



Workbook

Numerical Reasoning tests



Careers Service

Contents

Contents	3
1. Numeric Reasoning Questions.....	4
1.1. Tables of Data	4
1.2 Maths Problems	4
2. Questions on Data Tables	5
2.1 Solutions	14
2.2 Further Questions on Data Tables.....	23
2.3 Solutions	24
3. Typical Maths Questions with Solutions	29
3.1 Maths Questions to Try.....	33
3.2 Solutions - Maths Questions to Try	34
4. Question on Graphs.....	37
4.1 Solutions	38

1. Numeric Reasoning Questions

These notes cover two common types of question.

- Answering questions about tables of data.
- Solving maths problems.

1.1. Tables of Data

The first section is to do with reading information from tables of data and answering questions about the data. This section is split into three parts.

1. A series of example questions which you answer. There will be partial help plus guidance to answering the question using Hint Boxes. Typically you will fill in the required information in the Hint Boxes and then this should give you the answer.
2. Full answers to the questions so that you can compare your results.
3. Another new set of questions, without help, for you to do. Solutions are available.

Required Numeracy Skills

You will need the following skills in order to tackle these questions:

- Carefully reading of tables of data and understanding what is being asked of you.
- Comparing and making judgements on the basis of numerical values.
- Estimating values quickly to give ball-park figures on which decisions can be made.
- Using percentages and ratios – in particular how to convert ratios to percentages and also how to work out a percentage increase.

The Numeracy Refresher Booklet (NRB) covers all these important skills. For example you may have to brush up on fractions or decimals before you can confidently answer questions about percentages or even estimate ratios. Also you will have know how to write decimals to a given number of decimal places – the NRB covers this as well.

1.2 Maths Problems

You will be asked to solve problems which may involve the use of basic numeracy skills such as calculating and using percentages and/or ratios. So all the basic numeracy skills you needed for analysing data tables will also be used.

These problems are often stated in terms of “real-life” or context specific questions e.g. case-studies relevant to the employer’s core activities.

It is important to fully understand the question and what it is asking you to do before you start calculating. This type of question is often failed because the student is solving the wrong problem!

In these notes you are given sample questions together with solutions.

Finally you can then attempt a new set of questions – solutions are given.

Note that the Numeracy Refresher Booklet has been designed to help you with these types of questions as well as the data table questions.

2. Questions on Data Tables

The following table describes the income of various industries over a five year period. There are nine questions in this section and each has a Hint Box for you to fill in if you need help. The data table is repeated on each page.

The next section 2.1 contains full solutions to all these problems.

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 1.

Which industry had the largest increase in the euro amount of income from Year 3 to Year 4?

For example, Financial Services has an increase of $100-92=8$ billion euros from year 3 to year 4.

Answer:

Hint Box:

Write the increase (in billions of euros) and find the largest

Financial Services increase= 8 billion euros

Telecommunications increase=

Engineering increase=

Agriculture increase=

Manufacturing increase=

Transportation increase=

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 2.

Which industry had the smallest increase in the euro amount of income from Year 1 to Year 5?

For example, Financial Services has an increase of $110-70=40$ billion euros from year 1 to year 5.

Answer:

Hint Box

Write the increase (in billions of euros) and find the smallest.

Financial Services	increase= 40 billion euros
Telecommunications	increase=
Engineering	increase=
Agriculture	increase=
Manufacturing	increase=
Transportation	increase=

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 3.

Which industry experienced the largest percentage increase from Year 4 to Year 5?

This means that you have to work out the increase from Year 4 to Year 5 as a percentage of the income in Year 4 for each industry and find the biggest percentage increase. For example Telecommunications had an increase of 1 billion dollars from 23 billion euros in year 4 to year 5 and the percentage increase was therefore

$$\frac{1}{23} \times 100\% = 4.35\% \text{ to two decimal places.}$$

(Calculating to one decimal place would probably have been enough – but it could be possible that there were two industries with the same % increase to one decimal place)

Answer:

Hint Box

Find the percentage increase (in billions of euros) and find the largest

Financial Services	% increase=
Telecommunications	% increase= 4.35%
Engineering	% increase=
Agriculture	% increase=
Manufacturing	% increase=
Transportation	% increase=

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 4.

Which industry had the least percentage change from Year 1 to Year 4?

This means that you have to work out the increase from Year 1 to Year 4 as a percentage of the income in Year 1 for each industry and find the smallest percentage increase.

For example Manufacturing had an increase of $233 - 193 = 40$ billion euros from 193 billion euros in year 1 to 233 billion euros year 4 and the percentage increase was

therefore $\frac{40}{193} \times 100\% = 20.73\%$ to two decimal places.

(Once again taking two decimal places to be on the safe side.)

Answer:

Hint Box

Find the percentage change for each industry and find the least

Financial Services % increase=

Telecommunications % increase=

Engineering % increase=

Agriculture % increase=

Manufacturing % increase= 20.73%

Transportation % increase=

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 5.

a) If the trend in the Transportation industry were to continue, write down the most likely income for Year 6.

b) Which of the above industries would you find difficult to predict a reasonably accurate figure for Year 6?

As an example, consider Financial Services.

The differences between successive years are in billions of euro:

Year 2-Year 1	Year 3-Year 2	Year 4-Year 3	Year 5-Year 4
12	10	8	10

These fluctuate between 8 and 12, and have an average value of 10. Since there is no clear trend we can put forward an increase of 10 as a ball-park figure.

So the prediction would be $110 + 10 = 120$ for year 6.

But now consider Manufacturing, we find the differences between successive years:

Year 2-Year 1	Year 3-Year 2	Year 4-Year 3	Year 5-Year 4
5	8	27	34

There is no clear pattern – we need more data. So I would find this difficult to predict other than Year 6 would be an increase on Year 5.

a) Prediction for Transportation in Year 6?

b) Make an estimate for Year 6 for each industry– write a question mark if difficult to predict.

	Prediction for Year 6 (billions of euros)
Financial Services	120
Telecommunications	
Engineering	
Agriculture	
Manufacturing	?

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 6.

Which type of industry shows the steadiest rate of growth in income during this period of five years?

Some industries are growing by about the same amount every year. Find the one that in your opinion has the steadiest growth i.e. the increases from one year to another have the smallest fluctuation. For example Agriculture has the following differences from one year to the next:

<i>Year 2-Year 1</i>	<i>Year 3-Year 2</i>	<i>Year 4-Year 3</i>	<i>Year 5-Year 4</i>
2	0	4	21

There is a major fluctuation in the growth of income between years 4 and 5 and so the growth in Agriculture income over the five years is not particularly steady.

Industry with steadiest rate of growth:

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 7.

In how many instances did a type of industry make a gain of 10% or more over the previous year listed?

Looking at Financial Services we have

- Year 1 to Year 2 increase of 12 on 70 - more than 10% increase (10% of 70 is 7 and 12 is greater than 7)
- Year 2 to Year 3 increase of 10 on 82 - more than 10% increase (10% of 82 is 8.2 and 10 is greater than 8.2)
- Year 3 to Year 4 increase of 8 on 92 - less than 10% increase (10% of 92 is 9.2)
- Year 4 to Year 5 increase of 10 on 100 – a 10% increase (10% of 100 is 10)

So 3 instances of 10% increases or more for Financial Services.

Now do the same for the other industries. Try not to use a calculator as it is easy to work out 10% of any number – just move the decimal point to the left one place.

Fill in the following table.

For each industry, answer Y if growth is 10% or more, N if less than 10%. Then count the number of Y's and put in last column.

Do this for all industries.

Finally add up the numbers in the last column and put in the last cell in this column.

	Yr1-Yr2	Yr2-Yr3	Yr3-Yr4	Yr4-Yr5	10% or more
Financial Services	Y	Y	N	Y	3
Telecommunications					
Engineering					
Agriculture					
Manufacturing					
Transportation					
Total Number of instances of 10% or more increases					

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 8.

Find all industries that increased their respective incomes by more than 20% from Year 1 to Year 3.

For example, if we take Manufacturing, then we find the following information

	Growth in billions	% Increase
Manufacturing	13	$\frac{13}{193} \times 100\% = 6.7\%$

Note that I have worked out the % increase to one decimal place – there is no point in being more accurate.

Answer - Which Industries have more than a 20% increase?:

Hint Box:

Work out the percentage increases from Year 1 to Year 3 and choose those industries with increases over 20%.

Financial Services	% increase=
Telecommunications	% increase=
Engineering	% increase=
Agriculture	% increase=
Manufacturing	% increase= 6.7%
Transportation	% increase=

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 9.

Find the average income of each industry over the first three years and over the last four years. Your answers should be to two decimal places.

Working out averages for Financial Services

First three years, average = $\frac{(70 + 82 + 92)}{3} = 81.33$ to two decimal places.

Last four years, average = $\frac{(82 + 92 + 100 + 110)}{4} = 96.00$

You fill in the rest.

Hint Box:

Work out averages in billions of euros for Years 1, 2 and 3 and then work out averages for Years 2, 3, 4 and 5.

	First Three years	Last Four Years
	Average	Average
Financial Services	81.33	100.67
Telecommunications		
Engineering		
Agriculture		
Manufacturing		
Transportation		

2.1 Solutions

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 1.

Which industry had the largest increase in the euro amount of income from Year 3 to Year 4?

Taking away the Year 3 figures from the Year 4 figures we find:

Answer: Manufacturing (increase of 27 billion euros).

Hint Box:

Write the increase (in billions of euros) and find the largest

Financial Services increase= 8

Telecommunications increase= 2

Engineering increase= 5

Agriculture increase= 4

Manufacturing increase= 27 (the largest)

Transportation increase= 3

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 2.

Which industry had the smallest increase in the euro amount of income from Year 1 to Year 5?

Taking away the Year 1 figures from the Year 5 figures we find:

Answer: Telecommunications (increase of 7 billion euros).

Hint Box

Write the increase (in billions of euros) and find the smallest.

Financial Services	increase= 40
Telecommunications	increase= 7
Engineering	increase= 21
Agriculture	increase= 27
Manufacturing	increase= 74
Transportation	increase= 13

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 3.

Which industry experienced the largest percentage increase from Year 4 to Year 5?

For each industry you take away Year 4 from Year 5 figures to give the increase. Then you work out the increase as a percentage of the Year 4 figures – all to two decimal places.

Answer: Agriculture – increase of 65.63% (see box below)

Answer Box

Find the percentage increase (in billions of euros) and find the largest

$$\text{Financial Services} \quad \% \text{ increase} = \frac{110 - 100}{100} \times 100\% = 10\%$$

$$\text{Telecommunications} \quad \% \text{ increase} = \frac{24 - 23}{23} \times 100\% = 4.35\%$$

$$\text{Engineering} \quad \% \text{ increase} = \frac{54 - 49}{49} \times 100\% = 10.20\%$$

$$\text{Agriculture} \quad \% \text{ increase} = \frac{53 - 32}{32} \times 100\% = 65.63\%$$

$$\text{Manufacturing} \quad \% \text{ increase} = \frac{267 - 233}{233} \times 100\% = 14.59\%$$

$$\text{Transportation} \quad \% \text{ increase} = \frac{51 - 47}{47} \times 100\% = 8.51\%$$

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 4.

Which industry had the least percentage change from Year 1 to Year 4?

You have to work out the increase from Year 1 to Year 4 as a percentage of the income in Year 1 for each industry and find the smallest percentage increase.

I have worked out the percentages to two decimal places.

Answer: Manufacturing, increase of 20.73% years 1 to 4 (see below)

Answer Box

The percentage change from Year 1 to Year 4 for each industry is as follows

Financial Services	% increase=	$\frac{100 - 70}{70} \times 100\% = 42.86\%$
Telecommunications	% increase=	$\frac{23 - 17}{17} \times 100\% = 35.29\%$
Engineering	% increase=	$\frac{49 - 33}{33} \times 100\% = 48.48\%$
Agriculture	% increase=	$\frac{32 - 26}{26} \times 100\% = 23.08\%$
Manufacturing	% increase=	$\frac{233 - 193}{193} \times 100\% = 20.73\%$
Transportation	% increase=	$\frac{47 - 38}{38} \times 100\% = 23.68\%$

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 5.

a) If the trend in the Transportation industry were to continue, write down the most likely income for Year 6.

b) Which of the above industries would you find difficult to predict a reasonably accurate figure for Year 6?

In order to answer this question I have drawn up a table of the differences between successive years to see if I can spot any trends from which I can make predictions for Year 6. The last column gives the predicted increase from Year 5 to Year 6 based on these figures. Note that these are tentative figures and there may be more than one possible choice.

Table of Differences in Successive Years

Industry	Yr2-Yr1	Yr3-Yr2	Yr4-Yr3	Yr5-Yr4	Yr6-Yr5 prediction
Financial Services	12	10	8	10	10
Telecommunications	3	1	2	1	2 or 1
Engineering	7	4	5	5	5
Agriculture	2	0	4	21	?
Manufacturing	5	8	27	34	?
Transportation	3	3	3	4	4 or 3

Based on this table I would add the last column to the income for Year 5 to get the prediction for Year 6.

Answer:

a) Prediction for Transportation in Year 6? 55 or 54

Answer:

b) Make an estimate for Year 6 for each industry– write a question mark if difficult to predict.

Prediction for Year 6 (billions of euros)

Financial Services	110
Telecommunications	26 or 25
Engineering	59
Agriculture	?
Manufacturing	?

So we see that making predictions on Agriculture and Manufacturing is not possible without more data e.g. data on the economic cycle etc..

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 6.

Which type of industry shows the steadiest rate of growth in income during this period of five years?

In order to answer this question I use the table of the differences between successive years from the last question to see if there are any industries with steady increases in income over the five years. Apart from Agriculture and Manufacturing (which are industries subject to large fluctuations) the others are fairly consistent in their increases. However, Transportation does seem to have the steadiest rate of growth.

Table of Differences in Successive Years

Industry	Yr2-Yr1	Yr3-Yr2	Yr4-Yr3	Yr5-Yr4	Yr6-Yr5 prediction
Financial Services	12	10	8	10	10
Telecommunications	3	1	2	1	2 or 1
Engineering	7	4	5	5	5
Agriculture	2	0	4	21	?
Manufacturing	5	8	27	34	?
Transportation	3	3	3	4	4 or 3

Answer:

Industry with steadiest rate of growth: Transportation

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 7.

In how many instances did a type of industry make a gain of 10% or more over the previous year listed?

I have filled in the following table.

For each industry, I have answered Y if growth is 10% or more, N if less than 10%.

Then I have counted the number of Y's and put this number in the last column. This is done for all industries.

Finally I have added up the numbers in the last column and put the total in the last cell in this column.

	Yr1-Yr2	Yr2-Yr3	Yr3-Yr4	Yr4-Yr5	10% or more
Financial Services	Y	Y	N	Y	3
Telecommunications	Y	N	N	N	1
Engineering	Y	Y	Y	Y	4
Agriculture	N	N	Y	Y	2
Manufacturing	N	N	Y	Y	2
Transportation	N	N	N	N	0
Total Number of instances of 10% or more increases					12

So we have found that there are 12 instances of an increase of 10% from one year to the next across all industries.

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 8.

Find all industries that increased their respective incomes by more than 20% from Year 1 to Year 3.

We form a table of percentage increases for each of the industries for Years 1 to 3. Note all calculations are to one decimal place as there is no need to be any more accurate.

Answer - Which Industries?:

From box below we have

Financial Services

Telecommunications

Engineering

All have increased by more than 20% from Year 1 to Year 3

Answer Box:

These are the percentage increases from Year 1 to Year 3. All to one decimal place. I then chose those industries with increases over 20%.

Financial Services	% increase=	$\frac{92 - 70}{70} \times 100\% = 31.4\%$
Telecommunications	% increase=	$\frac{21 - 17}{17} \times 100\% = 23.5\%$
Engineering	% increase=	$\frac{44 - 33}{33} \times 100\% = 33.3\%$
Agriculture	% increase=	$\frac{28 - 26}{26} \times 100\% = 7.7\%$
Manufacturing	% increase=	$\frac{206 - 193}{193} \times 100\% = 6.7\%$
Transportation	% increase=	$\frac{44 - 38}{38} \times 100\% = 15.8\%$

INCOME OF INDUSTRIES (in billions of euros)					
Industry	Year 1	Year 2	Year 3	Year 4	Year 5
Financial Services	70	82	92	100	110
Telecommunications	17	20	21	23	24
Engineering	33	40	44	49	54
Agriculture	26	28	28	32	53
Manufacturing	193	198	206	233	267
Transportation	38	41	44	47	51

Question 9.

Find the average income of each industry over the first three years and over the last four years. Your answers should be to two decimal places.

Answer Box:

Averages in billions of euros for Years 1, 2 and 3.

Averages for Years 2, 3, 4 and 5.

All calculations to 2 decimal places.

	First Three years Average	Last Four Years Average
Financial Services	81.33	96.00
Telecommunications	19.33	22.00
Engineering	39	46.75
Agriculture	27.33	35.25
Manufacturing	199	226.00
Transportation	41	45.75

2.2 Further Questions on Data Tables

NEWSPAPER READERSHIP					
	Readership (millions)			Percentage of copies Sold in Year 3, by outlet	
Daily Newspapers	Year 1	Year 2	Year 3	Subscription	Newsagents
Daily Herald	5.2	4.9	4.4	52%	38%
Reynolds News	2.7	2.9	3.1	48%	40%
The Echo	6.2	6.1	5.2	32%	50%
Daily Planet	8.3	10.2	10.3	38%	49%
Daily Spin	9.7	9.9	10.5	25%	72%

Study this table of data and answer the questions below. Solutions can be found in the following pages.

The first three columns tell us how many readers (in millions) each newspaper has in each year. Each of these readers may have different ways of purchasing any one paper and also read the paper on a very regular or semi-regular basis. Another complicating factor is that a reader may read and subscribe to more than one of these papers. The last two columns give information on selling of copies in Year 3. For example, 52% of the Daily Herald copies were sold by subscription.

Questions

- What is the average readership for each newspaper over the three years?
- Has the total readership for the five newspapers fallen from Year 1 to Year 3?
Is the total readership in any year the same as the number of different people buying at least one of the five newspapers in that year?
- If the Daily Planet sold 75 million copies by Subscription in Year 3, how many copies (to the nearest hundred thousand) were sold at the newsagents in Year 3?
- If the Echo sold 220 million copies in total in Year 3, how many copies (to the nearest hundred thousand) were not sold at the newsagents nor were sold by Subscription?
- The Daily Herald, Reynolds News and Daily Spin are all part of the same group MadeUp which only has these three papers.
The sales in Year 3 were in the ratios
Daily Herald:Reynolds News:DailySpin= 2:3:4
The total paper sales in Year 3 for the MadeUp group were 873 million.
How many of each of the Made Up papers were sold in Year 3?
How many copies of the Daily Spin were bought at the newsagents in Year 3?
- Which newspaper showed the largest percentage change in readership from Year 1 to Year 2?
- Which newspaper showed the smallest percentage change in readership from Year 2 to Year 3?

2.3 Solutions

Question 1

What is the average readership for each newspaper over the three years?

Solution

We simply take the averages of the three years

	Yr1	Yr2	Yr3	
				All calculations to 2 decimal places
Daily Herald	5.2	4.9	4.4	Average=(5.2+4.9+4.4)/3= 4.83
Reynolds News	2.7	2.9	3.1	Average=(2.7+2.9+3.1)/3= 2.9
The Echo	6.2	6.1	5.2	Average=(6.2+6.1+5.2)/3= 5.83
Daily Planet	8.3	10.2	10.3	Average=(8.3+10.2+10.3)/3= 9.60
Daily Spin	9.7	9.9	10.5	Average=(9.7+9.9+10.5)/3= 10.03

Question 2

Has the total readership for the five newspapers fallen from Year 1 to Year 3?

Is the total readership in any year the same as the number of different people buying at least one of the five newspapers in that year?

Solution

Summing up the columns we have

	Year 1	Year 2	Year 3
Daily Herald	5.2	4.9	4.4
Reynolds News	2.7	2.9	3.1
The Echo	6.2	6.1	5.2
Daily Planet	8.3	10.2	10.3
Daily Spin	9.7	9.9	10.5
Total readership	32.1	34	33.5

So the total readership has increased from Year 1 to Year 3 and not fallen (although it has fallen from Year 2 to Year 3).

Is the total readership in any year the same as the number of different people buying at least one of the five newspapers in that year?

No – because you can have people buying more than one paper and so adding the readership figures together gives double counting of these people and so over estimates the number of different people reading one of the five papers.

Question 3

If the Daily Planet sold 75 million copies by Subscription in Year 3, how many copies (to the nearest hundred thousand) were sold at the newsagents in Year 3?

Solution

	Year 3 copies sold by subscription	% sold by subscription in Year 3	Number of copies sold In Year 3 (to three decimal places)
The Daily Planet	75 million	38%	$\frac{100}{38} \times 75 = 197.368$ million

The table above shows how to calculate the total number of copies sold in Year 3 given that 38% of the total is 75 million i.e. 197.368 million.

We still have to calculate the total sold at the newsagents in Year 3.

This is 49% of the total i.e. 49% of 197.368 million = $\frac{49}{100} \times 197.368 = 96.71$ million.

Hence to the nearest hundred thousand the total sold at the newsagents is 96.7 million.

Question 4

If the Echo sold 220 million copies in total in Year 3, how many copies (to the nearest hundred thousand) were not sold at the newsagents nor were sold by Subscription?.

Solution

This is the information the table gives us for Year 3

	% sold by Subscription	% sold at newsagents
The Echo	32%	50%

The percentage not sold by subscription nor at the newsagents is

$100\% - 32\% - 50\% = 100\% - 82\% = 18\%$

And 18% of 220 million is given by $\frac{18}{100} \times 220 = 39.6$ million. (This is to the nearest hundred thousand already).

Question 5

The Daily Herald, Reynolds News and Daily Spin are all part of the same group MadeUp which only has these three papers.

The sales in Year 3 were in the ratios

Daily Herald:Reynolds News:DailySpin= 2:3:4

The total paper sales in Year 3 for the MadeUp group were 873 million.

How many of each of the Made Up papers were sold in Year 3?

How many copies of the Daily Spin were bought at the newsagents in Year 3?

Solution

We use the ideas of shares. Since the ratios of the sales is 2:3:4 we can allocate $2+3+4=9$ shares and each share gets $873/9=97$ million.

The Daily Herald has 2 shares = $2 \times 97 = 194$ million copies bought in Year 3

The Reynolds News has 3 shares = $3 \times 97 = 291$ million copies bought in Year 3

The Daily Spin has 4 shares = $4 \times 97 = 388$ million copies bought in Year 3

Now according to the table 72% of the copies of the Daily Spin were bought at the newsagents in Year 3.

This is given by $\frac{72}{100} \times 388 = 279.36$ million copies in Year 3.

Question 6

Which newspaper showed the largest percentage change in readership from Year 1 to Year 2?

Solution

For each paper we find the percentage change from Year 1 to Year 2 by finding the difference of the readership Year 2-Year 1 as a percentage of Year 1 readership. Note that this can be a negative % difference – but we find the largest in magnitude as the answer (i.e. forget about the minus sign and only take account of how big a change we have). We take all our answers to two decimal places.

	Yr1	Yr2	%change Yr1 to Yr 2
Daily Herald	5.2	4.9	$\frac{4.9 - 5.2}{5.2} \times 100\% = -5.77\%$
Reynolds News	2.7	2.9	$\frac{2.9 - 2.7}{2.7} \times 100\% = 7.41\%$
The Echo	6.2	6.1	$\frac{6.1 - 6.2}{6.1} \times 100\% = -1.61\%$
Daily Planet	8.3	10.2	$\frac{10.2 - 8.3}{8.3} \times 100\% = 22.89\%$
Daily Spin	9.7	9.9	$\frac{9.9 - 9.7}{9.7} \times 100\% = 2.06\%$

We see that the Daily Planet has had the biggest % change from Years 1 to 2, an increase of 22.89% in the readership.

Question 7

Which newspaper showed the smallest percentage change in readership from Year 2 to Year 3?

Solution

For each paper we find the percentage change from Year 2 to Year 3 by finding the difference of the readership Year 3-Year 2 as a percentage of Year 2 readership. Note that this can be a negative % difference – but we find the largest in magnitude as the answer (i.e. forget about the minus sign and only take account of how large a change we have). We take all our answers to two decimal places.

	Yr 2	Yr 3	%change Yr2 to Yr 3
Daily Herald	4.9	4.4	$\frac{4.4 - 4.9}{4.9} \times 100 = -10.20\%$
Reynolds News	2.9	3.1	$\frac{3.1 - 2.9}{2.9} \times 100 = 6.90\%$
The Echo	6.1	5.2	$\frac{5.2 - 6.1}{6.1} \times 100 = -14.75\%$
Daily Planet	10.2	10.3	$\frac{10.3 - 10.2}{10.2} \times 100 = 0.98\%$
Daily Spin	9.9	10.5	$\frac{10.5 - 9.9}{9.9} \times 100 = 6.06\%$

The Daily Planet has gained 0.98% readers and this is the smallest percentage change from Year 2 to Year 3 amongst all the papers.

3. Typical Maths Questions with Solutions

Question 1

The selling price of a box of chocolates is £6.00.

This price was 20% greater than the cost to produce the box.

How much did it cost to produce a box and what was the profit?

Solution

Think of the cost to produce as being 100%. The price is 20% greater, so is 120% of the cost i.e. $\frac{120}{100}$ of the cost.

Hence the cost is $\frac{100}{120} = \frac{5}{6}$ of the price $= \frac{5}{6} \times £6 = £5$ pounds

The profit = selling price - cost = £6 - £5 = £1.

Question 2

A picture on a page was reduced on a copier to 60% of its original size, and this copy was then reduced by 20%.

What percentage of the size of the original picture was the final copy?

Solution

The picture area has been reduced by 60% or $\frac{3}{5}$ ($=60/100$).

So to find the new size we multiply the original size by $\frac{3}{5}$.

If we further reduce the new size by 20% $= \frac{1}{5}$ then we have to multiply again by

$\frac{1}{5}$ i.e. another we have $\frac{1}{5} \times \frac{3}{5} = \frac{3}{25}$ of the original size we started off with.

Now to express this as a percentage we multiply by 100 and we obtain:

$$\frac{3}{25} \times 100 = 12\%.$$

So the final copy is 12% of the original picture.

Question 3

In a certain department, 15% of the females and 25% of the males are working on a project. 60% of the department is female.

What percentage of the department is working on the project?

Solution

Idea is to add together the % of females in the department working on the project to the % of males in the department working on the project.

1. Females

60% of the department is female and 15% of females are working on the project, hence the proportion of females on the project is 15% of 60% of the workers.

In terms of fractions this is $\frac{15}{100} \times \frac{60}{100} = \frac{900}{10000} = \frac{9}{100}$ of the workers i.e. 9%.

Hence 9% of the departmental staff on the project are female.

2. Males

40% of the department is male and 25% are working on the project, hence the proportion of males on the project is 25% of 40% of the workers.

In terms of fractions this is $\frac{25}{100} \times \frac{40}{100} = \frac{1000}{10000} = \frac{1}{10}$ of the workers, which is 10%.

Hence 10% of the departmental staff on the project are male.

So the total percentage is 10%+9%=19% of the department working on the project.

Question 4

The total cost for three items of work on a car was £154.

These items were: overhaul of the carburettor, replace brake pads and a tune-up.

Overhaul of the carburettor cost twice as much as the tune-up.

Brake pads cost one-third as much as the carburettor overhaul.

What did the tune-up cost?

There are two solutions given. The first uses idea of shares and the second uses elementary algebra.

Solution 1

We are given that cost carburettor overhaul=cost of 2 tune-ups=cost of 3 brake pads

We can represent the ratios of the costs for the carburettor, tune-up and brake pads by giving cost shares to each of the repairs in the ratios 6:3:2 i.e.

Give the carburettor repair 6 shares, tune-up 3 shares and the brake pads 2 shares.

Then the relative costs are preserved.

Hence there are 6+3+2=11 shares to add up to £154.

So each share is worth £14 and the tune-up gets 3 shares i.e. **costs £42.**

(I could have given the carburettor 12 shares, tune-up 6 shares and the brake pads 4 shares – all that matters is that the ratios of the shares are the same.

In that case there would be 12+6+4=22 shares each worth £154/22=£7 and the tune-up with 6 shares would still cost 6 × £7 = £42)

Solution 2

We will work out the cost of tune-up as follows

Let C=cost of carburettor work, T=cost of a tune-up, B=cost of brake pads

We are given that $C = 2T$, and $C = 3B$. So $2T = 3B$ and $B = 2/3T$

$$\begin{aligned}\text{So cost repair work} = £154 &= C + T + B \\ &= 2T + T + 2/3T = (2 + 1 + 2/3)T \\ &= 11/3T\end{aligned}$$

$$\text{Hence } \frac{11}{3}T = £154 \text{ gives us } T = \frac{3}{11} \times £154 = £42 .$$

Question 5

A certain preparation consists of liquids x, y, and z in the proportion 5:2:1. How many gallons of the preparation can be made from a stock of materials consisting of 25 gallons of x, 20 gallons of y, and 8 gallons of z?

Solution

The proportions 5:2:1 have to be preserved.

So if we use G gallons of z then we must use 2G gallons of y and 5G gallons of x. to get 8G gallons of the preparation.

So G has to be as big as possible.

As we have 8 gallons of z, G is not more than 8.

As we have 20 gallons of y, 2G not more than 20 i.e. G not more than 10.

As we have 25 gallons of x, 5G not more than 25 i.e. G not more than 5.

So maximum value of G is 5 and we get $8 \times 5 = 40$ gallons of preparation.

Question 6

A product costing 60 cents per unit to produce had been selling at the average rate of 1,200,000 units per month.

After the product was improved, sales increased to an average of 2,000,000 units per month.

However, the new product cost five percent more to produce. If the manufacturer's selling price in each instance was 75 cents per unit, what was the manufacturer's added profit per month with the newer product?

Solution

First we work out the cost of production of the new product.

Original production cost = 60 cents per unit. Profit per unit was $75 - 60 = 15$ cents

Production cost of new product costs 5% more.

Now 5% of 60 = $\frac{1}{20}$ of 60 = 3 cents.

So new production costs = $60 + 3$ cents = 63 cents. Profit per unit is now $75 - 63 = 12$ cents.

Old profit = $1,200,000 \times 15 = 18,000,000$ cents = \$180,000

New profit = $2,000,000 \times 12 = 24,000,000$ cents = \$240,000

So added profit = $\$240,000 - \$180,000 = \$60,000$

3.1 Maths Questions to Try

Question 1

Bob has a salary of £35,000.

He has a tax allowance of £5,000 and he pays income tax at 22% on the rest.

He pays into a pension and for this his salary is deducted 8% after tax has been deducted.

How much does he get per month assuming that these are the only deductions?

Question 2

An item costs £8.50 to produce. How much should the manufacturer sell these items for if wants to realise a profit of 15% on these items? Give your answer to the nearest penny.

Question 3

In a department 10% of the staff have a salary of £20,000, 30% a salary of £25,000, 50% a salary of £30,000, 10% a salary of £40,000.

What is the average salary?

Question 4

Bob, Cilla and Terrence agree to buy ten pounds worth of lottery tickets with Bob contributing £5, Cilla £3 and Terence £2.

They agree that if they win anything with any of these tickets that it should be shared out in the same ratio as their contributions.

They win £250. How much does each get?

Question 5

A product costing 75 p per unit to produce had been selling at the average rate of 650,000 units per month.

After the product was improved, sales increased to an average of 1,000,000 units per month.

However, the new product cost 10 percent more to produce. If the manufacturer's selling price in each instance was 105p per unit, what was the manufacturer's added profit per month with the newer product?

Question 6

Same information as in Question 5.

It is projected that if a new marketing campaign for a year is focused on the improved product then sales could be further raised as follows.

£700,000 spent on the campaign then a further 150,000 units per month are projected to be sold in the first year of the campaign. After the campaign has finished it is projected that in the next year sales will be 1,100,000 per month and that in subsequent years sales will also be 1,100,000 per month.

Assuming that all prices and costs stay as they are, how many months are predicted from the beginning of the campaign before the cost of the campaign is recouped?

3.2 Solutions - Maths Questions to Try

Question 1

Bob has a salary of £35,000.

He has a tax allowance of £5,000 and he pays income tax at 22% on the rest.

He pays into a pension and for this his salary is deducted 8% after tax has been deducted.

How much does he get per month assuming that these are the only deductions?

Solution

Bob is taxed on £35,000-£5,000=£30,000

Income tax is at 22% on this £30,000 and is $\frac{22}{100} \times 30,000 = £6,600$.

Leaving £35,000-£6,600=£28,400.

8% of this is for his pension contributions i.e. $\frac{8}{100} \times 28,400 = £2,272$

The amount left is then £28,400-£2,272=£26,128

Take home per month is $\frac{26128}{12} = £2,177.33$ to nearest penny.

Question 2

An item costs £8.50 to produce. How much should the manufacturer sell these items for if wants to realise a profit of 15% on these items? Give your answer to the nearest penny.

Solution

The price of an item is 115% of the production cost in order to realise a profit of 15%.

This price is $\frac{115}{100} \times 8.5 = £9.775$. Clearly a decision has to be made whether the price is £9.77 or £9.78. Usually numbers such as 9.775 which end with a 5 are rounded up and so we take the price to be £9.78

Question 3

In a department 10% of the staff have a salary of £20,000, 30% a salary of £25,000, 50% a salary of £30,000, 10% a salary of £40,000.

What is the average salary?

Solution This is a weighted average.

The average value is given by

1. For each salary value multiplying the frequency (in fraction form) of the salary amongst the staff by the salary value.
2. For example, the salary £20,000 has a frequency of 10% which is $\frac{10}{100} = 0.1$ and we multiply this by £20,000.
3. Adding these numbers together across all salary values.

For this question we have 4 salary values and the weighted average is

$$\frac{10}{100} \times 20,000 + \frac{30}{100} \times 25,000 + \frac{50}{100} \times 30,000 + \frac{10}{100} \times 40,000$$
$$= £2000 + £7500 + £15000 + £4000$$
$$= £28,500$$

Question 4

Bob, Cilla and Terry agree to buy ten pounds worth of lottery tickets with Bob contributing £5, Cilla £3 and Terry £2.

They agree that if they win anything with any of these tickets that it should be shared out in the same ratio as their contributions.

They win £250. How much does each get?

Solution

Their agreement means that the winnings should go to Bob, Cilla, Terry in the ratio 5:3:2. Think of these as being shares in the winnings.

There are $5+3+2=10$ shares in all for the £250.

Hence each share is worth $£250/10=£25$.

So Bob gets 5 shares = £125, Cilla 3 shares = £75 and Terry 2 shares = £50.

Question 5

A product costing 75 p per unit to produce had been selling at the average rate of 650,000 units per month.

After the product was improved, sales increased to an average of 1,000,000 units per month.

However, the new product cost 10 percent more to produce. If the manufacturer's selling price in each instance was 105p per unit, what was the manufacturer's added profit per month with the newer product?

Solution

The improved product costs 10% more to produce.

$$\text{i.e. } 110\% \text{ of original production cost of } 75\text{p} = \frac{110}{100} \times 75 = 82.5\text{p}.$$

Profit was originally $105-75=30\text{p}$.

Now with the improved product the profit is $105-82.5=22.5\text{p}$

The original profit per month was $650,000 \times 30 = 19,500,000\text{p} = £195,000$

The new product profit per month is $1,000,000 \times 22.5 = 22,500,000\text{p} = £225,000$

Hence the added profit per month is $£225,000 - £195,000 = £30,000$

Question 6

Same information as in Question 5.

It is projected that if a new marketing campaign for a year is focused on the improved product then sales could be further raised as follows.

£700,000 spent on the campaign then a further 150,000 units per month are projected to be sold in the first year of the campaign. After the campaign has finished it is projected that in the next year sales will be 1,100,000 per month and that in subsequent years sales will also be 1,100,000 per month.

Assuming that all prices and costs stay as they are, how many months are predicted from the beginning of the campaign before the cost of the campaign is recouped?

Solution

The profit is 22.5p per item. In the first year there is projected to be an extra profit of $150,000 \times 12 \times 22.5p = 40,500,000p = £405,000$

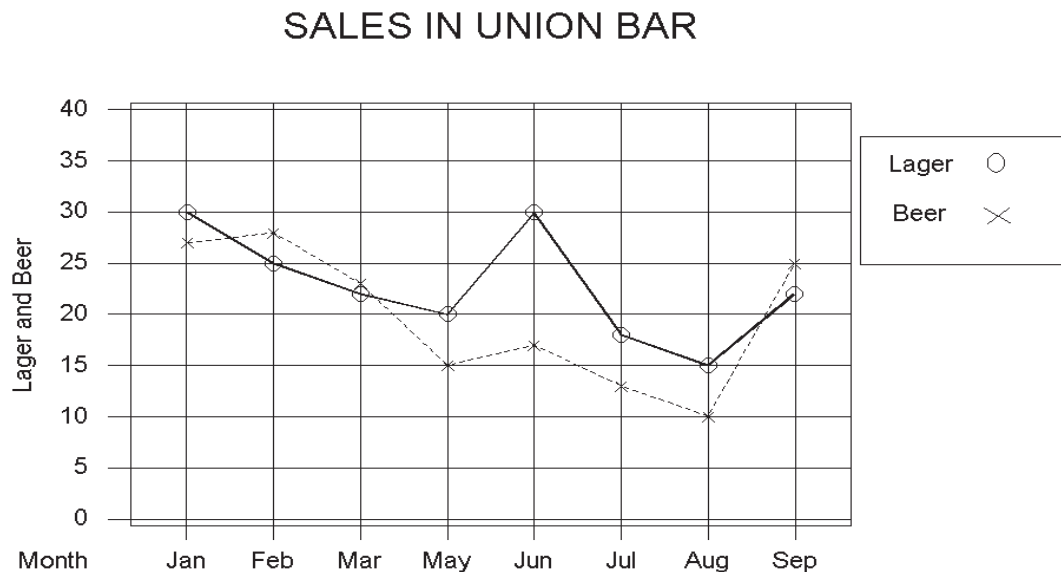
Each year after that the projected extra profit is

$100,000 \times 12 \times 22.5p = 27,000,000p = £270,000$ with each month contributing £22,500.

In the first two years it is projected that $£405,000 + £270,000 = £675,000$ will be recouped. A further £25,000 is needed and so it will take a further two months at £22,500 per month to recoup the full amount.

So it is predicted that in 26 months the campaign costs will be recovered and that an extra £22,500 of profit per month thereafter will be realised.

4. Question on Graphs



The above graph gives the sales of lager and beer per month in thousands of pints. The prices of lager and beer does vary from month to month. However, it is always the case that beer's price per pint is 20% more than that of lager's price per pint. Answer the following questions. Solutions are found in following pages.

Question 1

What was the selling price for lager in July if the income for lager in July was £28,800? What was the income from beer sales in July?

Question 2

If the selling price of beer was £2.10 per pint in January, what was the selling price of lager?

Also what was the total income in that month (both beer and lager)?

Question 3

If the total income for both beer and lager in May was £ 62,700 find the price charged for a pint of lager.

Question 4

Surprisingly, there is always a large amount of lager and beer consumed in October. Past years has shown that a good predictor for consumption of lager and beer in October is given by adding 50% on to the consumption of lager in September, and by adding 40% on to the consumption of beer in September.

What price for lager should the Union charge, based on this prediction, so that the predicted total income in October for beer and lager sales is £110,000?

4.1 Solutions

Question 1

**What was the selling price for lager in July if the income for lager was £28,800?
What was the income from beer sales in July?**

Solution

Looking at the graph we see that the 18,000 pints of lager were sold in July.

Hence the price per pint is $\frac{28,800}{18,000} = \frac{288}{180} = £1.60$

In order to find the income from beer sales we need to find the price of a pint of beer.
We know that this is 20% more than a pint of lager so:

So the price of a pint of beer = $\frac{120}{100} \times 1.60 = £1.92$

Looking at the graph we see that 13,000 pints of beer were sold in July.

So the income from beer sales is $13,000 \times 1.92 = £24,960$

Question 2

If the selling price of beer was £2.10 per pint in January, what was the selling price of lager?

Also what was the total income in that month (both beer and lager)?

Solution

The price of beer is 20% higher than that of lager, which means that to convert from the price of beer to lager we need to multiply by $100/120 = 5/6$.

Hence the price of lager in January is $\frac{100}{120} \times 2.10 = £1.75$.

Looking at the graph for January we see that

27,000 pints of beer

30,000 pints of lager

were sold.

Income from beer sales = $27,000 \times 2.10 = £56,700$

Income from lager sales = $30,000 \times 1.75 = £52,500$

Total income = $£56,700 + £52,500 = £109,200$

Question 3

If the total income for both beer and lager in May was £ 62,700 find the price charged for a pint of lager.

Solution

The graph tells us that
20,000 pints of lager and 15,000 pints of beer were sold in May.

Remember that beer is 20% more expensive than lager.

It follows that 15,000 pints of beer are in price terms the same as

$$\frac{120}{100} \times 15000 = 18,000 \text{ pints of lager.}$$

So the total income is obtained by selling the equivalent of
 $20,000 + 18,000 = 38,000$ pints of lager

Since the total income is £62,700, the price of a pint of lager is $\frac{62,700}{38,000} = £1.65$

Question 4

Surprisingly, there is always a large amount of lager and beer consumed in October. Past years has shown that a good predictor for consumption of lager and beer in October is given by adding 50% on to the consumption of lager in September, and by adding 40% on to the consumption of beer in September. What price for lager should the Union charge, based on this prediction, so that the predicted total income in October for beer and lager sales is £110,000?

Solution

The September consumption of lager was 22000 pints. Hence the predicted consumption in October is 150% of 22000 = $\frac{150}{100} \times 22000 = 33,000$ pints of lager.

The September consumption of beer was 25000 pints. Hence the predicted consumption in October is 140% of 25000 = $\frac{140}{100} \times 25000 = 35,000$ pints of beer.

Remember that the price of a pint of beer is 20% more than that of a pint of lager.

Convert the 35,000 pints of beer to the price equivalent number of pints of lager.

This is given by $\frac{120}{100} \times 35000 = 42,000$ pints of lager.

Hence the total predicted income for October is price equivalent to
 $33000 + 42000 = 75000$ pints of lager.

In order to achieve a predicted income of £110,000 we need to charge

$$\frac{110000}{75000} = £1.47 \text{ for a pint of lager on taking 2 decimal places}$$
