

# A New Method for Solving Fully Fuzzy Linear Programming with LR- type Fuzzy Numbers

M. M. Shamooshaki<sup>1</sup>, A. Hosseinzadeh<sup>1</sup>, S. A. Edalatpanah<sup>2,\*</sup>

<sup>1</sup>Department of Mathematics, Comprehensive Imam Hossein University, Tehran, Iran

<sup>2</sup>Department of Applied Mathematics, Tonekabon Branch, Islamic Azad University, Tonekabon, Iran

\*Corresponding author: saedalatpanah@gmail.com

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**Abstract** One of the most practicable subjects in recent studies is based on LR fuzzy number, which was defined and used by Dubois and Prade with some useful and easy approximation arithmetic operators on them. In this paper, a new method is proposed to find the fuzzy optimal solution of fully fuzzy Linear Programming (FFLP) problems with equality constraints. We use also LR-type fuzzy numbers and our model covers a wide class of fuzzy numbers, such as triangular fuzzy numbers. To illustrate our method, a numerical example is solved.

**Keywords:** fully fuzzy linear programming, LR-type fuzzy numbers, ranking function, lexicography

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## 1. Introduction

Linear Programming (LP) is one of the most important fields of operation research that, given its efficiency in different areas, has developed during recent decades. Linear programming is the process of taking various linear inequalities relating to some situation, and finding the "best" value obtainable under those conditions. Linear programming is in two forms: classical linear programming and fuzzy linear programming (FLP) in which the variables are assessed in a fuzzy manner. In conventional approaches, the value of parameters in LP models must be well defined and precise. However, in real world environment, this is not a realistic assumption.

In real world applications, certainty, reliability and precision of data is often illusory. The optimal solution of an LP only depends on a limited number of constraints; therefore much of the collected information has little impact on the solution. It is useful to consider the knowledge of experts about the parameters as fuzzy data. In such a situation the parameters of linear programming problems may be represented as fuzzy numbers (FLP). The FLP problems in which all the parameters as well as the variables are represented by fuzzy numbers are known as fully fuzzy linear programming (FFLP) problems. There are a lot of methods for solving FFLP problems. A max-min operator for converting a fuzzy decision making problem to its crisp equivalent was first proposed by Bellman and Zadeh (1979). The idea was adopted by several authors for solving FLP problems. Rommelfanger *et al.*, (1989), Fang and Hu (1999), Maleki *et al.*, (2000), Maleki (2002) and Khan *et al.*, (2010) are studies where the objective

functions, the decision variables, the technical coefficients and the constraints are fuzzy numbers respectively. Dehghan *et al.* (2006), proposed a fuzzy linear programming approach for finding the exact solution of fully fuzzy linear system of equations (FFLSE). Ebrahimnejad and Nasserri (2009) developed the complementary slackness theorem for solving FLP with fuzzy parameters. Lotfi *et al.* (2009), proposed a method to obtain the approximate solution of FFLP problems. By using their method, the solutions obtained are approximate and also it is very difficult to apply this method to find the fuzzy optimal solution of FFLP problems. Moreover Kumar *et al.* (2011) considered this problem and presented a new method for solving FFLP problems. However this method is limited to a certain class of fuzzy numbers, i.e. triangular fuzzy numbers and needed some corrections made by Saberi Najafi and Edalatpanah (2013).

In this paper, we propose a new method for solving FFLP problems by using LR- type fuzzy numbers, which include a wide class of FFLP problems. Moreover, our method does not have the disadvantages of the above methods and compared with other methods is more efficient.

## 2. Preliminaries

In this section some basic definitions, arithmetic operations and ranking function are reviewed, taken from Zimmermann (1978), Dubois and Prade (1980), Kaufmann (1998), and Kumar (2011).

**Definition 2.1.** (Kaufmann (1998)) The characteristic function  $\mu_M$  of a crisp set  $M \subseteq X$  assigns a value either 0 or 1 to each member in  $X$ . This function can be generalized to a function  $\mu_{\tilde{M}}$  such that the value assigned to the element of the universal set  $X$  fall within a specified

range i.e.  $\mu_{\tilde{M}} : X \rightarrow [0,1]$ . The assigned value indicate the membership grade of the element in the set M. The function  $\mu_{\tilde{A}}$  is called the membership function and the set  $\tilde{M} = \{(x, \mu_{\tilde{M}}(x)); x \in X\}$  Defined by  $\mu_{\tilde{M}}$  for each  $x \in X$  is called a fuzzy set.

**Definition 2.2.** (Kaufmann (1998)) A fuzzy set  $\tilde{M}$ , defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1.  $\mu_{\tilde{M}} : R \rightarrow [0,1]$  is continuous.
2.  $\mu_{\tilde{M}}(x) = 0$  for all  $x \in (-\infty, a] \cup [d, \infty)$ .
3.  $\mu_{\tilde{M}}(x)$  strictly increasing on [a,b] and strictly decreasing on [c, d].
4.  $\mu_{\tilde{M}}(x) = 1$  for all  $x \in [b, c]$ , where  $a < b < c < d$ .

**Definition 2.3.** (Zimmermann (1978)) A function, usually denoted by L or R, is a reference function of a fuzzy number if and only if  $L(x) = L(-x), L(0) = 1$  and L is nonincreasing on  $[0, +\infty)$ .

**Definition 2.4.** (Zimmermann (1978)) A fuzzy number  $\tilde{M}$  is of left, right-type (LR-type), if there exist reference functions L (for left), R (for right), and scalars  $\alpha > 0, \beta > 0$  with

$$\mu_{\tilde{M}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leq m \\ R\left(\frac{x-m}{\beta}\right), & x \geq m \end{cases}$$

The variable (m) is called value of  $\tilde{M}$ , which is a real number and  $\alpha$  and  $\beta$  are called the left and right spreads, respectively and  $\tilde{M}$  is denoted by  $(m, \alpha, \beta)_{LR}$ .

**Definition 2.5.** (Kumar (2011)) Two LR-type fuzzy numbers  $\tilde{M} = (m, \alpha, \beta)_{LR}$  and  $\tilde{N} = (n, \gamma, \delta)_{LR}$  are said to be equal if and only if  $m = n$  and  $\alpha = \gamma$  and  $\beta = \delta$ .

**Theorem 2.6.** (Kumar(2011)) Let  $\tilde{M} = (m, \alpha, \beta)_{LR}$ ,  $\tilde{N} = (n, \gamma, \delta)_{LR}$  be two fuzzy numbers of LR-type. Then, we have:

- (1)  $(m, \alpha, \beta)_{LR} \oplus (n, \gamma, \delta)_{LR} = (m+n, \alpha+\gamma, \beta+\delta)_{LR}$ ,
- (2)  $-(m, \alpha, \beta)_{LR} = (-m, \beta, \alpha)_{RL}$ ,
- (3)  $(m, \alpha, \beta)_{LR} \ominus (n, \gamma, \delta)_{RL} = (m-n, \alpha+\delta, \beta+\gamma)_{LR}$ .

**Definition 2.7.** (Dubois and Prade (1980)) A fuzzy number  $(m, \alpha, \beta)_{LR}$  is said to be non-negative fuzzy number if and only if  $m - \alpha \geq 0$ .

**Definition 2.8.** (Dubois and Prade (1980)) A fuzzy number  $(m, \alpha, \beta)_{LR}$  is said to be non-negative fuzzy number if and only if  $m - \alpha \leq 0$ .

**Theorem 2.9.** (Kumar (2011)) Let  $\tilde{M} = (m, \alpha, \beta)_{LR}$ ,  $\tilde{N} = (n, \gamma, \delta)_{LR}$  be two fuzzy numbers of LR-type. Then, we have:

- 1) For  $\tilde{M}, \tilde{N}$  are positive,

$$(m, \alpha, \beta)_{LR} \otimes (n, \gamma, \delta)_{LR} \approx (mn, m\gamma + n\alpha, m\delta + n\beta)_{LR}$$

- 2) For  $\tilde{M}$  negative,  $\tilde{N}$  positive,

$$(m, \alpha, \beta)_{RL} \otimes (n, \gamma, \delta)_{LR} \approx (mn, n\alpha - m\delta, n\beta - m\gamma)_{RL}$$

- 3) For  $\tilde{M}, \tilde{N}$  are negative,

$$(m, \alpha, \beta)_{LR} \otimes (n, \gamma, \delta)_{LR} \approx (mn, -m\delta - n\beta, -m\gamma - n\alpha)_{RL}$$

**Definition 2.10.** A ranking function (combining Zimmermann, (1978) and Kumar et.al (2011)), is a function  $\mathfrak{R} : F(R) \rightarrow R$ , where F(R) is a set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into a real line, where a natural order exists. Let  $\tilde{M} = (m, \alpha, \beta)_{LR}$  be a LR-type fuzzy number, then

$$\mathfrak{R}(\tilde{M}) = m + \frac{\beta - \alpha}{4}$$

where is equal with

$$\mathfrak{R}(\tilde{M}) = \int_{-\infty}^{\infty} x \mu_{\tilde{M}}(x) dx / \int_{-\infty}^{\infty} \mu_{\tilde{M}}(x) dx$$

### 3. FFLP Problem Formulation and Proposed Method for Solution

FFLP problems with  $m$  fuzzy equality constraints and  $n$  fuzzy variables can be formulated as follows:

$$\begin{aligned} &\text{Max (or Min) } \tilde{C}^T \otimes \tilde{x}, \\ &\text{s.t. } \tilde{M} \otimes \tilde{x} = \tilde{b}, \end{aligned} \tag{3.1}$$

$\tilde{x}$  is non-negative fuzzy number.

We know that,  $\tilde{c}^T = [\tilde{C}_j]_{1 \times n}$ ,  $\tilde{x} = [\tilde{X}_j]_{n \times 1}$ ,  $\tilde{M} = [\tilde{m}_{ij}]_{m \times n}$ ,  $\tilde{b} = [\tilde{b}_i]_{m \times 1}$ , and  $\tilde{C}_j, \tilde{X}_j, \tilde{m}_{ij}, \tilde{b}_i \in F(R)$  are all LR-type fuzzy numbers. Thus, FFLP (3.1) can be written as:

$$\begin{aligned} &\text{Max(or Min) } \sum_{j=1}^n (c_j, \xi_j, \zeta_j)_{LR} \otimes (x_j, \alpha_j, \beta_j)_{LR}, \\ &\text{s.t.} \\ &\sum_{j=1}^n (m_{i,j}, \delta_{i,j}, \gamma_{i,j})_{LR} = (b_i, \rho_i, \eta_i)_{LR} \quad \forall i = 1, 2, \dots, m, \end{aligned} \tag{3.2}$$

$(x_j, \alpha_j, \beta_j)_{LR}$  is non-negative fuzzy number.

where

$$\tilde{c}_j = (c_j, \xi_j, \zeta_j)_{LR}, \tilde{b}_i = (b_i, \rho_i, \eta_i)_{LR},$$

$$\tilde{m}_{ij} = (m_{ij}, \mu_{ij}, \nu_{ij})_{LR}$$

and

$$(n_{i,j}, \delta_{i,j}, \gamma_{i,j})_{LR} = (m_{ij}, \mu_{ij}, \nu_{ij})_{LR} \otimes (x_j, \alpha_j, \beta_j)_{LR}$$

We assume that all coefficient fuzzy numbers are positive. The similar procedure can be performed for the other cases.

Now, the steps of the proposed method are as follows:

**Step 1.** By using Theorem (2.9) we first calculate multiplication of two fuzzy numbers and drawing on Definition (2.5), then by using Definition (2.10), we convert the FFLP problem (3.2) into the following linear programming problem:

$$\text{Max(or Min)} \mathfrak{R} \left( \sum_{j=1}^n (c_j, \xi_j, \zeta_j)_{LR} \otimes (x_j, \alpha_j, \beta_j)_{LR} \right).$$

s. t.

$$\begin{aligned} \sum_{j=1}^n (n_{ij}) &= b_i \quad \forall i = 1, 2, \dots, m, \\ \sum_{j=1}^n (\delta_{ij}) &= \rho_i \quad \forall i = 1, 2, \dots, m, \\ \sum_{j=1}^n (\gamma_{ij}) &= \eta_i \quad \forall i = 1, 2, \dots, m, \\ x_j, \alpha_j, \beta_j &\geq 0 \quad \forall i = 1, 2, \dots, m. \end{aligned} \quad (3.3)$$

**Step 2.** Find the optimal solution of (3.3), say  $x_j^*, \alpha_j^*$  and  $\beta_j^*$ .

**Step 3.** Find the fuzzy optimal solution of FFLP problem (3.2), by putting  $\tilde{x}_j^* = (x_j^*, \alpha_j^*, \beta_j^*)_{LR}$ .

**Step 4.** Find the fuzzy optimal objective value by putting  $\tilde{x}_j^*$  in:

$$\sum_{j=1}^n (c_j, \xi_j, \zeta_j)_{LR} \otimes (x_j^*, \alpha_j^*, \beta_j^*)_{LR}.$$

Below, we solve an example which was already solved by Kumar et al. (2011) using our method with Mathematica 9 software.

**Example 3.1.**

Let us consider the following FFLP and solve it by the proposed method

$$\text{Max(or Min)} ((2, 1, 1)_{LR})\tilde{x}_1 \oplus ((3, 1, 1)_{LR})\tilde{x}_2$$

s.t.

$$\begin{aligned} ((1, 1, 1)_{LR})\tilde{x}_1 \oplus ((2, 1, 1)_{LR})\tilde{x}_2 &= (10, 8, 14)_{LR} \\ ((2, 1, 1)_{LR})\tilde{x}_1 \oplus ((1, 1, 1)_{LR})\tilde{x}_2 &= (8, 7, 13)_{LR} \end{aligned}$$

$\tilde{x}_1$  and  $\tilde{x}_2$  are non-negative LR-type fuzzy numbers,

In view of (3.3) and definition (2.10) and, we can write this FFLP as :

$$\text{Max } 2x_1 + 3x_2 + \frac{1}{4}(2(\beta_1 - \alpha_1) + 3(\beta_2 - \alpha_2))$$

s.t.

$$\begin{aligned} x_1 + 2x_2 &= 10 \\ x_1 + x_2 + \alpha_1 + 2\alpha_2 &= 8 \\ x_1 + x_2 + \beta_1 + 2\beta_2 &= 14 \\ x_1 + x_2 &= 8 \\ x_1 + x_2 + 2\alpha_1 + \alpha_2 &= 7 \\ x_1 + x_2 + 2\beta_1 + \beta_2 &= 13 \\ x_1, x_2, \alpha_1, \alpha_2, \beta_1, \beta_2 &\geq 0 \end{aligned}$$

The optimal solution of the LP problem is  $x_1 = 2$ ,

$$\alpha_1 = 0, \beta_1 = \frac{10}{3}, x_2 = 4, \alpha_2 = 1, \beta_2 = \frac{7}{3}.$$

Also, the optimal solution of the FFLP problem, obtained in Step 3, is  $\tilde{x}_1 = (2, 0, 2)_{LR}$  and  $\tilde{x}_2 = (4, 1, 3)_{LR}$ . Step 4 yields the optimal fuzzy objective value to be  $(16, 9, \frac{37}{2})_{LR}$ .

This problem is also solved by Kumar et.al (2011), for which the authors implemented the triangular fuzzy numbers. However, our method permits a great choice of auxiliary reference functions  $L$  and  $R$ , which covers the triangular fuzzy numbers as a special case. The optimal solution reached by Kumar was 17, while our optimal solution is 18.375. Our proposed method is also easy to implement compared to Dehghan et.al (2006), and Lotfi et.al(2009).

## 4. Conclusions

In this paper, a new method is developed to solve FFLP problems. This method is based on ranking function on LR -fuzzy numbers. We also solved a problem using rank function and mathematica softwar which was already solved by Kumar using TFR's. We observed, our method is far more efficient.

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