Ontario

Number Sense and Numeration, Grades 4 to 6

Volume 6
Decimal Numbers

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

Ontario Education excellence for all

2006

Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

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INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories behind good instruction that are elaborated on in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The guide comprises the following volumes:

- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

The present volume – Volume 6: Decimal Numbers – provides:

- a discussion of mathematical models and instructional strategies that support student understanding of decimal numbers;
- sample learning activities dealing with decimal numbers for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume also contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by "eLearning modules" that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pp. 37, 54, and 76).

Relating Mathematics Topics to the Big Ideas

> *The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the "big ideas" of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.*
>
> (*The Ontario Curriculum, Grades 1–8: Mathematics, 2005*, p. 4)

In planning mathematics instruction, teachers generally develop learning activities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- operational sense
- relationships
- representation
- proportional reasoning

Each of the big ideas is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful

for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping

students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The following table outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.

Characteristics of Junior Learners and Implications for Instruction		
Area of Development	**Characteristics of Junior Learners**	**Implications for Teaching Mathematics**
Intellectual development	Generally, students in the junior grades: • prefer active learning experiences that allow them to interact with their peers; • are curious about the world around them; • are at a concrete operational stage of development, and are often not ready to think abstractly; • enjoy and understand the subtleties of humour.	The mathematics program should provide: • learning experiences that allow students to actively explore and construct mathematical ideas; • learning situations that involve the use of concrete materials; • opportunities for students to see that mathematics is practical and important in their daily lives; • enjoyable activities that stimulate curiosity and interest; • tasks that challenge students to reason and think deeply about mathematical ideas.
Physical development	Generally, students in the junior grades: • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); • are concerned about body image; • are active and energetic; • display wide variations in physical development and maturity.	The mathematics program should provide: • opportunities for physical movement and hands-on learning; • a classroom that is safe and physically appealing.
Psychological development	Generally, students in the junior grades: • are less reliant on praise but still respond well to positive feedback; • accept greater responsibility for their actions and work; • are influenced by their peer groups.	The mathematics program should provide: • ongoing feedback on students' learning and progress; • an environment in which students can take risks without fear of ridicule; • opportunities for students to accept responsibility for their work; • a classroom climate that supports diversity and encourages all members to work cooperatively.
Social development	Generally, students in the junior grades: • are less egocentric, yet require individual attention; • can be volatile and changeable in regard to friendship, yet want to be part of a social group; • can be talkative; • are more tentative and unsure of themselves; • mature socially at different rates.	The mathematics program should provide: • opportunities to work with others in a variety of groupings (pairs, small groups, large group); • opportunities to discuss mathematical ideas; • clear expectations of what is acceptable social behaviour; • learning activities that involve all students regardless of ability.

(continued)

Characteristics of Junior Learners and Implications for Instruction		
Area of Development	**Characteristics of Junior Learners**	**Implications for Teaching Mathematics**
Moral and ethical development	Generally, students in the junior grades: • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask "why" questions; • begin to consider others' points of view.	The mathematics program should provide: • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas and evaluate the ideas of others.

(Adapted, with permission, from *Making Math Happen in the Junior Grades*. Elementary Teachers' Federation of Ontario, 2004.)

LEARNING ABOUT DECIMAL NUMBERS IN THE JUNIOR GRADES

Introduction

Comprehending decimal numbers is an important development in students' understanding of number. However, a deep understanding of decimal numbers can develop only when students have opportunities to explore decimal concepts concretely and pictorially, and to relate them to whole numbers and fractions.

PRIOR LEARNING

The development of whole number and fraction concepts in the primary grades contributes to students' understanding of decimal numbers. Specifically, students in the primary grade learn that:

- our number system is based on groupings of 10 – 10 ones make a ten, 10 tens make a hundred, 10 hundreds make a thousand, and so on;
- fractions represent equal parts of a whole;
- a whole, divided into 10 equal parts, results in tenths.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

Instruction that focuses on meaning, rather than on symbols and abstract rules, helps students understand decimal numbers and how they can be used in meaningful ways. In the junior grades, students gradually come to understand the quantity relationships of decimals to thousandths, relate fractions to decimals and percents, and perform operations with decimals to thousandths and beyond.

Developing a representational meaning for decimal numbers depends on an understanding of the base ten number system, but developing a *quantity* understanding of decimals depends on developing fraction sense. Students learn that fractions are *parts of a whole* – a convention developed to describe quantities less than one. This prior knowledge also helps students understand that decimals are numbers less than one whole. It is important to give students opportunities to determine for themselves the connections between decimals and fractions

with denominators of 10 and 100. That understanding can then be developed with other fractions (i.e., with denominators of 2, 4, 5, 20, 25, and 50).

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to decimal numbers, listed in the following table.

Curriculum Expectations Related to Decimal Numbers, Grades 4, 5, and 6

By the end of Grade 4, students will:	By the end of Grade 5, students will:	By the end of Grade 6, students will:
Overall Expectations	***Overall Expectations***	***Overall Expectations***
• read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100; • demonstrate an understanding of magnitude by counting forward and backwards by 0.1 and by fractional amounts; • solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.	• read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers; • demonstrate an understanding of magnitude by counting forward and backwards by 0.01; • solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.	• read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers; • solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies; • demonstrate an understanding of relationships involving percent, ratio, and unit rate.
Specific Expectations	***Specific Expectations***	***Specific Expectations***
• demonstrate an understanding of place value in whole numbers and decimal numbers from 0.1 to 10 000, using a variety of tools and strategies; • represent, compare, and order decimal numbers to tenths, using a variety of tools and using standard decimal notation; • read and represent money amounts to $100; • count forward by tenths from any decimal number expressed to one decimal place, using concrete materials and number lines; • add and subtract decimal numbers to tenths, using concrete materials and student-generated algorithms;	• represent, compare, and order whole numbers and decimal numbers from 0.01 to 100 000, using a variety of tools; • demonstrate an understanding of place value in whole numbers and decimal numbers from 0.01 to 100 000, using a variety of tools and strategies; • round decimal numbers to the nearest tenth, in problems arising from real-life situations; • demonstrate and explain equivalent representations of a decimal number, using concrete materials and drawings; • read and write money amounts to $1000; • count forward by hundredths from any decimal number expressed to two decimal places, using concrete materials and number lines;	• represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools; • demonstrate an understanding of place value in whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools and strategies; • add and subtract decimal numbers to thousandths, using concrete materials, estimation, algorithms, and calculators; • multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators; • multiply whole numbers by 0.1, 0.01, and 0.001 using mental strategies;

Curriculum Expectations Related to Decimal Numbers, Grades 4, 5, and 6		
By the end of Grade 4, students will:	**By the end of Grade 5, students will:**	**By the end of Grade 6, students will:**
Specific Expectations *(continued)* • add and subtract money amounts by making simulated purchases and providing change for amounts up to $100, using a variety of tools; • determine and explain, through investigation, the relationship between fractions (i.e., halves, fifths, tenths) and decimals to tenths, using a variety of tools and strategies.	***Specific Expectations*** *(continued)* • add and subtract decimal numbers to hundredths, including money amounts, using concrete materials, estimation, and algorithms; • multiply decimal numbers by 10, 100, 1000, and 10 000, and divide decimal numbers by 10 and 100, using mental strategies; • describe multiplicative relationships between quantities using simple fractions and decimals; • determine and explain, through investigation using concrete materials, drawings, and calculators, the relationship between fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100) and their equivalent decimal forms.	***Specific Expectations*** *(continued)* • multiply and divide decimal numbers by 10, 100, 1000, and 10 000 using mental strategies; • use estimation when solving problems involving the addition and subtraction of whole numbers and decimals, to help judge the reasonableness of a solution; • determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents; • represent relationships using unit rates.

(*The Ontario Curriculum, Grades 1–8: Mathematics, 2005*)

The sections that follow offer teachers strategies and content knowledge to address these expectations in the junior grades while helping students develop an understanding of decimals. Teachers can facilitate this understanding by helping students to:

- relate fractions and decimal numbers;
- compare and order decimal numbers;
- explore various strategies for decimal-number computations.

Relating Fractions and Decimal Numbers

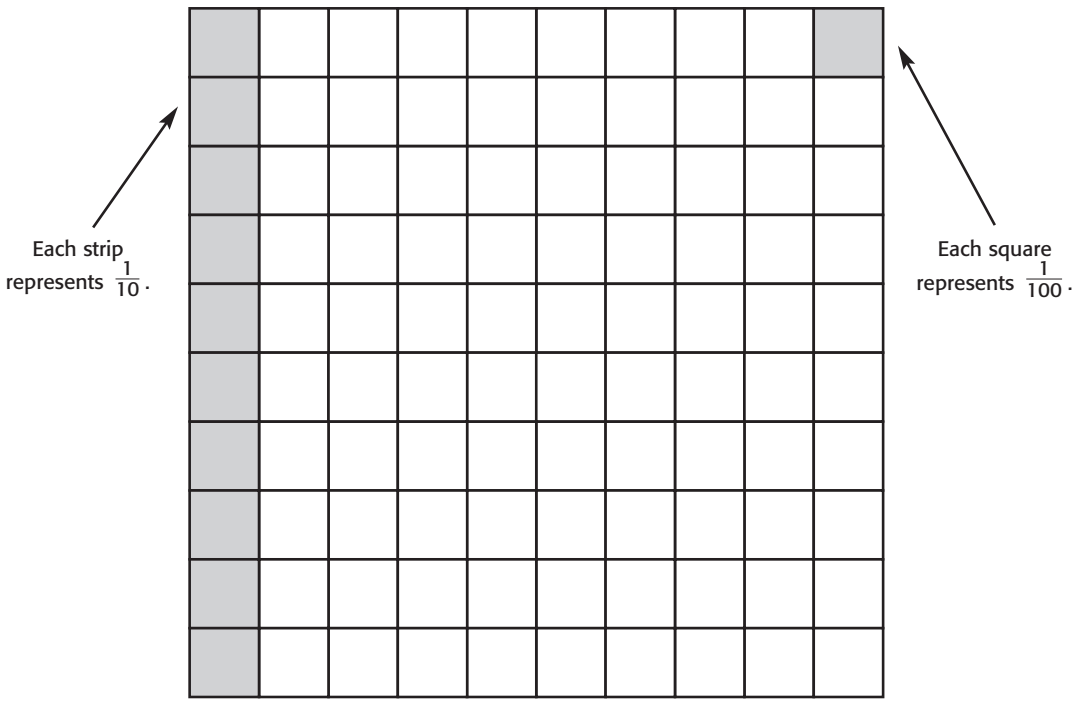
Although adults quickly recognize that 0.5 and 1/2 are simply different representations of the same quantity, children have difficulty connecting the two different systems – fractional representation and decimal representation. It is especially difficult for children to make the connection when they are merely told that the two representations are "the same thing". Teachers in the junior grades should strive to see and explain that both decimal numbers and fractions represent the same *concepts*. This involves more than simply pointing out to students that a particular fraction and its corresponding decimal represent the same quantity – it involves modelling base ten fractions, exploring and expanding the base ten number system, and making connections between the two systems.

MODELLING BASE TEN FRACTIONS

Students need time to investigate base ten fractions, which are fractions that have a denominator of 10, 100, 1000, and so on. Both area models and length models can be used to explore these fractions. Although set models can also be used, they become impractical when working with anything other than tenths.

Base ten blocks or 10 × 10 grids are useful models for working with tenths and hundredths.

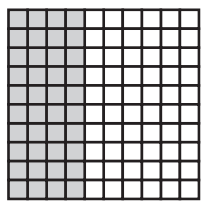
Strips-and-Squares Model



It is important for students to understand that the large square represents one whole, 10 strips make one whole, and 100 smaller squares make one whole. Students can work with blank 10 × 10 grids and shade sections in, or they can cut up coloured grids and place them on blank grids. Base ten blocks provide similar three-dimensional experiences.

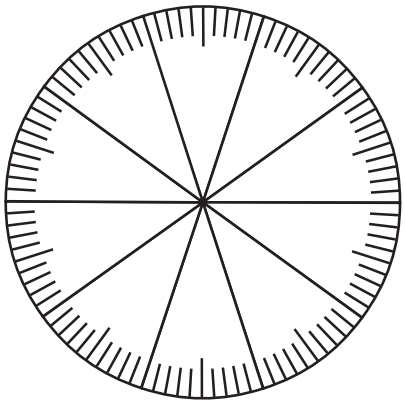
When giving students problems that use the strips-and-squares model, the aim should be to develop concepts rather than rules. Some examples include:

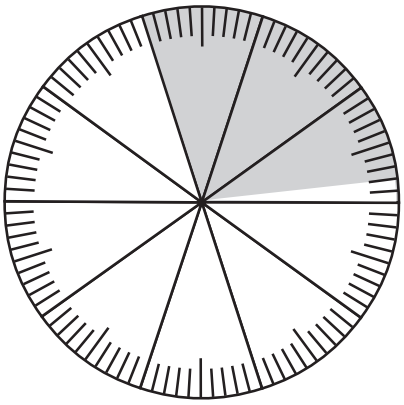
- Fiona rolled a number cube 10 times, and 6 of those times an even number came up. Represent the number of even rolls as a fraction, and show it on a 10 × 10 grid.
- Luis has 5 coins in his pocket. They total less than 1 dollar and more than 50 cents. How much money could he have? Write the amount as a fraction of a dollar, and shade the amount on a 10 × 10 grid.
- What fraction of the grid at right is shaded? Write two different fractions to show the amount. How are the fractions related?



A hundredths wheel (or decimal wheel) also serves as an excellent area model for tenths and hundredths. A hundredths wheel is divided into 10 sections, each divided further into 10 equal intervals. When a slit is cut along one radius and two wheels of different colours are placed together, the model can be used to show decimals and fractions of less than one.

Hundredths Wheel





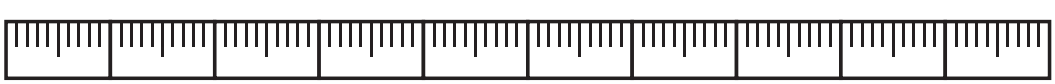
The wheel shows 0.28 or $\frac{28}{100}$.

This model will be familiar to students, as many have seen "pies" divided into thirds, fourths, tenths, and so on. Ignoring the smaller graduations, the hundredths wheel is simply a tenths wheel.

Money also provides a model for hundredths that students are very familiar with. It is important for teachers to make connections to the knowledge students bring to the classroom, but it is also important to know the limits of a particular model. Investigating tenths with money is not as meaningful for students, since in everyday language we rarely refer to 6 dimes as "six tenths of a dollar". Money amounts are usually represented to hundredths, but very rarely to one decimal place or three decimal places.

Length models are also useful for investigating base ten fractions. Paper strips can be divided into tenths, and metre sticks show both tenths (decimetres) and hundredths (centimetres). These concrete models transfer well to semi-concrete models, like number lines drawn with 10 or 100 divisions, and help students make quantity comparisons between decimals and base ten fractions.

Metre Stick Showing Tenths and Hundredths of the Whole



"Where would you place 0.6 on this number line? How about 0.06?"

I know 0.6 is 6/10, and the number line is divided into tenths (the larger lines), so 0.6 is the sixth large line. Each tenth is divided into 10 smaller pieces, so each of those is one hundredth, and 6 of the smaller lines are 0.06.

Number Line Showing Tenths and Hundredths of the Whole

The advantage of each of the models presented so far is that the whole *remains unchanged* – it is simply divided into smaller pieces to represent hundredths.

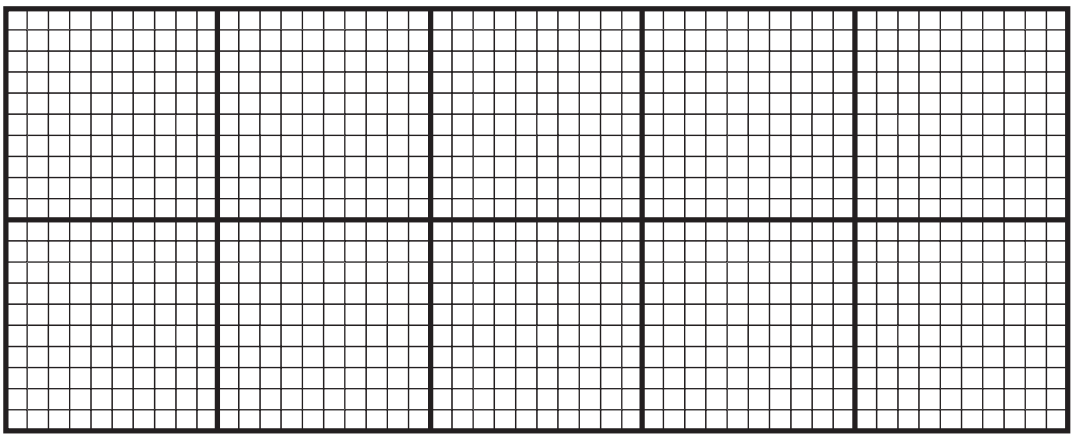
The metre stick is an excellent model for thousandths when it is marked with millimetre increments. The length of the whole does not change, and students can see that each interval can be further subdivided (decimetres into centimetres, centimetres into millimetres) while the whole always stays the same.

Although area and three-dimensional models can also be used to represent thousandths, teachers should note that "redefining a whole" can be very confusing for students. Activities with base ten blocks that frequently redefine the whole should be pursued only with students who have a firm grasp of the concept of tenths and hundredths.

When representing thousandths with base ten blocks, students must define the large cube as one. (With whole numbers, the cube represented 1000.) With the large cube as one, flats become tenths, rods become hundredths, and units become thousandths. Although this three-dimensional model offers powerful learning opportunities, students should not be asked to redefine wholes in this way before having many rich experiences with base ten fractions.

Illustrated below is an easier-to-understand area model for thousandths, in which 10 × 10 grids are joined into a group of 10 to form a new whole.

Area Model Showing Thousandths of the Whole



Although this two-dimensional model calls for the creation of a new whole – previously, with tenths and hundredths, the 10 × 10 grid was the whole – students readily can see that the model has grown larger to show the new whole. (Students have more difficulty understanding the model when using base ten blocks because the whole does not grow larger – the blocks are merely re-labelled when the whole changes.)

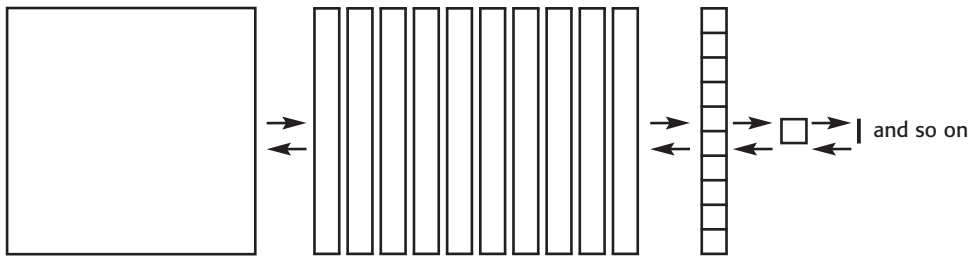
EXPANDING THE BASE TEN NUMBER SYSTEM

Many of the difficulties students have with decimal numbers stem from the fact that decimals are primarily taught as an extension of the place-value system. Understanding how fractional amounts can be represented as decimals in the base ten number system is a key junior-grade concept.

In the primary grades, students learn that the idea of "ten makes one" is crucial to our number system. Ten ones make a ten; 10 tens make a hundred, and so on. Students in the junior grades extend this idea to larger numbers, like hundred thousands and millions. They may find it more difficult to extend this concept to numbers of less than one without multiple experiences. Although students may have an understanding of whole numbers (ones, tens, hundreds, ...), they may misunderstand the pattern of tens to the right of the decimal numbers and think of the first decimal place as oneths, the next place as tenths, and so on. Also, they may have difficulty recognizing that a decimal number such as 0.234 is both 2 tenths, 3 hundredths, 4 thousandths, as well as 234 thousandths.

Area models are effective for demonstrating that ten-makes-one also works "going the other way". Base ten blocks are a three-dimensional representation of the strips-and-squares area model, which is shown below.

Area Model Showing Strips and Squares



Students' initial experiences with this model involve moving to the *left*: ten squares make one strip; ten of those strips make a bigger square; and so on. Each new region formed has a new name and its own unique place in the place-value chart. Ten ones make 1 ten; 10 tens make 1 hundred; 10 hundreds make 1 thousand; and so on.

Teachers can build on this experience by having students investigate "going the other way", which involves moving to the *right*. What happens if you take a square and divide it into ten equal strips? And what if you take one of those strips and divide it into ten smaller squares? Could you ever reach the smallest strip or square, or the largest strip or square?

Ultimately students should learn that this series involving ten-makes-one and one-makes-ten extends infinitely in both directions, and that the "pieces" formed when the whole is broken into squares or strips are special fractions (base ten fractions) – each with its own place in the place-value system.

The **decimal point** is a special symbol that separates the position of the whole-number units on the left from the position of the fractional units on the right. The value to the right of the decimal point is 1/10, which is the value of that place; the value two places to the right of the decimal point is 1/100, which is the value of that place; and so on.

Teachers can help students develop an understanding of the decimal-number system by connecting to the understandings that students have about whole numbers.

Example 1: Students Read and Write Number Patterns

Have students read and write numbers as follows:

- 222 000 two hundred twenty-two thousands
- 22 200 two hundred twenty-two hundreds
- 2220 two hundred twenty-two tens
- 222 two hundred twenty-two ones

Continue the pattern with decimals:

- 22.2 two hundred twenty-two tenths
- 2.22 two hundred twenty-two hundredths
- 0.222 two hundred twenty-two thousandths

Example 2: Using Different Number Forms

76 = 70 + 6

425 = 400 + 20 + 5 OR
4 hundreds + 2 tens + 5 ones OR
4 hundreds + 1 ten + 15 ones OR
3 hundreds + 12 tens + 5 ones

Extend to decimals:

0.56 = 0.5 + 0.06 OR
5 tenths + 6 hundredths

7.38 = 7 + 0.3 + 0.08 OR
7 + 3 tenths + 8 hundredths OR
7 + 2 tenths + 18 hundredths OR
6 + 13 tenths + 8 hundredths

Activities like those in the examples not only use patterning to develop concepts, but also encourage students to think about how numbers *greater than one* can be represented using different base ten fractions. For example, reading 22.2 as "two hundred twenty-two tenths" requires students to think about how many tenths there are in 2 (20), and how many tenths there are in 20 (200).

CONNECTING DECIMALS AND FRACTIONS

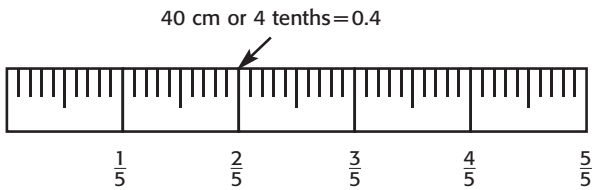
Students in the junior grades begin to work flexibly between some of the different representations for rational numbers. For example, if asked to compare 3/4 and 4/5, one strategy is to convert the fractions to decimal numbers. 3/4 is 0.75 (a commonly known decimal linked to money), and 4/5 can be thought of as 8/10, which converts to 0.8. These fractions, represented as decimal numbers, can now be easily compared.

When connecting the two different representations, it is important for teachers to help students make a *conceptual* connection rather than a *procedural* one. Conversion between both representations can (unfortunately) be taught in a very rote manner – "Find an equivalent fraction with tenths or hundredths as the denominator, and then write the numerator after the decimal point." This instruction will do little to help students understand that decimals are fractions. Instead, students need to learn that fractions can be *turned into* decimals, and vice versa.

Activities should offer students opportunities to use concrete base ten models to represent fractions as decimals and decimals as fractions. For example, consider the following.

"Use a metre stick to represent 2/5 as a decimal."

Students will have used this model to explore base ten fractions before being given this problem. Here is one student's solution:



I used the metre stick as one whole, or 1. To figure out where 2/5 was, I divided the stick into 5 equal parts. 100 cm ÷ 5 = 20 cm, so 2/5 is at the 40 cm mark. 40 cm is 4 tenths of the metre stick, or 0.4. So 2/5 can be written as 0.4.

Similar activities using strips and squares, or base ten blocks, also help students to make connections between the representations.

It is important for students to experience a range of problem types when making connections between decimals and fractions. Sample problem types with numbers *less than one* include:

- given the fraction, write the decimal equivalent;
- given the decimal, write the fraction equivalent;
- given a fraction and decimal, determine if they are equivalent representations.

Also, problems should involve determining fractional amounts *greater than one* (e.g., write the decimal equivalent for 2 72/100).

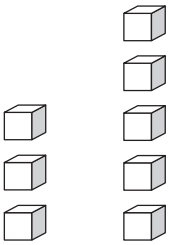
Comparing and Ordering Decimal Numbers

Shopping and measuring are real-life activities in which decimal numbers often need to be compared or ordered. Learning activities in which students compare and order decimal numbers not only develop practical skills, but also help to deepen students' understanding of place value in decimal numbers. Students can compare and order decimal numbers using models and reasoning strategies.

USING AREA MODELS OR BASE TEN BLOCKS

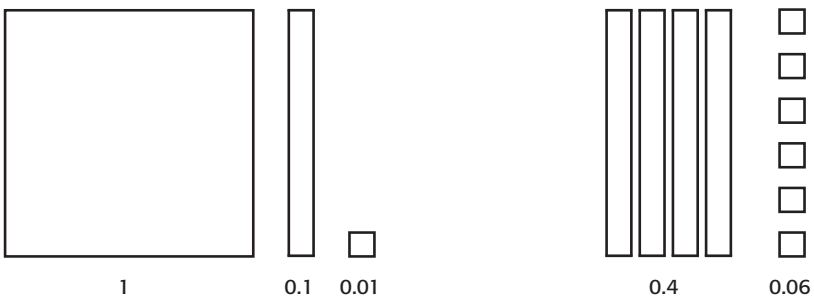
Concrete materials, such as fraction circles, 10 × 10 grids, and base ten blocks, allow students to compare and order decimal numbers. Models provide visual representations that show the relative size of the decimal numbers.

To compare 0.3 and 0.5 using base ten blocks, for example, the rod could be used to represent the whole, and the small cubes to represent tenths.



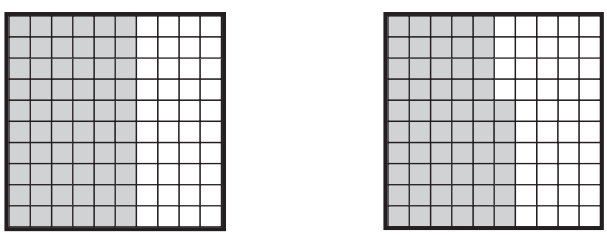
0.3 < 0.5

To compare 0.4 and 0.06, a strips-and-squares model could be used. A large square would become the whole; the strip, one tenth; and the smaller square, one hundredth.



0.4 > 0.06

10 × 10 grids allow students to colour or shade in strips and squares to compare decimal numbers. For example, students can use a 10 × 10 grid to compare 0.6 and 0.56:



0.6 > **0.56**

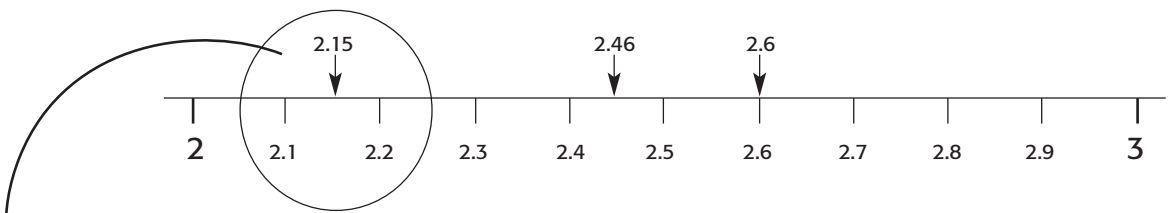
USING LENGTH MODELS

A metre stick is an excellent model for comparing decimal numbers. To compare 0.56 and 0.8, for example, students can use the centimetre and decimetre increments to locate each number on the metre stick. Each centimetre is 1/100 or 0.01 of the whole length. Fifty-six hundredths, or 0.56, is at the 56 cm mark; and eight tenths, or 0.8, is 8 dm or 80 cm.

Locating numbers on a number line extends the physical model and connects to students' prior learning with whole numbers. Before comparing and ordering decimals, students should have meaningful experiences with locating decimals on a number line. Some sample problems are:

- Draw a number line that starts at 0 and ends at 1. Where would you put 0.782? Why?
- On a number line that extends from 3 to 5, locate 4.25, and give reasons for your choice.
- 2.5 is halfway between 1 and 4. What number is halfway between 1 and 2.5? Use a number line and explain your reasoning.

Partial number lines can be used to order decimals as well. For example, to order 2.46, 2.15, and 2.6, students could draw a number line that extends from 2 to 3, then mark the tenths between 2 and 3, and then locate the decimal numbers.



To help students visualize hundredths and beyond, sections of the number line can be "blown up" or enlarged to show smaller increments.

Blowing up a section of this number line will allow students to model thousandths in a similar manner.

USING REASONING STRATEGIES

After students have had opportunities to compare decimal numbers using models and number lines, they can compare decimals using reasoning strategies that are based on their understanding of place value.

For example, to compare 3.45 and 3.7, students observe that both numbers have the same number of ones (3), and that there are 7 tenths in 3.7, but only 4 tenths in 3.45. Therefore, 3.45 is less than 3.7, even though there are more digits in 3.45. Teachers need to be cautious that this type of reasoning does not become overly procedural, however. Consider, on the following page, how the student is comparing 15.15 and 15.9, and demonstrates only a procedural knowledge of comparing decimals:

I looked at the tens, and they were the same. Then I looked at the ones, and they were the same. Then I looked at the tenths, and since 1 is less than 9, 15.15 is less than 15.9.

Students gain little understanding of quantity if they compare decimals by looking from digit to digit.

Students should apply whole-number reasoning strategies and use benchmarks. For example, when comparing 15.15 and 15.9, students should recognize that 15.15 is a little bigger than 15, and that 15.9 is almost 16, so 15.15 is the smaller number.

1/2 or 0.5 is an important benchmark as well. When asked to order 6.52, 5.9, 6.48, 6.23, and 6.7, students can use 6.5 as the halfway point between 6 and 7. Students should recognize that 5.9, 6.23, and 6.48 are all less than 6.5, and 6.52 and 6.7 are greater than 6.5.

Strategies for Decimal-Number Computations

Strategies for decimal-number computations can be found in Volume 2: Addition and Subtraction, Volume 3: Multiplication, and Volume 4: Division.

A Summary of General Instructional Strategies

Students in the junior grades benefit from the following instructional strategies:

- representing decimal numbers using a variety of models, and explaining the relationship between the decimal parts and the whole;
- discussing and demonstrating base ten relationships in whole numbers and decimal numbers (e.g., 10 ones make ten, 10 tenths make one, 10 hundredths make a tenth);
- using models to relate fractions and decimal numbers (e.g., using fraction strips to show that 2/10 = 0.2);
- comparing and ordering decimal numbers using models, number lines, and reasoning strategies;
- investigating various strategies for computing with decimal numbers, including mental and paper-and-pencil methods.

The Grades 4–6 Decimal Numbers module at www.eworkshop.on.ca provides additional information on developing decimal concepts with students. The module also contains a variety of learning activities and teaching resources.

eworkshop.on.ca

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Learning Activities for Decimal Numbers

Introduction

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to decimal numbers. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts in order to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or activity.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students' understanding of mathematical concepts.

HOME CONNECTION: This section is addressed to parents or guardians, and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.

Grade 4 Learning Activity
Decimal Game

OVERVIEW

In this learning activity, students play a game in which they shade tenths of fraction strips and express the shaded parts as fractions and decimal numbers. The activity helps students to understand decimal quantities and the meaning of decimal-number notation.

BIG IDEAS

This learning activity focuses on the following big ideas:

Quantity: Students explore decimal quantities by shading fraction strips divided into tenths.

Relationships: This learning activity allows students to see relationships between fractions and decimal numbers (e.g., both number forms can be used to represent tenths).

Representation: Students learn about decimal-number representations (e.g., the role of the decimal point). They also explore the idea that fractions and decimal numbers can represent the same quantity (e.g., 3/10 = 0.3).

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- represent, compare, and order decimal numbers to tenths, using a variety of tools (e.g., concrete materials such as paper strips divided into tenths and base ten materials, number lines, drawings) and using standard decimal notation;
- determine and explain, through investigation, the relationship between fractions (i.e., halves, fifths, tenths) and decimals to tenths, using a variety of tools (e.g., concrete materials, drawings, calculators) and strategies (e.g., decompose 2/5 into 4/10 by dividing each fifth into two equal parts to show that 2/5 can be represented as 0.4).

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- read, represent, compare, and order whole numbers to 10 000, decimal numbers to tenths, and simple fractions, and represent money amounts to $100.

ABOUT THE LEARNING ACTIVITY

TIME:
60 minutes

MATERIALS

- **Dec4.BLM1: Fraction Strip Divided Into Tenths**
- chart paper
- markers
- overhead transparency of **Dec4.BLM2: Fraction Strips**
- overhead projector
- **Dec4.BLM3: Tenths Game Board** (1 per student)
- ten-sided number cube (1 number cube per pair of students), or alternatively, spinners made from **Dec4.BLM4: Ten-Section Spinner**, a paper clip, and a pencil (1 per pair of students)
- a variety of manipulatives for representing tenths (e.g., fraction circles, counters, square tiles)
- **Dec4.BLM5: Cover the Tenths Game** (1 per student)

MATH LANGUAGE

- tenths
- decimal point
- decimal number
- fraction
- fraction strip
- representation
- greater than
- less than

INSTRUCTIONAL GROUPING:
pairs

INSTRUCTIONAL SEQUENCING

Before starting this learning activity, students should have had experience representing tenths as parts of whole objects and representing tenths using fraction notation (e.g., 4 tenths can be recorded as "4/10"). In this learning activity, students continue to explore the concept of tenths and learn that tenths can be represented as decimal numbers.

ABOUT THE MATH

In this learning activity, students review the concept of tenths as parts of a whole, and explore how tenths can be represented as fractions and as decimal numbers. The learning activity helps students to recognize that both notations (fraction and decimal number) represent the same quantity.

When students understand that tenths can be expressed as fractions and as decimals, they are able to recognize equivalent representations of the same number (e.g., 1/2 and 0.5), allowing them to choose the more useful representation in different situations. For example, it may be easier for some students to add 1/2 and 4/10 by thinking of them as 0.5 and 0.4.

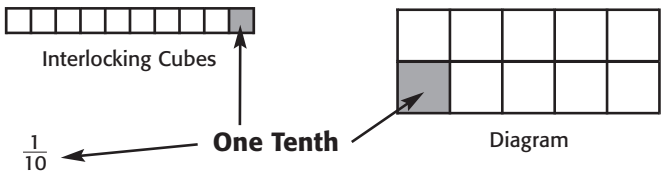
GETTING STARTED

Show students **Dec4.BLM1: Fraction Strip Divided Into Tenths**, and ask: "If this strip represents one whole, what is each part called? How do you know?"

Ask students to explain different ways to represent "tenths". For example, they might suggest using:

- concrete materials (e.g., snapping together 10 interlocking cubes in a row and recognizing that each cube is a tenth of the row);
- diagrams (e.g., drawing a shape, such as a circle, square, or rectangle, and dividing the shape into 10 equal parts);
- symbols (e.g., recording the fraction 1/10).

Record students' ideas on chart paper or the board using diagrams, symbols, and words.



Refer to each recorded representation and ask the following questions:

- "In this representation (concrete material, diagram, symbol), what does the whole look like?"
- "How do you know that this part (interlocking cube, section of the rectangle, number) is one tenth of the whole?"
- "How could you show 2 tenths? 3 tenths? 10 tenths?"
- "How could you show 11 tenths using interlocking cubes? A diagram? A fraction?"

Explain that decimal numbers can also represent tenths. Record "0.1" on the board, and explain that this decimal number is read as "one tenth". Discuss the following ideas:

- The decimal point separates the whole-number part of the number from the decimal-number part.
- The zero to the left of the decimal point shows that there are no ones.
- The place-value column to the right of the decimal point tells the number of tenths. For example, in 0.1, there is 1 tenth.

Ask students to suggest places where they have seen decimal points (e.g., prices, measurements, sports statistics).

Display an overhead transparency of **Dec4.BLM2: Fraction Strips**. To begin, show only the fraction strip with 4 tenths shaded. Ask:

- "If the strip represents 1 whole, what part of the strip is shaded?"
- "How do you know that 4 tenths is less than 1?"
- "How would you record 4 tenths as a fraction?"
- "How would you record 4 tenths as a decimal number?"

Record both the fraction (4/10) and the decimal number (0.4) below the fraction strip on the overhead transparency.

Continue the discussion by having students describe the other fraction strips (6 tenths, 8 tenths) on the overhead transparency. Ask them to give both the fraction and the decimal representations for each fraction strip. Label the fraction strips accordingly.

WORKING ON IT

Arrange students in pairs. Provide each student with a copy of **Dec4.BLM3: Tenths Game Board**. Give each pair of students a ten-sided number cube. (Alternatively, have them use **Dec4.BLM4: Ten-Section Spinner**). Explain that students will play a game that will allow them to represent tenths in different ways.

Explain the game procedures:

- The first player rolls the number cube. Whatever number is rolled, the student shades in that many sections of a fraction strip on his or her copy of **Dec4.BLM3: Tenths Game Board**. For example, if a 7 is rolled, the student shades in 7 sections of the strip. The student announces the number that is shaded ("seven tenths") and then records the number as a decimal number and as a fraction, below the shaded strip.
- The second player rolls the number cube and completes a section of his or her game board.
- Players continue to take turns.
- If a player rolls a number that he or she has already rolled, that player does not shade in a fraction strip.
- Players should check each other's game board as they are playing, to make sure that the numbers are being written correctly.
- The first player to complete his or her game board, by shading fraction strips and recording corresponding fraction and decimal numbers for one tenth through to ten tenths, wins the game.

Observe students while they play the game. Note whether they record appropriate fraction and decimal representations for each shaded fraction strip. Observe, as well, what students do when they roll a 10. (Do students have difficulty grasping that 1.0 is the same as ten tenths?)

Ask students questions such as the following:

- "What number does this fraction strip show?"
- "How can you record this number as a fraction? How can you record the number as a decimal number?"
- "How do you know that this fraction and this decimal number represent the same quantity?"
- "How can you represent ten tenths on the fraction strip? As a fraction? As a decimal number?"
- "How do you know what numbers you still need to roll?"

REFLECTING AND CONNECTING

Reconvene students after the game. Talk to them about how they represented tenths using diagrams (fraction strips), as decimal numbers, and as fractions. As an example, show a fraction strip with 6 tenths shaded in, and ask students to explain two ways to record the number. Record "6/10" and "0.6" on the board.

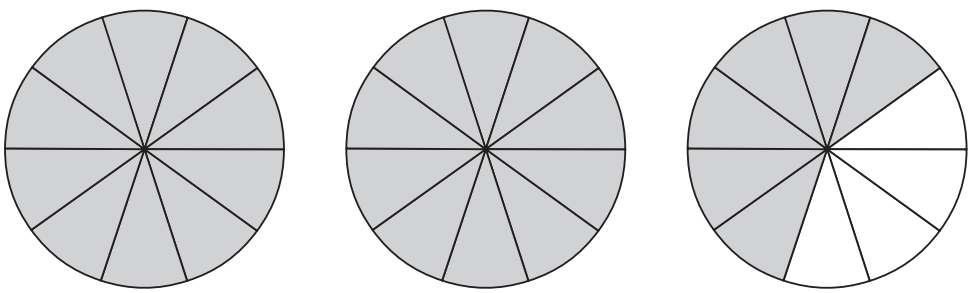
Ask students to explain what they learned about decimal numbers when they played the game. Record students' ideas on the board or chart paper. For example, students might explain the following ideas:

- Both 4/10 and 0.4 represent 4 tenths.
- The number of tenths is recorded to the right of the decimal point.
- If there is no whole-number part (i.e., the number is less than 1), 0 is recorded to the left of the decimal point.
- 10 tenths is the same as 1 whole.
- 10 tenths is recorded as 1.0.

Initiate a discussion about decimal-number quantities. Write "0.5" and "0.8" on the board, and ask students to identify the greater number. Have students explain why 0.8 is greater than 0.5. Listen for student responses like, "Because 8 is bigger than 5." Write 5.0 and 0.8 on the board as an illustration of why this explanation is not sufficient. Encourage students to draw a diagram (e.g., showing fraction strips) on the board to demonstrate that 0.8 is a greater quantity than 0.5.

As well, ask students to name numbers that are greater than 0.7 but less than 1.0. Have them model 0.8 and 0.9 using fraction-strip diagrams.

During the discussion, clearly model the use of the decimal point and its role in separating the part of the number that is a whole number from the part of the number that is less than one whole (decimal numbers). You may want to extend the discussion to include numbers like 2 and 6 tenths (2.6), or 3 and 6 tenths (3.6). Ask students to represent these numbers using manipulatives (e.g., fraction circles, counters, square tiles) or drawings.



2 and 6 tenths

ADAPTATIONS/EXTENSIONS

Students need a strong understanding of fractions before they will be able to grasp the concept that fractions and decimal numbers can represent the same quantity. Simplify the game by having students shade in the fraction strips according to the number shown on the number cube, and have them record only the fraction that represents the shaded portion of the fraction strip.

For students requiring a greater challenge, provide them with two six-sided number cubes instead of one ten-sided number cube. To complete their game card, students roll both number cubes, and then choose to add or subtract the numbers rolled to determine how many tenths to shade.

ASSESSMENT

Observe students as they play the game, and assess how well they:

- shade in fraction strips according to the number shown on the number cube;
- identify the shaded part of fraction strips (e.g., 3 shaded spaces shows "three tenths");
- record fractions and decimal numbers that represent the shaded part of fraction strips;
- explain that a fraction and a decimal number represent the same quantity.

HOME CONNECTION

Send home copies of **Dec4.BLM5: Cover the Tenths Game**. The game provides an opportunity for students and their parents/guardians to represent tenths as fractions and as decimal numbers.

LEARNING CONNECTION 1
Decimal Numbers Using Base Ten Blocks

MATERIALS

- overhead base ten blocks (flats, rods, small cubes) or regular base ten blocks (flats, rods, small cubes)
- overhead projector

This activity reinforces students' understanding of decimal-number quantities.

Show students a rod from a set of base ten blocks (use overhead blocks, if available) using an overhead projector. Ask: "What number does this rod represent?" Students will likely answer "10".

Ask: "Does the rod have to be 10? Could it represent 1?" Explain that the rod, for the purposes of the activity, will represent 1.

Display a flat using the overhead projector, and ask students, "If the rod represents 1, what number does the flat represent?" Ask students to explain how they know that the flat represents 10.

Next, show a small cube and ask students, "If the rod represents 1, what does the small cube represent?" Ask students to explain how they know that the small cube represents one tenth. Reinforce the idea that the rod is made up of 10 small cubes, so each small cube is one tenth of the rod.

Place two rods and a small cube on the overhead projector, and ask students to identify the number that is represented. On the board, record "two and one tenth", "2 1/10", and "2.1". Discuss how all three notations represent the quantity shown by the base ten blocks.

Repeat with several other numbers so that students are comfortable representing numbers using a place-value chart.

LEARNING CONNECTION 2
Decimal Number Grab Bags

MATERIALS

- **Dec4.BLM6: Place-Value Mat** (1 per pair of students)
- **Dec4.BLM7: Decimal Number Grab Bag Recording Sheet** (1 per pair of students)
- large paper bags containing base ten blocks – including 3 to 5 flats, 5 to 9 rods, and 10 to 20 small cubes (1 per pair of students)

Provide each pair of students with a copy of **Dec4.BLM6: Place-Value Mat**, a copy of **Dec4.BLM7: Decimal Grab Bag Recording Sheet**, and a large paper bag containing base ten blocks. Show that the bag contains base ten blocks, and explain that a rod represents 1, a flat represents 10, and a small cube represents 1 tenth.

Explain the activity:

- Pairs of students take turns "grabbing" (using both hands) a quantity of base ten blocks from the bag, and organizing the blocks on the place-value mat (trading 10 cubes for a rod, if necessary).
- Both students record a drawing, a fraction, and a decimal number to represent the number drawn from the bag.
- Students compare what they have recorded.
- Students return the materials to the bag after each turn.

As students work on the activity, ask questions such as the following:

- "What number did you grab from the bag?"
- "How can you represent this number using a drawing? A fraction? A decimal number?"
- "How does the place-value mat help you organize the number?"

There are several variations for the activity. For example, students could:

- compare each new number to the previous one by deciding if it is greater or less;
- challenge their partners to grab a number that is greater than or less than the previous number;
- add each new decimal number to the previous numbers.

LEARNING CONNECTION 3
Closest to Ten

MATERIALS

- six-sided number cubes (2 per pair of students)
- **Dec4.BLM8: Closest to Ten Recording Sheet** (1 per student)
- a variety of manipulatives for representing decimal numbers (e.g., fraction circles divided into tenths, base ten blocks, counters)

This game provides an opportunity for students to add and subtract decimal numbers to tenths.

Have students play the game in pairs. To begin, one player rolls two number cubes and uses the numbers rolled to create a decimal number containing a whole-number digit and a tenths

digit (e.g., after rolling a 5 and a 3, a player can create either 5.3 or 3.5). The second player then does the same. Each player records the numbers he or she created beside "Roll 1" on **Dec4.BLM8: Closest to Ten Recording Sheet**.

Players continue to create decimal numbers. Each time they record a number, players must either add the number to or subtract it from the previous number on **Dec4.BLM8: Closest to Ten Recording Sheet**. The resulting sum or difference is recorded in the appropriate space.

After five rolls, the player whose final number is closest to 10 wins. (The number may be less than or greater than 10.)

Encourage students to use manipulatives (e.g., fraction circles divided into tenths, base ten blocks, counters) to help them add or subtract their numbers.

Reconvene the class after students have played the game a few times. Discuss the game by asking questions such as the following:

- "What strategies did you use to add decimal numbers?"
- "What strategies did you use to subtract decimal numbers?"
- "How did you decide whether to add or subtract two numbers?"
- "How did you figure out who won the game?"

Provide an opportunity for students to play the game again, so that they can try the strategies they learned about during the class discussion.

LEARNING CONNECTION 4
Counting Tenths

MATERIALS

- calculators (1 per student)
- overhead calculator, if available

Note: Check that calculators have the memory feature required for this activity. Enter + .1 and then press the = key repeatedly. The display should show 0.2, 0.3, 0.4, and so on. If you are using the TI-15 calculator, you will have to use the OP1 or OP2 buttons.

Counting by tenths helps to build an understanding of decimal quantity and can reinforce an understanding of the relationship between tenths and the whole.

Provide each student with a calculator. Instruct students to enter + .1 in the calculator and ask them to read the number ("one tenth"). Next, have students press the = key and read the number. Have them continue to press the = key repeatedly and read the number on their calculators each time.

When students reach 0.9, have them predict what their calculators will show when they press the = key. (Some students may predict that the calculators will show 0.10.) Have students check their predictions by pressing =, and discuss how 1 represents one whole.

Have students continue counting with their calculators (1.1, 1.2, 1.3, ...). When students reach 1.9, have them predict the next number before continuing to count. They can continue counting by tenths until they reach 3 or 4.

Conclude the counting activity by asking the following questions:

- "How many tenths did you add to get from 1 to 2?" (10)
- "How many tenths are there altogether in 2?" (20)
- "How many tenths would you need to add to make 1.6 (2.8, 3.1) appear on your calculator?"

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on decimal concepts. On the homepage click "Toolkit". In the "Numeracy" section, find "Decimal Numbers (4 to 6)", and then click the number to the right of it.

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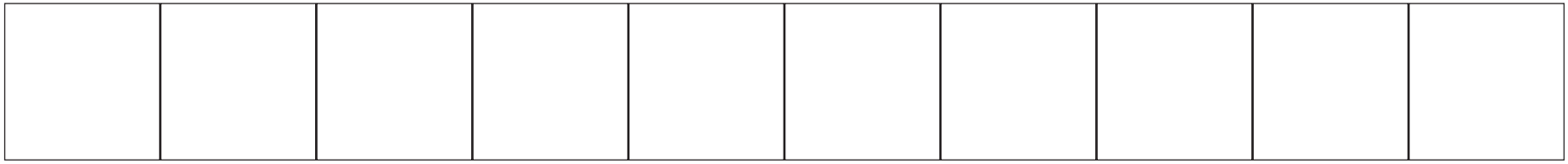
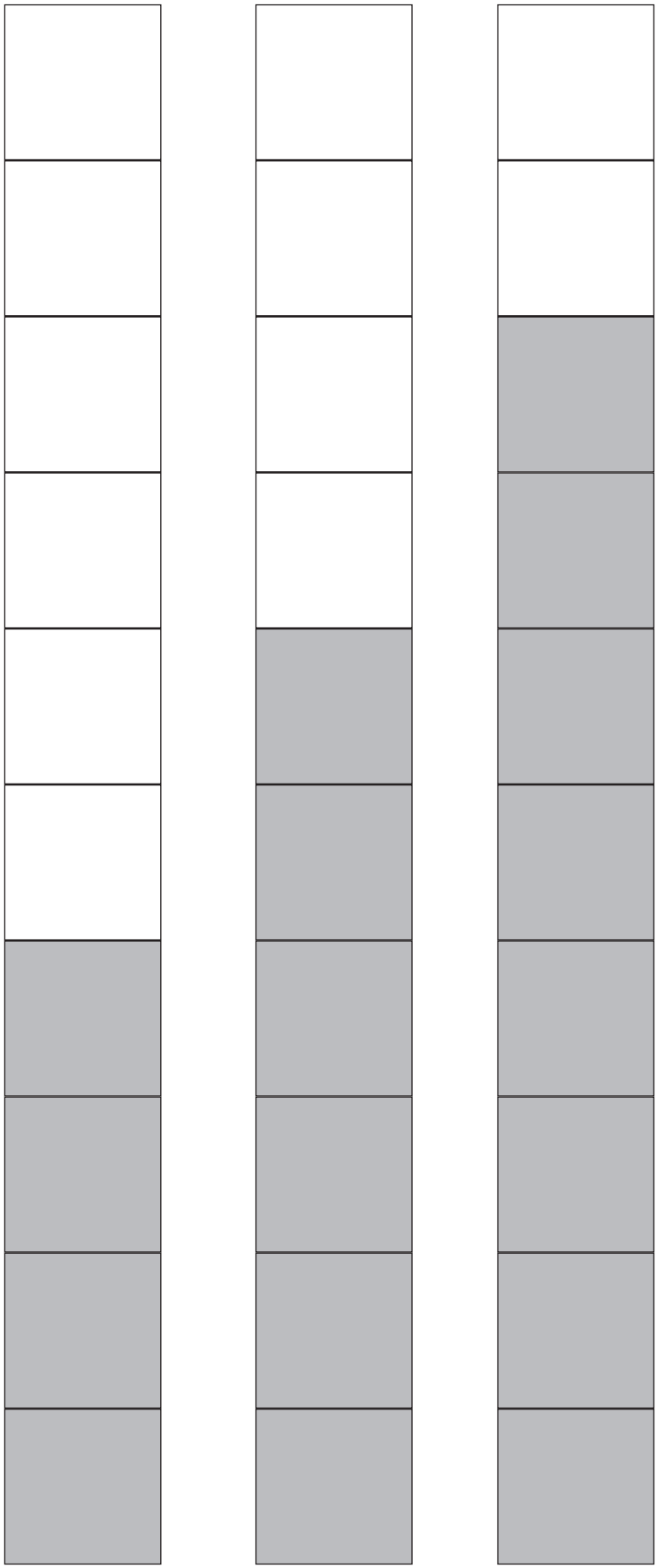
Fraction Strip Divided Into Tenths



Fraction Strips



Tenths Game Board

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

Fraction: ____________________

Decimal Number: ____________

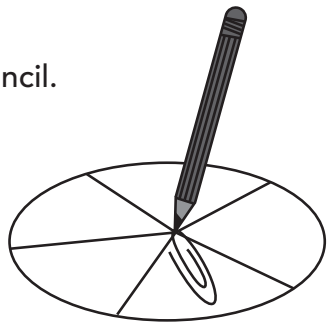
$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$

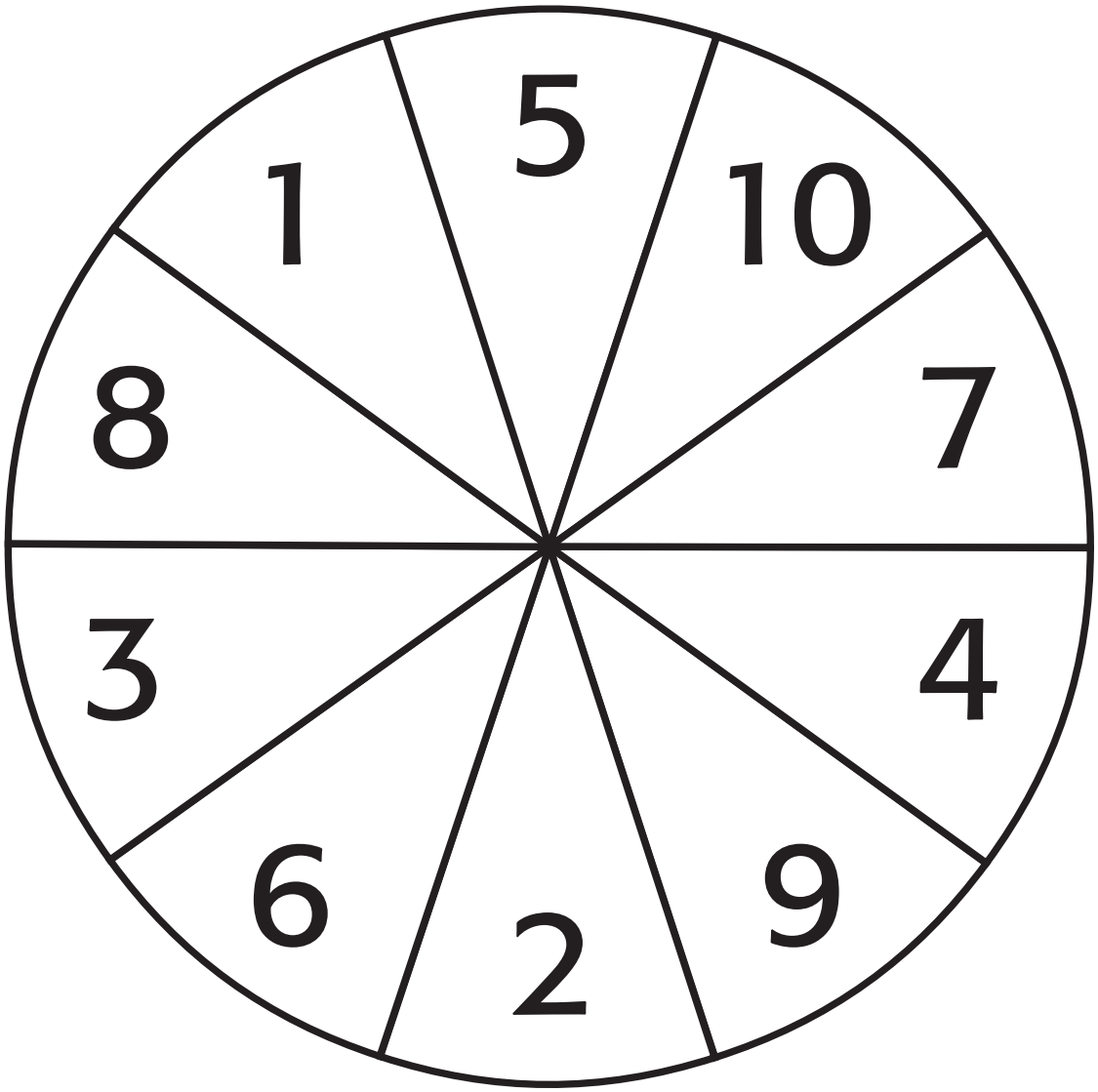
Fraction: ____________________

Decimal Number: ____________

Ten-Section Spinner

Make a spinner using this page, a paper clip, and a pencil.

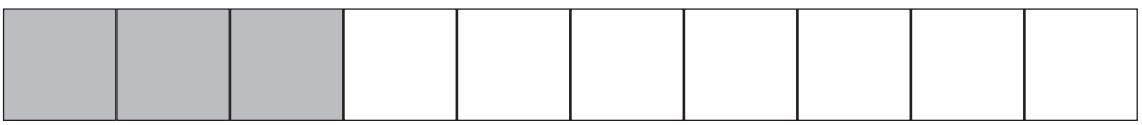




Cover the Tenths Game

Dear Parent/Guardian:

We have been learning that tenths can be written as fractions and as decimal numbers. For example, three tenths of the following rectangle is shaded. Three tenths can be written as 3/10 and 0.3.

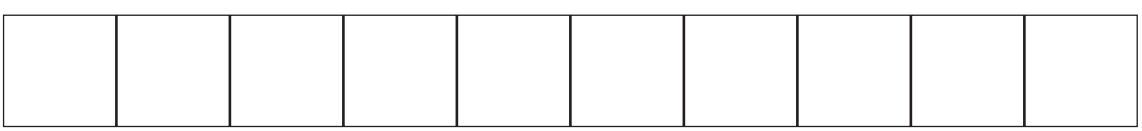


Here is a game to play with your child. You will need 10 small objects (e.g., buttons, coins, beans), a piece of paper, and a pencil.

- Players take turns covering spaces in the rectangle at the bottom of this page with small objects (one small object per space).
- At each turn, players decide whether to cover one, two, or three spaces.
- After placing the small objects in the spaces, players announce the total number of spaces that are covered and write the number of covered spaces as a fraction and as a decimal number on a piece of paper. For example, if 4 spaces are covered, the player would announce "4 tenths" and would record "4/10" and "0.4".
- The player who finishes covering all 10 spaces announces "10 tenths is 1 whole" and records 10/10 and 1.0. This player wins the game.

After playing the game several times, ask your child to explain strategies for winning the game.

Thank you for doing this activity with your child.



Place-Value Mat

Tens	Ones	Tenths

Decimal Number Grab Bag Recording Sheet

Drawing			Fraction	Decimal Number
Tens	Ones	Tenths		
			$34\frac{3}{10}$	34.3

Closest to Ten Recording Sheet

Game 1	Game 2
Roll 1 ..... _____	Roll 1 ..... _____
Roll 2 ..... _____	Roll 2 ..... _____
Sum or Difference _____	Sum or Difference _____
Roll 3 ..... _____	Roll 3 ..... _____
Sum or Difference _____	Sum or Difference _____
Roll 4 ..... _____	Roll 4 ..... _____
Sum or Difference _____	Sum or Difference _____
Roll 5 ..... _____	Roll 5 ..... _____
Final Sum or Difference _____	**Final Sum or Difference** _____

Grade 5 Learning Activity Number Books

OVERVIEW

In this learning activity, students play a game in which they match cards showing partially shaded 10 × 10 grids with corresponding fraction and decimal cards. The game helps to develop students' understanding of tenths and hundredths, and emphasizes the relationships between fractions and their equivalent decimal forms.

BIG IDEAS

This learning activity focuses on the following big ideas:

Quantity: In the Number Books game, students match fraction and decimal cards to quantities represented by partially shaded 10 × 10 grids.

Relationships: The game focuses on relationships between fractions and their equivalent decimal forms (e.g., 4/10 = 0.4; 25/100 = 0.25). It also helps students to recognize equivalencies between fractions (e.g., 3/10 = 30/100) and between decimal numbers (e.g., 0.3 = 0.30).

Representation: The game provides an opportunity for students to identify equivalent fraction-decimal number representations (e.g., 0.4 = 40/100 = 4/10 = 2/5).

Proportional reasoning: Students determine the relationship between simple fractions, fractions with a denominator of 10 or 100, and equivalent decimal numbers (e.g., 3/4 = 75/100 = 0.75).

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- demonstrate and explain the concept of equivalent fractions, using concrete materials (e.g., use fraction strips to show that 3/4 is equal to 9/12);
- demonstrate and explain equivalent representations of a decimal number, using concrete materials and drawings (e.g., use base ten materials to show that three tenths [0.3] is equal to thirty hundredths [0.30]);
- determine and explain, through investigation using concrete materials, drawings, and calculators, the relationship between fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100) and their equivalent decimal forms (e.g., use a 10 × 10 grid to show that 2/5 = 40/100, which can also be represented as 0.4).

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- read, represent, compare, and order whole numbers to 100 000, decimal numbers to hundredths, proper and improper fractions, and mixed numbers.

ABOUT THE LEARNING ACTIVITY

TIME:
60 minutes

MATERIALS

- overhead transparency of **Dec5.BLM1: Twenty-Five Hundredths**
- overhead projector
- overhead marker
- overhead transparency of **Dec5.BLM2: Sixty Hundredths**
- **Dec5.BLM3: 10×10 Grid**
- **Dec5.BLM4: 10×10 Grid Cards** (1 set of 9 cut-out cards per pair of students)
- **Dec5.BLM5a–c: Number Cards** (1 set of 27 cut-out cards per pair of students)
- a few transparencies of **Dec5.BLM3: 10×10 Grid**
- **Dec5.BLM6: Blank 10×10 Grid Cards**
- **Dec5.BLM7: Blank Cards**
- **Dec5.BLM8: Money Amounts as Decimal Numbers**

MATH LANGUAGE

- tenths
- hundredths
- representation
- equal/equivalent

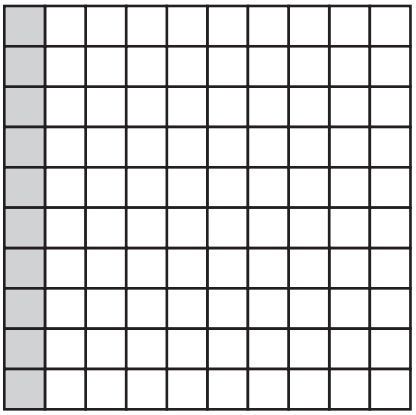
INSTRUCTIONAL GROUPING:
pairs

INSTRUCTIONAL SEQUENCING

Before starting this learning activity, students should have had experiences representing tenths as fractions (e.g., 4/10) and as decimal numbers (e.g., 0.4). This learning activity provides opportunities for students to extend their understanding of decimal numbers by exploring the meaning and representations of hundredths.

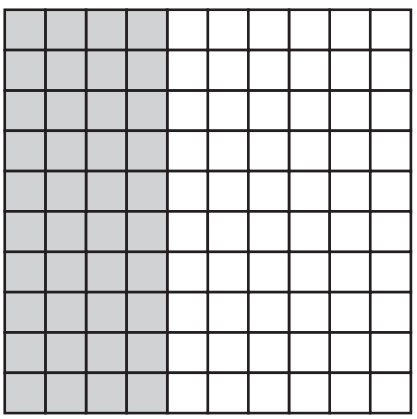
ABOUT THE MATH

Instructional activities with 10×10 grids help students to develop a sense of quantity about tenths and hundredths, and to grasp the relationship between one, tenths, and hundredths. The grid allows students to consider one column (or one row) as a tenth of the entire grid. The same row (or column) represents 10 hundredths of the grid.



The shaded part of the grid represents 1 tenth or 10 hundredths of the grid.

The 10 × 10 grid also helps students recognize relationships between fractions and decimal numbers.



40 hundredths

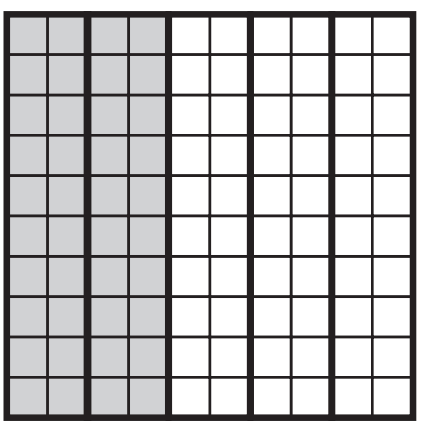
$\frac{40}{100}$

$\frac{4}{10}$

0.40

0.4

By dividing the grid into equal parts (e.g., into fifths, as illustrated in the following 10 × 10 grid), students can observe the relationship between decimal numbers and fractions with denominators other than 10 and 100.



Four tenths (0.4) of the grid is shaded.

Two fifths ($\frac{2}{5}$) of the grid is shaded.

$0.4 = \frac{2}{5}$

The focus in the following learning activity is on equal representations (e.g., the equivalence of 0.4, 40/100, 4/10, and 2/5). The notion that different numerical forms can represent the same quantity is a major development in students' understanding about numbers.

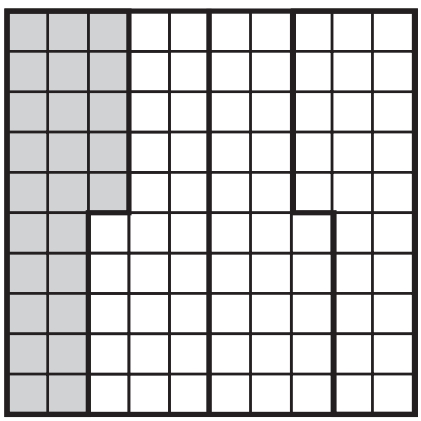
GETTING STARTED

Display an overhead transparency of **Dec5.BLM1: Twenty-Five Hundredths**. Discuss how the grid has 10 rows with 10 squares in each row, and establish that there are 100 squares in the grid. Refer to the grid as a "10 × 10 grid" and emphasize the idea that each square is one hundredth of the grid.

Ask: "What are different ways to describe the part of the grid that is shaded?" Provide time for students to think about the question individually, and then have them share their ideas with a partner. After partners have discussed the question, invite a few students to share their ideas with the whole class.

Discuss the following ideas:

- Since the grid is divided into 100 squares, each square is 1 hundredth of the grid, and 25 hundredths are shaded. Discuss different ways of recording the number:
  - twenty-five hundredths (in words)
  - 0.25 (as a decimal number)
  - 25/100 (as a fraction with a denominator of 100)
- The grid can be divided into four equal parts. The shaded part is one fourth of the grid. To illustrate this idea, use an overhead marker to outline four equal parts on the grid. Ensure that students recognize that all four parts are equal in size. (All parts have 25 squares.)



Review the meaning of 1/4 – the denominator represents the number of equal parts, and the numerator represents the number of parts being considered (in this case, the shaded part). Emphasize the notion that 0.25, 25/100, and 1/4 are different ways to represent the same quantity.

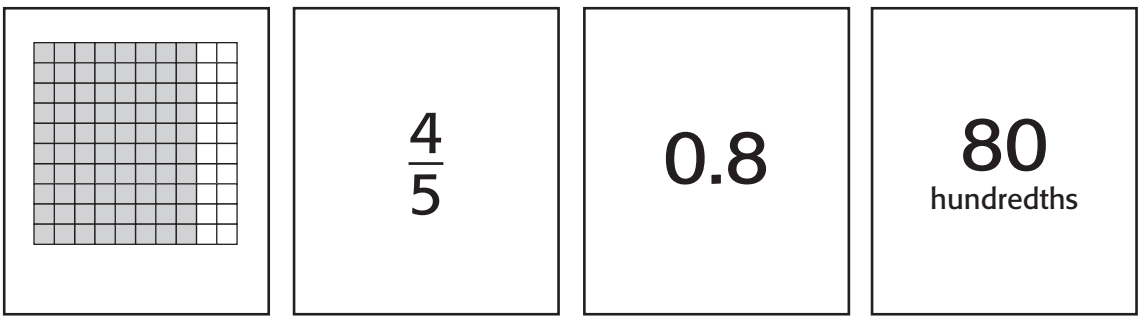
Display an overhead transparency of **Dec5.BLM2: Sixty Hundredths**. Have students work with a partner to discuss and record different ways to represent the shaded part of the grid.

Ask a few students to share their work. Discuss how the shaded part of the grid can be represented using words (sixty hundredths), decimal numbers (0.60, 0.6), and fractions (60/100, 6/10, 3/5). Use an overhead marker to highlight areas of the grid in order to demonstrate that 0.60 (sixty hundredths) is equal to 0.6 (six tenths), and that 60/100 is equal to 6/10 and 3/5.

Dec5.BLM3: 10 × 10 Grid is a blank grid, which can be partially shaded if students require more experience in determining equivalent number representations.

WORKING ON IT

Tell students that they will be playing a game called Number Books. Explain that the goal of the game is to be the player who obtains the most "number books". Explain that a complete number book consists of a 10 × 10 grid card and its matching fraction card, decimal card, and hundredths card.



Demonstrate the game by playing it with a volunteer.

- Each pair of players needs a deck of 10 × 10 grid cards (cut from **Dec5.BLM4: 10 × 10 Grid Cards**) and a deck of number cards (cut from **Dec5.BLM5a–c: Number Cards**). To prepare for the game, students draw six cards from the deck of 10 × 10 grid cards and display the cards, in a row, face up, between the players. One player deals four number cards to each player, and places the remaining number cards, face down, in a pile between the players.
- Players examine their hand of cards to find any number cards that match the displayed 10 × 10 cards. If they have one or more matching cards, they take turns placing the number cards face up, beside the corresponding 10 × 10 grid card. Players must lay down any matching cards they have in their hand. Players replace any cards they lay down by selecting cards from the pile of face-down cards; however, they may not play any of these cards until their next turn.
- If players are unable to lay down any cards during their turn, they place all four cards in their hand, face down, at the bottom of the number-card pile, and select another four number cards from the top of the pile. They may not, however, lay down any cards in the new hand until their next turn.
- A player who completes a book by laying down the fourth matching card may claim the four-card book and place it in front of him or her.
- When a four-card book is claimed, another 10 × 10 grid card is placed between the players. Players may lay down number cards beside this new card in an attempt to assemble another book.
- The game can be played for a specific amount of time (e.g., 20 minutes) or until all books have been assembled and claimed.

Have students play the game with a partner. As students play the game, ask them questions that focus on different ways to represent tenths and hundredths, and the relationship between these number forms:

- "What number cards will you need to complete this number book?"
- "Why do this fraction and this decimal number represent the same quantity?"

- "How do you know that 0.2 is the same as 20 hundredths?"
- "How do you know that this fraction (decimal number) matches the shaded part of this 10 × 10 grid?"

REFLECTING AND CONNECTING

Reconvene the class after students have played the game. Ask them to explain what they learned when they played the game. For example, students might discuss how the game helped them to recognize equivalent fractions and decimal numbers, and how it allowed them to better understand the quantity represented by fractions and decimal numbers.

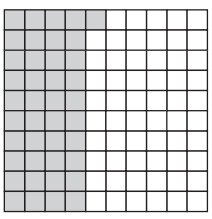
Record 40/100, 2/5, and 0.4 on the board. Ask students to explain why all three numbers represent the same amount. Allow students to use an overhead transparency of **Dec5.BLM3: 10 × 10 Grid** and overhead markers to demonstrate the quantity (i.e., 40 hundredths, 2 fifths, 4 tenths) represented by all three numbers.

Discuss the equivalency of other numbers:
- 50/100, 1/2, 0.5
- 30/100, 3/10, 0.3
- 75/100, 3/4, 0.75
- 80/100, 4/5, 0.8

ADAPTATIONS/EXTENSIONS

Simplify the game for students who have difficulty matching equivalent fractions and decimal numbers. For example, students might play the game using only the 10 × 10 grid cards and the fraction cards, or the 10 × 10 grid cards and the decimal cards.

An easier version of the game can also involve 10 × 10 grids and matching number cards, including fractions with denominators of 100, decimal numbers (hundredths only), and hundredths cards.



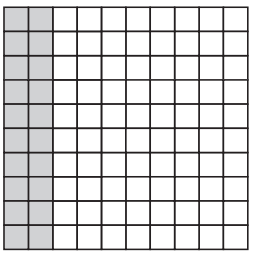
$\frac{41}{100}$

0.41

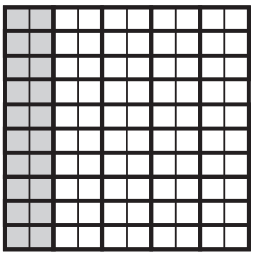
41
hundredths

Students or classroom volunteers can make the game cards using **Dec5.BLM6: Blank 10 × 10 Grid Cards** and **Dec5.BLM7: Blank Cards**.

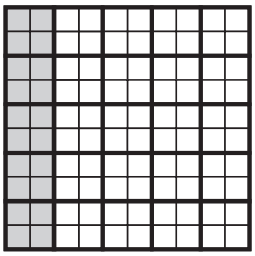
Challenge students by asking them to use a 10 × 10 grid to prove that the following fractions are equivalent: 20/100, 10/50, 5/25, 4/20, 2/10, 1/5.



The grid is divided into 100 equal parts; $\frac{20}{100}$ is shaded.



The grid is divided into 50 equal parts; $\frac{10}{50}$ is shaded.



The grid is divided into 25 equal parts; $\frac{5}{25}$ is shaded.

Have students use 10 × 10 grids to determine equivalent fractions for the following numbers:

- 25 hundredths
- 40 hundredths
- 60 hundredths
- 90 hundredths

ASSESSMENT

Observe students and converse with them as they play the game in order to assess how well they identify and explain equivalent forms of the same number (e.g., the equivalency of 0.5, 50/100, 5/10, 1/2).

Pose the following problem and write it on the board: "Seventy hundredths of Earth's surface is covered with water. What part of Earth's surface is covered by land?" Ask students to show a solution to the problem using fractions, decimal numbers, and diagrams, such as 10 × 10 grids.

Examine students' responses to assess how well they:

- explain that 30 hundredths of Earth's surface is covered by land;
- represent 30 hundredths using a fraction (30/100, 3/10), a decimal number (0.3), and a diagram (e.g., a shaded 10 × 10 grid).

HOME CONNECTION

Send home **Dec5.BLM8: Money Amounts as Decimal Numbers**. The game described in this Home Connection letter helps students to apply their understanding of decimal numbers to money representations.

LEARNING CONNECTION 1
Using a Decimal Wheel

MATERIALS

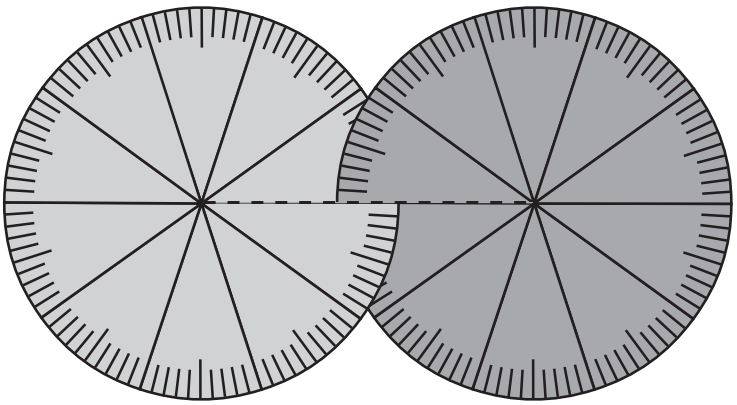
- copies of **Dec5.BLM9: Hundredths Wheel**, printed on two different colours of stiff paper (1 copy of each colour per student)
- scissors (1 pair per student)

Note: Copies of **Dec5.BLM9: Hundredths Wheel** could be cut out and laminated ahead of time to increase their durability.

A hundredths wheel is a tool with which students can model decimal numbers (tenths and hundredths). Provide each student with a copy of **Dec5.BLM9: Hundredths Wheel**. Discuss how the circle is divided into ten equal sections (indicating tenths), and how it is marked with 100 equal intervals around the edge (indicating hundredths).

Distribute a second copy of **Dec5.BLM9: Hundredths Wheel** (printed on a different colour of paper), and instruct students to assemble the decimal wheel as follows.

- Have students cut out both circles, and then cut along the dotted line to the centre of each circle.
- Have them slip the circles together at the cut edges, so that the wheels can be rotated to show the different parts of a whole.



Begin by asking students to use their decimal wheels to show one half, then one fourth, then three fourths. Continue by having students model different decimal numbers you give orally (e.g., 3 tenths, 6 tenths, 60 hundredths, 67 hundredths, 6 hundredths). For each decimal number, ask students questions such as the following:

- "Is the decimal number shown on your wheel more or less than 1/2?"
- "Is the decimal number closer to 1/2 or 3/4?"
- "How far from one whole is the decimal number?"

Have students use their decimal wheels to model fractions and decimal numbers you record on the board (e.g., 7/10, 23/100, 3/10, 3/100). Have students discuss the following ideas by referring to their decimal wheels:

- fraction-decimal number equivalencies (e.g., 7/10 = 0.7);
- the composition of fractions with denominators of 100 as tenths and as hundredths (e.g., 23/100 = 2/10 + 3/100);
- the composition of decimal numbers as tenths and hundredths (e.g., 0.37 = 0.3 + 0.07).

Students can use decimal wheels to add and subtract decimal numbers. Begin by having students solve simple addition problems. For example, to calculate 0.5 + 0.3, students can show 5 tenths on the wheel, and then add 3 tenths by rotating the wheel for three more tenths. When students are familiar with the use of the decimal wheel for addition, they can use it to solve more complex problems (e.g., 0.2 + 0.28).

LEARNING CONNECTION 2
Decimal Golf

MATERIALS

- six-sided number cubes (2 per pair of students)
- **Dec5.BLM10: Decimal Golf Score Cards** (1 per pair of students)
- a variety of manipulatives for representing decimal numbers (e.g., 10 × 10 grids, hundredths wheels, metre sticks)

Provide each pair of students with two number cubes and a copy of **Dec5.BLM10: Decimal Golf Score Card**. Ask students to examine the score card and draw their attention to the decimal number "par" that is given for each "hole number".

Explain the game:

- Players take turns rolling two number cubes and using the digits on the number cubes to create a two-digit number that includes a decimal number (e.g., after rolling a 2 and a 5, a player could create 0.25, 0.52, 2.5, or 5.2). Players should try to create the number that is closest to par (the target number for each hole). Players record the number in the appropriate section of the score card.
- After both players have recorded their numbers for each hole, they determine which player recorded the number that is closer to par.
- The player with the number closer to par circles his or her number on the score card. If players tie, they both circle their numbers.
- Players may use manipulatives (e.g., 10 × 10 grids, hundredths wheels, metre sticks) to prove that their number is closer to par than the other player's.
- The game continues until players have completed all nine holes. The winner is the player who has the most circled numbers.

eWORKSHOP CONNECTION

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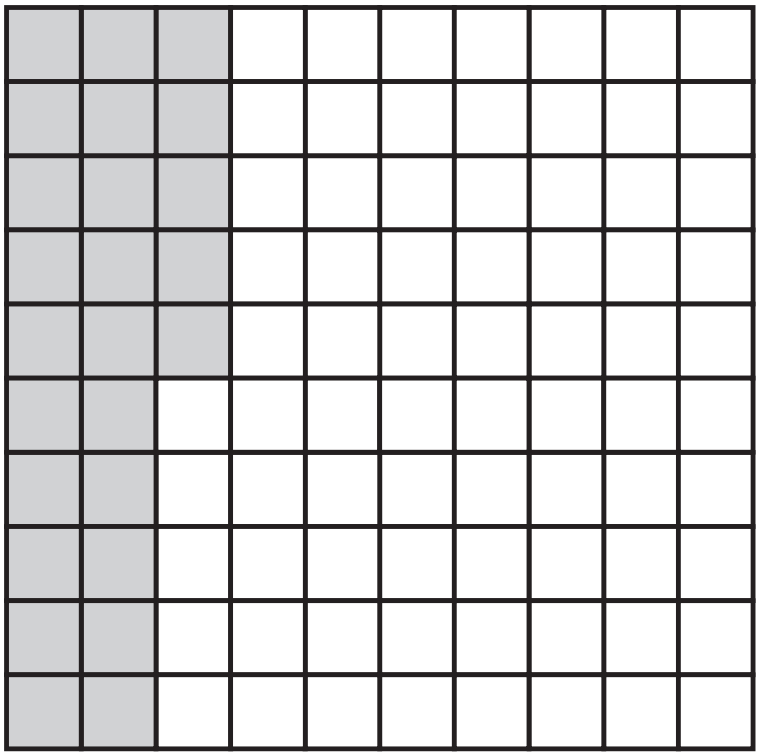
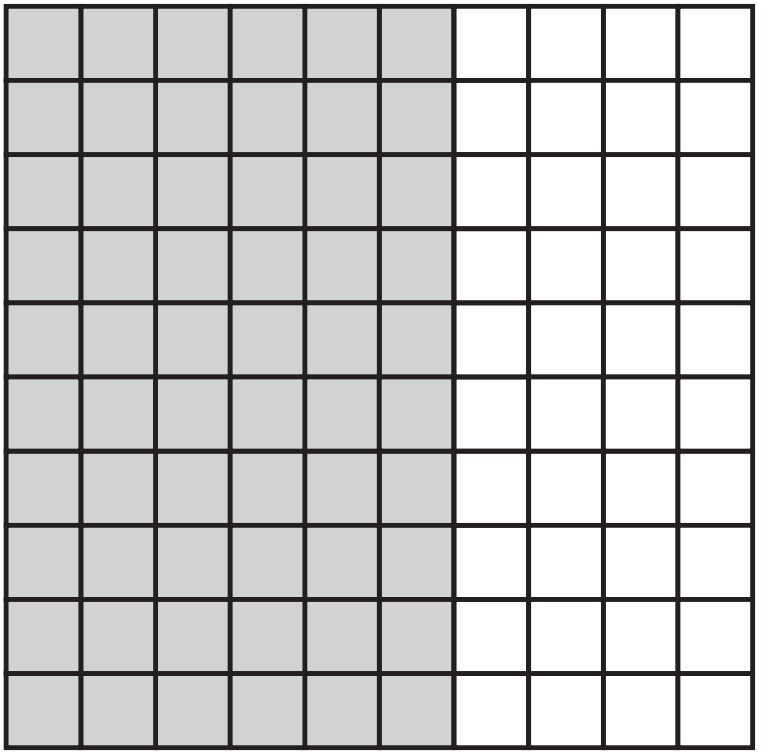
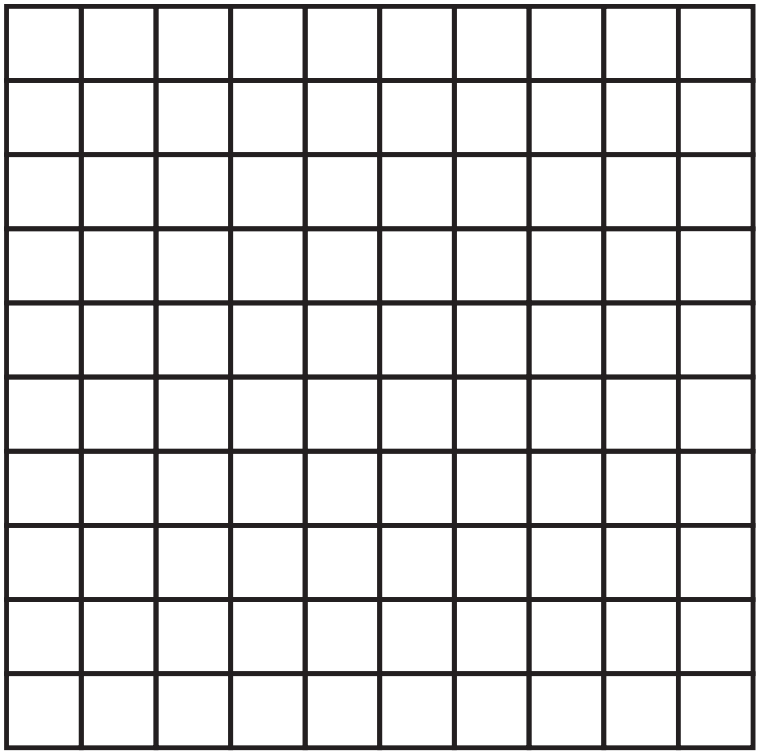
Twenty-Five Hundredths



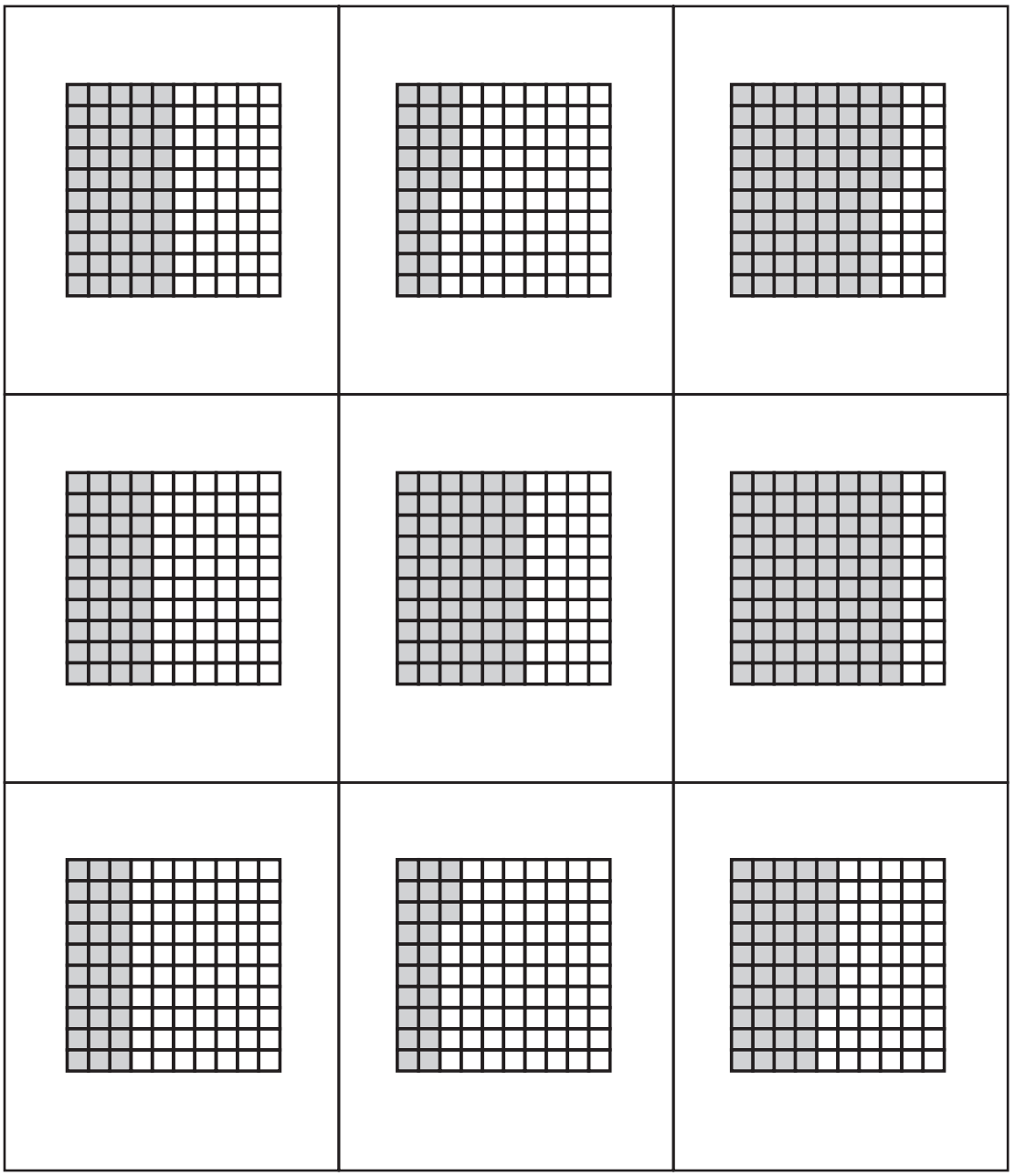
Sixty Hundredths



10×10 Grid



10×10 Grid Cards



Number Cards

$\frac{1}{2}$	0.5	50 hundredths
$\frac{1}{4}$	0.25	25 hundredths
$\frac{3}{4}$	0.75	75 hundredths

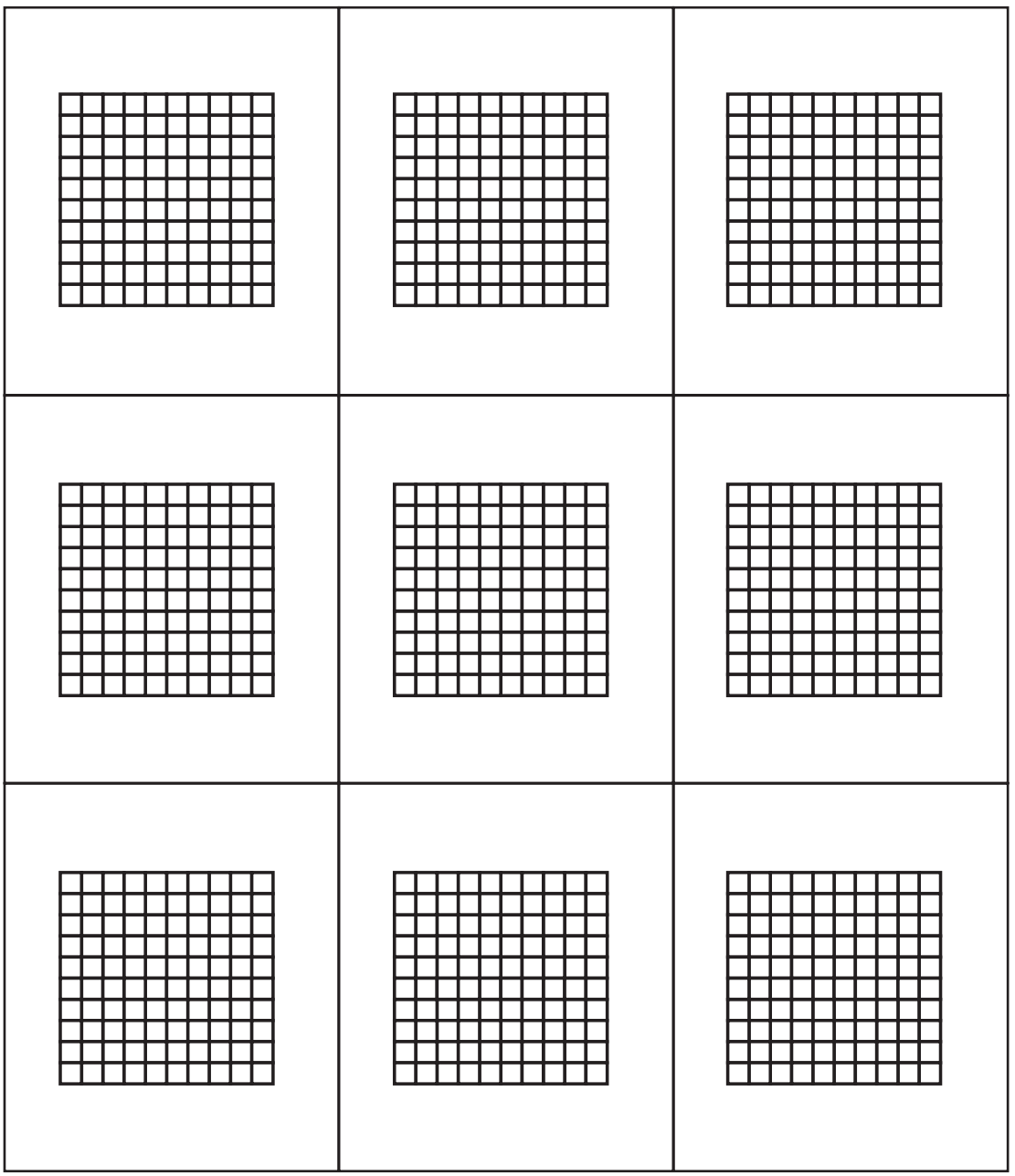
Number Cards

$\frac{2}{5}$	0.4	40 hundredths
$\frac{3}{5}$	0.6	60 hundredths
$\frac{4}{5}$	0.8	80 hundredths

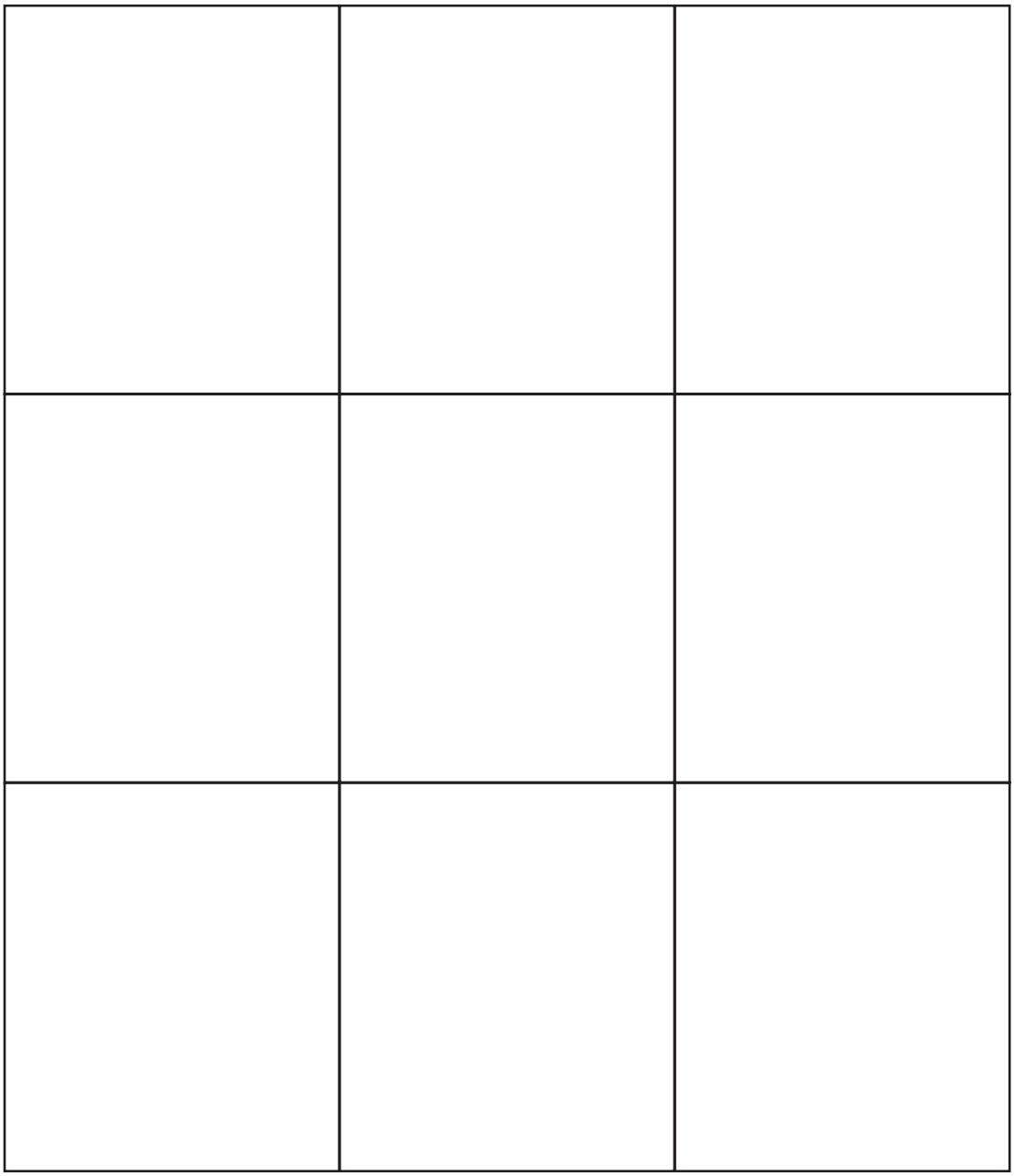
Number Cards

$\frac{3}{10}$	0.3	30 hundredths
$\frac{23}{100}$	0.23	23 hundredths
$\frac{47}{100}$	0.47	47 hundredths

Blank 10×10 Grid Cards



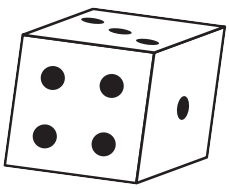
Blank Cards



Money Amounts as Decimal Numbers

Dear Parent/Guardian:

We have been learning about decimal numbers in math class.



Money amounts can be written as decimal numbers.

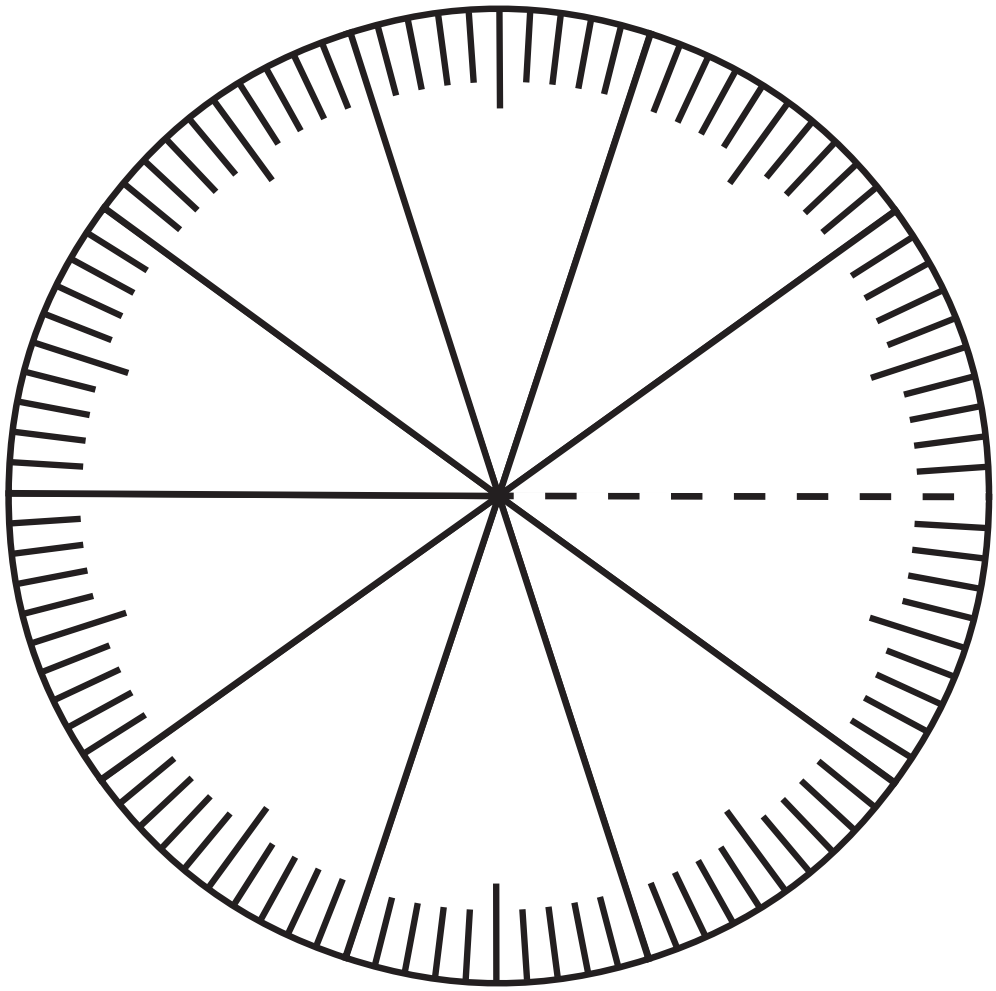
- 1¢ is 1 hundredth of a dollar. 1¢ can be written $0.01.
- 10¢ is 10 hundredths of a dollar. 10¢ can be written $0.10.
- 23¢ is 23 hundredths of a dollar. 23¢ can be written $0.23.

Play this game with your child. You will need a number cube (die) to play the game.

- Each player takes a turn rolling the number cube twice. The number on the first roll shows the number of dimes, and the number on the second roll shows the number of pennies. The player records the money amount as a decimal number. For example, by rolling a 3 and then a 5, a player has 3 dimes and 5 pennies, and records $0.35.
- After both players have recorded their decimal numbers, they decide which player has the greater money amount. The player with "more money" earns a point.
- The first player to earn 10 points wins the game.

Thank you for helping your child to understand decimal numbers.

Hundredths Wheel



Decimal Golf Score Cards

Hole Number	1	2	3	4	5	6	7	8	9
Par	0.25	1.0	0.15	0.5	0.64	3.5	0.58	1.3	0.22
Player One: _____									
Player Two: _____									

Hole Number	1	2	3	4	5	6	7	8	9
Par	0.25	1.0	0.15	0.5	0.64	3.5	0.58	1.3	0.22
Player One: _____									
Player Two: _____									

Grade 6 Learning Activity
The Contest

OVERVIEW

In this learning activity, students are presented with a scenario about a contest involving three classes who donate to a food bank. Students are asked to justify the decision about the winning class, based on data about student participation expressed as a fraction, a decimal number, and a percent.

BIG IDEAS

This learning activity focuses on the following big ideas:

Quantity: Students compare fractional quantities expressed as a fraction, a decimal number, and a percent.

Relationships: Students investigate the relationships among fractions, decimal numbers, and percents by determining equivalent representations (e.g., 3/4 = 0.75 = 75%), and by comparing numbers (e.g., 4/5 is greater than 75%).

Representation: Students learn that fractional quantities can be expressed as fractions, decimal numbers, and percents, and that these number forms can represent the equivalent quantities (e.g., 1/2, 0.5, 50%).

Proportional reasoning: This learning activity provides opportunities for students to use proportional reasoning as they determine equivalent fractions, decimal numbers, and percents.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);
- represent, compare, and order fractional amounts with unlike denominators, including proper and improper fractions and mixed numbers, using a variety of tools (e.g., fraction circles, Cuisenaire rods, drawings, number lines, calculators) and using standard fractional notation;
- determine and explain, through investigation using concrete materials, drawings, and calculators, the relationships among fractions (i.e., with denominators of 2, 4, 5, 10, 20, 25, 50, and 100), decimal numbers, and percents (e.g., use a 10 × 10 grid to show that 1/4 = 0.25 or 25%).

These specific expectations contribute to the development of the following **overall expectations**.

Students will:

- read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;
- demonstrate an understanding of relationships involving percent, ratio, and unit rate.

ABOUT THE LEARNING ACTIVITY

TIME:
60 minutes

MATERIALS

- **Dec6.BLM1: The Contest** (1 copy per student)
- sheets of chart paper or large sheets of newsprint (1 sheet per pair of students)
- markers (a few per pair of students)
- **Dec6.BLM2: 10×10 Grids** (several copies for the class)
- calculators
- a variety of manipulatives for representing fractions, decimal numbers, and percents (e.g., base ten blocks, counters, fraction circles)
- sheets of paper or math journals (1 per student)
- **Dec6.BLM3: Fractions and Percents in Ads** (1 per student)

MATH LANGUAGE

- fraction
- decimal number
- percent
- fractional number
- equivalent
- representation

INSTRUCTIONAL GROUPING:
pairs

INSTRUCTIONAL SEQUENCING

Before starting this learning activity, students should have had experiences exploring the meaning of "hundredth" and representing hundredths as fractions and decimal numbers. They should also have been introduced to basic ideas about percent (e.g., that *percent* means "per 100") and should have had opportunities to represent percents on 10×10 grids. This learning activity allows students to recognize relationships among fractions, decimal numbers, and percent.

ABOUT THE MATH

This learning activity reinforces students' understandings of fractions, decimal numbers, and percents, and of the connections among the three number forms. It helps students to understand that fractions, decimal numbers, and percents involve part-whole relationships:

- In a fraction, the whole is divided into equal parts.
- In a decimal number, the whole is divided into tenths, hundredths, thousandths, and so on.
- In a percent, the whole is comprised of 100 parts or 100%.

Fractions, decimal numbers, and percents can represent the same quantities (e.g., 1/2, 0.5, and 50% all represent "half"). By relating fractions, decimal numbers, and percents, students can determine equivalent forms (e.g., 4/5 = 0.8 = 80%) and can compare numbers (e.g., 3/5 is less than 80%).

Note: Decimal numbers such as 0.6 or 3.25 are often read as "point six" (or "decimal six") and "three point two five". To connect decimal numbers to their meaning, it is helpful to read 0.6 as "six tenths" and 3.25 as "three and twenty-five hundredths".

GETTING STARTED

Tell students that they will be examining a difficult situation that faced a Grade 6 class in another school. Provide each student with a copy of **Dec6.BLM1: The Contest**, and read the page together. Discuss the situation, and remind students that the Grade 4 class understands fractions, that the Grade 5 class also understands decimal numbers, but that neither grade has studied percents. If students have questions about the number of students in each class, ask them to think about whether they need this information to solve the problem.

WORKING ON IT

Have students work with a partner. Provide each pair with markers and a sheet of chart paper or a large sheet of newsprint. Make available copies of **Dec6.BLM2: 10 × 10 Grids**, calculators, and a variety of manipulatives for representing fractions, decimal numbers, and percents (e.g., base ten blocks, counters, fraction circles). Encourage students to use these materials.

Remind students that their explanation must be clear enough for Grade 4 and Grade 5 students to understand, and suggest that they express their ideas using words, numbers, and/or diagrams.

As students work, observe their strategies, and ask them to fully explain their thinking. It may be necessary to encourage some students to add more detail to their explanations, or to try explaining their ideas in different ways.

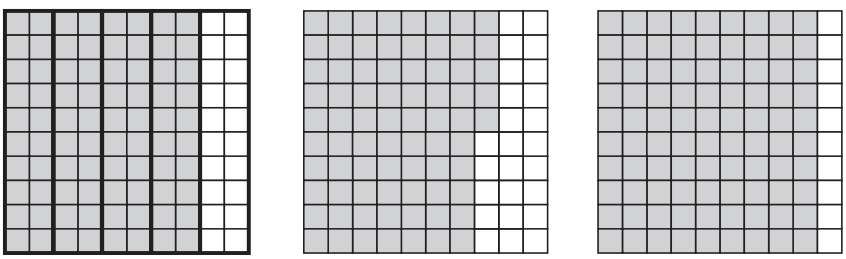
Pose the following questions to students as they are working:

- "Do you agree that the Grade 6 class won the contest? How do you know?"
- "How can you clearly show that the Grade 6 class won the contest?"
- "How did you show your ideas so that Grade 4 students will understand your explanation?"

STRATEGIES STUDENTS MIGHT USE

USING 10 × 10 GRIDS

Students might shade 10 × 10 grids to represent and compare 4/5, 0.75, and 90%.

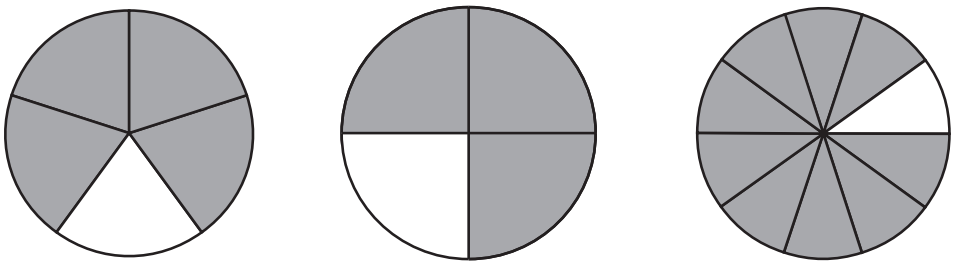


$\frac{4}{5}$ 0.75 90%

(continued)

USING MANIPULATIVES OR DIAGRAMS

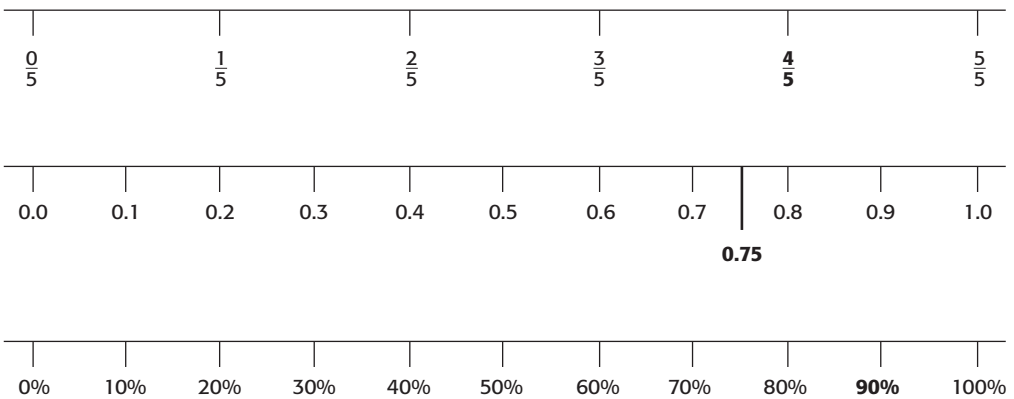
Students might use manipulatives (e.g., fraction circles) or diagrams to represent and compare the numbers.



$\frac{4}{5}$ $0.75 = \frac{3}{4}$ $90\% = \frac{9}{10}$

USING NUMBER LINES

Students might compare the numbers by locating them on number lines.



THINKING OF THE WHOLE AS 100

Students might think of the numbers in terms of a whole of 100. (Although 100 is not a realistic class size, it provides a common denominator for the fractional numbers from all three classes.)

Grade 4: $\frac{4}{5}$ of 100 students is 80 students.
Grade 5: 0.75 of 100 students is 75 students.
Grade 6: 90% of 100 students is 90 students.

REPRESENTING THE NUMBERS AS FRACTIONS, DECIMAL NUMBERS, AND PERCENTS

Students might represent the numbers in all three forms in order to compare them.

	Grade 4: $\frac{4}{5}$	Grade 5: 0.75	Grade 6: 90%
Fraction	$\frac{4}{5}$ or $\frac{16}{20}$	$\frac{75}{100}$, $\frac{3}{4}$, or $\frac{15}{20}$	$\frac{90}{100}$, $\frac{9}{10}$, or $\frac{18}{20}$
Decimal Number	0.80	0.75	0.90
Percent	80%	75%	90%

REFLECTING AND CONNECTING

Reconvene the class and ask a few pairs of students to present their work to the class. Select pairs who used various approaches so that students can observe and discuss different ways of showing relationships among fractions, decimal numbers, and percents.

Focus the discussion on the concept that fractions, decimal numbers, and percents are all different ways to represent fractional numbers. Emphasize the idea that all three number forms represent parts of a whole – 5 fifths represents the whole in 4/5; 100 hundredths represents the whole in 0.75; and 100% represents the whole in 90%. Record "4/5", "0.75", and "90%" on the board, and ask questions such as the following:

- "What does 4/5 mean? What does 5 as the denominator mean? What does 4 as the numerator mean? Does the fraction mean that there are only 5 students in the class, and that 4 students donated to the food bank? How many fifths would there be if the entire class donated to the food bank?"
- "What does 0.75 mean? Does the decimal number mean that there were 100 students in the class? How many hundredths would there be if the entire class donated to the food bank?"
- "What does 90% mean? Does the percent mean that there were 100 students in the class? If the entire class donated to the food bank, what percent would that be?"

Extend students' thinking by asking the following questions. Encourage students to use 10 × 10 grids and manipulatives (e.g., base ten blocks, counters, fraction circles) to explain their answers.

- "What fraction would the Grade 4 class need in order to win? How can you figure that out?"
- "What decimal number would the Grade 5 class need in order to win? How do you know?"
- "Which class had the lowest portion of students participating? How do you know?"
- "Suppose the Grade 5 and the Grade 6 classes tied with the Grade 4 class. What decimal number and what percent are equal to 4/5?"
- "Suppose the Grade 4 and the Grade 6 classes tied with the Grade 5 class? What fraction and what percent are equal to 0.75?"
- "Suppose the Grade 4 and the Grade 5 classes tied with the Grade 6 class? What fraction and what decimal number are equal to 90%?"

To conclude the learning activity, have students write a response to the following question on a sheet of paper or in their math journals:

> "What portion of the students in each class did not donate to the food bank? Express the portion of students who did not participate in different ways. Use diagrams to support your explanation."

ADAPTATIONS/EXTENSIONS

In this learning activity, students need to understand the meaning of fractions, decimal numbers, and percents. If students have difficulty dealing with all three number forms at the same time, provide a problem in which students need to compare numbers expressed in one form (i.e., only fractions, or only decimal numbers, or only percents). Encourage students to use materials such as 10 × 10 grids, fraction models, and base ten blocks to help them compare the numbers. As well, carefully consider student groupings. Place struggling students with classmates who will support them as they participate in and learn from the activity.

To extend the learning activity, provide students with the number of students in each class:

- 25 students in the Grade 4 class
- 28 students in the Grade 5 class
- 30 students in the Grade 6 class

Have students determine the number of students in each class who did and did not donate to the food bank.

ASSESSMENT

Observe students to assess how well they:

- represent fractions, decimal numbers, and percents using diagrams and manipulatives;
- explain that all three number forms (fractions, decimal numbers, percents) represent fractional parts of a whole;
- explain relationships among fractions, decimal numbers, and percents;
- compare fractions, decimal numbers, and percents.

Use the assessment information to help you make decisions about subsequent learning activities that will further students' understanding of the relationships among fractions, decimal numbers, and percents.

HOME CONNECTION

Send home copies of **Dec6.BLM3: Fractions and Percents in Ads**. This letter encourages parents/guardians to discuss with their children the meaning of fractions and percents in sales ads.

LEARNING CONNECTION 1
Number Clothes Line

MATERIALS

- string or cord (approximately 2 to 3 m long)
- clothes pegs
- number cards (e.g., index cards) labelled "0" and "1"
- number cards (e.g., index cards) with a fraction, decimal number, or percent written on each card. (Sample numbers: 1/2, 2/3, 3/4, 9/10, 0.2, 0.33, 0.45, 0.59, 0.8, 1%, 10%, 25%, 28%, 95%, 100%)

Suspend a string or a cord in the classroom, and explain to students that it represents a number line. Use clothes pegs to attach number cards labelled "0" and "1" at opposite ends of the string. Show one of the number cards (fraction, decimal number, or percent), and ask students to discuss with a partner the position of the number on the line relative to 0 and 1. Ask a few students to explain where the number should be placed on the line. When the class agrees on an appropriate spot, attach the number card to the line using a clothes peg.

Provide each pair of students with a number card. Ask partners to discuss where their card should be placed on the number line. Ask each pair, one at a time, to show their number card to the class and then attach the card to the number line. Have students explain their rationale for positioning the card in its spot. Remind students that they need to determine the position of their number in relation to all numbers that have already been attached to the number line.

After all cards have been attached to the number line, ask students to examine the order of the numbers to ensure that they are arranged from least to greatest value. Have students justify any adjustments they would like to make.

This activity could be done in one day, or it could occur over several days by selecting a number card each day and discussing its position on the number line.

LEARNING CONNECTION 2
Fraction-Decimal-Percent Connectors

MATERIALS

- cards cut from **Dec6.BLM4: Fraction-Decimal-Percent Connector Cards**
- **Dec6.BLM5: Fill-in-the-Blank Connector Cards** (2 copies per pair of students)

Give a card from **Dec6.BLM4: Fraction-Decimal-Percent Connector Cards** to each student. Ask all students to stand. Begin by having the student with the starred card read his or her question, and then sit down. The student with the correct answer on his or her card reads the response, and then asks the question on his or her card and sits down. The activity continues until a student reads the last card and sits down. (The answer to the question on the last card is 3/4, which is the number on the first card. Extra cards can be created within the chain of cards as needed.)

The activity can be repeated after collecting and shuffling the cards, and redistributing the cards to students. Students might enjoy the challenge of improving the time it takes to complete the circuit.

Students can work with a partner to make their own cards using copies of **Dec6.BLM5: Fill-in-the-Blank Connector Cards**. Repeat the activity using the student-made cards.

LEARNING CONNECTION 3
Equivalent Number Triplets

MATERIALS

- **Dec6.BLM6: Equivalent Number Triplets** (1 per pair of students)
- index cards (30 per pair of students)
- **Dec6.BLM7: Find the Equivalent Number Triplets** (1 per pair of students)

This game reinforces the concept that a fractional number can be represented as a fraction, decimal number, or percent.

Have students prepare game cards. Provide each pair of students with a copy of **Dec6.BLM6: Equivalent Number Triplets** and 30 index cards. Have pairs record (or glue cut-outs of) each number from **Dec6.BLM6: Equivalent Number Triplets** onto a separate index card.

Explain that the game involves collecting sets of three number cards showing an equivalent fraction, decimal number, and percent. Explain that the goal of the game is to be the first player to collect two sets of cards.

To begin, one player shuffles the cards and deals six cards to each player. The remaining cards are placed face down in a pile between the two players. Players take turns drawing a card from the pile. If a player chooses to keep the card drawn from the pile, he or she must discard a card from his or her hand and place it face down at the bottom of the pile.

The game continues until one player collects two three-card sets.

As an extension, have pairs of students complete **Dec6.BLM7: Find the Equivalent Number Triplets**. Tell students that they are given a fraction, a decimal number, or a percent for each set of numbers, and that they must record the other two equivalent representations. After students complete the blackline master, they can prepare a set of game cards using the numbers on the worksheet.

LEARNING CONNECTION 4
Concentration

MATERIALS

- **Dec6.BLM6: Equivalent Number Triplets** (1 per pair of students)
- index cards (30 per pair of students)

Have students prepare game cards. Provide each pair of students with a copy of **Dec6.BLM6: Equivalent Number Triplets** and 30 index cards. Have pairs record (or glue cut-outs of) each of the numbers from **Dec6.BLM6: Equivalent Number Triplets** onto a separate index card. (Cards may have already been prepared for the previous learning connection.)

To play the game in pairs, one player shuffles the cards and places them face down in an array. Then the second player flips over three cards, trying to find cards that show an equivalent fraction, decimal number, and percent. If the player reveals a set of equivalent numbers, he or she keeps the set of cards and takes another turn. If the player does not find a complete set of matching cards, he or she flips the cards face down again, and the first player takes a turn.

The player with the most sets of cards at the end of the game wins.

LEARNING CONNECTION 5
The Greater Decimal Number

MATERIALS

- spinners made with **Dec6.BLM8: 0–9 Spinner**, a paper clip, and a pencil (1 per pair of students)
- sheets of paper (1 per student)
- manipulatives for representing decimal numbers (e.g., base ten blocks, metre sticks)

Provide each pair of students with the materials needed to make a spinner (a copy of **Dec6.BLM8: 0–9 Spinner**, a pencil, and a paper clip). Each player begins by drawing seven spaces for a "blank" number, such as the following, on a sheet of paper:

_____ _____ _____ _____ • _____ _____ _____

Players take turns spinning the spinner and recording the numeral shown on the spinner in one of the spaces of the number. Once a numeral has been recorded, players may not erase it or record it in a different space. When players have filled all seven spaces in their number, they compare them to determine which player has the greater number. Players may use manipulatives (e.g., base ten blocks, metre sticks) to help them compare the numbers. The player with the greater number wins the game.

After students have played the game a few times, discuss the strategies they used to help them create the greatest possible number.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on decimal concepts. On the homepage, click "Toolkit". In the "Numeracy" section, find "Decimal Numbers (4 to 6)", and then click the number to the right of it.

The Contest

The situation

Ms. MacIntosh's Grade 6 class is holding a contest to encourage students in the school to donate to the local food bank. The class with the most students that contribute wins a pizza party.

All of the junior-grade classes have decided to enter. The Grade 6 students know that each class has a different number of students, so to be fair, they have asked each class to report the number of students contributing to the food bank as a portion of the whole class. The class with the greatest portion will win.

The results

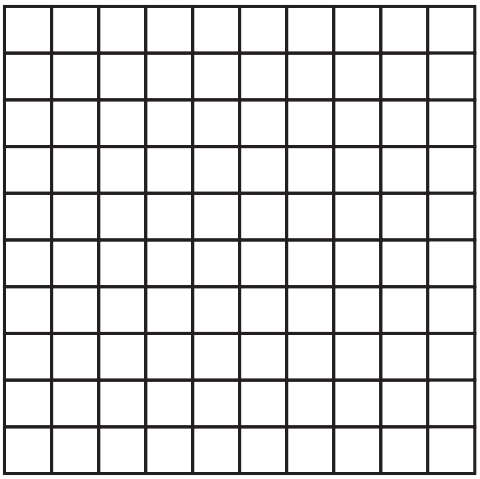
- The Grade 4 class is learning about fractions and has reported that 4/5 of their students contributed to the food bank.
- The Grade 5 class is learning about decimal numbers and has reported that 0.75 of their students contributed to the food bank.
- The Grade 6 class is learning about percents and has reported that 90% of their students contributed to the food bank.

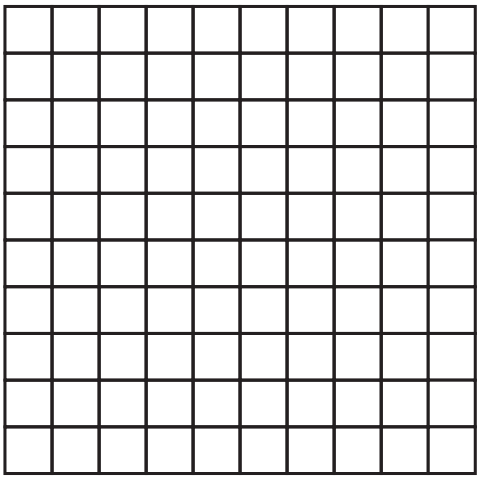
The Grade 6 students have decided that their own class is the winner. But they are concerned that the other classes might think that the decision is incorrect and that the Grade 6 class is dishonest.

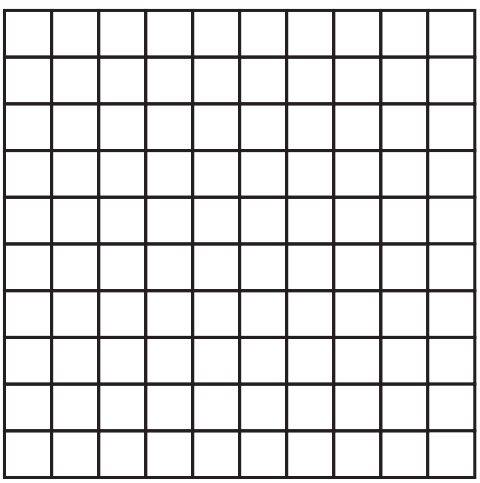
Your challenge

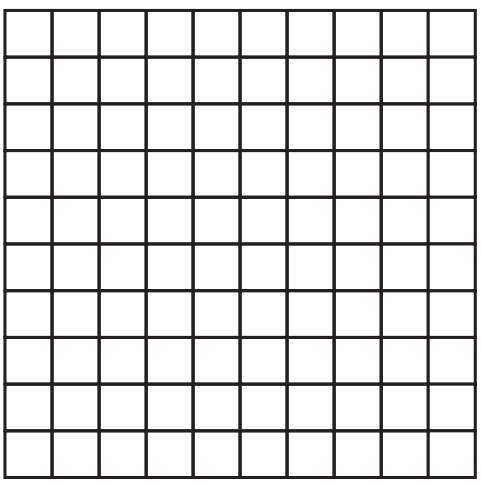
First, determine whether the Grade 6 students' decision is correct. Then prepare an explanation for the Grade 4 and Grade 5 classes that will help these students understand the decision, so they won't think that the older students are cheating. You may use any classroom materials and manipulatives to help you explain your ideas. Show your explanation clearly on the paper given to you.

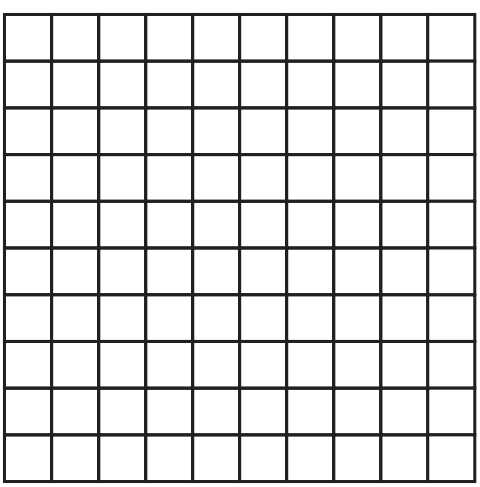
10×10 Grids

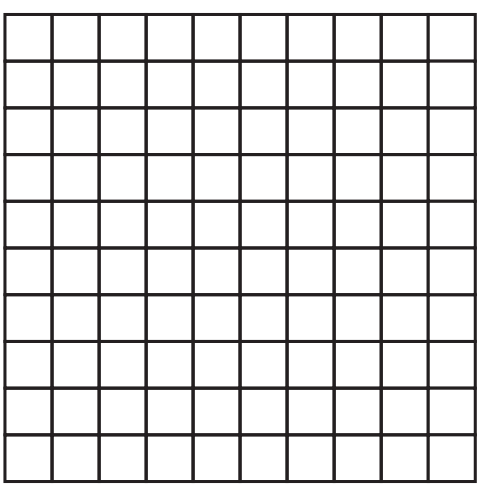












Fractions and Percents in Ads

Dear Parent/Guardian:

We are learning about fractions and percents.

Fractions and percents are often used in newspapers and flyer ads to describe sales.

All items $\frac{1}{2}$ price!	Up to 25% off!

Save 10%!

With your child, find ads with fractions and percents. Ask your child to explain the discount described by the fraction or percent.

Ask other questions such as the following:

- "What would you pay for an item that is regularly $28, if it is 1/2 price?"
- "What does it mean if items are 'up to 25% off'?"
- "How could you figure out, in your head, what you would save if an item that regularly costs $35 is discounted by 10%?"
- "Which store(s) is offering the best discount?"
- "Which store(s) is offering the least discount?"
- "What would this ad say if a fraction were used instead of a percent?"
- "Do most stores use fractions or percents in their ads? Why do you think they do?"

Thank you for discussing fractions and percents with your child.

Fraction-Decimal-Percent Connector Cards

I have $\frac{3}{4}$. * Who has a decimal equivalent to $\frac{1}{2}$?	I have 0.5. Who has a percent equivalent to $\frac{1}{5}$?	I have 20%. Who has a decimal equivalent to $\frac{1}{4}$?	I have 0.25. Who has a fraction equivalent to 60%?
I have $\frac{3}{5}$. Who has a percent equivalent to $\frac{1}{4}$?	I have 25%. Who has a fraction equivalent to 90%?	I have $\frac{9}{10}$. Who has a decimal equivalent to 45%?	I have 0.45. Who has a percent equivalent to 0.56?
I have 56%. Who has a fraction equivalent to 20%?	I have $\frac{1}{5}$. Who has a decimal equivalent to $\frac{3}{5}$?	I have 0.6. Who has a percent equivalent to $\frac{1}{10}$?	I have 10%. Who has a fraction equivalent to 11%?
I have $\frac{11}{100}$. Who has a decimal equivalent to $\frac{7}{10}$?	I have 0.7. Who has a percent equivalent to $\frac{1}{2}$?	I have 50%. Who has a fraction equivalent to 80%?	I have $\frac{4}{5}$. Who has a decimal equivalent to $\frac{1}{5}$?

I have 0.2. Who has a percent equivalent to 0.7?	I have 7%. Who has a fraction equivalent to 0.07?	I have $\frac{7}{100}$. Who has a percent equivalent to 0.3?	I have 30%. Who has a decimal equivalent to 7%?
I have 0.07. Who has a fraction equivalent to 30%?	I have $\frac{3}{10}$. Who has a decimal equivalent to 99%?	I have 0.99. Who has a percent equivalent to 0.04?	I have 4%. Who has a fraction equivalent to 0.25?
I have $\frac{1}{4}$. Who has a percent equivalent to $\frac{4}{5}$?	I have 80%. Who has a decimal equivalent to 4%?	I have 0.04. Who has a percent equivalent to $\frac{9}{10}$?	I have 90%. Who has a fraction equivalent to 0.75?

Fill-in-the-Blank Connector Cards

I have _____. * Who has _____ _____ ?	I have _____. Who has _____ _____ ?	I have _____. Who has _____ _____ ?
I have _____. Who has _____ _____ ?	I have _____. Who has _____ _____ ?	I have _____. Who has _____ _____ ?
I have _____. Who has _____ _____ ?	I have _____. Who has _____ _____ ?	I have _____. Who has _____ _____ ?
I have _____. Who has _____ _____ ?	I have _____. Who has _____ _____ ?	I have _____. Who has _____ _____ ?

Equivalent Number Triplets

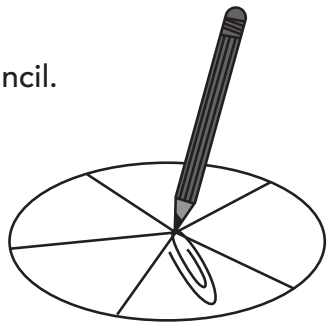
Fraction	Decimal Number	Percent
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{3}{10}$	0.3	30%
$\frac{2}{5}$	0.4	40%
$\frac{3}{5}$	0.6	60%
$\frac{7}{10}$	0.7	70%
$\frac{4}{5}$	0.8	80%

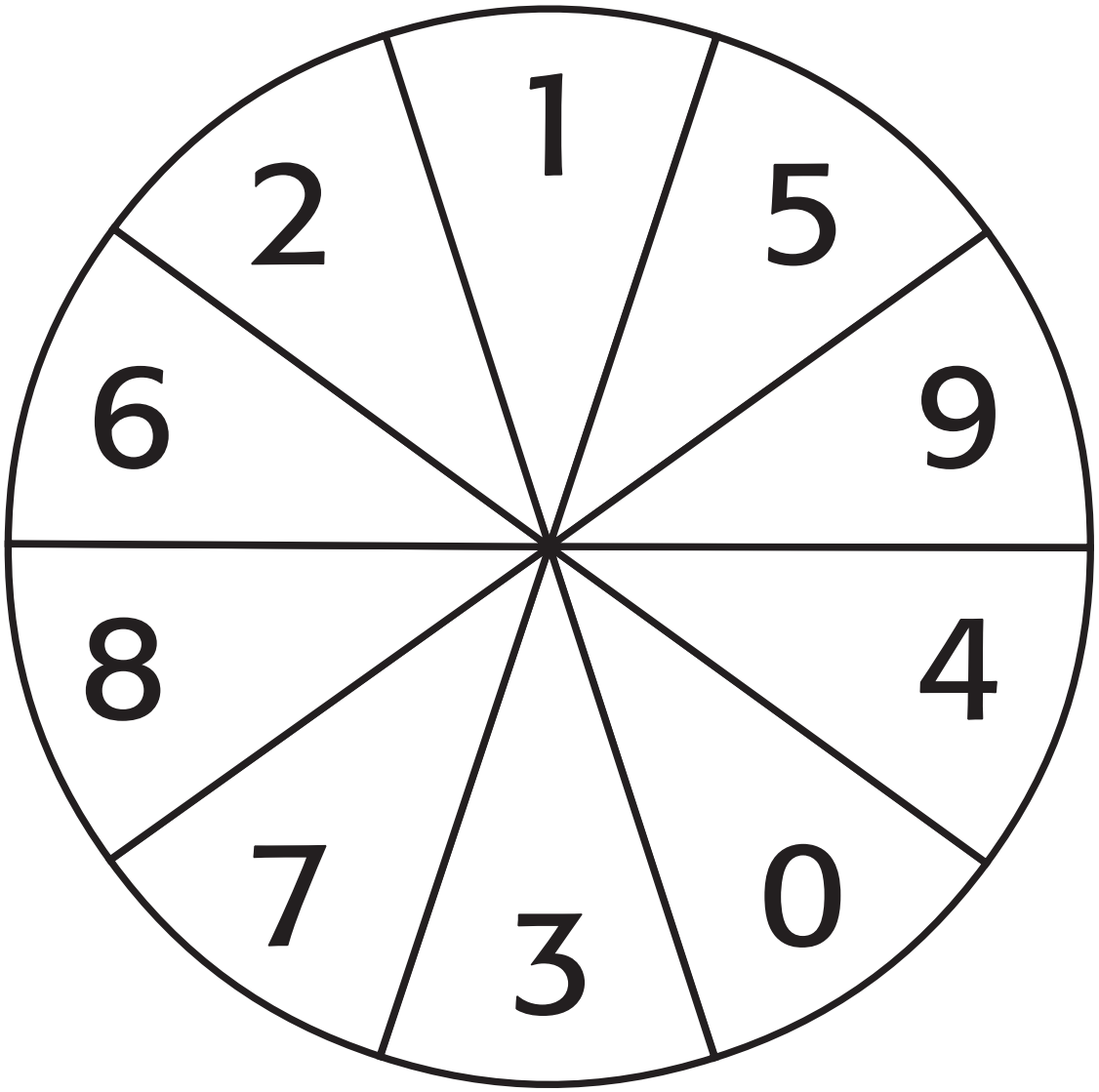
Find the Equivalent Number Triplets

Fraction	Decimal Number	Percent
$\frac{1}{2}$		
	0.25	
		75%
$\frac{1}{10}$		
	0.2	
		30%
$\frac{2}{5}$		
	0.6	
		70%
$\frac{4}{5}$		

0–9 Spinner

Make a spinner using this page, a paper clip, and a pencil.





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