Analysis and Application of Quadratic Linearization to the Control of Permanent Magnet Synchronous Motor

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Abstract: The introduction of state equation model of the electrical machine based on direct and quadrature axes variables has paved the way for powerful control theories to be brought to bear on the problem of control of electric machines. Exact linearization has been applied to the control of permanent magnet synchronous motor (PMSM). In this paper, application of approximate linearization is proposed for the control of PMSM. Since the PMSM model is essentially quadratic, quadratic linearization is considered for the application. Conditions on the coordinate transformation and state feedback are derived for the linearization of a four dimensional permanent magnet (PM) machine model. The proposed linearization technique does not introduce singularities in the system as in the case of exact linearization. Also, to account for higher order nonlinearities including unmodelled dynamics, the linear zing transformations are adaptively tuned. Simulation studies verify the theoretical results presented. Hardware/software implementation is carried out to verify the effectiveness of the linearization technique proposed. The linearization technique proposed can also be applied to other types of electrical motors.

Keywords: Nonlinear systems; Permanent magnet motor; Quadratic Linearization; State feedback.

1. Introduction

Permanent magnet (PM) machines, particularly at low power range, are widely used in the industry because of their high efficiency. They have gained popularity in variable frequency drive applications. The merits of the machine are elimination of field copper loss, higher power density, low rotor inertia and a robust construction of the rotor [1].

A dynamic model of a PM machine using direct and quadrature axis variables such as in [1] paves the way for powerful control theories to be brought to bear on the problem of control of PM machine. Linearization which is a system-theoretic method of control is applied in this paper.

Zribi and Chiasson [2] proposed exact linearization for position control of PM stepper motor. Zhu et al [3] have combined exact-linearization with a state observer for rotor position and speed. In Wu et al [4], a two-input, two-output PMSM model is linear zed using differential geometric method. Jun Zhang et al [5] discuss decoupling control applied to PMSM using exact linearization. Bodson and Chiasson [6] have applied exact linearization to the control of electric motors including PMSM. Typically, exact input-output linearization involves deriving state feedback in terms of the inverse of a matrix of state variables, which is assumed to exist. A practical difficulty may arise when the matrix tends to be singular during the course of machine operation.In the dynamic feedback linearization method, proposed in [7], a singularity involving the rotor flux is introduced besides additional complexities involved.

Poincare derived what are known as homological equations for approximate linearization of autonomous differential systems as given in [8]. Krener [9] extended Poincare's work to include control input. Approximate linearization does not suffer from the singularity issue mentioned above. Since PMSM can be adequately described by a quadratic model during

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normal operation[1], quadratic linearization [10,11] of PMSM is proposed in this paper.

Being an approximation technique, quadratic linearization introduces third and higher order terms into the system. These together with unmodelled dynamics present in PMSM can be accounted for by tuning the linear zing transformations similar to the method proposed by Narendra et al [12]. With this modification, approximate linearization can be made equivalent to exact linearization without the singularity issue being involved.

The main contribution of the paper is the application of approximate linearization and in particular, quadratic linearization to PMSM. The original results of the paper which have not been reported elsewhere are the following:

1. Necessary and sufficient conditions for quadratic linearization of a class of two-input control affine system have been derived. PMSM model is shown to belong to this class.
2. Stability analysis is carried for the first time in this paper for the class of system considered both before and after linearization.
3. Verification of proposed theory using hardware implementation is included in the paper for the first time.

To summarize the rest of the paper, in section II, background material on quadratic linearization is given. In section III, the main theoretical result on quadratic linearization of PMSM is stated and proved. In section IV, tuning rules for linear zing transformation are derived for least square error minimization of the linear zed system output with respect to the output of a linear canonical system. In section V, simulation studies are carried out using MATLAB/SIMULINK to verify the theoretical results. In section VI, implementation of PMSM machine control using the proposed technique is described. In section VII, the paper is concluded.

2. Background

Consider a single input control affine system of the form [10, 11]

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax}+\boldsymbol{Bu}+\boldsymbol{f}^{(2)}(\boldsymbol{x})+\boldsymbol{f}^{(3)}(\boldsymbol{x})+\ldots+\boldsymbol{f}^{(m)}(\boldsymbol{x})+\cdots+\boldsymbol{g}^{(1)}(\boldsymbol{x})\boldsymbol{u}+\cdots+\boldsymbol{g}^{(m-1)}(\boldsymbol{x})\boldsymbol{u}+\cdots \tag{1}$$

where A and B are matrices in the controller normal form

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{pmatrix}; \boldsymbol{B} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \tag{2}$$

A is a $n \times n$ matrix and B is a $n \times 1$ matrix. $\boldsymbol{x} = [x_1 \quad x_2 \quad \cdots \quad x_n]^T$ and u is a scalar input. $\boldsymbol{f}^{(m)}(\boldsymbol{x}), \boldsymbol{g}^{(m-1)}(\boldsymbol{x})$ are homogeneous vector polynomials of order m and $(m-1)$ respectively, $m = 2,3,\cdots$

In order to cancel the quadratic term of the system, change of coordinate and feedback of the following form is considered, as given in [10,11]

$$\boldsymbol{y} = \boldsymbol{x} + \phi^{(2)}(\boldsymbol{x}) \tag{3}$$

$$\boldsymbol{u} = \{1+\beta^{(1)}(\boldsymbol{x})\}\boldsymbol{v} + \alpha^{(2)}(\boldsymbol{x}) \tag{4}$$

where $\phi^{(2)}(\boldsymbol{x})$ and $\alpha^{(2)}(\boldsymbol{x})$ are vector and scalar quadratic polynomials respectively and $\beta^{(1)}(\boldsymbol{x})$ is a scalar linear polynomial. y and v are the transformed (new) state and input respectively.

Applying the transformations (3) and (4), (1) can be reduced to

$$\dot{y} = Ay + Bv + O(y,v)^{(3)} \tag{5}$$

where $O(y,v)^{(3)}$ represents terms of degree greater than or equal to 3, provided the following homological equations (6) and (7) as given in [13], are satisfied. $O(y,v)^{(3)}$

$$-A\phi^{(2)}(x) + B\alpha^{(2)}(x) + f^{(2)}(x) + \frac{\partial\phi^{(2)}(x)}{\partial x}Ax = 0 \tag{6}$$

$$B\beta^{(1)}(x)v + \frac{\partial\phi^{(2)}(x)}{\partial x}Bv + g^{(1)}(x)v = 0; \forall v \tag{7}$$

$\phi^{(2)}(x)$, $\alpha^{(2)}(x)$ and $\beta^{(1)}(x)$ can be derived by solving (6) and (7).

3. Quadratic Linearization of PMSM

A. Machine Model

The PM machine model given in Bose [1] and Pillay and Krishnan [14] can be derived as below

$$\dot{x} = Ax + Bu + f^{(2)}(x) \tag{8}$$

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [\theta \quad \omega_r \quad i_q \quad i_d]^T; u = [u_1 \quad u_2]^T = [u_q \quad u_d]^T$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1.5p\lambda_{af}/J & 0 \\ 0 & -\lambda_{af}p/L_q & -R/L_q & 0 \\ 0 & 0 & 0 & -R/L_d \end{bmatrix}; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/L_q & 0 \\ 0 & 1/L_d \end{bmatrix}; f^{(2)}(x) = \begin{bmatrix} 0 \\ 1.5p(L_d - L_q)i_q i_d/J \\ -L_d p\omega_r i_d/L_q \\ L_q p\omega_r i_q/L_d \end{bmatrix}$$

where u_q, u_d, i_q, i_d represent the quadrature and direct axis voltages and currents respectively and θ, ω_r represent rotor position and rotor speed respectively. λ_{af} is the flux induced by the permanent magnet of the rotor in the stator phases. L_d, L_q are the direct and quadrature inductances respectively. R is the stator resistance, p is the number of pole pairs and J is the system moment of inertia.

Model (8) has to be first reduced to Brunovsky form [15, 16] of (2) before quadratic linearization of (3) and (4) can be applied. For this, linear transformations as given in (9) due to Kuo [17] is used.

$$x = \begin{bmatrix} a_1c_1 & 0 & 0 & 0 \\ 0 & a_1c_1 & 0 & 0 \\ 0 & 0 & c_1 & 0 \\ 0 & 0 & 0 & a_4 \end{bmatrix} y$$

$$u = \begin{bmatrix} 1 & 0 \\ 0 & a_4/c_2 \end{bmatrix} u' + \begin{bmatrix} 0 & -a_1a_2 & -a_3 & 0 \\ 0 & 0 & 0 & -a_4^2/c_2 \end{bmatrix} y \tag{9}$$

where

$$a_1 = \frac{1.5p\lambda_{af}}{J}; a_2 = \frac{-\lambda_{af}p}{L_q}; a_3 = \frac{-R}{L_q}; a_4 = \frac{-R}{L_d}; c_1 = \frac{1}{L_q}; c_2 = \frac{1}{L_d} \tag{10}$$

Using the linear transformations (9) and (10), (8) can be reduced to Brunovsky form for two inputs (11) as below (where $\boldsymbol{x}, \boldsymbol{u}, \boldsymbol{A}$ and $\boldsymbol{B}$ are retained for simplicity of notation).

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{f}^{(2)}(\boldsymbol{x}) \tag{11}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; \boldsymbol{f}^{(2)}(\boldsymbol{x}) = \begin{bmatrix} 0 \\ \boldsymbol{C}_1 \boldsymbol{x}_3 \boldsymbol{x}_4 \\ \boldsymbol{C}_2 \boldsymbol{x}_2 \boldsymbol{x}_4 \\ \boldsymbol{C}_3 \boldsymbol{x}_2 \boldsymbol{x}_3 \end{bmatrix}; \boldsymbol{C}_1 = \frac{1.5\boldsymbol{p}(\boldsymbol{L_d} - \boldsymbol{L_q})\boldsymbol{a}_4}{\boldsymbol{J}\boldsymbol{a}_1}; \boldsymbol{C}_2 = \frac{-\boldsymbol{L_d}\,\boldsymbol{p}\boldsymbol{a}_1\boldsymbol{a}_4}{\boldsymbol{L_q}}; \boldsymbol{C}_3 = \frac{\boldsymbol{L_q}\boldsymbol{c}_1^2}{\boldsymbol{L_d}\boldsymbol{a}_4}$$

$\boldsymbol{a}_1, \boldsymbol{a}_4$ and $\boldsymbol{c}_1$ are as defined in (10).

Remark 1: The quadratic linearization for the case of single input given in section II can be extended to the case of two inputs in a straight forward way to the model (11). This is considered next.

B. Conditions for Quadratic Linearization

*Theorem 1***:** Consider a four dimensional two input system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{f}^{(2)}(\boldsymbol{x}) \tag{12}$$

$$\boldsymbol{x} = [\boldsymbol{x}_1 \quad \boldsymbol{x}_2 \quad \boldsymbol{x}_3 \quad \boldsymbol{x}_4]^{\boldsymbol{T}}; \boldsymbol{u} = [\boldsymbol{u}_1 \quad \boldsymbol{u}_2]^{\boldsymbol{T}}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; \boldsymbol{f}^{(2)}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{f}_1^{(2)}(\boldsymbol{x}) \\ \boldsymbol{f}_2^{(2)}(\boldsymbol{x}) \\ \boldsymbol{f}_3^{(2)}(\boldsymbol{x}) \\ \boldsymbol{f}_4^{(2)}(\boldsymbol{x}) \end{bmatrix}$$

Then using the transformations [18]

$$\boldsymbol{y} = \boldsymbol{x} + \phi^{(2)}(\boldsymbol{x}) \tag{13}$$

$$\boldsymbol{u} = (\boldsymbol{I}_2 + \beta^{(1)}(\boldsymbol{x}))\boldsymbol{v} + \alpha^{(2)}(\boldsymbol{x}) \tag{14}$$

where

$$\boldsymbol{y} = [\boldsymbol{y}_1 \quad \boldsymbol{y}_2 \quad \boldsymbol{y}_3 \quad \boldsymbol{y}_4]^{\boldsymbol{T}};$$

$$\phi^{(2)}(\boldsymbol{x}) = \left[\phi_1^{(2)}(\boldsymbol{x}) \quad \phi_2^{(2)}(\boldsymbol{x}) \quad \phi_3^{(2)}(\boldsymbol{x}) \quad \phi_4^{(2)}(\boldsymbol{x})\right]^{\boldsymbol{T}};$$

$$\beta^{(1)}(\boldsymbol{x}) = \left[\beta_{\boldsymbol{i},\boldsymbol{j}}^{(1)}(\boldsymbol{x})\right]_{\boldsymbol{i},\boldsymbol{j}=1,2}; \boldsymbol{v} = [\boldsymbol{v}_1 \quad \boldsymbol{v}_2]^{\boldsymbol{T}} \; \alpha^{(2)}(\boldsymbol{x}) = \left[\alpha_1^{(2)}(\boldsymbol{x}) \quad \alpha_2^{(2)}(\boldsymbol{x})\right]^{\boldsymbol{T}},$$

system (12) can be quadratic linearized if and only if

$$\frac{\partial \boldsymbol{f}_1^{(2)}(\boldsymbol{x})}{\partial \boldsymbol{x}_4} = 0 \text{ and } \frac{\partial^2 f_1^{(2)}(x)}{\partial x_3^2} = 0 .$$

Proof: See Appendix.

*Remark 2***:** In the state feedback given by (14), the old input (u) is expressed in terms of new input (v) and state (x) and can be implemented as such without the need for inversion of a matrix. Thus, the issue of possible singularity as in the case of exact linearization does not arise in the approach proposed.

Corollary 1:

The PMSM model given by (11) is quadratic linearizable using the transformations (13) and (14).

Proof: Since $f_1^{(2)}(x)=0,$ the conditions for Theorem 1 are satisfied. Hence the result.

C. Derivation of Linearization Transformations

In this section, linear zing transformations (13) and (14) are derived in their simplest form for ease of implementation. Consider the normal form of PMSM model given in (11). Choosing the arbitrary function $\phi_1^{(2)}(\boldsymbol{x})=0$ and noting that $\boldsymbol{f}_1^{(2)}(\boldsymbol{x})=0,$ $(A.8)$ yields $\phi_2^{(2)}(\boldsymbol{x})=0$. $\phi_3^{(2)}(\boldsymbol{x})$ is constructed from $(A.3)$, for $i=2$ as

$$\phi_3^{(2)}(\boldsymbol{x})=\boldsymbol{f}_2^{(2)}(\boldsymbol{x})=\boldsymbol{C}_1\boldsymbol{x}_3\boldsymbol{x}_4 \tag{15}$$

$\phi_4^{(2)}(\boldsymbol{x})$ can be chosen for simplicity as $\phi_4^{(2)}(\boldsymbol{x})=0$. Hence

$$\phi^{(2)}(\boldsymbol{x})=\begin{bmatrix}0\\0\\\boldsymbol{C}_1\boldsymbol{x}_3\boldsymbol{x}_4\\0\end{bmatrix} \tag{16}$$

$\alpha^{(2)}(\boldsymbol{x})$ can be then be derived from $(A.4)$ as

$$\alpha^{(2)}(\boldsymbol{x})=\begin{bmatrix}-\boldsymbol{C}_2\boldsymbol{x}_2\boldsymbol{x}_4\\-\boldsymbol{C}_3\boldsymbol{x}_2\boldsymbol{x}_3\end{bmatrix} \tag{17}$$

$\beta^{(1)}(\boldsymbol{x})$ can be derived from $(A.6)$ as

$$\beta^{(1)}(\boldsymbol{x})=-\begin{bmatrix}\boldsymbol{C}_1\boldsymbol{x}_4 & \boldsymbol{C}_1\boldsymbol{x}_3\\0 & 0\end{bmatrix} \tag{18}$$

Using transformations (13) and (14), where $\phi^{(2)}(\boldsymbol{x})$, $\alpha^{(2)}(\boldsymbol{x})$ and $\beta^{(1)}(\boldsymbol{x})$ are as derived in (16) – (18), system (11) reduces to

$$\dot{\boldsymbol{y}}=\boldsymbol{A}\boldsymbol{y}+\boldsymbol{B}\boldsymbol{v}+\boldsymbol{O}(\boldsymbol{y},\boldsymbol{v})^{(3)} \tag{19}$$

where $\boldsymbol{O}(\boldsymbol{y},\boldsymbol{v})^{(3)}$ represents third and higher order terms.

4. Tuning

A. Tuning Formulae

When the PMSM model is quadratic linear zed, third and higher order terms are introduced into the system by the process of quadratic linearization even though the system model is not assumed to possess such higher order nonlinearity originally (refer (8) and (11)). This problem is approached in two ways. The first approach is theoretical leading to what is called 'generalized quadratic linearization' which seeks to remove the second order nonlinearity in the model while at the same time cancelling all third and higher order terms introduced. The generalized quadratic linearization is shown to be applicable to a class of control affine systems [18,19]. PMSM and induction motors, for example, belong to the class.

The second approach which is empirical and applicable to PMSM model is discussed in this section. To account for higher order nonlinearities (introduced during quadratic linearization), and the unmodelled dynamics, tuning of the linear zing transformations against an actual PM machine on the lines similar to those used by Levin and Narendra[12] is proposed. Figure 1 shows the block diagram for tuning. $\boldsymbol{N}_1$ represents state feedback transformation (14) and $\boldsymbol{N}_2$ represents coordinate transformation (13). The coefficients in the linear zing transformations $\boldsymbol{N}_1$ and $\boldsymbol{N}_2$ are updated based on the error between the outputs of quadratic linear zed system and a linear canonical form of the machine model (normal form) [20]. $\boldsymbol{V}$ Corresponds to the input and $y(m)$ represents the output at the m^{th} iteration. $\hat{y}(m)$ corresponds to the output of the linear normal form for the same input.

Referring to Figure 1, it is required to propagate error first to $\boldsymbol{N}_2$ and through the PMSM machine to $\boldsymbol{N}_1$**.** Since the error cannot be propagated through the actual machine with an unknown model, the assumed PMSM model (11) is used instead.

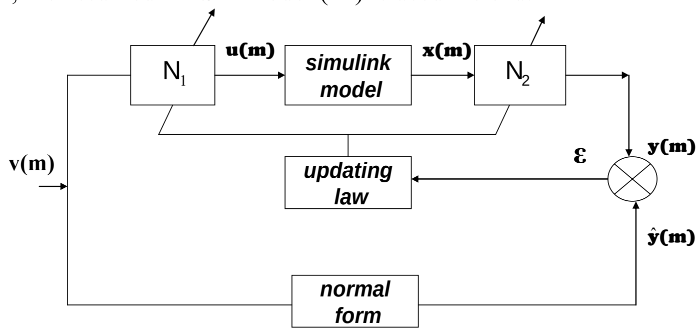

Figure 1. Block diagram for tuning of transformation

At any step 'm', squared error (E) in Figure 1 can be calculated as

$$\boldsymbol{E} = (\varepsilon^{\boldsymbol{T}}\varepsilon)^{1/2} = \left[(\boldsymbol{y}-\hat{\boldsymbol{y}})^{\boldsymbol{T}}(\boldsymbol{y}-\hat{\boldsymbol{y}})\right]^{1/2} \tag{20}$$

where $\varepsilon = \begin{bmatrix}\varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4\end{bmatrix}^{\boldsymbol{T}}$ and $\varepsilon = \boldsymbol{y} - \hat{\boldsymbol{y}}$

To tune N_1 and N_2 independently, the transformation matrices can be redefined as

$$\phi^{(2)}(\boldsymbol{x}) = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{C}_1\boldsymbol{x}_3\boldsymbol{x}_4 \\ 0 \end{bmatrix} \tag{21}$$

and

$$\beta^{(1)}(\boldsymbol{x}) = -\begin{bmatrix} \boldsymbol{C}_1^{'}\boldsymbol{x}_4 & \boldsymbol{C}_1^{'}\boldsymbol{x}_3 \\ 0 & 0 \end{bmatrix} \tag{22}$$

where $\boldsymbol{C}_1$ and $\boldsymbol{C}_1^{'}$ can be separately tuned. $\alpha^{(2)}(\boldsymbol{x})$ is not varied.

It is easily derived that N_2 can be tuned using update formula for $\boldsymbol{C}_1$ as

$$\boldsymbol{C}_1(\boldsymbol{m}) = \boldsymbol{C}_1(\boldsymbol{m}-1) - \rho_1\Delta\boldsymbol{C}_1(\boldsymbol{m}); 0 < \rho_1 < 1 \tag{23}$$

where $\boldsymbol{m}$ corresponds to the updating step, ρ_1 corresponds to the accelerating factor and

$$\Delta\boldsymbol{C}_1 = \frac{\partial\boldsymbol{E}}{\partial\boldsymbol{C}_1} = \frac{\partial\boldsymbol{E}}{\partial\boldsymbol{y}}\frac{\partial\boldsymbol{y}}{\partial\boldsymbol{C}_1} = \frac{\varepsilon_3}{\boldsymbol{E}}\boldsymbol{x}_3\boldsymbol{x}_4 \tag{24}$$

For updating $\boldsymbol{N}_1$, it is as summed that the steady state of the model (11) is reached within the tuning period and the tuning rule for $\boldsymbol{C}_1^{'}$ is derived as

$$\boldsymbol{C}_1^{'}(\boldsymbol{m}) = \boldsymbol{C}_1^{'}(\boldsymbol{m}-1) - \rho_2\Delta\boldsymbol{C}_1^{'}(\boldsymbol{m}); 0 < \rho_2 < 1 \tag{25}$$

where m corresponds to the updating step, ρ_2 correspond to the accelerating factor and

$$\Delta C_1{}' = \frac{(v_1 x_4 + v_2 x_3)}{E}\left(\frac{\varepsilon_2}{C_2 x_4} + \frac{\varepsilon_3 C_1 x_3 + \varepsilon_4}{C_2 x_2}\right) \tag{26}$$

B. Stability Analysis

In order to check the asymptotic stability of (11), for autonomous system we put $\boldsymbol{u} = 0$, which results in

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f}^{(2)}(\boldsymbol{x}) \tag{27}$$

Assuming A is strictly Hurwitz as given in Fang et al [21] and Greenberg [22], there exist, symmetric and positive – definite matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$ which satisfy

$$\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -2\boldsymbol{Q} \tag{28}$$

Consider the following candidate Lyapunov function

$$\boldsymbol{V}(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{x} \tag{29}$$

The time derivative of $V(x)$ is given by

$$\dot{\boldsymbol{V}}(\boldsymbol{x}) = -\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{Q}\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{f}^{(2)}(\boldsymbol{x}) + \frac{1}{2}(\boldsymbol{f}^{(2)}(\boldsymbol{x}))^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{x} \tag{30}$$

It can be verified that $\boldsymbol{f}^{(2)}(\boldsymbol{x})$ is locally Lipschitz, thus, there exists a positive constant M such that

$$\left\|\boldsymbol{f}^{(2)}(\boldsymbol{x}) - \boldsymbol{f}^{(2)}(0)\right\| \le \boldsymbol{M}\|\boldsymbol{x}\| \tag{31}$$

Then

$$\dot{\boldsymbol{V}}(\boldsymbol{x}) \le -\|\boldsymbol{Q} - \boldsymbol{M}\boldsymbol{P}\|\left\|\boldsymbol{x}^2\right\| \tag{32}$$

On that account if the condition

$$\boldsymbol{Q} > \boldsymbol{M}\boldsymbol{P} \tag{33}$$

is satisfied, $x = 0$ is an asymptotically stable equilibrium.

This means that (11) is asymptotically stable under certain conditions. In order to check asymptotic stability of system (19), which results after quadratic linearization, for autonomous system we put $\boldsymbol{v} = 0$, which results in

$$\dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{y} + \boldsymbol{O}(\boldsymbol{y})^{(3)} \tag{34}$$

Assuming A is strictly Hurwitz as given in Fang et al [21] and Greenberg [22], there exist, symmetric and positive – definite matrices P and Q which satisfy

$$\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} = -2\boldsymbol{Q} \tag{35}$$

Consider the following candidate Lyapunov function

$$\boldsymbol{V}(\boldsymbol{y}) = \frac{1}{2}\boldsymbol{y}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{y} \tag{36}$$

The time derivative of $V(y)$ is given by

$$\dot{\boldsymbol{V}}(\boldsymbol{y}) = -\boldsymbol{y}^{\boldsymbol{T}}\boldsymbol{Q}\boldsymbol{y} + \frac{1}{2}\boldsymbol{y}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{O}(\boldsymbol{y})^{(3)} + \frac{1}{2}(\boldsymbol{O}(\boldsymbol{y})^{(3)})^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{y} \tag{37}$$

Assuming that $O(y)^{(3)}$ is locally Lipschitz, thus, there exists a positive constant M_0 such that

$$\left\| \boldsymbol{O}(\boldsymbol{y}_1)^{(3)} - \boldsymbol{O}(\boldsymbol{y}_2)^{(3)} \right\| \le \boldsymbol{M}_0 \left\| \boldsymbol{y}_1 - \boldsymbol{y}_2 \right\| \tag{38}$$

for y_1, y_2 in a neighborhood R_0 containing the origin.
Then, the time derivative of $V(y)$ is given by

$$\dot{\boldsymbol{V}}(\boldsymbol{y}) \le -\left\| \boldsymbol{Q} - \boldsymbol{M}_0 \boldsymbol{P} \right\| \left\| \boldsymbol{y}^2 \right\| \tag{39}$$

On that account if the condition

$$\boldsymbol{Q} > \boldsymbol{M}_0 \boldsymbol{P} \tag{40}$$

is satisfied, $y = 0$ is an asymptotically stable equilibrium. Stability analysis of PMSM model follows as a special case.

5. Simulation Results

Application of coordinate and state feedback to linearized the PMSM model is simulated using MATLAB/SIMULINK [23]. Effectiveness of the tuning of the transformations is demonstrated through simulation results. A sample of the experimental data is given in this section. A larger set of the experimental data in [24] confirm the conclusions obtained.

The objective of simulation is to investigate the open loop steady state gain (ω_r versus u_q) of the PMSM model under different operating conditions before and after linearization and to verify if the system behaves like a linear system after linearization. Also, dynamic responses of the system is are studied for variations in reference speed and load conditions for uniformity of response as is characteristic of a linear system. Effectiveness of tuning is also investigated in on similar lines by obtaining dynamic responses of the system for different set points before and after tuning.

A. Quadratic Linearization

For the Interior PMSM, parameters are taken as follows:
Stator resistance R= 0.15ΩΩ; q- axis Inductance L_q = 1.2mH; d-axis Inductance L_d = 0.76mH; Flux induced in magnets λ = 0.013125 Wb; Moment of Inertia J = 0.0008 kg m^2; Friction factor B = 1 N-msec; No. of pole pairs p = 4.

Figure 2 shows the Simulink model of the PMSM which is constructed using speed block, torque block and control circuit as given in [23]. u_q and u_d are taken as inputs to the motor.

The PM model in figure 2 is especially configured for the IPM where $L_d \ne L_q$ (blocks shown in cyan colour are not included).

Figure 3 shows the linearization of PMSM (blocks shown in cyan colour are not included). L_1 and L_2 blocks in Figure 3 include the linear transformations (9) and N_1 and N_2 represent the nonlinear transformations (14) and (13). Prior to linearization, the open loop steady state gain of ω_r versus u_q of the PMSM model is investigated and it is observed that the open loop steady state gain of ω_r versus u_q (keeping u_d constant) varies under different operating conditions giving a standard deviation of over 50% of the average value of the gain. After linearization, the static gain variation corresponded to a standard deviation of a little over 3% of the average value of the gain thus verifying improved static linearity.

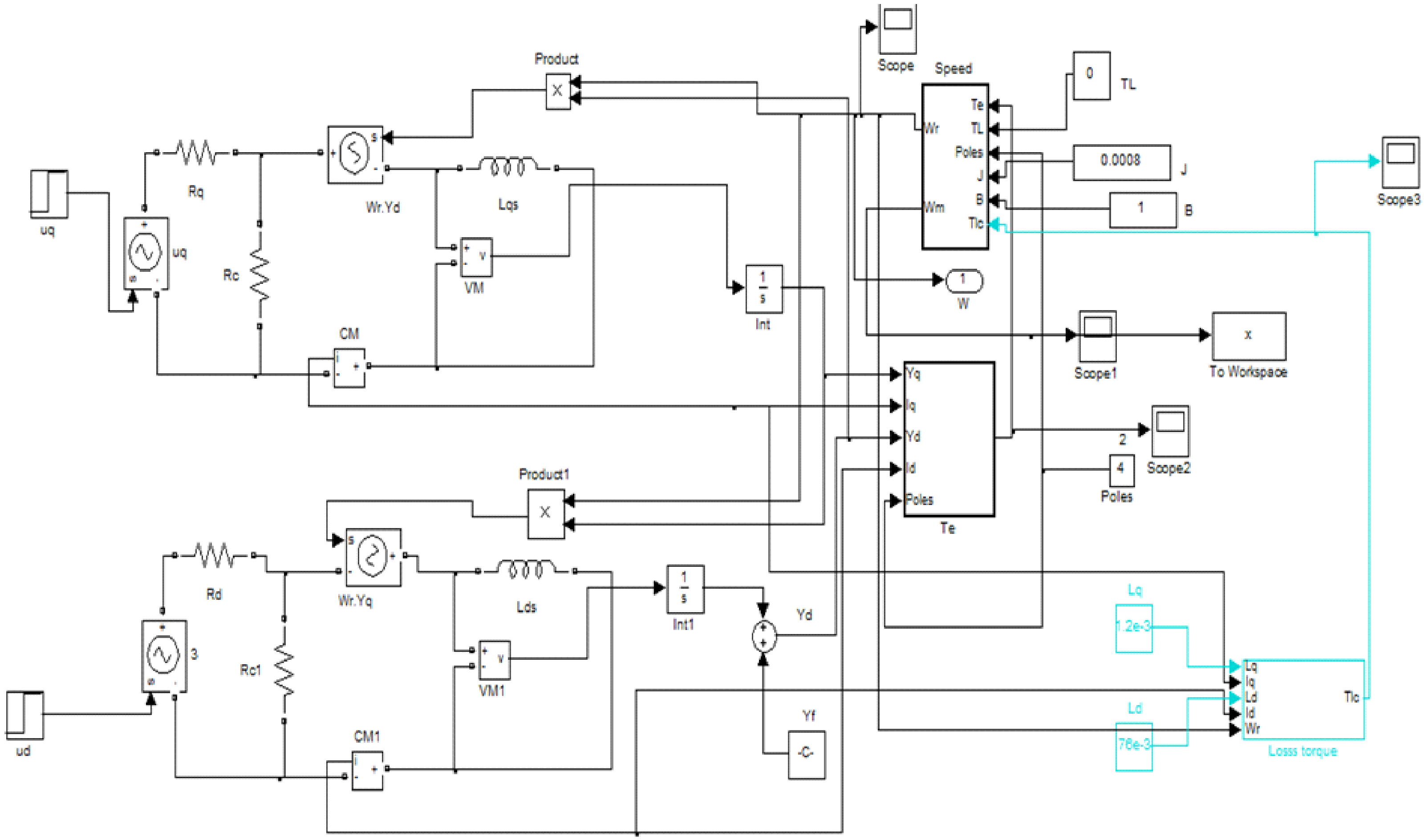

Figure 2. PMSM with u_q and u_d as inputs

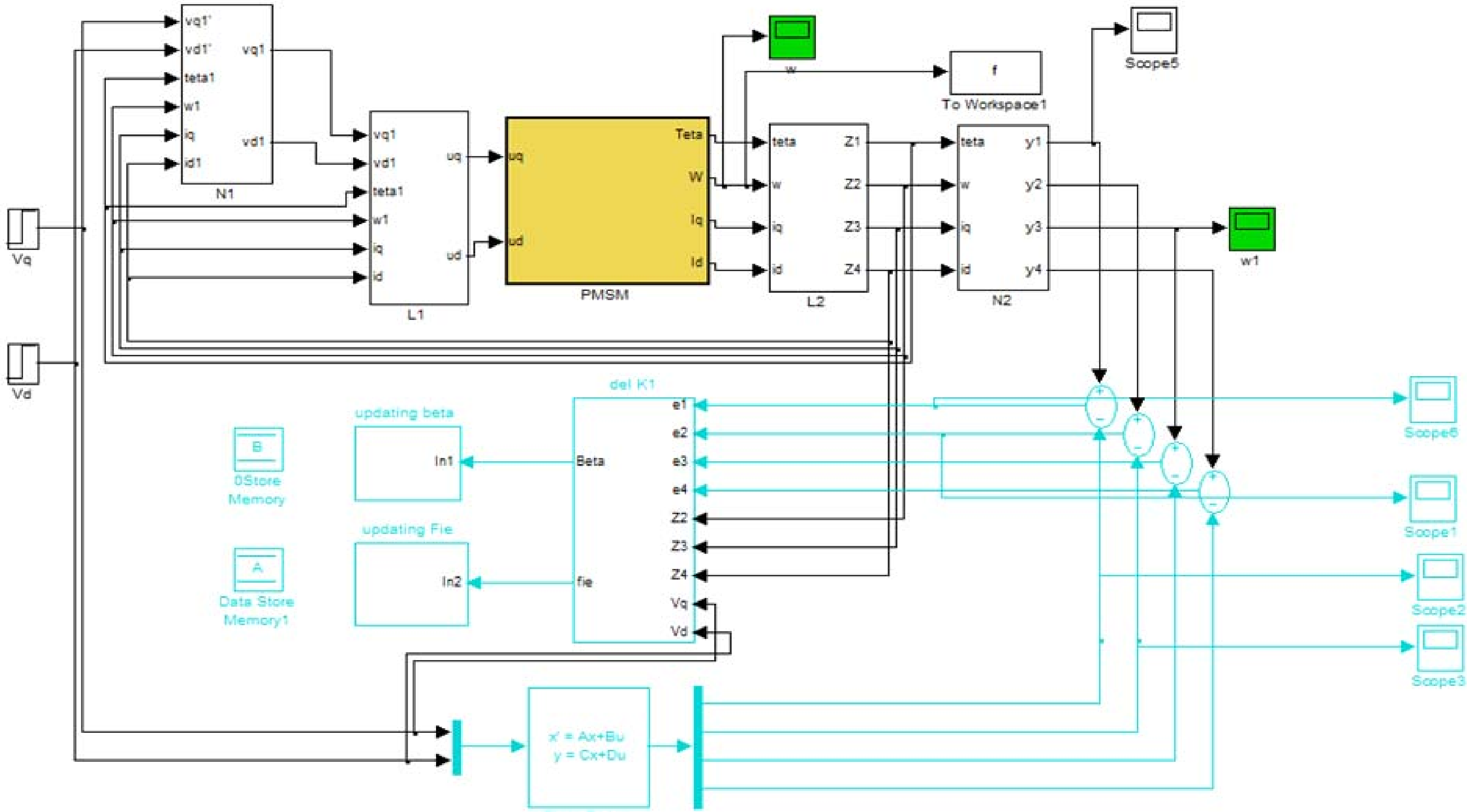

Figure 3. Linearization of PMSM model

Combined closed loop responses of angular speed of the motor after linearization and before linearization are shown for reference speed of 120 rad/s in Figure 4, assuming zero load torque. The response after linearization is uniform over the reference input range as expected of a linear system. Also, the settling time of angular speed is considerably reduced. Figure5 shows the combined closed loop speed responses when the load is varied after and before linearization for the same reference speed of 120 rad/s. Again, while the response is not uniform before linearization, the response after linearization is uniform as can be expected of a linear system. Here k_p and k_i represent proportional constant and integral constant of the controller respectively.

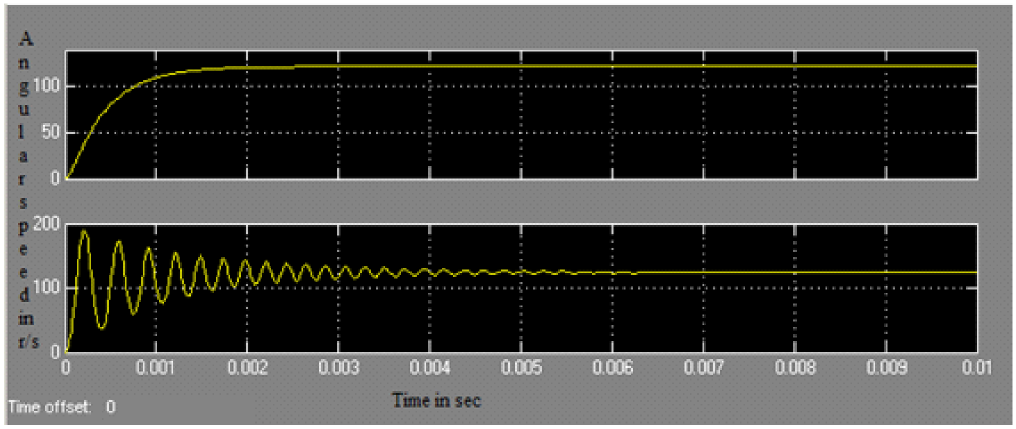

Figure 4. Combined Time response of angular speed in closed loop after and before linearization when set speed =120 rad/s; $k_p = 50; k_i = 2$

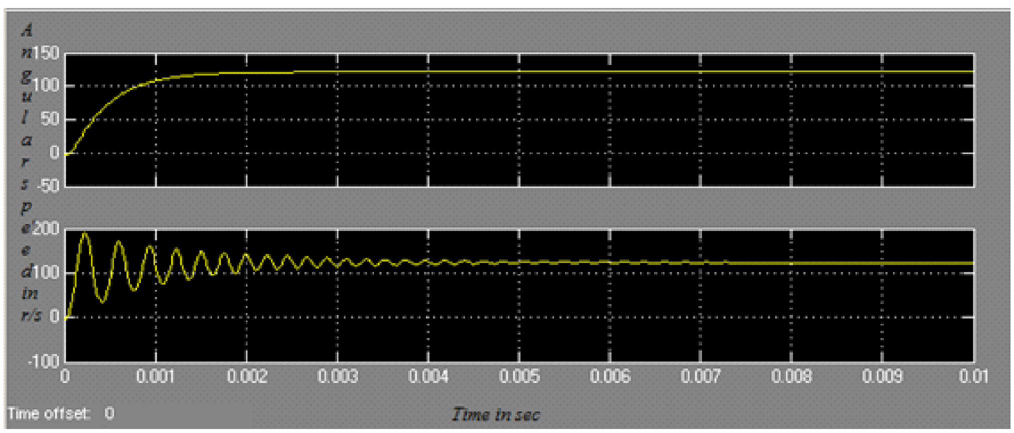

Figure 5. Combined Time Response of angular speed in closed loop after and before linearization when set speed = 120 rad/s T = 1.5 N-m; $k_p = 50; k_i = 2$

B. Simulation of Tuning

To simulate higher order loss, core loss is included in the Simulink model. The core loss caused by the permanent magnet (PM) flux and armature reaction flux, is a significant component of the total loss of a PMSM. The net core loss P_{lc} as given by Ramin Monajemy[25], for PMSM is computed as follows:

$$P_{lc} = \left[\frac{1.5\omega_r^2 (L_q i_q)^2}{R_c} + \frac{1.5\omega_r^2 (\lambda_{af} + L_d i_d)^2}{R_c} \right] \tag{41}$$

where R_c represents core loss resistance, λ_{af} represents magnet flux linkage, P_{lc} represents core loss and ω_r denotes rotor electrical speed. The mechanical torque equation including core losses is given by (42), where T_l represents load torque and T_{lc} represents torque due core loss.

$$T_e = J\frac{d\omega_r}{dt} + T_l + T_{lc} \tag{42}$$

$$T_{lc} = \frac{P_{lc}}{\omega_r} \tag{43}$$

PMSM Simulink model including core loss is given in Figure 2. The blocks representing core loss is indicated in cyan colour in Figure 2. The loss torque block represents the core loss. The tuning of the transformation is carried out as per update laws given in Equations (23) and (25) and is included in Figure 3 (indicated by blocks in cyan colour). The tuning is stopped after the error $E \le 0.01$.

Prior to tuning, the open loop steady state gain of y_2 versus v_1 of the PMSM model after linearization including core loss is investigated and it is observed that the open loop steady state gain of y_2 versus v_1 varies under different operating conditions giving a standard deviation of over 70% of the average value of the gain. After tuning, the static gain variation corresponded to a standard deviation of a little over 2% of the average value of the gain thus verifying the effect of tuning. The combined closed loop response after tuning and before tuning is shown in Figure 6. The closed loop response before tuning is highly non-uniform when compared to the uniform response after tuning. The settling time of angular speed is also reduced considerably after tuning. This verifies that the tuning can effectively cancel the higher order nonlinearity.

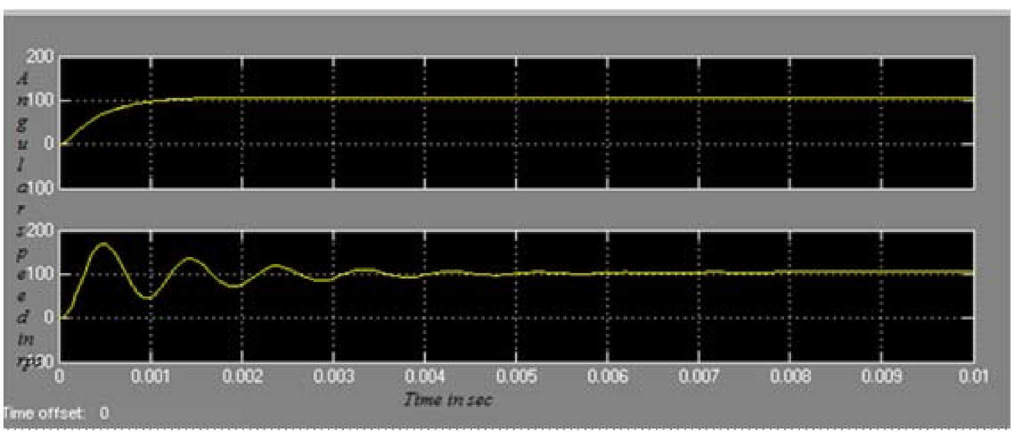

Figure 6. Combined Time response of angular speed in closed loop after and before tuning when set speed = 110 rad/s; $k_p = 50; k_i = 2$

C. Hardware Implementation

Hardware implementation is done using a PMSM machine to verify the effectiveness of the linearization technique proposed. Surface Mounted Permanent Magnet motor is used for the hardware implementation. The proposed system of Figure 7 was implemented. TMS320F2812 DSP controller operating with a clock speed of 150 MHz was used to carry out the implementation of Clarke's and inverse Clarke's transformations, Park's and inverse Park's transformations[26], linear zing transformation, PI controller, and inverter switching for speed control. A three phase insulated gate bipolar transistor (IGBT) power module was used for the inverter, which was supplied at a DC link supply voltage of 325 V. An incremental encoder (@2000 pulses/rev) was used to calculate the rotor speed and to determine the initial position of rotor position (θ).

Figures 8 and 9 show closed loop speed responses before and after linearization under reference input change for a given load. Figures 10 and 11 show the closed loop speed responses before linearization when a load is applied and released respectively. Figures 12 and 13 show the corresponding responses after linearization. The responses shown in Figures 8 to 13 indicate speed in rpm and time in seconds.

It is seen from these figures that the dynamic responses of speed for step change in speed reference is smoother and more uniform for cases after linearization, when compared to the cases before linearization. Also it is seen that there are spikes in the responses before linearization. The dynamic responses of speed for load variations are also smoother and more uniform for the cases after linearization, when compared to the cases before linearization. The results presented here are necessarily samples of a more comprehensive set of experimental data obtained. It is verified by experimental results that a uniform dynamic response under a fixed controller can be obtained for the linear zed system for different load conditions and set point variations, in contrast to the case before linearization.

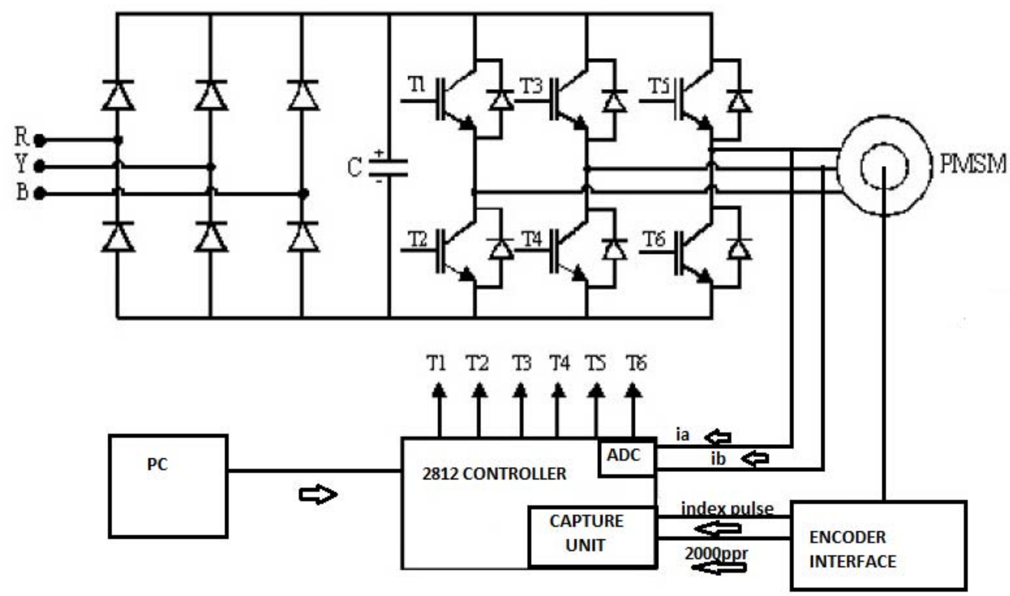

Figure 7. Hardware Implementation Diagram

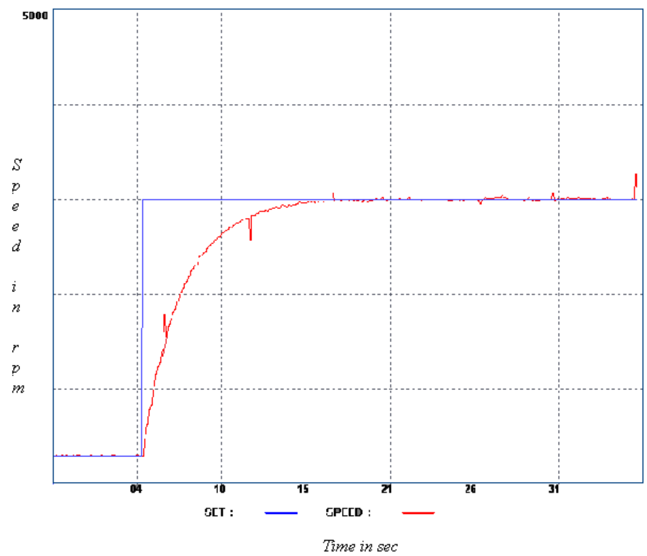

Figure 8. Speed control before linearization when load is 3 kg and sets peed is 3000 rpm

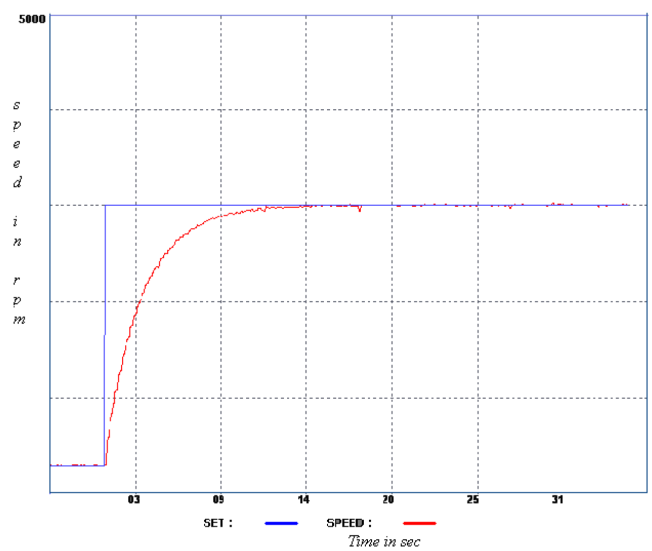

Figure 9. Speed control after linearization when load is 3 kg and set speed is 3000 rpm

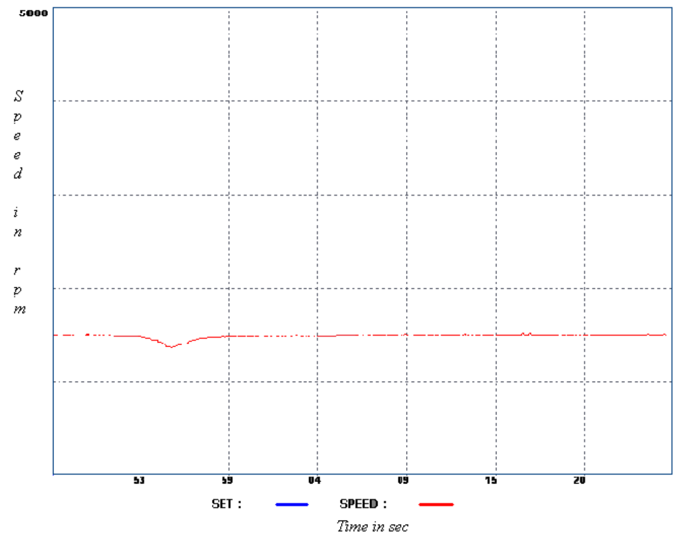

Figure 10. Speed control before linearization when speed is 1500 rpm and 1 kg is applied

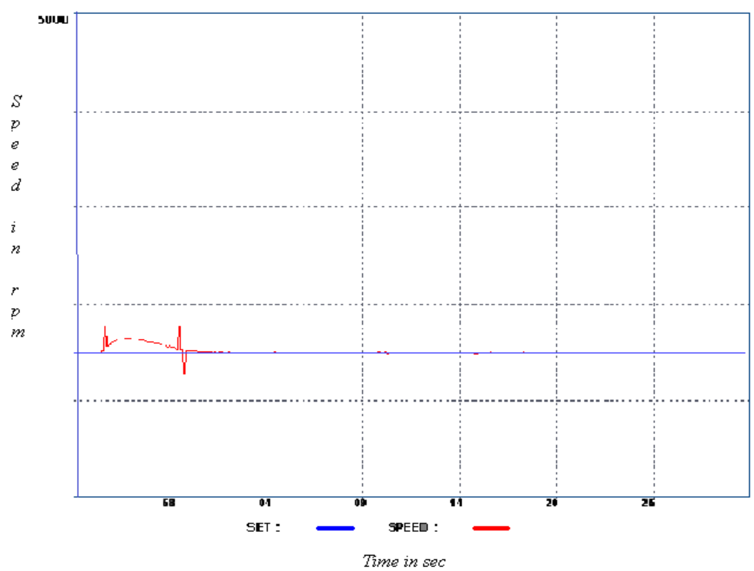

Figure 11. Speed control before linearization when speed is 1500 rp mand 1 kg is released

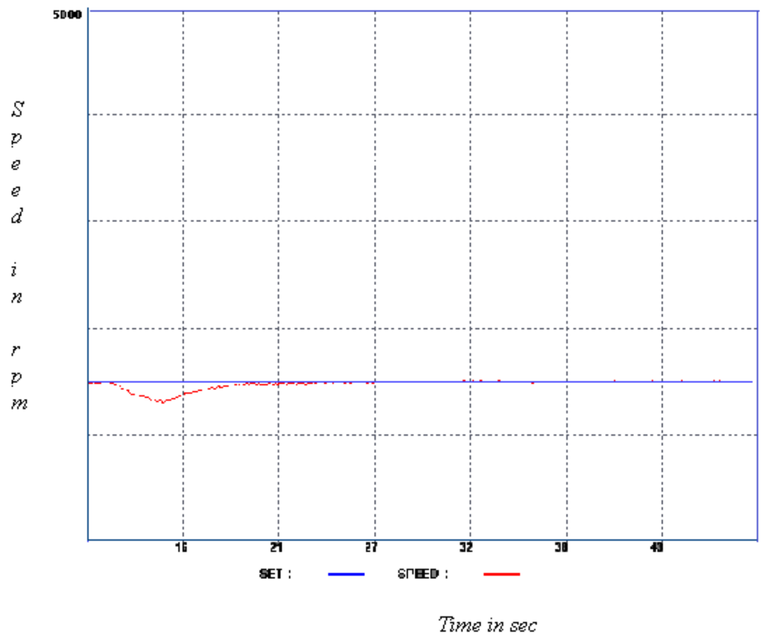

Figure 12. Speed control after linearization when speed is 1500 rpm and 1 kg is applied

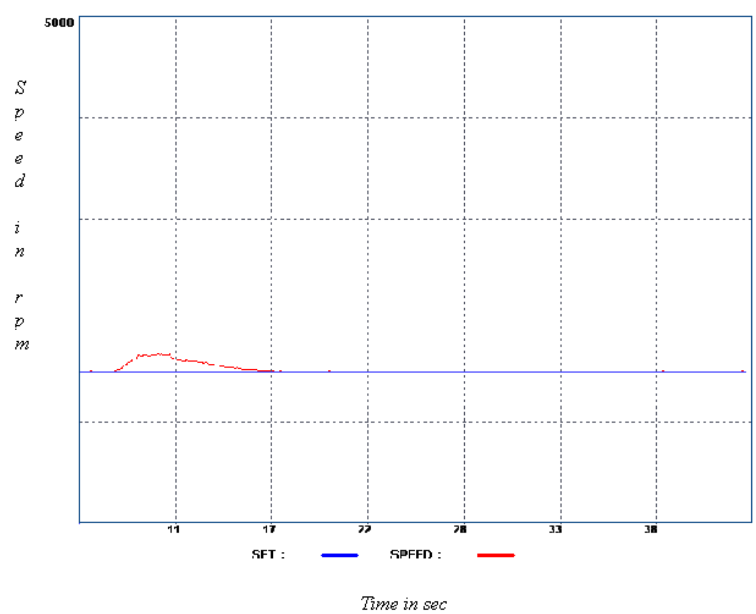

Figure 13. Speed control after linearization when speed is 1500 rpm and 1 kg is released

6. Conclusion

Necessary and sufficient conditions for quadratic linearizability of a class of control affine systems are derived. PMSM model is shown to belong to this class. Linearzing transformations for PMSM model is derived. The necessary and sufficient conditions derived specifically for the PMSM model are new. Simulation studies are carried out using MATLAB/SIMULINK to verify quadratic linearization of PMSM. Also by simulating core loss, as an example of higher order nonlinearity, it is shown that the linear zing transformations can be tuned against a canonical linear form of the machine to cancel the nonlinearity. Finally, a practical implementation of quadratic linearization on an actual PMSM machine is carried out with the help of DSP system together with PMSM and associated circuits. Experimental results are obtained which verify the theoretical results presented.

The approximate linearization technique is proposed to be extended to other types of electrical machines, such as, induction motor, wound synchronous motor etc. Necessary and sufficient conditions for quadratic linearization are to be derived for the other machine models as well. Necessary and sufficient conditions for quadratic linearization of a class of two-input control affine system which has been derived can be applied to any control affine system with quadratic nonlinearity, for example, heat exchanger process involving product of mass flow and temperature or mass transfer process involving product of mass flow and concentration.

Appendix

Proof of Theorem 1

Proof: For the special class of system of the form (12), homological equations (6) and (7) can be rewritten as

$$-\boldsymbol{A}\phi^{(2)}(\boldsymbol{x})+\boldsymbol{B}\alpha^{(2)}(\boldsymbol{x})+\boldsymbol{f}^{(2)}(\boldsymbol{x})+\frac{\partial\phi^{(2)}(\boldsymbol{x})}{\partial\boldsymbol{x}}\boldsymbol{A}\boldsymbol{x}=0 \qquad (\boldsymbol{A}.1)$$

$$\boldsymbol{B}\beta^{(1)}(\boldsymbol{x})\boldsymbol{v}+\frac{\partial\phi^{(2)}(\boldsymbol{x})}{\partial\boldsymbol{x}}\boldsymbol{B}\boldsymbol{v}=0;\forall\boldsymbol{v} \qquad (\boldsymbol{A}.2)$$

Substitute for A, B and $f^{(2)}(x)$ from (12) into (A.1), to get

$$-\phi_{\boldsymbol{i}+1}^{(2)}(\boldsymbol{x})+\boldsymbol{f}_{\boldsymbol{i}}^{(2)}(\boldsymbol{x})+\sum_{\boldsymbol{j}=1,2}\frac{\partial\phi_{\boldsymbol{i}}^{(2)}(\boldsymbol{x})}{\partial\boldsymbol{x}_{\boldsymbol{j}}}\boldsymbol{x}_{\boldsymbol{j}+1}=0;\boldsymbol{i}=1,2 \qquad (\boldsymbol{A}.3)$$

$$\alpha_{\boldsymbol{k}}^{(2)}(\boldsymbol{x})+\boldsymbol{f}_{\boldsymbol{i}}^{(2)}(\boldsymbol{x})+\sum_{\boldsymbol{j}=1,2}\frac{\partial\phi_{\boldsymbol{i}}^{(2)}(\boldsymbol{x})}{\partial\boldsymbol{x}_{\boldsymbol{j}}}\boldsymbol{x}_{\boldsymbol{j}+1}=0;\boldsymbol{i}=3,4;\boldsymbol{k}=(\boldsymbol{i}-2) \qquad (\boldsymbol{A}.4)$$

By inspection, (A.2) can be reduced to

$$\left\{\frac{\partial\phi_{\boldsymbol{i}}^{(2)}(\boldsymbol{x})}{\partial\boldsymbol{x}_{\boldsymbol{j}}}\right\}_{\substack{\boldsymbol{i}=1,2\\ \boldsymbol{j}=3,4}}=0 \qquad (\boldsymbol{A}.5)$$

and

$$\beta^{(1)}(\boldsymbol{x})+\left\{\frac{\partial\phi_{\boldsymbol{i}}^{(2)}(\boldsymbol{x})}{\partial\boldsymbol{x}_{\boldsymbol{j}}}\right\}_{\boldsymbol{i},\boldsymbol{j}=3,4}=0 \qquad (\boldsymbol{A}.6)$$

(A.4) and (A.6) containing arbitrary terms $\alpha_k^{(2)}(x)$ and $\beta^{(1)}(x)$ respectively, can be satisfied for arbitrary $\phi_i^{(2)}(x); i=3,4$.

Also, $\phi_1^{(2)}(x)$ being arbitrary, can be chosen such that

$$\frac{\partial\phi_1^{(2)}(\boldsymbol{x})}{\partial\boldsymbol{x}_{\boldsymbol{j}}}=0;\boldsymbol{j}=3,4 \qquad (\boldsymbol{A}.7)$$

Since $\phi_3^{(2)}(x)$ is arbitrary, as mentioned above, (A.3) can be satisfied for $i=2$. This together with the substitution of (A.7) into (A.5), results in (A.8) and (A.9) which are equivalent to homological equations (A.1) and (A.2).

$$-\phi_2^{(2)}(x)+f_1^{(2)}(x)+\sum_{j=1,2}\frac{\partial\phi_1^{(2)}(x)}{\partial x_j}x_{j+1}=0 \qquad (A.8)$$

$$\frac{\partial\phi_2^{(2)}(x)}{\partial x_j}=0;\ j=3,4 \qquad (A.9)$$

To prove necessity, differentiating $(A.8)$, with respect to $x_k; k=3,4$ and denoting $\frac{\partial\psi_j^{(2)}}{\partial x_k}$ as $\psi_{j,x_k}^{(2)}$ and $\frac{\partial}{\partial x_l}\frac{\partial\psi_j^{(2)}}{\partial x_k}$ as $\psi_{j,x_k,x_l}^{(2)}$ and using (A.9), one can have

$$\phi_{2,x_3}^{(2)}(x)=f_{1,x_3}^{(2)}(x)+\sum_{j=1,2}\phi_{1,x_j,x_3}^{(2)}x_{j+1}+\phi_{1,x_2}^{(2)}(x)=0 \qquad (A.10)$$

and

$$\phi_{2,x_4}^{(2)}(x)=f_{1,x_4}^{(2)}(x)+\sum_{j=1,2}\phi_{1,x_j,x_4}^{(2)}x_{j+1}=0 \qquad (A.11)$$

Using (A.7), (A.10) and (A.11) can be written as

$$\phi_{2,x_3}^{(2)}(x)=f_{1,x_3}^{(2)}(x)+\phi_{1,x_2}^{(2)}(x)=0 \qquad (A.12)$$

$$\phi_{2,x_4}^{(2)}(x)=f_{1,x_4}^{(2)}(x)=0$$

That is

$$f_{1,x_4}^{(2)}(x)=0 \qquad (A.13)$$

Further by differentiating (A.12) with respect to x_3, one can get

$$\phi_{2,x_3,x_3}^{(2)}=f_{1,x_3,x_3}^{(2)}+\phi_{1,x_2,x_3}^{(2)}=0$$

Using (A.7), it follows that

$$f_{1,x_3,x_3}^{(2)}=0 \qquad (A.14)$$

The necessity of the result thus follows.

To prove sufficiency, it is required to show that assuming (A.13) and (A.14), $\phi_2^{(2)}(x)$ given by (A.8) has to satisfy (A.9).

Differentiating (A.8) with respect to x_3 and rearranging

$$\phi_{2,x_3}^{(2)}(x)=f_{1,x_3}^{(2)}(x)+\sum_{j=1,2}\phi_{1,x_j,x_3}^{(2)}x_{j+1}+\phi_{1,x_2}^{(2)}(x) \qquad (A.15)$$

Using (A.7), (A.15) becomes

$$\phi_{2,x_3}^{(2)}(x)=f_{1,x_3}^{(2)}(x)+\phi_{1,x_2}^{(2)}(x) \qquad (A.16)$$

Expanding the linear form $f_{1,x_3}^{(2)}(x)$ as

$$f_{1,x_3}^{(2)}(x)=f_{1,13}x_1+f_{1,23}x_2+f_{1,33}x_3+f_{1,34}x_4 \qquad (A.17)$$

and using (A.13) and (A.14), (A.17) reduces to

$$\boldsymbol{f}^{(2)}_{1,x_3}(\boldsymbol{x}) = \boldsymbol{f}_{1,13}\boldsymbol{x}_1 + \boldsymbol{f}_{1,23}\boldsymbol{x}_2 \qquad (\boldsymbol{A}.18)$$

Substituting (A.18) into (A.16) yields

$$\phi^{(2)}_{2,x_3}(\boldsymbol{x}) = \boldsymbol{f}_{1,13}\boldsymbol{x}_1 + \boldsymbol{f}_{1,23}\boldsymbol{x}_2 + \phi^{(2)}_{1,x_2}(\boldsymbol{x}) \qquad (\boldsymbol{A}.19)$$

As per (A.7), $\phi_1^{(2)}(x)$ which can be arbitrary, is to be chosen as a function of x_1 and x_2 only. Choosing the derivative $\phi^{(2)}_{1,x_2}(\boldsymbol{x})$ as

$$\phi^{(2)}_{1,x_2} = -\boldsymbol{f}_{1,13}\boldsymbol{x}_1 - \boldsymbol{f}_{1,23}\boldsymbol{x}_2$$

yields from (A.19) that

$$\phi^{(2)}_{2,x_3}(\boldsymbol{x}) = 0 \qquad (\boldsymbol{A}.20)$$

Differentiating (A.8) with respect to x_4 and rearranging, one can get

$$\phi^{(2)}_{2,x_4}(\boldsymbol{x}) = \boldsymbol{f}^{(2)}_{1,x_4}(\boldsymbol{x}) + \phi^{(2)}_{1,x_1,x_4}\boldsymbol{x}_2 + \phi^{(2)}_{1,x_2,x_4}\boldsymbol{x}_3 \qquad (\boldsymbol{A}.21)$$

According to the assumption (A.7) and using (A.13), it follows from (A.21) that $\phi^{(2)}_{2,x_4}(\boldsymbol{x}) = 0$. Hence ($A$.9) is satisfied **.** Hence the sufficiency condition is proved.

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