small or vanishing Planck scale fluctuations in macroscopic or mesoscopic regions and consequences thereof. Our analysis led to *strong anticorrelation result* but, as we saw, not necessarily to a *long-range* anticorrelation (see, however, the last example of the harmonic crystal where exactly this happens to be the case). We now add another aspect in form of the *holographic principle.* We employ it in the simple form typically used in various recent thought experiments. That is, for simplicity reasons, we only deal with situations where a *space-like holographic bound* is supposed to hold (cf. [10]).

As in the literature, cited above, we assume a given volume, V, to be divided into $N = V/l_p^3$ or $N \sim V$ cells. We make the asumption that each cell can store a finite amount of information, in the simplest case one bit, represented by an internal state labelled by the numbers ± 1.

If these microscopic states can be independently chosen or more realistically, as in the preceding section, are only finitely or short-ranged correlated, we get an *information storage capacity*

$$I \sim V \tag{36}$$

as it prevails in ordinary physics. The holographic principle claims that on the Planck scale we have instead in certain situations a behavior

$$I \sim V^{2/3} \text{ (surface area) or } I \sim l^2 \tag{37}$$

if l is the diameter of V.

In [3] it is argued that such a behavior would naturally lead to the length-fluctuation result reported there, i.e.

$$\delta l \gtrsim l_p (l/l_p)^{1/3} \tag{38}$$

We want to show in the following that this is, in our view, not the case and that a different scenario is more plausible.

In a first step we show that the holographic principle alone does not! imply strong negative correlations among the microscopic degrees of freedom. This property is rather the consequence of our above *small-fluctuation result*. If, for example, our microscopic degrees of freedom are positively but long-range correlated in V, this would, on the one hand, diminish the information storage capacity (all spins or most of them are typically almost aligned in a given microscopic fluctuation pattern in V), but an averaging over such states would produce *large global fluctuations* proportional to (some fractional power of) V. But such large fluctuations are not observed as was argued in the preceding sections.

Conclusion 5.1 *The holographic principle as such entails long-range correlations among the microscopic degrees of freedom in V (positive or negative ones). The small-fluctuation result, on the other hand, entails strongly negative correlations (but not necessarily of long range!). Taken together both principles entail strongly negatively correlated long-ranged fluctuations!*

In [3] or [4] the holographic principle comes into the play by arguing that the number of degrees of freedom in the volume V is on the one hand $l^3/\delta l^3$ and on the other hand l^2/l_p^2, with δl the minimal uncertainty of a length measurement of l (cf. the discussion at the beginning of this paper). Ng argues that δl is at the same time the minimal length which can be resolved in V, put differently, which can be attributed a physical meaning. He then relates the two expressions to each other as

$$l^3/\delta l^3 \lesssim l^2/l_p^2 \tag{39}$$

and gets

$$\delta l \gtrsim (l_p^2 \cdot l)^{1/3} \tag{40}$$

We however provided arguments in the preceding sections that these expressions are not! logically related.

We showed above that we may have $\delta l \sim l_p$ and hence in principle (neglecting interactions among the cells) an available number of degrees of freedom, $N = l^3/l_p^3$, without running into a logical contradiction. The reason is that in our view the information in V is not! stored in single more or less uncorrelated bits of size l_p^3 or possibly varying size δl^3, but rather in a strongly correlated pattern, extending over the full volume V. That is to say, a configuration on the surface of V can be freely chosen (so to speak), leaving us with a total of roughly $2^{l^2/l_p^2}$ different surface states, i.e. $I = l^2/l_p^2$, each of which induces, due to long-ranged anti-correlations a more or less unique configuration within V, extending this respective surface configuration. As to this point see also the remarks in section 7 of [27].

This point of view has various consequences and ramifications also for *black hole physics* which we only briefly mention (see the beautiful Galiean trialogue, [26]). Suffice it to remark that, adopting this point of view, there is no real problem in combining individual length fluctuations of roughly Planck size, l_p, with an information storage capacity going only with the surface area of V.

We further note that the unique dependence of a volume state on a corresponding surface state is not entirely unusual outside ordinary (statistical) physics. Consider, for example, the *Dirichlet property* employed in elliptic boundary value problems. With L an elliptic partial differential operator and $g(x)$ a configuration on the boundary, ∂V, of V, there exists under fairly weak conditions a unique solution, $f(x)$, in V, i.e. $Lf = 0$, extending smoothly to the boundary function, $g(x)$. If we discretize this problem we get roughly an example with $I \sim |\partial V|$, where it is understood that we label the volume states by the uniquely associated surface states.

What we have said may provide a clue as to the more hidden reasons why the holographic principle does not hold in the regime of the physics of more ordinary length and energy scales. In ordinary quantum statistical mechanics, for example, or when studying the *asymptotic distribution of eigenvalues* of elliptic partial differential operators in a finite volume under selfadjoint boundary condition (cf. e.g. [28]), the number of states below a certain energy threshold E is proportional to the phase-space volume (see below). In this scenario we typically work with a fixed (Hamilton) operator, L (i.e. including fixed boundary conditions, for example $f = 0$ on ∂V) and study the eigenvalue problem

$$L f_i = \lambda_i f_i \; , \; \lambda_i \leq E \tag{41}$$

with all f_i being in principle physically admissible states.

We note that in statistical mechanics or theories of many degrees of freedom in general we have to deal with Hamiltonians, describing the interaction of many constituents (their number being typically proportional to the ordinary geometric volume). In this case V does *not* denote the ordinary geometric volume the system is occupying but the generalized *phase-space volume* which is a region having a huge dimension and is rather of the order e^V if V denotes the ordinary geometric volume. The information content is then of the order $\log e^V = V$ (cf. e.g. [30]).

Statistical mechanics tells us that one can in many cases regard a subsystem, enclosed in the subvolume V, as being to some extent independent of the ambient system which may be treated as a heat bath, by neglecting the usually short ranged boundary effects or, rather, incorporating them in a statistical manner. More specifically, one makes for example a *random phase approximation* in order to arrive at a *canonical partition function* with respect to the subsystem contained in V (see e.g. [29]). In such a scenario, i.e. macroscopically excited states in the interior, the entropy is typically proportional to the geometric volume. We note that a similar behavior prevails if we study the *entanglement-entropy* of macroscopically excited eigenstates of a Hamiltonian (describing the interaction of many degrees of freedom) restricted to a subvolume (cf. [30]). That is, we can make the following observation.

Observation 5.2 *In ordinary quantum statistical mechanics we study a subsystem contained in a subvolume, V, by neglecting to a certain extent the different microscopic boundary states the system can occupy, or, more specifically, by taking them only into account in a statistical way in, say, the canonical partition function, while we treat the bulk Hamiltonian as a fixed operator (i.e. fixed boundary conditions). We hence regard the possible correlations between the interior of V and the heat bath outside as both sufficiently weak and irregular. We assume however that the internal degrees of freedom, that is, the eigenstates of H, can in principle all be excited as their energy differences are assumed to be relatively small (this latter point being important in our view). This yields a relation like $I \sim V$.*

Things however may change dramatically if the subsystem cannot really be assumed to be separated from the ambient space due to very *long range* correlations between interior and exterior. We now have to deal with a truely *open subsystem*. In that case varying boundary conditions have a strong effect on the interior of the system and can no longer be emulated in a simple statistical overall manner, quite to the contrary, in *extreme* situations each possible boundary state may induces a particular state in the interior.

Observation 5.3 *In this latter case we have to change our working hypothesis. Phrased in the language of statistical mechanics, we are no longer allowed to deal, on the one hand, with a fixed Hamiltonian, H, i.e. fixed boundary conditions together with the full sequence of its possible eigenstates, and incorporate the boundary fluctuations in the canonical partion function. Instead of that we rather may have to work with a fixed formal Hamilton operator for the interior of a given region supplemented with varying boundary conditions or, stated differently, a particular Hamiltonian for each boundary state. But it may now happen that we only see the respective ground states of this class of Hamiltonians being excited (or some few of the lowest lying of them) as the higher excited states may turn out to have a very high energy and are virtually not excited. In our Dirichlet example we thus may be allowed to take only the solutions*

$$Lf = 0 \cdot f \ , \ f = g \ on \ \partial V \tag{42}$$

into account but now have to cover the full set of different possible boundary conditions (or, rather, a countable set of typical ones).

Such a scenario would lead to an area-like behavior of entropy or information content.

Remark: We note that in supersymmetry breaking and other model theories it is sometimes argued that the super-partners or higher excited modes cannot be seen at ordinary energies due to their supposed huge masses.

6 Conclusion

We want to conclude this paper with a comment concerning seemingly related findings in ordinary physics described by the notion of entanglement-entropy (see [30] and the references given there). We already mentioned that macroscopically excited eigenstates of Hamiltonians describing the interaction of many degrees of freedom lead in the generic case to partial states on subvolumes having an entropy which is proportional to the volume while groundstates, away from criticallity, that is in the regime of short-range correlations, in the general case have an entanglement-entropy which goes with the area of the boundary of the subvolume (note that there exist examples, i.e. groundstates of product form, where it happens to be zero).

The holographic principle, on the other hand, deals with the maximally possible entropy or information storage capacity of a region. That means, we have to include also the highly excited states in our considerations. But our above remark shows that in ordinary physics these lead usually to a volume-behavior of entanglement-entropy. Therefore we reach the following conclusion.

Conclusion 6.1 *The area law of the holographic principle cannot be understood within the context of ordinary (statistical) physics but needs prerequisites as described in the preceding sections, thus leading to a kind of generalized statistical mechanics.*

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