No.	Museum No.	Individual Table			Remarks
		Principal Number	Type	End	
163	YBC 9854	8,20	A	a-rá 50 6,56,40	19 written 20-lá-1. Cf. p. 24.
		8	A	Destroyed.	
164	CBS 29.15.76	6,40	Destr.	[a-rá 50 5,33],20	19 written 20-lá-1.
		6	A	Destroyed.	
		5	A	a-rá 50 4,10	
		4	A	Destroyed.	
		3,45	Destr.	[a-rá 50] 3,7,30	
		3,20	Destr.	Destroyed.	
165	CBS 8379	3,45	Not A.	50 3,7,30 [3,45] a-rá 3,45 1[4,3,45]	19 written 20-lá-1. Cf. p. 24.
		3,20	Not A.	Destroyed.	
166	CBS 8247	3,20	Not A.	[50] 2,46,40	Error: [20] 1,6,20(!). 19 written 20-lá-1. Small fragment; reverse(?) destroyed.
		3	C	Destroyed.	
167	CBS 29.13.174	2	A	Destroyed.	19 written 20-lá-1. Reverse: metrological table (measures of length) and blank space.
		1,40	A	Destroyed.	
168	CBS 29.15.478	1,40	C	Destroyed.	Very badly preserved; readings very doubtful. Reverse type. Obverse: two single multipl. tables for 15 (cf. p. 22 No. 98).
		1,30	Not A.	50 [1,15]	
		1,20	C	Destroyed.	

§ 4. Squares, Cubes, and Varia

e. Squares and Square Roots; Cube Roots

The canonical system apparently includes a table of squares. This is indicated by the fact that at least some of the combined tables contain a table of squares at the end.[93] Cubes are not preserved on any combined multiplication table but are so closely related to the tables of squares that there is no need to treat them separately. There is only one recognizable difference in the material available today: although we have tables of squares (16 tables) and square roots (17 tables), only tables for cube *roots* (6 tables) are hitherto attested.

The following numbers continue those listed in MKT I Chapter I § 4.

[93] E.g., Nos. 101 and 102 (MKT I p. 35).

No. 28. CBS 7369 rev. IV. Table of squares at the end of a combined multiplication table (cf. p. 30 No. 152) followed by a table of measures of length (kùš and GAR).

1	a-rá	1	1
2	a-rá	2	4
	etc.		
[11]	a-rá	11	2,1

The following is destroyed, but it is plausible that the table extended to 1,0.

No. 29. CBS 8266. Single table of square roots.

From	15-e	30 íb-si$_8$	to	[24,4]-e	38 íb-si$_8$
	16,1-e	31 íb-si$_8$		25,21-e	39 íb-si$_8$
	17,4-e	32 íb-si$_8$		26,40-e	40 íb-si$_8$
	etc.				

The last three lines are on the reverse, the remainder of which is blank.

No. 30. Plimpton 318. Single table of square roots.

From	15	30	to	54,9	57
	16,1	31		56,4	5[8]
	17,4	32		5[8],1	59
	etc.			1,2,3,2,1	1,2,1

The numbers in the last line are the squares of 1,1,1 and 1,1.[94]

No. 31. CBS 8270. Fragment containing five lines of a table of square roots. Reverse empty. Terminology destroyed.

Only the left-hand side of the fragment is preserved:

45,[4	corresponding to	52
46,4[9		53
48,3[6		54
50,25		55
52,[1]6		56

No. 31a. W 1923–366, published by Van der Meer, Syllabaries, No. 156. Six-sided prism, of which most of Col. V and all of Col. VI are destroyed. The first two columns and almost all of Col. III contain two metrological tables of measures of length.

The last four lines of Col. III, all of Col. IV, and what is left of Col. V constitute a table of square roots. The table begins

1-e 1 íb-si$_8$
4-e 2 íb-si$_8$

and continues in this fashion through Col. IV, which ends with

41,40-e 50 íb-si$_8$.

The first seven lines of Col. V are missing, but can easily be restored as having contained the squares of the integers between 51 and 57 inclusive. The few lines which are preserved in Col. V read as follows:

56,[4-e 58 íb-si$_8$]
58,1[-e 59 íb-si$_8$]

1[-e 1 íb-si$_8$]

1,2,1[-e 1,1 íb-si$_8$]
1,4,4[-e 1,2 íb-si$_8$]
1,6,9[-e 1,3 íb-si$_8$]
1,8,16[-e 1,4 íb-si$_8$]
1,10,15[sic][-e 1,5 íb-si$_8$]
1,12,36[-e 1,6 íb-si$_8$]
1,14,49[-e 1,7 íb-si$_8$].

The rest of the text is broken.

[94] Cf. MKT III p. 52.

The following errors are to be noted:

Col. IV 25:	10,41-e 29 íb-si$_8$ should read	14,1-e etc.
28:	17-e 32 íb-si$_8$	17,4-e etc.
38:	29,14-e 42 íb-si$_8$	29,24-e etc.
Col. V 8′:	1,10,15[-e 1,5 íb-si$_8$]	1,10,25-e etc.

No. 32. CBS 8165. Fragment of a single table. Obverse cube roots beginning with

1-e 1 ba-si$_8$
8-e 2 ba-si$_8$
27-e 3 ba-si$_8$.

Only parts of seven lines are preserved. The reverse is inscribed with scattered, half-erased numbers which are obviously connected with the calculation of cubes, e.g., 2,13,20 (= 20^3).

The six preceding tables all belong to the standard type of reciprocal and multiplication tables. They give the squares for all integers from 1 to 60 or the corresponding square or cube roots, and are frequently distributed on two tablets, as illustrated by the examples given above. The following text belongs to a more extensive type, and is closely related to a tablet found at Kiš and tentatively dated to the Persian period.[95]

No. 33. CBS 1535. Not from Nippur. Table of squares. Three badly damaged columns on the obverse, one on the reverse. Column I undoubtedly began with 1 a-rá 1 1; the first preserved lines are:

[5,30 a-rá	5,30	30,1]5
[6 a-rá	6	3]6
[6,30 a-rá]	6,30	42,15
[7 a-rá]	7	49
[7,30 a-r]á	7,30	56,15
	etc.	

The second column contains the squares from [18 a-rá 18 5,24] to [33,30 a-rá 3]3,30 [18,42],15, continued in Column III from [34 a-rá 34 19,16] to 46,30 a-rá 46,30 36,2,15. The concluding part from 47 [a-rá 47] 36,[49] to [1 a-rá 1 1] only fills the major part of one column of the reverse (end destroyed); the remainder of the reverse is blank.

Among the squares of this type, seven cases occur where a "zero" is called for. Unfortunately, only two of these numbers are not totally destroyed, namely,

24,30 a-rá 24,30 10,.,15
44,30 a-rá 44,30 33,0,15.

The sign which we transcribe here by "." looks

[95] Cf. MKT I p. 72 No. 27 and p. 73 note 21. A more precise dating to the end of the eighth century B.C. would be possible if, as seems likely, the person mentioned in the colophon turns out to be the same as the owner of the tablet published by Langdon VT Plate IV (p. 25).

exactly like the ordinary sign for 10, but no sign at all is given for "0" in the second case. The square of 42,30 is given as 30,.6,15 where "." again stands for a sign written like the sign for 10; but in this case the function of this special sign is not to indicate a "zero", but simply to separate 30 from 6 to prevent a mistaken reading 36.[96] The only other case where a unit should be separated from a ten-group of a higher order is destroyed. Hence this text sheds no new light on the problem of the use of a sign for zero before the Seleucid period.

The square of 51,30 is erroneously given as 43,12,15 instead of 44,12,15.

The following two texts in all probability belong to the Old-Babylonian period.

The obverse of **YBC 7294** ($6\frac{1}{2}$ by $5\frac{1}{2}$ cm.) has a line drawn parallel to the longer side and slightly left of the center, and contains three numbers written in a large hand:

2,30	6,15
2,30	

The explanation of these numbers is simple: $2{,}30^2 = 6{,}15{,}0$. The reverse is uninscribed.

Another tablet of exactly the same type is **YBC 10801**, which measures $7\frac{1}{2}$ by 6 cm. The line on the obverse is drawn to the left of the center, but this time parallel to the shorter side. The numbers, which are written large, read:

4,35	21,25
4,35	

As before, the numbers are explained by the fact that $4{,}35^2 = 21{,}0{,}25$. It is interesting to note that, as is usual in Old-Babylonian texts, the presence of the internal "zero" is not indicated in any way in the text. The reverse is again uninscribed.

For a more extensive text of a similar type, cf. MKT III p. 51.

f. Logarithms

Tablets which contain tables of exponents a^n, where n is an integer between 2 and 10, and a is one of the numbers 9, 16, 1,40, 3,45 (note that all of these are squares), are known.[96a] We now have an Old-Babylonian tablet which answers the question: to what power must a certain number a be raised in order to yield a given number? This problem is identical with finding the *logarithm* to the base a of a given number.

[96] Cf. Neugebauer [4].

[96a] Cf. MKT I pp. 77ff., Neugebauer [6], and Neugebauer, Vorlesungen, pp. 199–202.

One side of the text in question (**MLC 2078**) is destroyed except for slight traces; all edges are preserved. On the other side and on the left margin appears the following:

1 [1]15-e 2 íb-si$_8$
[2]30-e 4 íb-si$_8$
[3]45-e 8 íb-si$_8$
[4]1-e 16 íb-si$_8$
[5]*ga-mi-ru-um* níg (or: 4) *i du*(?) *uk* PI(?) . . . *ma*(?) . . . *i du*(?) *uk*(?)

2 [6]2-e 1 íb-si$_8$
[7]4-e 2 íb-si$_8$
[8]8-e 3 íb-si$_8$
[9]16-e 4 íb-si$_8$
[10]32-e 5 íb-si$_8$
[11]1,4-e 6 íb-si$_8$

Left Edge: 1,16[96b]-e 32 íb-si$_8$
1,30-e 1,4 íb-si$_8$

The meaning of the numbers in No. 1 is clearly

$$16^{0;15} = 2$$
$$16^{0;30} = 4$$
$$16^{0;45} = 8$$
$$16^{1} = 16,$$

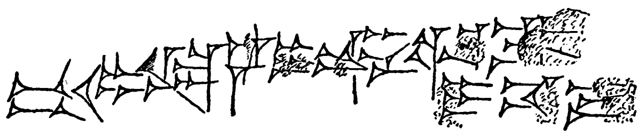
or, in other words,

$$0;15 = \log_{16} 2$$
$$0;30 = \log_{16} 4$$
$$0;45 = \log_{16} 8$$
$$1 = \log_{16} 16,$$

It is also evident that the two lines on the left edge are the direct continuation of this group, namely,

$$16^{1;15} = 32 \qquad \text{or} \qquad 1;15 = \log_{16} 32$$
$$16^{1;30} = 1{,}4 \qquad\qquad 1;30 = \log_{16} 1{,}4.$$

Line 5 (see the drawing below) may give some indica-



tion of the fact that the lines on the left edge are to be read after line 4, but we are unable to grasp what is said. It is, however, equally possible that line 5 may have served as an introductory heading to No. 2.

The second group of numbers, No. 2, means

$$2 = 2^1 \qquad\qquad 1 = \log_2 2$$
$$4 = 2^2 \qquad\qquad 2 = \log_2 4$$
$$8 = 2^3 \qquad \text{or} \qquad 3 = \log_2 8$$
$$\text{etc.} \qquad\qquad \text{etc.}$$
$$1{,}4 = 2^6 \qquad\qquad 6 = \log_2 1{,}4.$$

[96b] Sic, instead of 1,15.

The fact that the term íb-si$_8$ is used in both groups confirms the conclusion reached on the basis of the material published in MKT that íb-si$_8$ (or ba-si$_{(8)}$) not only means square root (or cube root) but is also used in connection with arithmetical operations in general where numbers are to be found which satisfy certain conditions[96c], e.g., in our case, $a^x = b$. The general character of the term is underlined in the present text by its use in two opposite directions: in No. 1 to indicate the number b, in No. 2 the logarithm x.

The new "logarithmic" tables are the logical supplement to the exponential tables mentioned at the beginning. Both exhibit a knowledge of the basic laws of operating with exponents. In a comparison with our concept of logarithm, the only missing element is the selection of a common base and the tabulation for constant intervals, which would be needed if the tables were to be used for practical computations in general. It is accordingly clear that the Old-Babylonian mathematicians were very close to an important discovery but failed to take the final, essential step. The starting-point for the whole problem is in all probability to be found in the computation of interest, which is dealt with in several previously published texts.[96d]

g. Varia, Fragments

Various tablets containing isolated numbers, obviously written down in the course of some calculation,[97] are preserved. One such note is made on CBS 3551:

7,36
7,36 íb(?)-si$_8$(?)
57,47,36

The last line should read 57,45,36, the square of 7,36. Many isolated numbers are written on CBS 7356. Numbers written in a disorderly manner within square fields are to be found on CBS 1215. CBS 7360 and CBS 7362 are very small fragments with a few number signs.

Another example, CBS 29.15.481, deals with areas, as is shown by the occurrence of uš "length" and the surface measure SAR. The following is a tentative transcription of the obverse:

40	2	20	uš	SI(?)	
40	30				
12	SAR	9(?)	erim(?)	. . .	15,42
50		. . .	uš	40(?)	
42	55		SI(?)	1,20	
$\frac{1}{2}$	SAR	$6\frac{1}{3}$	. . .	$26\frac{1}{2}$	
30	7		. . . .		
30	6		. . . .		
$\frac{1}{3}$	. . . .		. . . .		

The last section of the badly preserved reverse reads:

15 50 1 50 7 20 24 10
54 2 40 8 45 . . .
1 24 4 23 20

The following are fragments of multiplication tables: CBS 7363 (50 or 48 or 45), CBS 7359 (45 or 44,26,40), CBS 7358 (40?), CBS 29.15.499 (9?).

Three small fragments in the Metropolitan Museum are from the Seleucid (or a slightly earlier) period. The first fragment seems to be part of a larger table of reciprocals although we were able to find satisfactory restorations for no more than two lines:

MM 86.11.408. No edge preserved.

1) traces
2) blank
3)
4) 10
5) 5,24
6) 37,3,13
7) 6,4,41,14
8) [1,33,18,43,12 38,3]4,48,53,2[0]
9) [. . . (?) . . .]9,34,11(?)
10) [1,40 36]
11) [1,49,13,36 32,57,32],20,37,30
12) [. . . (?) . . .]2,9,30

The following two fragments may even belong to astronomical texts:

MM 86.11.406. No edge preserved.

1) traces
2) . . . 46(?) 20 . . .
3) . . . 3 10 30 9 20 . . . .
4) . . . 3 55 . . . . .
5) . . 32 54(?) . . .
6) . . . . 32 . . . .

MM 86.11.407. No edge preserved.

1) . . . 10(?) 50 . . .
2) . . . 10 28 . . . .
3) . . . 10 1 6 . . . .
4) . . . . 1 11(?) . . .

[96c] Cf. Neugebauer, Vorlesungen, pp. 199ff. and Neugebauer [1] pp. 201ff., especially p. 203b.

[96d] See MKT III p. 83 under "Zinseszins."—It is worth noting that the words "30-e 4 íb-si$_8$", which appear in line 2 of our present text, also occur in Strassburg 366 obv. 4 (Neugebauer MKT I p. 257; Thureau-Dangin TMB p. 205). Unfortunately, although the problem in Strassburg 366 is perfectly preserved, the precise character of the question which is asked and the reasons for the calculations which are carried out remain obscure; it is quite possible that the subject is interest on money.

[97] Examples are given MKT I pp. 80ff.