Final Year Project

The Mechanics of the Trebuchet

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1. Abstract

The trebuchet was an immense counterpoise siege engine that became the artillery weapon of choice throughout the middle ages. This project's aim was to model the mechanics of one such engine by use of an Euler method as opposed to the more common Lagrangian form. While the mechanics of second complexity level have been modelled quite successfully using Excel, once further complexity and greater degrees of freedom are introduced the difficulty increases substantially. For the basic seesaw, the optimum conditions were found to be a 4:1 arm length ratio, a 100:1 mass ratio, and an angle of release of approximately 37.49º. Preliminary results have also been acquired for the more complicated trebuchet with sling model, these are also presented here.

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2. Introduction

The trebuchet is a specific type of siege engine, the next evolutionary stage in the generic term of catapult. While previous catapults, such as the onager, were a family of torsion siege engines (i.e. they relied upon stored energy in the form of tension for their power, much like a bow) the trebuchet and its direct predecessor, the mangonel, were counterpoise engines, drawing their power from a counterweight.

As their name suggests, "siege engines" had the sole purpose of breaking down castle walls, and consequentially would be required to launch vast projectiles towards the target. Not only this, but their range would also need to be greater than that of a defending archer, lest the operators be in danger. *"Modern experiments suggest that a trebuchet, the most advanced medieval siege engine, with an arm of approximately 15m in length would have been capable of throwing a 135kg projectile a distance of 275m, this would require a counterpoise of approximately 10 tonnes"* [3].

The primary advantages of the trebuchet were its accuracy and relative rate of fire. A weapon may be capable of launching immense missiles, but if it cannot be aimed it will prove highly unsuccessful. The trebuchet was, intrinsically, no more accurate than other siege engines. However, the motion of fire was so smooth that after launch it remained stationary, allowing the engineers to adjust the trajectory. This was a significant improvement on previous engines, most notably the onager whose very name translates to "*Wild Ass*" [4].

The trebuchet is very much in vogue at present, with numerous websites [14] featuring building varying scales of replicas, to computer modelling and engineering competitions. The machine has even made it so far as Hollywood with trebuchets having a prominent (for a machine at least) role in several feature films, namely *Lord of the Rings: The Return of the King* and *Kingdom of Heaven*, as shown in Figures 1 and 2 respectively.

Figure 1: Trebuchet depicted in LotR [15]

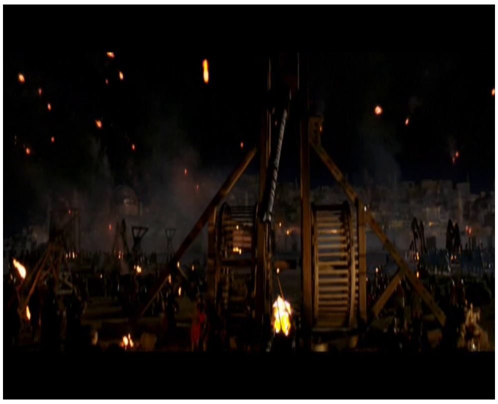

Figure 2: Trebuchet shown in KoH [16]

Due to the machine's popularity, many educational organisations have constructed their own models for various Design and Physics lessons. Figure 3 is of the trebuchet from

Charterhouse School, Godalming. The key points and distances have been labelled, and this is the convention for notation throughout this project.

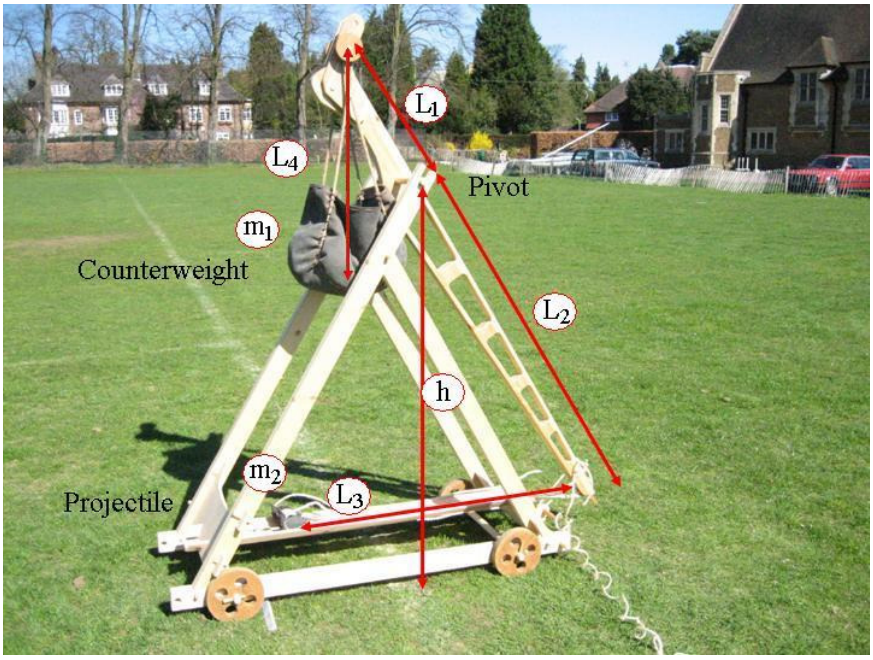

Figure 3: The Charterhouse trebuchet; where L_1 and L_2 are the distances from the pivot to the counterweight and projectile slings respectively, L_3 and L_4 the sling lengths for the counterweight and projectile respectively, h the height of the pivot off the ground and m_1 and m_2 the counterweight and projectile masses respectively.

3. Modelling

The computational modelling stages have attempted to recreate each machine's performance; the only exception is that forces of friction have been neglected. In this way the full potential of the machine could be examined, looking at the "best case" scenario for each trebuchet. While friction was ignored on the modelling of the trebuchet, the effects of drag were taken into account when calculating the range from the launch velocity and angle of release provided.

The calculation steps are described in Figure 4 and the spreadsheets themselves are presented as appendices A-C.

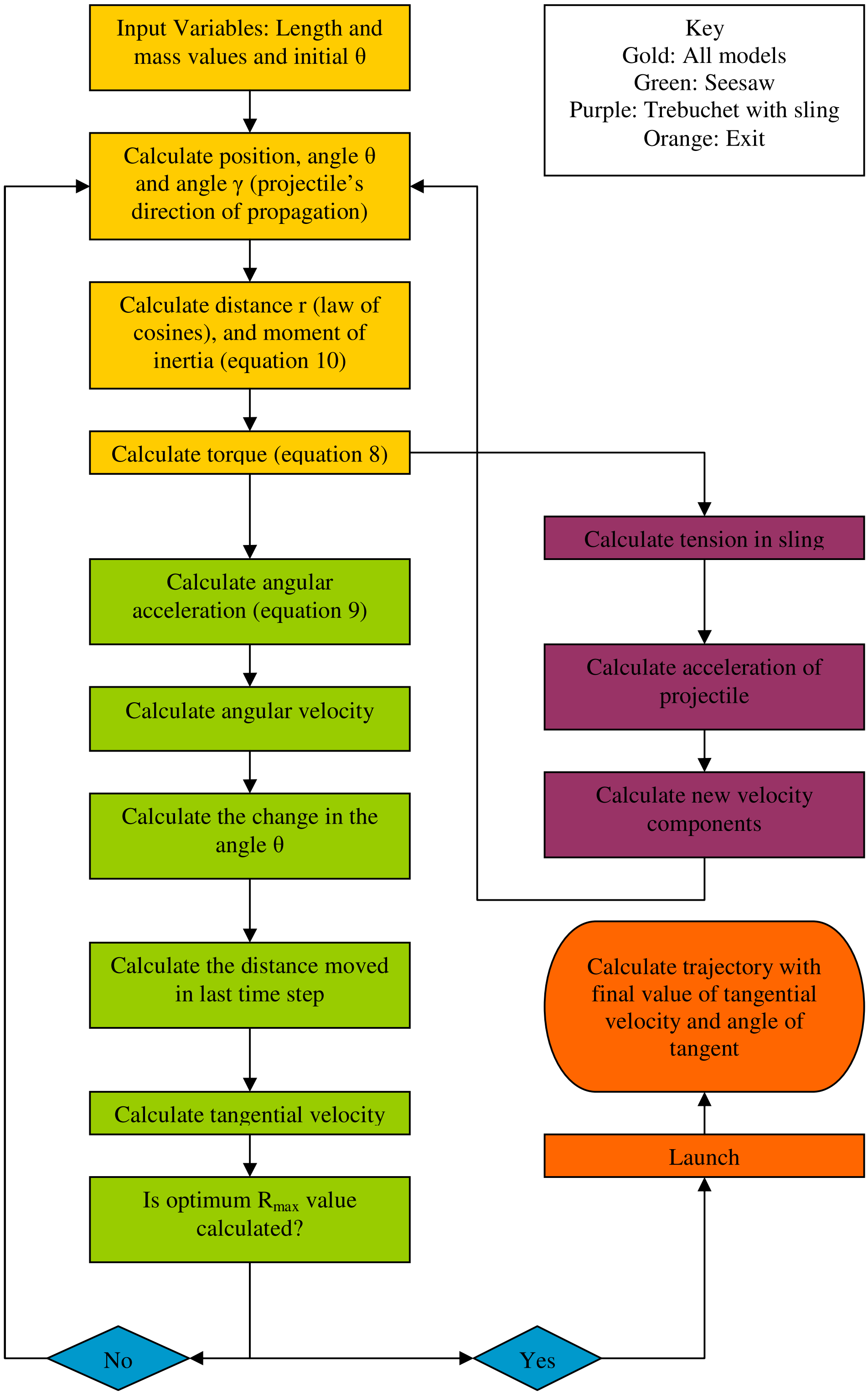

Figure 4: Graphical representation of the calculation steps as used in appendices A & B, calculations with unspecified equations are trivial. The final calculated values of velocity and release angle are then used with the calculations shown in appendix C.

3.1 The Perfect Engine

A catapult, that is, the generic term for medieval artillery, may be modelled with varying degrees of complexity. Specifically here counterpoise engines were examined, those that derive their power from using a counterweight. The counterweight has a certain amount of gravitational potential energy; a portion of this is then converted into kinetic energy for the projectile. The most perfect siege engine would convert 100% of the counterweight's potential energy into kinetic energy for the projectile using Equation 1.

$$m_1 g\Delta h = \frac{1}{2} m_2 v^2$$ **Equation 1**

This is the best possible situation for the projectile. If the siege engine's range is considered, again assuming perfect conditions, i.e. no air resistance, then the maximum range is achieved if this 100% conversion of energy is launched at a release angle, γ, of 45°, as is shown in Figure 5.

Figure 5: Trajectory plot of projectile with no air resistance[i], where v is the launch velocity, and the angle of release is represented by the angle γ

The flight time, t[ii], can be calculated with Equation 2:

$$t = \frac{v \sin \gamma}{g}$$ **Equation 2**

The maximum range is then given by Equation 4:

$$R_{\max} = v_x t$$ **Equation 3**

$$R_{\max} = \frac{2v^2 \sin \gamma \cos \gamma}{g}$$ **Equation 4**

Then using the launch velocity, v, from re-arranging Equation 1, the equation becomes:

$$R_{\max} = \frac{4m_1 \Delta h \sin \gamma \cos \gamma}{m_2}$$ **Equation 5**

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[i] Note that trajectory is symmetrical where air resistance is not taken into account
[ii] Where t is the time to reach maximum height and return to ground

The maximum range is achieved when the launch angle, γ, is 45º, consequentially Equation 5 for R_{max} becomes:

$$R_{\max} = \frac{2m_1 \Delta h}{m_2} \qquad \textbf{Equation 6}$$

It should be noted that for the perfect engine, the maximum available potential energy is converted into kinetic energy, thereby achieving the highest possible launch velocity. However, for other machines, a compromise between launch velocity and angle of release must be considered and hence Equation 5 is the more accurate. This is explained more fully in section 3.2.

The interesting point here is that for maximum range, only the relative masses are required, and that the length of the beam is irrelevant. This would seem to imply that the mass ratio determines what range can potentially be achieved, leaving the characteristics of the trebuchet itself, i.e. the arm length ratio, to determine the efficiency of the machine.

The results in Table 1 display the maximum range for perfect versions of the engines considered at later stages of modelling. The “Britannica” trebuchet is looking at a real trebuchet which would fire a projectile of 135kg. The “Standard” trebuchet was determined through examining the optimal conditions in section 3.2. The “Charterhouse” series is concerned with the Charterhouse trebuchet in section 4 and the different combinations of counterweight and projectile masses available. The purpose is to provide a comparison value and thus establish the efficiency of engine in each stage of development.

Trebuchet	Mass Ratio	Maximum Range (m)
Britannica	100:1.35	628.54
Standard	100:1	282.84
Charterhouse 1	75:1	94.40
Charterhouse 2	100:1	125.87
Charterhouse 3	125:1	157.33
Charterhouse 4	26:1	32.83
Charterhouse 5	35:1	43.78
Charterhouse 6	43:1	54.72

Table 1: Perfect Engine R_{max} results

Perfect conditions are unlikely; there will be friction to take into account at each point of contact, there is recoil present, due to unused kinetic energy and air resistance should be taken into account. Therefore, a siege engine’s effectiveness really depends on its efficiency. How much of that gravitational potential energy is converted into kinetic energy for the projectile? As far as modelling is concerned, the complexities lie purely in the calculation of the projectile’s launch angle and velocity. Once these values are known it is relatively straightforward to calculate the projectile’s range. Of course, greater accuracy can be introduced at this stage too; the projectile’s trajectory itself can be calculated, taking into account the drag effective on the projectile.

3.1.1 Efficiency Calculations

The perfect engine calculations provide the maximum possible range for a set of counterweight and projectile masses by analysing the respective energies. It is these values that will be used to calculate a specific trebuchet's efficiency with Equation 7.

$$\varepsilon = \frac{R_{calc}}{R_{\max}} \times 100$$ **Equation 7**

Where the efficiency of an engine, ε, is proportional to the range calculated by that trebuchet complexity level, R_{calc}, and inversely proportional to the range of those masses in the perfect engine, R_{max}. The efficiency is converted into a percentage for ease of analysis. For experimental results, the calculated range, R_{calc}, is merely replaced by a measured range, R_{meas}. This method is used throughout the project to calculate efficiency.

3.2 The Seesaw Trebuchet

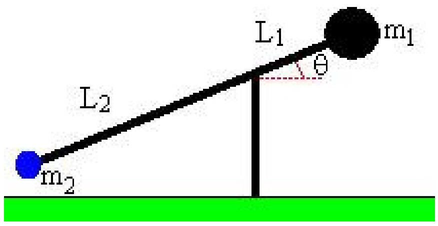

Figure 6: The Seesaw Trebuchet

The most basic trebuchet is based on a simple seesaw, shown in Figure 6. The counterweight positioned at one end of the beam and the projectile on the other. By calculating the torque produced on the arm by each mass, the angular acceleration can therefore be calculated. As the torque of each mass is produced by the effect of gravity on them, the forces on the arm will vary throughout the beam's motion, and consequentially so too, will the torque and angular acceleration.

$$\tau = m_1 g l_1 \cos\theta - m_2 g l_2 \cos\theta$$ **Equation 8**

$$\alpha = \frac{\tau}{I}$$ **Equation 9**

$$I = \sum_{i=1}^{N} m_i r_i^2$$ **Equation 10**

In this case the net torque, τ, is due to the difference between the torques of the masses, Equation 8. The angular acceleration, α, requires the net torque, τ, and the moment of inertia, I, to be known. A system's moment of inertia being the sum of all the component masses, m_i, present in that system multiplied by the square of their respective distances from the rotational axes, r_i^2.

Due to the use of an Euler method, it may be assumed that the equations of motion will hold true for each individual time step. Although the acceleration of the projectile will vary on its path until launch, each time step is small enough[iii] that it should be acceptable to assume the acceleration remains constant for that brief period. As the projectile progresses along its arc, a tangential velocity, or launch velocity for a given release angle, is continually calculated.

Based on the background research [2, 3, 11], the trebuchet engineers were able to crudely determine the angle of release, therefore an assumption has been introduced that the release angle can be stated. In this way the variables are reduced and the launch velocity is simply read off when the angle of release, γ, has been reached.

3.2.1 Investigating Arm length Ratio

This being one of the initial tests of the equations, performed at the start of the project, keeping the numbers simple and the quantity of results data to analyse minimal, seemed appropriate. The choice of a 10:1 mass ratio was purely arbitrary. Hence a 10kg counterweight and a 1kg projectile were used. This can be seen in Figure 7.

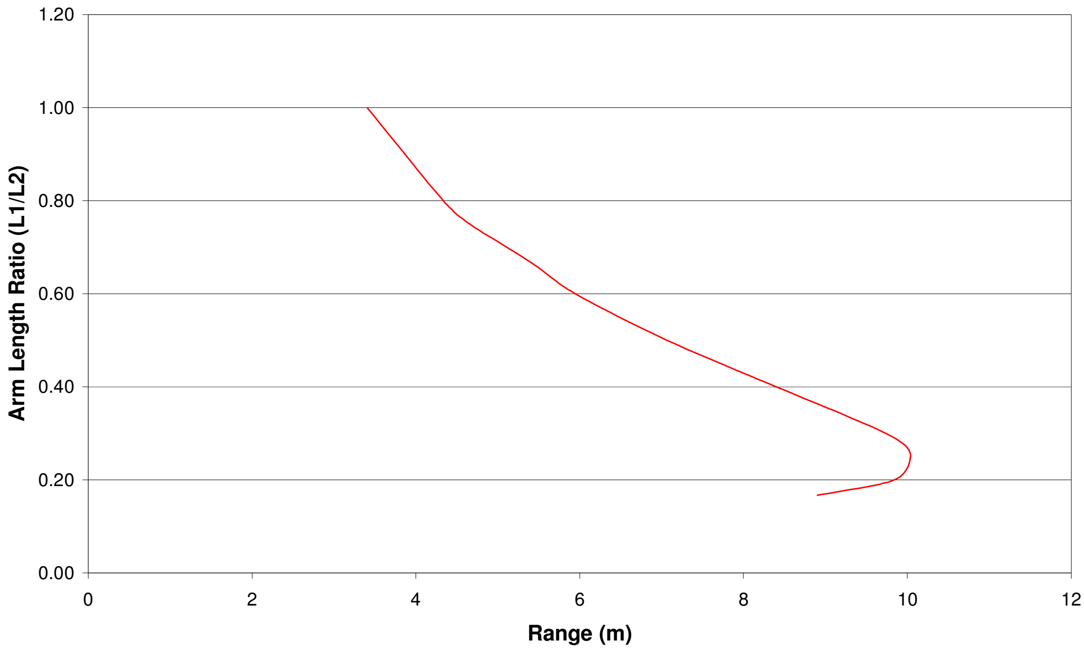

Figure 7: The effect of arm length ratio on projectile range

From this point on the standard set-up for my modelled trebuchet was to have the optimum 4:1 arm length ration. Using this new standard, the effect of the respective masses was also considered.

[iii] For the trebuchet spreadsheets Δt was taken as 0.0005s

3.2.2 Investigating Counterweight to Projectile Mass Ratio

Various counterweights were used for a projectile of 1kg; Figure 8 displays the results of the investigation into optimum mass ratio conditions.

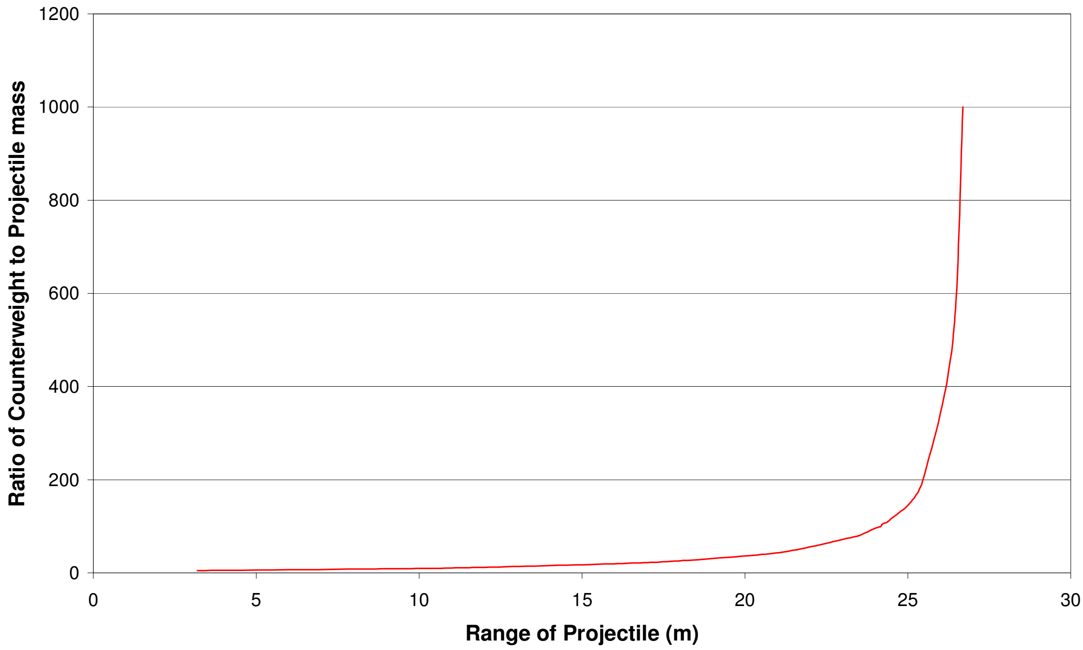

Figure 8: The effect of mass ratio on projectile range

From calculations using the Perfect Engine model, it was expected that the range would simply be a linear relation with the mass ratio. As Figure 8 depicts, initially this is true, a small increase in counterweight mass will have a large effect on the range. There comes a point where this no longer applies, and it takes a huge increase in counterweight mass to have any significant change in range[iv]. The new standard values will therefore be a counterweight of 100kg and a projectile of 1kg. This mass ratio is comparable to that investigated in "*Siege engine dynamics*" [2] where a counterweight mass of 10000kg and a projectile mass of 100kg were used.

3.2.3 Investigating the Optimum Launch Angle

The next stage of the project was to find the optimum launch angle for a seesaw trebuchet. Using the now established values for L_1, L_2, m_1 and m_2 the release angle will be varied in order to find the effect this has on the range. From the perfect engine, there is the statement that the optimum angle of release is 45°, thus providing the maximum range. However, now a real engine is considered, the angle of release and launch velocity become inextricably linked. The further the distance which the projectile has to accelerate over, the greater its tangential velocity. Thus the maximum tangential velocity will be when the counterweight is at the lowest point on its arc, i.e. the beam is vertical.

[iv] Note that mass ratio and range still remain proportional

However, the angle of release at this point will be 0°; obviously this will not provide the maximum range. So, there must be some optimum "compromise" where the angle of release is closer to 45° but the launch velocity is still high. These results are shown in Figure 9.

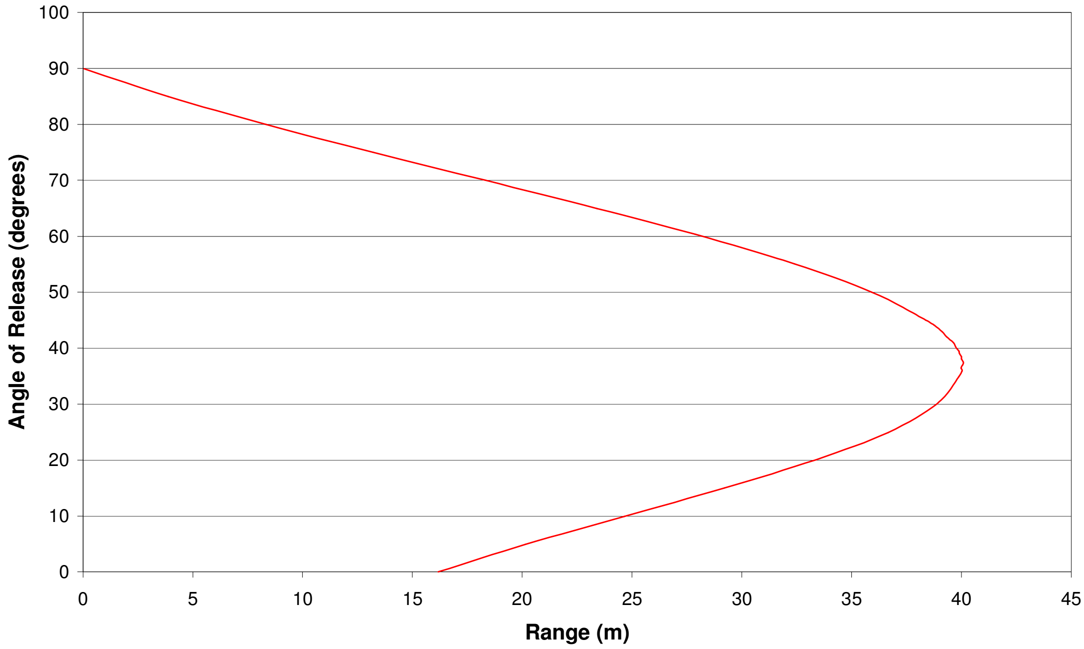

Figure 9: Determining the optimum launch angle

The optimum range is not achieved when the theoretical optimum angle of 45° is used but it is found to be approximately 37.49°. This situation reaching the best compromise between angle of release and launch velocity.

3.2.4 Investigating Efficiency

There are obviously several points that will cause the efficiency to decrease. The perfect engine has the benefit of having both the maximum velocity and the optimum launch angle with which to acquire its maximum range. The seesaw trebuchet must make a compromise between the two. However, there is a discrepancy between the maximum range for the perfect engine and the maximum range if the seesaw was a perfect engine. As well as the variation of velocity with release angle there is another variable to take into account.

For the perfect engine the value of Δh remains constant for a fixed L_1. However, as the release angle for the seesaw trebuchet varies depending on the optimal conditions, Δh is not necessarily fixed. A release angle of less than 45° will mean that the value of Δh must be greater than for the equivalent perfect engine. This discrepancy equates to an increase of approximately 2.5% in the maximum range, and it is these increased ranges that are used for the efficiency calculations. The results of this compromise can be seen in Table 2.

Seesaw Trebuchet	Range Calculated with Seesaw model (m)	Perfect Engine Maximum Range (m)	Efficiency (%)
Britannica	115.59	645.25	17.91
Standard	38.88	290.36	13.39
Charterhouse 1	11.08	96.91	11.43
Charterhouse 2	11.52	129.21	8.92
Charterhouse 3	11.72	161.51	7.26
Charterhouse 4	8.89	33.71	26.37
Charterhouse 5	9.68	44.94	21.54
Charterhouse 6	10.19	56.18	18.14

Table 2: The Efficiency of the Seesaw Trebuchet

The modelled seesaw trebuchet is quite inefficient. It is interesting to note the quite extensive variation of efficiency for the "Charterhouse series", where the only variables are the two masses. There appears to be a relation that as the ratio between the counterweight and projectile masses increase, the efficiency of the machine decreases. This is especially apparent when only considering the Charterhouse series. The results in Figure 8 would seem to corroborate this hypothesis, where the benefit due to an increased mass ratio is only applicable to a certain degree. Once the region of 100-200:1 is reached the benefit of a further increase in mass ratio is negligible.

3.3 Calculating the Projectile Trajectory

The calculation for R_{max}, using Equation 5, while perfectly valid, does not take into account any resistive forces, i.e. drag on the projectile due to air. Once the trebuchet model has calculated a launch velocity and angle of release, it is then possible to plot an accurate trajectory including the effects of air resistance.

An Euler method was used to calculate the trajectory, recording a position and velocity components at each time interval. By this method we consider that the projectile's acceleration varies throughout its flight. Trajectory models which neglect the effect of air resistance show only a variation in the vertical velocity of the projectile, from the initial v_y to zero before returning to v_y, meanwhile, the horizontal component of velocity, v_x, would remain constant. Successive recalculation of the horizontal and vertical velocity components as well as the direction of propagation of the projectile was required. As stated in section 3.2, the use of an Euler method allows the use of the equations of motion despite acceleration not being constant over the entire system.

The trajectory model has introduced the effect of air resistance as an additional acceleration on the projectile. This acceleration is calculated through Newton's second law with the resistive force given by the drag equation shown below:

$$D = \frac{\rho v^2 A C_d}{2} \qquad \textbf{Equation 11}$$

Equation 11 [8, 9] relies heavily on the projectile's velocity, v. Obviously a faster moving projectile must move through more air per second, and thus the force of drag will be higher than for a slower moving projectile. The value of the density of air, ρ, was taken to be 1.225kgm^{-3} [v][12]. The reference area, A, is not necessarily equal to the cross section, however, as it is related to it, this seemed an acceptable assumption to make. The drag coefficient is a specific value for a shape. Typically, a sphere has a drag coefficient of 0.47 [10], which is the value used for the calculations.

The height of the projectile's launch was also incorporated into the calculations, an improvement on the R_{max} equation, which assumes that the projectile's initial and final position y-component are equal to zero.

To find the optimum conditions, v and γ, Equation 5 was used. The R_{max} value was calculated for each iteration and then the maximum value was found. The launch angle and angle of release corresponding to this maximum R_{max} value were then entered into the trajectory model in order to calculate the true flight path.

3.4 Trebuchet with Hinged Counterweight

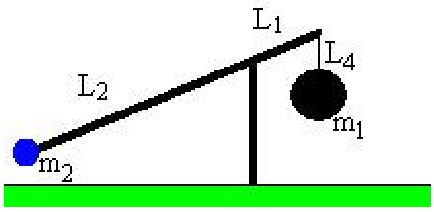

Figure 10: The trebuchet with hinged counterweight

This next model would include the addition of a hinged counterweight suspended from the end of the beam L_1, on a sling of length L_4. As torque is proportional to the mass' perpendicular distance from the pivot, the effect of this additional degree of freedom is that as the distance from the counterweight to the pivot now increases throughout the trebuchet's motion, so too, will the torque.

This should counteract the reduction in torque and, consequentially, angular acceleration once the beam has passed the maximum value at the horizontal. There will still be a factor increasing the trebuchet's torque.

Although this extra torque will naturally increase the range of the trebuchet, the improvement gained from the hinged counterweight should be significantly less than that gained by introducing a sling. Therefore, rather than focusing on one minor aspect, work proceeded onto the next stage of the project.

[v] At 288.16K density of air is 1.2250514kgm^{-3} [12]

3.5 The Trebuchet with Sling

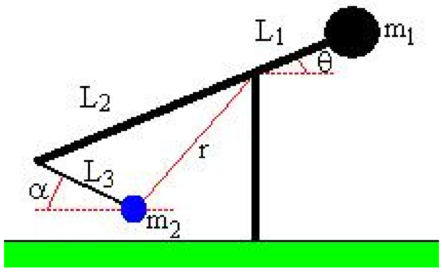

Figure 11: The trebuchet with sling

With the introduction of the sling, demonstrated in Figure 11, the projectile no longer follows the straightforward circular arc of the beam. Initially the projectile is constrained to a trough until there is a great enough vertical component of tension to counteract the force due to gravity, the projectile's weight. The projectile then begins to follow an arc before it is snapped back before being launched, as described in Figure 12.

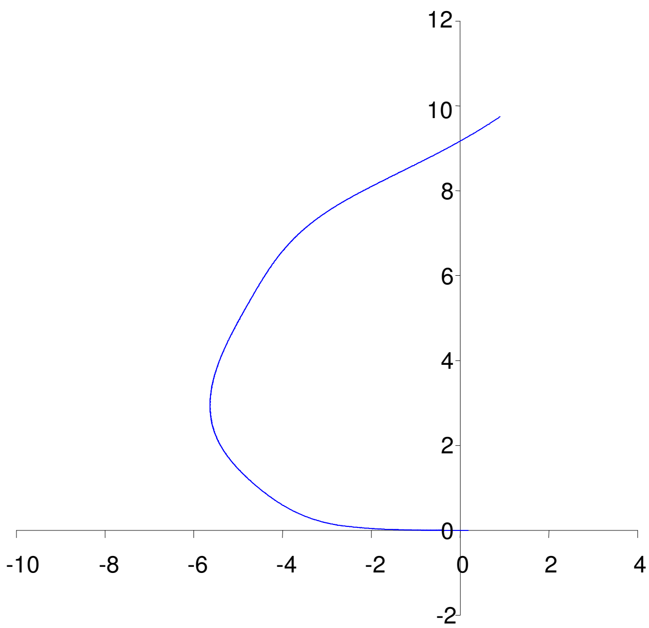

Figure 12: Plot of projectile around trebuchet with sling

To further compound the problems, there is no longer the situation of a straight L_2:L_1 ratio. A new value must be considered, the distance from the pivot to the projectile, r, as shown in Figure 11. Any factor dependant upon the projectile's distance from the pivot, will now vary throughout the projectile's motion about the trebuchet. This effectively replaces L_2, certainly with regards to the moment of inertia and therefore the subsequent values calculated. The distance r will now be changing throughout the projectile's motion along its path, from a minimum distance of $L_2 - L_3$, to a maximum of $L_2 + L_3$.

3.5.1 The Angle α

The complexity with this model was in calculating the position of the projectile once it had left the trough. It was necessary to calculate the height, A_y, in order to calculate α. However, once α reaches 90°, i.e. the sling becomes vertical, due to using the sine function there resulted an error. This was solved by calculation instead through the tangent function.

3.5.2 The Effect of the Sling

The addition of a sling not only introduces another degree of freedom, but also increases the number of variables. The effect of the sling length, L_3, on the system must be considered. The results in Table 3 are for a trebuchet with the standard values as defined in section 3.2, but with the addition of a sling of various lengths.

Trebuchet	Sling length (m)	Range calculated with sling model (m)	Perfect Engine maximum Range (m)	Efficiency (%)
Standard 1	3	210.92	290.36	72.64
Standard 2	2.5	195.85	290.36	67.45
Standard 3	3.5	202.94	290.36	69.89
Standard 4	2	141.77	290.36	48.83

Table 3: Effect of sling length on trebuchet efficiency

Again there would appear to be an optimum condition for the relative length of L_3, although to truly analyse this characteristic of the trebuchet it is necessary to have a generic spreadsheet for the trebuchet with sling and to get results for more combinations.

4. Experimental Results

The Charterhouse Trebuchet

A trebuchet housed at Charterhouse School, Godalming, Surrey was available for experiments. The trebuchet available was slightly more complex than those modelled, having an additional degree of freedom in the use of a hinged counterweight, although the effects of friction will outweigh any benefit. Still frames of video taken of this experiment are included in Appendix D.

The arm length ratio was different to the established standard of 4:1, instead, the ratio of $L_2:L_1$ for the Charterhouse trebuchet was 3:1. Two separate projectiles were used during the experiment, a hockey ball (m_2 = 158g) and a croquet ball (m_2 = 463g). These were each tested with counterweight masses of 12, 16 and 20 kg[vi]. This provided counterweight to projectile mass ratios of approximately 75:1, 100:1 and 125:1 for the lighter projectile, and 26:1, 35:1 and 43:1 for the heavier projectile.

Each set of variables was tested 5 times, to detect anomalous results and to test the trebuchet's famed accuracy. From operating the machine its smoothness of motion was apparent.

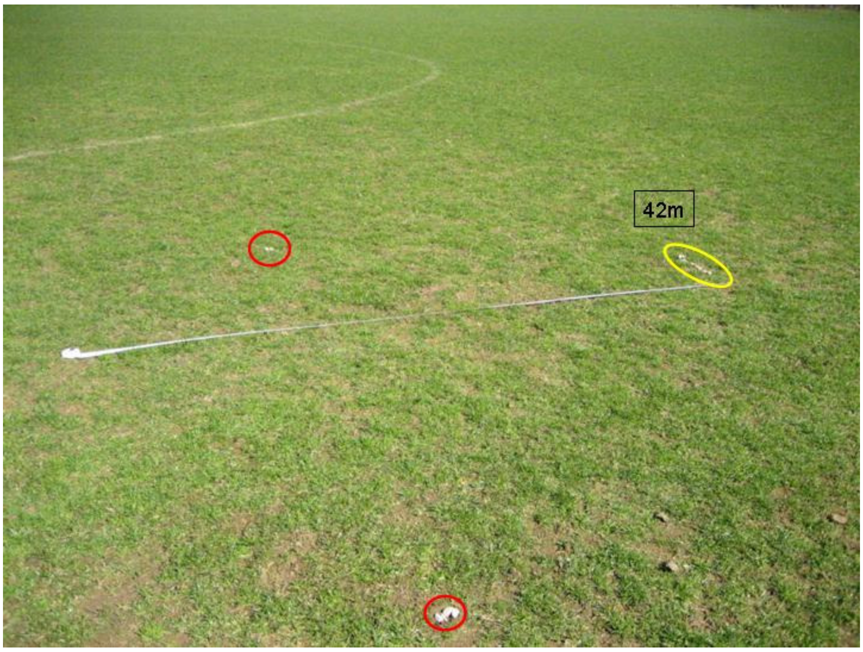

Figure 13: Landing positions of projectile

The firing range itself was marked out for intervals of 3 metres, signified by the yellow circle in Figure 13, and while the trebuchet operator launched the projectile, a "spotter" would mark the landing point of each projectile, as shown by the red circles in Figure 13.

The range for each launch was recorded and evaluated in Table 4. The standard deviations seem quite high, especially for the lighter projectile, which would seem to contradict the historical reports of the trebuchet's fabled accuracy. However, there were some unexplained "misfires" throughout the course of the experiment. This was possibly due to a twisted sling introducing a further degree of freedom to the projectile. The fact that one of the sets of results, Charterhouse 5, had in fact a very low standard deviation implies the inaccuracy of results is more to do with the inexperience of the trebuchet handlers, rather than being the fault of the machine. Also, the trajectory of the projectile was quite high so this may be due to the effect of the little wind present as the lighter projectile would be more susceptible to this, and also had the higher standard deviations.

[vi] The lighter hockey ball results are classed as "Charterhouse 1-3" and the heavier croquet ball results are classed as "Charterhouse 4-6"

Trebuchet	CW Mass (kg)	Average Range (m)	Standard Deviation	Theoretical Range (m)	Efficiency (%)
Charterhouse 1	12	28.75	1.374	96.91	29.67
Charterhouse 2	16	37.78	1.822	129.21	29.24
Charterhouse 3	20	41.31	2.753	161.51	25.58
Charterhouse 4	12	13.31	1.073	33.71	39.48
Charterhouse 5	16	20.18	0.2843	44.94	44.90
Charterhouse 6	20	27.93	1.098	56.18	49.72

Table 4: Experimental results

As described by the perfect engine results, the smallest mass ratio provides the shortest range and the greatest mass ratio provides the longest range. The varying efficiency however, prevents the experimental data matching the theory exactly.

The correlation with these results and the model for the seesaw trebuchet is interesting. Both exhibit a curious variation of efficiency with respect to mass ratio. This is contrary to the earlier deduction, that the mass ratio plays little or no role in determining the efficiency of the engine. However, while the modelled trebuchet showed that the trebuchet's efficiency was inversely proportional to its mass ratio, the experimental results imply that there is in fact an optimum mass ratio for the efficiency of a machine. This is again supported by the data in Figure 8, the beneficial effect of an increased counterweight mass being offset by a rapid reduction in efficiency. This efficiency curve is displayed in Figure 14.

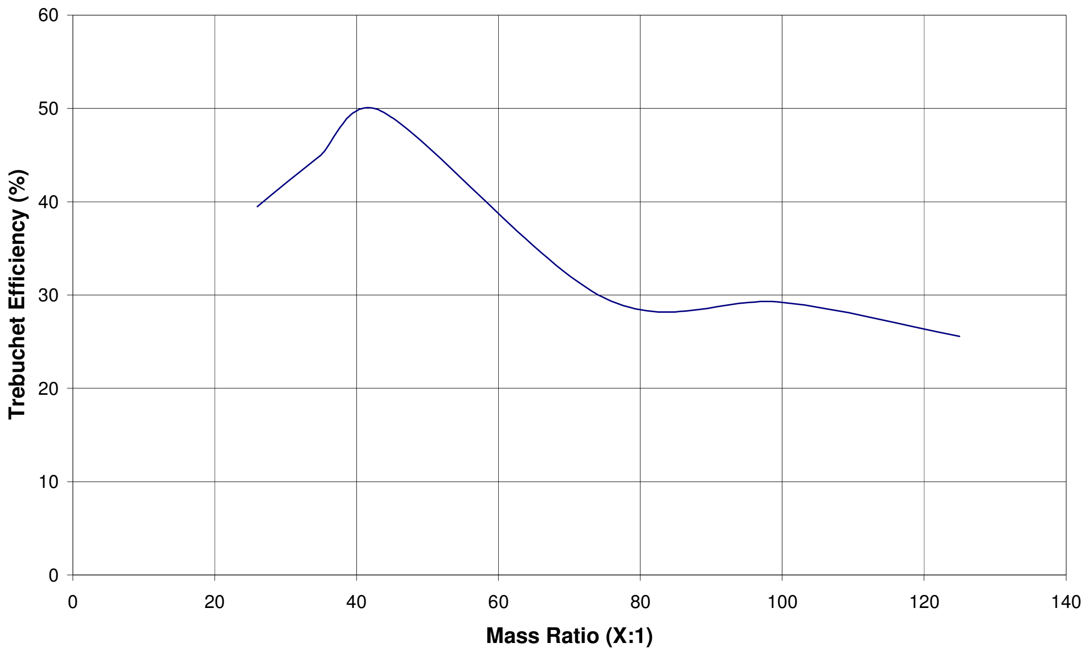

Figure 14: Efficiency against mass ratio curve for experimental data

Although Figure 14 shows a peak in trebuchet efficiency around the 43:1 mark, this requires further investigation as no results were taken for ratios from 43:1 - 75:1. To truly achieve reliable data on this issue, more mass ratios would need to be tested.

5. Analysis

The efficiency of different engines will now be compared. The only experimentally examined trebuchet was the Charterhouse trebuchet. The efficiency of that engine was therefore not only due to the mechanics of the engine, as was the case with the models, but also due to the frictional forces inherent in the machine.

The comparison in Table 5 looks at 4 different engines of varying complexity and their respective efficiencies. As the efficiency of a trebuchet is dependant upon both the mass ratio as well as the arm length ratio, the average efficiency was used to examine the type of machine rather than to confuse the issue with the characteristics, which are investigated elsewhere in this project.

Type of trebuchet	Average Efficiency (%)	Standard deviation
Perfect engine	100	0
Seesaw trebuchet	16.18	6.95
Trebuchet with sling	64.7	10.79
Charterhouse trebuchet	36.43	9.72

Table 5: Comparing efficiencies of the different models

The perfect engine obviously is the idealised model, with 100% conversion of potential energy into kinetic energy. The calculation lacks accuracy however, due to its dependence on the Δh term, which leads to a 2.5% error in the R_{max} calculation. The other models should, and do, all have efficiencies less than that for the perfect engine. The seesaw trebuchet, our most simple attempt at modelling the trebuchet's mechanics is, as expected the least efficient model. The Charterhouse trebuchet utilises all historic methods of increasing range; the sling, a hinged counterweight and wheels. This additional complexity still is not enough to offset the effects of friction, which substantially reduce the efficiency of the machine. As expected the trebuchet with sling provides the most efficient model here, having the benefit of both additional degrees of freedom and frictionless materials. These results compare well to those in "Trebuchet Mechanics" [11], with a seesaw efficiency of 11% and trebuchet with sling efficiency of 83%[vii].

[vii] The average efficiency of the sling was greatly reduced due to the effect of the lowest valued result, however, the trend still remains of the this being the most efficient model considered

6. Conclusion

The trebuchet is a highly sensitive mechanism, even the so-called "perfect engine" was limited by a factor of Δh, albeit by only 2.5%. The seesaw trebuchet has had optimum values calculated and defined as:

- $L_2:L_1$ equal to 4:1
- $m_1:m_2$ approximately 100:1 before efficiency begins to decrease
- Release angle of approximately 37.49°

The trebuchet with sling is obviously an improvement on the basic seesaw, with even the poorest result acquired more than tripling the range of the standard trebuchet. At present it appears the optimum sling length would be roughly 75% of the length of L_2, however, without further tests this is little more than speculation.

Of course, these were determined in the absence of friction, which as Table 5 shows, greatly affects the engine's efficiency.

7. Further Work

The next stage for this project would be to develop the spreadsheet for the trebuchet with sling further, so that one set of equations would cover a trebuchet of any dimensions. It would also prove interesting to further investigate the length ratio for the trebuchet with sling, in order to find the optimum conditions regarding L_2 and L_3 with respect to L_1. The next stage would then be to acquire some results for the effect of adding a hinge to the counterweight, and to investigate whether this alters the optimum ratio for the seesaw trebuchet due to the now varying ratio of length. A further investigation to explain the physics behind this mass ratio relationship with the trebuchet efficiency would also be another avenue to explore.

8. Acknowledgements

The author would like to acknowledge the support of the project supervisor, Dr D. Faux, as well as Stephen Hearn of Charterhouse school for providing the trebuchet.

9. References

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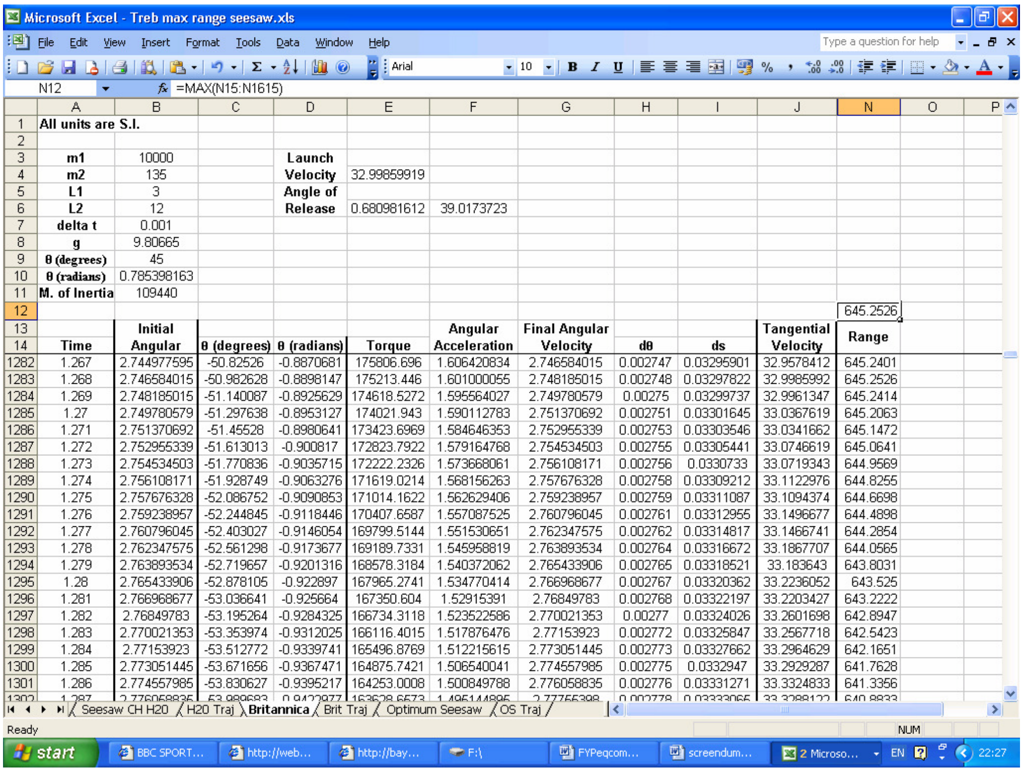
10. Appendices

Appendices A – C are screen dumps of the spreadsheets used for the modelling section of this project. Due to the complexity and size of the files in some cases, not all information was possible to show. The key areas have been selected as described by Figure 4. Appendix D displays still images from video of the Charterhouse trebuchet.

10.1 Appendix A – Seesaw Trebuchet Calculations



Microsoft Excel - Treb max range seesaw.xls

N12 =MAX(N15:N1615)

	A	B	C	D	E	F	G	H	I	J	N
1	All units are S.I.										
2											
3	m1	10000		Launch							
4	m2	135		Velocity	32.99859919						
5	L1	3		Angle of							
6	L2	12		Release	0.680981612	39.0173723					
7	delta t	0.001									
8	g	9.80665									
9	θ (degrees)	45									
10	θ (radians)	0.785398163									
11	M. of Inertia	109440									
12											645.2526
13		Initial				Angular	Final Angular			Tangential	Range
14	Time	Angular	θ (degrees)	θ (radians)	Torque	Acceleration	Velocity	dθ	ds	Velocity	
1282	1.267	2.744977595	-50.82526	-0.8870681	175806.696	1.606420834	2.746584015	0.002747	0.03295901	32.9578412	645.2401
1283	1.268	2.746584015	-50.982628	-0.8898147	175213.446	1.601000055	2.748185015	0.002748	0.03297822	32.9985992	645.2526
1284	1.269	2.748185015	-51.140087	-0.8925629	174618.5272	1.595564027	2.749780579	0.00275	0.03299737	32.9961347	645.2414
1285	1.27	2.749780579	-51.297638	-0.8953127	174021.943	1.590112783	2.751370692	0.002751	0.03301645	33.0367619	645.2063
1286	1.271	2.751370692	-51.45528	-0.8980641	173423.6969	1.584646353	2.752955339	0.002753	0.03303546	33.0341662	645.1472
1287	1.272	2.752955339	-51.613013	-0.900817	172823.7922	1.579164768	2.754534503	0.002755	0.03305441	33.0746619	645.0641
1288	1.273	2.754534503	-51.770836	-0.9035715	172222.2326	1.573668061	2.756108171	0.002756	0.0330733	33.0719343	644.9569
1289	1.274	2.756108171	-51.928749	-0.9063276	171619.0214	1.568156263	2.757676328	0.002758	0.03309212	33.1122976	644.8255
1290	1.275	2.757676328	-52.086752	-0.9090853	171014.1622	1.562629406	2.759238957	0.002759	0.03311087	33.1094374	644.6698
1291	1.276	2.759238957	-52.244845	-0.9118446	170407.6587	1.557087525	2.760796045	0.002761	0.03312955	33.1496677	644.4898
1292	1.277	2.760796045	-52.403027	-0.9146054	169799.5144	1.551530651	2.762347575	0.002762	0.03314817	33.1466741	644.2854
1293	1.278	2.762347575	-52.561298	-0.9173677	169189.7331	1.545958819	2.763893534	0.002764	0.03316672	33.1867707	644.0565
1294	1.279	2.763893534	-52.719657	-0.9201316	168578.3184	1.540372062	2.765433906	0.002765	0.03318521	33.183643	643.8031
1295	1.28	2.765433906	-52.878105	-0.922897	167965.2741	1.534770414	2.766968677	0.002767	0.03320362	33.2236052	643.525
1296	1.281	2.766968677	-53.036641	-0.925664	167350.604	1.52915391	2.76849783	0.002768	0.03322197	33.2203427	643.2222
1297	1.282	2.76849783	-53.195264	-0.9284325	166734.3118	1.523522586	2.770021353	0.00277	0.03324026	33.2601698	642.8947
1298	1.283	2.770021353	-53.353974	-0.9312025	166116.4015	1.517876476	2.77153923	0.002772	0.03325847	33.2567718	642.5423
1299	1.284	2.77153923	-53.512772	-0.9339741	165496.8769	1.512215615	2.773051445	0.002773	0.03327662	33.2964629	642.1651
1300	1.285	2.773051445	-53.671656	-0.9367471	164875.7421	1.506540041	2.774557985	0.002775	0.0332947	33.2929287	641.7628
1301	1.286	2.774557985	-53.830627	-0.9395217	164253.0008	1.500849788	2.776058835	0.002776	0.03331271	33.3324833	641.3356

Seesaw CH H20 / H20 Traj / Britannica / Brit Traj / Optimum Seesaw / OS Traj

10.2 Appendix B – Trebuchet with Sling Calculations

Microsoft Excel - Std sling treb.xls

AC4

	A	B	C	D	F	G	H	I	J	L	M	Z	AC	AD
1	**Unless stated otherwise all units are S.I.**													
2														
3	m1	100	θ (degrees)	45						Launch				
4	m2	1	θ (radians)	0.785398						Velocity	51.94213			
5	L1	1								Angle of				
6	L2	4								Release	0.674462	38.64381219		
7	L3	3								These values simply taken from				
8	delta t	0.0005								corresponding cells of max theoretical range				
9	g	9.80665												
10	h	3									Range discrepancy due to effect of extra delta h			
11														
12														309.665472
13–14	**Time (s)**	**V_x (ms^{-1})**	**V_y (ms^{-1})**	**θ (degrees)**	**B_x**	**B_y**	**Ay**	**Ax**	**α (degrees)**	**Moment of Inertia**	**r**	**Tangential Velocity**	**Angle of release**	**Theoretical Range**
15	0	0	0	45	-2.82843	0.171573	0	0.166662648	3.278589	109.0278	3.004626	0.018299236	176.8202	0
16	0.0005	-0.06185059	0	44.99991	-2.82843	0.171579	0	0.166631723	3.278742	109.0278	3.004624	0.036598482	176.8208	-2.385E-05
17	0.001	-0.12370134	0	44.99974	-2.82844	0.171586	0	0.166569872	3.278923	109.0277	3.004621	0.054897739	176.822	-7.151E-05
18	0.0015	-0.18555228	0	44.99948	-2.82845	0.171599	0	0.166477096	3.279257	109.0277	3.004616	0.073197007	176.8238	-0.0001429
19	0.002	-0.24740357	0	44.99913	-2.82847	0.171616	0	0.166353394	3.279701	109.0277	3.004609	0.091496285	176.8261	-0.0002381
20	0.0025	-0.30925533	0	44.99869	-2.82849	0.171637	0	0.166198767	3.280258	109.0276	3.0046	0.109795574	176.8291	-0.0003568
21	0.003	-0.37110766	0	44.99817	-2.82852	0.171663	0	0.166013213	3.280925	109.0276	3.00459	0.128094875	176.8326	-0.0004989
22	0.0035	-0.4329607	0	44.99756	-2.82855	0.171693	0	0.165796733	3.281704	109.0275	3.004578	0.146394188	176.8367	-0.0006644
23	0.004	-0.49481455	0	44.99686	-2.82858	0.171728	0	0.165549325	3.282594	109.0274	3.004564	0.164693514	176.8414	-0.000853
24	0.0045	-0.55666933	0	44.99607	-2.82862	0.171767	0	0.165270991	3.283595	109.0273	3.004549	0.182992853	176.8467	-0.0010644
25	0.005	-0.61852517	0	44.9952	-2.82866	0.17181	0	0.164961728	3.284708	109.0272	3.004532	0.201292207	176.8526	-0.0012986
26	0.0055	-0.68038216	0	44.99424	-2.82871	0.171857	0	0.164621537	3.285933	109.0271	3.004513	0.219591578	176.8591	-0.0015551
27	0.006	-0.74224044	0	44.9932	-2.82876	0.171909	0	0.164250417	3.287269	109.027	3.004493	0.237890966	176.8662	-0.0018338
28	0.0065	-0.80410011	0	44.99206	-2.82882	0.171965	0	0.163848367	3.288717	109.0268	3.004471	0.256190375	176.8738	-0.0021342
29	0.007	-0.8659613	0	44.99084	-2.82888	0.172025	0	0.163415386	3.290276	109.0267	3.004447	0.274489806	176.8821	-0.0024561
1996	0.9905	22.74010097	15.1525	-126.621	2.386086	6.210389	9.595	0.680307391	63.25248	143.9563	6.629952	61.26051254	5.889556	61.6379437
1997	0.991	22.77099101	15.20889	-126.754	2.393495	6.204868	9.6026	0.691692886	63.39508	144.0722	6.638693	61.31289592	5.980566	62.5194509
1998	0.9915	22.80181123	15.26552	-126.886	2.400889	6.199333	9.6102	0.703093792	63.53764	144.189	6.64748	61.36557811	6.071443	63.3970875
1999	0.992	22.83256043	15.32239	-127.018	2.408266	6.193784	9.6179	0.714510072	63.68017	144.3065	6.656314	61.41855973	6.162185	64.2708302
2000	0.9925	22.8632374	15.3795	-127.15	2.415627	6.18822	9.6255	0.725941691	63.82266	144.4248	6.665195	61.47184139	6.252791	65.1406559

projectile | Sling working | Trajectory | Standard Trebuchet | Ty | Angle | Vel time

10.3 Appendix C – Trajectory Calculations

Microsoft Excel - Trajectory.xls

B2 =Britannica!F6

	A	B	C	D	E	F	G	H	I	J	K
1	**All units are S.I.**										
2	**Launch Angle (deg)**	39.0173723									
3	**Launch velocity (u)**	32.99859919			**ρ**	1.225					
4	**Launch Angle (rad)**	0.680981612			**C_d (Sphere)**	0.47			**density**		
5	**Vert v**	20.77446597	**h**	2	**A**	0.162091			**stone**	2750	kg/m^3
6	**g**	9.80665							**volume**	0.049091	m^3
7	**m2**	135									
8	**Delta t**	0.01							**radius**	0.227146	m
9											
10–11	**Flight Time (s)**	**x velocity (m/s)**	**y velocity (m/s)**	**angle of projectio**	**x drag (N)**	**y drag (N)**	**x drag acc (ms^{-2})**	**y drag acc (ms^{-2})**	**x position**	**y position**	
12	0.00	25.64	20.77	39.02	30.67	20.14	0.23	0.15	0	2	
13	0.01	25.64	20.67	38.89	30.67	19.95	0.23	0.15	0.25635	2.206251	
14	0.02	25.63	20.58	38.75	30.66	19.75	0.23	0.15	0.512678	2.411507	
15	0.03	25.63	20.48	38.62	30.66	19.56	0.23	0.14	0.768983	2.615768	
16	0.04	25.63	20.38	38.49	30.65	19.37	0.23	0.14	1.025265	2.819034	
17	0.05	25.63	20.28	38.35	30.65	19.18	0.23	0.14	1.281524	3.021305	
18	0.06	25.62	20.18	38.22	30.64	19.00	0.23	0.14	1.537761	3.22258	
19	0.07	25.62	20.08	38.08	30.63	18.81	0.23	0.14	1.793975	3.422862	
436	4.24	24.71	-21.02	-40.38	28.49	20.61	0.21	0.15	106.705	1.06851	
437	4.25	24.71	-21.12	-40.52	28.49	20.81	0.21	0.15	106.952	0.856834	
438	4.26	24.71	-21.22	-40.66	28.48	21.01	0.21	0.16	107.1991	0.644162	
439	4.27	24.70	-21.32	-40.79	28.48	21.20	0.21	0.16	107.4461	0.430494	
440	4.28	24.70	-21.42	-40.93	28.47	21.40	0.21	0.16	107.6931	0.215829	
441	4.29	24.70	-21.52	-41.06	28.47	21.60	0.21	0.16	107.9401	0.000168	
442	4.30	24.70	-21.62	-41.19	28.46	21.80	0.21	0.16	108.1871	0	
443	4.31	24.70	-21.72	-41.33	28.46	22.00	0.21	0.16	108.434	0	
444	4.32	24.69	-21.82	-41.46	28.45	22.21	0.21	0.16	108.6809	0	
445	4.33	24.69	-21.92	-41.59	28.45	22.41	0.21	0.17	108.9278	0	
446	4.34	24.69	-22.01	-41.72	28.44	22.61	0.21	0.17	109.1747	0	
447	4.35	24.69	-22.11	-41.85	28.44	22.82	0.21	0.17	109.4216	0	
448	4.36	24.68	-22.21	-41.98	28.43	23.03	0.21	0.17	109.6684	0	
449	4.37	24.68	-22.31	-42.11	28.43	23.23	0.21	0.17	109.9152	0	
450	4.38	24.68	-22.41	-42.24	28.42	23.44	0.21	0.17	110.162	0	

Britannica | Trajectory

10.4 Appendix D – Still Frames of the Charterhouse trebuchet