Introduction to Decision Making Methods

János Fülöp
Laboratory of Operations Research and Decision Systems,
Computer and Automation Institute, Hungarian Academy of Sciences

1. Decision Making Process

"Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on." (Harris (1980))

According to Baker et al. (2001), decision making should start with the identification of the decision maker(s) and stakeholder(s) in the decision, reducing the possible disagreement about problem definition, requirements, goals and criteria. Then, a general decision making process can be divided into the following steps:

Step 1. Define the problem

"This process must, as a minimum, identify root causes, limiting assumptions, system and organizational boundaries and interfaces, and any stakeholder issues. The goal is to express the issue in a clear, one-sentence *problem statement* that describes both the initial conditions and the desired conditions." Of course, the one-sentence limit is often exceeded in the practice in case of complex decision problems. The problem statement must however be a concise and unambiguous *written* material agreed by all decision makers and stakeholders. Even if it can be sometimes a long iterative process to come to such an agreement, it is a crucial and necessary point before proceeding to the next step.

Step 2. Determine requirements

"Requirements are conditions that any acceptable solution to the problem *must* meet. Requirements spell out what the solution to the problem *must* do." In mathematical form, these requirements are the constraints describing the set of the feasible (admissible) solutions of the decision problem. It is very important that even if subjective or judgmental evaluations may occur in the following steps, the requirements must be stated in exact quantitative form, i.e. for any possible solution it has to be decided unambiguously whether it meets the requirements or not. We can prevent the ensuing debates by putting down the requirements and how to check them in a written material.

Step 3. Establish goals

"Goals are broad statements of intent and desirable programmatic values.... Goals go beyond the minimum essential *must have's* (i.e. requirements) to *wants* and *desires*." In mathematical form, the goals are objectives contrary to the requirements that are constraints. The goals may be conflicting but this is a natural concomitant of practical decision situations.

Step 4. Identify alternatives

"Alternatives offer different approaches for changing the initial condition into the desired condition." Be it an existing one or only constructed in mind, any alternative must meet the requirements. If the number of the possible alternatives is finite, we can check one by one if it meets the requirements. The infeasible ones must be deleted (screened out) from the further consideration, and we obtain the explicit list of the alternatives. If the number of the possible alternatives is infinite, the set of alternatives is considered as the set of the solutions fulfilling the constraints in the mathematical form of the requirements.

Step 5. Define criteria

"Decision criteria, which will discriminate among alternatives, must be based on the goals. It is necessary to define discriminating criteria as objective measures of the goals to measure how well each alternative achieves the goals." Since the goals will be represented in the form of criteria, every goal must generate at least one criterion but complex goals may be represented only by several criteria.

It can be helpful to group together criteria into a series of sets that relate to separate and distinguishable components of the overall objective for the decision. This is particularly helpful if the emerging decision structure contains a relatively large number of criteria. Grouping criteria can help the process of checking whether the set of criteria selected is appropriate to the problem, can ease the process of calculating criteria weights in some methods, and can facilitate the emergence of higher level views of the issues. It is a usual way to arrange the groups of criteria, subcriteria, and sub-subcriteria in a tree-structure (UK DTLR (2001)).

According to Baker et al. (2001), criteria should be

- able to discriminate among the alternatives and to support the comparison of the performance of the alternatives,
- complete to include all goals,
- operational and meaningful,
- non-redundant,
- few in number.

In some methods, see Keeney and Raiffa (1976), non-redundancy is required in the form of independency.

We mention that some authors use the word attribute instead of criterion. Attribute is also sometimes used to refer to a measurable criterion.

Step 6. Select a decision making tool

There are several tools for solving a decision problem. Some of them will be briefly described here, and references of further readings will also be proposed. The selection of an appropriate tool is not an easy task and depends on the concrete decision problem, as well as on the objectives of the decision makers. Sometimes 'the simpler the method, the better" but complex decision problems may require complex methods, as well.

Step 7. Evaluate alternatives against criteria

Every correct method for decision making needs, as input data, the evaluation of the alternatives against the criteria. Depending on the criterion, the assessment may be objective (factual), with respect to some commonly shared and understood scale of measurement (e.g. money) or can be subjective (judgmental), reflecting the subjective assessment of the evaluator. After the evaluations the selected decision making tool can be applied to rank the alternatives or to choose a subset of the most promising alternatives.

Step 8. Validate solutions against problem statement

The alternatives selected by the applied decision making tools have always to be validated against the requirements and goals of the decision problem. It may happen that the decision making tool was misapplied. In complex problems the selected alternatives may also call the attention of the decision makers and stakeholders that further goals or requirements should be added to the decision model.

2. Single criterion vs. multiple criteria, finite number of alternatives vs. infinite number of alternatives

It is very important to make distinction between the cases whether we have a single or multiple criteria. A decision problem may have a single criterion or a single aggregate measure like cost. Then the decision can be made implicitly by determining the alternative with the best value of the single criterion or aggregate measure. We have then the classic form of an optimization problem: the objective function is the single criterion; the constraints are the requirements on the alternatives. Depending on the form and functional description of the optimization problem, different optimization techniques can be used for the solution, linear programming, nonlinear programming, discrete optimization, etc. (Nemhauser et al. (1989)).

The case when we have a finite number of criteria but the number of the feasible alternatives (the ones meeting the requirements) is infinite belongs to the field of multiple criteria optimization. Also, techniques of multiple criteria optimization can be used when the number of feasible alternatives is finite but they are given only in implicit form (Steuer, R. E. (1986)).

This brief survey focuses on decision making problems when the number of the criteria and alternatives is finite, and the alternatives are given explicitly. Problems of this type are called *multi-attribute decision making problems*.

3. Multi-attribute decision making methods

Consider a multi-attribute decision making problem with m criteria and n alternatives. Let $\mathbf{C}_1,\ldots,\mathbf{C}_m$ and $\mathbf{A}_1,..,\mathbf{A}_n$ denote the criteria and alternatives, respectively. A standard feature of multi-attribute decision making methodology is the *decision table* as shown below. In the table each row belongs to a criterion and each column describes the performance of an alternative. The score a_{ij} describes the performance of alternative $\mathbf{A}_j$ against criterion $\mathbf{C}_i$. For the sake of simplicity we assume that a higher score value means a better performance since any goal of minimization can be easily transformed into a goal of maximization.

As shown in decision table, weights $w_1,...,w_m$ are assigned to the criteria. Weight w_i reflects the relative importance of criteria $\mathbf{C}_i$ to the decision, and is assumed to be positive. The weights of the criteria are usually determined on subjective basis. They represent the opinion of a single decision maker or synthesize the opinions of a group of experts using a group decision technique, as well.

The values $x_1,...,x_n$ associated with the alternatives in the decision table are used in the MAUT methods (see below) and are the final ranking values of the alternatives. Usually, higher ranking value means a better performance of the alternative, so the alternative with the highest ranking value is the best of the alternatives.

		x_1	·	·	x_n
		$\mathbf{A}_1$	·	·	$\mathbf{A}_n$
w_1	$\mathbf{C}_1$	a_{11}	·	·	a_{m1}
·	·	·	·	·	·
·	·	·	·	·	·
w_m	$\mathbf{C}_m$	a_{m1}	·	·	a_{mn}

Table 1. The decision table

Multi-attribute decision making techniques can partially or completely rank the alternatives: a single most preferred alternative can be identified or a short list of a limited number of alternatives can be selected for subsequent detailed appraisal.

Besides some monetary based and elementary methods, the two main families in the multi-attribute decision making methods are those based on the Multi-attribute Utility Theory (MAUT) and Outranking methods.

The family of MAUT methods consists of aggregating the different criteria into a function, which has to be maximized. Thereby the mathematical conditions of aggregations are examined. This theory allows complete compensation between criteria, i.e. the gain on one criterion can compensate the lost on another (Keeney and Raiffa (1976)).

The concept of outranking was proposed by Roy (1968). The basic idea is as follows. Alternative $\mathbf{A}_i$ outranks $\mathbf{A}_j$ if on a great part of the criteria $\mathbf{A}_i$ performs at least as good as $\mathbf{A}_j$ (concordance condition), while its worse performance is still acceptable on the other criteria (non-discordance condition). After having determined for each pair of alternatives whether one alternative outranks another, these pairwise outranking assessments can be combined into a partial or complete ranking. Contrary to the MAUT methods, where the alternative with the best value of the aggregated function can be obtained and considered as the best one, a partial ranking of an outranking method may not render the best alternative directly. A subset of alternatives can be determined such that any alternative not in the subset be outranked by at least one member of the subset. The aim is to make this subset as small as possible. This subset of alternatives can be considered as a shortlist, within which a good compromise alternative should be found by further considerations or methods.

3.1 Cost-benefit analysis

Cost-benefit analysis (CBA) is a worldwide used technique in decision making. CBA evaluates the costs and benefits of the alternatives on monetary basis. Recently, attempts have been made to incorporate the environmental impacts within CBA to improve the quality of environmental decision making. Although advances have been made, problems persist in applying CBA to environmental issues, including the monetary valuation of environmental impacts (UK DTLR (2001)).

On the other hand, CBA has great attractions as a tool for guiding public policy:

- “it considers the gains and losses to all members of the society on whose behalf the CBA is being undertaken;
- it values impacts in terms of a single, familiar measurement scale - money - and can therefore in principle show that implementing an alternative is worthwhile relative to doing nothing;
- the money values used to weight the relative importance of the different impacts are based on people's preferences generally using established methods of measurement.”

Despite its limitations, CBA can be efficiently integrated into complex methods of environmental decision making. See Munda (1996) how CBA can be integrated into environmental assessment and US EPA (2000) for guidelines on economic analysis including cost and benefit analysis.

3.2 Elementary methods

These elementary approaches are simple and no computational support is needed to perform the analysis These methods are best suited for problems with a single decision maker, few alternatives and criteria that is rarely characteristic in environmental decision making (Linkov et al. (2004)).

3.2.1 Pros and cons analysis

Pros and cons analysis is a qualitative comparison method in which good things (pros) and bad things (cons) are identified about each alternative. Lists of the pros and cons are compared one to another for each alternative. The alternative with the strongest pros and weakest cons is preferred. It requires no mathematical skill and is easy to implement. (Baker et al. (2001)).

3.2.2 Maximin and maximax methods

The maximin method is based upon a strategy that tries to avoid the worst possible performance, maximizing the minimal performing criterion. The alternative for which the score of its weakest criterion is the highest is preferred. The maximin method can be used only when all criteria are comparable so that they can be measured on a common scale, which is a limitation (Linkov et al. (2004)).

3.2.3 Conjunctive and disjunctive methods

These methods require satisfactory rather than best performance in each criterion. The conjunctive method requires that an alternative must meet a minimal performance threshold for all criteria. The disjunctive method requires that the alternative should exceed the given threshold for at least one criterion. Any alternative that does not meet the conjunctive or disjunctive rules is deleted from the further consideration. These screening rules can be used to select a subset of alternatives for analysis by other, more complex decision making tools (Linkov et al. (2004)). Screening by conjunctive and disjunctive rules can also be applied in Step 2 (Determine requirements) of the decision making process (see Section 1).

3.2.4 Lexicographic method

In the lexicographic method criteria are ranked in the order of their importance. The alternative with the best performance score on the most important criterion is chosen. If there are ties with respect to this criterion, the performance of the tied alternatives on the next most important criterion will be compared, and so on, till a unique alternative is found (Linkov et al. (2004)).

3.3 MAUT methods

In most of the approaches based on the Multi-attribute Utility Theory (MAUT), the weights associated with the criteria can properly reflect the relative importance of the criteria only if the scores a_{ij} are from a common, dimensionless scale. The basis of MAUT is the use of utility functions. Utility functions can be applied to transform the raw performance values of the alternatives against diverse criteria, both factual (objective, quantitative) and judgmental (subjective, qualitative), to a common, dimensionless scale. In the practice, the intervals [0,1] or [0,100] are used for this purpose. Utility functions play another very important role: they convert the raw performance values so that a more preferred performance obtains a higher utility value. A good example is a criterion reflecting the goal of cost minimization. The associated utility function must result in higher utility values for lower cost values.

It is common that some normalization is performed on a nonnegative row in the matrix of the a_{ij} entries. The entries in a row can be divided by the sum of the entries in the row, by the maximal element in the row, or by a desired value greater than any entry in the row. These normalizations can be also formalized as applying utility functions.

3.3.1 Simple multiattribute rating technique (SMART)

SMART is the simplest form of the MAUT methods. The ranking value x_j of alternative $\mathbf{A}_j$ is obtained simply as the weighted algebraic mean of the utility values associated with it, i.e.

$$x_j = \sum_{i=1}^{m} w_i a_{ij} / \sum_{i=1}^{m} w_i, \quad j = 1,...,n.$$

Besides the above simple additive model, Edwards (1977) also proposed a simple method to assess weights for each of the criteria to reflect its relative importance to the decision. First, the criteria are ranked in order of importance and 10 points are assigned to the least important criterion. Then, the next-least-important criterion is chosen, more points are assigned to it, and so on, to reflect their relative importance. The final weights are obtained by normalizing the sum of the points to one. However, as Edwards and Barron (1994) pointed out, the comparison of the importance of attributes is meaningless if it does not reflect the range of the utility values of the alternatives as well. They proposed a variant named SMARTS (SMART using Swings) that in the course of the comparison of the importance of the criteria also considers the amplitude of the utility values, i.e. the changes from the worst utility value level to the best level among the alternatives. See also Barron and Barrett (1996) for further techniques.

3.3.2 Generalized means

In a decision problem the vector $\mathbf{x}=(x_1,...,x_n)$ plays a role of aggregation taking the performance scores for every criterion with the given weight into account. This means that the vector $\mathbf{x}$ should fit into the rows of the decision matrix as well as possible. Mészáros and Rapcsák (1996) introduced an entropy optimization problem to find the vector $\mathbf{x}$ of best fit. They pointed out that the optimal solution is a positive multiple of the vector of the weighted geometric means of the columns, consequently, with

$$w = \sum_{i=1}^{m} w_i ,$$

the values

$$x_j = \prod_{i=1}^{m} a_{ij}^{w_i / w}, \quad i = 1,...,n$$

constitute a reasonable and theoretically established system of ranking values. By introducing another entropy optimization problem, based on another measure of fitting, the weighted algebraic means (used also in SMART and additive linear models) were obtained as best fitting ranking values.

Mészáros and Rapcsák (1996) also proposed to determine the rating values in the form of generalized mean:

$$x_j = f^{-1}(\sum_{i=1}^{m} \frac{w_i}{w} f(a_{ij})), \quad i = 1,...,n ,$$

where f is a strictly monotone real function. This wide class of means also includes the weighted algebraic and geometric means with $f(t)=t$ and $f(t)=\log(t)$, respectively.

3.3.3 The Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) was proposed by Saaty (1980). The basic idea of the approach is to convert subjective assessments of relative importance to a set of overall scores or weights. AHP is one of the more widely applied multiattribute decision making methods. We follow here the summary of UK DTRL (2000) on the AHP.

The methodology of AHP is based on pairwise comparisons of the following type 'How important is criterion $\mathbf{C}_i$ relative to criterion $\mathbf{C}_j$?' Questions of this type are used to establish the weights for criteria and similar questions are to be answered to assess the performance scores for alternatives on the subjective (judgmental) criteria.

Consider how to derive the weights of the criteria. Assume first that the m criteria are not arranged in a tree-structure. For each pair of criteria, the decision maker is required to respond to a pairwise comparison question asking the relative importance of the two. The responses can use the following nine-point scale expressing the intensity of the preference for one criterion versus another

1= Equal importance or preference.

3= Moderate importance or preference of one over another.

5= Strong or essential importance or preference.

7= Very strong or demonstrated importance or preference.

9= Extreme importance or preference.

If the judgement is that criterion $\mathbf{C}_j$ is more important than criterion $\mathbf{C}_i$, then the reciprocal of the relevant index value is assigned.

Let c_{ij} denote the value obtained by comparing criterion $\mathbf{C}_i$ relative to criterion $\mathbf{C}_j$. Because the decision maker is assumed to be consistent in making judgements about any one pair of criteria and since all criteria will always rank equally when compared to themselves, we have $c_{ij}=1/c_{ij}$ and $c_{ii}=1$. This means that it is only necessary to make $^1/_2 m(m - 1)$ comparisons to establish the full set of pairwise judgements for m criteria. The entries c_{ij}, $i,j=1,\ldots,m$ can be arranged in a *pairwise comparison matrix* C of size mxm.

The next step is to estimate the set of weights that are most consistent with the relativities expressed in the comparison matrix. Note that while there is complete consistency in the (reciprocal) judgements made about any one pair, consistency of judgements between pairs, i.e. $c_{ij}c_{kj}= c_{ik}$ for all i,j,k, is not guaranteed. Thus the task is to search for an m-vector of the weights such that the mxm matrix W of entries w_i/w_j will provide the best fit to the judgments recorded in the pairwise comparison matrix C. Several of techniques were proposed for this purpose.

Saaty's original method to compute the weights is based on matrix algebra and determines them as the elements in the eigenvector associated with the maximum eigenvalue of the matrix. The eigenvalue method has been criticized both from prioritization and consistency points of view and several other techniques have been developed. A number of other methods are based on the minimization of the distance between matrices C and W. Some of these approaches give the vector

w directly or by simple computations, some other ones require the solution of numerically difficult optimization problems. One of these approaches, the logarithmic least squares method, results in a straightforward way of computing vector w: calculate the geometric mean of each row in the matrix C, calculate the sum of the geometric means, and normalize each of the geometric means by dividing by the sum just computed (Saaty and Vargas (1984)). See Gass and Rapcsák (2004) for further references on distance-minimizing methods and a new approach based on singular value decomposition.

In the practice the criteria are often arranged in a tree-structure. Then, AHP performs a series of pairwise comparisons within smaller segments of tree and then between sections at a higher level in the tree-structure.

Similarly to calculation of the weights for the criteria, AHP also uses the technique based on pairwise comparisons to determine the relative performance scores of the decision table for each of the alternatives on each subjective (judgemental) criterion. Now, the pairwise questions to be answered ask about the relative importance of the performances of pairs of alternatives relating the considered criterion. Responses use the same set of nine index assessments as before, and the same techniques can be used as at computing the weights of criteria.

With the weights and performance scores determined by the pairwise comparison technique above, and after further possible normalization, alternatives are evaluated using any of the decision table aggregation techniques of the MAUT methods. The so-called additive AHP uses the same weighted algebraic means as SMART, and the multiplicative AHP is essentially based on the computation of the weighted geometric means.

A number of specialists have voiced a number of concerns about the AHP, including the potential internal inconsistency and the questionable theoretical foundation of the rigid 1-9 scale, as well as the phenomenon of rank reversal possibly arising when a new alternative is introduced. On the same time, there have also been attempts to derive similar methods that retain the strengths of AHP while avoiding some of the criticisms. See Triantaphyllou, E. (2000) and Figueira et al. (2004) for state-of-art surveys and further references.

3.4 Outranking methods

The principal outranking methods assume data availability broadly similar to that required for the MAUT methods. That is, they require alternatives and criteria to be specified, and use the same data of the decision table, namely the a_{ij}'s and w_i's.

Vincke (1992) provides an introduction to the best known outranking methods; see also Figueira et al. (2004) for state-of-art surveys. Here, the two most popular families of the outranking methods, the ELECTRE and the PROMETHEE methods will be briefly outlined.

3.4.1 The ELECTRE methods

The simplest method of the ELECTRE family is ELECTRE I.

The ELECTRE methodology is based on the concordance and discordance indices defined as follows. We start from the data of the decision matrix, and assume here that the sum of the weights of all criteria equals to 1. For an ordered pair of alternatives $(\mathbf{A}_j,\mathbf{A}_k)$, the concordance index c_{jk} is the sum of all the weights for those criteria where the performance score of $\mathbf{A}_j$ is least as high as that of $\mathbf{A}_k$, i.e.

$$c_{jk} = \sum_{i:\, a_{ij} \geq a_{ik}} w_i, \quad j,k = 1,..,n,\ j \neq k.$$

Clearly, the concordance index lies between 0 and 1.

The computation of the discordance index d_{jk} is a bit more complicated: d_{jk}=0 if a_{ij}>a_{ik}, i=1,...,m, i.e. the discordance index is zero if $\mathbf{A}_j$ performs better than $\mathbf{A}_k$ on all criteria,. Otherwise,

$$d_{jk} = \max_{i=1,..,m} \frac{a_{ik} - a_{ij}}{\max\limits_{j=1,..,n} a_{ij} - \min\limits_{j=1,..,n} a_{ij}}, \quad j,k = 1,..,n,\ j \neq k\ ,$$

i.e. for each criterion where $\mathbf{A}_k$ outperforms $\mathbf{A}_j$, the ratio is calculated between the difference in performance level between $\mathbf{A}_k$ and $\mathbf{A}_j$ and the maximum difference in score on the criterion concerned between any pair of alternatives. The maximum of these ratios (which must lie between 0 and 1) is the discordance index.

A concordance threshold c^* and discordance threshold d^* are then defined such that $0<d^*<c^*<1$. Then, $\mathbf{A}_j$ outranks $\mathbf{A}_k$ if the $c_{jk}>c^*$ and $d_{jk}<d^*$, i.e. the concordance index is above and the discordance index is below its threshold, respectively.

This outranking defines a partial ranking on the set of alternatives. Consider the set of all alternatives that outrank at least one other alternative and are themselves not outranked. This set contains the promising alternatives for this decision problem. Interactively changing the level thresholds, we also can change the size of this set.

The ELECTRE I method is used to construct a partial ranking and choose a set of promising alternatives. ELECTRE II is used for ranking the alternatives. In ELECTRE III an outranking degree is established, representing an outranking creditability between two alternatives which makes this method more sophisticated (and, of course, more complicated and difficult to interpret). See Figueira et al (2004) for more details and further members of the ELECTRE family.

3.4.2 The PROMETHEE methods

The decision table is the starting point of the PROMETHEE methodology introduced by Brans and Vincke (1985) and Brans et al. (1986). The scores a_{ij} need not necessarily be normalized or transformed into a common dimensionless scale. We only assume that, for the sake of simplicity, a higher score value means a better performance. It is also assumed that the weights w_i of the criteria have been determined by an appropriate method (this is not a part of the PROMETHEE methods), furthermore, $\sum_{i=1}^{m} w_i = 1$. Here, following Brans and Mareschal (1994), we give a brief review of the PROMETHEE methods.

In order to take the deviations and the scales of the criteria into account, a preference function is associated to each criterion. For this purpose, a preference function $P_i(\mathbf{A}_j,\mathbf{A}_k)$ is defined, representing the degree of the preference of alternative $\mathbf{A}_j$ over $\mathbf{A}_k$ for criterion $\mathbf{C}_i$. We consider a degree in normalized form, so that
$0 \leq P_i(\mathbf{A}_j,\mathbf{A}_k) \leq 1$ and
$P_i(\mathbf{A}_j,\mathbf{A}_k) = 0$ means no preference or indifference,
$P_i(\mathbf{A}_j,\mathbf{A}_k) \approx 0$ means weak preference,
$P_i(\mathbf{A}_j,\mathbf{A}_k) \approx 1$ means strong preference, and
$P_i(\mathbf{A}_j,\mathbf{A}_k) = 1$ means strict preference.

In most practical cases $P_i(\mathbf{A}_j,\mathbf{A}_k)$ is function of the deviation $d = a_{ij} - a_{ik}$, i.e. $P_i(\mathbf{A}_j,\mathbf{A}_k) = p_i(a_{ij} - a_{ik})$, where p_i is a nondecreasing function, $p_i(d)=0$ for $d \le 0$, and $0 \le p_i(d) \le 1$ for $d > 0$. A set of six typical preference functions was proposed by Brans and Vincke (1985) and Brans et al. (1986). The simplicity is the main advantage of these preferences functions: no more than two parameters in each case, each having a clear economical significance.

A multicriteria preference index $\pi(\mathbf{A}_j,\mathbf{A}_k)$ of $\mathbf{A}_j$ over $\mathbf{A}_k$ can then be defined considering all the criteria:

$$\pi(\mathbf{A}_j,\mathbf{A}_k) = \sum_{i=1}^{m} w_i P_i(\mathbf{A}_j,\mathbf{A}_k).$$

This index also takes values between 0 and 1, and represents the global intensity of preference between the couples of alternatives.

In order to rank the alternatives, the following precedence flows are defined:

Positive outranking flow:

$$\phi^+(\mathbf{A}_j) = \frac{1}{n-1}\sum_{k=1}^{n} \pi(\mathbf{A}_j,\mathbf{A}_k).$$

Negative outranking flow:

$$\phi^-(\mathbf{A}_j) = \frac{1}{n-1}\sum_{k=1}^{n} \pi(\mathbf{A}_k,\mathbf{A}_j).$$

The positive outranking flow expresses how much each alternative is outranking all the others. The higher $\phi^+(\mathbf{A}_j)$, the better the alternative. $\phi^+(\mathbf{A}_j)$ represents the *power* of $\mathbf{A}_j$, its *outranking* character.

The negative outranking flow expresses how much each alternative is outranked by all the others. The smaller $\phi^-(\mathbf{A}_j)$, the better the alternative. $\phi^-(\mathbf{A}_j)$ represents the *weakness* of $\mathbf{A}_j$, its *outranked* character.

The PROMETHEE I partial ranking

$\mathbf{A}_j$ is *preferred to* $\mathbf{A}_k$ when $\phi^+(\mathbf{A}_j) \ge \phi^+(\mathbf{A}_k)$, $\phi^-(\mathbf{A}_j) \le \phi^-(\mathbf{A}_k)$, and at least one of the inequalities holds as a strict inequality.

$\mathbf{A}_j$ and $\mathbf{A}_k$ are *indifferent* when $\phi^+(\mathbf{A}_j)=\phi^+(\mathbf{A}_k)$ and $\phi^-(\mathbf{A}_j)=\phi^-(\mathbf{A}_k)$.

$\mathbf{A}_j$ and $\mathbf{A}_k$ are *incomparable* otherwise.

In this partial ranking some couples of alternatives are comparable, some others are not. This information can be useful in concrete applications for decision making.

The PROMETHEE II complete partial ranking

If a complete ranking of the alternatives is requested by the decision maker, avoiding any incomparabilities, the *net outranking flow* can be considered:

$$\phi(\mathbf{A}_j) = \phi^+(\mathbf{A}_j) - \phi^-(\mathbf{A}_j).$$

The PROMETHEE II complete ranking is then defined:

$\mathbf{A}_j$ is *preferred to* $\mathbf{A}_k$ when $\phi(\mathbf{A}_j) > \phi(\mathbf{A}_k)$, and

$\mathbf{A}_j$ and $\mathbf{A}_k$ are *indifferent* when $\phi(\mathbf{A}_j) = \phi(\mathbf{A}_k)$.

All alternatives are now comparable, the alternative with the highest $\phi(\mathbf{A}_j)$ can be considered as best one. Of course, a considerable part of information gets lost by taking the difference of the positive and negative outranking flows.

PROMETHEE V: Optimization under constraints

Optimization under constraints is a typical problem of operations research. The problem of finding an optimal selection of several alternatives, given a set of constraints, belongs to this field. PROMETHEE V extends PROMEHEE II to this selection problem. The objective is to maximize the total net outranking flow value of the selected alternatives meanwhile they are feasible to the constraints. Binary variables are introduced to represent whether an alternative is selected or not, and integer programming techniques are applied to solve the optimization problem.

The GAIA visual modelling method

The set of alternatives can be represented by *n* points in the *m*-dimensional space, where *m* is the number of criteria. As the number of criteria is usually greater than two, it is impossible to have a clear vision of these points. GAIA offers a visualization technique by projecting the points on a two-dimensional plane, where the plane is defined so that as few information as possible gets lost by the projection. The GAIA plane provides the decision maker with a powerful tool for the analysis of the differentiation power of the criteria and their conflicting aspects. Clusters of similar alternatives as well as incomparability between two alternatives are clearly represented. The projection of the vector of the weights of criteria suggests the direction, where the most promising alternatives can be found on the plane.

The methodology applied in GAIA appeared earlier in statistics as a visualization tool under the name of principal components analysis. See Rapcsák (2004) for the mathematical background of the methodology.

Strengthening PROMETHEE with ideas of AHP

Some ideas of AHP can also be applied in the PROMEETHE methodology. Recently, Macharis et al. (2004) proposed to use the pairwise comparison technique of AHP to determine the weights of the criteria. Similarly, the use of the tree-structure to decompose the decision problem into smaller parts can also be beneficial.

4. Group decision making

Group decision is usually understood as aggregating different individual preferences on a given set of alternatives to a single collective preference. It is assumed that the individuals participating in making a group decision face the same common problem and are all interested in finding a solution.

A group decision situation involves multiple actors (decision makers), each with different skills, experience and knowledge relating to different aspects (criteria) of the problem. In a correct method for synthesizing group decisions, the competence of the different actors to the different professional fields has also to be taken into account.

We assume that each actor considers the same sets of alternatives and criteria. It is also assumed that there is a special actor with authority for establishing consensus rules and determining voting powers to the group members on the different criteria. Keeney and Raiffa (1976) call this entity the Supra Decision Maker (SDM). The final decision is derived by aggregating (synthesizing) the opinions of the group members according to the rules and priorities defined by the SDM.

There are several approaches to extend the basic multiattribute decision making techniques for the case of group decision. Some earlier MAUT methods of group decision are reviewed by Bose et al. (1997). Here we present the method applied in the WINGDSS software (Csáki et al. (1995)).

Consider a decision problem with l group members (decision makers) $\mathbf{D}_1,\ldots,\mathbf{D}_l$, n alternatives $\mathbf{A}_1,..,\mathbf{A}_n$ and m criteria $\mathbf{C}_1,\ldots,\mathbf{C}_m$. In case of a factual criterion the evaluation scores must be identical for any alternative and any decision maker, while subjective (judgmental) criteria can be evaluated differently by each decision maker. Denote the result of the evaluation of decision maker $\mathbf{D}_k$ for alternative $\mathbf{A}_j$ on the criterion $\mathbf{C}_i$ by a_{ij}^k. Assume that the possible problem arising from the different dimensions of the criteria has already been settled, and the a_{ij}^k values are the result of proper transformations.

The individual preferences on the criteria are expressed as weights: let the weights of importance $w_i^k \geq 0$ be assigned at criterion $\mathbf{C}_i$ by decision maker $\mathbf{D}_k$, i=1,...,m; k=1,...,l.

The different knowledge and priority of the group members are expressed by voting powers both for weighing the criteria and qualifying (scoring) the alternatives against the criteria. For factual criteria only the preference weights given by the decision makers will be revised at each criterion by the voting powers for weighing. However, in case of subjective criteria, not only the weights but also the a_{ij}^k values will be modified by the voting powers for qualifying.

Let $V(w)_i^k$ denote the voting power assigned to $\mathbf{D}_k$ for weighing on criterion $\mathbf{C}_i$, and $V(q)_i^k$ the voting power assigned to $\mathbf{D}_k$ for qualifying (scoring) on criterion $\mathbf{C}_i$, i=1,...,m; k=1,...,l.

The method of calculating the group utility (group ranking value) of alternative $\mathbf{A}_j$ is as follows:

For each criterion $\mathbf{C}_i$, the individual weights of importance of the criteria will be aggregated into the group weights W_i:

$$W_i = \frac{\sum_{k=1}^{l} V(w)_i^k w_i^k}{\sum_{k=1}^{l} V(w)_i^k}, \quad i = 1,\ldots,m .$$

The group qualification Q_{ij} of alternative $\mathbf{A}_j$ against criterion $\mathbf{C}_i$ is:

$$Q_{ij} = \frac{\sum_{k=1}^{l} V(q)_i^k a_{ij}^k}{\sum_{k=1}^{l} V(q)_i^k}, \quad i = 1,\ldots,m,\ j = 1,\ldots,n .$$

The group utility U_j of $\mathbf{A}_j$ is determined as the weighted algebraic mean of the aggregated qualification values with the aggregated weights:

$$U_j = \frac{\sum_{i=1}^{m} W_i Q_{ij}}{\sum_{i=1}^{m} W_i}, \quad j = 1,\ldots,n .$$

In addition to the weighted algebraic means used in the above aggregations, WINGDSS also offers the weighted geometric mean, but generalized means can also be applied (see subsection 3.3.2). Csáki et al. (1995) also describes the formulas for computing in the case when the criteria are given in a tree-structure.

The best alternative of group decision is the one associated with the highest group utility. A *correct* group utility function for cardinal ranking must satisfy the axioms given in Keeney (1976). The utility function computed by the WINGDSS methodology is appropriate in this respect.

The approach of the Analytic Hierarchy Process can also be extended to group decision support (Dyer and Forman (1992)), see also Lai et al. (2202) for a recent application and further references. Since the AHP is based on pairwise comparison matrices, the key question is how to synthesize the individual pairwise comparison matrices of the group members. Aczél and Saaty (1983) showed that under reasonable assumptions (reciprocity and homogeneity) the only synthesizing function is the geometric mean. Another approach was proposed by Gass and Rapcsák (1998) for synthesizing group decisions in AHP. It consists of the aggregation of the individual weight vectors determined by singular value decomposition, taking the voting powers of the group members also into account.

Of course, the extensions of the outranking methods for group decision support have also been developed. Macharis et al. (1998) presents a PROMETHEE procedure for group decision support. Another method, based on ELECTRE methodology, was proposed by Leyva-López and Fernández-González (2003) for group decision support.

5. Sensitivity analysis

Some values of the multiattribute decision models are often subjective. The weights of the criteria and the scoring values of the alternatives against the subjective (judgmental) criteria contain always some uncertainties. It is therefore an important question how the final ranking or the ranking values of the alternatives are sensitive to the changes of some input parameters of the decision model.

The simplest case is when the value of the weight of a single criterion is allowed to vary. For additive multiattribute models, the ranking values of the alternatives are simple linear functions of this single variable and attractive graphical tools can be applied to present a simple sensitivity analysis to a user (Forman and Selly (2001)).

For a wide class of multiattribute decision models Mareschal (1988) showed how to determine the stability intervals or regions for the weights of different criteria. These consist of the values that the weights of one or more criteria can take without altering the results given by the initial set of weights, all other weights being kept constant. Wolters and Mareschal (1995) proposed a linear programming model to find the minimum modification of the weights required to make a certain alternative ranked first.

Triantaphyllou and Sanchez (1997) presented an approach of a more complex sensitivity analysis with the change of the scores of the alternatives against the criteria, as well.

A general and comprehensive methodology was presented by Mészáros and Rapcsák (1996) for a wide class of MAUT models where the aggregation is based on generalized means, including so the additive and multiplicative models as well. In this approach the weights and the scores of the alternatives against the criteria can change simultaneously in given intervals. The following questions were addressed:

- What are the intervals of the final ranking values of the alternatives with the restriction that the intervals of the weights and scores are given?

- What are the intervals of the weights and scores with the restriction that the final ranking of the alternatives does not change?
- Consider a subset of alternatives whose ranking values are allowed to change in an interval. In what intervals are the weights and scores allowed to vary, and how will these modifications effect the ranking values of the entire set of alternatives?

Mészáros and Rapcsák (1996) pointed out that these questions lead to the optimization of linear fractional functions over rectangles and proposed an efficient technique to solve these problems. Some of the results of Mészáros and Rapcsák (1996) were recently extended by Ekárt and Németh (2005) for more general decision functions.

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