MODIFIED MEAN CURVATURE MOTION FOR MULTISPECTRAL ANISOTROPIC DIFFUSION

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ABSTRACT

This paper introduces a new anisotropic diffusion algorithm for enhancing and segmenting multispectral image data. The algorithm is based upon *mean curvature motion*. Using a *modified image gradient* computation, the diffusion method is further improved by allowing the control of feature scale, and the sensitivity to heavy-tailed noise is eliminated. For comparison, a vector distance dissimilarity method is introduced and extended for multi-scale processing. The experiments on remotely sensed imagery and color imagery demonstrate the performance of the algorithms in terms of image entropy reduction and impulse elimination as well as visual quality.

I. INTRODUCTION

Anisotropic diffusion [5] is a selective smoothing technique that effectively provides intra-region smoothing and inhibits inter-region smoothing. Thus, anisotropic diffusion is useful as a precursor to image segmentation and the dual problem of edge detection. Several versions of the diffusion algorithm (e.g., [2], [5]) have been proposed for single band imagery. This paper addresses the problem of enhancing multispectral (color or remotely sensed) imagery that has been corrupted by additive noise.

In [2], Alvarez, Lions, and Morel introduced a modification of the original anisotropic diffusion mechanism. Instead of simultaneously smoothing in multiple directions, diffusion proceeds only in the direction orthogonal to the local image gradient with their approach. The partial differential equation (PDE) that models this method has been studied for its property of *mean curvature motion* (MCM). It can be shown that the level sets of the image diffuse at a rate proportional to the mean curvature.

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The standard anisotropic diffusion algorithms [2], [5] are applicable only to single-band intensity images. Though shortcomings exist, some advances in developing anisotropic diffusion algorithms for operation on multispectral images, such as color images, have been put forward. Recent work in this area includes contributions by Sapiro and Ringach [6], Chambolle [3], and Acton and Landis [1]. We will contrast the MCM approach of [3] and [6] with the dissimilarity measure approach of [1] and [8]. Furthermore, we will extend these two basic solutions with a "modified gradient" solution.

In anisotropic diffusion, the rate of smoothing is dependent upon the local value of the *diffusion coefficient*. In general, the diffusion coefficient is a continuous, nonincreasing function of the local image gradient magnitude. Typical implementations of the diffusion coefficient are extremely sensitive to impulses with high gradient magnitudes. So, one potential method to improve the performance of anisotropic diffusion on noisy images is to compute the image gradients using a filtered version of the image as in [7]. In this paper, we investigate the extension of this idea to multispectral anisotropic diffusion algorithms.

First, we review the basic MCM algorithm and present the modified approach. We also introduce vector distance dissimilarity (VDD) diffusion for multispectral imagery. The performance of MCM, VDD and the improved versions of MCM and VDD are compared in two example applications.

II. MODIFIED GRADIENT MEAN CURVATURE MOTION

Let a continuous multispectral image be represented as a function $\bar{u}(x^1, x^2): \Re^2 \to \Re^m$. The diffusion of the image can be defined by the *mean curvature motion* PDE [2]:

$$\frac{\partial \bar{u}(x^1, x^2, t)}{\partial t} = f(\lambda_+, \lambda_-) \frac{\partial^2 \bar{u}(x^1, x^2, t)}{\partial^2 \theta_-} \qquad (1)$$

where θ_- is the direction perpendicular to the image gradient, and f is the diffusion coefficient (to be defined shortly). The strength of this approach lies in the fact that diffusion of the image does not occur in the direction of maximum rate of change (which will be perpendicular to image edges), but occurs only in the direction of minimum rate of change. So, the image will be smoothed in a direction parallel to edges, but not across edges. Eq. (1) is also attractive because of its property of *mean curvature motion* (MCM) -- the level sets of the solution to this equation move in the normal direction (corresponding to the gradient) with a rate proportional to their mean curvature [2].

For diffusion on m-band multispectral data, define the intensity vector $\bar{u}(\bar{x}) = (u^1(\bar{x}), u^2(\bar{x}), \ldots u^m(\bar{x}))$, and the partial derivative term with respect to x^h as $\bar{u}_h(\bar{x}) = (\partial u^1/\partial x^h, \partial u^2/\partial x^h, \ldots \partial u^m/\partial x^h)$. The rate of change in the multispectral image can be given by $\sum_{h=1}^{2}\sum_{k=1}^{2} g_{hk}\,dx^h dx^k$, where $g_{hk}(\bar{x}) = \bar{u}_h(\bar{x}) \cdot \bar{u}_k(\bar{x})$ (a vector product), and $\sum_{h=1}^{2} dx^h dx^h = 1$. To implement MCM, we need the direction of the rate of change in addition to the magnitude. Two possible solutions, given in [4], are $\theta_0 = (1/2)\arctan(2g_{12}/(g_{11} - g_{22}))$ and $\theta_1 = \theta_0 + \pi/2$. Now, let θ_+ be the angle of the direction of the maximum rate of change, and let λ_+ be the maximum rate of change. Similarly, let θ_- be the angle of the direction of the minimum rate of change, and let λ_- be the minimum rate of change.

With the terms defined, the MCM diffusion of (1) can be implemented. The diffusion coefficient in (1) controls the rate of smoothing and is a decreasing function of the difference (λ_+ - λ_-). One possible implementation is given by

$$f(\lambda_+, \lambda_-) = \exp\left(- \frac{(\lambda_+ - \lambda_-)}{k^2} \right) \tag{2}$$

where k is a gradient magnitude threshold. Diffusion is implemented on each multispectral band separately.

Unfortunately, the MCM algorithm is sensitive to impulse noise. In fact, the MCM technique will actually enhance, not eliminate, outliers in the image. To overcome this limitation, we modify the MCM algorithm. In this modified algorithm, an *estimate* image is computed at each step for each multispectral band. The smoothed estimate images are used in the computation of the image gradients for diffusion. For Modified Gradient MCM (MGMCM), we use Gaussian-smoothed images to compute the partial derivative terms used in (1):

$$\frac{\partial^2 \bar{u}(x^1, x^2, t)}{\partial^2 \theta_-} = \frac{\partial^2 \bar{s}(x^1, x^2, t)}{\partial^2 \theta_-} \tag{3}$$

where $\bar{s} = \bar{u} * g(\sigma)$ and $g(\sigma)$ is a 2-D Gaussian kernel of standard deviation σ. Selection of σ depends on the scale desired in the enhanced image; hence, this improved PDE is capable of computing a true scale-space for the original multispectral image. Note that the diffusion mechanism operates on the unfiltered multispectral bands – only the derivatives are computed from the filtered imagery.

The Gaussian filter is not the only possible scale-generating kernel in this paradigm. Successful results have been reported for morphological filters [7]. One possible morphological method uses the open-close filter, $\bar{s} = (\bar{u} \circ \bar{B}) \bullet \bar{B}$, where scale is determined by the size (and shape) of the structuring element $\bar{B}$.

III. MODIFIED VECTOR DISTANCE DISSIMILARITY

We are contrasting the MCM algorithm with another multispectral anisotropic diffusion algorithm based on the following PDE [5]

$$\frac{\partial \bar{u}}{\partial t} = \mathrm{div}[\bar{c}\nabla\bar{u}] \tag{4}$$

where $\bar{u}(x, y, t)$ is the image intensity at location (x,y) and time t, div is the divergence operator, and $\bar{c}$ represents the diffusion coefficients. With this method (which is designed for single-band intensity images), gradient magnitudes and diffusion coefficients are usually computed for each pixel with respect to four neighbors (n, s, e, w). A discrete version of (4) is given by:

$$\begin{aligned} u(x, y, t+1) &= u(x, y, t) \\ &+ \gamma[c_n(x, y, t)\nabla u_n(x, y, t) + c_s(x, y, t)\nabla u_s(x, y, t) \\ &+ c_e(x, y, t)\nabla u_e(x, y, t) + c_w(x, y, t)\nabla u_w(x, y, t)] \end{aligned} \tag{5}$$

where $\nabla u_n(x,y,t) = u(x,y+1,t) - u(x,y,t)$ and $c_n(x,y,t) = \exp\left[- \frac{(\nabla u_n(x,y,t))^2}{k^2} \right]$ and so on for the other three directions of diffusion.

A particular color (or multispectral n-tuple) may be considered a vector in three (or n-) space. We can define gradient approximations for multispectral images in terms of the distance between two multispectral vectors, or *dissimilarity operators* [1],[8]. For example, with an RGB image, the gradient term in the "northern" direction may be defined in the following Euclidean form:

$$|\nabla u_n(x,y,t)| = \sqrt{\begin{aligned}&(R(x,y+1,t)-R(x,y,t))^2\\ &+(G(x,y+1,t)-G(x,y,t))^2\\ &+(B(x,y+1,t)-B(x,y,t))^2\end{aligned}} \quad (6)$$

where R, G, and B are the red, green, and blue intensities, respectively. The gradient magnitude terms for the other three directions of diffusion are defined similarly. Because these terms are used in the diffusion coefficient only, the gradient magnitude (not the sign) is needed. Substitution of these multispectral gradient terms into the diffusion coefficients used in (5) constitutes the VDD algorithm.

Of course, the VDD is limited by the same inability to remove impulse noise and small scale features as with the standard MCM algorithm. So, a modified gradient VDD (MGVDD) approach can be implemented by simply applying a scale generating filter prior to computation of the local image gradient magnitudes. As will be seen in the results, the modified gradient technique has significant performance advantages.

IV. RESULTS

Figures 1 - 6 demonstrate the performance of the MCM, MGMCM, VDD and MGVDD algorithms on a SPOT multispectral image of the Seattle area. Figures 3 and 4 show the results of performing 50 iterations of the MCM and the MGMCM algorithms, respectively, on the noisy image. One may observe that the MCM image in Figure 3 appears noisy, while the MGMCM is enhanced and is void of corruptive noise. The MGMCM image does not contain the small clusters of pixels, caused by noise; thus, the MGMCM result is useful for the segmentation and edge detection processes used in the interpretation of the remotely sensed imagery. Qualitatively, the VDD result (Figure 5) is not able to reject outliers nor does it enhance the noisy image. Although the MCM result is markedly better than the VDD result, the MGMCM result is comparable in visual quality to the MGVDD result (Figure 6).

The same experiment is repeated for a color imaging application, as shown in Figures 7-12. The RGB image of the Castle is corrupted with Gaussian-distributed additive noise and the MCM, MGMCM, VDD and MGVDD results are provided. Again, the modified gradient methods outperform the standard MCM and VDD methods. However, one may observe the superior edge preservation of the MGVDD approach with this man-made scene (Figure 12).

One quantitative indication of smoothing performance is the extent of reduction (or increase!) in image entropy. Tables I and II give the image entropy for the two examples, showing a slight reduction for MGMCM over MCM. Accordingly, the MGVDD result provides reduced entropy as compared to the VDD result. Another measure of effectiveness is impulse elimination. Impulses in an image are locally extreme statistical outliers and are typically due to noise. For this application, an impulse can be defined as a pixel with value differing from the values of each of its four-connected neighbors by a magnitude that is equal to or above the standard deviation of the additive noise. Tables III and IV reveal that the MGMCM method removes over 80% of impulses in both examples. The MGVDD removes nearly all of the impulses. Finally, Table V details the computational complexity of the four algorithms in terms of operations per pixel for each update. The operations are classified as additions, multiplication operations, exponential operations, trigonometric operations and square root calculations. Due to the overhead associated with computing the diffusion direction, the MCM methods are more expensive computationally.

We conclude that the use of the modified gradients significantly improves the noise elimination for the MCM and VDD diffusion techniques on multispectral data. The improvement in smoothing and noise reduction is demonstrated by the improvements in image quality, image entropy reduction and impulse elimination.

Table I: Entropy for the Seattle Example

Image	Entropy
Original	4.258
Noisy	7.688
MCM	7.369
MGMCM	7.260
VDD	7.682
MGVDD	7.047

Table II: Entropy for the Castle Example

Image	Entropy
Original	4.366
Noisy	7.542
MCM	7.317
MGMCM	7.269
VDD	7.525
MGVDD	7.195

Table III: Impulse Count for the Seattle Example

Image	Impulse Count
Original	120
Noisy	160
MCM	105
MGMCM	28
VDD	153
MGVDD	14

Table IV: Impulse Count for the Castle Example

Image	Impulse Count
Original	9
Noisy	172
MCM	57
MGMCM	23
VDD	198
MGVDD	3

Table V: Computational Complexity
(in operations per image pixel per iteration)

Algorithm	Add.	Mult.	Exp.	Trig.	Sq. Rt.
MCM	82	81	1	5	1
MGMCM	106	108	1	5	1
VDD	24	17	2	0	0
MGVDD	54	44	2	0	0

(Add. = additions/subtractions/comparisons; Mult. = multiplication/division operations; Exp. = exponential calculations; Trig. = trigonometric operations; Sq. Rt. = square root calculations.)

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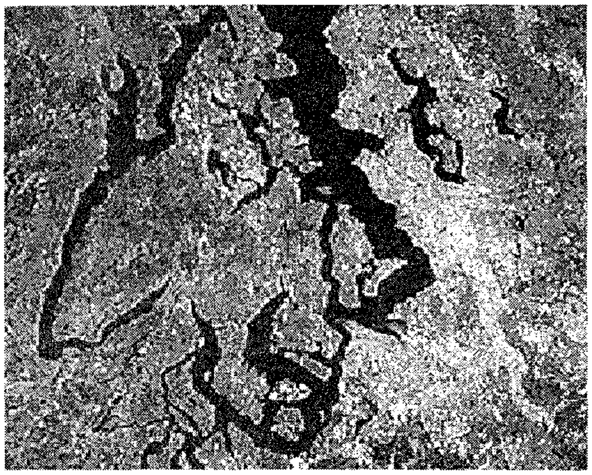

Figure 1. Original Seattle Image

Figure 2. Image with 15dB additive noise

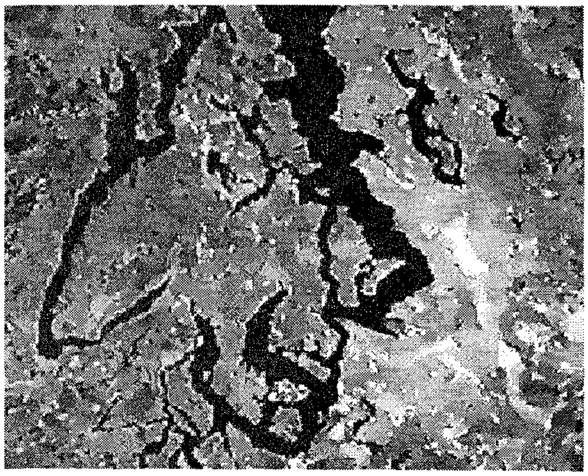

Figure 3. MCM Result

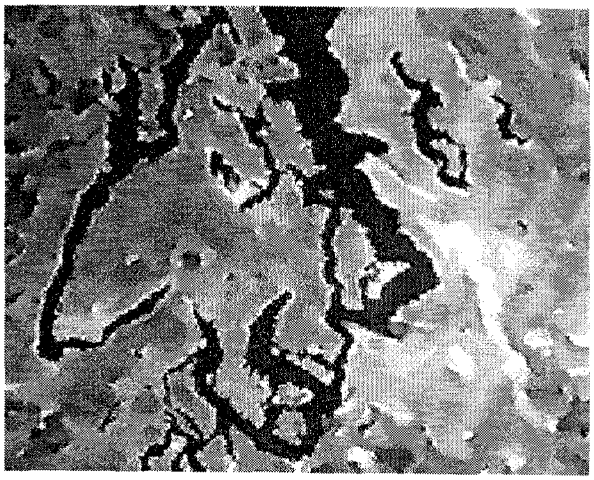

Figure 4. MGMCM Result

Figure 5. VDD Result

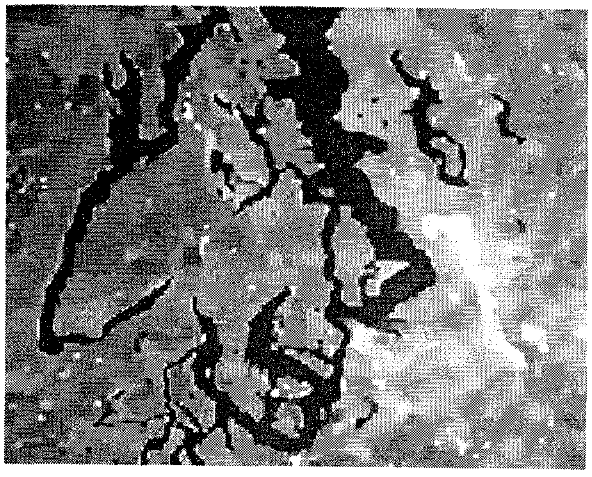

Figure 6. MGVDD Result

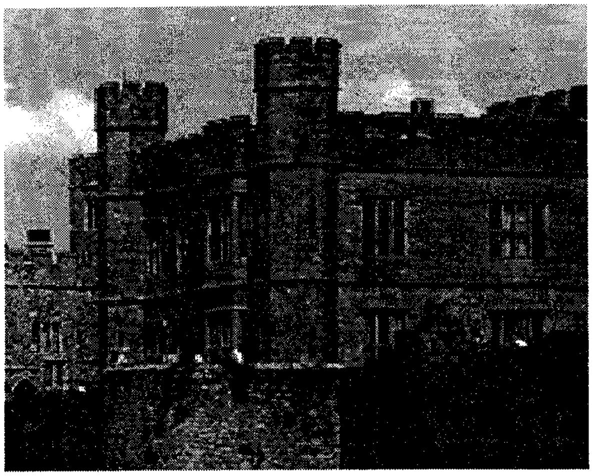

Figure 7. Original Castle Image

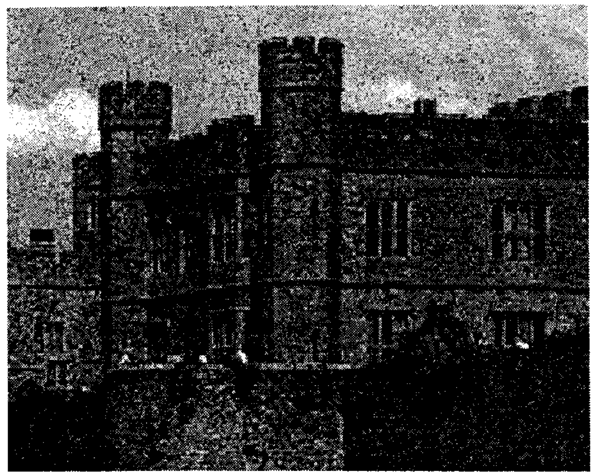

Figure 8. Image with 15dB additive noise

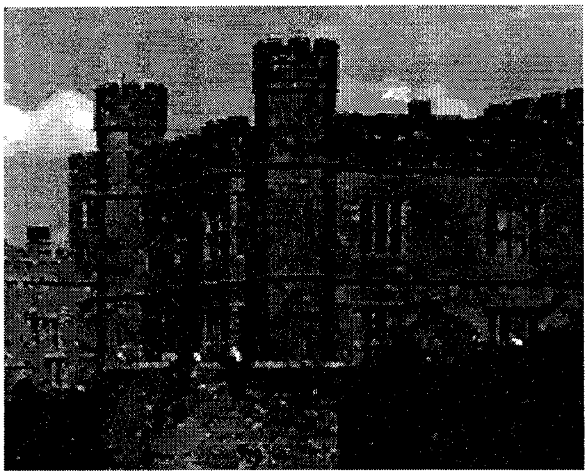

Figure 9. MCM Result

Figure 10. MGMCM Result

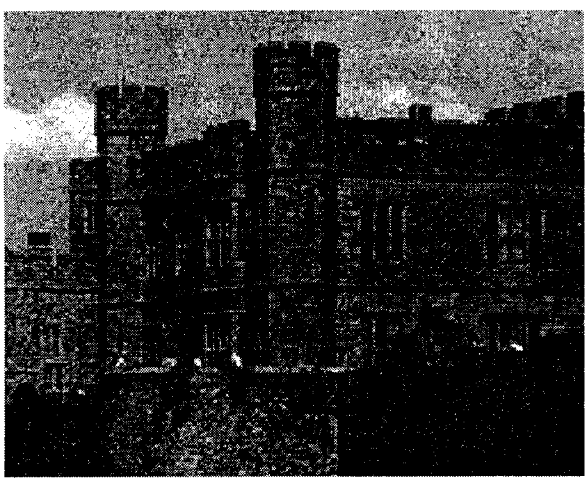

Figure 11. VDD Result

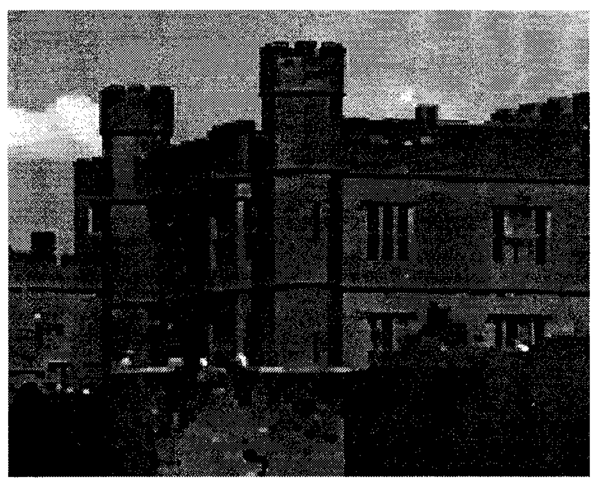

Figure 12. MGVDD Result