Monitoring Software Reliability using Statistical Process Control: An Ordered Statistics Approach

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ABSTRACT

The nature and complexity of software have changed significantly in the last few decades. With the easy availability of computing power, deeper and broader applications are made. It has been extremely necessary to produce good quality software with high precession of reliability right in the first place. Olden day's software errors and bugs were fixed at a later stage in the software development. Today to produce high quality reliable software and to keep a specific time schedule is a big challenge. To cope up the challenge many concepts, methodology and practices of software engineering have been evolved for developing reliable software. Better methods of controlling the process of software production are underway. One of such methods to assess the software reliability is using control charts. In this paper we proposed an NHPP based control mechanism by using order statistics with cumulative quantity between observations of failure data using mean value function of exponential distribution.

General Terms

Software reliability, Software quality, Six Sigma, Control Charts, PDF, CDF

Keywords: Ordered Statistics, Statistical Process Control (SPC), Exponential Distribution, Control Limits, software reliability, software quality

1. INTRODUCTION

As computer applications became more diverse and spread through almost every area of everyday life, reliability became a very important characteristic for software, since it is a matter of economy. To produce a software having reliability, it is necessary to measure and control its reliability. To do this, a number of models have been developed; new models try to make better predictions. Software reliability represents a user oriented view of software quality. It relates directly to operation rather than design of the program, and hence it is dynamic. For this reason software reliability is interested in failures occurrence and not faults in a program

1.1. Software reliability Modeling

The probability that a given program will work as intended by the user, i.e., without failures in a specified environment and for a specified duration can be termed as software reliability [1][2]. The aim of software engineer is to increase this probability and make it one if possible. To do this one must measure the reliability of the software. A commonly used approach for measuring software reliability is by using an analytical model whose parameters are generally estimated from available data on software failures. Reliability quantities have been defined with respect to time, although it is possible to define them with respect to other variables. We have taken inter failures time data of Musa(1975) which are random values. In reliability study there are two characteristics of a random process: 1) the probability distribution of the random variables, i.e., Poisson and 2) the variation of the process with time. A random process whose probability distribution varies with time is called non homogeneous. For the random process for time variation we can define two functions, the mean value function m(t), as the average cumulative failures associated with each time point and the failure intensity function λ, as the rate of change of mean value function. When there are changes in the software i.e. software corrections occur it is called non homogeneous process.

Let M(t) be the random process representing the number of failures experienced by time t, then the mean value function is defined by $\mu(t) = E[m(t)]$. i.e. the expected number of failures at time t. the failure intensity function of the $m(t)$ process is the instantaneous rate of change of the expected number of failures with respect to time or $\lambda(t) = \frac{d\mu(t)}{dt}$. [3]

2. ORDERED STATISTICS

Let X denote a continuous random variable with Probability Density Function (PDF) f(x) and Cumulative Distribution Function (CDF) F(x), and let $(X_1, X_2, …, X_n)$ denote a random sample of size n drawn on X. The original sample observations may be unordered with respect to magnitude. A transformation is required to produce a corresponding ordered sample. Let $(X_1, X_2, …, X_n)$ denote the ordered random sample such that $X_1 < X_2 < … < X_n$; then $(X_1, X_2, …, X_n)$ are collectively known as the order statistics derived from the parent X. The various distributional characteristics can be known from Balakrishnan and Cohen [4]. The inter-failure time data represent the time lapse between every two consecutive failures. On the other hand if a reasonable waiting time for failures is not a serious problem, we can group the inter-failure time data into non

overlapping successive sub groups of size 4 or 5 and add the failure times within each sub group. For instance if a data of 100 inter-failure times are available we can group them into 20 disjoint subgroups of size 5. The sum total in each subgroup would denote the time lapse between every 5th order statistics in a sample of size 5. In general for inter-failure data of size 'n', if r (any natural no) less than 'n' and preferably a factor n, we can conveniently divide the data into 'k' disjoint subgroups (k=n/r) and the cumulative total in each subgroup indicate the time between every rth failure. The probability distribution of such a time lapse would be that of the rth ordered statistics in a subgroup of size r, which would be equal to rth power of the distribution function of the original variable m (t).The whole process involves the mathematical model of the mean value function and knowledge about its parameters. If the parameters are known they can be taken as they are for the further analysis, if the parameters are not know they have to be estimated using a sample data by any admissible, efficient method of distribution. This is essential because the control limits depend on mean value function, which intern depends on the parameters. If software failures are quite frequent keeping track of inter-failure is tedious. If failures are more frequent order statistics are preferable.[5]

2.1. Model Description

Considering failure detection as a non homogenous Poisson process with an exponentially decaying rate function, the expected number of failures observed by time t is given by $m(t) = a(1 - e^{-bt})$ and the failure rate by $\lambda(t) = m'(t)$. To calculate the parameter values and control limits using Order Statistics approach, we considered exponential distribution [8]. The mean value function of exponential distribution is

$$m(t) = a(1 - e^{-bt})$$

In order to group the inter-failure time data into non overlapping successive sub groups of size r the mean value function can be written as

$$m(t) = a(1 - e^{-bt})^r$$

$$m(s_k) = [a(1 - e^{-bs_k})]^r$$

$$m'(s_k) = a^r r (1 - e^{-bs_k})^{r-1} b e^{-bs_k} \qquad 2.1.1$$

The likelihood function L can be written as

$$L = e^{-m(s_n)} \prod_{k=1}^{n} m'(s_k) \qquad 2.2.2$$

Substituting eq-2.1.1 in eq-2.2.2 we can write

$$L = e^{-m(s_n)} \prod_{k=1}^{n} a^r r (1 - e^{-bs_k})^{r-1} b e^{-bs_k}$$

$$\log L = -m(s_n) + \sum_{k=1}^{n} [\log a^r + \log b + \log r + \log e^{-bs_k} + \log(1 - e^{-bs_k})^{r-1})] \qquad 2.2.3$$

$$m(s_n) = [a(1 - e^{-bs_n})]^r \qquad 2.2.4$$

Substitute equation 2.2.4 in 2.2.3 we get

$$\log L = -[a(1 - e^{-bs_n})]^r + \sum_{k=1}^{n} [\log a^r + \log b + \log r + \log e^{-bs_k} + \log(1 - e^{-bs_k})^{r-1})] \qquad 2.2.5$$

$$\frac{\partial \log L}{\partial a} = 0,$$
$$a^r = \frac{n}{(1 - e^{-bs_n})^r} \qquad 2.2.6$$

$$\frac{\partial \log L}{\partial b} = 0,$$
$$a^r r s_n e^{-bs_n} (1 - e^{-bs_n})^{r-1} + \frac{n}{b} + 0 \qquad - \sum_{k=1}^{n} s_k + (r - 1) \sum_{k=1}^{n} \frac{s_k e^{-bs_k}}{(1 - e^{-bs_k})} \qquad 2.2.7$$

Substitute equation 2.2.6 in 2.2.7 we obtain the following equation

$$g(b) = \frac{n r s_n e^{-bs_n}}{(1 - e^{-bs_n})} + \frac{n}{b} - \sum_{k=1}^{n} s_k + (r - 1) \sum_{k=1}^{n} \frac{s_k e^{-bs_k}}{(1 - e^{-bs_k})} \qquad 2.2.8$$

Derivate with respect to b of equation 2.2.8 we obtain

$$g'(b) = \frac{-n r s_n^2 e^{-bs_n}}{(1 - e^{-bs_n})^2} - \frac{n}{b^2} - (r - 1) \sum_{k=1}^{n} \frac{(s_k^2 e^{-bs_k})}{(1 - e^{-bs_k})^2} \qquad 2.2.9$$

2.2. Parameter estimation and Control limits

Parameter estimation is a statistical method trying to estimate parameters based on inter failures time data which is based on ordered statistics. For the given observations using equations 2.2.8 and 2.2.9 the parameters 'a' and 'b' are computed by using the popular Newton Rapson method A program written in C was used for this purpose. [3]

Based on the time between failures data given in Table-1, we compute the software failure process through mean value control chart. We use cumulative time between failures data for software reliability monitoring through SPC. The parameters obtained from Goel-Okumoto model applied on the given time domain data are as follows:

Table 1: Parameter estimates and their control limits of 4 and 5 order Statistics

	Order	$\hat{a}$	$\hat{b}$
Data Set of Table 2	**4th**	2.415117	0.000099
	5th	1.933309	0.000114

' $\hat{a}$ ' and ' $\hat{b}$ ' are ordered statistics of parameters and the values can be computed using analytical method for the given time between failures data shown in Table 1. Using values of 'a' and 'b' we can compute $m(t)$. Now equate the *pdf* of m*(t) to* 0.00135, 0.99865, and 0.5 and the respective control limits are given by

$$T_U = (1 - e^{-bt}) = 0.99865$$

$$T_C = (1 - e^{-bt}) = 0.5$$

$$T_L = (1 - e^{-bt}) = 0.00135$$

These limits are convert at $m(t_u)$, $m(t_c)$ and $m(t_L)$ are given by

$$m(t_u) = 33.3512569382986,$$

$$m(t_c) = 16.6981710073481,$$

$$m(t_L) = 0.04508506100108$$

They are used to find whether the software process is in control or not by placing the points in Mean value chart shown in figure-1.and figure-2. A point below the control limit $m(t_L)$ indicates an alarming signal. A point above the control limit $m(t_u)$ indicates better quality. If the points are falling within the control limits it indicates the software process is in stable. [6]

STATISTICAL PROCESS CONTROL

Statistical process control is the application of statistical methods to provide the information necessary to continuously control or improve processes throughout the entire lifecycle of a product [7]. SPC techniques help to locate trends, cycles, and irregularities within the development process and provide clues about how well the process meets specifications or requirements. They are tools for measuring and understanding process variation and distinguishing between random inherent variations and significant deviations so that correct decisions can be made about whether to make changes to the process or product. One of such primary statistical technique used to assess process variation is the control chart. [8]

2.3. Control Chart

The control chart displays sequential process measurements relative to the overall process average and control limits. The upper and lower control limits establish the boundaries of normal variation for the process being measured. Variation within control limits is attributable to random or chance causes, while variation beyond control limits indicates a process change due to causes other than chance, a condition that may require investigation. [7] The upper control limit (UCL) and lower control limit (LCL) give the boundaries within which observed fluctuations are typical and acceptable There are many different types of control charts, pn, p, c, etc., [8], [9],[10]

2.4. Developing Control Chart

Given the n inter-failure data the values of m(t) at T_c, T_u, T_L and at the given n inter-failure times are calculated. Then successive differences of m(t)'s are taken, which leads to n-1 values. The graph with the said inter-failure times 1 to n-1 on X-axis, the n-1 values of successive differences m(t)'s on Y-axis, and the 3 control lines parallel to X-axis at $m(T_L)$, $m(T_U)$, $m(T_C)$ respectively constitutes mean value chart to assess the software failure phenomena on the basis of the given inter-failures time data.

2.5. Illustration

The procedure of a mean value chart for failure software process will be illustrated with an example here. Table 1 show the time between failures of software product reported by Musa (1975) [11].

Table 2: Software failure data reported by Musa (1975) [11]

Fault	Time	Fault	Time	Fault	Time	Fault	Time	Fault	Time	Fault	Time	Fault	Time	Fault	Time
1	3	18	120	35	227	52	21	69	529	86	860	103	108	120	22
2	30	19	26	36	65	53	233	70	379	87	983	104	0	121	75
3	113	20	114	37	176	54	134	71	44	88	707	105	3110	122	482
4	81	21	325	38	58	55	357	72	129	89	33	106	1247	123	5509
5	115	22	55	39	457	56	193	73	810	90	868	107	943	124	100
6	9	23	242	40	300	57	236	74	290	91	724	108	700	125	10
7	2	24	68	41	97	58	31	75	300	92	2323	109	875	126	1071
8	91	25	422	42	263	59	369	76	529	93	2930	110	245	127	371
9	112	26	180	43	452	60	748	77	281	94	1461	111	729	128	790
10	15	27	10	44	255	61	0	78	160	95	843	112	1897	129	6150
11	138	28	1146	45	197	62	232	79	828	96	12	113	447	130	3321
12	50	29	600	46	193	63	330	80	1011	97	261	114	386	131	1045
13	77	30	15	47	6	64	365	81	445	98	1800	115	446	132	648
14	24	31	36	48	79	65	1222	82	296	99	865	116	122	133	5485
15	108	32	4	49	816	66	543	83	1755	100	1435	117	990	134	1160
16	88	33	0	50	1351	67	10	84	1064	101	30	118	948	135	1864
17	670	34	8	51	148	68	16	85	1783	102	143	119	1082	136	4116

Table 3:4th order cumulative faults and their m(t) successive difference.

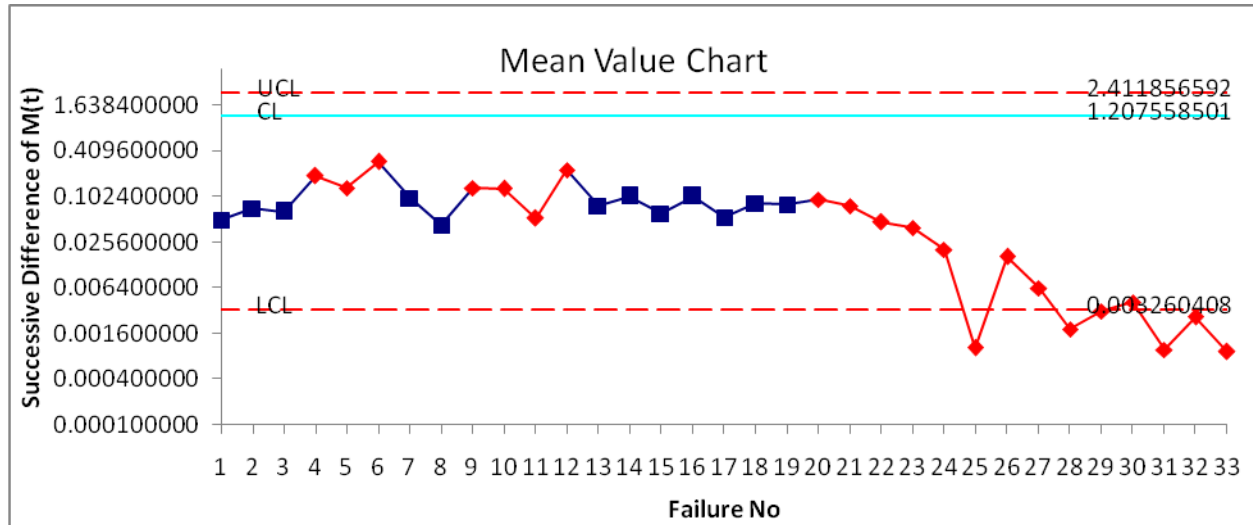| Fault	4-order Cumulative Faults	m(t)	Successive Difference's of m(t)	Fault	4-order Cumulative Faults	m(t)	Successive Difference's of m(t)
1	227	0.053669607	0.050189929	18	16358	1.936901718	0.083134596
2	444	0.103859536	0.070964302	19	18287	2.020036314	0.079829413
3	759	0.174823838	0.064912354	20	20567	2.099865727	0.09363885
4	1056	0.239736192	0.191343658	21	24127	2.193504577	0.077302531
5	1986	0.431079851	0.131004192	22	28460	2.270807108	0.046687097
6	2676	0.562084043	0.296000509	23	32408	2.317494206	0.039546532
7	4434	0.858084551	0.097761832	24	37654	2.357040738	0.020362766
8	5089	0.955846384	0.042703058	25	42015	2.377403504	0.001034693
9	5389	0.998549441	0.132378119	26	42296	2.378438197	0.016427908
1	6380	1.13092756	0.12873383	27	48296	2.394866105	0.006274847
11	7447	1.259661391	0.053077534	28	52042	2.401140952	0.001810038
12	7922	1.312738924	0.22760806	29	53443	2.40295099	0.003163639
13	10258	1.540346985	0.075916166	30	56485	2.406114629	0.004113047
14	11175	1.616263151	0.102288223	31	62651	2.410227676	0.000973225
15	12559	1.718551373	0.061080293	32	64893	2.411200901	0.00261935
16	13486	1.779631666	0.103253053	33	76057	2.413820251	0.000925177
17	15277	1.882884719	0.054016999	34	88682	2.414745429	

Fig-1: Mean Value Chart of 4th order data set

Table: 4: 5th order cumulative faults and their m(t) successive difference

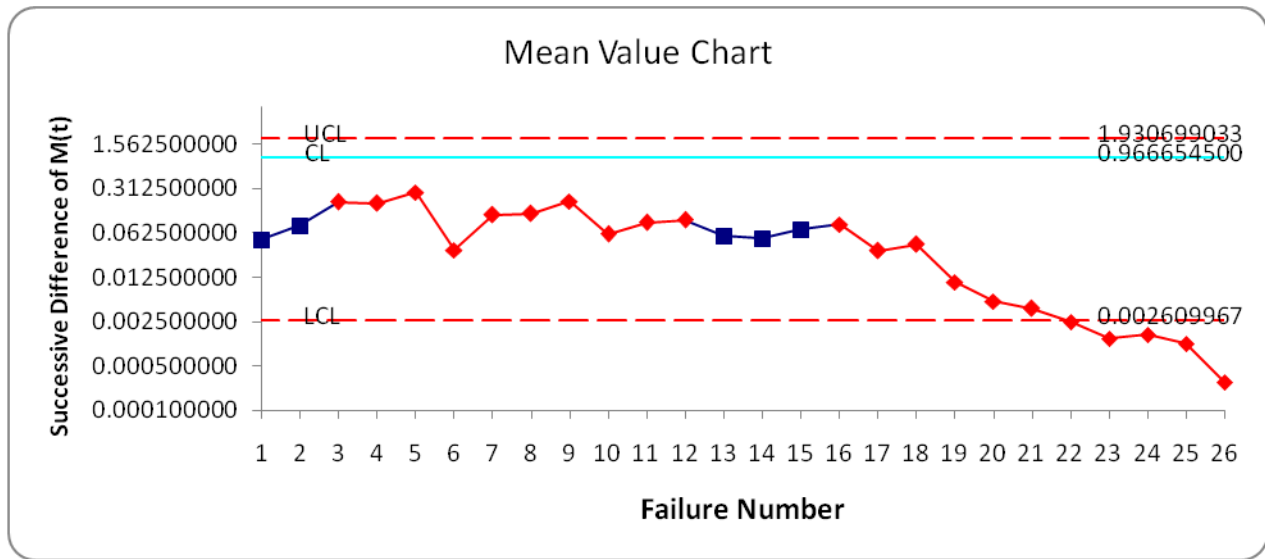| Fault	5-order Cumulative	m(t)	Successive Difference's Of m(t)'s	Fault	5-order Cumulative	m(t)	Successive Difference's Of m(t)'s
1	342	0.073925386	0.04791294	15	17758	1.62782472	0.050149302
2	571	0.121838326	0.080156008	16	20567	1.677974022	0.069965628
3	968	0.201994334	0.189702018	17	25910	1.74793965	0.084558265
4	1986	0.391696352	0.183547444	18	29361	1.832497915	0.032788872
5	3098	0.575243796	0.270820129	19	37642	1.865286786	0.041557817
6	5049	0.846063925	0.033556388	20	42015	1.906844603	0.010389187
7	5324	0.879620314	0.11950945	21	45406	1.91723379	0.005154027
8	6380	0.999129764	0.125362531	22	49416	1.922387817	0.004007073
9	7644	1.124492295	0.196749357	23	53321	1.92639489	0.002484134
10	10089	1.321241652	0.059242997	24	56485	1.928879023	0.001341437
11	10982	1.380484649	0.090964117	25	62661	1.930220461	0.001561037
12	12559	1.471448766	0.100355072	26	74364	1.931781497	0.001125183
13	14708	1.571803838	0.056020882	27	84566	1.93290668	0.00027658
14	16185	0.073925386	0.04791294				

Fig 2: Mean Value Chart of 5th order data set

3. CONCLUSION

The Mean value charts of Fig 1 and 2 have shown out of control signals i.e. below LCL. By observing Mean value charts, we identified that failures situation is detected at an early stages. The early detection of software failure will improve the software reliability. When the control signals are below LCL, it is likely that there are assignable causes leading to significant process deterioration and it should be investigated. Hence, we conclude that our control mechanism proposed in this chapter giving a positive recommendation for its use to estimate whether the process is in control or out of control.

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