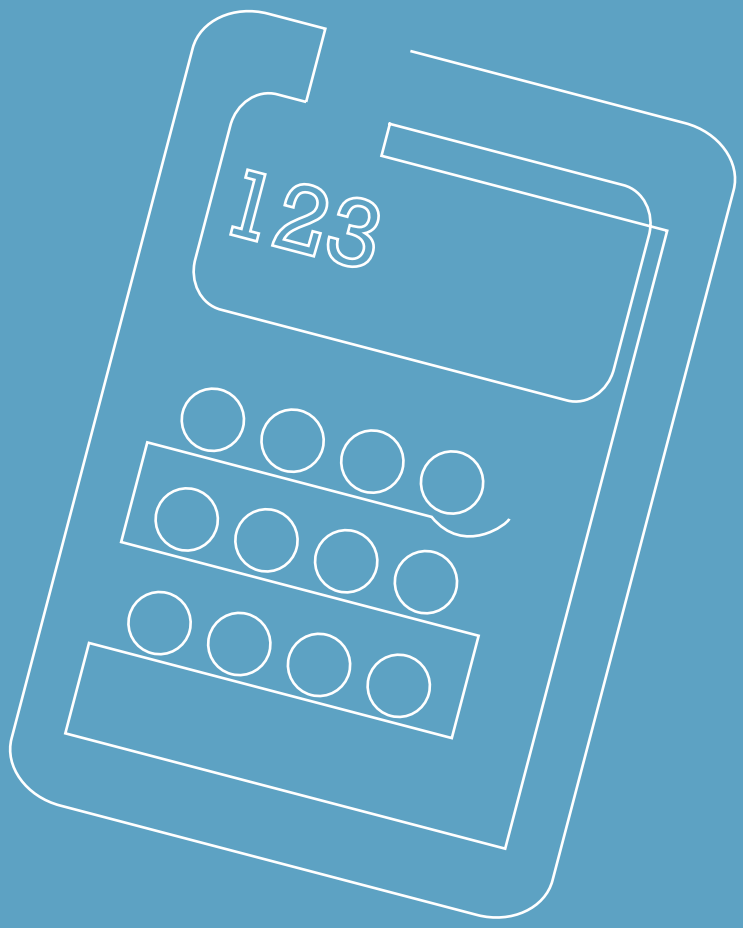
PRIMARY

Mathematics

Guidelines for Teachers of Students with

MILD

General Learning Disabilities



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National Council for Curriculum and Assessment
An Chomhairle Náisiúnta Curaclaim agus Measúnachta

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Rationale and introduction

Mathematics gives students the language through which they can interpret, analyse, describe, make predictions, and solve problems in everyday life.

It allows them to participate in a wide range of mathematical experiences and relationships both in school and in daily living.

Rationale

The aims and objectives of the Primary School Curriculum, Mathematics are valid for all students. However, not all students learn mathematics in an even and predictable manner. The abstract and conceptual nature of mathematics poses particular challenges to students with mild general learning disabilities. The teacher has a pivotal role in mediating the objectives of the Primary School Curriculum, Mathematics for students with mild general learning disabilities.

To meet the needs of students with mild general learning disabilities, greater emphasis is placed on the social, rather than the creative and aesthetic value of mathematics without excluding those important aspects. There will be particular emphasis on managing money, understanding timetables and using measures in everyday life situations. The acquisition of these skills must be prioritised in order to equip the student to participate fully and independently in society.

Introduction

A level of proficiency in basic mathematics is needed to cope independently and effectively with everyday living including telling the time, shopping, reading timetables, cooking, measuring, and so forth.

These guidelines aim to provide teachers with an understanding of the particular barriers to learning mathematics that students with mild general learning disabilities may encounter, and to provide some strategies that they can employ in planning mathematical experiences for their students, whether they are in mainstream classes or in special schools. As students with mild general learning disabilities may be learning their mathematics in many different settings it is important that the teacher initially identifies the point at which the individual students are operating. The section *'Before you start'* offers a quick checklist for teachers to identify a starting point within the Primary School Curriculum, Mathematics for individual students, many of whom will be able to access this material. Additional advice for teaching

mathematics to students with mild general learning disabilities is provided here with the caveat that the age and stage of development of the student provides the basis for mathematics learning for these students.

The aims of the Primary School Curriculum, Mathematics are:

- to develop a positive attitude towards mathematics and an appreciation of both its practical and its aesthetic aspects
- to develop problem-solving abilities and a facility for the application of mathematics to everyday life
- to enable the child to use mathematical language effectively and accurately
- to enable the child to acquire an understanding of mathematical concepts and processes to his/her appropriate level of development and ability
- to enable the child to acquire proficiency in fundamental mathematical skills and in recalling basic number facts.

The abstract and conceptual nature of mathematics poses particular challenges to students with mild general learning disabilities. The Primary School Curriculum, Mathematics stresses the importance of active learning, thus providing opportunities for students to manipulate, touch, and see objects as they develop their understanding of mathematical concepts. Learning within a group in which students are encouraged to talk about and explain how or why they did something will also support development of their own thinking about mathematics. An integrated approach to mathematics will help students to understand the relevance of mathematics in their daily lives.

It is important that the teacher is fully aware of the difficulties, both personal and academic, encountered by students with mild general learning disabilities. Personal difficulties are very often underpinned by a poor self-image brought about by a long-term sense of failure.

Failure may be one outcome of low intellectual ability, and can lead to slow progress. Slow progress is further aggravated by poor memory. Language and reading difficulties can confound students' difficulties with mathematics. Poor academic progress contributes to the poor self-image and lack of motivation, which in turn may lead to withdrawal or maladjustment of students. Low expectations can inhibit the student's effort and performance contributing to a cycle of failure. Teachers of students with mild general learning disabilities must strive to break that cycle by providing opportunities for students to experience success with mathematics.

Particular issues in mathematics for students with mild general learning disabilities

Many students with mild general learning disabilities require a structured approach to mathematics. Opportunities to practice mathematics skills and concepts enable students to consolidate their learning. Direct teaching, using explicit strategies, is essential as some students may acquire inappropriate or incorrect strategies from incidental learning. While many students learn by working things out for themselves or observing how others work, when knowledge or skills are being used in a new context it is important to support students by making their learning explicit, since transfer of learning does not always take place automatically.

Students with a mild general learning disability are not always proficient at using the skills and knowledge they have already acquired if the features of the new task are significantly different. They may have difficulty extracting the key features of a task and ignoring the less important ones. For example, they may focus on the numbers in a problem but not consider what they are being asked to do; hence they often just add all the numerals without considering the purpose. They may also focus on incidental information and fail to notice the salient feature of the topic. For example, they may be able to count by rote but fail to understand the use of numbers as labels for quantities *(I am 7, I live in number 7, she has 7 sweets).*

Features of the Primary School Curriculum, Mathematics

The following features of the Primary School Curriculum are relevant to students with mild general learning disabilities.

- **Early mathematical activities** are aimed at encouraging more work in pre-mathematical activities to develop concepts before commencing formal number work. These activities are particularly beneficial for students with mild general learning disabilities. Teachers can frequently revisit these activities in age-appropriate settings.
- The introduction of **number limits** encourages the consolidation of number facts and the development of the concept of place value. This is reflected in the reduction in the use of complex calculations and in the presentation of the same concept in different ways.
- The emphasis on accurate use of **mathematical language** and understanding of symbols will contribute to a greater understanding of mathematics for all students. Students with mild general learning disabilities will need frequent opportunities to consolidate their understanding of both symbols and mathematical language.
- **Calculators** have been introduced at fourth class, but it is essential that students develop good **estimation skills** from the earliest stages if they are to use them efficiently. This is particularly true of students with mild general learning disabilities, who may confuse the meaning of arithmetical signs or have conceptual weaknesses in some areas.
- The increased emphasis on the **use of manipulatives** (concrete materials) throughout the school, including the senior classes, means that students who may still need the support of materials will not be perceived as being different from their peers.
- The Primary School Curriculum also emphasises **real-life problem-solving**, using checkable answers and activities based in the student's environment. Encouraging students to engage with open-ended problems such as making scarves for teddy, building a house using a limited number of bricks, or working out how to spend a sum of money on food for a party will help them to realise that there are many ways to solve a problem, and that sometimes there is more than one 'right' answer.
- The introduction of **'fun areas**' such as chance and mathematical trails allows for differentiated approaches, which can include all learners in exciting mathematical activities. On a trail some students may be looking for one digit numbers, while others may be seeking up to three digit numbers or adding numbers together. For further ideas see Primary School Curriculum: Mathematics, Teacher Guidelines (Mathematical trails).
- The use of a **broad range of assessment tools** is essential in the accurate identification of students' strengths and needs. This is particularly true in mathematics where students may need to acquire certain skills or concepts before proceeding to more complex learning.
- The emphasis on using a **variety of methods of recording students' progress** encourages differentiation of response, which recognises different learning styles. Some students may be able to give an answer verbally while others would benefit from producing a diagram or drawing. All students should have the opportunity to present their work in a variety of ways.
- In the measures strand **answers should be verifiable** where possible to encourage understanding and the development of personal benchmarks. This can provide support to students with mild general learning disabilities. Hand-weighing and pouring activities using a variety of shapes of container will assist in this area. If students measure and label objects such as bookshelves, desks, etc in the classroom, they can build up visual benchmarks. For example, a student can stand beside the bookshelf and see that he/she is taller than a metre, or that the teacher is shorter than the door, which is more than two metres.
- Increased emphasis on **integration and on linkage in strands** of the Primary School Curriculum, Mathematics means that students can come to see mathematics as relevant and connected to their own lives. Frequent reference by the teacher to mathematical elements in the course of learning is essential, for example measuring the distance from the classroom to the office using a trundle wheel in geography, using time words in history (before, long ago, a year), measuring jumps or distances run in PE.

School planning

No school plan will be complete without taking due cognisance of the needs of students with mild general learning disabilities. With proper planning for differentiation to meet the needs of students with mild general learning disabilities, they can achieve success and participate fully in the curriculum at their own level.

Curriculum planning

While students with mild general learning disabilities are expected to partake fully in the curriculum they will not be expected to do the same tasks at the same level as the more able students. For these students, learning will concentrate on the areas of the curriculum that provide them with essential life skills, for example money, time, and measurement.

Students with mild general learning disabilities will need more opportunities to use concrete materials and engage in concrete tasks rather than working from a textbook where their weakness in language would further aggravate their arithmetical difficulties. As their concentration span is short, they benefit most from tasks which are short and varied.

Using the same mathematical language and methodology throughout the school benefits all students, especially students with mild general learning disabilities. This is especially true in such difficult areas as subtraction with renaming, and in the teaching of fractions and decimals. For example, in doing the algorithm 9-3, if one teacher says *'nine take three'* and another says *'three from nine'*, this will cause considerable confusion to the weaker student when she/he moves from one teacher to another or from class teacher to resource teacher.

It follows, therefore, that it is important to also keep parents informed at regular intervals about what and how students are learning and their general progress. As the student with mild general learning disabilities requires the caring help of both parent and teacher in order to make satisfactory progress, it is important that parents and teachers have opportunities to meet and discuss the student's progress.

Regular communication with the class teacher and support teacher are also important as each teacher is aware of how the student is progressing in the classroom and in learning support.

Good teaching practice needs regular assessment. Assessment should be informative and enable the teacher to plan for further learning, and report on the student's progress. Teacher designed tests can support the student by allowing for a certain amount of success. When failure occurs, the student will benefit from a discussion of where she/he went wrong and reassurance that making mistakes is part of the learning process. Students with mild general learning disabilities should be provided with a test suited to their abilities and stage of development. As students with mild general learning disabilities may be at different stages of development, different teacher-designed tests and individual learning profiles may be necessary to best support their learning.

Organisational planning

Organisational planning involves consideration of resource requirements, home-school policy, and a policy on homework.

As already noted, planning will ensure that the class teacher, support teacher, and the parent are working as a cohesive unit in supporting the student with general learning disabilities. For example, parents could spend time with the student using mathematical games that enhance the learning of basic facts. Some parents may not be familiar with an algorithm, for example decomposition, and try and teach the student using *'borrow payback'*, thus creating confusion. It is advised that teachers communicate with parents, at the beginning of the school year as to the approaches and methodologies used, so as to enable parents to support the student's learning in the home context.

Students with mild general learning disabilities very often have poor short-term/long-term memory. Homework should be used as an opportunity for reinforcing topics covered recently and in the past and should be appropriate to the child's level of progress and achievement. These students need constant reinforcement.

Classroom planning

Classroom planning for each school setting – special school, special class, and mainstream class – will differ according to the needs of particular students at any given time. These students progress at a slower rate than mainstream students.

Since the Primary School Curriculum, Mathematics is sequential and dependent on knowledge gained, they may experience great difficulty in keeping abreast of their fellow students, although they may be able to partake in the majority of mathematical activities in the class. If, for example, the teacher is teaching a new concept or new algorithm he/she must be careful that the calculations are simple, thus ensuring that the lack of number facts on the part of a student with mild general learning disabilities does not interfere with his/her acquisition of the concept. For example when using addition with renaming, 23+38 is a suitable example but 89+98 is not, since in the second case the student will have great difficulty calculating 8+9 and will not be able to concentrate on the concept involved. The situation is starker when it comes to multiplication. For example, 25×3 is easily managed, whereas 49×8 is likely to bring about failure in the case of the student with mild general learning disabilities.

Classroom planning for these students will involve close co-operation between the resource teacher and the class teacher in order to ensure that there is continuity and reinforcement in each other's work.

While providing special suitable work for these students the teacher should avail of any opportunity to allow participation in whole class work. Project work may provide such an opportunity. If, for example, the students are making a model of the school, the more able students can do the complex calculations relating to scale while students with mild general learning disabilities can be gainfully employed in measurement.

Mathematical knowledge can be reinforced in other areas of the curriculum and such opportunities should not be missed when dealing with the student with mild general learning disabilities. For example, number and measure can be reinforced when doing running, jumping, etc. in physical education.

Some aspects of the teaching of mathematics need particular emphasis in the case of students with mild general learning disability. These include

- mathematical language
- symbols
- materials
- worksheets and textbooks
- using ICT.

Mathematical language

Mathematics must be seen as a language with its own vocabulary of words, symbols and tools that are used in particular circumstances.

Many students confuse mathematical language with 'ordinary' language. They say *'He's bigger than me'* when they mean older, or *'My table is longer than his'* when they mean wider. It is important to teach mathematical language to the students, and to reinforce it on a daily basis. A list of mathematical words can be kept either by the teacher or by the students themselves (for example, taped into a copybook), and they can tick them off as they learn to use them appropriately and accurately, either orally or in writing. As already noted, keeping parents informed of the words being used and the importance of practicing them frequently will help the student to make use of these words in real contexts.

Symbols

Some students may have difficulty with mathematical symbols. They will often call the plus sign *'add'*, and others will use *'and'* or *'plus'*. As already noted, consistency of approach is vital and this is especially true if the class contains students who have come from different schools or classes where they may have developed misunderstandings in relation to some of the symbols. This is particularly relevant if the student is moving to a new class or being integrated with another class for mathematics. It is important that parents and all staff who are in contact with the student are aware of the terminology being used. In the classroom, charts with the symbol, word equivalents, and an example can help students to reinforce a correct interpretation of the symbol.

+	plus and add	3 + 4 = 7

Plus and equals symbols are introduced first, followed by the minus sign and, subsequently, the greater than and less than signs (see symbols chart in the teacher guidelines for mathematics in the Primary School Curriculum).

Materials

Age-appropriateness

It is very important that the materials used are age-appropriate. Brightly coloured counters and interlocking cubes may be appropriate for younger students, but they often come to associate these with the infant classes. Using football cards or coins can make a counting activity more age-appropriate for older students. Pretending to be the man on the turnstile totting up the takings after a match can make counting more real to an older student. *'Jobs'* that involve counting can be incorporated into the mathematics class and are usually good motivators, for example counting out information notes for distribution to other classes, having a class sale of work for charity, costing and buying foodstuff for cookery classes, or examining the cost of popular clothes such as jeans or favourite chocolate bars.

Distributing milk, books, or pencils can be very valuable activities that support the modelling of language such as *'Are there enough pencils for all your group? How many cartons do you usually need for your group? Is there anyone absent today? How many do you need today?'* The teacher can manipulate this actively by not giving enough items to the student and asking how many more are needed. During break times informal discussion about lunch items can include such questions as, *'Who has the most sandwiches?'* or *'Whose carton of juice contains the most milk?'*

Accessibility

Another important feature of using materials in the classroom is that they must be stored in ways that make them easily accessible to students. Clearly labelled and colour-coded boxes, a routine tidy-up session, and procedures to maintain clean and neat materials will encourage students to use the materials appropriately and responsibly. Having a mathematics table where the materials needed for the lesson are easily accessible is important and can be a role of responsibility for different students. It can be changed as topics change, for example *'Our measures table'*, *'Sets of four'*, etc.

Worksheets and textbooks

It is important that worksheets and textbooks are used carefully with students who have a mild general learning disability. Some mainstream textbooks move too quickly through the content areas and therefore should be seen as a complement to other means of instruction rather than the only source. With these students it is particularly important to maintain a balance between computational and problem-solving tasks and between the use of concrete and symbolic representation.

In using worksheets and textbooks, the following questions should be considered:

- Is there too much text in the worksheet/textbook? (This could inhibit the ability of poor readers to engage with the mathematical content.)
- Is there a lot of *'clutter'* on the page? (Students are often distracted by *'busy'* pages containing too much unimportant information or illustration.)
- Are the spaces given for the answer large enough for students who may have poor motor control or write their numbers very large?
- Are worksheets that are intended for homework clear in their instructions so that parents know what the student is supposed to be doing?
- Do worksheets/textbooks offer a variety of task types, for example practical tasks, open-ended investigations, puzzles, games, project-based work, and word problems?
- Do they present material in different ways for different purposes and learning styles, for example pictorially, diagrammatically, and using minimal language as appropriate?
- Is the print size appropriate to the developmental stage of the student?
- Over time, do worksheets/textbooks offer an appropriate balance between the strands?
- Do they offer opportunities for collaborative or group work, for example active investigation and talk-about sections, and integration opportunities that focus on other areas of the curriculum?
- Do they reflect the interests and environment of the student?
- Do they consider the social aspects of mathematics, for example shopping, measuring for a purpose, life-skills?
- Are computational activities carefully graded, offering appropriate opportunities to consolidate learning at each level?

Making sure that worksheets and textbooks are age-appropriate is extremely important, since students are usually very aware that a particular book is being used by a younger brother/sister or by a more junior class within the school, and this can seriously impact on their self-esteem.

Using ICT

Many software programmes can be used at different levels within the one group or class. Valuable teacher time can be taken up in establishing the correct starting point for a particular student. Colour codes and symbolic representations taped to the front of software boxes can help, and a card with clear instructions can be given to the individual student. Once students have practiced this procedure, they will be able to locate and load the software themselves, and an older student can either help or supervise. It is helpful to keep symbols constant and, where possible, include the students in the choice of how information can be presented symbolically. This activity is also part of the advice provided in the *Primary School Curriculum: Mathematics, Teachers Guidelines.*

Assessment for learning

Students should have opportunities to experience different assessment methods from an early age, and play an active part in their own assessment. Early identification of difficulties in mathematics is essential if the students are to achieve their full potential.

Assessment should be seen as a positive rather than negative experience that will help them in their future learning, and clear, constructive feedback to students is necessary if they are to play an active part in their own future learning. Good communication is essential between the teachers and parents, other teachers and professionals who come into contact with students regarding each student's progress, strengths and future learning goals.

An important issue in assessment is the evaluation of a planned test with regard to its aims and its suitability for the students for whom it is intended. The language of the test is crucial. If the language is too difficult or wordy for students who have a reading difficulty the test may not accurately reflect their mathematical ability.

Teachers should use a variety of assessment tools that take account of different learning styles. When planning a sequence of learning outcomes for a particular student or group it is necessary to keep in mind how progress will be assessed. Too often a pen and paper test at the end of a unit of work is used. However, it is possible to assess progress by observing, for example, how a student handles money in the class shop, how accurately a student can continue a pattern, or can make a shape with his/her body. When assessing student progress in the various areas of mathematics teachers should use a variety of assessment tools, including teacher observation, teacher-designed tasks and tests, portfolios, projects, work samples, and criterion reference tests. See the assessment section in the Introduction to the Primary School Curriculum for further details on assessment techniques.

Approaches and methodologies

The Primary School Curriculum recognises that students learn within a social context and are active participants in the learning process. They naturally investigate and rely on their own methods of working out mathematical problems and therefore need to experiment and discuss in a supportive environment that enables them to build on and develop their ideas.

Students with mild general learning disabilities need to build up personal support structures which they can use when faced with a new situation or problem. They need to learn *'how to learn',* how to verbalise what the problem is, and how they intend to go about solving it.

In the classroom situation, it may be good for the more able student and the less able student alike if peer teaching is introduced. The more able student can be gainfully employed in helping his/her less able peer in reinforcing algorithms, learning basic facts, and in providing reinforcement of the concepts already learned.

Difficulties with numbers

Students with mild general learning disabilities find it very difficult to memorise basic facts. Consequently, number is an area that presents multiple difficulties. This has a debilitating effect on their success rate as very often they may have an algorithm done correctly but the answer is wrong because of lack of knowledge of basic facts. In order to overcome this difficulty, the teacher should

- choose figures that are easy to calculate when teaching a new concept or algorithm
- allow students to use a calculator or a one hundred square when calculating more difficult basic facts
- if part of an algorithm is correct, mark the part that is correct and discuss the error in the remainder part with the student.

Concrete materials

When teaching new algorithms it is important (where possible) to use concrete materials, for example, in difficult areas like decomposition or fraction or decimal concepts. Students should have opportunities to use such materials until the concept is well grounded. When mistakes occur after the concrete materials are removed, they must be reintroduced to the student during the remediation process.

Learning basic facts

Learning basic facts is very often a long and tedious task for the student with mild general learning disabilities due to possible short-term and long-term memory difficulties. However, these difficulties can be alleviated by

- providing the student with strategies for learning basic facts
- using some of the large variety of table games available, for example dominoes
- using computer games to reinforce knowledge of basic facts.

It is of crucial importance to match the work given with the student's ability to do it, in order to ensure that the student will experience success.

Potential areas of difficulty

The potential areas of difficulty are described in order to outline the possible implications for learning and suggest possible differentiated teaching strategies. Students with mild general learning disabilities are more like their peers than unalike. They have the same range of interests, the same need for affirmation and success, and exhibit a wide range of learning styles. Not all students with mild general learning disabilities will exhibit all of the potential areas of difficulty, but it may be helpful for teachers to understand the implications that these may have for their teaching and, consequently, for the student's learning.

During his/her early years a student with mild general learning disabilities may have had restricted experiences in comparison to his/her more able peer. Restricted mobility, poor motor control, or poor understanding may have resulted in fewer opportunities to use the language of mathematics – for example counting stairs, judging distances or heights, and learning mathematical rhymes. Teachers should be aware that they may not have learned these things incidentally.

Some students with mild general learning disabilities will have difficulties in problem-solving due to inherent limitations in their ability to abstract and generalise. Their ability to work independently is constrained by poor attention span and retention. Hence, the learning process needs to be broken down into short sequential steps and work done needs to be reinforced through over-learning and repetition.

The difficulties experienced by these students can lead to poor self-esteem and a fear of failure. These combined can lead to a sense of helplessness that results in the student constantly seeking help or refusing to proceed with even the simplest of tasks. Students may get trapped in the *'I can't do maths'* syndrome. It is important that such students experience success as often as possible and participate in fun mathematical activities if they are to overcome this feeling. The setting of realistic learning targets by teacher and student may help in the achievement of success and the return of confidence. It is also important for students to realise that making mistakes is an integral part of the learning process, and that they should not be discouraged by their mistakes.

While difficulties in numeracy and literacy often overshadow the student's learning experience, it is important to provide a wide variety of learning tasks in order to allow students to show their skills and develop confidence in other areas of the curriculum. The use of the calculator, for example, can enable the student to sometimes bypass arithmetical difficulties and work successfully on other mathematical topics.

Students need constant encouragement. They need to be involved in their own learning and have opportunities to discuss their difficulties. Students have valuable insights into their own learning needs and this should not be ignored. Teachers should help students towards an awareness of what their learning needs are, in the context of what they can do rather than what they cannot do. Effective teaching builds on students' strengths rather than focusing on their weaknesses.

The tables that follow outline characteristics that may be associated with a mild general learning disability, examine their implications for the learning of mathematics, and suggest a range of strategies that may assist the student.

Sometimes it is the way in which the material is presented to the student that creates a barrier to learning. Using a variety of approaches and methodologies will facilitate different learning styles in any given class group.

Before you start

The following lists are not comprehensive but should serve as a guide in assessing where a student is in relation to mathematics. It is suggested that individual teachers or groups of teachers develop their own profiling system that will insure that instruction is tailored to meet their students' needs and levels of attainment. Teachers can use this information to ask

- Is the student's reading level inhibiting his/her ability to engage with the mathematical elements?
- If so, can the material be presented orally, diagrammatically, pictorially?

When an observational picture of the students has been created they can be grouped accordingly or provided with individual work. However, it is very important to include large group or full class work occasionally, so that students can learn from seeing how other students approach problems.

All students will benefit from a variety of teaching styles and classroom activities. Students with mild general learning disabilities will particularly benefit if the teacher is aware of their strengths and weaknesses before embarking on a new topic. The following table outlines some observations that the teacher may note about the student. Such informal assessment can assist the teacher in selecting suitably differentiated methods for the class, and may prove useful in providing feedback to parents and students.

A generic list for informal teacher observation of students

Pre-test using a simple task in an area you are introducing

- What is the state of readiness of the student to do this task in terms of knowledge, skills, and attitudes?
- What learning strategies are being used by the student?
- Can the student describe what he/she is doing and how he/she is doing it?
- Does the student understand why he/she is doing this activity?
- Is the student's reading level inhibiting his/her ability to engage with the task?
- Is the student's numeracy level inhibiting his/her ability to engage with the task?
- Does the student have any misconceptions about the task?

Independence

- How does the student cope when left alone with a task?
- Does he/she give up easily, ask for help, begin the next task, sit inactively, or become disruptive?
- Does the student clearly understand what to do if he/she has a problem or is finished work earlier than others?
- Is the student organised in his/her work? Does the student check if he/she has the correct equipment, for example pencil, pen, ruler, graph paper?
- Does he/she have other difficulties, such as hearing loss, poor vision, poor motor control, or hyperactivity, which may need to be considered?
- Can the student present work in a way that can be understood by others?

Group work

- Does the student appear to learn better in a group, alone, or in pair-work?
- Can he/she take turns and listen to other students' responses?
- Can he/she present work in a clear and coherent manner on behalf of the group?

Instructions

- Does the student listen to and understand instructions?
- Can the student read instructions?
- Can he/she follow instructions given by the teacher?
- Can she/he follow more than one instruction effectively?
- Can he/she make appropriate responses to the instructions?
- Is the student clear about routines for setting out materials and clearing up after an activity?

Choosing and using the right materials

- Does he/she choose appropriate support materials, for example number line or objects for counting, addition, and subtraction?
- Does he/she use materials appropriately, for example use a ruler correctly to measure, use a number strip for counting on, use blocks and push away those already counted, use a number square efficiently, use a calculator appropriately (i.e. make decisions about when its use is necessary)?

Readiness for mathematics

- Can the student classify, compare, and order objects and pictures?
- Does he/she use labelling words efficiently when questioned? ('*Can you find a teddy for me? What can you see? What is this?*')
- Does he/she respond to different aspects of a mathematical problem, for example functions, attributes, differences, and similarities? ('*Can you find all the red objects for me, please? Can you point out the red circles?*')
- Can the student understand negatives? ('*Simon says do not stand up. Find all the blocks which are not red.*')
- Does he/she reason, for example predict, project, identify cause and effect, and suggest alternatives? ('*Why? What if I …?*')

Addressing potential areas of difficulty for students with mild general learning disabilities

▲ Potential area of difficulty	= Implications for learning
The student may have a short-term memory difficulty.	Retention of number facts can be a problem.

+ Possible strategies

- Encourage the use of visual clues to aid memory.
- Use number rhymes and songs.
- Provide the student with strategies for remembering facts such as doubles, near doubles, etc.
- Practise estimation skills so that a calculator can be used efficiently.
- Work on making number operations automatic through fun games such as table darts.

▲ Potential area of difficulty	= Implications for learning
The student may have a short attention span, lack of concentration, and lack of application.	The student finds it difficult to stay on task.

+ Possible strategies

- Provide shorter tasks with clear rewards for staying on task, such as computer use or game time.
- Keep periods of instruction short and to the point, and recap frequently.
- Provide short work sessions with achievable goals. Encourage the student to become aware of the difficulty and to try to *'beat their target'* in staying on task.
- Use teacher observation efficiently and note achievements, strengths, and preferred learning styles for use in planning future work.
- Use classroom management which focuses on contingent praise and encouragement (i.e. rewarding the task behaviour and not just the answer). This includes accepting 'good' answers that may not be necessarily correct, for example '*Mary found an interesting way of doing that problem; let's see how it works*'. The use of open-ended problems that have different possible answers can help to develop a positive attitude to problem-solving.

▲ Potential area of difficulty	= Implications for learning
The student may have difficulty in understanding mathematical concepts/abstractions.	The student finds mathematics particularly difficult and has difficulty with counting numbers, place value, and with understanding what is happening when using the four operations.

+ Possible strategies

- Frequent practice of the concepts to be learned should be varied by use of games, ICT, and real-life problems relevant to the student's experience. Make the learning fun by using funny names, silly scenarios, or unlikely settings, for example:
  'Tommy went to Mars for his holiday and met five aliens. They each had four arms. How many arms had the aliens altogether? Draw the aliens. What if they each had eight arms?'

▲ Potential area of difficulty	= Implications for learning
The student may have difficulties with spatial awareness; he/she may not have had opportunities to play with puzzles, blocks, etc.	The student may have difficulty organising materials, may display left/right confusion in recording, may not recognise shapes if inverted, or constantly lose items in the room.

+ Possible strategies

- Plenty of work with three dimensional objects will be needed, with particular attention being paid to the language of spatial awareness, for example up/down, over/under, shape words.
- Using jigsaws, block puzzles, and tangrams in a fun way can help students in this area.
- Work on awareness of own personal space, especially in PE, left and right, distance from, etc.
- Maintain consistent organisation patterns in the classroom.
- Use visual cues for location and direction on charts or table top.
- Give oral cues related to the student's own position, for example on your door side.
- Allow students to remain in the same seating position for group work.

▲ Potential area of difficulty	= Implications for learning
The student may have difficulty applying previously learned knowledge.	The student may find it difficult to use a skill or concept in another setting such as measuring in geography or science.

+ Possible strategies

- Draw the student's attention to what is happening: *'This is just like the measuring we did last week. What did we use to measure our books? How did we place the ruler?'*
- Consciously reinforce mathematical concepts in other areas of the curriculum, for example sorting and classifying in science, space and shape in art (printing 2-D shapes, both randomly and in sequences/ patterns).

▲ Potential area of difficulty	= Implications for learning
The student may have difficulty with transfer to real life.	The student does not use mathematics in real situations. For example, he/she does not use any of the four operations when buying goods in a shop, does not see the need to measure when cooking, does not recognise shapes in the environment.

+ Possible strategies

- Use real-life objects and coins in play situations.
- Discuss how pocket money is spent.
- If possible, provide opportunities to handle money in a real shop or school shop.
- Make sure that parents are aware of the importance of counting, handling money, etc. at home, for example setting the table, dividing up food, sharing equally, weighing for cooking, or measuring when doing DIY.

▲ Potential area of difficulty	= Implications for learning
The student may have difficulty with visual sequencing.	The student can't copy from the board or from a book, or has difficulty with sequencing numbers, mirror writing, etc.

+ Possible strategies

- Start with tracing exercises, using tracing paper.
- Teach students how to 'chunk' information (for example copy only one part of the sum at a time), and how to check that it is correct.
- Use number rhymes and songs to reinforce sequences.
- Use visual cues, for example ✕ start here

▲ Potential area of difficulty	= Implications for learning
The student may experience confusion with signs and symbols.	The student does not *'read'* symbols and asks questions like 'Is this an add sum?'

+ Possible strategies

- Use charts and discussion of everyday symbols, and relate these to mathematical symbols.
- Encourage students to verbalise what they should do first, for example look for and identify the symbol, or apply the correct symbol if it is a written problem.

▲ Potential area of difficulty	= Implications for learning
The student may display poor vocabulary/other language difficulties.	The student cannot follow complex sentences or multiple meanings, may process only part of the instruction, for example when told to put a red circle around all the big things he/she may process only 'circle' and 'things'. The student finds it difficult to verbalise what he/she doing in mathematics, or to relate the vocabulary of mathematics to real-life situations.

+ Possible strategies

- Identify and specifically target mathematical language, ensuring that it is reinforced in different settings and in other areas of the curriculum, for example location words (on, in, under, etc.) in PE, drawing games that involve following instructions containing target words (draw a square in the middle of your page, put a blue triangle beside/under/to the right of the square.
- Be clear in communicating to both students and parents the language that is being covered each week, for example using a note in a copy or a wall chart to list the 'mathematics' words of the week'.

▲ Potential area of difficulty	= Implications for mathematics
The student may experience reading difficulties. (For writing difficulties, see the section on communication and language.)	Reading difficulties can prevent students from engaging with mathematics. They may be capable of completing the mathematical task but become frustrated and confused by printed words.

+ Possible strategies

- Provide alternative forms of problems using visual presentation of material.
- Ask the student to pick out the parts of the problem that he/she can read, and to focus on what information is relevant. There is often a lot of redundant information in a written problem.
- Avoid presenting the student with pages of textbook problems by giving modified worksheets or verbally delivered instructions, for example *'Mary has six sweets and she gives her brother four; how many has she left?'* Encourage the student to use drawings to 'write' down the important features, for example
  Mary has ✎ ✎ ✎ ✎ ✎ ✎
  'Take away four by crossing them out. How many are left?'

▲ Potential area of difficulty	= Implications for mathematics
The student has difficulty in following instructions.	The student becomes confused when faced with more than one instruction at a time.

+ Possible strategies

- Get the student to repeat the instruction(s).
- Use short, clear instructions or pictorial cues, e.g. a picture of a copybook on the blackboard or on a card.
- Use cue sheets, for example:
  ' Take out a copy and pencil (picture).
  What kind of problem is it?
  What do I need to know?
  What do I do next?'
- Give clear guidance on how and when assistance will be given by the teacher/other students during the lesson.

▲ Potential area of difficulty	= Implications for mathematics
The student may be overwhelmed by the learning process.	The student becomes overwhelmed when presented with new information or skills, and consequently cannot learn.

+ Possible strategies

- Vary the materials given to a group—some using number strips to five while others are using them to ten, where appropriate.
- Adapt the teaching style, for example use more discussion at the beginning and at the end of the lesson to help both teacher and student to understand how learning takes place.
- Introduce variety in the responses required. The same activity can often be done with a group or the class, but some students can be required to answer orally, some by using symbolic representation or some by using a pictorial response, for example
  ❁+ ❁ ❁ = ❁❁❁
- Vary the requirements of the task. One group or individual may only have to do six of the calculations whereas another may have to do ten or more. Set personal targets for the students so that they do not feel that others are getting less to do than they are.

Selecting content

When planning for a class, group or individual it is useful to have a checklist of elements you wish to include. Some suggestions are given in the following example. In order to maximise the cross-curricular elements of mathematics two further planning grids, with suggestions, are given in examples 2 and 3.

EXAMPLE 1

☑ Does your mathematical programme contain these elements?

- ☐ activities that *link* with other areas of mathematics (for example, using money to teach tens and units)
- ☐ direct teaching of mathematical *Language*
- ☐ consideration of *age-appropriateness*
- ☐ a *balance* between class, group, and individual teaching
- ☐ *direct teaching* of skills such as turn-taking or active listening, and good work habits such as having materials ready, knowing what to do when finished, etc.
- ☐ activities that *integrate* with other areas of the curriculum
- ☐ real-life *problem-solving* opportunities
- ☐ variety in the *materials* to be used
- ☐ direct teaching of mathematical *strategies*
- ☐ *methods of recording* progress such as the student's own log books, portfolios of completed work, individual progress charts with clearly set individual targets.

AIMS
What do you want to do? (strand/topic)
What particularr area do you want to focus on? (language, skill, concept?)

OBJECTIVES
What learning outcomes do you expect for each student or group, including both skills and concepts? (*These should form the basis for assessment and inform decisions about what the next target should be.)*

Planning cross-curricular work in mathematics

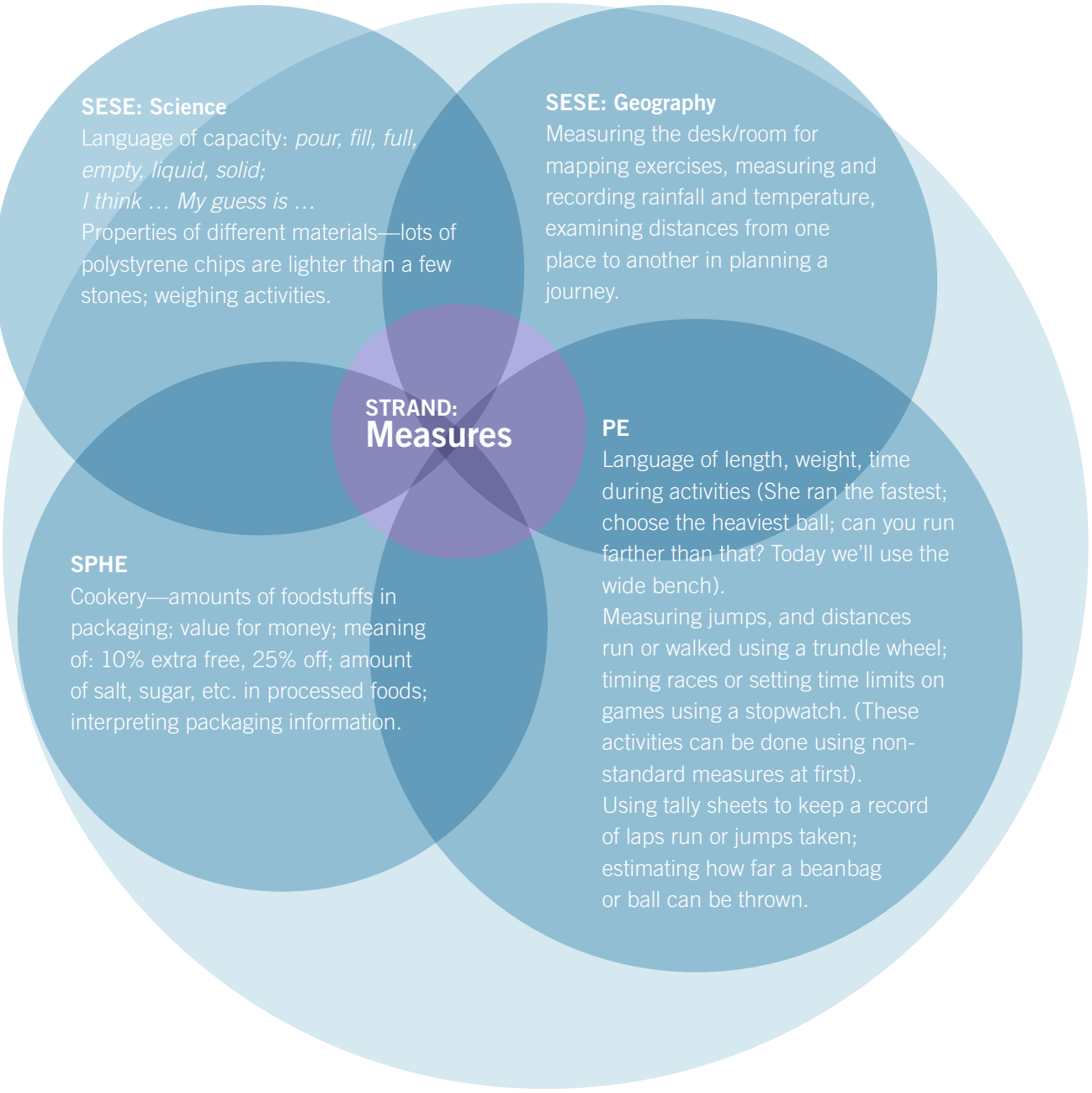
ORGANISATIONAL ISSUES
Can you enlist the help of other teachers/parents/ assistants/students?
Is the necessary equipment easily available and are the students familiar with its use?
Can the topic be taught to the whole class, but with differentiated elements as outlined in the section on differentiation?

ASSESSMENT
How will you record the students' progress?
How will you use this information to plan your next step?
How will you reinforce the skills/concepts in other areas?

EXAMPLE 2

Planning cross-curricular work in mathematics

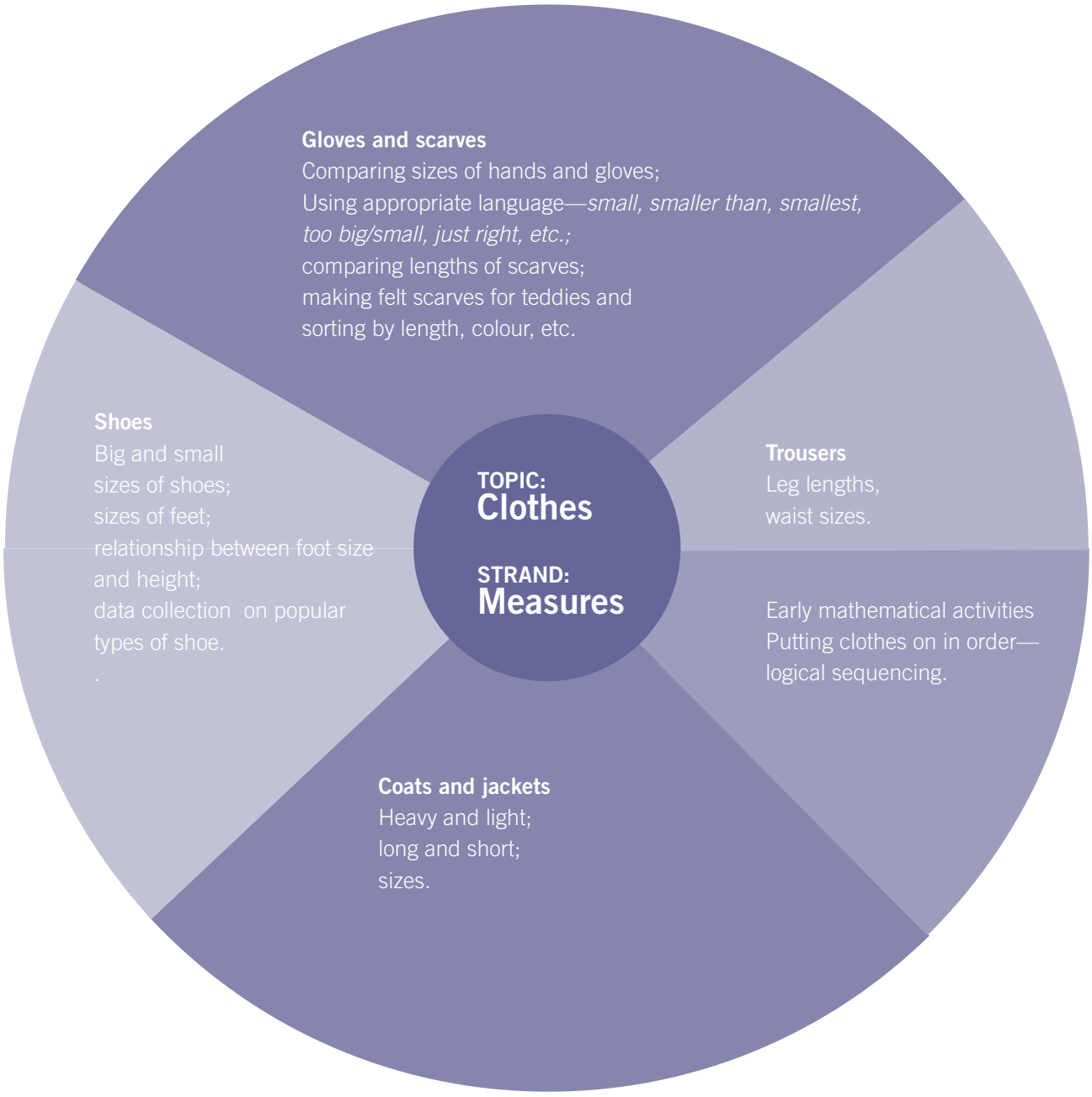
This is not an exhaustive list and would need to be adapted to suit the ages and ability levels of the students. The focus for the teacher should be on identifying the language to be reinforced and to consciously use it across subjects. It is very important to draw the students' attention to the fact that measuring in geography or science is just the same as in mathematics.



EXAMPLE 3

Planning topic work in mathematics

Below is an example of how an area of mathematics (measures) can be explored through topic work. The age-appropriateness of the activities must be considered at all times. Computer software packages and books which support the vocabulary are easily available.



Strategies for teaching students with mild general learning disabilities

General strategies

The strategies outlined in this section use readily available concrete materials, allow students to experiment while learning new concepts, and encourage students to discuss their ideas on working out mathematical problems. In using the strategies that follow, the following approach should be taken:

(i) The teacher demonstrates the algorithm on the board while using the concrete materials, interacting with the students and posing questions as outlined in the individual strategies.
(ii) The students apply the algorithm using concrete materials until the steps are internalised and the concept is well grounded.
(iii) The students practice using the algorithm without concrete materials.
(iv) A student is guided to return to the use of concrete materials when problems occur.

The following list of general strategies may prove helpful in the teaching and learning of new facts, concepts, algorithms, and skills.

- Draw out important facts through classroom discussion.
- Try to plan for the consistent use of language and methodology throughout the school and, if possible, in the home.
- Encourage the student to work at a slow pace and in a very structured way. Avoid great leaps, taking very small, carefully thought out steps one at a time.
- Provide opportunities for the student to talk about a problem and to say how he/she can go about solving it.
- Encourage students to practice and apply the skill of estimation as often as possible.
- Provide the student with strategies for learning basic facts, and use a variety of table games and computer games to reinforce this knowledge.
- Use wall charts and display boards to show the facts and concepts that are relevant to the current classroom work.
- Provide regular opportunities for the student to use concrete materials until a new concept is well grounded.
- If a student has difficulties with a new concept and needs remediation, encourage him/her to return to using concrete materials.
- Choose figures that are easy to calculate when teaching a new concept or algorithm.
- Use materials that can be deconstructed where appropriate, for example using a bundle of ten lollipop sticks allows the students to see the ten single units, unlike a ten euro note. Initial use of lollipop sticks or Unifix cubes followed by the use of money means that the student is moving from a concrete to a more abstract use of materials.
- Use manageable materials where appropriate, for example money is far more manageable than lollipop sticks for larger numbers (hundreds), and cheques may be appropriate for even larger numbers (thousands).
- The students should only record the algorithm in the correct mathematical format when they are very familiar with the algorithm and even then only with easily managed numbers.

Diagnosing student difficulties

The diagnosis of student difficulties is one of the most important skills that a teacher can develop. The table below analyses some of the more common student errors and suggests what may be the cause of the errors. It is included here to assist the teacher in the diagnosis of errors so that a suitable remediation strategy can be undertaken.

Arithmetic operation or concept	Example of error	Further explanation of error and possible reason for error
Counting	Twenty-eight, twenty-nine, twenty-ten, twenty-eleven, etc.	Problems with matching, counting, and place value
Writing numbers	Writing 12000500 for twelve thousand five hundred	Poor concept of large numbers and place value
Addition	24 +38 52	Not carrying Poor concept of place value
Addition	23 +44 77	Carrying when it does not apply Poor concept of place value
Addition	24 +58 712	Not making ten Poor concept of place value Poor estimation
Addition	24 +58 19	Adding anticlockwise Poor concept of place value
Addition	26 27 +38 81	Carrying one instead of two Poor concept of place value
Subtraction	64 – 28 44	Saying 4 from 8
Subtraction	73 – 25 58	Forgetting to ‘cross out’ Poor concept of decomposition
Subtraction	^{7}~~8~~7 – 25 52	‘Crossing out’ when there is no need Poor concept of decomposition
Subtraction	^{1}~~2~~^{1}0^{1}0 – 1 4 5	Failure to understand what is happening
Subtraction	^{3}~~4~~^{9}~~0~~^{9}~~0~~^{9}~~0~~ – 1 2 4 5 2 7 5 4	Poor concept of decomposition

Arithmetic operation or concept	Example of error	Further explanation of error and possible reason for error
Multiplication	34 x3 92	Not carrying
Multiplication	23 x3 79	Carrying when not applicable
Multiplication	25 x3 35	Adding the one carried on to the two Does not multiply by three Poor concept of multiplication
Multiplication	25 x3 615	Putting down the fifteen, does not carry Poor understanding of place value
Multiplication	29 x3 77	Carrying one instead of two Problems with place value
Multiplication	28 x3 102	Carrying four instead of two Does not understand place value
Multiplication	28 x3 76	Saying three eights are sixteen Does not know tables/basic facts
Multiplication	20 x3 63	Saying three zeros are three Problem with zero
Multiplication	302 x3 96	Failure to understand the function of zero
Division	2)58 24	'Not carrying one Poor concept of division
Division	3)75 22	Adding one instead of ten to the units Problems with place value
Division	2)418 24	Ignoring the one Not recording zero Inability to estimate Poor concept of division
Division	2)418 29	Not recording zero Inability to estimate
Decimals	2.35 4 940	Not putting in the decimal point Inability to estimate Understanding of decimals not adequately developed
Decimals	2.315 is bigger than 3.26	Choosing 2.315 based on the 'length' of the number rather than using knowledge of place value Understanding of decimals not adequately developed
Measure: changing units	2.42km = 242m	Concept of measurement not adequately developed

The strategies that follow are designed to assist students who have difficulties such as those outlined in the table above including poor concept of place value, poor understanding of the relevant algorithm, and difficulty remembering number facts.

Examples of strategies

Each group of strategies covers a range of different levels on a topic from a particular strand unit. A list of necessary materials and resources is provided and an overview of the type of methodology employed in the strategies is included. Specific class levels are not stated as different students may benefit from these strategies at different stages.

Strand: Number

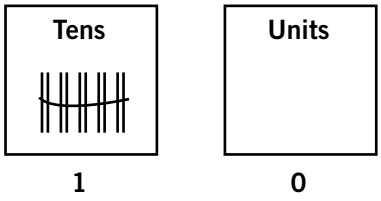
STRATEGIES FOR TEACHING PLACE VALUE

Materials and resources: Lollipop sticks, elastic bands to group the lollipop sticks into bundles, a selection of square pieces of cardboard measuring approximately 20cm×20cm for place value mats.

Methodology: The strategies outlined below involve the use of concrete materials and teacher-student dialogue.

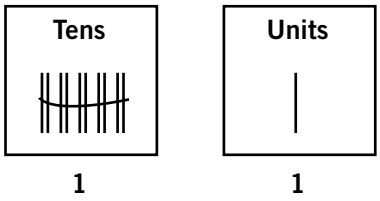
STRATEGY: Place value mats

The teacher shows bundles of lollipop sticks to the students and asks them to find out how many there are in each bundle. They discover that there are ten in each. Then they are asked to count out ten single lollipop sticks on their desks. The teacher asks them to make a ten and then poses the question 'How many are left?' The bundle of ten is placed on the tens' mat.



The following teacher-student dialogue draws out the concept of place value.

Teacher	Student
How many tens in the tens' place?	One.
We write one in the tens' place.	
How many units in the units' place?	None.
We write zero in the units' place. Now we all write the number 10.	
Take the elastic off the ten and add one more.	
How many have we now?	Eleven.
If I make a ten how many will be left?	One.
Put them on their mats. Write the number eleven.	Eleven makes one ten and one unit.



The same approach can be used to demonstrate numbers up to nineteen. For twenty, the dialogue should lead to the fact that twenty makes two tens with no units left over, giving 20. All numbers up to 99 can be demonstrated in this way.

STRATEGY: Pick up a number

Student understanding of the concept of number and place value can be assessed by asking the student to pick up a given number of lollipop sticks. For example, if the student is asked to pick up 21 lollipop sticks does the student:

- pick up a bundle of 10, another bundle of 10 and 1 single stick. [Correct]
- count out 21 single sticks. [Correct, but now ask the student to pick up 21 sticks in the quickest possible way]
- pick up a bundle of 10, another bundle of 10, (saying *'that is 20'*) and a third bundle of 10 (saying *'that is 21'*). [Incorrect, encourage the student to count the lollipop sticks to discover that they have 30, not 21, and to try again?]

Strand: Number

STRATEGIES FOR TEACHING COUNTING

Materials and resources: Lollipop sticks, unifix cubes, money, circular or square card, or paper for number line steps.

Methodology: The strategies outlined below involve the use of concrete materials, teacher-student dialogue, and discovery learning.

STRATEGY: Walking the number line

Some students will have difficulties recognising numbers and understanding the ordinality of numbers, for example 4 comes after 3 and 5 comes after 4. Rather than present the number line as a straight line on paper or on the board, it may be useful to present it as a series of separate discs or squares as shown in the diagram below:

Initially, large versions of the discs can be placed on the floor by the students. Having to replace the discs in their correct order each day provides reinforcement of the positions of the numbers for the students. Students can now be asked to walk, skip, or jump along the number line.

Teacher	Student
'Stand on the number 4. *Now take one step forward.* *Where are you now?'*	5
Take another step forward. Where are you now?	6

After considerable time has been spent walking the number line students can be asked hypothetical questions.

Teacher	Student
'If you are at 6 and you take one step forward, where are you now?'	7

This can be reinforced with some simple written exercises on adding 1.

This strategy can be developed by asking the student to take two steps forward leading to simple written exercises on adding 2.

Students then construct their own number line on their desks using smaller discs provided by the teacher. For a student experiencing difficulties, the number line can be shortened and then extended as the student achieves proficiency. In order to 'wean' the student off the number line some of the discs can be inverted so that the numbers are no longer visible.

The number of inverted discs can be gradually increased as the student progresses until eventually the student is able to perform the operation without the aid of a number line.

STRATEGY: Counting on

Before the number line is totally removed as an aid, the students must be taught the skills of counting on using their fingers. The teacher places two sets of objects on the table, for example a group of 8 objects and a group of 3 objects. The student is asked to hold up the number of fingers corresponding to the smaller set of objects (in this case 3 fingers). As the teacher moves one object from the smaller set into the larger set, she/he presses the first of the three raised fingers saying, *'8 and another 1 makes 9'.* While moving the second object, she/he holds the second finger saying, *'9 and another 1 makes 10'.* Finally, as the third object is moved, the third raised finger is held and the teacher says, *'10 and another 1 makes 11'.* The aim is that the student will eventually be able to follow this procedure independently. The reverse process applies when teaching students to subtract from numbers greater than ten.

STRATEGY: Twenty-ten

To overcome the difficulty that some students experience with counting beyond twenty-nine, for example twenty-nine, twenty-ten, twenty-eleven, etc., try teaching counting, place value, and writing numbers concurrently. Engaging in activities such as making the numbers with lollipop sticks, unifix cubes, or money, or placing them on place value mats, and learning to write the numbers will improve the students' understanding and help to accelerate the learning process.

STRATEGY: Which is bigger?

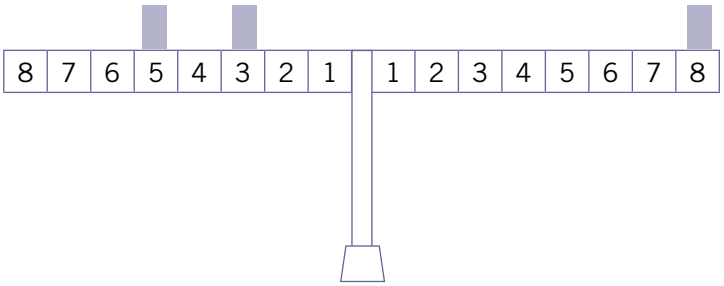
19 lollipops may appear to be more than 21 lollipops especially when the units are spread out as in the diagram below.

19 **21**

It may be necessary for students to count both sets of sticks in order to confirm that 21 is bigger than 19. It may help students to make pairs of numbers such as 12 and 21, 24 and 42, 25 and 52, etc. In this case physical appearance will more than likely be enough to indicate which is the greater number.

STRATEGY: Number balance

Using the number balance, students can discover a range of number facts for themselves. For example, by placing a certain weight on number 8 on the right-hand side of the balance and a similar weight on both of the numbers 5 and 3 on the left-hand side the two sides will balance.



This will also occur if the weights on the left-hand side are placed on 7 and 1, or 6 and 2, or 4 and 4, but it will not happen with any other combination of numbers. In this way students can discover 'the story of 8'. A discovery may be more easily retained by the student than a list of facts taught directly by the teacher.

Strand: Number

STRATEGIES FOR TEACHING ADDITION (NUMBER FACTS)

Materials and resources: hundred square, lollipop sticks.

Methodology: The strategies outlined below are designed to reduce to a minimum the number of addition facts that must be memorised. If all of the strategies outlined below are successfully utilised very few facts remain to be committed to memory. The usefulness of the strategies will vary according to the ability of the student, but mastery of even a few of the strategies should be of benefit. A very similar approach can be taken to assist the student in the memorisation of subtraction facts. A student who has great difficulty in memorising the number facts can be provided with the basic facts in the form of a hundred square permanently taped onto his/her desk.

STRATEGY: Start with the larger number

It is easier to start with the larger number and add on the smaller number. For example, 8+4 is easier to work out by counting on than by attempting 4+8.

STRATEGY: Addition is commutative

Plenty of practice using concrete materials to show that addition is commutative (3+4 is the same as 4+3) will halve the number of addition facts that a student needs to memorise.

STRATEGY: Adding zero, one, and two

Adding zero to a number gives the number itself.
Adding one to a number gives the next number.
Adding two to a number gives the number after the next number.

STRATEGY: Adding on 10

Using concrete materials such as lollipop sticks students can see the pattern that develops when numbers are added to 10.

For example: 10+1 = one ten + one unit = 11
10+2 = one ten + two units = 12
10+3 = one ten + three units = 13

STRATEGY: Doubles

The twelve double facts (1+1=2, 2+2=4, and so on to 12+12=24) should be committed to memory at an early age. This should be possible if one or two facts are taken at a time. With the aid of flash cards and oral repetition, both individually and as a group, students can progress at their own pace, moving onto a new fact once previous facts are well established.

Once these facts are memorised, facts such as 3+4 can be treated as one more than 3+3, giving 3+4 = 7. Similarly, 5+7 can be treated as two more than 5+5, giving 5+7 = 12. The double facts can also be used to find one less and two less.

STRATEGY: Nines and elevens

9+6 is one less than 10+6 = 16, giving 9+6 = 15. It may also be useful to note that when adding on nine to a number (called the addend), where the answer is in the teens, the addend is the result of adding the tens digit to the units digit. This pattern can be seen in the example below.

9+6 = 15	1+5 = 6
9+7 = 16	1+6 = 7
9+8 = 17	1+7 = 8

Similarly, when adding eleven to a number, it can be treated as one more than adding ten. And 11+4 is one more than 10+4 = 14, giving 11+4 = 15. Also adding eleven is very similar to adding one, and some concrete work with lollipop sticks will demonstrate the pattern to the students.

STRATEGY: Adding twelve

Adding twelve to a single digit is very similar to adding two. Concrete work with lollipop sticks will demonstrate the pattern to the students. For example, 12+9 and 12+11 can be obtained by adding one less and one more respectively than when adding twelve to ten.

STRATEGY: Use of games

Reinforcement of simple number facts can be achieved by playing simple card games such as Snap, Multo, and Old Maid, and by the appropriate use of suitable computer programmes. The introduction of quizzes can add an element of competition to the learning of number facts and can improve student motivation.

Strand: Number

STRATEGIES FOR TEACHING ADDITION

Materials and resources: lollipop sticks, magnetic board.

Methodology: Many of the mistakes made by students carrying out the addition algorithm are made due to poor understanding of the concepts involved. The following strategies suggest concrete materials and teacher-student dialogue to explain and practice the addition algorithm.

STRATEGY: To carry or not to carry

A student who insists on carrying when it is not appropriate can be asked to make the numbers with lollipop sticks. When they can see that there is not enough to make a ten, this can be linked to the fact that we do not carry. In addition, a list of addition sums can be given to the student with the instruction to circle the ones that involve carrying. The student can then concentrate on whether or not carrying is appropriate without actually having to do the full calculation.

STRATEGY: Magnetic board

While many students can cope with addition without renaming, many will experience difficulty with addition with renaming unless it is done with concrete materials. In the following example, the student is asked to make two numbers and combine them (with renaming) using concrete materials representing tens and units displayed on a magnetic board. The teacher-student dialogue is described first.

Teacher	Student
Make twenty-three.	
Make thirty-eight.	(Figure 1).
Add the units. *How many units are there?*	 *Eleven* (Figure 2).
Have we enough to make a ten?	*Yes.*
Make a ten and place it in the tens' place.	
How many will be left?	*One.*
We write one in the units' place.	
We write one in the tens' place *for the one ten we made.*	(Figure 3).
Now add the tens. *How many tens are there?*	 *Six.*
We write six in the tens' place.	(Figure 4).
How many altogether?	*Sixty-one.*

Figure 1

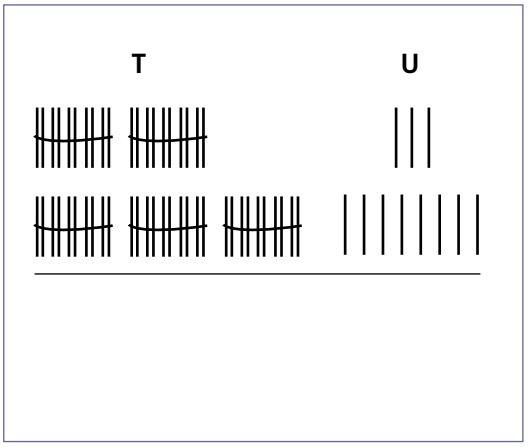


Figure 2

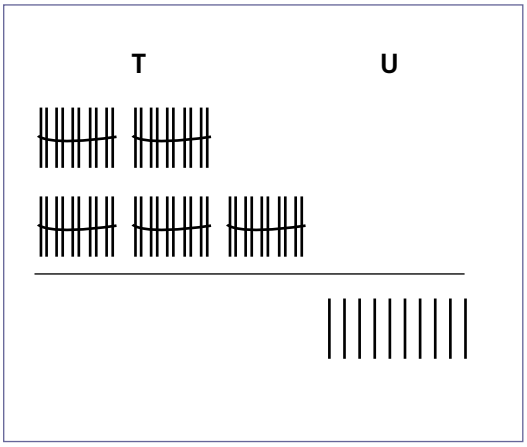


Figure 3

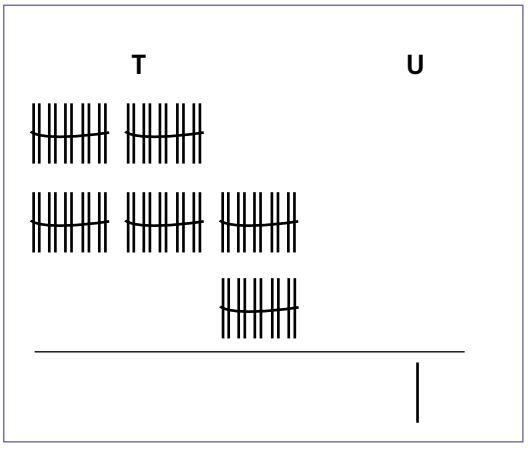
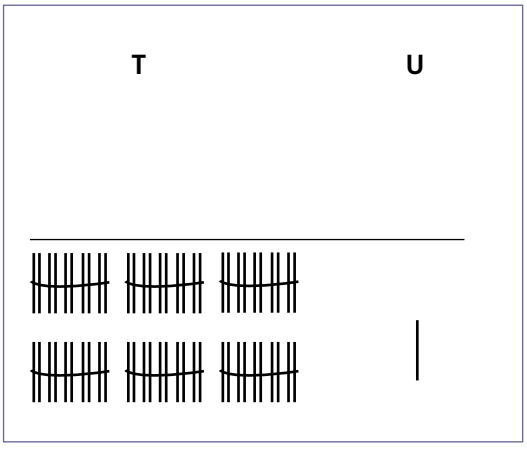


Figure 4



Strand: Number

STRATEGIES FOR TEACHING SUBTRACTION

Materials and resources: lollipop sticks, magnetic board, place value mats, pretend euro notes and coins to represent hundreds, tens, and units.

Methodology: Subtraction with renaming can be very problematic for the student. In particular the language of subtraction can vary from person to person. Planning for consistent language and methodology in the school and in the home can greatly assist the learning process. The following strategies suggest concrete materials and teacher-student dialogue to explain and practice the subtraction algorithm

STRATEGY: Magnetic board

Subtraction with renaming can cause difficulty for students unless it is done with concrete materials. In the following example, the student is asked to make two numbers and subtract them (with renaming) using concrete materials representing tens and units displayed on a magnetic board. The teacher-student dialogue is first described for the algorithm.

Teacher	Student
Make fifty-one.	
Three from one we cannot take.	(Figure 1).
Break up a ten. *How many tens and units do we have now?*	*We now have four tens and eleven units* (Figure 2).
Take three from eleven. *What does that leave?*	*Three from eleven leaves eight.*
Take two from four. *What does that leave?*	*Two from four leaves two* (Figure 3).
How many do we have left altogether?	*Twenty eight* (Figure 4).

Figure 1

51
- 23

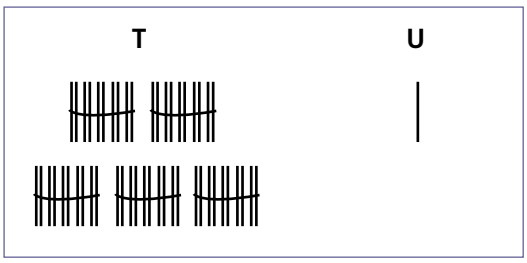


Figure 2

$^{4}\not{5}^{1}1$
- 2 3

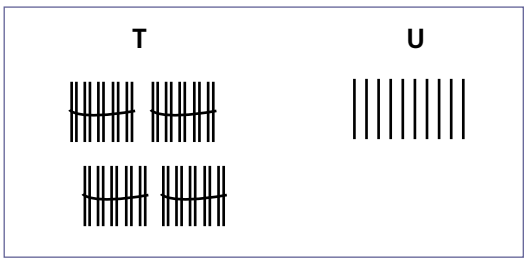


Figure 3

$^{4}\not{5}^{1}1$
- 2 3
2 8

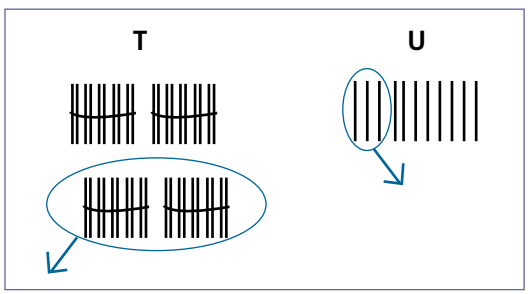


Figure 4

$^{4}\not{5}^{1}1$
- 2 3
2 8

STRATEGY: Subtracting larger numbers

Using money instead of lollipop sticks can make larger numbers easier to handle. Money is convenient to use and is familiar and meaningful to the student. Larger amounts of money, for example thousands, can be represented by a cheque. The following example can be done on a magnetic board or on place value mats:

400
- 155

Teacher	Student
Make four hundred.	
Zero take five we cannot do.	(Figure 5).
We need to break up a ten but there is no ten there. *We break up a hundred.* *How many tens and units are there now?*	*There are ten tens and no units* (Figure 6).
We break up a ten. *How many tens and units are there now?*	*There are nine tens and ten units* (Figure 7).
Ten take five leaves? *Nine take four leaves?* *Three take two leaves?*	*Five.* *Five.* *One.*
How many do we have left altogether?	*One hundred and fifty five* (Figure 8).

Figure 5

H	T	U
		

Figure 6

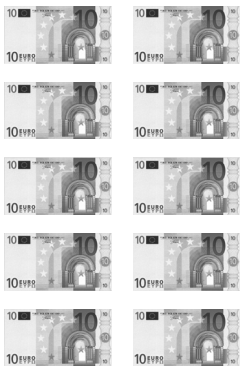
H	T	U
		

Figure 7

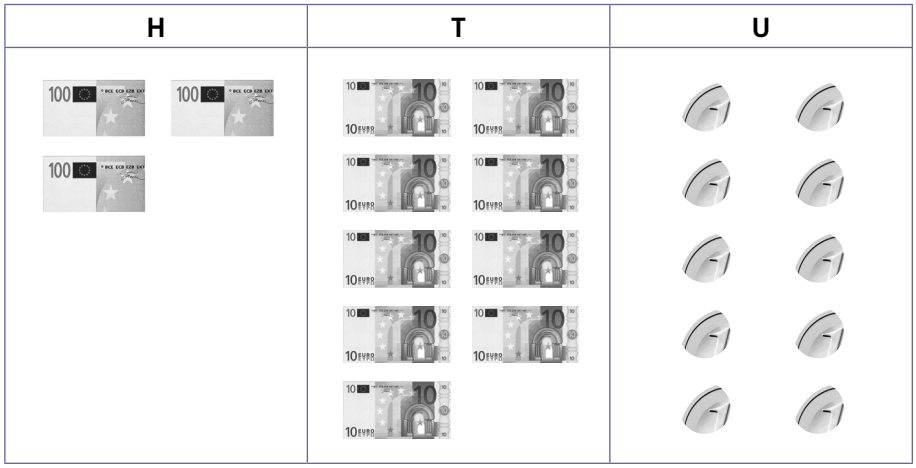
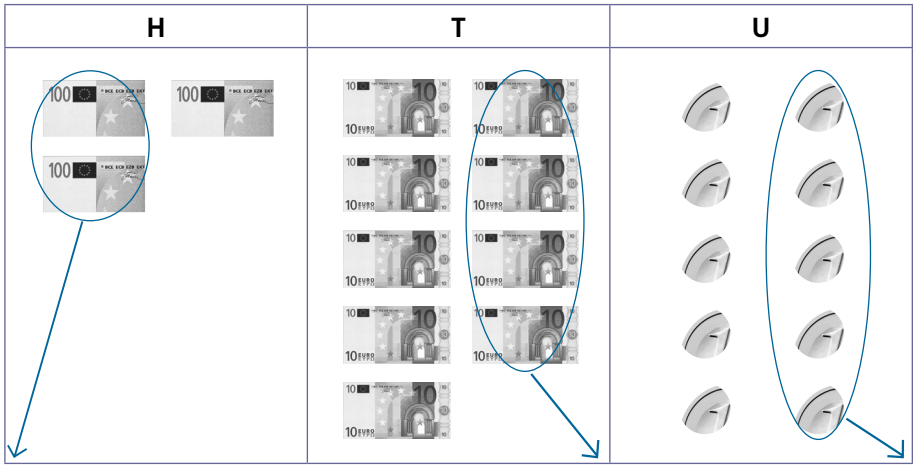


Figure 8



Strand: Number

STRATEGIES FOR TEACHING MULTIPLICATION

Materials and resources: lollipop sticks, magnetic board, place value mats, pretend euro notes and coins to represent hundreds, tens, and units.

Methodology: Students with mild general learning disabilities will benefit from the use of concrete materials when being introduced to multiplication.

STRATEGY: Single digit by single digit

Lollipop sticks can be used to introduce multiplication as repeated addition, as shown below:

3 + 3 + 3 + 3 = 4 x 3 =12

5 + 5 = 2 x 5 =10

STRATEGY: Two digits by one digit

Teacher	Student
Make three twenty fours.	(Figure 1).
What do three fours make?	*Twelve.*
How many tens is that? *How many units are left?* *Write two in the units' place and put the ten in the tens' place.*	*One ten.* *Two units.* (Figure 2).
Now, in the 'tens' place what do three twos make? *And the one ten we made gives?*	*Six.* *Seven.*
So what do we have altogether?	*Seventy-two* (Figure 3).

Figure 1

24
x 3

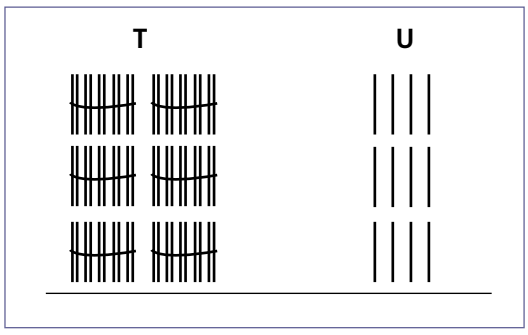


Figure 2

24
x 3
2

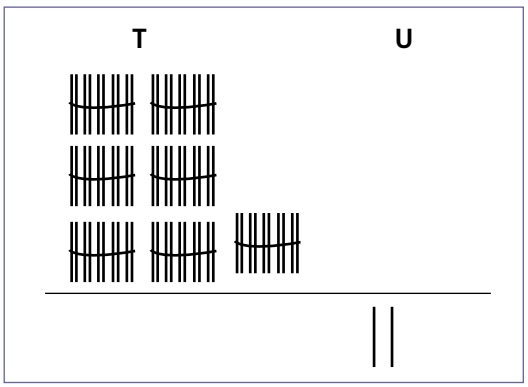
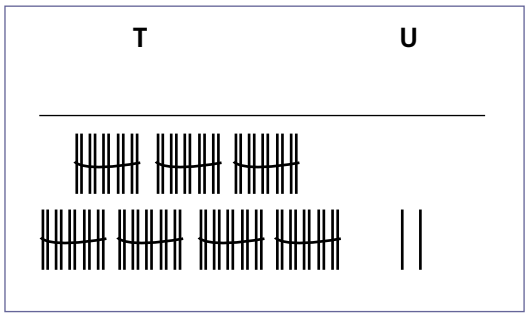


Figure 3

24
x 3
72



Strand: Number

STRATEGIES FOR TEACHING DIVISION

Materials and resources: lollipop sticks, magnetic board, place value mats, pretend euro notes and coins to represent hundreds, tens, and units.

Methodology: There are two concepts involved in division. One is sharing. (For example, if 12 apples are shared between 3 students how many apples will each student get?) The other is repeated subtraction. (For example, how many 3s are in 12?) The strategies presented below vary from initial division activities to division with regrouping. Once again the emphasis is on the use of concrete materials. Students record the algorithm only when they have become very familiar with the steps involved.

STRATEGY: Early division activities

Before being introduced to the concept of division, students should have plenty of practice counting up in twos, threes, fours, and so on using lollipop sticks, beads, and other concrete materials. This skill can be reinforced through written exercises such as the one below.

Fill the missing numbers into the boxes:

2, 4, ☐, 8, 10, ☐

5, 10, ☐, 20, ☐, 30

Practice through oral counting can provide further reinforcement. It may take a long time to achieve proficiency in the more difficult numbers, 6, 7, 8 and 9, but students may still learn short division through tasks based on the easier numbers.

STRATEGY: Division as groups

Dialogues such as the one shown in the table below accompanied by suitable work with concrete materials will follow the initial division activities.

Teacher	**Student**
Count out six lollipop sticks.	
Put them into groups of two.	
How many groups of two are there in six?	*Three.*
How many twos in six?	*Three.*
We write it like this: $2\overline{)6}$ 3	
Count out twelve lollipop sticks.	
This time we want to find out how many threes in twelve. So we put them into groups of three.	
How many groups are there?	*Four.*
Now many threes in twelve?	*Four.*
We write it like this: $3\overline{)12}$ 4	

STRATEGY: Division as sharing

When moving on to dividing a two-digit number by a single digit number it may be better to use the concept of sharing in order to get over the difficulty caused by place value, especially when renaming is involved. The following example shows the procedure that may be followed, and the type of dialogue that may be encouraged:

Teacher	Student
We want to share €75 between these three students.	
Make €75 with money.	(Figure 1)
First we share the tens. *How many €10 can we give to easch people*	*Two tens each* (Figure 2)
How many tens are left?	*One*
We write: 3) $\underline{75}$ 2	
How will we share out the last 10? Will we get a scissors and cut it into three equal parts?	*No way!* *Go to the bank and change the 10 note for ten one-euro coins*
We had five units and we made ten more units. How many units do we have now?	*Fifteen* (Figure 3)
We write: 3) $\underline{7^{1}5}$ 2	
Now we share the units. How many one-euro coins can we give to each pupil?	*Five each* (Figure 4)
We write: 3) $\underline{7^{1}5}$ 2 5	
How much money does each pupil get altogether?	*Twenty five euro each* (Figure 5)

Figure 1

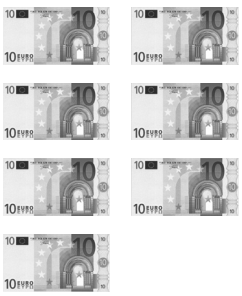
T	U
	

Figure 2

Figure 3

T	U

Figure 4

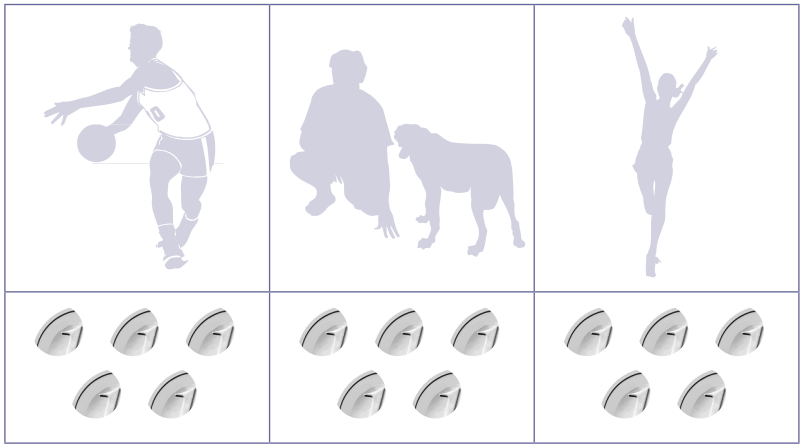
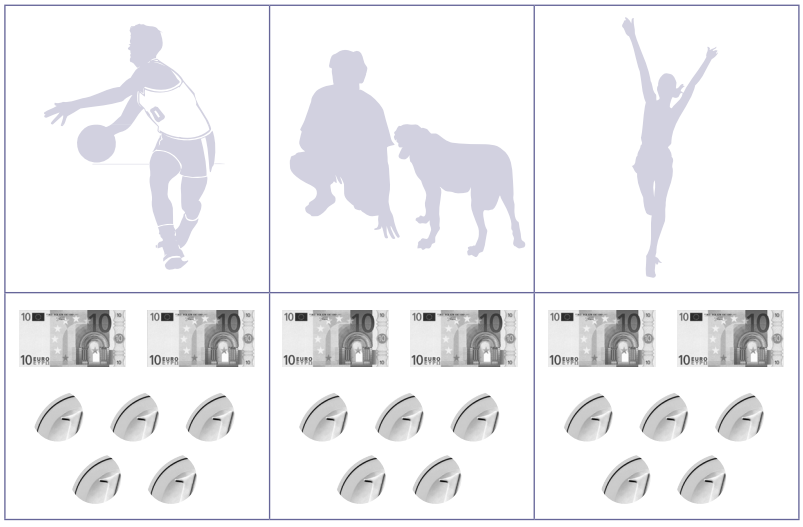


Figure 5



Strand: Number

STRATEGIES FOR TEACHING FRACTIONS

Materials and resources: paper for paper-folding activities, various shapes and fractions of these shapes (circle, rectangle), lollipop sticks.

Methodology: Fractions are part of a new number system and consequently the learning of fractions is challenging for most students. It is vital that students use concrete materials for the introduction of these new concepts. The language of fractions must also be introduced, internalised, and verbalised by the students. The strategies below concentrate on verbal exchange and use of concrete materials to introduce various fractional concepts.

STRATEGY: Introducing the language of fractions

The dialogue below takes place while students have access to a number of different paper shapes or other suitable materials that can be split into halves.

Teacher	Student
If you divide something into two equal parts, what do you call each part?	*A half.*
A half is written like this: V	
How do you get a half of something?	*You divide it into two equal parts.*
If you eat a half of a cake what part have you left?	*You still have a half left.*
Put them into groups of two.	

STRATEGY: Equivalence of fractions

The same procedure as above is followed with thirds and quarters, but when a quarter is defined and the students are able to tell the teacher how to get a quarter or three quarters, the shapes representing halves and quarters are placed side-by-side and the students are asked what they can discover from this.

They should understand that a half is the same as two quarters. They can now be encouraged to open a discovery chart at the back of their copies. This chart can be used to record information discovered by the students in the dialogues. This method of discovering the equivalence of fractions is more meaningful to students than presenting them with an overcrowded fraction wall.

STRATEGY: Finding a fraction of a number

The language of fractions is a vital part of this concept. The use of lollipop sticks allows the following approach to be taken:

'To find a third of six lollipop sticks we divide the lollipop sticks into three equal parts. This gives two sticks in each part. So one third of six is two.'

In finding a multiple fraction of a set it may be useful to first find the unit fraction, and then to stagger, physically, the multiple fractions as shown in the diagram overleaf. The act of putting the various unit fractions together can help to consolidate the concept.

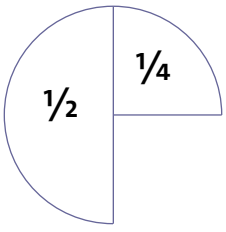
⅓ of 12 *Take 12 lollipop sticks and divide them into three equal parts.*		⅓ of 12 is 4
⅔ of 12 *We count the first third and the second third* (Note the staggering)		⅔ of 12 is 8
¼ of 12 *Take 12 lollipop sticks and divide them into four equal parts.*		¼ of 12 is 3
¾ of 12 *We count the first quarter and the second quarter and the third quarter.* (Note the staggering)		¾ of 12 is 9

Getting a multiple fraction of a set directly, without having first got a unit fraction of a set, is more difficult. It should not be attempted until the above procedure is well established. Dialogue based on finding multiple fractions of a cake could proceed like this:

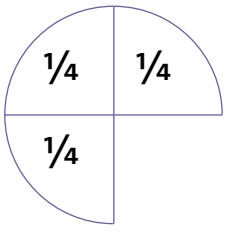
'To get ⅓ of a cake we first divide it into three equal parts to make thirds. Then we put two of the parts together to make two-thirds.'
'To get ¼ of a cake we first divide it into four equal parts to make quarters. Then we put three of the parts together to make three-quarters.'

STRATEGY: Addition and subtraction of fractions
Once again, concrete materials such as shapes on a magnetic board are a useful aid to grasping the necessary concepts. However, addition and subtraction of fractions should not be attempted until the students are very familiar with the equivalence of fractions. The need for a common name or 'common denominator', when adding and subtracting fractions, can be compared to asking, 'What is one horse plus two donkeys? It is one animal plus two animals, which is three animals, animals being the name that suits both?' Similarly, when adding thirds and quarters the name that suits both is found by counting up in threes and fours until a common name (twelve) is found.

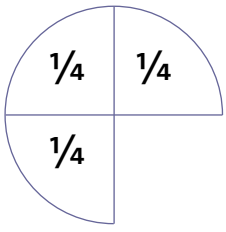
The following example shows how fraction shapes can be used in the addition of simple fractions. The approach for more complex fractions and for subtraction is similar.



$\frac{1}{2} + \frac{1}{4}$



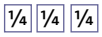
$= \frac{3}{4}$



$= \frac{2}{4} + \frac{1}{4}$

STRATEGY: Given a fraction, find the whole

Perhaps the most difficult fraction concept to teach is when the student is given a multiple fraction of a set and asked to find the whole set, for example three-quarters of a number is 30, find the whole number. Most students find this difficult because they associate a quarter with dividing by four. The following approach which uses the unitary method may help:

	=	30c	Three pencils cost 30c.
	=	10c	One pencil costs 10c.
	=	40c	Four pencils cost 40c.
	=	30	Three quarters is 30.
	=	30	Three quarters is 30.
	=	10	One quarter is 10.
	=	40	Four quarters or one whole is 40.

Fractions are a difficult concept to teach, but with the help of concrete materials, good teaching methodology, and careful pacing, many students with mild general learning disabilities can experience success.

Strand: Number

STRATEGIES FOR TEACHING DECIMALS

Materials and resources: pencils, crayons, and other countable objects, lollipop sticks (units and tenths), elastic bands to group the lollipop sticks into bundles, place value mats, measuring and weighing instruments, items of various weights, magnetic board, Dienes blocks, and magnetised Dienes blocks.

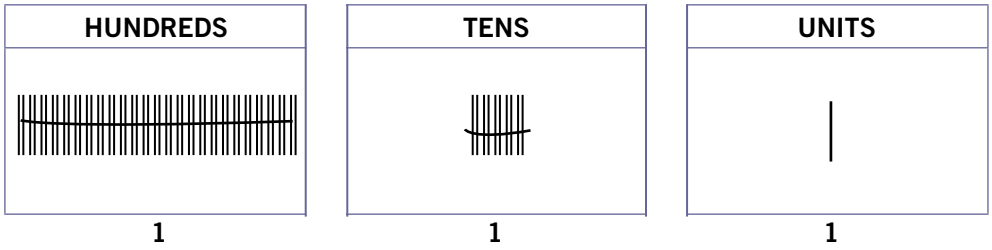
Methodology: The strategies outlined below involve the use of concrete materials and teacher/student dialogue to reinforce the fundamental concept that decimals involve bits, pieces, or fractions of whole numbers. Building on the already understood concept of place value, the strategies also suggest linking with decimals in other areas of the curriculum, for example measure.

STRATEGY: Introducing decimals

Understanding of decimals requires a solid grasp of the concept of place value. Revision of place value is a good starting point for introducing decimals. This can lead on to group discussion on the size of various objects in the class, for example:

This pencil is longer than that pencil. It is twice as long.
The small pencil is two times shorter than the long pencil.
The larger box of crayons holds more than the small box.
It holds three times as many crayons.
The small box holds three times less than the large box.

When the students have internalised this language the teacher makes one hundred and eleven with lollipop sticks, places them on the place value mats, and writes the number one hundred and eleven as shown in the diagram.



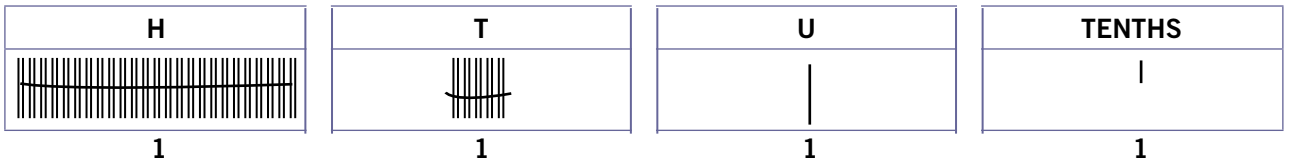
The following teacher-student dialogue draws out the concept of decimals:

Teacher	**Student**
Which is bigger, the ten or the unit? (holding up the ten and the unit)	*The ten is bigger.* *The ten is ten times bigger than the unit.*
Which is bigger, the hundred or the ten? (holding up the hundred and the ten)	*The hundred is bigger.* *The hundred is ten times bigger than the ten because it takes ten tens to make a hundred.*
So the ten is ten times bigger than the unit and the hundred is ten times bigger than the ten. What comes next?	*Something that is ten times bigger.* *Every time we move to the left it gets ten times bigger.* *One thousand is ten times bigger than one hundred.*

The procedure is then reversed.

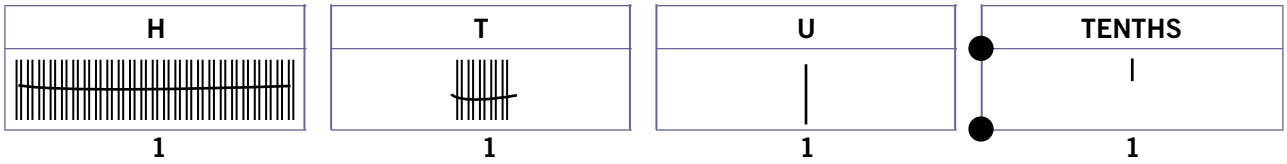
Teacher	**Student**
Which is smaller, the hundred or the ten? (holding up the hundred and the ten)	*The ten is smaller.* *The ten is ten times smaller than the hundred.*
Which is smaller, the ten or the unit? (holding up the ten and the unit)	*The unit is smaller.* *The unit is ten times smaller than the ten.*
So the ten is ten times smaller than the hundred and the unit is ten times smaller than the ten. *What comes next?*	*Something that is ten times smaller.* *Every time we move to the right it gets ten times smaller.*

Before establishing what is ten times smaller than the unit some 'scaffolding' is required. One possible approach is to show a unit and a half of a unit of a lollipop stick showing that the half is two times smaller than the unit. Then show a unit and a quarter of a lollipop stick to demonstrate that a quarter is four times smaller than a unit. Finally establish that a tenth is ten times smaller than a unit. Introduce a tenth as a new 'number in the family' and explain that it is given its own placeholder.



Return to the hundreds tens and units to reinforce. *Ten times smaller than a hundred is ten. Ten times smaller than ten is a unit and ten times smaller than a unit is one tenth.*

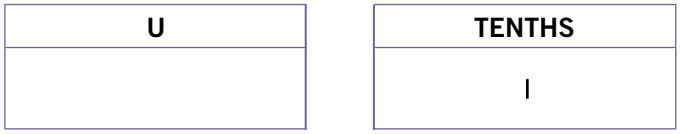
Teacher	Student
So the ten is ten times smaller than the hundred and the unit is ten times smaller than the ten. What comes next?	*Ten times smaller than a unit is one tenth.*
Which is the odd one out?	*The tenth is the odd one out because it is only a piece or a fraction. All of the others are made up of whole lollipop sticks.*
Because it's the odd one out, the tenth is separated from the rest by a decimal point. Anything to the left of the decimal point is made up of whole things. Anything to the right of the decimal point is made up of pieces, bits or fractions.	



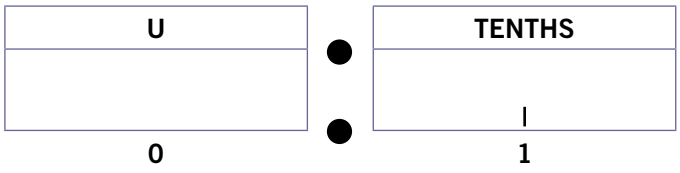
This dialogue and work with concrete materials serves to establish the function of the decimal point as separating whole things from pieces of things.

STRATEGY: Writing simple fractions as decimals

The teacher draws the units and tenths placeholders on the magnetic board and, using lollipop sticks and tenths of lollipop sticks with magnetic backing, asks a student to pick up one tenth and place it in the tenths' place. Then the following discussion takes place.



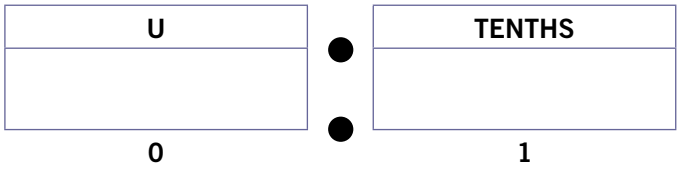
Teacher	Student
How many units do we have?	*None.*
We write zero in the units' place.	
How many tenths do we have?	*One.*
We write one in the tenths' place.	
What separates the whole thing from the pieces of things?	*The decimal point.*
We write the decimal point between the units and the tenths.	



The same language and methodology is used for $\frac{2}{10}$, $\frac{3}{10}$, ..., $\frac{9}{10}$ and so on.

STRATEGY: Writing simple decimals as fractions

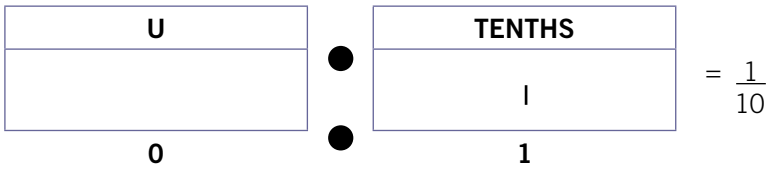
The student is asked to pick up 0.1 lollipop sticks and the following diagram is show on the magnetic board.



From the board, the student can see that 0.1 means no units and one tenth. He/she picks up one tenth of a lollipop stick and places it in the tenths placeholder.

Teacher-student dialogue then takes place.

Teacher	Student
I asked you to pick up 0.1 lollipop sticks. What did you pick up?	*One tenth of a lollipop stick.*
So, 0.1 is one tenth. *We write:* $0.1 = \frac{1}{10}$	



$= \frac{1}{10}$

The same language and methodology is used for 1.2, 1.3, 2.4, and so on.

STRATEGY: Using decimals when measuring length

After students have learned about decimals and about measure, further reinforcement can take place by linking these two concepts. The following exercise involves rewriting metres as centimetres and assumes understanding of simple relationships between fractions of a metre and centimetres:

$\frac{1}{2}$ m = 50cm $\frac{1}{4}$ m = 25cm

$\frac{1}{10}$ m = 10cm $\frac{3}{10}$ m = 30cm

The teacher draws a line on the board measuring 1.1m and explains that 1.1m means one full metre and one tenth of a metre, and is the same as 1,100cm. After further examples, students are asked to draw 1.4m, 1.7m, 2.1m, and so on.

STRATEGY: Using decimals when measuring weight

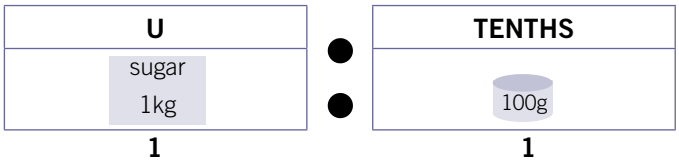
This exercise assumes understanding of simple relationships between fractions of a kilogram and grams:

$\frac{1}{2}$ kg = 500g $\frac{1}{4}$ kg = 250g

$\frac{1}{10}$ kg = 100g $\frac{3}{10}$ kg = 300g

Using a selection of 1kg bags and smaller bags weighing 100g, the students are asked to pick up 1.1kg. The work can be displayed as follows:

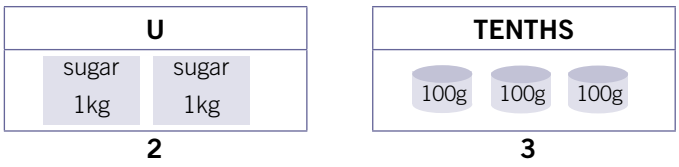
1.1kg = 1kg + $\frac{1}{10}$ kg



= 1,000g + 100g = 1,100g

When rewriting grams as kilograms students are asked to pick up 2300g in the quickest possible way and to place the weights in the placeholders. The work can be displayed as follows:

2,300g = 2kg +300g



= 2.3kg

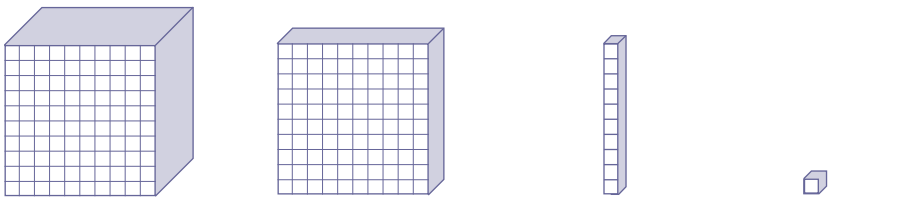
A similar approach can be taken with other units of measure.

STRATEGY: Operations involving decimals

The magnetic pieces used when teaching fractions can also be used as concrete aids for the addition, subtraction, multiplication, and division of decimals. Using whole circles to represent units and tenths of a circle to represent tenths, the algorithm used follows that already outlined in the section on fractions.

STRATEGY: Using Dienes blocks to explain hundredths and thousandths

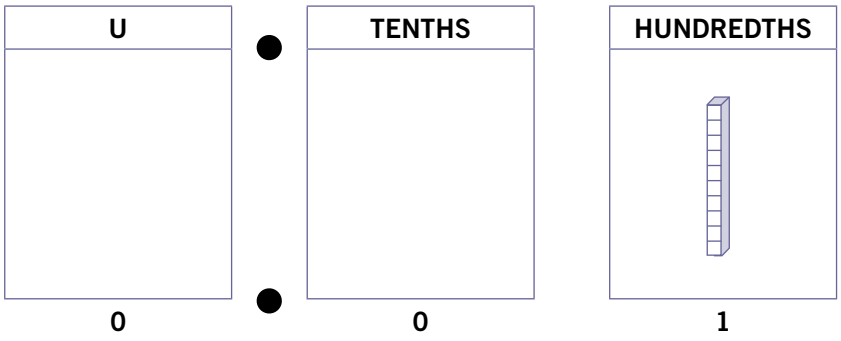
Dienes blocks and magnetised Dienes blocks can be used to make the concepts of hundredths and thousandths more concrete. The Dienes block measuring 10 cubes by 10 cubes by 10 cubes is used to represent one unit. The 10 by 10 piece represents one tenth. The 1 by 10 piece represents one hundredth and a single cube represents one thousandth.



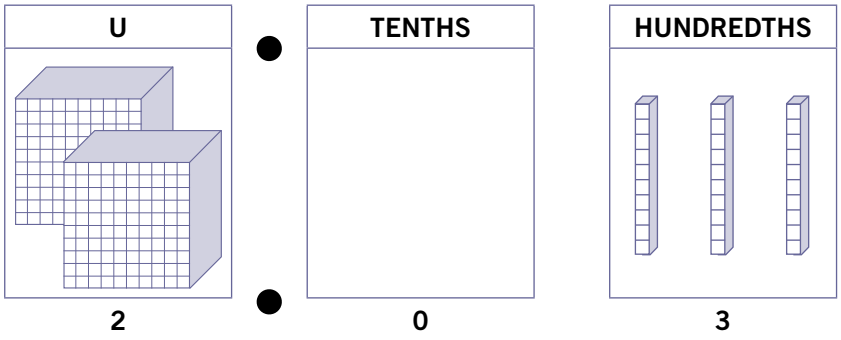
one unit one tenth one hundreth one thousandth

The language and methodology used for introducing tenths can be used for introducing hundredths and thousandths. Eventually, students present their work as shown in the following diagrams.

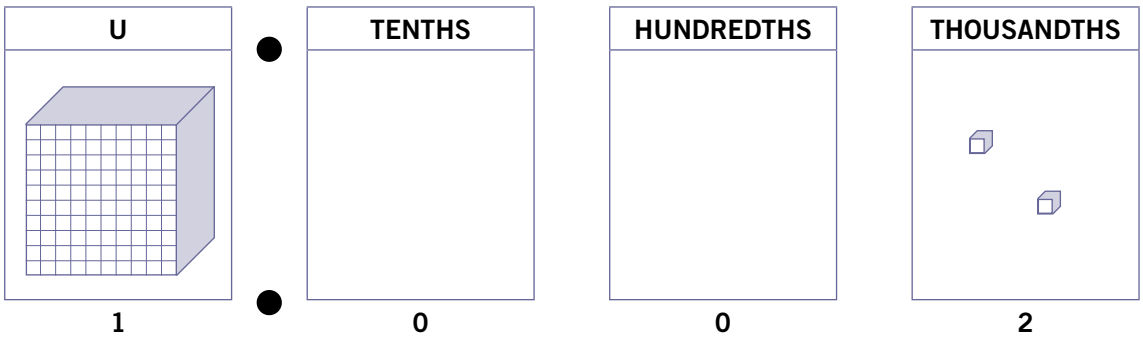
$\frac{1}{100}$ =



$2\frac{3}{100}$ =



$1\frac{2}{1,000}$ =



Strand: Measures

STRATEGIES FOR TEACHING LENGTH AND WEIGHT

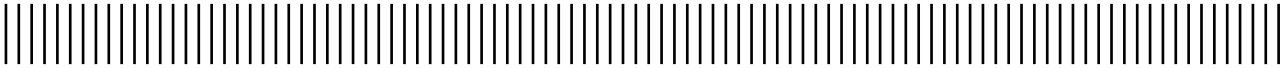
Materials and resources: lollipop sticks, place value mats, measuring and weighing instruments, string, items of various weights.

Methodology: The strategies presented here focus on areas of difficulty with length and weight and, in particular, on the use of decimals in measurement.

STRATEGY: Parts of a hundred and a thousand

Familiarity with fractions of a hundred and a thousand will help the student to deal with decimals in measure. A concrete representation of fractions of a hundred could be provided using lollipop sticks, and appropriate dialogue will reinforce the concepts.

The first diagram shows one hundred lollipop sticks.



The second diagram shows one hundred lollipop sticks divided into two equal parts. Each part is a half. A half of one hundred is fifty.

Here the one hundred is divided into four equal parts. Each part is a quarter of a hundred. A quarter of a hundred is twenty-five. Two quarters are fifty. Three-quarters are 75.

Each part here is one tenth of a hundred, because the hundred is divided into ten equal parts. One tenth of a hundred is ten, two tenths are twenty, three tenths are thirty, and so on.

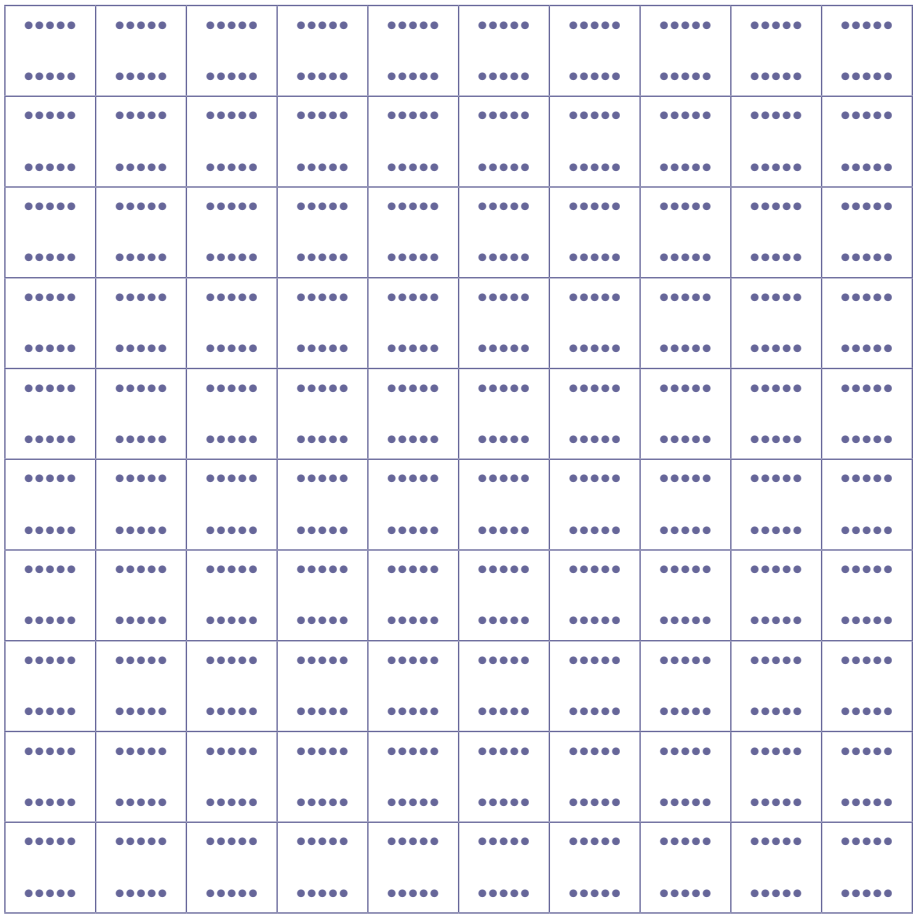
Finally, the lollipop sticks can be divided into one hundred equal parts. Each part is one hundredth of a hundred. One hundredth of a hundred is one. Two hundredths of a hundred is two, three hundredths of a hundred is three, and so on.

This work can be reinforced with prominently displayed wall charts containing facts that are particularly important for the understanding of decimals in measurement.

$\frac{1}{10}$ of 100 = 10

$\frac{1}{100}$ of 100 = 1

A visual approach is also necessary when dealing with fractions of a thousand. A wall chart with one thousand dots (or other small symbols or pictures) can be displayed.



Alternatively, students can build up such a picture by attaching boxes of ten dots to a display board until one thousand is formed. This will help students to have a mental image of one thousand as a quantity.

Relevant fractions of one thousand are discussed as outlined already for one hundred, and charts are displayed to highlight appropriate facts.

$\frac{1}{10}$ of 1,000 = 100

$\frac{1}{100}$ of 1,000 = 10

$\frac{1}{1,000}$ of 1,000 = 1

STRATEGY: Decimal place value chart

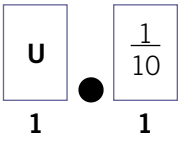
The following chart can be on permanent display in the classroom:

TH	H	T	U		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$
				●			

The students use this to see that 0.1 means one tenth, 2.3 means two units and three tenths, 0.01 means one hundredth, and so on.

STRATEGY: Drawing lines

Drawing lines of various lengths can reinforce the use of decimals in measure. The use of metres and centimetres illustrates parts of a hundred. Similarly, the use of centimetres and millimetres can reinforce the concept of tenths. The following simple example indicates the kind of approach that could be taken when drawing a line that measures 1.1m.



1.1m means one full metre and one tenth of a metre. This is drawn on the board and the following calculation is discussed.

1.1m = 1 metre + ½ of a metre = 100cm + 10cm = 110cm

This work continues so that students understand that each example contains a certain number of whole metres and a 'bit' or a 'part' of a metre.

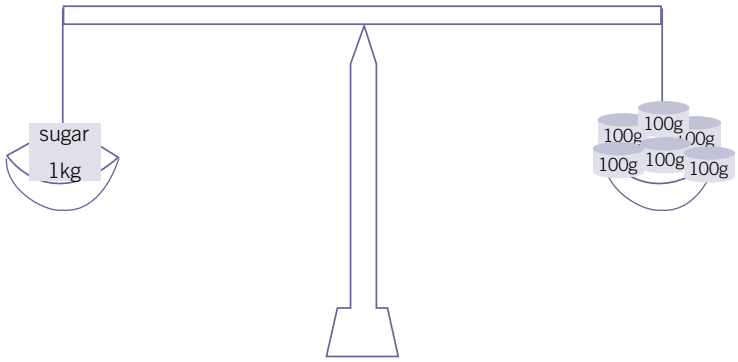
STRATEGY: How long is a piece of string?

Cutting and measuring pieces of string can provide a concrete example of various decimal operations. The table below outlines some examples:

Operation	Working with string
2.4m + 1.8m	Measure and cut two pieces of string with these lengths. Put them end to end and measure the whole length.
2.4m - 1.8m	Measure a piece of string of length 2.4 m. Measure 1.8m in from one end of it and cut it off. Measure the piece that is left.
2.4 x 3	Cut three pieces of string 2.4m long. Place them end to end and measure the whole length.
2.4m ÷ 2	Measure a piece of string of length 2.4m. Fold it so that the two ends meet and cut the string into two equal pieces. Measure one piece.

STRATEGY: A weight off your mind

Parts of a thousand can be demonstrated concretely with weight or capacity. Examination of the weights on the balance below and appropriate dialogue lead to the fact that one tenth of a kilogram is one hundred grams.



Given a decimal representation of a weight in kilograms, students can be asked to select the correct items, place them in the placeholders, and write the result in grams as outlined already in the decimal strategies.

Strand: Measures

STRATEGIES FOR TEACHING MONEY

Materials and resources: money (cent and euro coins and notes), unfix cubes, other resources appropriate for shopping role-playing.

Methodology: The strategies outlined below cover practical addition and subtraction experiences when using money, understanding the value of coins and notes, knowing what coins or notes are to be handed in when paying for goods at various prices, and knowing what change to expect.

STRATEGY: Pairs

After coin recognition has been achieved, reinforcement of 'the story of ten' can precede simple shopping activities. The number balance strategy has already been outlined. This card game further reinforces the pairs of numbers that combine to give ten.

Each pair of students has eleven cards as shown. J is the Joker.

1	2	3	4	5	6	7	8	9	10	J

The cards are shuffled and one player gets six cards and the other five. Both students discard all matching pairs, for example 8 + 2, 6 + 4. Then the player with the least number of cards picks from the person with the most. This continues until one person is left with the Joker. The other person is the winner.

After other suitable reinforcement activities students commit the story of ten to memory.

STRATEGY: Shopping with cent

The table below shows a simple shopping activity based on the story of ten. Suitable items can be named in the item column.

Item	Cost	Change out of 10c
	8c	2c
	5c	☐
	7c	☐
	6c	☐
	3c	☐
	2c	☐
	4c	☐
	1c	☐
	10c	☐

Students can also practice giving exact change for amounts not greater than 10c. Some examples are given in the table below.

Item	Cost	Exact coins
	2c	○ (2c) ○○ (1c + 1c)
	4c	○○ (2c + 2c) ○○○○ (1c + 1c + 1c + 1c)
	5c	○ (5c) ○○○ (2c + 2c + 1c) ○○○○○ (1c + 1c + 1c + 1c + 1c)

A similar approach can be taken with the story of 20. Students take part in a variety of activities using concrete materials such as unfix cubes until the pairs of numbers that combine to make twenty are memorised. It is important that the similarities between the story of ten and the story of twenty are discussed explicitly. Simple shopping activities with 20c follow. These exercises are repeated for 30c, 40c, 50c, 60c, 70c, 80c, and 90c.

STRATEGY: Shopping with euro

If real or play notes and coins are not available, ten stacks of ten unfix cubes can be used to represent one euro. Here the students meet the same combination of numbers again, for example:
spend one ten and there are nine tens (90) left.

1 matches with 9.
Spend thirty (3 tens) and there are seven tens left (70).
3 matches with 7.

After reinforcement of these facts, simple shopping activities follow:

Hand in	Cost	Change
€1	10c	
€1	30c	
€1	60c	

The tables below suggest how students might deal with more complicated calculations in stages.

Hand in	Cost	Change	
€1	15c	Spend 10c first Spend 5c next	90c left 85c change
€1	64c	Spend 60c first Spend 4c next	40c left 36c change

Item	Cost	Hand in	Change	
	€4.62	€5	Spend €4 first Spend 60c next Spend 2c next	€1 left 40c left 38c left
	€2.48	€10	Spend €2 first Spend 40c next Spend 8c next	€8 left €7.60 left €7.52 left

Where possible, students should have access to euro coins and notes when engaged in shopping activities. This will aid students in their progression to eventual memorisation of the relevant number facts, and will help to situate their money skills in the appropriate real-life context.

Exemplar 1: **Mathematics**

Strand: Algebra | **Strand unit:** Extending patterns | **Level:** Junior infants

Materials/resources	Learning targets	Language
Multi-links, spools, buttons, cubes, blocks, counters, shells, care bears, beads, nuts, toy cars, toy animals, bottle tops, 2-D and 3-D shapes, laces for threading.	Students will extend their knowledge of pattern according to colour, shape, and size. They will - copy pattern - continue/finish pattern - devise their own patterns.	First, next, after, need more, repeat, again, second, last, comes after, loud, soft

Methodology

Direct teaching with the group can be used to examine simple patterns. Beads on a string with a fixed beginning can ensure that students are able to add to one end only: *blue, red, blue, red; What comes next?*

- → Students copy the pattern using beads and string. Talk and discussion will heighten student awareness of the pattern. Students describe the pattern. Repeat, using a variety of materials, colours, shapes, and sizes.
- → Students copy a pattern using concrete materials, following a pictorial representation on card. Repeat using variety of materials/variety of cards.
- → Students continue/finish a pattern (colour, shape, and size).
- → Students create their own pattern (colour, shape, and size).
- → Students separate the colours into bowls, taking turns.

Linkage/integration

Visual Arts

- Pattern-making with gummed paper shapes
- Sponge prints, for example hand/foot prints using different coloured paints
- Examining patterns on sweets, for example liquorice allsorts
- Decorating the witch's house (Hansel and Gretel) with patterned sweets
- Making Granny's pattern bedspread (Little Red Riding Hood)
- Making Humpty Dumpty's wall
- Making Mary, Mary Quite Contrary's garden (bluebells and cockle shells all in a row).

Music

- Percussion instruments:
  drum beat, then xylophone, then triangle, then repeat
  (music pattern will become more complex as students improve).
- Voice: loud sound, soft sound; loud, soft; loud, soft
- Clapping activities
- Song: 'Head, shoulders, knees and toes'.

Exemplar 1: **Mathematics**

Development

Maths trail: 'Looking at patterns' (walls, wire, paving slabs, railings, flower beds).

Differentiation

If students are older, use other appropriate objects to form patterns, for example football jerseys, euro coins, attribute blocks. The content of the exemplar may be spread over several lessons, depending on the ability of the students.

Assessment

- Has the student difficulty organising a pattern (two colours)?
- Can the student extend a pattern?
- Can the student create a pattern?
- Can the student name the primary colours?
- Can the student recognise and name a circle, triangle, square, rectangle?
- Does the student concentrate well on the task?
- Is hand dominance fully established?

Exemplar 2: **Mathematics**

Strands: Number, Measure, Shape and space, Data	**Thematic approach:** Birthday	**Level:** Senior infants

Materials/resources	Learning targets	Language
Paper plates (large and small), paper cups, coloured candles, balloons, patterned wrapping paper, lemonade, one-hand analogue clock, calendar, large circle, triangle, square, rectangle, height chart, Unifix cubes, building blocks.	Further consolidation of concepts and language introduced in junior infants—o'clock, 2-D shapes, circles, triangles, squares, numerals 1 to 5.	Full/empty, large/small, more/less, heavy/light, young/old, pour, tall, as tall as, short, shorter than, yesterday, now, February, spring, dark, night, adults, usually, enough. Language focus will depend on the ability of the students.

Methodology

Use a direct teaching method, where appropriate, with teddy-bear's birthday as a starting point. *'Which month is it now? Which season? What age is the teddy? five? six? What age will he be next year? Will he be older or younger next year? His birthday is today. What day is it today? How many people are coming to the party? Count the boys. How many boys? Count the girls. How many girls? How many candles should we put on the cake? Allow one student to count out the correct number of candles. What colours are the candles? Point to each candle. What colour is it? How many balloons should we tie to the Wendy House door? How many balloons are red? How many are not red?'*

Linkage/integration

'What shape is the birthday cake (perhaps made from a circular or square biscuit tin)? *What time does teddy usually go to bed at? After tea-time? Show the time on a clock.'*

Music

- Sing 'Happy Birthday'. Clap hands five times. Tap knees six times (and repeat).

Visual Arts

- Examine a pattern on wrapping paper. Make wrapping paper. Draw a picture of the birthday party.
- Colour in a cake in the shape of a circle/square/rectangle/triangle.
- Make birthday cards. Make 'I am 5/6/7' badges.
- Using Plasticine, make two sausages for each plate.

Language

- See vocabulary above.

SPHE

- Care should be taken with matches/candles.
- Feelings: happiness (We are happy when we get nice birthday presents).
- The students are older this year than last year. What can the students do now that they could not do last year? Skip? Ride a bike? Sing a new song?

Exemplar 2: **Mathematics**

Development

- Students can set the table for the party. *How many cups are needed? How many plates?*
- Reinforce the concept of 'five'. Students build a tower of blocks, with Unifix cubes to represent ages 5/6/7. '*Is the 7 tower more than the 5 tower? (Yes, a little more.) Is the 5 tower less than the 7 tower? (Yes, a little less.)*'

Differentiation

Use clear, precise instructions with activities. There may be a need to repeat instructions several times. If the students are older make the 'party' more age-appropriate. For less able students, use closed questions. *(Is this glass full? Is this balloon red?).* Spread the content over several lessons. Broaden the content to include brown, purple, silver, and gold for those students who know the primary colours. Narrow the content to one colour for less able students.

Assessment

- Can the student distinguish between circle, square, rectangle, triangle?
- Can the student colour in shapes with reasonable accuracy?
- Can the student identify the numeral 5 (6, 7)?
- Can the student build a tower of 5 (6, 7)?
- Can the student associate the symbol 5 (6, 7) with the number group/tower?
- Has the student conservation of number to 5 (6, 7)?

Teacher observation: Does the student reverse numerals when writing?

Exemplar 3: **Mathematics**

Strand: Measures | **Strand unit:** Time | **Level:** Senior infants

Materials/resources	Learning targets	Language
One-hand analogue clock, two-hand analogue clock, various egg timers (1 minute, 3 minute), calendar, time/season pictures and charts.	Developing an understanding of the concept of time. Telling the time in one-hour intervals. Developing a 'time' vocabulary.	After, before, morning, afternoon, evening, night, yesterday, today, tomorrow, breakfast-time, lunch-time, tea-time, dinner-time, bed-time, play-time, start, stop, slow, slower, slowest, fast, faster, fastest.

Methodology

- Filling in a daily time chart: Today is _____. Tomorrow will be _____. Yesterday was _____.
- Filling in a birthday chart: reference to months, seasons, changes in clothes with the seasons.
- Advent calendar.
- Referring to pictorial representation of times of the day.
- Making a photographic record of students and some of their activities over time, for example planting seeds, plants in bloom, putting up the tree before Christmas, Christmas presents.
- Establishing what happens next:
  the students helping the teacher to re-tell favourite stories, or predicting what will happen in stories the students are not familiar with.
- Making a candle-clock with candle and pins (Pins fall as the candle burns.)
- Using the one-handed clock to teach the time in one-hour intervals as soon as students can read the numerals to 12.

Linkage/integration

Mathematics: Number
- 3 o' clock
- I will be 7 years old next year.

Music:
- Playing fast and slow pieces.

PE
- Running quickly, walking slowly.
- Game: 'What time is it Mr. Wolf?'

SESE
- Planting bean seeds and observing them over time.
- Evergreens, winter, Christmas, hibernation, bulbs, spring, St Patrick's Day, Easter, summer flowers, summer holidays, leaf fall, autumn, Halloween, harvest time.

Language
- Nursery Rhymes: Hickory Dickory Dock, Twinkle, Twinkle, Little Star.
- Story-Time: Owl Babies (Mark Waddel).

SPHE
- Feelings: afraid of the dark.

Exemplar 3: **Mathematics**

Differentiation

If students are older use PE activities involving an egg-timer or stop-watch. '*Can you skip/hop/bounce the ball for two minutes?'*

Assessment

- Can the student tell an activity which takes a short/long time?
- Can the student sequence two/three picture cards?
- Can the student understand yesterday/today/tomorrow?
- Can the student understand 'before' and 'after?'

Exemplar 4: **Mathematics**

Strand: Number | **Strand unit:** Place value | **Level:** First class

Materials/resources	Learning targets	Language
Abacus, Unifix cubes, counters, lollipop sticks, sets of counting objects, beads, toys, shells, 100 squares.	Students will identify groups of objects in sets of tens, and in tens and units. Students will assemble sets of tens and units using concrete materials, including money. Students will record number stories pictorially.	One less than, one more than, equals, equal to, tens, units, abacus, tens and units, more, before, after.

Methodology

Using lollipop sticks or Unifix cubes, students place ten lollipop sticks in a row and count them. *Now I have 10.* Students learn to place the bundle of ten on the 'tens' mat saying, *'I have one ten and no units. Mammy gave me another 1. Now I have 11'.* Students use the materials then record pictorially. Extend this skill to 99. Using lollipop sticks/ Unifix cubes in a similar way, students build up numbers to 99. Skip counting by tens, using 100 square. Establish a pattern. Work with an abacus, for example 54 = 5 tens and 4 units.

Linkage/integration

Money/shopping activities.
Algebra: substitute numerals on 100 square with a, b, c; what number goes at a? b? c?
Operations: counting in 5s, 10s.
Construct number stories based on counting in 10s. Link to ordinal numbers. '*My birthday is on the 3rd. My birthday is on the 31st.'*

PE

- Skip counting to ball bouncing, skipping games.

Music

- Body percussion in beats of ten—clapping, stamping, tapping.
- Song: 'There were 10 in the bed …'

Visual Arts

- Making/modelling groups of ten objects.

Exemplar 4: **Mathematics**

Development

Use number dice in snakes and ladders and other board games. Use snakes and ladders game for visual reinforcement of 100 square. The first student to get to 100 wins.

Bank game: cent coins and a pair of dice. The students roll the dice, add the numbers obtained, and take that amount in cent. They 'bank' amounts of ten cent for a 10c coin. The winner is the one who gains ten 10c coins first.

Differentiation

Students who are kinaesthetic learners build towers, or write using rice on the tray (or sandpaper/felt numerals). Trace, say, write.
Less able students may work only to 50 or less initially, depending on ability.

Assessment

Can/does the student

- match a numeral to a set (0–99)
- understand 10 units = one ten
- understand 46 = 4 tens and 6 units
- build towers in tens
- record a simple number story pictorially
- understand one more than 20 is 21
- understand one less than 20 is 19
- tap out to 20
- tap out to 30?

Exemplar 5: **Mathematics**

Strand: Measures	**Strand unit:** Money/time	**Thematic approach:** The jumble sale	**Level:** Third and fourth class

Materials/resources	Learning targets	Language
Money (using real coins). Items for jumble sale.	*The students will understand* - *simple addition/subtraction, and money calculations involving 'what change will I get?'* - rounding off 99c to €1 and €1.99 to €2.	How much?, How many?, amount, value, altogether, more, most, sell, pay, bought, euro, cent.

Methodology

Preparation talk and discussion:

- → Students each have five coins in their purses/wallets. How many different total amounts can they list?
- → The students make lucky dips for the jumble sale, using a combination of small items, for example pencils 15c, erasers 10c, sweets 20c, stickers 15c. *How much will each lucky dip cost? What change will they give/get from €1?*
- → '*I buy a book for 50c and a puzzle for 25c; how much do I spend? How much change do I get from €1? Estimate first. Will it be more or less than €1?*'
- → '*I bought a toy for 99c and a watch for €1.99. How much did I spend?*' Encourage students to adjust these amounts and to change their answer accordingly. Why are so many items in shops priced this way? Discuss.

Linkage/integration

Mathematics: Time

- Date of jumble sale, time of jumble sale.

Visual Arts

- Making jumble sale invitations.
- Making 'for sale' posters.

Exemplar 5: **Mathematics**

Development

- Use of calculator in money calculations.
- Examine in a practical way how 25c + 75c = 50c + 50c.

Differentiation

Some students will have difficulty understanding that 2.6 euro = €2.60.
'Sellers' may wish to use their calculators when they are totalling a string of amounts. They will need to understand 46c must be entered as 0.46 on the calculator.

Assessment

Can the student

- add simple amounts
- understand 'change'
- use a calculator in simple money calculations
- understand rounding up, for example €1.99 to €2?

Exemplar 6: **Mathematics**

Strand: Number | **Strand unit:** Percentages | **Level:** Fifth and sixth class

Materials/resources	Learning targets	Language
Percentage boards, Diene's blocks, squared paper, calculators, catalogues (clothes, travel, music), newspapers.	Students will understand benchmark percentages: 10%, 25%, 50%.	Cent, euro, American cent, century, centimetre, centipede, per cent, percentage, loss.

Methodology

- Talk and discussion.
- Percentages in daily life: orange (100% pure); beans (99% fat free).
- Revision of previous knowledge using percentage boards.
- Money (50% of €1 = 50c, 25% of €1 = 25c).
- Based on catalogues, students can discuss, estimate and calculate discounts of 50% during a sale. This can be followed by discounts of 25%, 10%.
- Using a 100 square, students fill in percentage amounts: 50%, 25%, 10%.

Linkage/integration

Mathematics
- Fractions, decimal fractions.

Geography
- Human environments.

Visual Arts
- Students look for discounted prices in newspapers and make their own sales posters advertising discounts in a clothes/music/travel shop.

Exemplar 6: **Mathematics**

Development

- Establish key points of reference: $\frac{8}{17}$ is close to $\frac{8}{16}$ which is $\frac{1}{2}$ or 50%.
- *If I got* $\frac{40}{80}$ *of my spellings correct in a test, what percentage would that be?*
- Multiple-choice questions: *'If I got* $\frac{10}{20}$ *marks for history what percentage would that be: 10%, 25%, 50%?'*
- '*A computer game costs €120. If I get 25% off, how much do I have to pay?*'
- Games: percentage dominoes.

Differentiation

- Use of calculators to check answers.
- Estimation (50% of 415 is roughly about 207, discuss.)
- Convert percentages to fractions ($50\% = \frac{50}{100} = \frac{1}{2}$).
- Convert fractions to percentages: multiply by 100 ($\frac{1}{3} \times \frac{100}{1} = 33\frac{1}{3}\%$).
- The student reports on some of the above as if explaining it to someone younger.

Assessment

Does the student

- understand the concept of per cent
- use some estimation/mental computation strategies
- convert a fraction to a percentage (and vice versa) using a calculator
- understand the use of per cent in human environments
- convert a fraction to a percentage (and vice versa) without using a calculator?

Appendix

Overview of content — Mathematics

Strands	Strand units
Early mathematical activities	• classifying • matching • comparing • ordering

Number	• counting
	• comparing and ordering
	• place value
	• operations
	  - addition
	  - subtraction
	  - multiplication
	  - division
	• fractions
	• decimals

Pattern and sequence	• number patterns and sequences • number sentences
Shape and space	• spatial awareness • 2-D and 3-D shapes • symmetry • lines and angles
Measures	• length • weight • capacity • time • money
Data	• recognising and interpreting data • chance

Checklist for assessing student's skills development

Can the student

Early mathematical activities	✔
Classify → attend to and participate in the matching of 3-D shapes	
→ classify objects on the basis of one attribute (colour, shape, size, and texture)	
→ classify objects on the basis of two attributes	
→ select from an assortment of objects, one that serves the same function as a given object	
→ classify socially related objects	
→ select from an assortment of objects, one similar to a given object	
Match → match pairs of identical objects in one-to-one correspondence	
→ match sets of identical objects in one-to-one correspondence	
→ match equivalent sets of concrete objects in one-to-one correspondence	
→ match non-equivalent sets of concrete objects in one-to-one correspondence	
Compare → compare objects according to length, width, height, size, and weight	
Order → order objects according to length, size, weight, or height?	

Can the student

Algebra - Pattern and sequence	✔
→ attend to repeated sounds and actions	
→ respond to repeated sounds, patterns, and movements	
→ imitate repeated sounds, patterns, and movements	
→ initiate and create his/her own repetitive sounds and movements	
→ attend to the sequencing of two or three familiar routines	
→ identify some of the patterns in daily routines	
→ match sets of identical objects in one-to-one correspondence	
→ correctly sequence two or three events	
→ demonstrate understanding of first, next, last	
→ follow the correct sequence in carrying out activities	
→ correctly sequence pictures that depict familiar activities	
→ use familiar 2-D and 3-D objects to copy/continue patterns (in colour, shape, or size)	
→ observe and talk about patterns	
→ copy, continue, and extend patterns?	

Can the student

Number	✔
→ demonstrate understanding of one as opposed to a lot	
→ demonstrate understanding of some and more	
→ demonstrate understanding of one-to-one correspondence	
→ rote count to a given number	
→ show an awareness of numerals	
→ show awareness of number in the environment	
→ show awareness of number in stories	
→ show awareness that number names come in a fixed order	
→ listen to, respond to, and participate in number rhymes, stories, and games	
→ talk with understanding about numbers of personal significance	
→ show understanding that numbers are used for counting	
→ trace/copy numerals, without a consistent error such as writing particular numbers backwards or upside down	
→ order numerals	
→ respond to the language of ordinal number: first, last	
→ count accurately the number of objects in a set, counting each object only once (noting maximum number counted accurately)	
→ match symbols to sets	
→ identify a set which has more/less objects	
→ estimate the number of objects in a set	
→ identify the empty set and recognise zero	
→ combine sets of objects	
→ partition sets of objects	
→ use the symbols + and = to construct simple word sentences involving addition	
→ use the symbols – and = to construct simple word sentences involving subtraction	
→ use mental strategies, for example counting on	
→ recognise number patterns, for example counting in twos, fives, tens	
→ count in twos, fives, tens	
→ count in twos, fives, tens, starting at any number	
→ show understanding of the mathematical signs needed for a task	
→ identify and record place value	
→ solve simple one-step oral problems?	

Can the student

Space and spatial awareness	✔
→ show an awareness of the position of his/her own body and body parts in space	
→ demonstrate movement in different parts of the body and how the body can move in space	
→ attend to and respond to the language of movement and positioning	
→ follow instructions related to movement and positioning	
→ use the language of movement and positioning	
→ observe and describe people and objects in different positions in space (on, under, far away, beside, etc.)	
→ give and follow simple directions	
→ follow instructions to position themselves in relation to others	
→ recognise right and left in real situations	
Shape	✔
→ attend to and participate in the identification of 3-D shapes	
→ attend to and participate in the matching of 3-D shapes	
→ combine 3-D shapes to make other shapes	
→ sort 3-D shapes (regular and irregular)	
→ attend to and participate in the matching of 2-D shapes	
→ attend to and participate in the sorting of 2-D shapes	
→ combine 2-D shapes to make other shapes or pictures	
→ sort 2-D shapes	
→ trace and copy 2-D shapes	
→ name 2-D shapes (list ones known)?	

Can the student

Measures - length	✔
→ show an awareness of the concept of length through the use of appropriate vocabulary (long, short, longer than, shorter than, etc.)	
→ identify which of two objects is long/short	
→ compare and order two objects according to length or height	
→ estimate and measure length using non-standard units	
Measures - weight	✔
→ show an awareness of the concept of weight through the use of appropriate vocabulary (heavy, light, heavier than, lighter than, etc.)	
→ identify which of two objects is heavy/light	
→ compare and order two objects according to weight	
→ estimate and measure weight using non-standard units	
Measures - capacity	✔
→ show an awareness of the concept of capacity through the use of appropriate vocabulary (full, empty, nearly full, nearly empty, etc.)	
→ identify which of two objects is full/empty	
→ compare and order two containers according to capacity	
→ estimate and measure capacity using non-standard units?	

Can the student

Measures - money	✔
→ demonstrate understanding that money is necessary to pay for goods	
→ sort and match coins	
→ demonstrate understanding that some coins are worth more than others	
→ recognise some coins and notes	
→ use the vocabulary of money (buy, sell, how much?, coins)	
→ calculate simple bills	
→ use money for social purposes with or without help	
Measures - time	✔
→ show an understanding of time as related to self	
→ show an awareness of specific times at school (break, music time)	
→ show awareness of day and night	
→ show awareness of the daily patterns of familiar events	
→ sequence pictures of daily events	
→ recognise the present time, for example, today	
→ identify things that happened in the recent past	
→ show understanding that things will happen in the future	
→ use the language of time to discuss events	
→ name the days of the week?	

Can the student

Data - collecting and processing	✔
→ sort by putting similar objects together	
→ make a set of objects with a given property	
→ sort and classify sets of objects by one criterion	
→ compare two objects, identifying similarities and differences	
→ sort and classify (with help) sets of objects by up to two criteria	
Data - representing and interpreting	✔
→ represent data using students and objects	
→ represent data using pictures	
→ interpret data which is represented by real objects or pictures?	

Overview of skills development

The skills that the student should develop are evident throughout the Primary School Curriculum, Mathematics. Opportunities should be provided for the student to acquire the knowledge, concepts, and skills required for everyday living and for use in other subject areas:

- applying and problem-solving
- communicating and expressing
- integrating and connecting
- reasoning
- implementing
- understanding and recalling.

The overview of skills development which follows is taken from the *Teacher Guidelines, Mathematics*. The mainstream class levels have been retained to show the developmental sequence. However, students with mild general learning disabilities may need to spend longer at different stages or to have the transferability of the skills explicitly identified for them.

Skills development

Class levels in the mainstream curriculum					
Infant classes					
Problem-solving	**Communicating and expressing**	**Integrating and connecting**	**Reasoning**	**Implementing**	**Recalling**
• select appropriate materials and processes for mathematical tasks • select and apply appropriate strategies to complete tasks or solve problems • recognise solutions to problems	• listen to and discuss other students' descriptions/ explanations • discuss/record using diagrams, pictures and symbols	• understand the mathematical ideas behind procedures	• make guesses and carry out experiments to test them • recognise and create mathematical patterns and relationships	• execute procedures efficiently	• recall and use terminology and facts

Class levels in the mainstream curriculum					
First and second classes					
Problem-solving	**Communicating and expressing**	**Integrating and connecting**	**Reasoning**	**Implementing**	**Recalling**
• select appropriate materials and processes for mathematical tasks/applications • apply concepts and processes in a variety of contexts	• listen to and discuss other students' descriptions/ explanations • discuss/record using diagrams, pictures and symbols	• understand the mathematical ideas behind procedures	• make guesses and carry out experiments to test them • recognise and create mathematical patterns and relationships	• execute procedures efficiently	• recall and use terminology and facts

Class levels in the mainstream curriculum					
Third and fourth classes					
Problem-solving	**Communicating and expressing**	**Integrating and connecting**	**Reasoning**	**Implementing**	**Recalling**
• analyse problems and plan an approach to solving them • select and use a variety of strategies to complete tasks/ projects or solve problems • evaluate solutions to problems	• discuss and explain the processes used/results of mathematical activities/ projects/ problems • discuss and record processes and results using a variety of methods • discuss problems presented verbally or diagramatically/ carry out analyses	• connect informally acquired mathematical ideas/ processes with formal mathematical ideas/ processes • understand the connections between mathematical procedures and concepts • represent mathematical ideas and processes in different modes: verbal, pictorial, diagrammatic, symbolic	• make informal deductions • explore and create mathematical patterns and relationships • reason systematically in a mathematical context	• execute standard procedures efficiently with a variety of tools	• recall and use terminology, facts and definitions

Class levels in the mainstream curriculum					
Fifth and sixth classes					
Problem-solving	**Communicating and expressing**	**Integrating and connecting**	**Reasoning**	**Implementing**	**Recalling**
• reflect upon and evaluate solutions to problems	• discuss and explain processes used and results in an organised way • discuss problems/carry out analyses		• search for, investigate and create mathematical patterns and relationships		