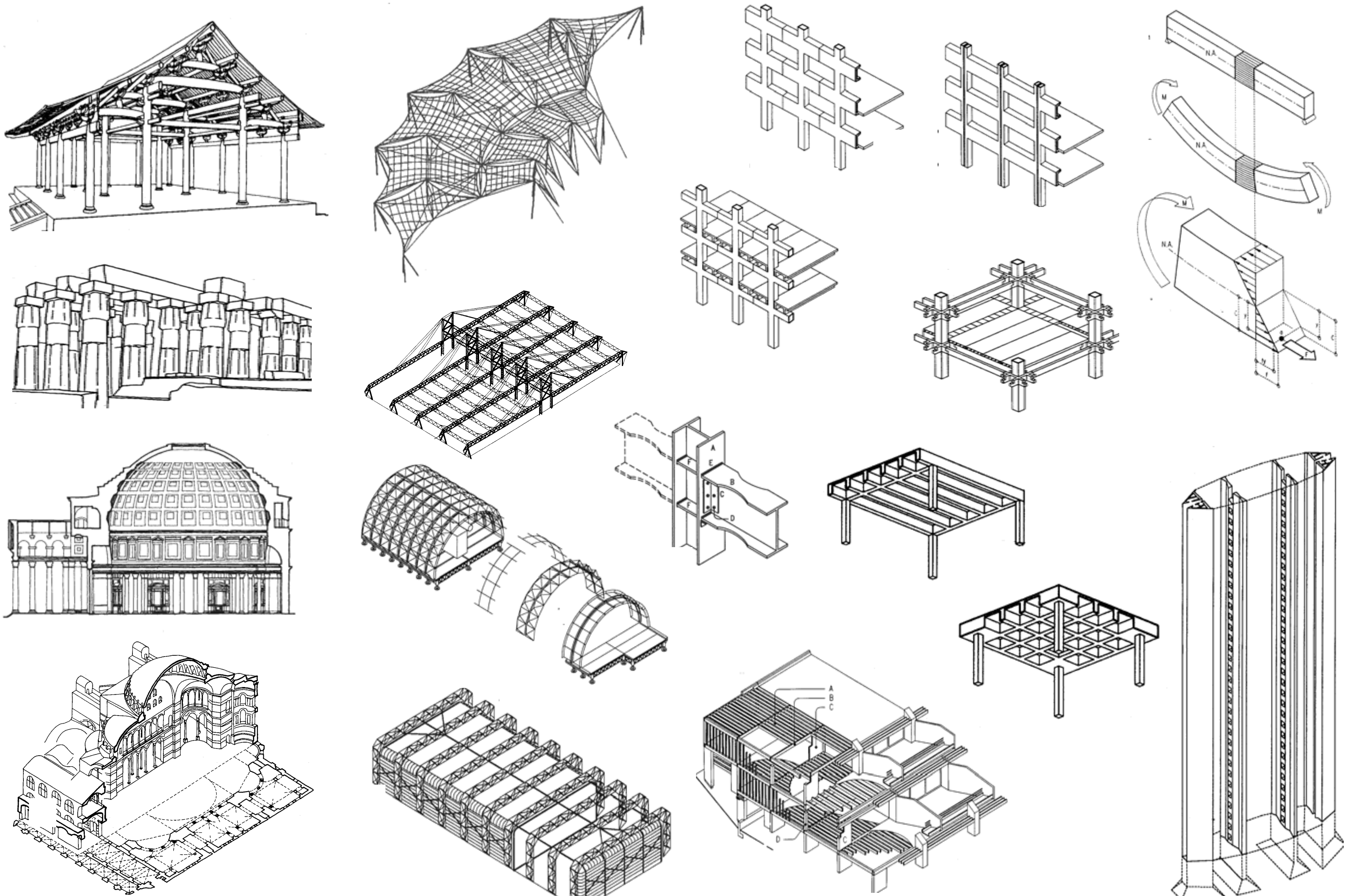

Structures in Architecture

G G Schierle

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Excerpts

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About the book

Structures not only support gravity and other loads, but are essential to define form and space. To design structures in synergy with form and space requires creativity and an informed intuition of structural principles. The objective of this book is to introduce the principles as foundation of creative design and demonstrate successful application on many case studies from around the world. Richly illustrated, the book clarifies complex concepts without calculus yet also provides a more profound understanding for readers with an advanced background in mathematics. The book also includes structural details in wood, steel, masonry, concrete, and fabric to facilitate design of structures that are effective and elegant. Many graphs streamline complex tasks like column buckling or design for wind and seismic forces. The graphs also visualize critical issues and correlate US with metric SI units of measurement. These features make the book useful as reference book for professional architects and civil engineers as well as a text book for architectural and engineering education. The book has 612 pages in 24 chapters.

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To My Family

Units

SI * units (metric)			Conversion factor **	US units		
		Remark				Remark
Length						
Millimeter	mm		25.4	Inch	in	
Centimeter	cm	10 mm	30.48	Foot	ft	12 in
Meter	m	1000 mm	0.9144	Yard	yd	3 ft
Kilometer	km	1000 m	1.609	Mile	mi	5280 ft
Area						
Square millimeter	mm^2		645.16	Square in	in^2	
Sq. centimeter	cm^2	100 mm^2	929	Square foot	ft^2	144 in^2
Square meter	m^2	1 Mil	0.835	Sq. yard	yd^2	9 ft^2
Hectar	ha	10000 m^2	2.472	Acre	Acre =	4840 yd^2
Volume						
Cubic millimeter	mm^3		16387	Cubic inch	in^3	
Cubic centimeter	cm^3	1 k mm^3	28317	Cubic foot	ft^3	
Cubic meter	m^3	1 Mil cm^3	0.7646	Cubic yard	yd^3	
Liter	l	0.001 m^3	0.264	Gallon	US gal = 3.785 liter	
Mass						
Gram	g		28.35	Ounce	oz	
Kilogram	kg	1000 g	0.4536	Pound	Lb, #	16 oz
Tonn	t	1000 kg	0.4536	Kip	k	1000 #
Force / load						
Newton	N		4.448	Pound	Lb, #	
Kilo Newton	kN	1000 N	4.448	Kip	k	1000 #
Newton/ meter	N/m		14.59	Pound/ ft	plf	
Kilo Newton/ m	kN/m		14.59	Kip/ ft	klf	1000 plf
Stress						
Pascal= N/m^2	Pa		6895	Pound/ in^2	psi	
Kilo Pascal	kPa	1000 Pa	6895	Kip / in^2	ksi	1000
Fabric stress						
Newton / m	N/m		175	Pound/ in	Lb/in	Fabric
Load / soil pressure						
Pascal	Pa	1000 Pa	47.88	Pound/ ft^2	psf	
Moment						
Newton-meter	N-m		1.356	Pound-foot	Lb-ft, #'	
Kilo Newton-m	kN-m	1000 N-	1.356	Kip-foot	k-ft, k'	1000#'
Temperature						
Celcius	°C		.55(F-32)	Fahrenheit	°F	
Water freezing		0°C	=	32°F		
Water boiling		100°C	=	212°F		

* SI = System International (French - designation for metric system)

** Multiplying US units with conversion factor = SI units
Dividing SI units by conversion factor = US units

Prefixes

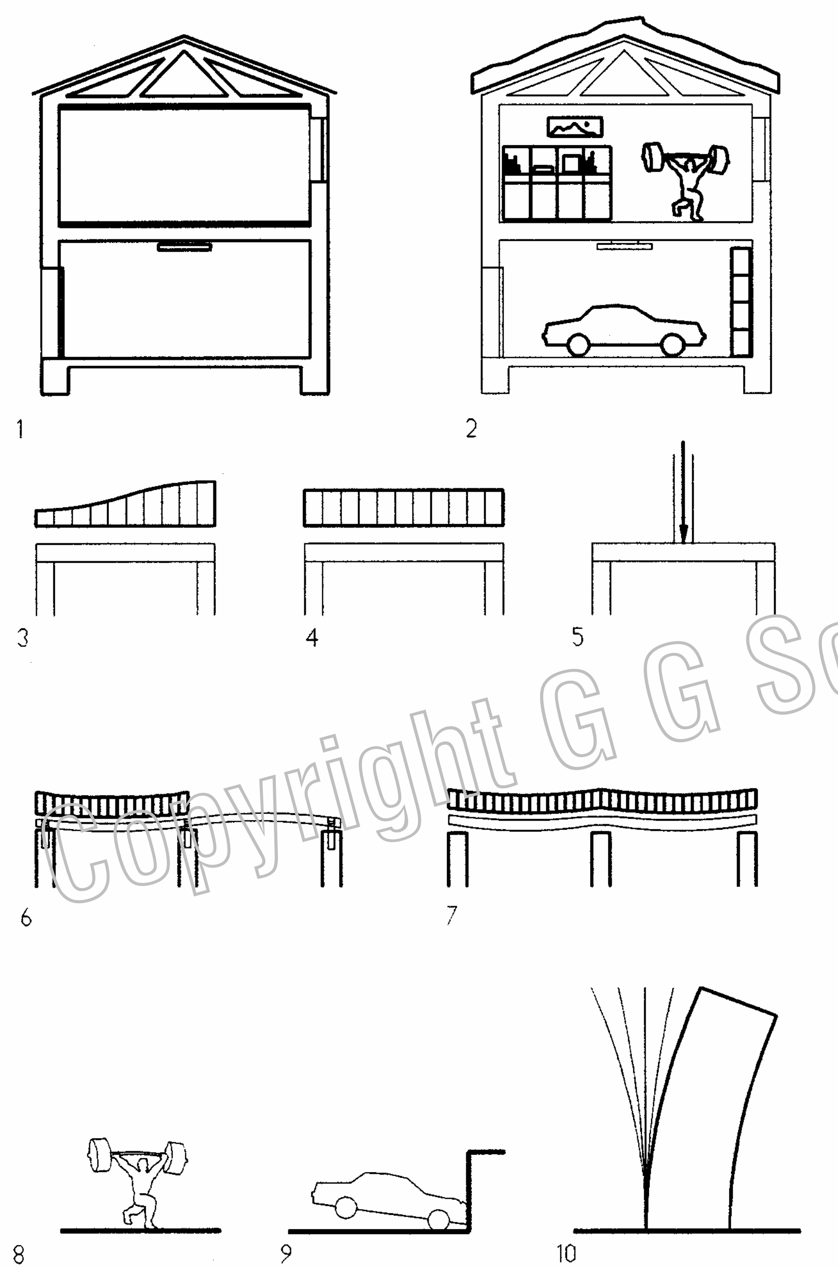
Prefix	Factor
Micro-	0.000001
MIli-, m	0.00001
Centi-	0.01
Deci-	0.1
Semi-, hemi-, demi-	0.5
Uni-	1
Bi-, di-	2
Tri-, ter-	3
Tetra-, tetr-, quadr-	4
Pent-, penta-, quintu-	5
Sex-, sexi-, hexi-, hexa-,	6
Hep-, septi-,	7
Oct-, oct-, octa-, octo-	8
Non-, nona-	9
Dec-, deca-, deci, deka-	10
Hect-, hector-	100
Kilo-, k	1,000
Mega-, M	1,000,000
Giga-, G	1,000,000,000
Tera-	1,000,000,000,000

Contents

2

Load

Understanding loads on buildings is essential for structural design and a major factor to define structural requirements. Load may be static, like furniture, dynamic like earthquakes, or impact load like a car hitting a building. Load may also be man-made, like equipment, or natural like snow or wind load. Although actual load is unpredictable, design loads are usually based on statistical probability. Tributary load is the load imposed on a structural element, like a beam or column, used to design the element. All of these aspects are described in this chapter.



Introduction

Structures resist various loads (gravity, seismic, wind, etc.) that may change over time. For example, furniture may be moved and wind may change rapidly and repeatedly.. Loads are defined as ***dead load*** (DL) and ***live load*** (LL); ***point load*** and ***distributed load***; ***static***, ***impact***, and ***dynamic load***, as shown at left.

1 Dead load: structure and permanently attached items (table 21.)
2 Live load: unattached items, like people, furniture, snow, etc (table 2.2)
3 Distributed load (random – snow drift, etc.)
4 Uniform load (uniform distribution)
5 Point load (concentrated load)
6 Uniform load on part of a beam is more critical than full load
7 Negative bending over support under full load reduces positive bending
8 Static load (load at rest)
9 Impact load (moving object hitting a structure)
10 Dynamic load (cyclic loads, like earthquakes, wind gusts, etc.)

Classification as DL and LL is due to the following considerations:

- Seismic load is primarily defined by dead load
- Dead load can be used to resist overturning under lateral load
- Long term DL can cause material fatigue
- DL deflection may be compensated by a *camper* (reversed deflection)
- For some elements, such as beams that span more than two supports partial load may be more critical than full load; thus DL is assumed on the full beam but LL only on part of it

Lateral load (load that acts horizontally) includes:

- Seismic load (earthquake load)
- Wind load
- Soil pressure on retaining walls

Other load issues introduced::

- ***Tributary load*** (load acting on a given member)
- ***Load path*** (the path load travels from origin to foundation)

Dead Load

Dead load is the weight of the structure itself and any item permanently attached to it, Dead load defines the mass of buildings for seismic design. Table 2.1 give the weight of materials to define building mass. Approximate dead loads are:

- Wood platform framing: 14 psf
- Wood platform framing with lightweight concrete: 28 psf
- Steel framing with concrete deck: 94 to 124 psf

Table 2.1. Material weight		
Weight by volume	US units	SI units
Masonry / concrete / etc.	pcf	kg/dm^3
Brick	120	1.92
Concrete masonry units (CMU)	100	1.60
Light-weight CMU	60	0.96
Concrete	150	2.40
Vermiculite concrete	25 - 60	0.40-0.96
Gravel / sand	90-120	1.44-1.92
Soil	75-115	1.20-1.84
Water at 4° C	62.4	1.00
Metals		
Aluminum	165	2.64
Cast iron	450	7.21
Steel	485	7.77
Stainless steel	492-510	7.88-8.17
Copper	556	8.91
Lead	710	11.38
Stone		
Granite / slate	175	2.81
Lime stone / marble	165	2.64
Sandstone	150	2.40
Wood		
Cedar	22	0.35
Douglas fir	34	0.55
Oak	47	0.75
Pine, white	25	0.40
Redwood	28	0.45

Table 2.1. Material weight - continued		
Weight by area	psf	Pa
Gypsum board, 5/8" (16 mm)	2.5	120
Stucco, 7/8" (22 mm)	8	383
Acoustic tile, ½"	0.8	38
Ceramic tile, 1/4" (6.3 mm)	2.5	120
Glass		
Sheet glass, 1/8" (3 mm)	1.5	72
Sheet glass, ¼" (6 mm)	3	144
Glass block, 4" (102 mm)	20	958
Roof material		
Built-up roof	7	335
Clay / concrete tiles	13-20	622-959
Metal	1-3	48-144
Shingles	3	144
Single ply roof / fabric roof	1-2	48-06
Industrial fabric (PVC , fiber glass)	1-2	48-96
Steel floor constructions		
Steel deck / concrete slab, 6" (15 mm)	40-60	1915-2873
Suspended ceiling	2	96
Floor finish	2	96
Steel framing (varies with height)	10-40	479-1915
Partitions (required by code)	20	958
Total	94 - 124	3543-5458
Wood platform framing		
Wood platform framing + floor / ceiling	14	670
Light-weight concrete option	14	670
Total (with and without concrete)	14 - 28	1341

IBC table 1607.1 excerpts. Minimum uniform live load			
Use or Occupancy		Pounds / ft²	kPa
Category	Description	psf	kPa
Access floors	Office use	50	2.39
	Computer use	100	4.79
Auditoria and Assembly areas	Fixed seating	50	2.39
	Movable seating	100	4.79
	Stage	125	5.99
Garages	Storage and repair	100	4.79
	Private	50	2.39
Hospitals	Wards and rooms	40	1.92
Libraries	Reading room	60	2.87
	Stack room	125	5.99
Manufacturing	Light	75	3.59
	Heavy	125	5.99
Offices		50	2.39
Printing plants	Press room	150	7.18
	Composing, etc	100	4.79
Residential	Basic floor area	40	1.92
	Exterior balconies	60	2.87
Reviewing stands, etc..		100	4.79
Schools	Classrooms	40	1.92
Sidewalks and driveways	Public access	250	11.97
Storage	Light	125	5.99
	Heavy	250	11.97
Stores		100	4.79
Pedestrian bridges		100	4.79

Live Load

IBC table 1607.1 defines live loads for various occupancies. Except for live load >100 psf (4.79 kPa) these loads may be reduced for large tributary areas as follows:

$R = r (A - 150)$
$R = r (A – 14)$ [for SI units]
Reductions R shall not exceed

- 40% for horizontal members
- 60 % for vertical members
- $R = 23.1 (1+ D/L)$
  where
  R = reduction in percent
  r = 0.08 for floors
  A = tributary area in square foot (m²)
  D = dead load
  L = Unreduced live load per square foot (m²)

Roof Load

Roof loads are defined by IBC

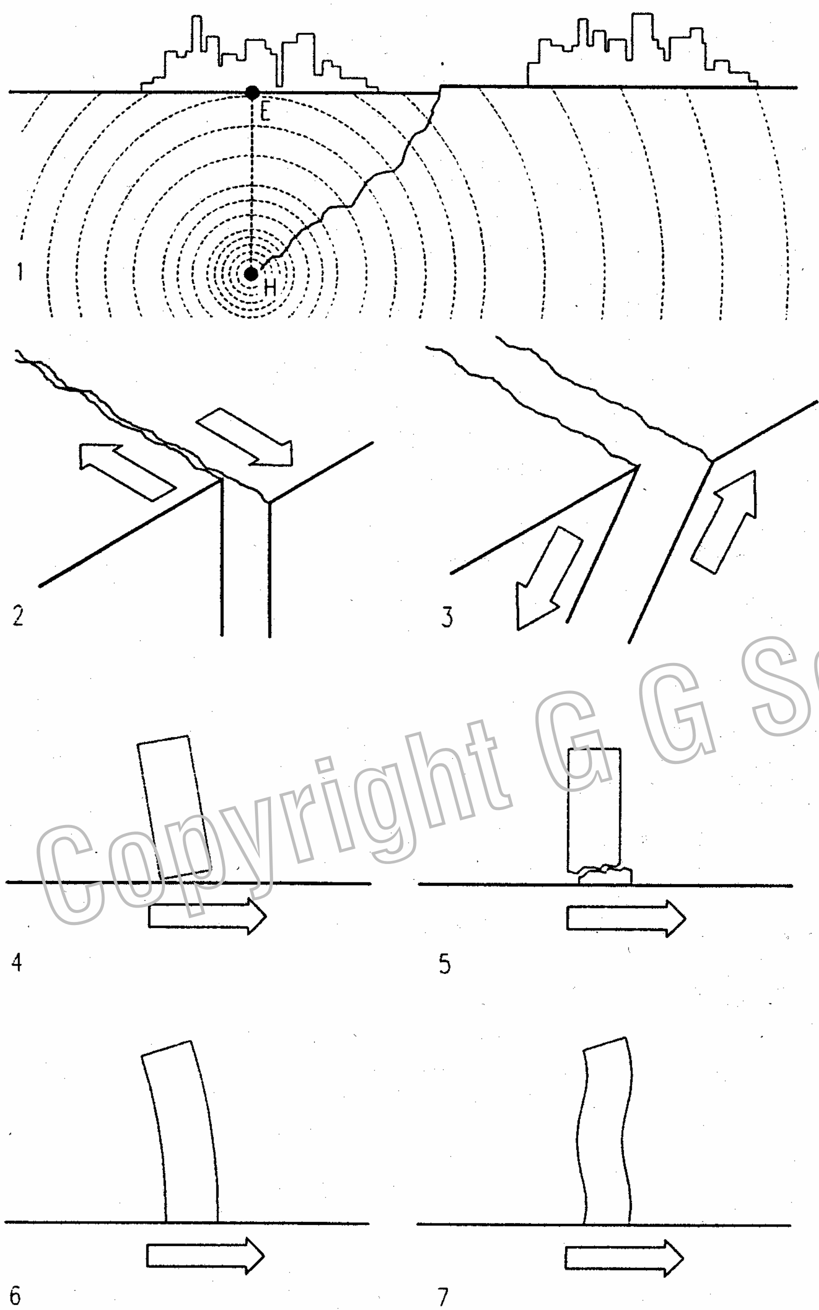
- Wind load per IBC 1609
- Snow load per IBC 1608
- Minimum roof loads:

Roof type	psf	Pa
Awnings and canopies	5	240
Green houses	10	479
Landscaped roofs (soil + landscaping as DL)	20	958
General flat, pitched, and curved roofs	L_r	L_r

$L_r = 20R_1 / R_2$
where

$12 < L_r < 20$		
$0.58 < L_r < 0.96$	for SI units	
$R_1 = 1$	for A ≤ 200 sq. ft.	for A ≤ 19 m²
$R_1 = 0.6$	for A ≥ 600 sq. ft.	for A ≥ 56 m²
$R_2 = 1$	for slopes ≤ 1:3	≤ 18°
$R_2 = 0.6$	for slopes ≥ 1:1	≥ 45°

A = tributary area
See IBC1607.11 for other R_1 and R_2 factors



Seismic load

Earthquakes cause horizontal and vertical ground shaking. The horizontal (lateral) shaking is usually most critical on buildings. Earthquakes are caused by slippage of seismic fault lines or volcanic eruption. Fault slippage occurs when the stress caused by differential movement exceeds the soil shear capacity. Differential movement occurs primarily at the intersection of tectonic plates, such as the San Andreas fault which separates the pacific plate from the US continental plate. Earthquake intensity is greatest after a long accumulation of fault stress. Seismic waves propagate generally in radial patterns, much like a stone thrown in water causes radial waves. The radial patterns imply shaking primarily vertical above the source and primarily horizontal with distance. The horizontal shaking usually dominates and is most critical on buildings. Although earthquakes are dynamic phenomena, their effect mat be treated as equivalent static force acting at the base of buildings. This lateral force, called *base shear*, is basically governed by Newton's law:

$f = m\ a$ (force = mass x acceleration)

Base shear is dampened by ductility, a structure's capacity to absorb energy through elastic deformation. Ductile structures deform much like flowers in the wind, yet brittle (non-ductile) structures sustain greater inertia forces. Steel moment resisting frames are ductile, though some shear walls are brittle. In earthquake prone areas seismic base shear as percentage of mass is approximately:

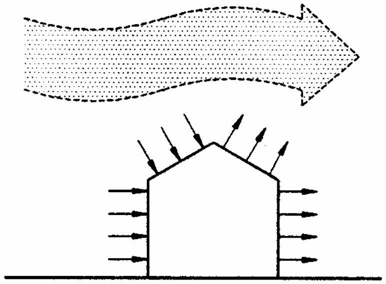
- ~ 4 % for tall ductile moment frames
- ~ 10 % for low-rise ductile moment frames
- ~ 15 % for plywood shear walls
- ~ 20 to 30 % for stiff shear walls

Seismic design objectives:

- Minimize mass
- Maximize ductility

1. Fault rupture / wave propagation
   (predominant vertical above rupture, lateral at distance)
2 Lateral slip fault
3 Thrust fault
4 Building overturning
5 Base shear
6 Bending deformation (first mode)
7 Bending deformation (higher mode)

E *Epicenter* (earthquake source above ground)
H *Hyper center* (actual earthquake source under ground)



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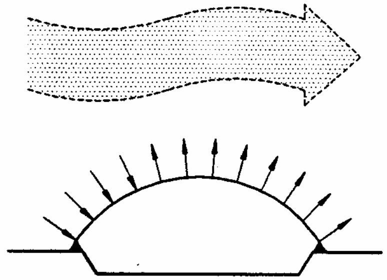

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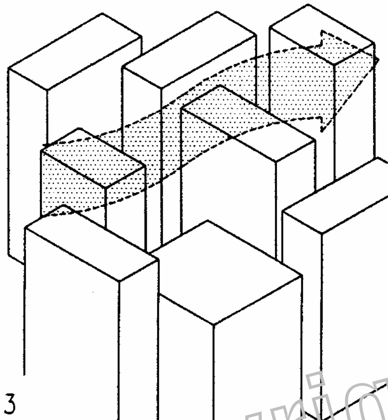

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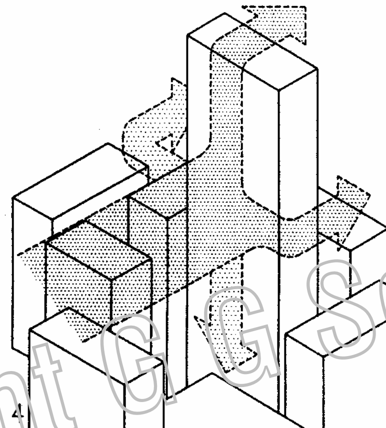

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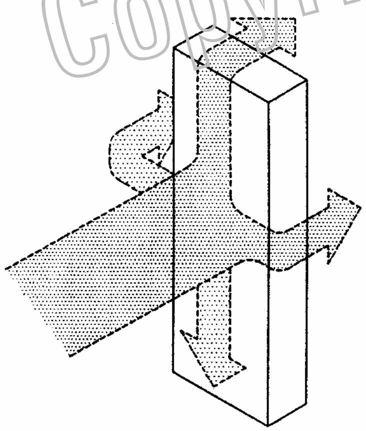

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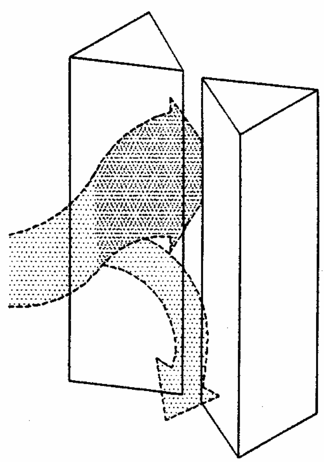


6

Wind load (see also *Lateral Force Design)*

Wind load generates lateral forces, much like earthquakes. But, though seismic forces are dynamic, wind load is usually static, except gusty wind and wind on flexible structures. In addition to pressure on the side facing the wind (called ***wind side***), wind also generates suction on the opposite side (called ***lee side***) as well as uplifting on roofs. Wind pressure on buildings increases with increasing velocity, height and exposure. IBC Figure 16-1 gives wind velocity (speed). Velocity wind pressure (pressure at 33 feet, 10 m above the ground) is defined by the formula

$$q = 0.00256\ V^2\ (H/33)^{2/7}$$

Where

q = velocity pressure in psf	[1 psf = 47.9 Pa]
V = velocity in mph	[1 mph = 1.609 km/h]
H = height in feet	[1 ft = 0.305 m]

The actual wind pressure P is the velocity pressure q multiplied by adjustment factors based on empirical data from wind tunnel tests, tabulated in code tables. The factors account for type of exposure, orientation, and peak pressure along edges, roof ridge, and method of analysis. IBC defines three exposures and two methods:

- *Exposure B* (sites protected by buildings or a forest)
- *Exposure C* (open sites outside cities)
- *Exposure D* (sites near an ocean or large lake)
- *Method 1: Normal force method*
- *Method 2: Projected area method*

Depending on location, height, and exposure, method 2 pressures range from 10-110 psf (0.5 to 5 kPa). This is further described in *Lateral Force Design*

Design objectives for wind load:

- Maximize mass to resist uplift
- Maximize stiffness to reduce drift

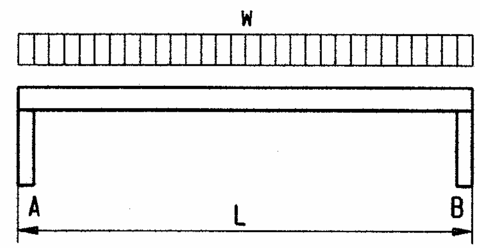
1 Wind load on gabled building (left pressure, right suction)
2 Wind load on dome or vault (left pressure, right suction)
3 Buildings within cities are protected by other buildings
4 Tall building exposed to full wind pressure
5 Wind on wide façade is more critical than on narrow facade
6 Building forms increase wind speed

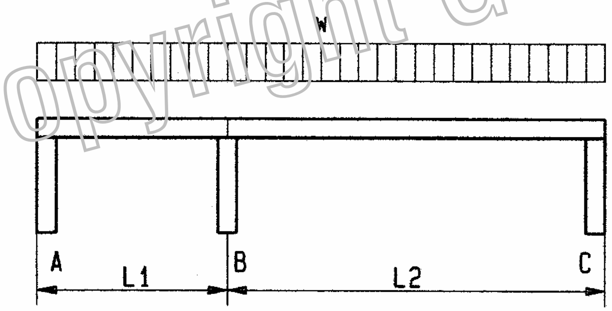
Tributary load and load path

Tributary load is the load acting on any element, like a beam, column, slab, wall, foundation, etc. Tributary load is needed to design / analyze any element.

Load path is the path any load travels from where it originates on a structure to where it is ultimately resisted (usually the foundation). It is essential to define the tributary load.

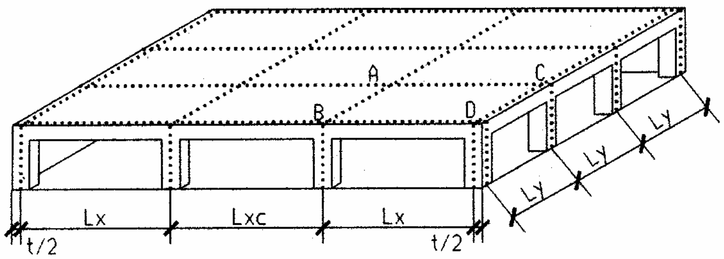
The following examples illustrate tributary load and load path

1 

2 

1 Simple beam / 2 columns
Assume
Uniform beam load w = 200 plf
Beam span L = 30'
Find
Load path: beam / column
Tributary load: Reactions at columns A and B
$R_a = R_b = R = w L/2$
R = 200x30/2 = 3000#
Convert pounds to kip
R = 3000#/1000 — R = 3.0 k

2 Two beams / three columns
Assume
Uniform beam load w = 2 klf
Beam spans L1 = 10', L2 = 20'
Find
Load path: beam / column
Tributary load: Reactions at columns A, B, C
R_a = 2x10/2 — R_a = 10 k
R_b = 2(10+20)/2 — R_b = 30 k
R_c = 2x20/2 — R_c = 20 k



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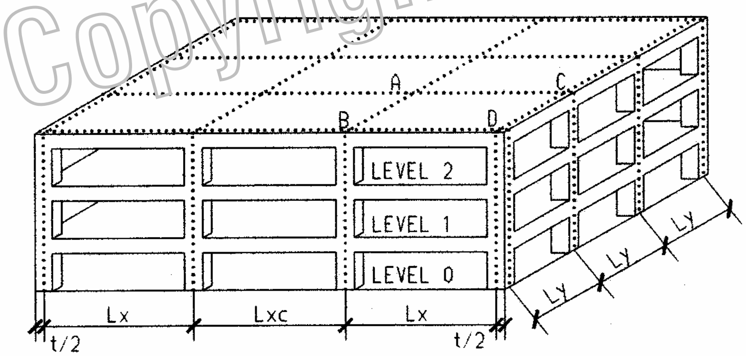


2

1 One-story concrete structure

Assume

Roofing	3 psf	
Ceiling	2 psf	
10" con. slab	125 psf	(150 pcf x10"/12")
DL	130 psf	
LL	20 psf	
Σ	150 psf	

Lx = 30', Lxc = 34', Ly = 25'
Columns, 12"x12" (t=12", t/2 = 6" = 0.5')
Column reactions A, B, C, D

R_a = 150 psf (30+34)/2 (25)	R_a = 120,000 #
R_b = 150 (30+34)/2 (25/2+0.5)	R_b = 62,400 #
R_c = 150 (30/2+0.5) (25)	R_c = 58,125 #
R_d = 150 (30/2+0.5) (25/2+0.5)	R_d = 30,225 #

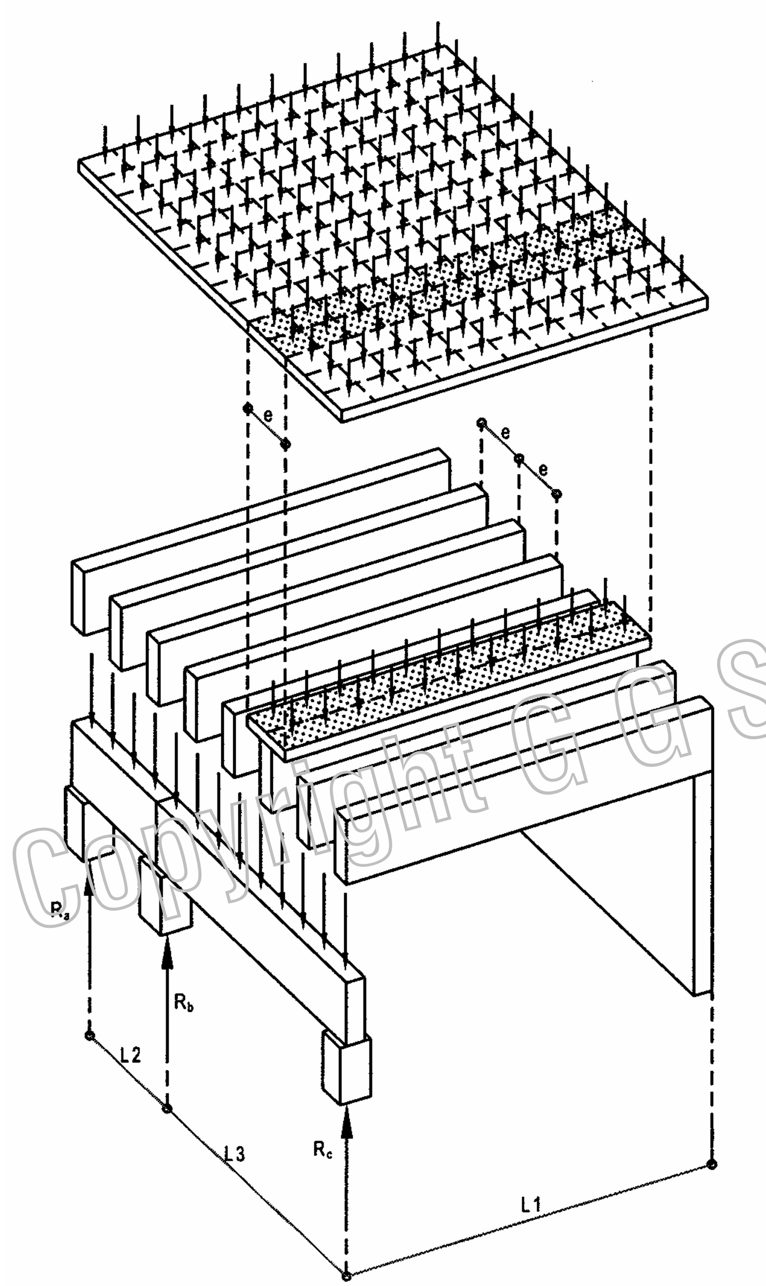
2 Three-story concrete structure

Assume

Roof DL	130 psf	
Roof LL	20 psf	
Roof Σ	150 psf	
Floor DL	150 psf	(includes columns, etc.)
Floor LL	50 psf	(Office)
Floor Σ	200 psf	

Lx = 30', Lxc = 34', Ly = 25'
Columns, 2'x2' (t =2', t/2 =1')

Column reactions at level 2		w = 150 psf)
R_a = 150 psf (30+34)/2 (25) =	150 x 800	R_a = 120,000 #
R_b = 150 (30+34)/2 (25/2+1) =	150 x 432	R_b = 64,800 #
R_c = 150 (30/2+1) (25) =	150 x 400	R_c = 60,000 #
R_d = 150 (30/2+1) (25/2+1) =	150 x 216	R_d = 32,400 #
Column reactions at level 1,	w = 150+200	w = 350 psf
R_a = 350 (800)		R_a = 280,000 #
R_b = 350 (432)		R_b = 151,200 #
R_c = 350 (400)		R_c = 140,000 #
R_d = 350 (216)		R_d = 75,600 #
Column reactions at level 0,	w = 150+200+200	w = 550 psf
R_a = 550 (800)		R_a = 440,000 #
R_b = 550 (432)		R_b = 237,600 #
R_c = 550 (400)		R_c = 220,000 #
R_d = 550 (216)		R_d = 118,800 #



Deck / joist / beam / column

Assume

Uniform load w = 80 psf

Joist spacing e = 2’

Joist span L_1 = 12’

Beam spans

L_2 = 10’

L_3 = 20’

Find load path and tributary load

Load path: plywood deck / joist / beam / columns

Tributary loads:

Uniform joist load

$w_j = w\ e$ = 80 psf x 2’ w_j = 160 plf

Beam load (assume uniform load due to narrow joist spacing)

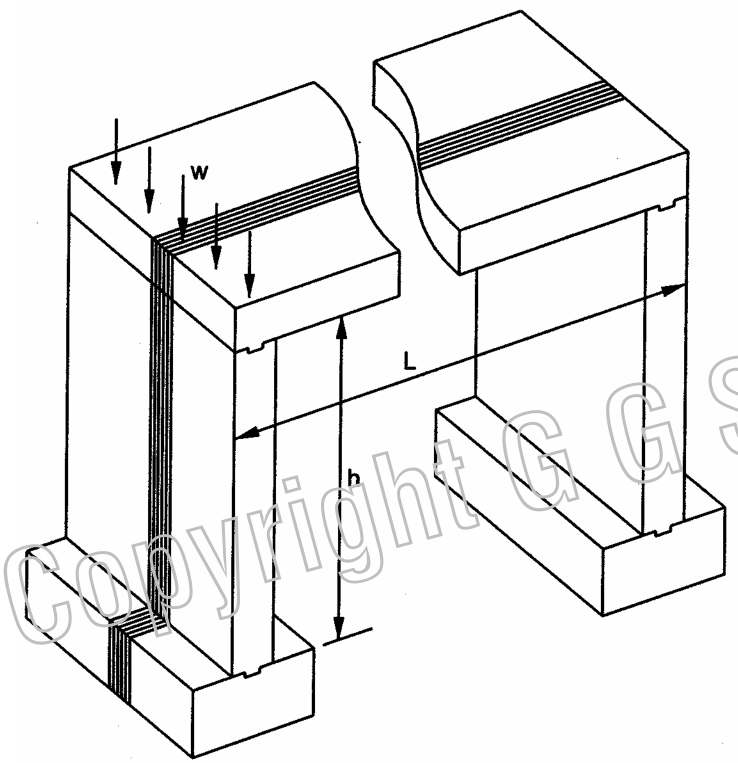
w_b = 80 psf $L_1/2$ = 80 psf x 12’ /2 w_b = 480 plf

Column reaction

$R_a = w_b\ L_2 / 2$ = 480 plf x 10 /2 R_a = 2,400 #

$R_b = w_b\ (L_2+L_3)/2$ = 480 (10+20) / 2 R_b = 7,200 #

$R_c = w_b\ L_3 / 2$ = 480 x 20 / 2 R_c = 4,800 #



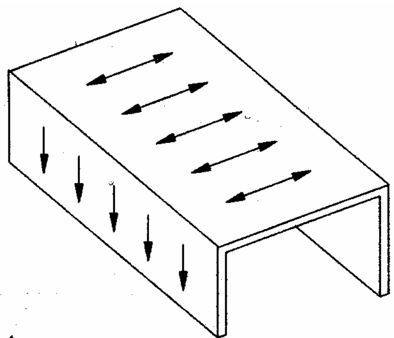
Concrete slab / wall / footing / soil

Allowable soil pressure 2000 psf (for stiff soil)
Concrete slab, 8" thick
Slab length L = 20'
Wall height h = 10'

DL =	100 psf	(150 pcf x 8"/12")
<u>LL =</u>	<u>40 psf</u>	(apartment LL)
Σ =	140 psf	

8" CMU wall, 10' high at 80 psf
(CMU = Concrete Masonry Units)
(8" nominal = 7 5/8" = 7.625" actual)
Concrete footing 2' x 1' at 150 pcf
nalyze a 1 ft wide strip (1 meter in SI units)

Slab load	
w = 140psf x 20 / 2	w = 1400 #
CMU wall load	
w = 80 psf x 10'	w = 800 #
Footing load	
w = 150 pcf x 2'x 1'	w = 300 #
Total load on soil	
P = 1400 + 800 + 300	P = 2500 #
Soil pressure	
f = P/A = 2500 # / (1'x2')	f =1250 psf
	1250 < 2000, ok



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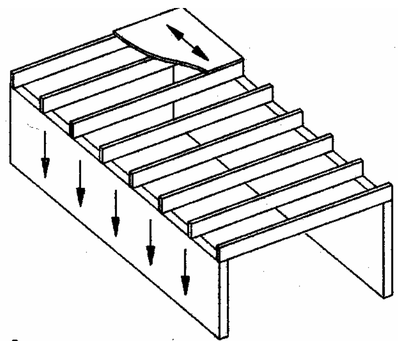

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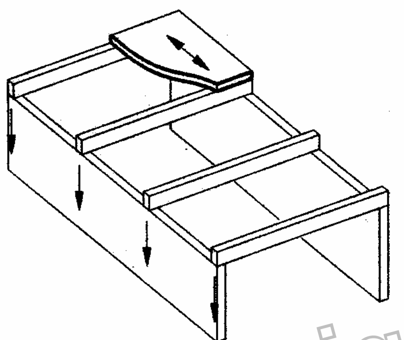

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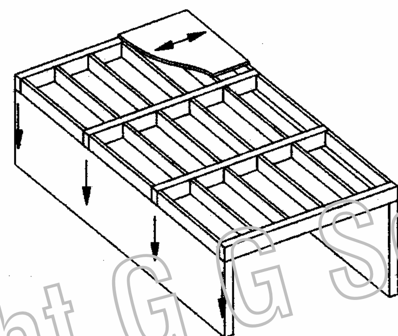

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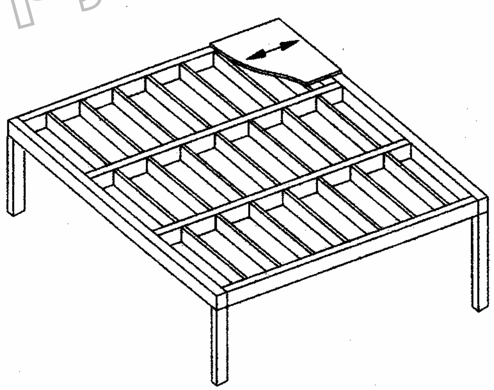

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1 **Concrete slab / wall**

Concrete slab t = 8”, span L = 20’

LL =	50 psf
DL =	120 psf
Σ =	170 psf

Slab load on wall

w = 170 psf x 20’/2 — w =1700 plf

2 **Joist roof / wall**

Plywood roof deck, 2x12 wood joists at 24”, span L = 18’

LL =	30 psf
DL =	20 psf
Σ =	50 psf

Roof load on wall (per linear foot of wall length)

w = 50 psf x 18’/2 — w = 450 plf

3 **Concrete slab / beam / wall**

Concrete slab t = 5”, span L =10’, beam span L = 30’

LL =	20 psf
DL =	70 psf (assume beam DL lumped with slab DL)
Σ =	90 psf

Beam load w = 90 psf x10’ / 1000 — w = 0.9 klf

Wall reaction R = 0.9 klf x 30’ / 2 — R = 13.5 k

4 **Concrete slab on metal deck / joist/ beam**

Spans: deck L = 8’, joist L = 20’, beam L=40’

LL =	40 psf
DL =	60 psf (assume joist and beam DL lumped with slab DL)
Σ =	100 psf

Joist load w = 100 psf x 8’ / 1000 — w = 0.8 klf

Beam point loads P = 0.8 klf x 20’ — P = 16 k

Wall reaction R = 4 P /2 = 4 x 16 k / 2 — R = 32 k

Note: wall requires pilaster to support beams

5 **Concrete slab on metal deck / joist/ beam / girder**

Spans: deck L = 5’, joist L = 20’, beam L=40’, girder L = 60’

LL = 50 psf

DL = 50 psf (assume joist/beam/girder DL lumped with slab DL)

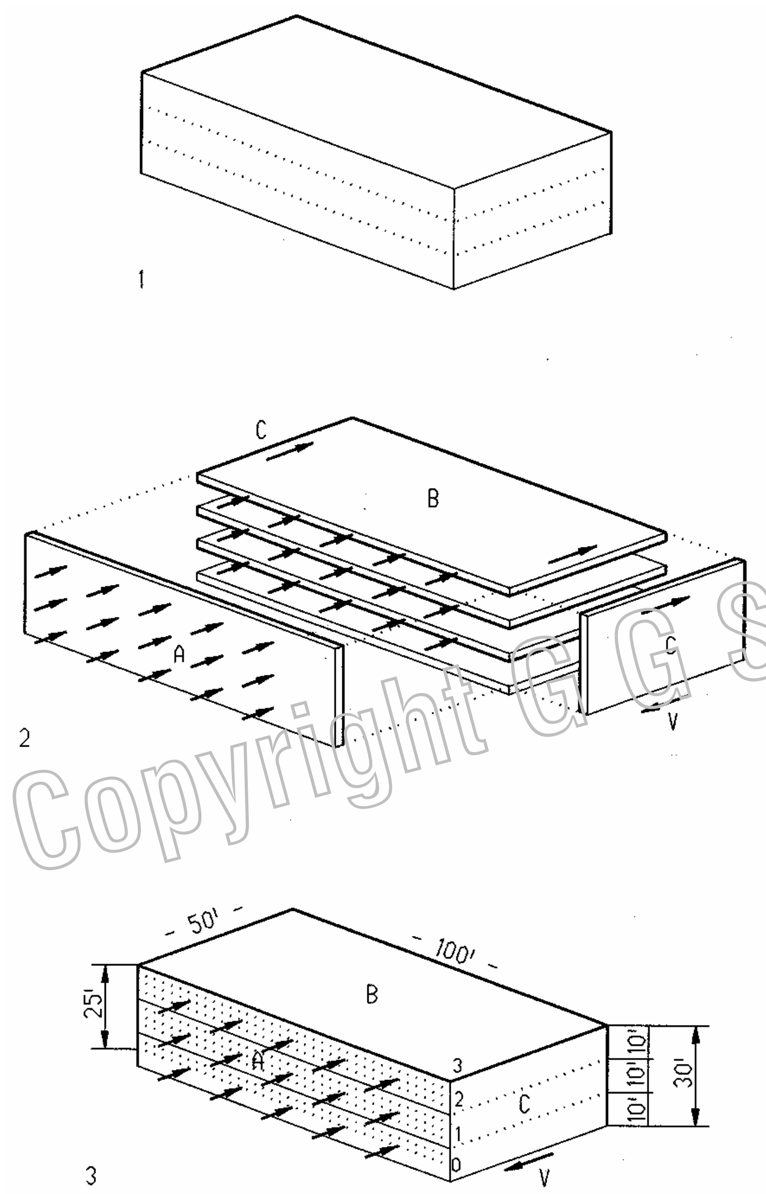
Σ =100 psf

Uniform joist load w = 100 psf x 5’/1000 — w = 0.5 klf

Beam point loads P = 0.5 klf x 20’ — P = 10 k

Girder point loads P = 7 x 10 k/2 — P = 35 k

Column reaction R = (100 psf/1000) x 40’ x 60’/4 — R = 60 k



Wind load resisted by shear wall

1 Three-story building
2 Exploded visualization
3 Dimensions

A Wind Wall
B Diaphragms
C Shear walls

Assume:
Building dimensions as shown in diagram
Wind pressure P = 20 psf

Find load path and tributary load

Load path
Wind wall > diaphragms > shear walls > footings

Note:
Floor and roof diaphragms act like beams to transfer load from wind wall to shear wall

Tributary loads

Roof diaphragm	
V3 = 20 psf x 100' x 5'/1000	V3 = 10 k
Level 2 diaphragm	
V2 = 20 psf x 100' x 10'/1000	V2 = 20 k
Level 1 diaphragm	
V1 = 20 psf x 100' x 10'/1000	V1 = 20 k

Shear walls

Level 2 shear walls	
V2 = 10 k / 2	V2 = 5 k
Level 1 shear wall	
V = (10 k + 20) k / 2	V1 = 15 k
Level 0 shear walls	
V0 = (10 k + 20 k + 20) k / 2	V0 = 25 k

Note:
- Floor and roof diaphragms resist half the load from above and below
- Floor and roof diaphragms transfer load from wind wall to shear walls
- The 2 shear walls resist each half of the diaphragm load from above

3

Basic Concepts

This chapter on basic concept introduces:

- Structural design for:
  - Strength
  - Stiffness
  - Stability
  - Synergy
- Rupture length (material properties, i.e., structural efficiency)
- Basic structure systems
  - Horizontal structures
  - Vertical / lateral structures for:
    - o Gravity load
    - o Lateral load

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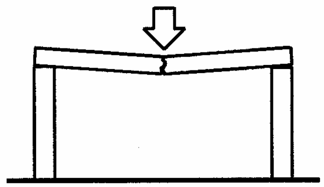

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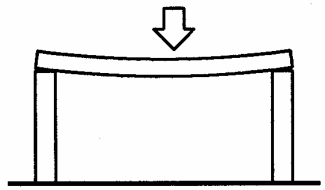

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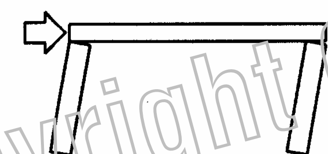

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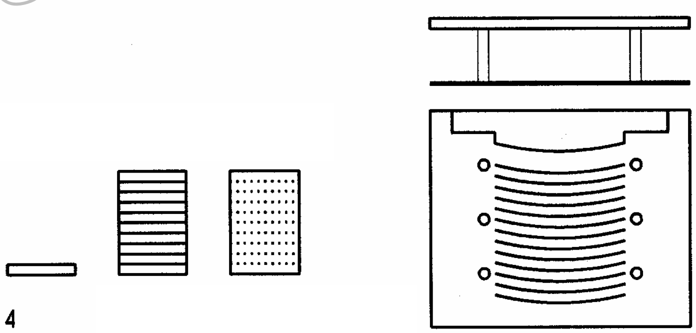


Strength, Stiffness, Stability, Synergy

Structures must be designed to satisfy three Ss and should satisfy all four Ss of structural design – as demonstrated on the following examples, illustrated at left.

1 Strength to prevent breaking

2 Stiffness to prevent excessive deformation

3 Stability to prevent collapse

4 Synergy to reinforce architectural design, described on two examples:
Pragmatic example: Beam composed of wooden boards
Philosophical example: Auditorium design

Comparing beams of wooden boards, b = 12" wide and d = 1" deep, each. Stiffness is defined by the Moment of Inertia, $I = b\,d^3 / 12$

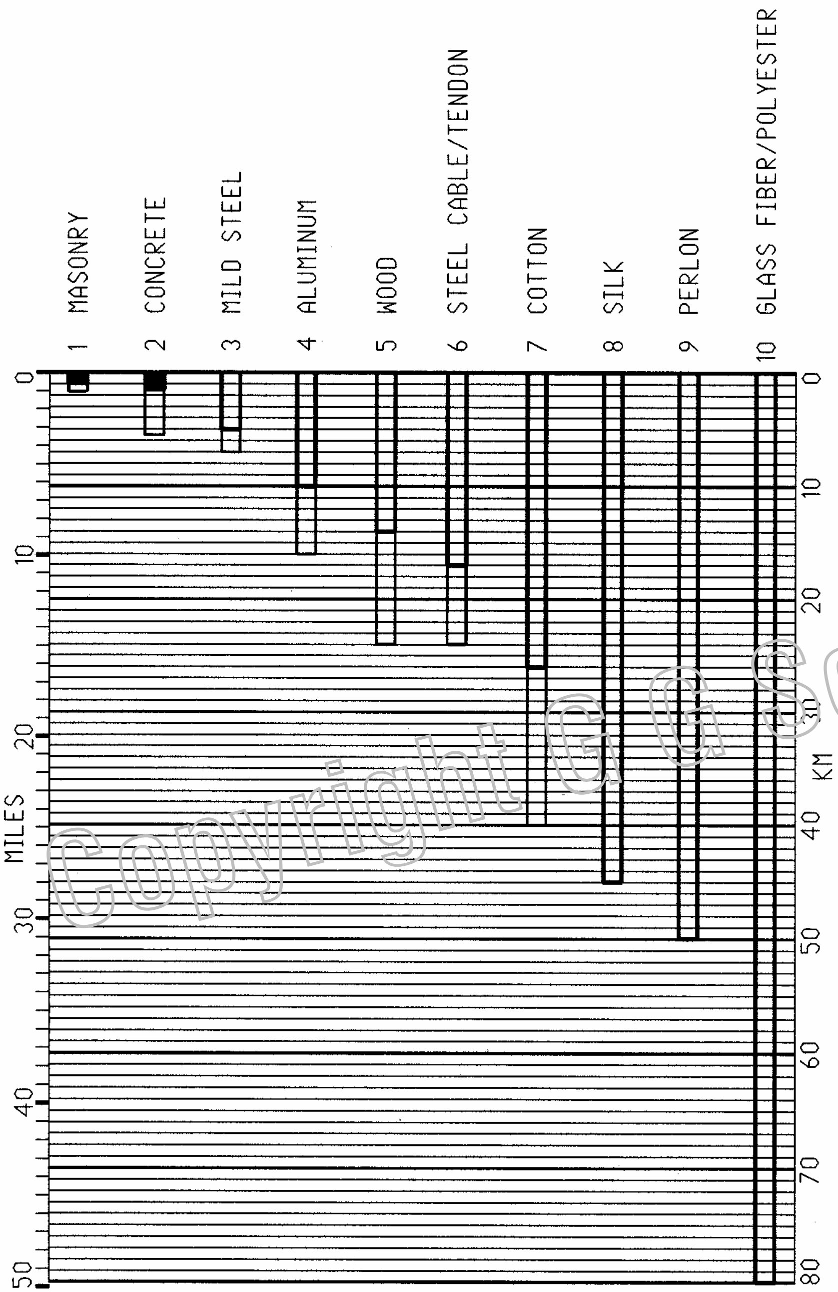
1 board, $I = 12x1^3/12$	$I = 1$
10 boards $I = 10\,(12x1^3/12)$	$I = 10$
10 boards glued $I = 12x10^3/12$	$I = 1000$

Strength is defined by the *Section* modulus, $S = I / (d/2)$

1 board, $S = 1/o.5$	$S = 2$
10 boards, $S = 10/0.5$	$S = 20$
10 boards, glued, $S = 1000/5$	$S = 200$

Note:
The same amount of material is 100 times stiffer and 10 times stronger when glued together to transfer shear and thereby engage top and bottom fibers in compression and tension (a system, greater than the sum of its parts). On a philosophical level, structures can strengthen architectural design as shown on the example of an auditorium:

- Architecturally, columns define the circulation
- Structurally, column location reduces bending in roof beams over 500% !



Rupture length

Rupture length is the maximum length a bar of constant cross section area can be suspended without rupture under its weight in tension (compression for concrete & masonry).

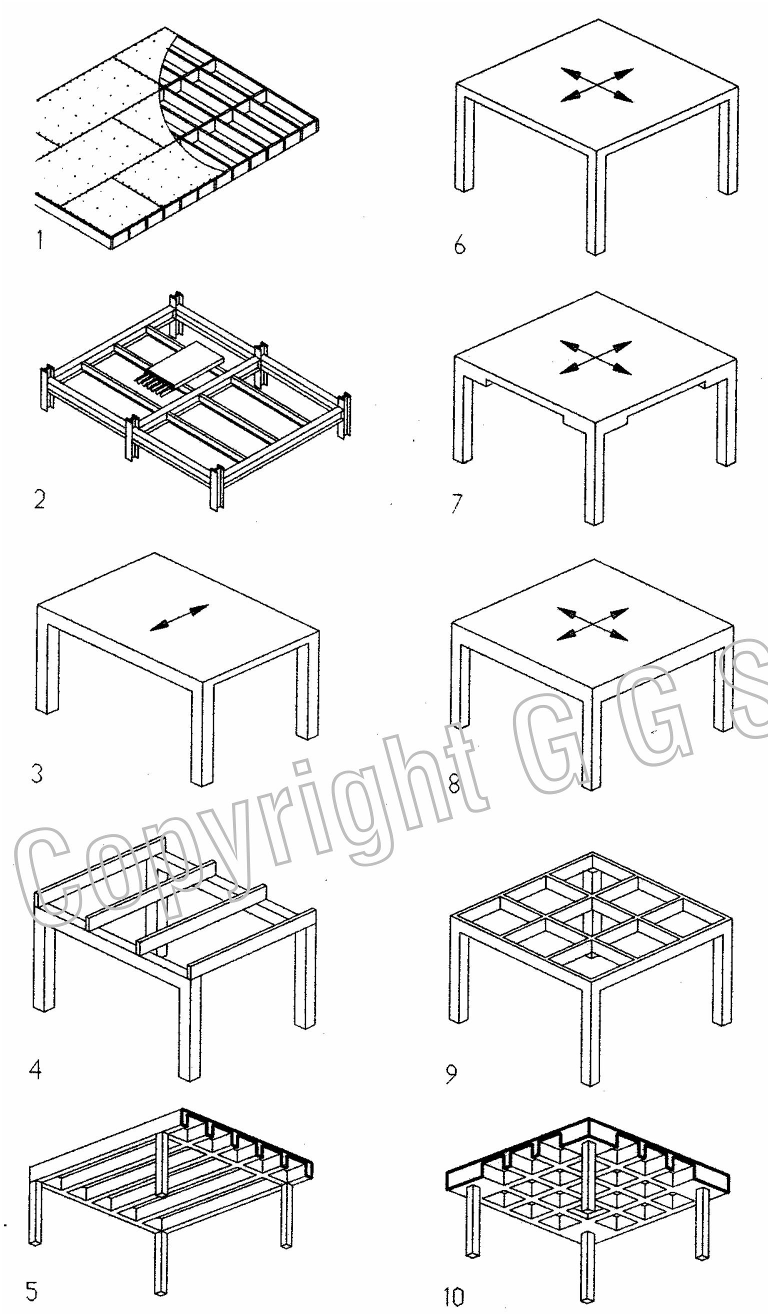
Rapture length defines material efficiency as strength / weight ratio:

$$R = F / \lambda$$

R = rupture length
F = breaking strength
λ = specific gravity (self weight)

Rupture length, is of particular importance for long-span structures. The depth of horizontal span members increases with span. Consequently the weight also increases with span. Therefore the capacity of material to span depends on both its strength and weight. This is why lightweight material, such as glass fiber fabrics are good for long-span structures. For some material, a thin line extends the rupture length to account for different material grades.

The graph data is partly based on a study of the Light weight Structures Institute, University Stuttgart, Germany



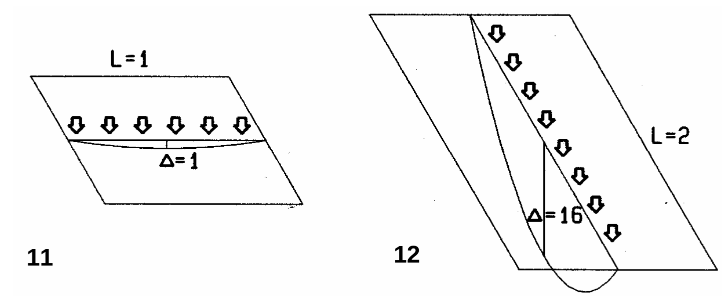
Horizontal structures

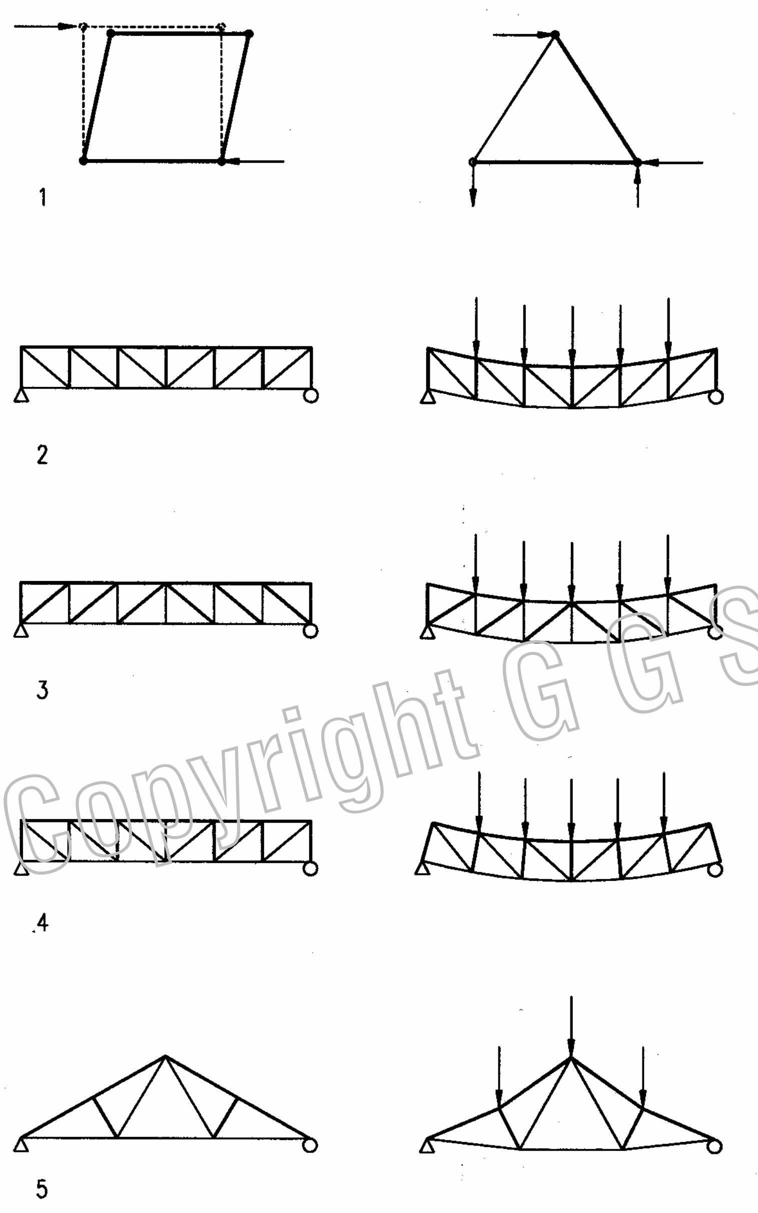
Horizontal systems come in two types: one way and two way. Two way systems are only efficient for spaces with about equal span in both directions; as described below. The diagrams here show one way systems at left and two way systems at right

1 Plywood deck on wood joists
2 Concrete slab on metal deck and steel joists
3 One way concrete slab
4 One way beams
5 One way rib slab
6 Two way concrete plate
7 Two way concrete slab on drop panels
8 Two way concrete slab on edge beams
9 Two way beams
10 Two way waffle slab
11 Deflection Δ for span length L1
12 Deflection Δ=16 due to double span L2 = 2 L1

Note:

Deflection increases with the fourth power of span. Hence for double span deflection increase 16 times. Therefore two way systems over rectangular plan are ineffective because elements that span the short way control deflection and consequently have to resist most load and elements that span the long way are very ineffective.

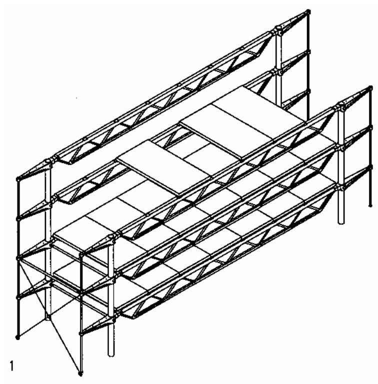




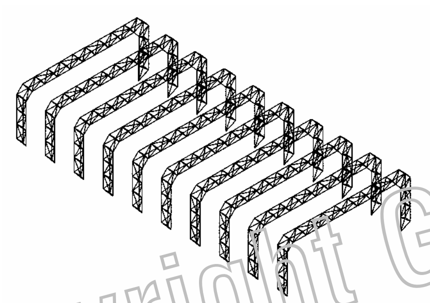
Trusses

Trusses support load much like beams, but for longer spans. As the depth and thus dead weight of beams increases with span they become increasingly inefficient, requiring most capacity to support their own weight rather than imposed live load. Trusses replace bulk by triangulation to reduce dead weight.

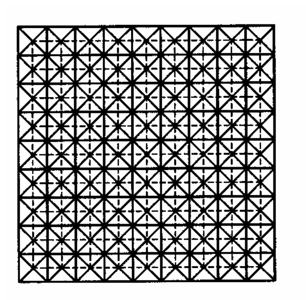
1 Unstable square panel deforms under load.
Only triangles are intrinsically stable polygons
2 Truss of triangular panels with inward sloping diagonal bars that elongate in tension under load (preferred configuration)
3 Outward sloping diagonal bars compress (disadvantage)
4 Top chords shorten in compression
Bottom chords elongate in tension under gravity load
5 Gable truss with top compression and bottom tension

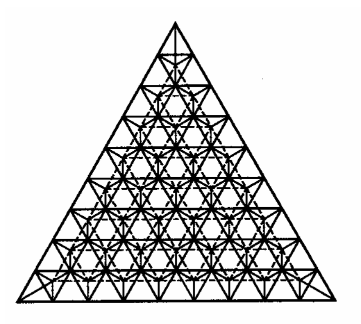


Warren trusses
Pompidou Center, Paris by Piano and Rogers



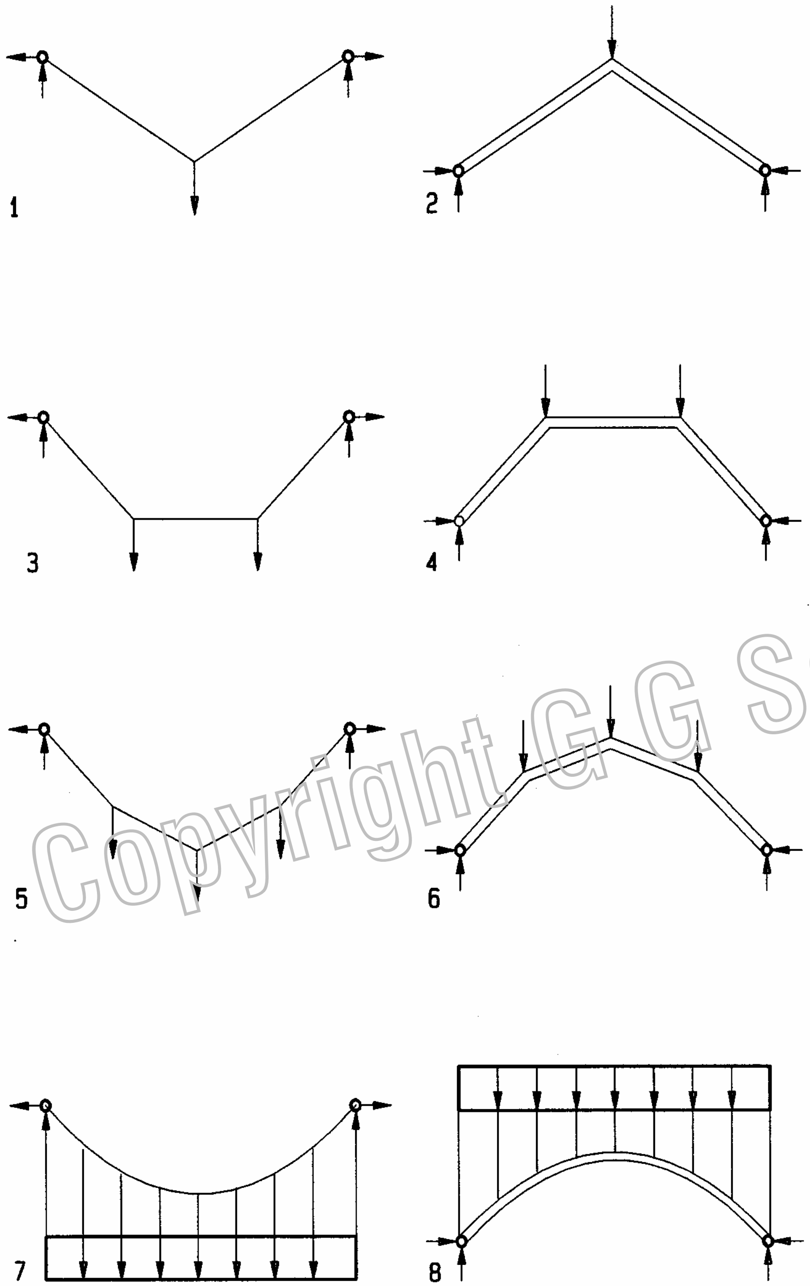
Prismatic trusses
IBM Sport Center by Michael Hopkins
(Prismatic trusses of triangular cross section provide rotational resistance)





Space trusses
Square and triangular plan

Note:
Two way space trusses are most effective if the spans in the principle directions are about equal, as described for two-way slabs above. The base modules of trusses should be compatible with plan configuration (square, triangular, etc.)



Funicular structures

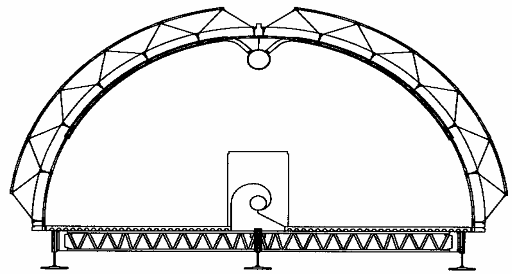
The funicular concept can be best described and visualized with cables or chains, suspended from two points, that adjust their form for any load in tension. But funicular structures may also be compressed like arches. Yet, although funicular tension structures adjust their form for pure tension under any load, funicular compression structures may be subject to bending in addition to compression since their form is rigid and not adaptable. The funicular line for tension and compression are inversely identical; the form of a cable becomes the form of an arch upside-down. Thus funicular forms may be found on tensile elements.

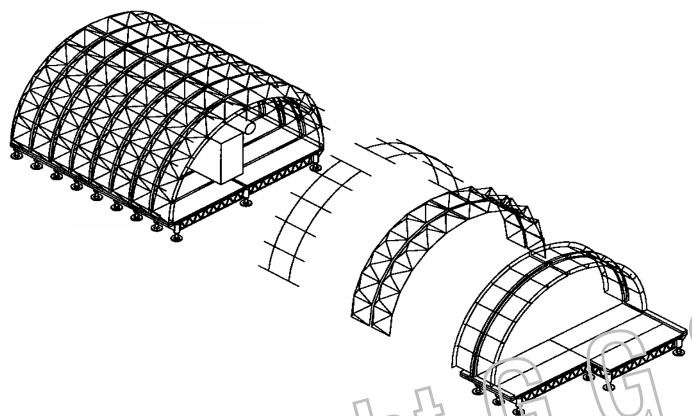
1 Funicular tension triangle under single load
2 Funicular compression triangle under single load
3 Funicular tension trapezoid under twin loads
4 Funicular compression trapezoid under twin loads
5 Funicular tension polygon under point loads
6 Funicular compression polygon under point load
7 Funicular tension parabola under uniform load
8 Funicular compression parabola under uniform load

Vault

IBM traveling exhibit by Renzo Piano

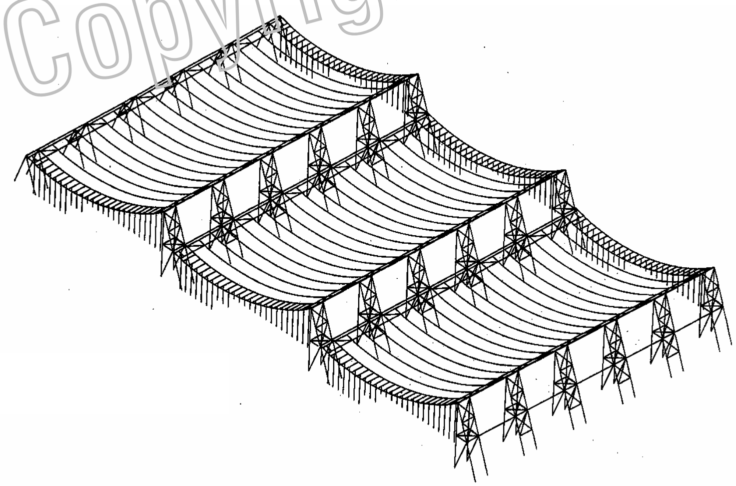
A series of trussed arches in linear extrusion form a vault space. The trussed arches consist of wood bars with metal connectors for quick assembly and disassembly as required for the traveling exhibit. Plastic panels form the enclosing skin. The trussed arches provide depth and rigidity to accommodate various load conditions

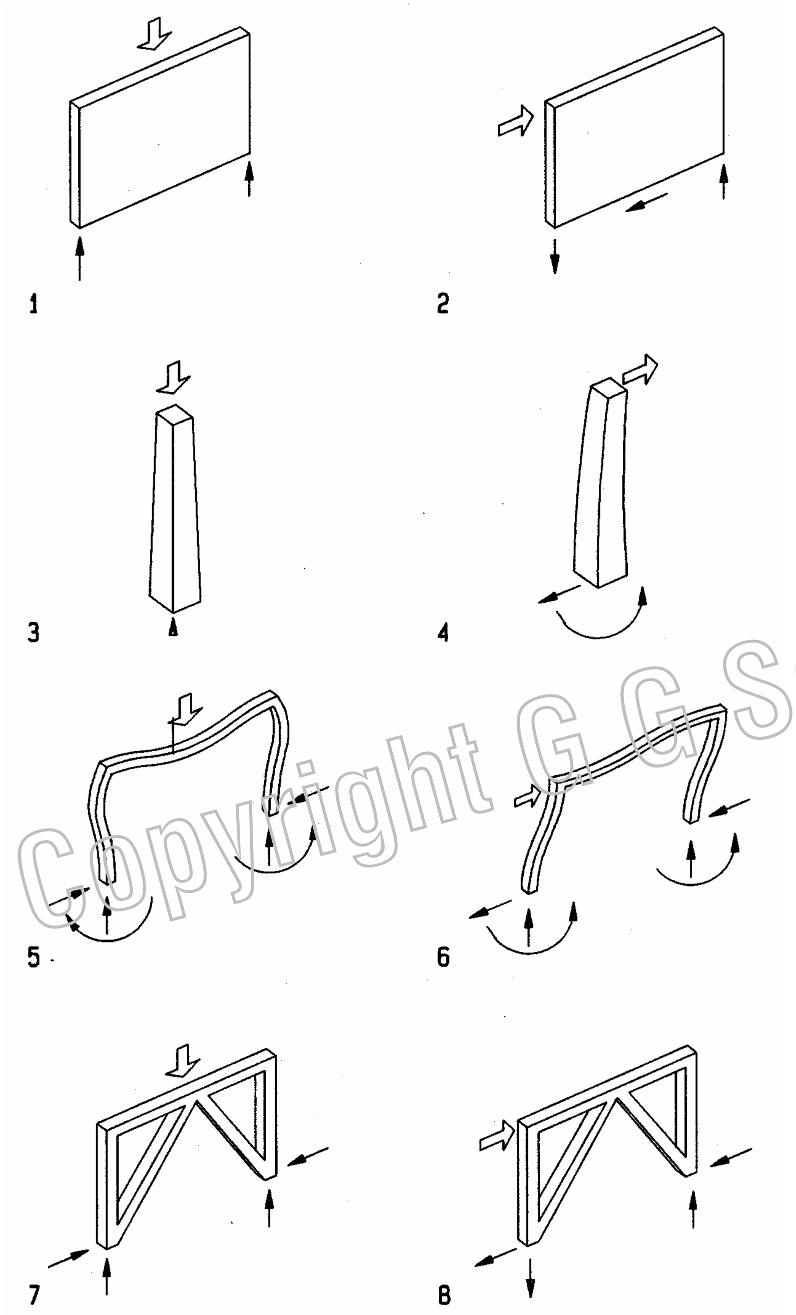




Suspension roof

Exhibit hall Hanover by Thomas Herzog



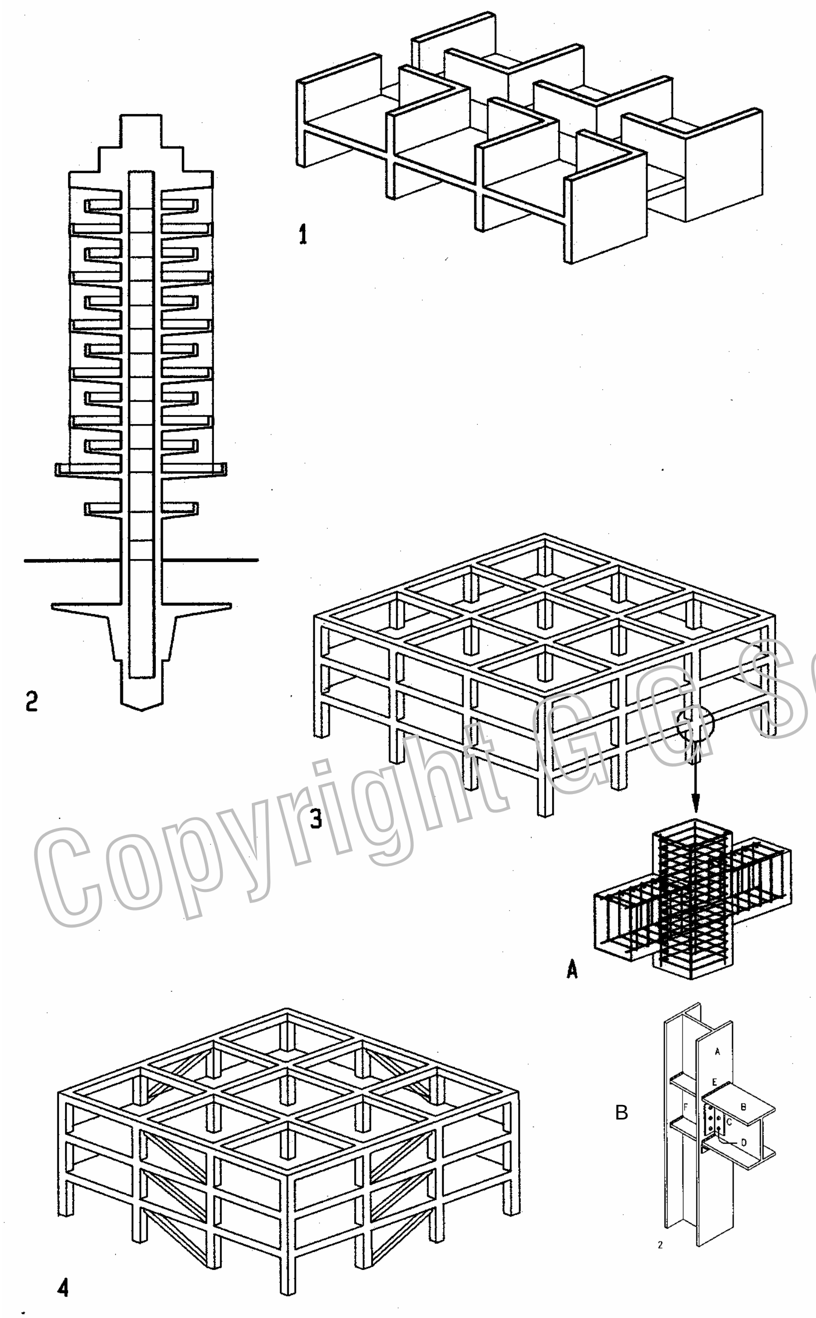


Vertical structures

Vertical elements

Vertical elements transfer load from roof to foundation, carrying gravity and/or lateral load. Although elements may resist only gravity or only lateral load, most are designed to resist both. Shear walls designed for both gravity and lateral load may use gravity dead load to resist overturning which is most important for short walls. Four basic elements are used individually or in combination to resist gravity and lateral loads

1 Wall under gravity load
2 Wall under lateral load (shear wall)
3 Cantilever under gravity load
4 Cantilever under lateral load
5 Moment frame under gravity load
6 Moment frame under lateral load
7 Braced frame under gravity load
9 Braced frame under lateral load



Vertical systems

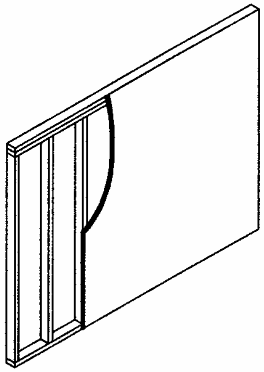
Vertical systems transfer the load of horizontal systems from roof to foundation, carrying gravity and/or lateral load. Although they may resist gravity or lateral load only, most resist both, gravity load in compression, lateral load in shear. Walls are usually designed to define spaces and provide support, an appropriate solution for apartment and hotel buildings. The four systems are:

1 Shear walls (apartments / hotels)
2 Cantilever (Johnson Wax tower by F L Wright)
3 Moment frame
4 Braced frame

A Concrete moment resistant joint
Column re-bars penetrate beam and beam re-bars penetrate column)

B Steel moment resistant joint
(stiffener plates between column flanges resist beam flange stress)

Vertical / lateral element selection criteria		
Element	Advantages	Challenges
Shear wall Architectural criteria Structural criteria	Good for apartments/hotels Very stiff, good for wind resistance	Inflexible for future changes Stiffness increases seismic forces
Cantilever Architectural criteria Structural criteria	Flexible planning Around cantilever Ductile, much like a tree trunk	Must remain in future changes Too flexible for tall structures
Moment frame Architectural criteria Structural criteria	Most flexible, good for office buildings Ductile, absorbs seismic force	Expensive, drift may cause problems Tall structures need additional stiffening
Braced frame Architectural criteria Structural criteria	More flexible then Shear walls Very stiff, good for Wind resistance	Less flexible than moment frame Stiffness increases seismic forces



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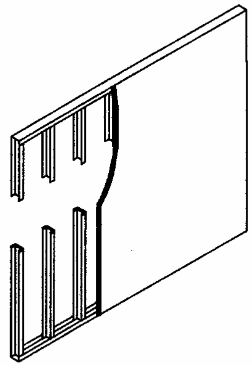

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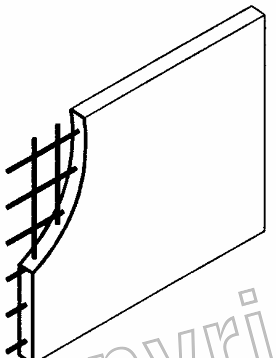

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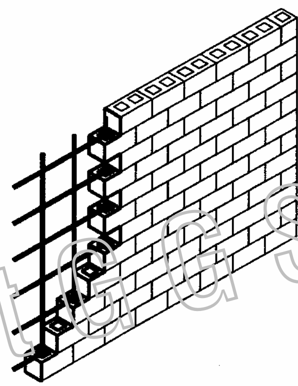

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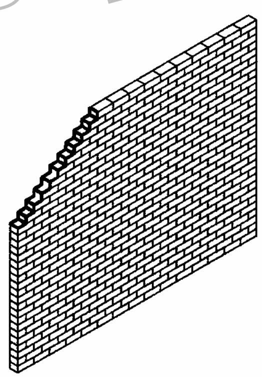

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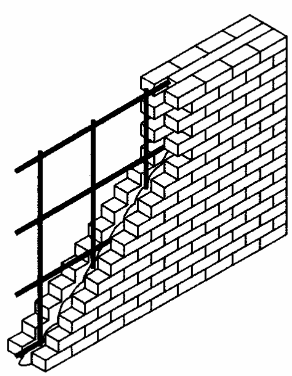

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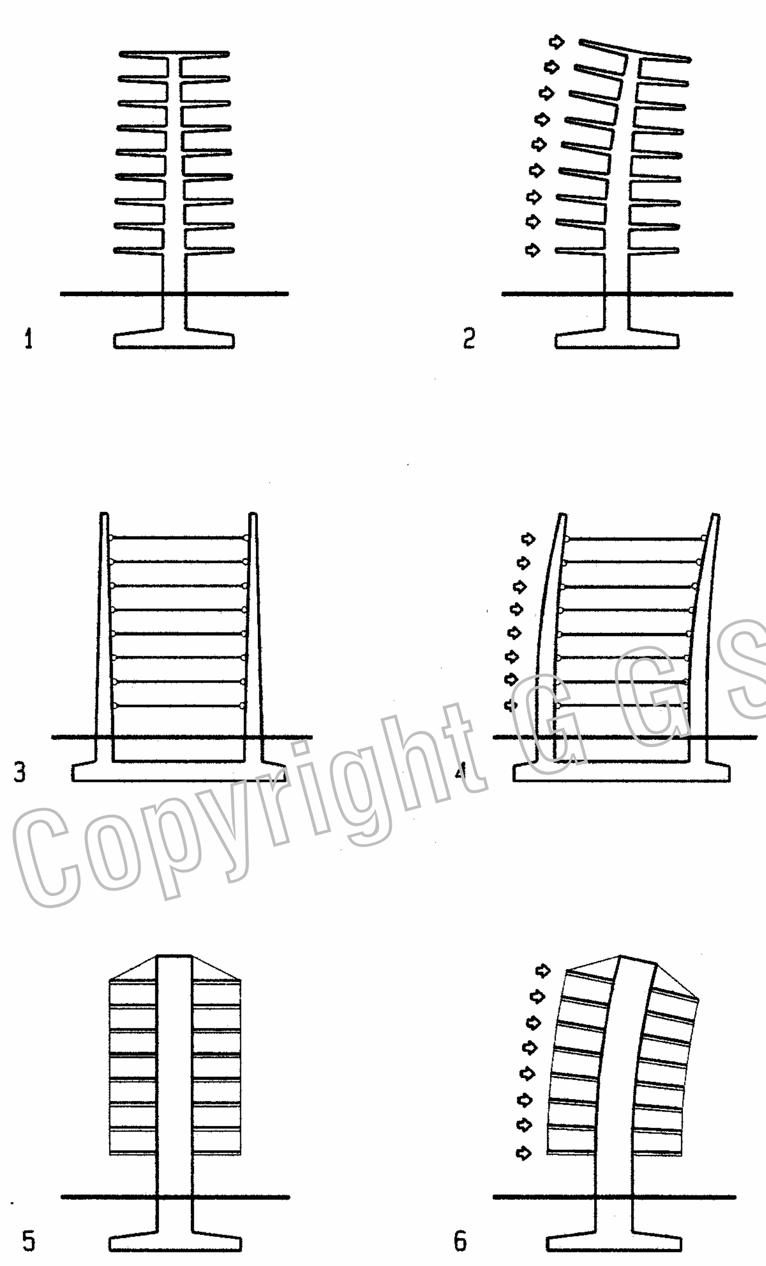
Shear walls

As the name implies, shear walls resist lateral load in shear. Shear walls may be of wood, concrete or masonry. In the US the most common material for low-rise apartments is light-weight wood framing with plywood or particle board sheathing. Framing studs, spaced 16 or 24 inches, support gravity load and sheathing resists lateral shear. In seismic areas concrete and masonry shear walls must be reinforced with steel bars to resist lateral shear.

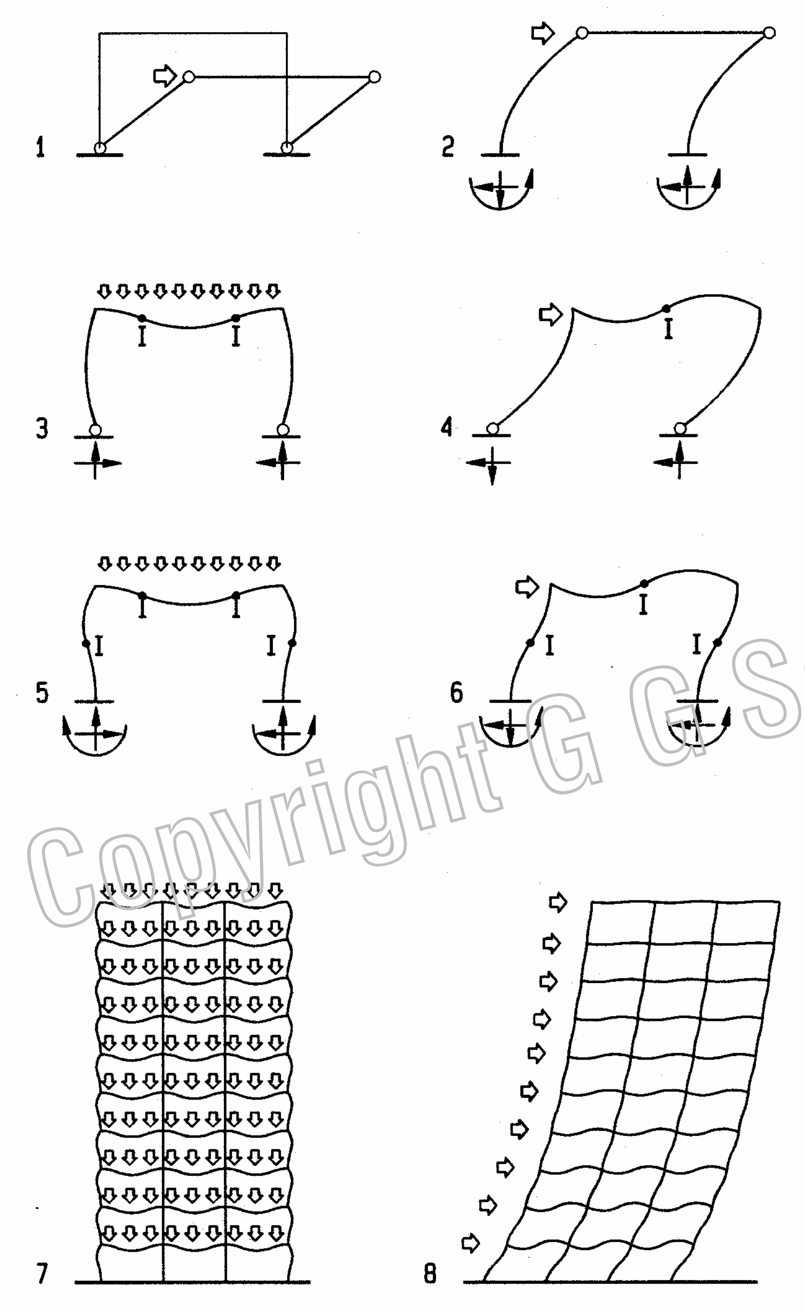
1 Wood shear wall with plywood sheathing
2 Light gauge steel shear wall with plywood sheathing
3 Concrete shear wall with steel reinforcing
4 CMU shear wall with steel reinforcing
5 Un-reinforced brick masonry (not allowed in seismic areas)
8 Two-wythe brick shear wall with steel reinforcing



Cantilevers

Cantilevers resist lateral load primarily in bending. They may consist of single towers or multiple towers. Single towers act much like trees and require large footings like tree roots to resist overturning. Bending in cantilevers increases from top down, justifying tapered form in response.

1 Single tower cantilever
2 Single tower cantilever under lateral load
3 Twin tower cantilever
4 Twin tower cantilever under lateral load
5 Suspended tower with single cantilever
6 Suspended tower under lateral load



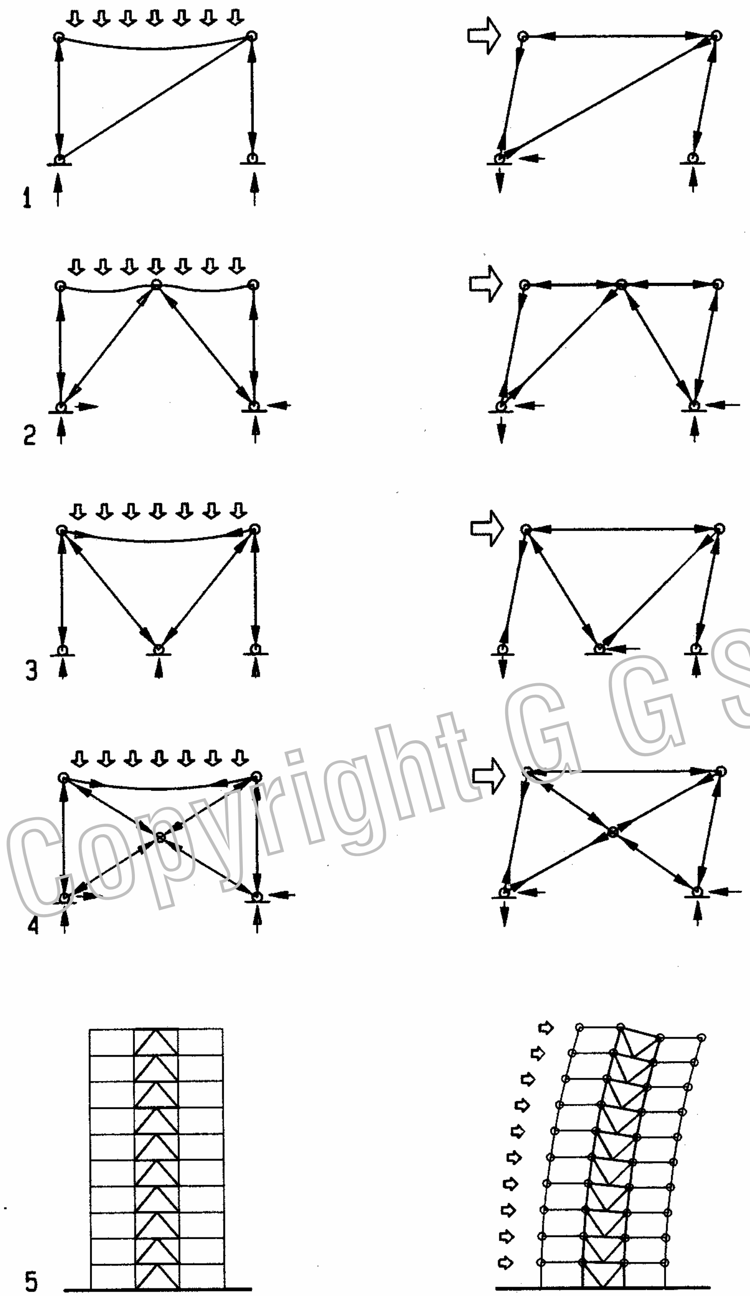
Moment frames

Moment frames resist gravity and lateral load in bending and compression. They are derived from post-and beam portals with moment resisting beam to column connections (for convenience referred to as moment frames and moment joints). The effect of moment joints is that load applied to the beam will rotate its ends and in turn rotate the attached columns. Equally, load applied to columns will rotate their ends and in turn rotate the beam. This mutual interaction makes moment frames effective to resist lateral load with ductility. Ductility is the capacity to deform without breaking, a good property to resist earthquakes, resulting in smaller seismic forces than in shear walls and braced frames. However, in areas with prevailing wind load, the greater stiffness of shear walls and braced frames is an advantage. The effect of moment joints to resist loads is visualized through amplified deformation as follows:

1 Portal with pin joints collapses under lateral load
2 Portal with moment joints at base under lateral load
3 Portal with moment beam/column joints under gravity load
4 Portal with moment beam/column joints under lateral load
5 Portal with all moment joints under gravity load
6 Portal with all moment joints under lateral load
7 High-rise moment frame under gravity load
8 Moment frame building under lateral load

I Inflection points (zero bending between negative and positive bending

Note:
Deformations reverse under reversed load



Braced frames

Braced frames resist gravity load in bending and axial compression, and lateral load in axial compression and tension by triangulation, much like trusses. The triangulation results in greater stiffness, an advantage to resist wind load, but increases seismic forces, a disadvantage to resist earthquakes. Triangulation may take several configurations, single diagonals, A-bracing, V-bracing, X-bracing, etc., considering both architectural and structural criteria. For example, location of doors may be effected by bracing and impossible with X-bracing. Structurally, a single diagonal brace is the longest, which increases buckling tendency under compression. Also the number of costly joints varies: two for single diagonals, three for A- and V-braces, and five joints for X-braces. The effect of bracing to resist load is visualized through amplified deformation as follows:

1 Single diagonal portal under gravity and lateral loads
2 A-braced portal under gravity and lateral load
3 V-braced portal under gravity and lateral load
4 X-braced portal under gravity and lateral load
5 Braced frame building without and with lateral load

Note:
Deformations and forces reverse under reversed load

Part II

Mechanics

Mechanics, as defined for the study of structures, is the behavior of physical systems under the action of forces; this includes both statics and dynamics.

Dynamics is the branch of mechanics that deals with the motion of a system of material particles under the influence of forces. Dynamic equilibrium, also known as kinetic equilibrium, is the condition of a mechanical system when the kinetic reaction of all forces acting on it is in dynamic equilibrium.

Statics is the branch of mechanics that deals with forces and force systems that act on bodies in equilibrium as described in the following.

4

Statics

Statics is the branch of mechanics that deals with forces and force systems that act on bodies in equilibrium. Since buildings are typically designed to be at rest (in equilibrium), the subject of this book is primarily focused on statics. Even though loads like earthquakes are dynamic they are usually treated as equivalent static forces.

Force and Moment

Force is an action on a body that tends to:

- change the shape of an object or
- move an object or
- change the motion of an object

US units: # (pound), k (kip)
SI units: N (Newton), kN (kilo Newton)

Moment is a force acting about a point at a distance called *lever arm*
M =P L (Force x lever arm)
The lever arm is measured normal (perpendicular) to the force.
Moments tend to:

- rotate an object or
- bend an object (bending moment)

US units: #' (pound-feet), k' (kip-feet), #" (pound-inch), k" (kip-inch)
SI units: N-m (Newton-meter), kN-m (kilo-Newton-meter)

1 Gravity force (compresses the pyramid)
2 Pulling force (moves the boulder)
3 Moment = force times lever arm (M = P L)

A Point about which the force rotates
L Lever arm
M Moment
P Force

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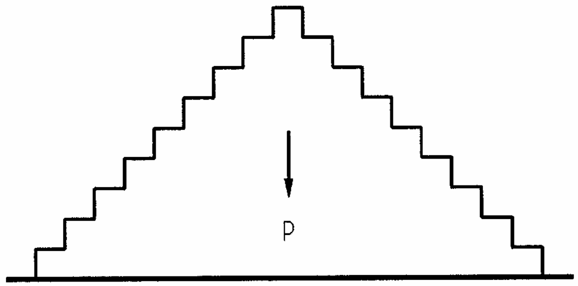

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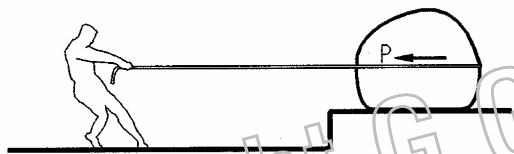

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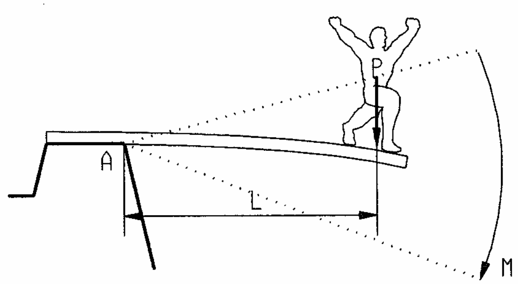

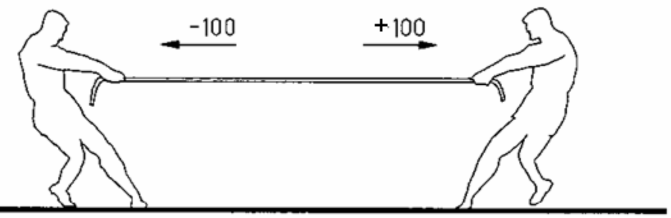

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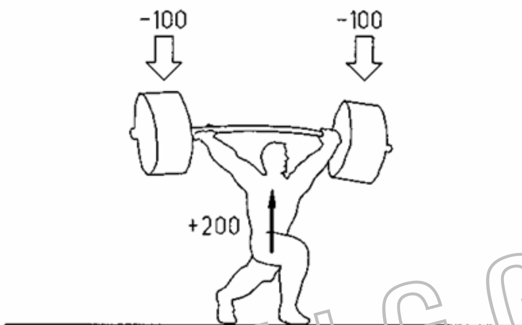

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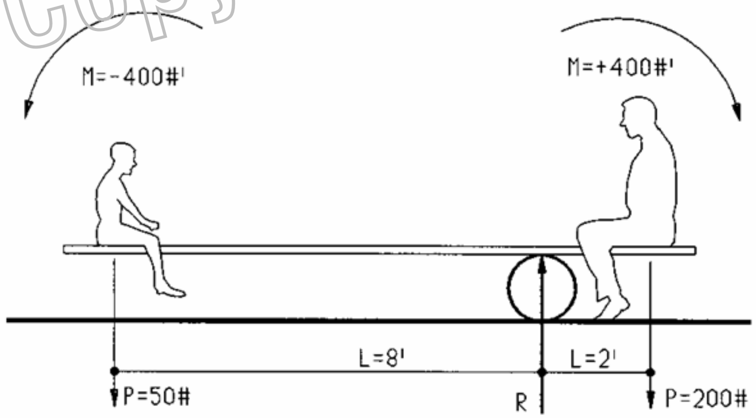

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Static Equilibrium

For any body to be in static equilibrium, all forces and moments acting on it must be in equilibrium, i.e. their sum must equal zero. This powerful concept is used for static analysis and defined by the following three equations of statics:

$\Sigma H = 0$ (all horizontal forces must equal zero)
$\Sigma V = 0$ (all vertical forces must equal zero)
$\Sigma M = 0$ (all moments must equal zero)

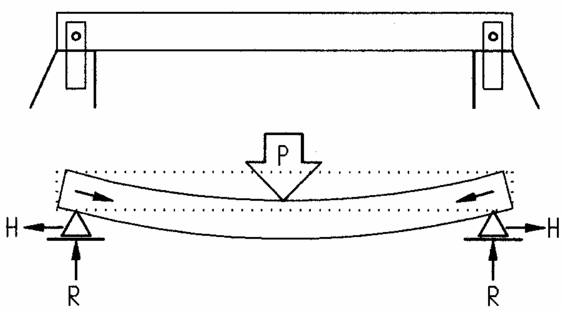
The equilibrium equations are illustrated as follows:

1 Horizontal equilibrium: pulling left and right with equal forces, mathematically defined as
$\Sigma H = 0 = +100 - 100 = 0$

2 Vertical equilibrium: pushing up with a force equal to a weight, mathematically defined as:
$\Sigma V = 0 = -2 \times 100 + 200 = 0$

3 Moment equilibrium: balancing both sides of a balance board, mathematically defined as:
$\Sigma M = 0 = -50\#(8') + 200\#(2') = -400 + 400 = 0$

Much of this book is based on the three equilibrium equations.



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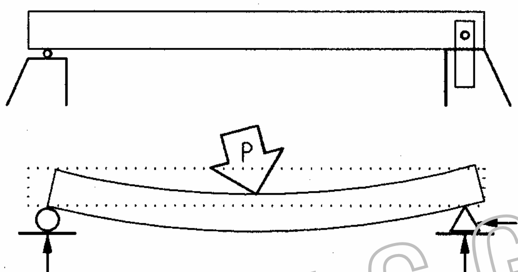

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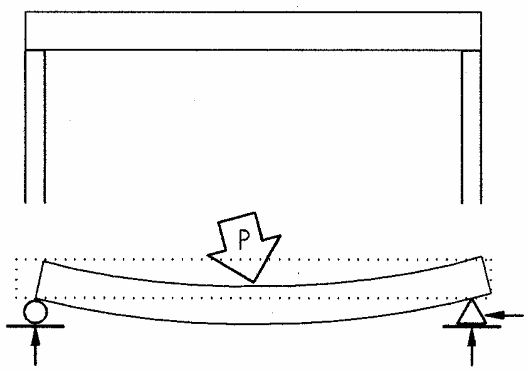

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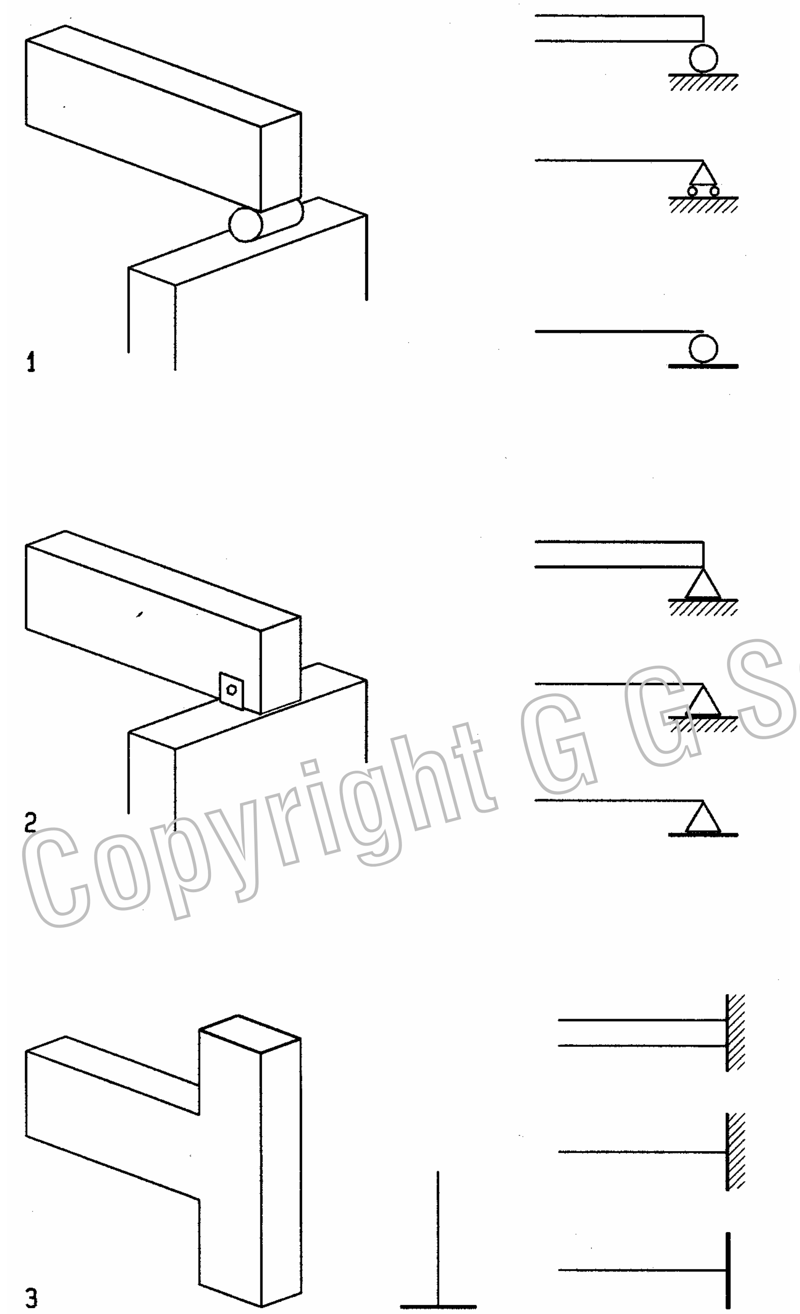
Supports

For convenience, support types are described for beams, but apply to other horizontal elements, like trusses, as well. The type of support affects analysis and design, as well as performance. Given the three equations of statics defined above, ΣH=0, ΣV=0, and ΣM=0, beams with three unknown reactions are considered *determinate* (as described below) and can be analyzed by the three static equations. Beams with more than three unknown reactions are considered *indeterminate* and cannot be analyzed by the three static equations alone. A beam with two pin supports (1 has four unknown reactions, one horizontal and one vertical reaction at each support. Under load, in addition to bending, this beam would deform like a suspended cable in tension, making the analysis more complex and not possible with static equations.

By contrast, a beam with one pin and one roller support (2) has only three unknown reactions, one horizontal and two vertical. In bridge structures such supports are quite common. To simplify analysis, in building structures this type of support may be assumed, since supporting walls or columns usually are flexible enough to simulate the same behavior as one pin and one roller support. The diagrams at left show for each support on top the physical conditions and below the symbolic abstraction.

1 Beam with fixed supports at both ends subject to bending and tension
2 Simple beam with one pin and one roller support subject to bending only
3 Beam with flexible supports, behaves like a simple beam

Simple beams, supported by one pin and one roller, are very common and easy to analyze. Designations of roller- and pin supports are used to describe the structural behavior assumed for analysis, but do not always reflect the actual physical support. For example, a pin support is not an actual pin but a support that resists horizontal and vertical movement but allows rotation. Roller supports may consist of *Teflon* or similar material of low friction that allows horizontal movement like a roller.

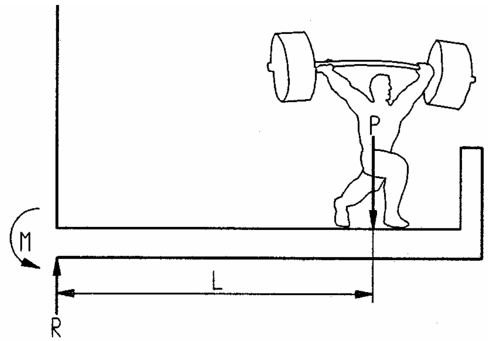


Support symbols

The diagrams show common types of support at left and related symbols at right. In addition to the pin and roller support described above, they also include fixed-end support (as used in steel and concrete moment frames, for example).

Support types				
		Degrees of freedom		
	Support type	Horizontal movement	Vertical movement	Rotation
1	Roller	Free	Fixed	Free
2	Pin	Fixed	Fixed	Free
3	Rigid	Fixed	Fixed	Fixed

1



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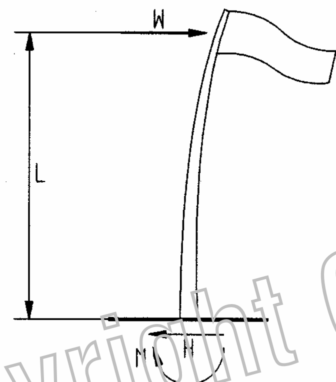

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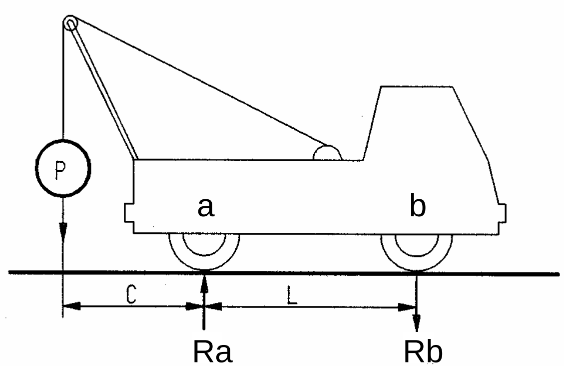


Reactions

Support reactions for asymmetrical loads and/or supports are computed using the equations of statics, $\Sigma H=0$, $\Sigma V=0$, and $\Sigma M=0$. The following examples illustrate the use of the three equations to find reactions.

1 **Weight lifter on balcony**

Assume:
P = 400#, L = 6'

$\Sigma V = 0 \uparrow +$
$R - P = 0$
$R = P$ — R = 400 #
$\Sigma M = 0, \circlearrowright +$
$P L - M = 0$
$M = P L = 400 \times 6$ — M = 2,400 #'

2. **Flag pole**

Assume:
H = 80# (wind load on flag)
L = 20'

$\Sigma H = 0 \rightarrow +$
$W - H = 0$
$H = W$ — H = 80 #
$\Sigma M = 0 \circlearrowright +$
$W L + M = 0$
$M = - W L = - 80 \times 20$ — M = -1,600 #'

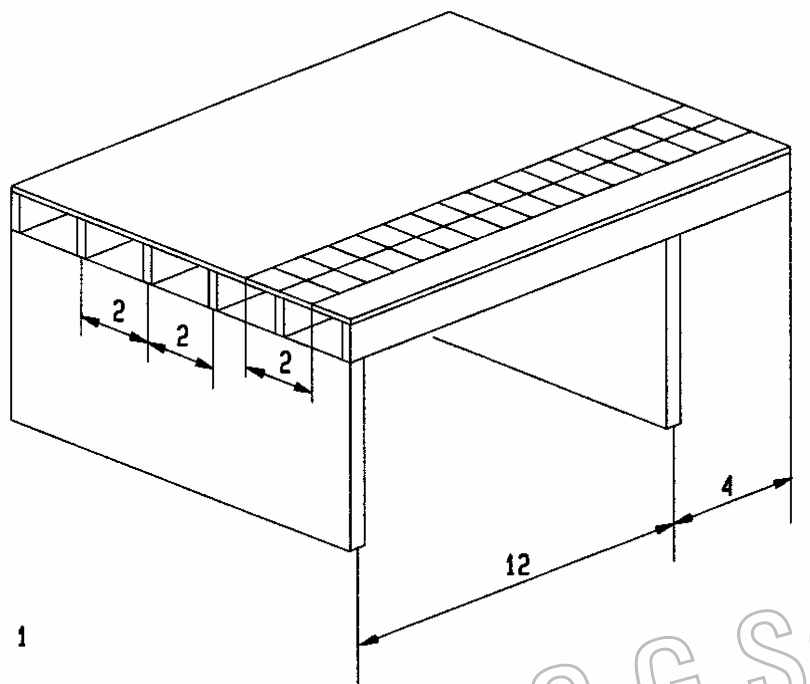
Note:
The negative moment implies, the positive moment arrow must be reversed

3 **Tow truck**

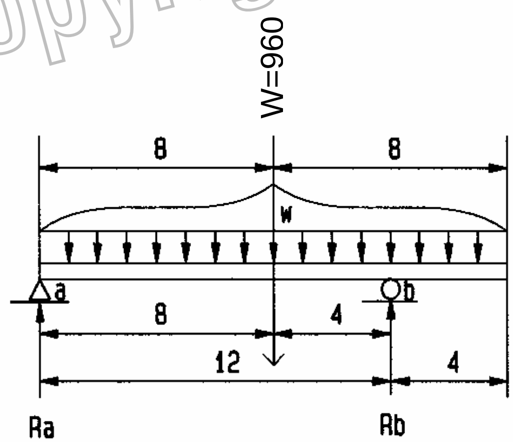
Assume:
P = 2k, C = 7', L = 10'

$\Sigma M_a = 0 \circlearrowright +$
$R_b L - P C = 0$
$R_b = P C / L = 2 \times 7 / 10$ — $R_b = 1.4k$
$\Sigma M_b = 0 \circlearrowright +$
$R_a L - P (C+L) = 0$
$R_a = P (C+L) / L = 2 (7+10) / 10$ — $R_a = +3.4k$
Check $\Sigma V = 0$
$\Sigma V = 0 = +3.4 - 1.4 - 2$ — $\Sigma V = 0$

Note:
The lever arm is always perpendicular to load
R_b pointing downward is provided by the truck weight



1



2

Beam reactions

To find reactions for asymmetrical beams:

- Draw an abstract beam diagram to illustrate computations
- Use $\Sigma M = 0$ at one support to find reaction at other support
- Verify results for vertical equilibrium

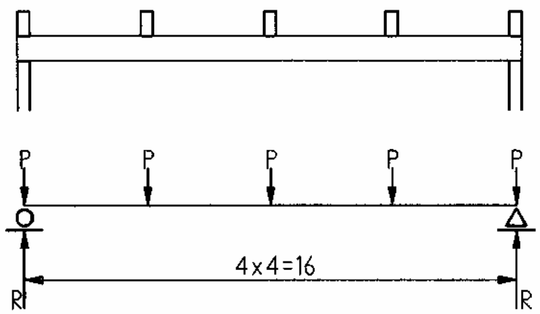
1 Floor framing
2 Abstract beam diagram

Assume:
DL = 10 psf
LL = 20 psf
Σ = 30 psf

Uniform beam load:
w = 30 psf x 2’ — w = 60 plf
For convenience, substitute total beam load W for uniform load w at its centroid
Total beam load
W = w L = 60 (12+4) — W = 960 #

Support reactions:
$\Sigma M_b = 0$ ↻+
$12 R_a - 4 W = 0$
$R_a = 4 \times 960 / 12$ — $R_a = 320$ #
$\Sigma M_a = 0$ ↻+
$8 W - 12 R_b = 0$
$12 R_b = 8 \times 960$
$R_b = 8 \times 960 / 12$ — $R_b = 640$ #
Check $\Sigma V = 0$ ↑+
$R_a + R_b - W = 320 + 640 - 960 = 0$ — $\Sigma V = 0$

Alternate method (use uniform load directly)
Support reactions:
$\Sigma M_b = 0$ ↻+
$12 R_a - 4 \times 60$ plf x 16’ = 0
$R_a = 4 \times 60 \times 16 / 12$ — $R_a = 320$ #
$\Sigma M_a = 0$ ↻+
$8 \times 60 \times 16 - 12 R_b = 0$
$12 R_b = 8 \times 60 \times 16$
$R_b = 8 \times 60 \times 16 / 12$ — $R_b = 640$ #
Check $\Sigma V = 0$ ↑+
$R_a + R_b - W = 320 + 640 - 960 = 0$ — $\Sigma V = 0$

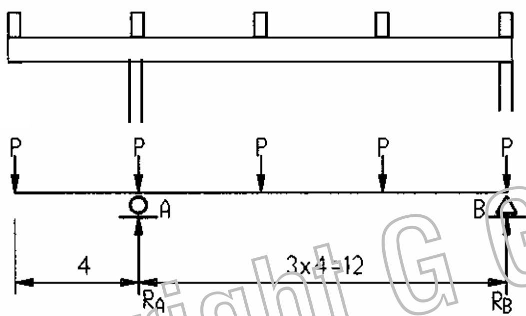


1

1 Simple beam with point loads

Assume: $P = 1.2k$

$R = 5\,P / 2 = 5 \times 1.2 / 2$ $\qquad R = 3\ k$



2

2 Beam with overhang and point loads

Assume: $P = 2k$

$\Sigma M_b = 0 \circlearrowright +$

$12\,R_a - 2x16 - 2x12 - 2x8 - 2x4 = 0$

$R_a = (32 + 24 + 16 + 8) / 12$ $\qquad R_a = 6.67\ k$

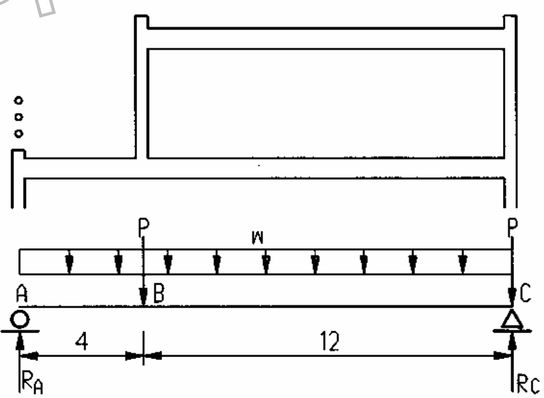
$\Sigma M_a = 0 \circlearrowright +$

$-12\,R_b - 2x4 + 2x4 + 2x8 + 2x12 = 0$

$R_b = (2 \times 8 + 2 \times 12) / 12$ $\qquad R_b = 3.33\ k$

Check $\Sigma V = 0 \uparrow +$

$6.67 + 3.33 - 5 \times 2$ $\qquad \Sigma V = 0$



3

3 Beam with uniform load and point load (wall)

Assume: $w = 100$ plf, $P = 800\#$

$\Sigma M_c = 0 \circlearrowright +$

$16\,R_a - 100x16x8 - 800x12 = 0$

$Ra = (100x16x8 + 800x12) / 16$ $\qquad R_a = 1{,}400\ \#$

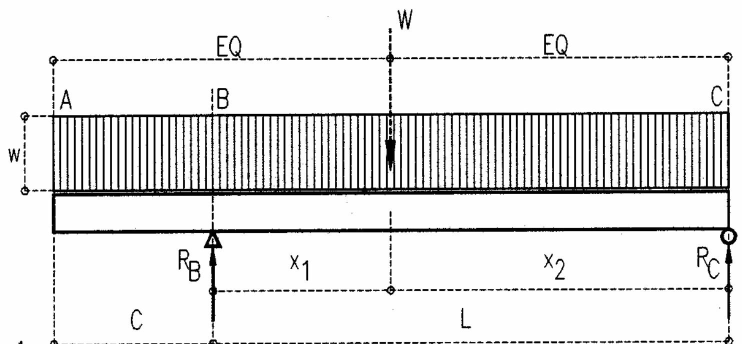
$\Sigma M_a = 0 \circlearrowright +$

$-16\,R_c + 100x16x8 + 800 \times 4 + 800 \times 16 = 0$

$R_c = (100x16x8 + 800x4 + 800\,x16) / 16$ $\qquad R_c = 1800\ \#$

Check $\Sigma V = 0 \uparrow +$

$1400+1800-100x16-800-800$ $\qquad \Sigma V = 0$



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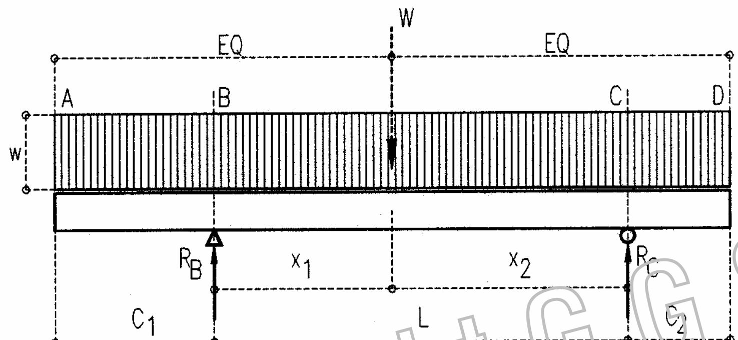

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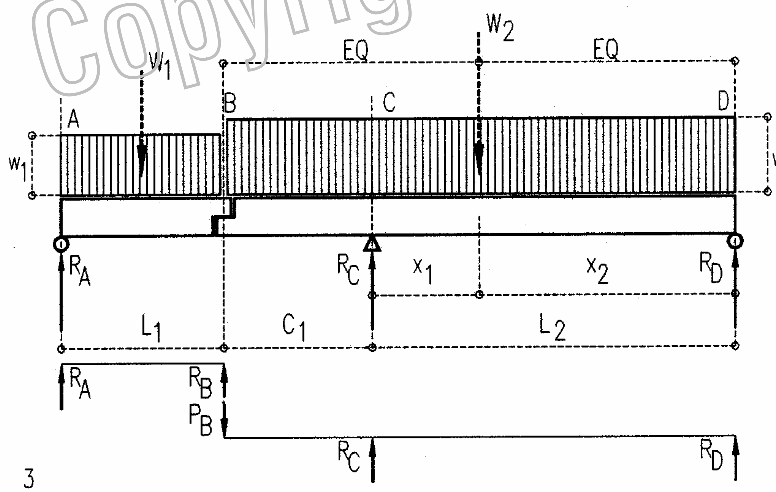


3

1 Beam with overhang

Assume: w = 300 plf, C = 3', L = 15'

X1 = 6, X2 = 9'

$\Sigma M_c = 0$ ↻+ $\Sigma M_b = 0$ ↻+

Check $\Sigma V = 0$ ↑+

$\Sigma V = 0$

2 Beam with two overhangs

Assume: w = 200 plf. C1 = 5', C2 = 3', L = 12

X1 = 5', X2 = 7'

$\Sigma M_c = 0$ ↻+

$12 R_b - 200 \times 20 \times 7 = 0$

$R_b = 200 \times 20 \times 7 / 12$ $R_b = 2333$ #

$\Sigma M_b = 0$ ↻+

$200 \times 20 \times 5 - 12 R_c - 0$

$R_c = 200 \times 20 \times 5 / 12$ $R_c = 1667$ #

Check $\Sigma V = 0$ ↑+

$2333 + 1667 - 200 \times 20$ $\Sigma V = 0$

3 Twin beams (treat as 2 beams, due to separation pin joint at b)

Simple left beam: w1 = 100 plf, L1 = 10'

$R_a = R_b = 100 \times 10 / 2$ $R_a = R_b = 500$ #

Right beam: w2 = 150 plf, C1 = 8', L2 = 20'

X1 = 6', X2 = 14', $P_b = R_b = 500$ #

$\Sigma M_d = 0$ ↻+

$20 R_c - 150 \times 28 \times 14 - 500 \times 28 = 0$

$R_c = (150 \times 28 \times 14 + 500 \times 28) / 20$ $R_c = 3640$ #

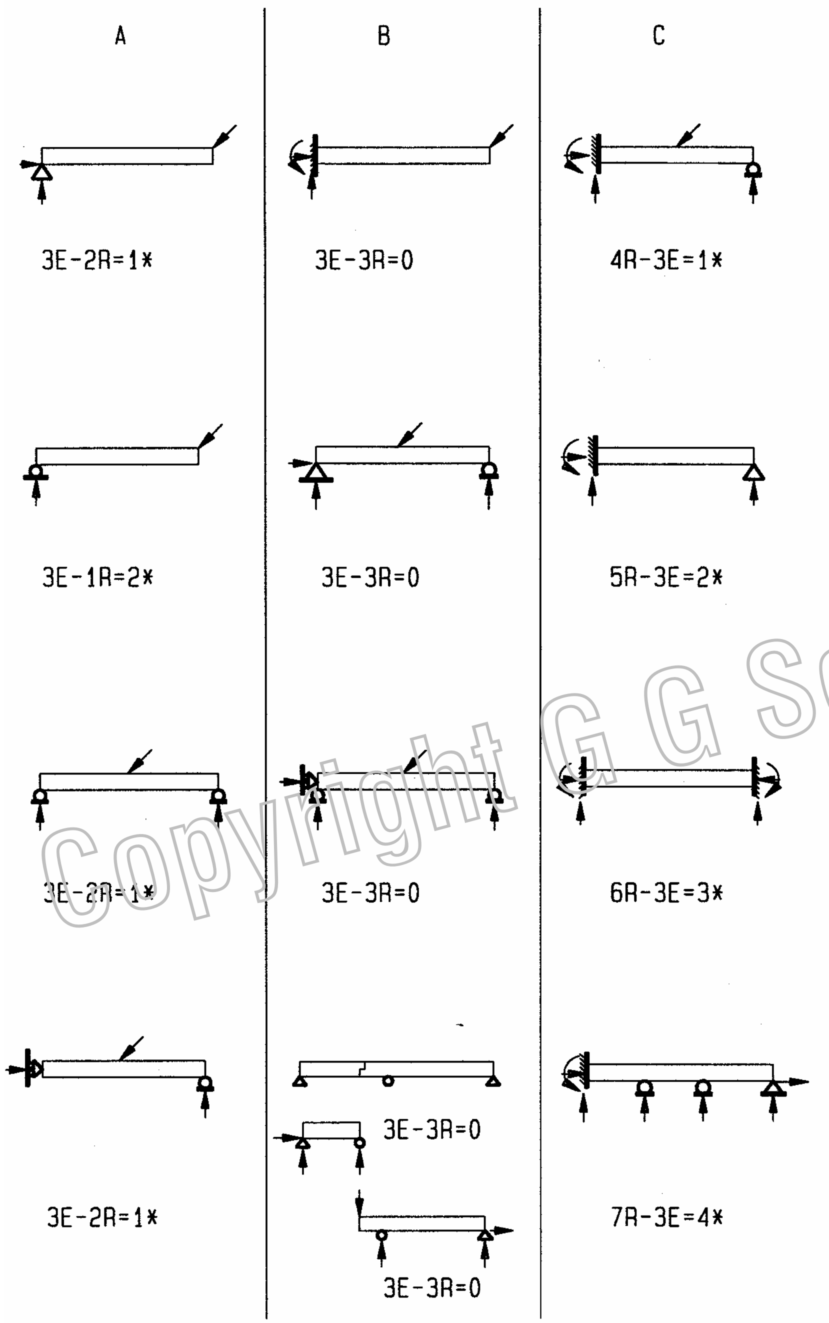
$\Sigma M_c = 0$ ↻+

$150 \times 28 \times 6 - 500 \times 8 - 20 R_d = 0$

$R_d = (150 \times 28 \times 6 - 500 \times 8) / 20$ $R_d = 1060$ #

Check $\Sigma V = 0$ ↑+

$3640 + 1060 - 150 \times 28 - 500$ $\Sigma V = 0$



Static Determinacy

Static determinacy has theoretical and practical implications. It determines if a structure can be analyzed by statics or requires another theory. Practically, it defines the degree of redundancy. Increased redundancy provides added safety. Static determinacy is defined by one of three conditions:

- Unstable: are unstable and must be avoided
- Determinate: have less redundancy but can be analyzed by statics
- Indeterminate: have most redundancy but cannot be analyzed by statics

Given the practical and theoretical implications it is critical to determine the static determinacy for any structure concept before proceeding with design. The following means define static determinacy for beams, trusses, and frames.

Beam determinacy

Given the three equations of statics, $\Sigma H = 0$, $\Sigma V = 0$, and $\Sigma M = 0$, static determinacy for beams is defined as follows:

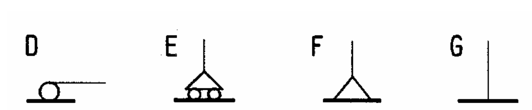
- Unstable $R < E$
- Determinate: $R = E$
- Indeterminate: $R > E$

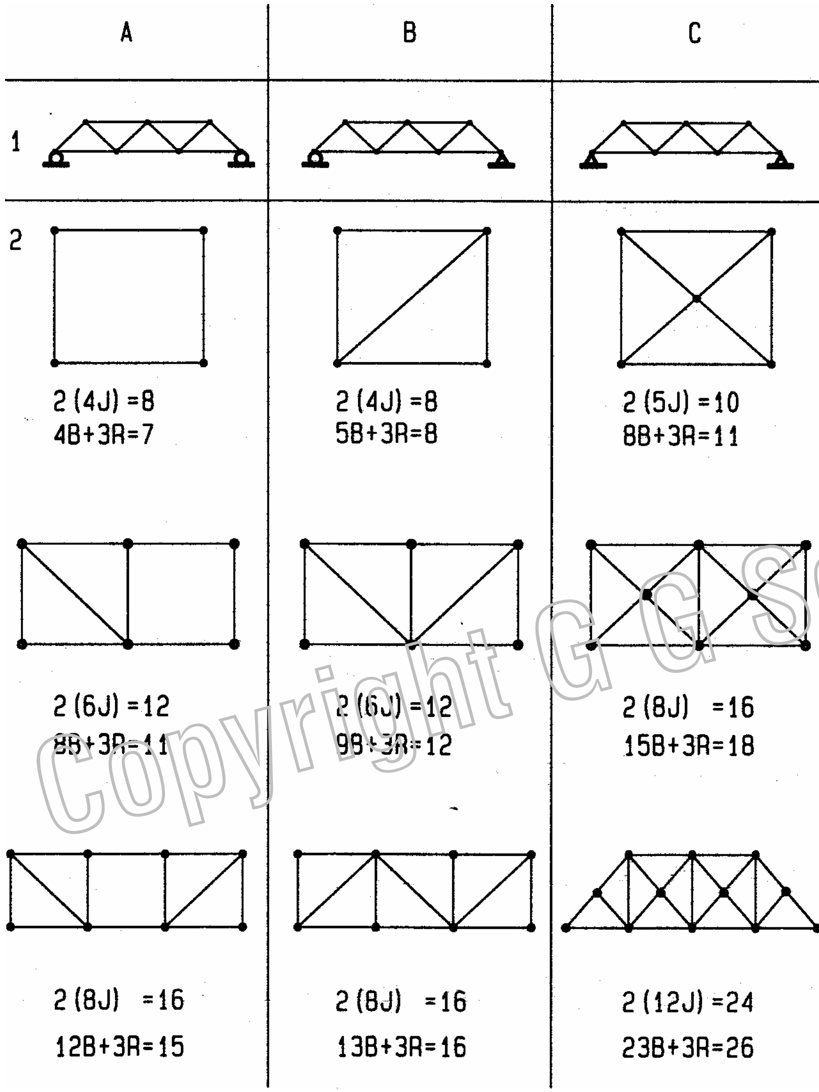
E = number of Equations (3)
R = number of Reactions

A Unstable beams
B Determinate beams
C Indeterminate beams
D Roller support
E Alternate roller support
F Pin support
G Rigid support (moment resisting)

Note:
Roller supports have one unknown reaction (vertical)
Pin supports have two unknown reactions (horizontal and vertical)
Rigid support has three unknown reactions (horizontal, vertical, rotational)
The degree of indeterminacy (a measure of redundancy) is computed as:

R – E = degree of indeterminacy





Truss determinacy

A Unstable trusses
B Determinate trusses
C Indeterminate trusses

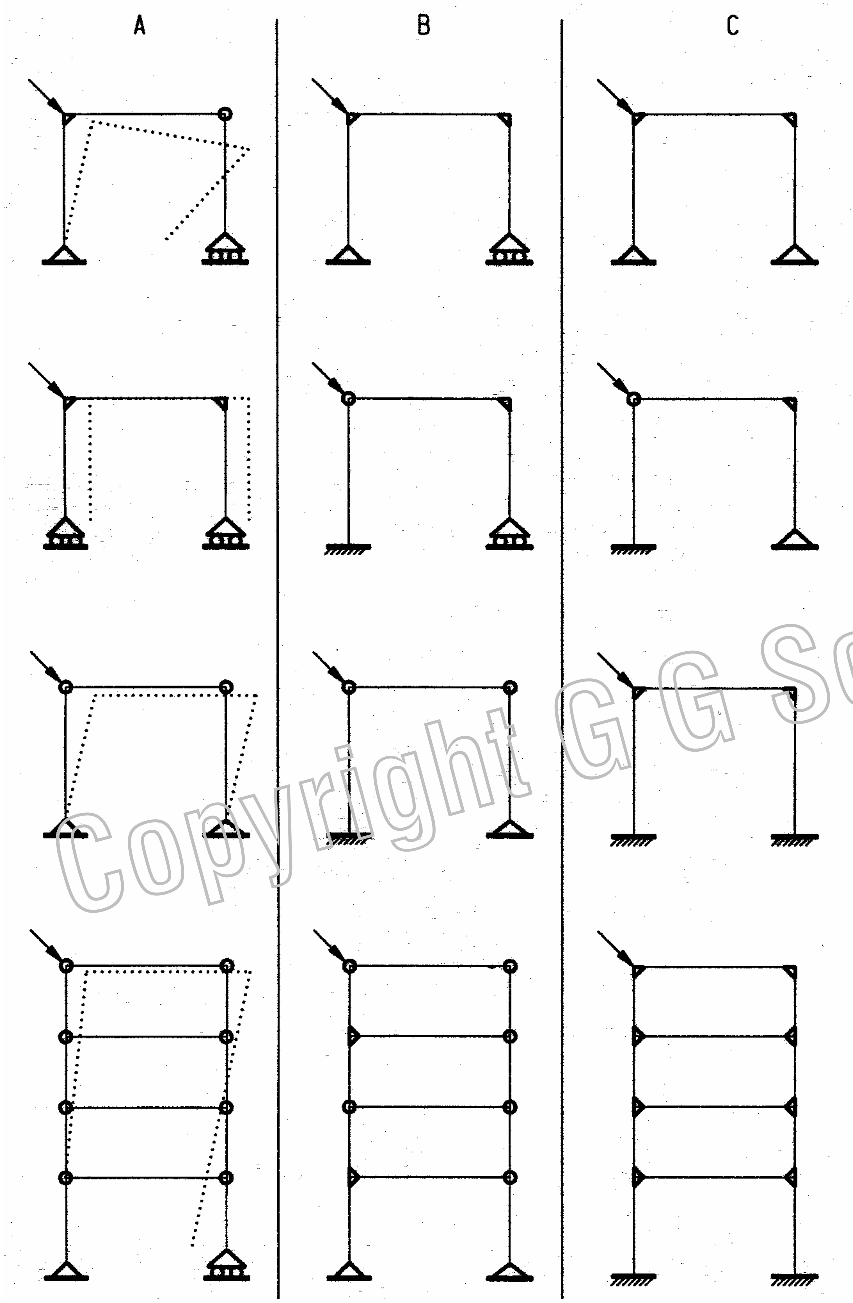
1 External determinacy (support reactions)
2 Internal determinacy (bar forces)

External determinacy is defined as for beams described above. For internal determinacy the moment equation, ΣM=0, cannot be used since trusses have pin joints to be statically determinate. Internal determinacy is defined as follows:
Each bar represents one unknown reaction and each joint has two equations for analysis, ΣH=0, ΣV=0. The moment equation ΣM=0 cannot be used since determinate trusses have pin joints. Thus internal determinacy is defined as:

- Unstable: B + R < 2 J
- Determinate: B + R = 2 J
- Indeterminate: B + R > 2 J

B = number bars
J = number of joints
R = number external reactions

Note:
The degree of indeterminacy is computed as:
B + R – 2 J = degree of indeterminacy



Frame determinacy

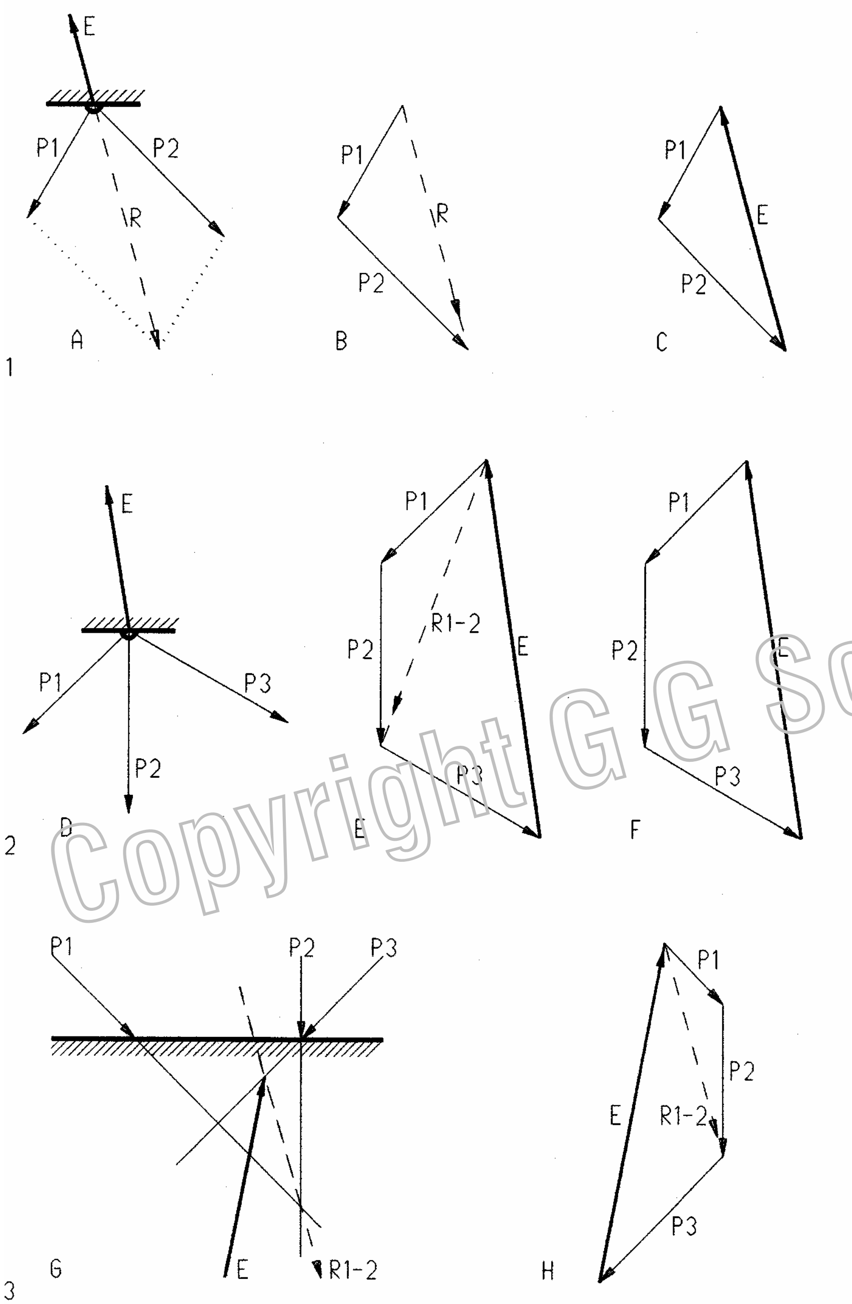
Static determinacy for frames is more complex than for beams or trusses and there is no simple formula to define it. However an intuitive process may be used, starting with external determinacy as follows:

1 A frame supported by pin and roller is externally determinate like a beam but internally unstable if internal joints are pins
5 Making the internal joints moment resistant makes the frame determinate
9 Removing a degree of freedom makes a determinate frame indeterminate

A similar process may be applied to multi-story frames as follows:

- One rigid joint at every second story stabilizes adjacent joints and makes the frame determinate
- Additional rigid joints makes a determinate frame indeterminate

A Unstable frames
B Determinate frames
C Indeterminate frames
D Roller support
E Alternate roller support
F Pin support
G Fixed support (moment resistant)
H Pin joint
I Rigid joint (moment resistant)



Vector Analysis

First used by Leonardo da Vinci, graphic vector analysis is a powerful method to analyze and visualize the flow of forces through a structure. However, the use of this method is restricted to statically determinate systems. In addition to forces, vectors may represent displacement, velocity, etc. Though only two-dimensional forces are described here, vectors may represent forces in three-dimensional space as well. Vectors are defined by *magnitude, line of action,* and *direction*, represented by a straight line with an arrow and defined as follows:

Magnitude is the vector length in a force scale, like 1” =10 k or 1 cm=50 kN

Line of Action is the vector slope and location in space

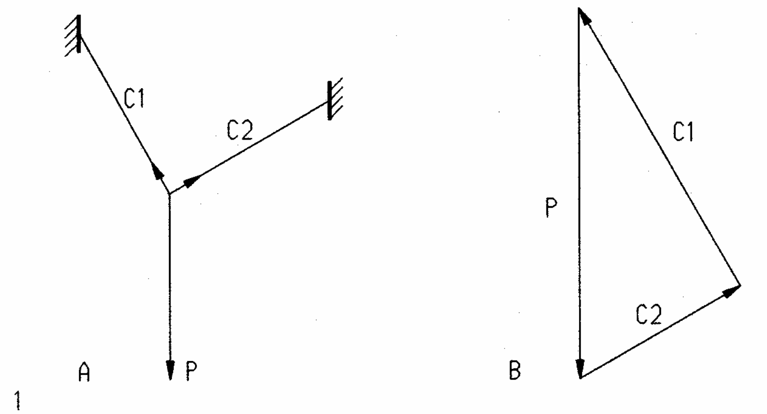
Direction is defined by an arrow pointing in the direction of action

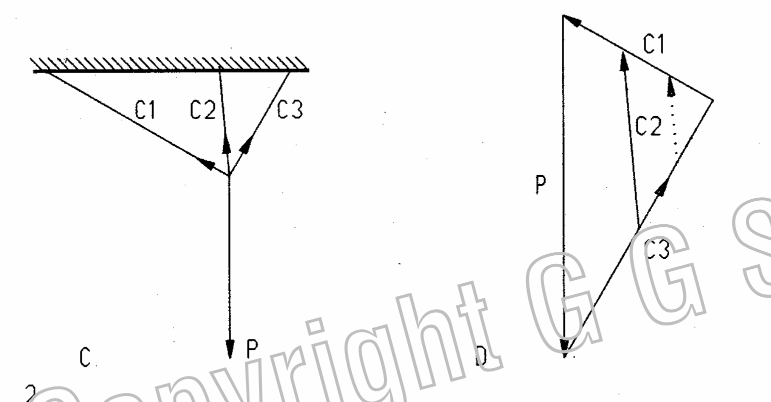
1 Two force vectors P1 and P2 acting on a body pull in a certain direction. The *resultant* R is a force with the same results as P1 and P2 combined, pulling in the same general direction. The resultant is found by drawing a force parallelogram [A] or a force triangle [B]. Lines in the vector triangle must be parallel to corresponding lines in the vector plan [A]. The line of action of the resultant is at the intersection of P1 / P2 in the vector plan [A]. Since most structures must be at rest it is more useful to find the *equilibriant* E that puts a set of forces in equilibrium [C]. The equilibriant is equal in magnitude but opposite in direction to the resultant. The equilibriant closes a force triangle with all vectors connected head-to-tail. The line of action of the equilibriant is also at the intersection of P1/P2 in the vector plan [A].

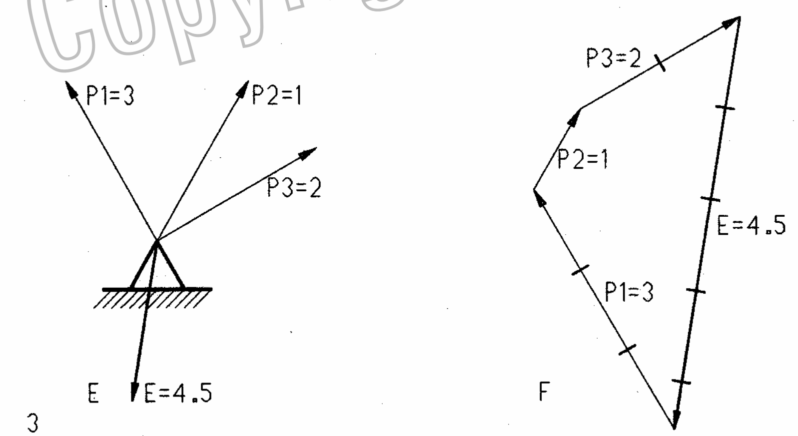
2 The equilibriant of three forces [D] is found, combining interim resultant R1-2 of forces P1 and P2 with P3 [E]. This process may be repeated for any number of forces. The interim resultants help to clarify the process but are not required [F]. The line of action of the equilibriant is located at the intersection of all forces in the vector plan [D]. Finding the equilibriant for any number of forces may be stated as follows:

The equilibriant closes a force polygon with all forces connected head-to-tail, and puts them in equilibrium in the force plan.

3 The equilibriant of forces without a common cross-point [G] is found in stages: First the interim resultant R1-2 of P1 and P2 is found [H] and located at the intersection of P1/P2 in the vector plan [G]. P3 is then combined with R1-2 to find the equilibriant for all three forces, located at the intersection of R1-2 with P3 in the vector plan. The process is repeated for any number of forces.







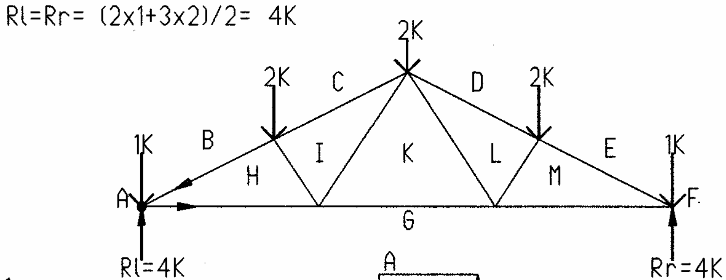
Vector components

Vector components have the same effect on a body as the initial vector. Thus components relate to a vector as two vectors relate to a resultant or equilibriant.

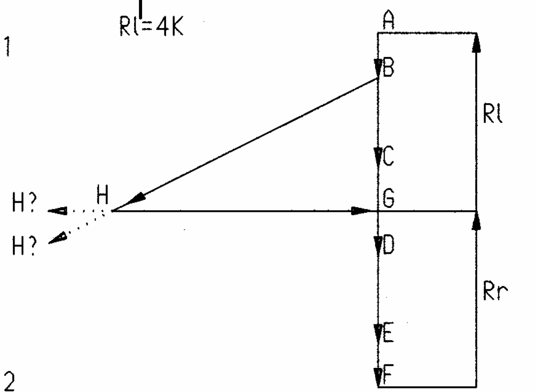
1 The component forces C1 and C2 in two cables supporting a load P are found by drawing a force triangle [B] with corresponding lines parallel to the those in the vector plan [A].

2 Forces in more than two cables supporting a load P are indeterminate [C] and cannot be found by graphic vector method since there is infinite number of solutions [D]. A problem with more than two unknown force components requires consideration of relative cable stiffness (cross-section area, length, and stiffness). Hence we can state:
Only two components can be found by graphic vector method

3 This example demonstrates graphic vector analysis: Forces are drawn on a vector plan with line of action and direction [E]. The magnitude may be written on each vector or the vector may be drawn at a force scale. A force polygon [F] is drawn next at a force scale, such as 1" = 1k. For good accuracy, the force scale should be as large as space permits. The line of action of the equilibriant (or resultant) is then transposed into the vector plan at the intersection of all force vectors [E].



1



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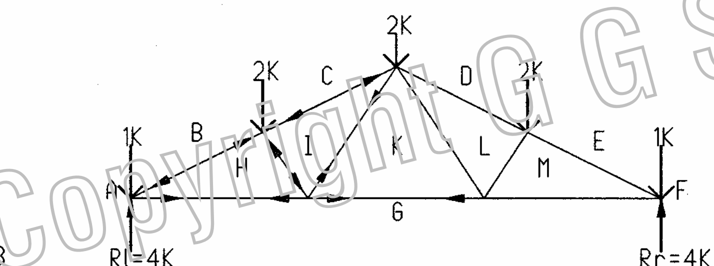

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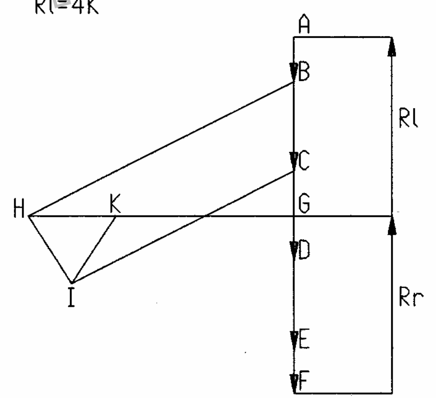


4

BH= -6.7K
HG= +6.0K
CI= -5.6K
IH= -1.8K
IK= +1.8K
KG= +4.0K

5

Truss Analysis

Graphic truss analysis (*Bow's Notation*) is a method to find bar forces using graphic vectors as in the following steps:

A Draw a truss scaled as large as possible (1) and compute the reactions as for beams (by moment method for asymmetrical trusses).

B Letter the spaces between loads, reactions, and truss bars. Name bars by adjacent letters: bar BH between B and H, etc.

C Draw a force polygon for external loads and reactions in a force scale, such as 1"=10 pounds (2). Use a large scale for accuracy. A closed polygon with head-to-tail arrows implies equilibrium. Offset the reactions to the right for clarity.

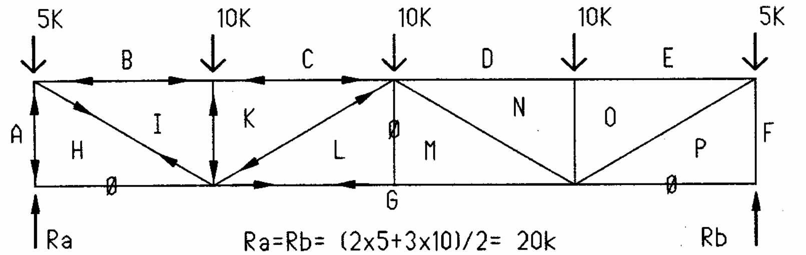
D Draw polygons for each joint to find forces in connected bars. Closed polygons with head-to-tail arrows are in equilibrium. Start with left joint ABHG. Draw a vector parallel to bar BH through B in the polygon. H is along BH. Draw a vector parallel to bar HG through G to find H at intersection BH-HG.

E Measure the bar forces as vector length in the polygon.

F Find bar tension and compression. Start with direction of load AB and follow polygon ABHGA with head-to-tail arrows. Transpose arrows to respective bars in the truss next to the joint. Arrows pushing toward the joint are in compression; arrows pulling away are in tension. Since the arrows reverse for adjacent joints, draw them only on the truss but not on the polygon.

G Draw equilibrium arrows on opposite bar ends; then proceed to the next joint with two unknown bar forces or less (3). Draw polygons for all joints (4), starting with known loads or bars (for symmetrical trusses half analysis is needed).

1 Truss diagram
2 Force polygon for loads, reactions, and the first joint polygon
3 Truss with completed tension and compression arrows
4 Completed force polygon for left half of truss
5 Tabulated bar forces (- implies compression)



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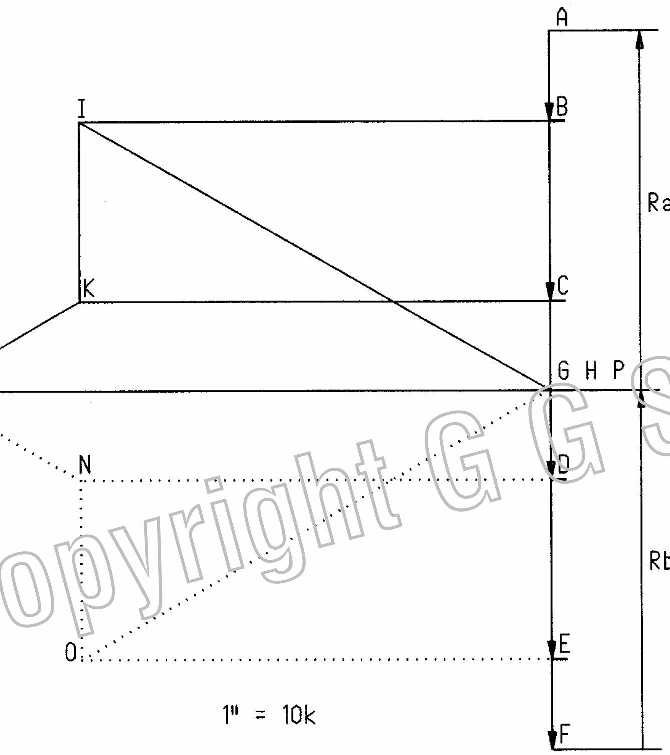
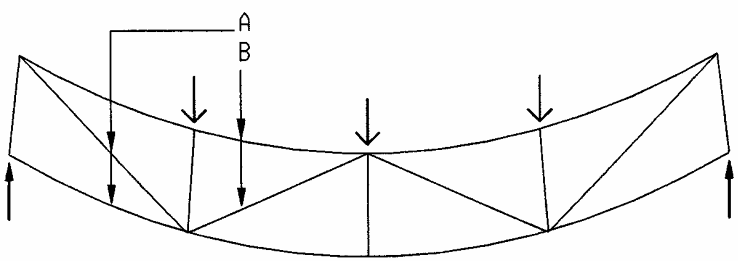


2

HA= -20K
BI= -26K
IH= +30K
HG= 0K
IK= -10K
CK= -26K
KL= -10K
LG= +35K
LM= 0K

3



4

Truss Example

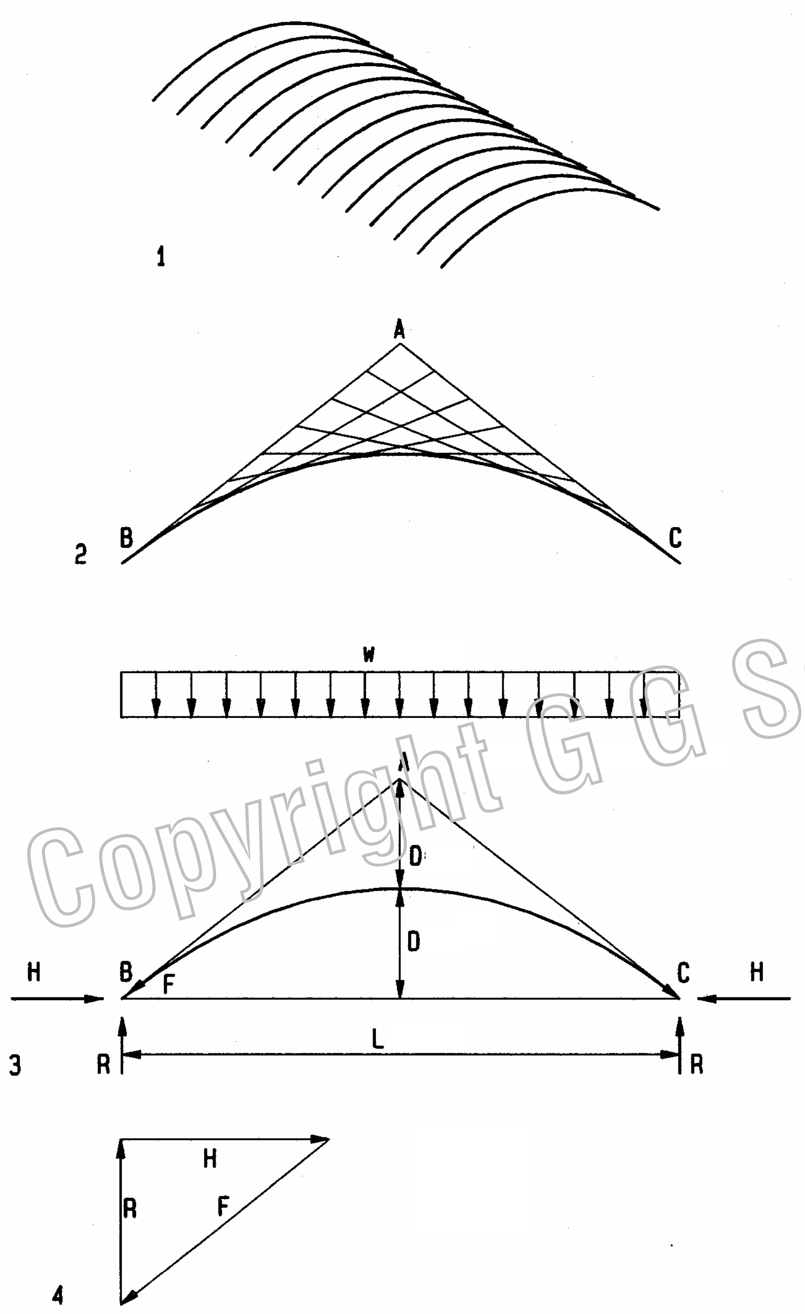
Some trusses have bars with zero force under certain loads. The example here has zero force in bars HG, LM, and PG under the given load. Under asymmetrical loads these bars would not be zero and, therefore, cannot be eliminated. Bars with zero force have vectors of zero length in the equilibrium polygon and, therefore, have both letters at the same location.

Tension and compression in truss bars can be visually verified by deformed shape (4), exaggerated for clarity. Bars in tension will elongate; bars in compression will shorten. In the truss illustrated the top chord is in compression; the bottom chord is in tension; inward sloping diagonal bars in tension; outward sloping diagonal bars in compression.

Since diagonal bars are the longest and, therefore, more likely subject to buckling, they are best oriented as tension bars.

1 Truss diagram
2 Force polygon
3 Tabulated bar forces (+ implies tension, - compression)
4 Deformed truss to visualize tension and compression bars

A Bar elongation causes tension
B Bar shortening causes compression



Funicular

Graphic vector are powerful means to design funicular structures, like arches and suspension roofs; providing both form and forces under uniform and random loads.

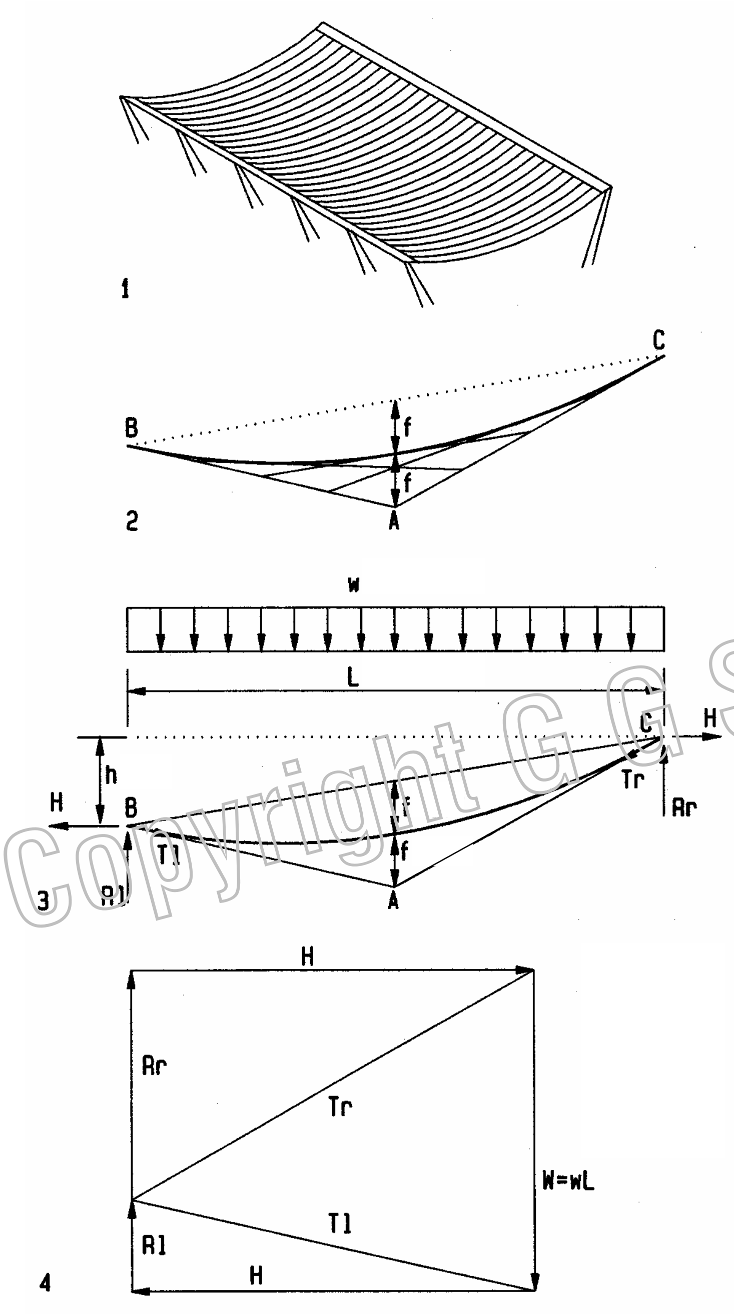
Arch

Assume:
Arch span L = 150, arch spacing e = 20'
DL = 14 psf
LL = 16 psf
Σ = 30 psf

Uniform load
w = 30 psf x 20' / 1000 — w = 0.6 klf
Vertical reactions
R = w L /2 = 0.6 x 150 / 2 — R = 45 k
Draw vector polygon, starting with vertical reaction R
Horizontal reaction — H = 56 k
Maximum arch force (diagonal vector parallel to arch tangent) — F = 72 k

1 Arch structure
2 Parabolic arch by graphic method
Process:
Draw AB and AC (tangents of arch at supports)
Divide tangents AB and AC into equal segments
Lines connecting AB to AC define parabolic arch envelope
3 Arch profile
Process:
Define desired arch rise D (usually D = L/5)
Define point A at 2D above supports
AB and AC are tangents of parabolic arch at supports
Compute vertical reactions R = w L /2
4 Equilibrium vector polygon at supports (force scale: 1" = 50 k)
Process:
Draw vertical vector (reaction R)
Complete vector polygon (diagonal vector parallel to tangent)
Measure vectors (H = horizontal reaction, F = max. arch force)

Note:
The arch force varies from minimum at crown (equal to horizontal reaction), gradually increasing with arch slope, to maximum at the supports.



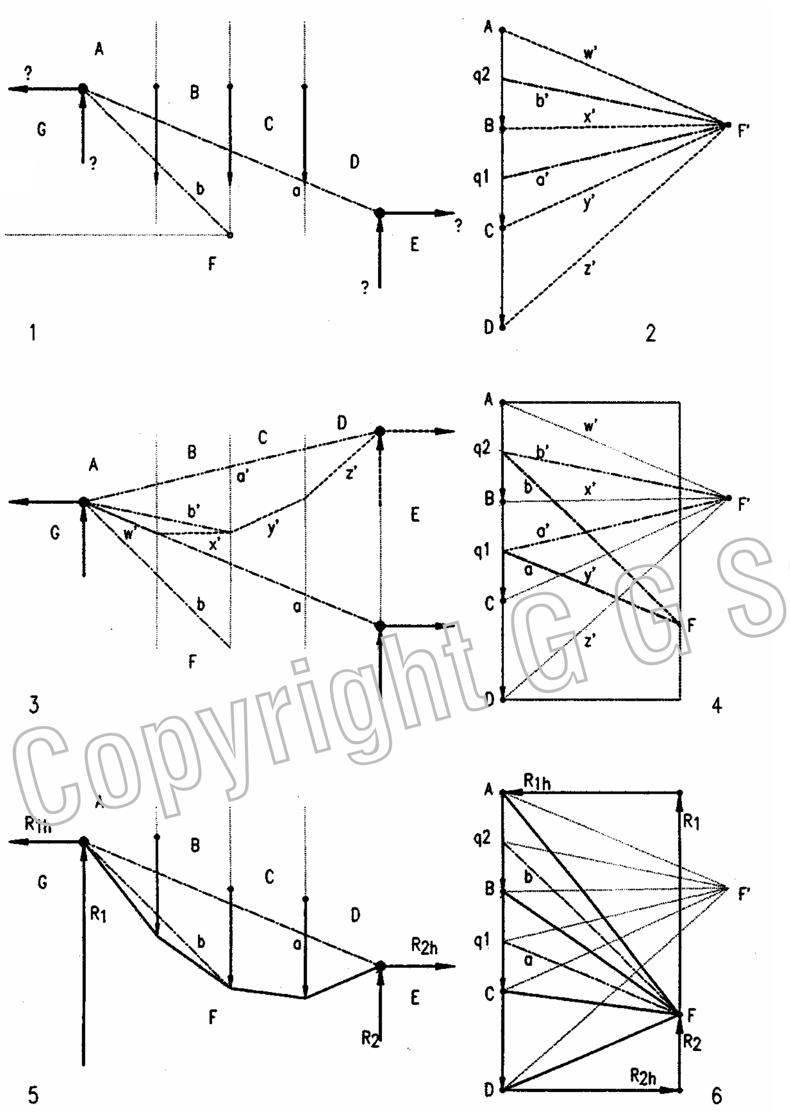
Suspension roof

Assume:
Span L = 300, cable spacing e = 10’, cable sag f = 30’, height difference h = 50’
DL = 14 psf
<u>LL = 16 psf</u>
Σ = 30 psf

Uniform load	
w = 30 psf x 10’ / 1000	w = 0.3 klf
Total load	
W = w L = 0.3 x 300	R = 90 k
Draw vector polygon, starting with total load W	
Horizontal reaction	H = 113 k
Vertical reactions	
Left reactions	Rl = 26 k
Right reaction	Rr = 64 k
Cable tension	
At left support	Tl =115 k
At right support (maximum)	Tr = 129 k

1 Cable roof structure
2 Parabolic cable by graphic method
Process:
Draw AB and AC (tangents of cable at supports)
Divide tangents AB and AC into equal segments
Lines connecting AB to AC define parabolic cable envelop
3 Cable profile
Process:
Define desired cable sag f (usually f = L/10)
Define point A at 2f below midpoint of line BC
AB and AC are tangents of parabolic cable at supports
Compute total load W = w L
4 Equilibrium vector polygon at supports (force scale: 1” = 50 k)
Process:
Draw vertical vector (total load W)
Draw equilibrium polygon W-Tl-Tr
Draw equilibrium polygons at left support Tl-H-Rl
Draw equilibrium polygons at right support Tr-Rr-H
Measure vectors H, Rl, Rr at force scale

Note:
This powerful method finds five unknowns: H, Rl, Rr. Tl. Tr
The maximum cable force is at the highest support



Random load funiculars

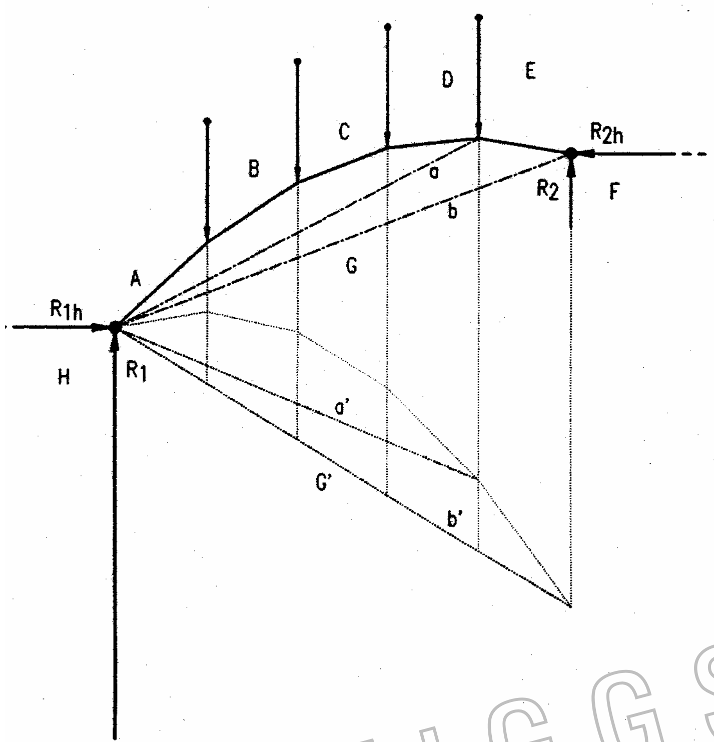
To find funicular form and forces under random load is similar to finding reactions as described above, with one major difference, it requires a two step approach. In step one a polygon is drawn using an arbitrary pole. In step two the polygon is redrawn after finding the correct pole location as intersection of two defining vectors. This method may be used do find form and forces for suspension cables and arches under various load conditions.

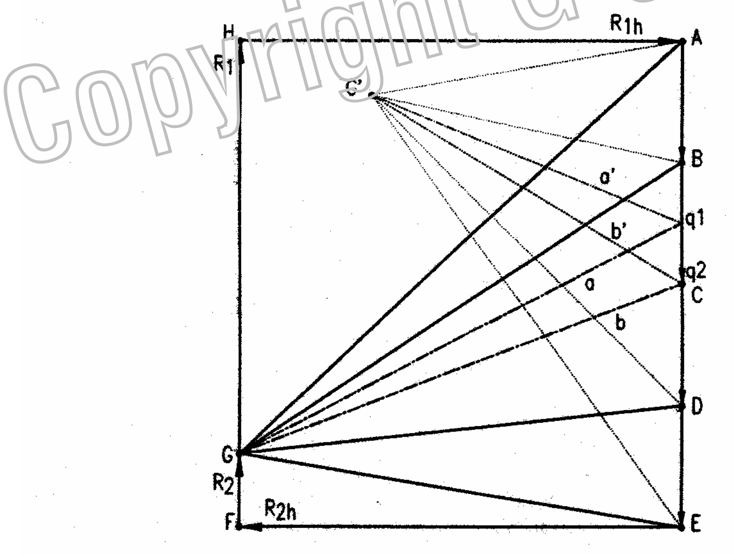
Suspension cable with random load

1 Vector plan of loads
2 Trial polygon of arbitrary pole F'
3 Vector plan based on arbitrary pole polygon
4 Arbitrary trial polygon with real pole at intersection of a and b
5 Corrected vector plan
6 Corrected vector polygon

Process:
Draw vectors AB, BC, etc. for all loads in trial polygon
Select an arbitrary pole F' in polygon
Draw polar vectors AF', BF', etc. for all loads in polygon
Draw parallel polar vectors at intersection of respective load in plan
Transpose trial closing line a' from [plan to polygon to find q1
Transpose trial closing line b' from plan to polygon to find q2
Define desired locations of right support in plan
Define desired locations of cable sag at intersection of any load in plan
Transpose closing line a between supports from plan to q1 in polygon
Transpose closing line b of sag from plan to q2 in polygon
Intersection of closing lines a and b in polygon define correct pole F
Draw correct polar vectors AF, Bf, etc. for all loads in polygon
Draw parallel polar vectors at intersection of respective load in plan
The corrected vectors will intersect with closing lines a and b in plan
Complete support equilibrium in polygon:
    Left support: AF- R1-R1h
    Right support: DF- R2h-R2
Measure vector lengths in force scale to complete the process

Note:
The process is based on equilibrium at both supports and intersections of all loads with the cable.





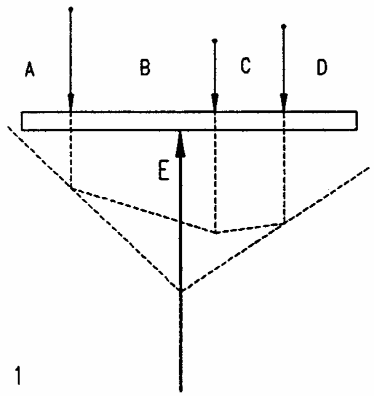
Arch with random load

The process for arches is similar to cables described above, but with forces reversed from tension to compression and the polygon pole on the opposite side

Top: vector plan
Bottom: vector polygon
Process:

- Draw vectors AB, BC, etc. for all loads in trial polygon
- Select an arbitrary pole G' in the polygon
- Draw polar vectors AG', BG', etc. for all loads in polygon
- Draw parallel polar vectors at intersection of respective loads in plan
- Transpose trial closing line a' from [plan to polygon to find q1
- Transpose trial closing line b' from plan to polygon to find q2
- Define desired locations of right support
- Define desired locations of arch rise at intersection of any load
- Transpose closing line a between supports from plan to q1 in polygon
- Transpose closing line b of arch rise from plan to q2 in polygon
- Intersection of closing lines a and b in polygon define correct pole G
- Draw correct polar vectors AG, BG, etc. for all loads in polygon
- Draw parallel polar vectors at intersection of respective load in plan
- The corrected vectors will intersect with closing lines a and b in plan
- Complete support equilibrium in polygon:
  - Left support: AG- R1-R1h
  - Right support: EG- R2h-R2
- Measure vector lengths in force scale to complete the process

Note:
The process is based on equilibrium at both supports and at all intersections of loads with the arch.



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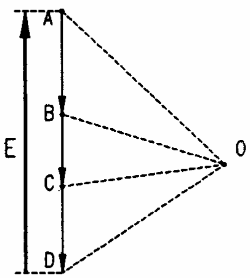

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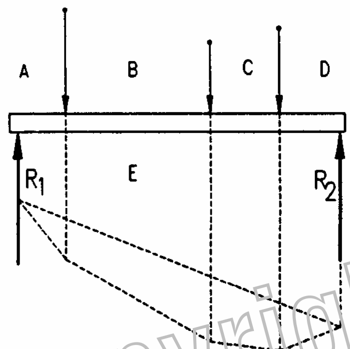

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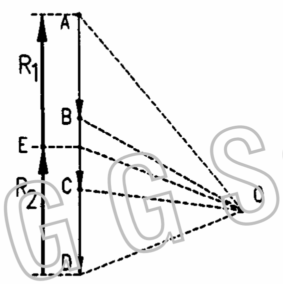

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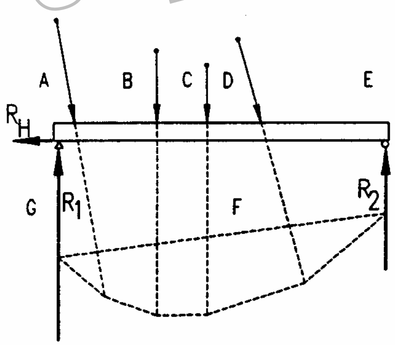

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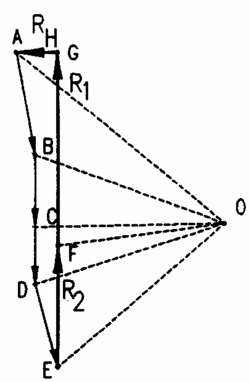

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Vector reactions

Reactions put loads in equilibrium and therefore can be found by graphic vector analysis by similar method used to find equilibriant. The process is illustrated on three examples. For convenience vectors are defined in a vector plan as for truss analysis. For example, the vector between A and B in the plan extends from A to B in the polygon.

1 Vector *plan* of loads and equilibriant
2 Equilibrium *polygon* of loads and equilibriant
Process:
Draw vectors A-B, B-C, etc. for all loads in polygon
Select an arbitrary pole O in polygon
Draw polar vectors A-O, B-O, etc. for all loads in polygon
Draw parallel polar vectors at intersection of respective load in plan
Draw and measure equilibriant E to close polygon
Draw equilibriant E at intersection of A-O and D-O in plan

Note:
The equilibriant E puts vectors in equilibrium

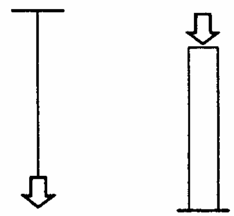
3 Vector plan for two reactions
4 Vector polygon for two reactions
Process:
Draw vectors A-B, B-C, etc. for all loads in polygon
Select an arbitrary pole O in polygon
Draw polar vectors A-O, B-O, etc. for all loads in polygon
Draw parallel polar vectors at intersection of respective load in plan
Draw closing vector E from R1 to R2 in plan
Closing vector E in polygon defines reactions R1 and R2
Measure Reactions R1 an R2

5 Random load vector plan
6 Random load vector polygon
Process:
Draw vectors A-B, B-C, etc. for all loads in polygon
Select an arbitrary pole O in polygon
Draw polar vectors A-O, B-O, etc. for all loads in polygon
Draw parallel polar vectors at intersection of respective loads in plan
Draw closing vector F from R1 to R2 in plan
Draw parallel vector F in polygon to define length of R1 and R2
Draw and measure reaction R1, R1, H to close the polygon

5

Strength
Stiffness
Stability

This chapter introduces the theory and examples of strength, stiffness, and stability described n the following sections: Force types; force vs. stress; allowable stress; axial stress; shear stress; principle stress and Mohr's circle; torsion; strain; Hooke's law; Poisson's ratio; creep, elastic modulus; thermal strain; thermal stress; and stability.



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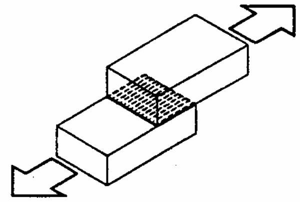

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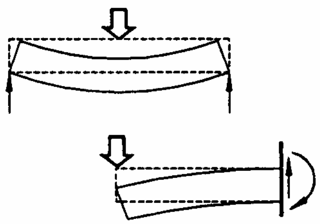

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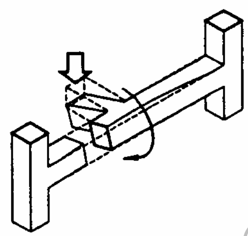

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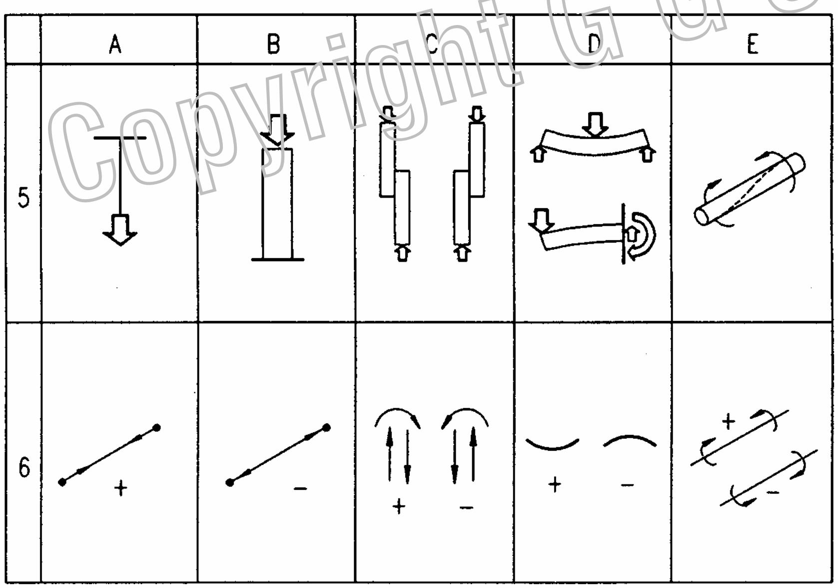


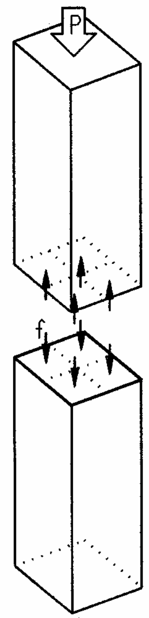
Force types

Forces on structures include tension, compression, shear, bending, and torsion. Their effects and notations are tabulated below and all but bending and related shear are described on the following pages. Bending and related shear are more complex and further described in the next chapter.

Type of forces			
Force type	Action	Symbol	Notation
Tension	Elongates	Internal reaction arrows	+
Compression	Shortens	Internal reaction arrows	-
Shear	Sliding force	Arrow couple	Clockwise couple +
Bending	Elongates one side shortens other side	Concave and convex arcs	Concave arc + Convex arc -
Torsion	Twists	Bar with arrows	Right-hand-rule +

1 Axial force (tension and compression)
2 Shear
3 Bending
4 Torsion
5 Force actions
6 Symbols and notations

A Tension
B Compression
C Shear
D Bending
E Torsion



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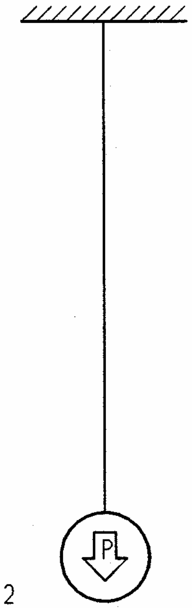

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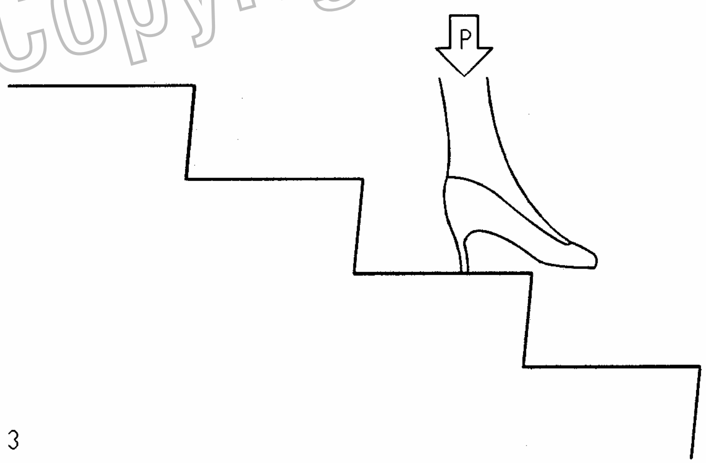

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Force vs. stress

Force and stress refer to the same phenomena, but with different meanings. Force is an external action, measured in absolute units: # (pound), k (kip); or SI units: N (Newton), kN (kilo Newton). Stress is an internal reaction in relative units (force/area), measured in psi (pound per square inch), ksi (kip per square inch); or SI units: Pa (Pascal), kPa (kilo Pascal). Axial stress is computed as:

$f = P / A$
where
f = stress
P = force
A = cross section area

Note: stress can be compared to allowable stress of a given material.

- **Force** is the load or action on a member
- Stress can be compared to allowable stress for any material, expressed as:
  $F \geq f$ (Allowable stress must be equal or greater than actual stress)
  where
  F = allowable stress
  f = actual stress

The type of stress is usually defined by subscript:

F_a, f_a (axial stress, capital F = allowable stress)
F_b, f_b (bending stress, capital F = allowable stress)
F_v, f_v (shear stress, capital F = allowable stress)

The following examples of axial stress demonstrate force and stress relations:

1 **Wood column** (compression)
Assume: Force P = 2000#, allowable stress F = 1000 psi
A = 2 x 2 = 4 in^2 (cross section area)
Stress f = P / A = 2000# / 4 f = 500 psi
1000 > 500, ok

2 **Steel rod** (tension)
Assume: P = 6 k, ½" rod, F_a = 30 ksi
Cross section area A = $\pi r^2 = (0.5/2)^2 \pi$ A = 0.2 in^2
Stress f = P / A = 5 k / 0.2 f = 25 ksi
25 < 30, ok

3 **Spiked heel on wood stair** (compression)
Assume: P = 200# (impact load), A = 0.04 in^2, F_a= 400 psi
Stress f = P / A = 200 / 0.04 f = 5000 psi
5000 >> 400. NOT ok

Note: The heel would sink into the wood, yield it and mark an indentation

Allowable stress

Allowable stress is defined by a material's ***ultimate strength*** or ***yield strength*** and a ***factor of safety***. Building codes and trade associations provide allowable stress for various materials and grades of materials, which may also depend on duration of load. Allowable wood stress also depends on temperature, moisture content, size, and if a member is single or repetitive, like closely spaced joists. Relevant factors regarding allowable stress are briefly introduced here and further described later in this chapter.

Ultimate strength is the stress at which a test specimen breaks under load. Ultimate strength varies by material, such as wood, steel, masonry, or concrete, as well as grades of each material.

Yield strength is the point where a material under load changes from elastic to plastic deformation. Elastic deformation allows the material to return to its unstressed length after the load is removed; by contrast plastic deformation is permanent.

Factor of safety accounts for uncertainty regarding consistency of material quality, type of stress (tension, compression, shear, bending) and actual load conditions. The factor of safety is defined differently for different materials. For example, for steel the factor of safety is based on yield strength, for concrete on the specified compressive strength (breaking strength). The tables at left give some typical allowable stresses.

Wood

Base values for *Douglas Fir-Larch* 2"-4" (5-10 cm) thick, 2" (5 cm) or wider for allowable stress: bending (F_b), tension (F_t), compression (F_c), compression normal to grain ($F_{C\perp}$), horizontal shear (F_v), and elastic modulus (E).

Grade	F_b	F_t	F_c	$F_{c\perp}$	F_v	E	units
Select	1,450	1000	1,700	625	95	1,900,000	psi
Structural:	*9,998*	*6,895*	*11,722*	*4,309*	*655*	*13,100,500*	*kPa*
No. 1:	1,000	675	1,450	625	95	1,700,000	psi
	6,895	*4,654*	*9,998*	*4,309*	*655*	*11,721,500*	*kPa*
No. 2:	875	575	1300	625	95	1,600,000	psi
	6,033	*3,965*	*8,964*	*4,309*	*655*	*11,032,000*	*kPa*

Steel

Table of yield stress (F_y); ultimate strength (F_u); allowable stress for bending (F_b), compression (F_c), tension (F_t), shear (F_v); and elastic modulus (E)

Steel grade	F_y	F_u	F_b	F_c F_t	F_v	E	ksi
ASTM A36	36	58-80	22	22	14.5	29,000	ksi
	248	*400-550*	*150*	*150*	*100*	*200,000*	*MPa*
ASTM A572	50	65	30	30	20	29,000	ksi
	345	*450*	*210*	*210*	*140*	*200,000*	*MPa*

Masonry

Allowable compressive stress F_a, for masonry with special inspection is 25% of specified strength f'_m by the working stress method; reduced for slenderness. Specified Compressive strength f'_m is based on compressive strength of masonry units and mortars type M, S, N.

Type	Concrete masonry (ksi)				Clay brick masonry (ksi)					
Unit strength	1.9	2.8	3.75	4.8	4	6	8	10	12	14
f'_m (M or S)	1.5	2	2.5	3	2	2.7	3.35	4	4.7	5.3
f'_m (N)	1.35	1.85	2.35	2.8	1.6	2.2	2.7	3.3	3.8	4.4

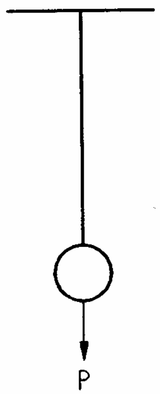
Type	Concrete masonry (MPa)				Clay brick masonry (MPa)					
Unit strength	13	19	26	33	28	41	55	69	83	97
f'_m (M or S)	10	14	17	21	14	19	23	28	32	37
f'_m (N)	9	13	16	19	11	15	19	23	26	30

Concrete

By working stress method, allowable stresses are based on compressive strength f'_c. Typical compressive strengths range from 2 to 6 ksi (14 to 41 MPa)

Allowable compressive stress		$0.40\ f'_c$
Allowable compressive bending stress		$0.45\ f'_c$
Allowable shear stress without reinforcing:	beam	$1.1\ f'_c{}^{1/2}$
	joist	$1.2\ f'_c{}^{1/2}$
	footing & slab on grade	$2.0\ f'_c{}^{1/2}$

Note: For concrete strength design method see chapter 8.



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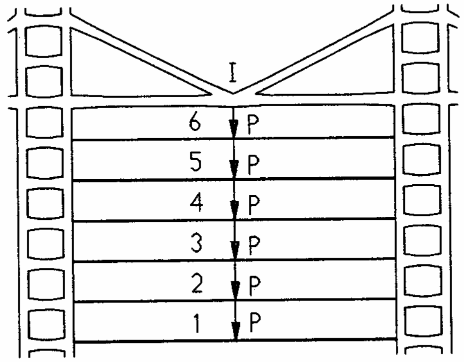

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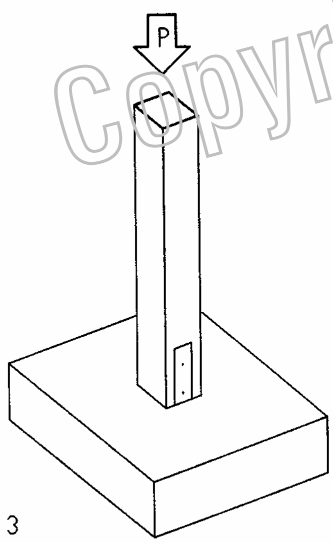

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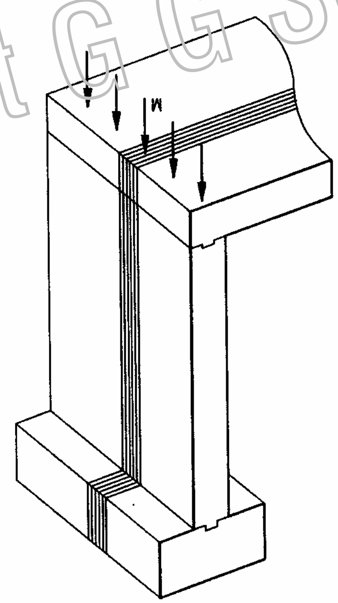

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Axial stress

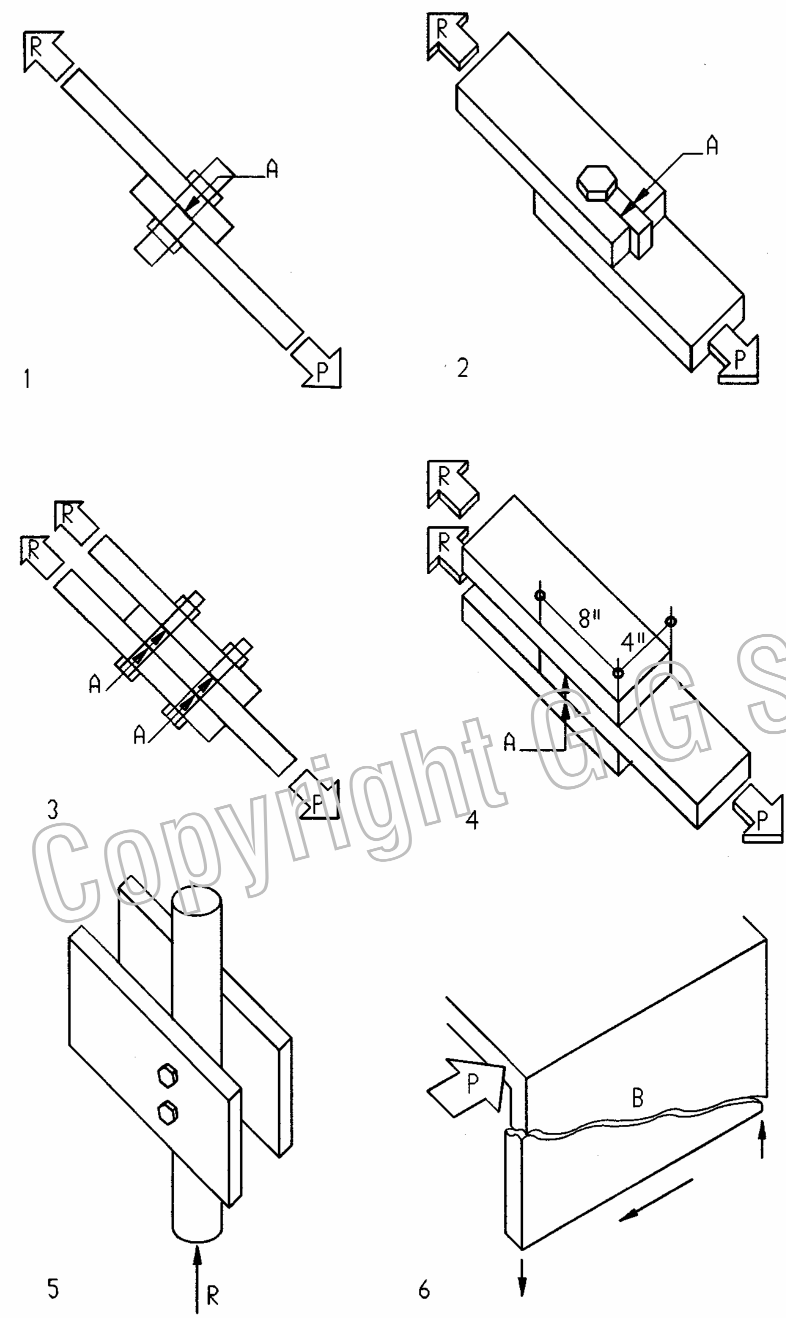
Axial stress acts in the axis of members, such as columns. Axial tension is common in rods and cables; wile axial compression is common in walls and columns. The following examples illustrate axial design and analysis. Analysis determines if an element is ok; design defines the required size. The equation, $f_a = P/A$, is used for analysis. The equation $A = P/F_a$, is used for design. Allowable stress, F_a, includes a factor of safety.

1 Crane cable design
Assume: P = 12 k, Fa = 70 ksi
Find required cable size
Metallic cross section A_m (cables are about 60% metallic)
$A_m = P / F_a = 12\text{ k} / 70\text{ ksi}$ — $A_m = 0.17\text{ in}^2$
Gross cable area
$A_g = A_m / 0.6 = 0.17 / 0.6$ — $A_g = 0.28\text{ in}^2$
Cable size
$\phi = 2\,(A/\pi)^{1/2} = 2\,(0.28 / \pi)^{1/2} = 0.6''$ — use $\phi = 5/8''$

2 Suspension hanger analysis (Hong Kong-Shanghai bank)
Assume: load per floor P=227 k, F_a=30 ksi, level 1 A = 12 in^2, level 6 A = 75 in^2
Hanger stress
Level 1: $f_a = P / A = 227 / 12$ — $f_a = 19\text{ ksi} < 30$
Level 6: $f_a = 6\,P / A = 6 \times 227 / 75$ — $f_a = 18\text{ ksi} < 30$

3 Post/footing analysis
Assume: P = 12,000 #, 3'x3'x2' footing at 150 pcf, 4x4 post (3.5"x3.5" actual)
Allowable post stress F_a = 1000 psi, allowable soil pressure F_s = 2000 psf
Post stress
P/A = 12,000 # / (3.5"x 3.5") — $f_a = 980\text{ psi} < 1000$
Soil pressure
f_s = P/A = (12,000 # + 3'x3'x2'x150 pcf)/ (3'x3') — $f_s = 1633\text{ psf} < 2000$

4 Slab/wall/footing, analyze a 1' wide strip
Assume: allowable wall stress F_a = 360 psi; allowable soil pressure F_s=1500 psf
Concrete slab, t =8" thick, L = 20' span
CMU wall, h =10', DL = 80 psf, t = 8" nominal (7 5/8" = 7.625" actual)
Slab load
100 psf DL+ 40 psf LL — DL+LL =140 psf
Load at wall base
P = 140psf (20'/2) + 80 psf (10') — P = 2,200 #
Wall stress
f_a = P / A = 2200 # / (12"x7.625") — $f_a = 24\text{ psi} < 360$
Load on soil
P = 2200 + 150 pcf x 2' x 1' — P = 2,500 #
Soil pressure
f_s = 2,500 # / (1' x2') — $f_s = 1250\text{ psf} < 1500$



Shear stress

Shear stress occurs in many situations, including the following examples, but also in conjunction with bending, described in the next chapter on bending. Shear stress develops as a resistance to sliding of adjacent parts or fibers, as shown on the following examples. Depending on the number of shear planes (the joining surface [A] of connected elements) shear is defined as single shear or double shear.

A Shear plane
B Shear crack

1 Single shear
Assume: P = 3 k = 3000 #, 2"x4" wood bars with ½" bolt of Fv = 20 ksi
Shear area (bolt cross section)
$A = \pi r^2 = \pi (0.5/2)^2$ — $A = 0.2 \ in^2$
Shear stress fv = P / A = 3/ 0.2 — fv = 15 ksi < 20

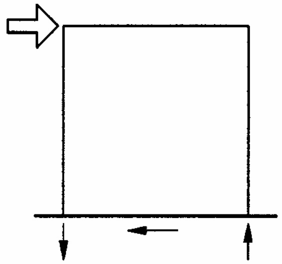
2 Check end block (A)
Assume: Block length 6", wood Fv = 95 psi, all other as above
End block shear area A = 2x 2" x 6" — $A = 24 \ in^2$
Shear stress fv = P / A = 3000# / 24 — fv =125 psi > 95
NOT ok
Required block length e = 125 x 6"/ 95 = 7.9: — use e = 8

3 Double shear
Assume: P = 22 k, 2 5/8" bolts of Fv = 20 ksi
Shear area $A = 4\pi r^2 = 4 \pi (0.625/2)^2$ — $A = 1.2 \ in^2$
Shear stress fv = P / A = 22 / 1.2 — fv = 18 ksi < 20

4 Double shear, glued
Assume: P = 6000 #, Wood bars, Fv = 95 psi
Shear area A = 2 x 4" x 8" — $A = 64 \ in^2$
Shear stress Fv = P / A = 6000 / 64 — fv =94 psi < 95

5 Twin beam double shear
Assume: P = R = 12 k, 2 ½" bolts, Fv = 20 ksi
Shear area $A = 4\pi r^2 = 4 \pi (0.5/2)^2$ — $A = 0.79 \ in^2$
Shear stress fv = P / A = 12 / 0.79 — fv = 15 ksi < 20

6 Shear wall
Assume: P = 20 k, 8" CMU wall, t = 7.625", L = 8', Fv = 30 psi
Shear area A = 7.625" x 12" x 8 ' — $A = 732 \ in^2$
Shear stress fv = P / A = 20,000 # / 732 — fv = 27 psi < 30



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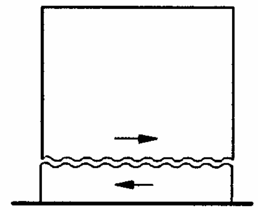

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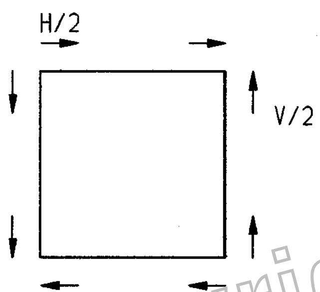

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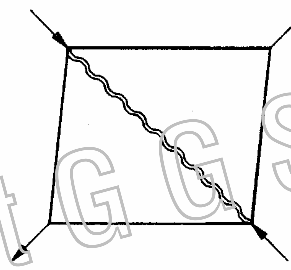

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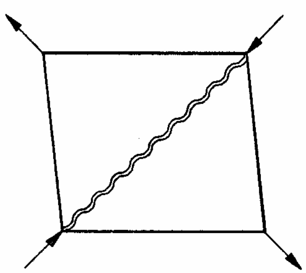

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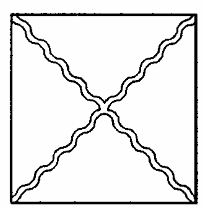

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Shear cracks

Shear cracks are diagonal, even for horizontal seismic forces. Although intuitive assumption suggest horizontal cracks for lateral load. The diagonal cracks are described, considering a square wall element subject to lateral load.

1 Lateral load generates a horizontal reaction at the base
The horizontal force couple tends to rotate the wall clockwise
A counterclockwise couple provides rotational equilibrium, $\Sigma M = 0$.

2 Incorrect intuitive assumption of horizontal shear crack

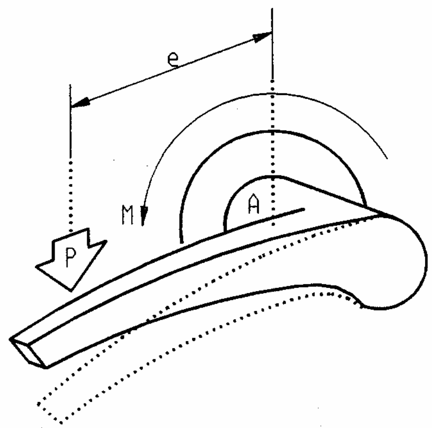
3 The horizontal and vertical couples presented as two vectors each

4 Combined shear vectors at each corner yield diagonal vectors
The diagonal vectors yield compression in one direction
The diagonal vectors yield tension in the opposite direction
The tension generates cracks

5 Reversed earthquake shaking generates cracks in opposite direction

6 Typical X-cracks caused by earthquakes

X-cracks caused by Northridge Earthquake



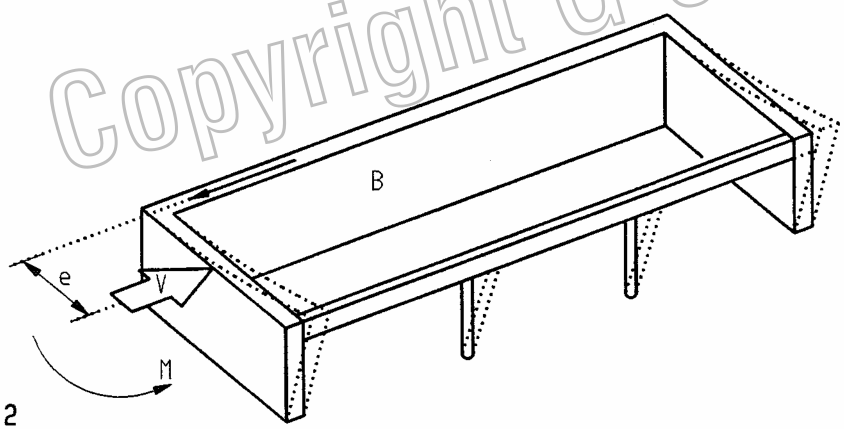
1

Torsion

Torsion is very common in machines but less common in building structures. The examples here include a small detail and an entire garage.

1 Door handle
Assume: P = 10 #, e = 3”

Torsion moment M
M = P e = 10 x 3 M = 30 #’



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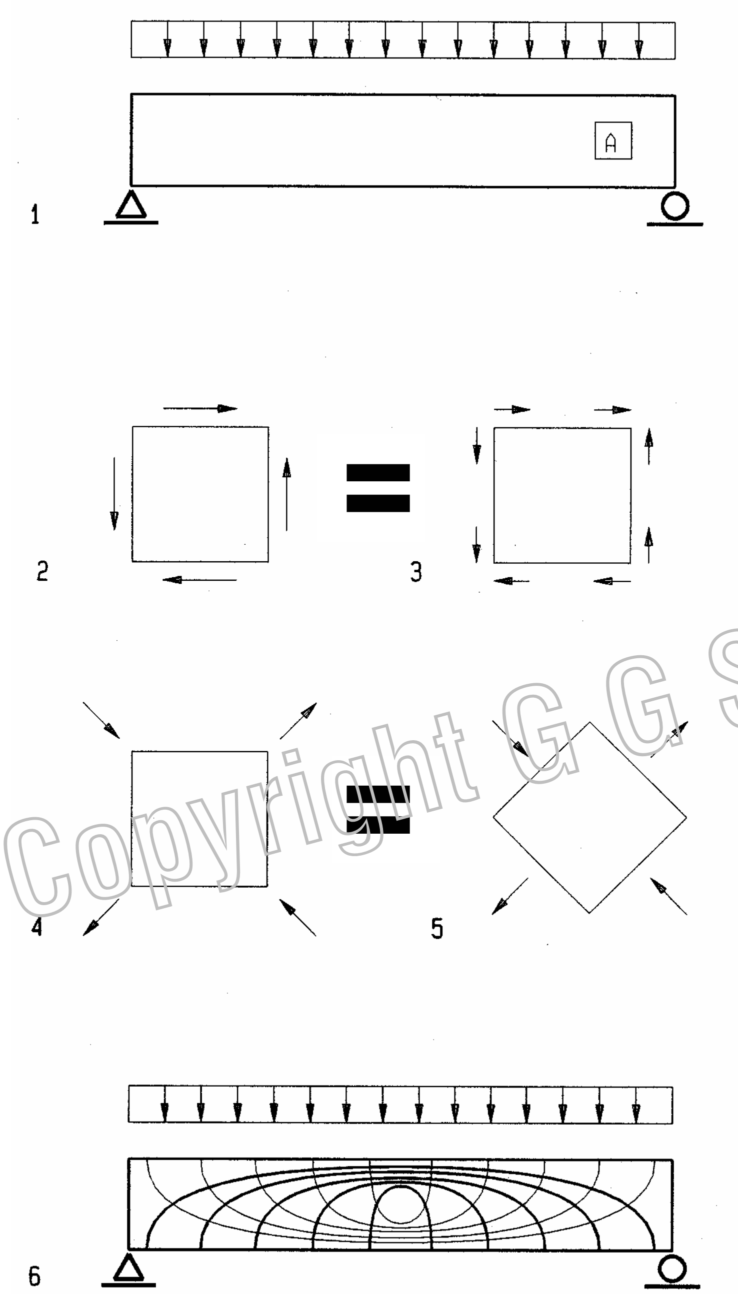
2 Tuck-under parking
Assume: Shear e =10’, base shear V = 12 k

Torsion moment M
M = V e = 12 k x 10’ M = 120 k’

Note:
The torsion moment is the product of base shear v and lever arm e, the distance from center of mass to center of resistance (rear shear wall).
In the past, torsion of tuck-under parking was assumed to be resisted by cross shear walls. However, since the Northridge Earthquake of 1994 where several buildings with tuck-under parking collapsed, such buildings are designed with moment resistant beam/column joints at the open rear side.



Principle stress

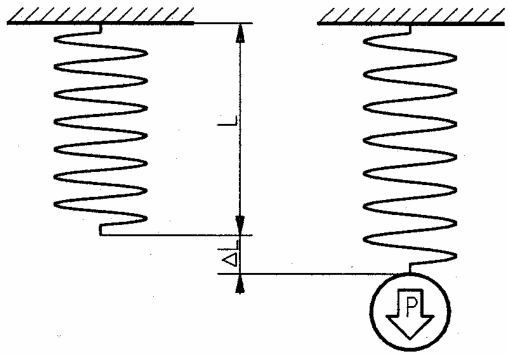
Shear stress in one direction, at 45 degrees acts as tensile and compressive stress, defined as *principle stress*. Shear stress is zero in the direction of principle stress, where the normal stress is maximum. At any direction between maximum principle stress and maximum shear stress, there is a combination of shear stress and normal stress. The magnitude of shear and principle stress is sometimes required for design of details. Professor Otto Mohr of Dresden University develop 1895 a graphic method to define the relationships between shear stress and principle stress, named ***Mohr's Circle***. Mohr's circle is derived in books on mechanics (Popov, 1968).

Isostatic lines

Isostatic lines define the directions of principal stress to visualize the stress trajectories in beams and other elements. Isostatic lines can be defined by experimentally by photo-elastic model simulation or graphically by Mohr's circle.

1 Simple beam with a square marked for investigation
2 Free-body of square marked on beam with shear stress arrows
3 Free-body square with shear arrows divided into pairs of equal effect
4 Free-body square with principal stress arrows (resultant shear stress vectors)
5 Free-body square rotated 45 degrees in direction of principal stress
6 Beam with isostatic lines (thick compression lines and thin tension lines)

Note:
Under gravity load beam shear increases from zero at mid-span to maximum at supports. Beam compression and tension, caused by bending stress, increase from zero at both supports to maximum at mid-span. The isostatic lines reflect this stress pattern; vertical orientation dominated by shear at both supports and horizontal orientation dominated by normal stress at mid-span. Isostaic lines appear as approximate tension "cables" and compression "arches".



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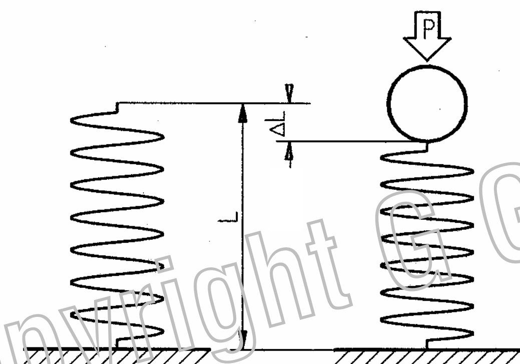

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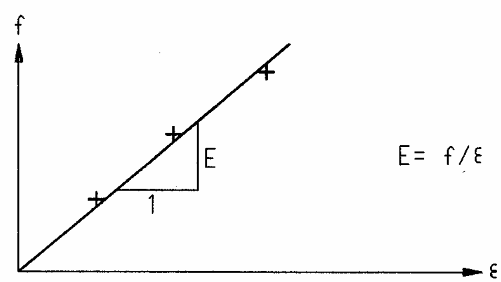

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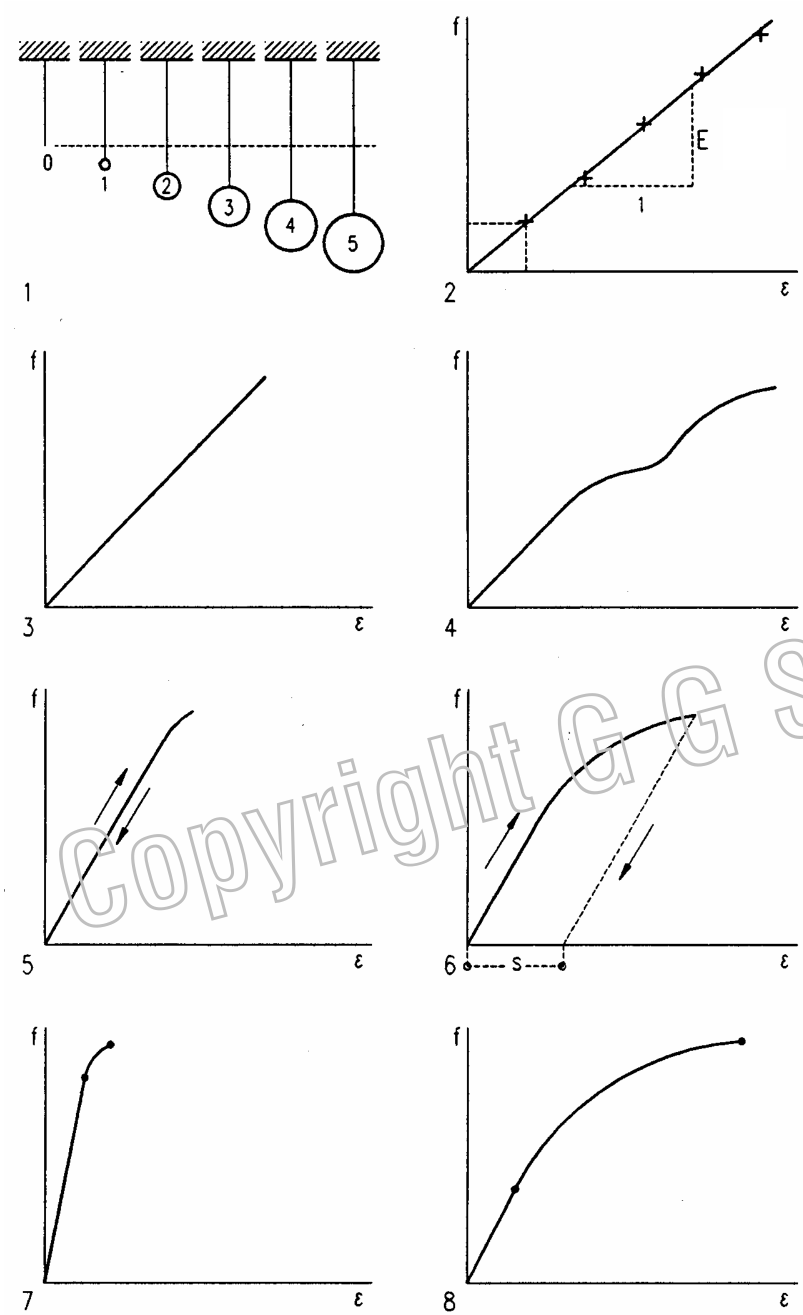
Strain

Strain is a deformation caused by stress, or change in temperature, described later. Strain may elongate or shorten a solid, depending on the type of stress.

Hooke's law

Material expands and contracts under tension and compression, respectively. The stress/strain relationship, called *Hooke's law* after the English scientist Robert Hooke, who discovered it in the 17th century, has since been confirmed by many empirical tests. The Hooke's law assumes isotropic material (equal properties in any direction). The stress/strain relation is visualized here by a spring, as substitute for rods as used in testing machines, to amplify the deformation.

1 Elongation due to tension
2 Shortening due to compression
3 Stress / strain graph
L Unstressed length
ΔL Strain (elongation or shortening under load)
P Applied load
ε Unit strain Epsilon ($\varepsilon = \Delta L/L$)
E Elastic modulus $E = f / \varepsilon f$ Stress $f = P / A$
A Cross section area of assumed rod



Stress/strain relations

Although stress/strain tests may be done for any materials, for convenience the following test description assumes a steel rod. After measuring the unstressed length, load is applied and the strain recorded. The load is then incrementally increased and all related elongations recorded on a Cartesian graph, strain on the horizontal axis, and stress on the vertical axis. The recorded measure points are connected by a line. A straight line implies linear stress/strain relations; a curved line implies non-linear relations. Most structural materials are linear up to the proportional limit, and non-linear beyond that point. If the rod returns to its original length after the load is removed, the material is considered *elastic;* if it remains deformed it is considered *plastic*. The remaining deformation is the *permanent set*. Rubber is an elastic material; clay a plastic material. Some materials, such as steel, are *elastic-plastic*, i.e., up to the *elastic limit* steel is elastic; beyond the elastic limit it is plastic. The transition from elastic to plastic strain is also called *yield point*. Materials which deform much and absorb energy before breaking are considered *ductile*; materials which break abruptly are considered *brittle*. Mild steel is considered a ductile material; concrete is usually brittle.

1 Test loads 1 to 5 kip
2 Stress-strain graph (horizontal axis = strain, vertical axis = stress)
3 Linear material (linear stress/strain relation
4 Non-linear material (non-linear stress /strain relation)
5 Elastic material (returns to original size if unloaded, like rubber)
6 Plastic material (remains permanently deformed like clay)
7 Brittle material (breaks abruptly)
8 Ductile material (deforms and absorbs energy before breaking)

Elastic modulus

E   E = f / ε = Elastic Modulus (defines material stiffness)
f   Stress
ε Unit strain ($\varepsilon = \Delta L/L$)
S   Permanent set (remaining strain after stress is removed)

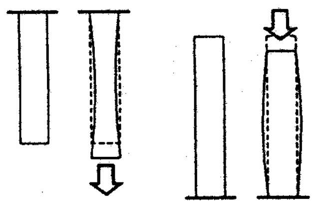
Derivation of working equation to compute strain:

$$\frac{\Delta L}{L} = \varepsilon = \frac{f}{E} = \frac{P}{AE}$$ solving for ΔL

$$\Delta L = PL / AE$$

The equation is used to compute strain due to load. It shows that strain:

- Increases with increasing P and L
- Increases inversely with A and E



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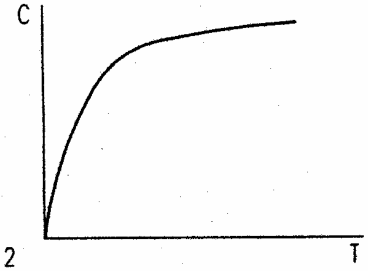

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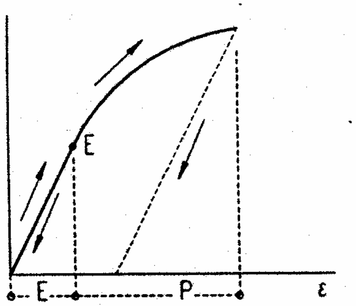

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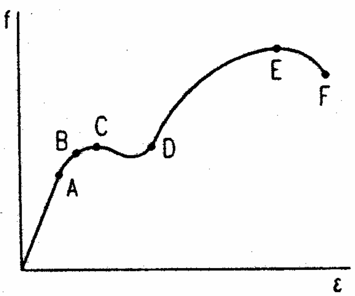

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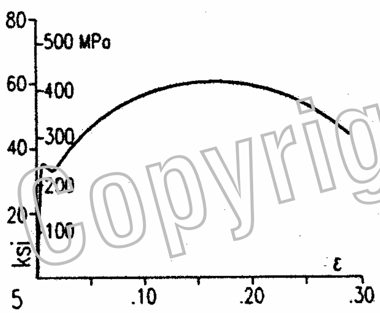

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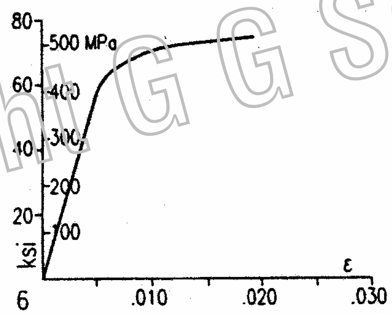

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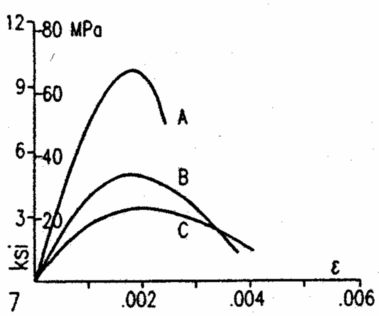

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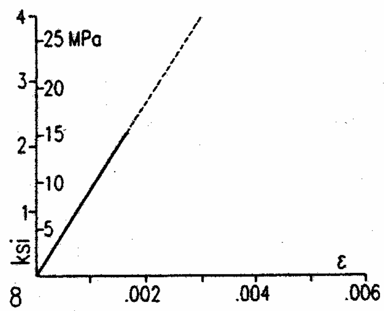

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Poisson's ratio

Poisson's ratio is named after French scientist *Poisson* who defined it 1807 as ratio of lateral strain / axial strain. All materials shrink laterally when elongated and expand when compressed. Poisson's ratio is defined as:

ν = lateral strain / axial strain

Based on empirical tests, Poisson's ratio for most materials is in the range of 0.25 to 0.35; only rubber reaches 0.5, the maximum for isotropic material.

Creep

Creep is a time dependent strain, most critical in concrete where it is caused by moisture squeezed from pores under stress. Creep tends to diminish with time. Concrete creep may exceed elastic strain several times, as demonstrated by Case Study 9 of Northridge Earthquake failures (Schierle, 2002). Yet much research is needed to provide design data and guidelines regarding creep.

Elastic modulus

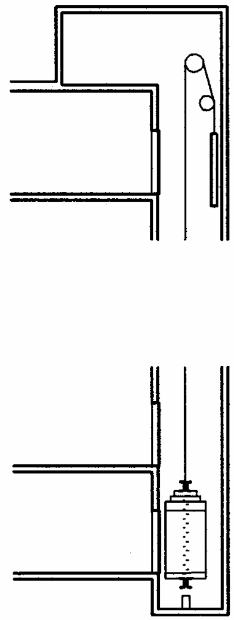
The elastic modulus E, also called modulus of elasticity or Young's modulus Y, after English scientist Young, who defined it 1807. The term elastic modulus is actually a misnomer since it defines stiffness, the opposite of elasticity.

Note:

Since $E=f/\varepsilon$ and ε is a ratio without units, the elastic modulus has the same units as stress

1 Poisson's ratio effect
2 Creep deformation (C = creep, T = time)
3 Elastic / plastic stress / strain curve (E = elastic range, P = plastic range)
4 Abstract steel graph (A = *proportional limit*, B = *elastic limit*, C = *yield point*, CD = *yield plateau*, E = *ultimate strength*, F = *breaking point*)
5 Mild steel stress / strain curve
6 High strength steel stress / strain curve
7 Concrete stress / strain curve (compressive strengths: A=9 ksi, B=4 ksi, C=3 ksi)
8 Stress / strain of linearly elastic wood

Allowable stress vs. elastic modulus (typically about 1:1000 ratio)		
Material	Allowable stress (psi)	Elastic modulus (psi)
Wood	1,400	1,400,000
Steel	30,000	30,000,000
Masonry	1,500	1,500,000
Concrete	3,000	3,000,000



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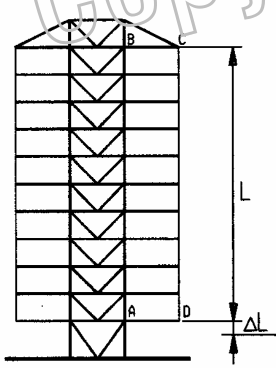

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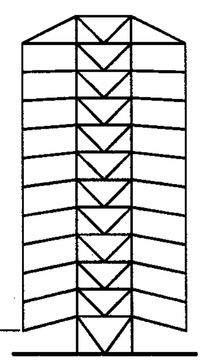

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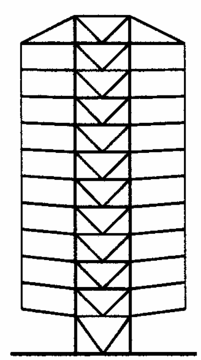

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Strain examples

Elevator cables

Assume

4 cables ϕ ½" each, 60% metallic areaBreaking strength Fy = 210 ksi

Allowable stress (210 ksi / 3)	Fa = 70ksi
Elastic Modulus	E = 16,000ksi
L = 800' each	
P = 8k	
Metallic area	
Am = 4 π r^2 = 4 x .6 π $(0.5/2)^2$	Am = 0.47in^2
Stress	
f = P / A = 8 / 0.47	f = 17ksi
	17 < 70, ok
Elongation under load	
ΔL = PL / AE	
ΔL = 8k x 800" x 12" / (0.47x16000)	ΔL = 10"

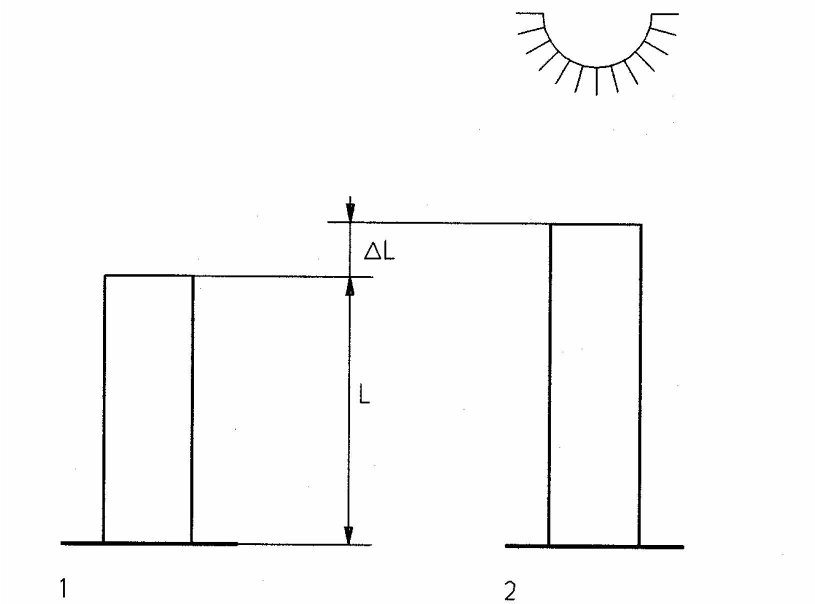
2 Suspended building

3 Differential strain

Assume	
10 stories @ 14' = 10x14'x12"	L = 1680"
Average column stress	f = 18 ksi
Average strand stress	f = 60 ksi
Elastic modulus (steel)	E = 29,000 ksi
Elastic modulus (strand)	E = 22,000 ksi
ΔL = PL/AE, since f = P/A $\rightarrow$ ΔL = f L/E	
Column strain	
ΔL = 18ksi x 1680" / 29000	ΔL = 1"
Strand strain	
ΔL = 60ksi x 1680" / 22000	ΔL = 4.6"
Differential settlement	ΔL = 5.6"

4 Shorten hangers under DL to reduce differential strain, or prestress strands to reduce ΔL by half

Note: Differential strain is additive since both strains are downwards
To limit differential strain, suspended buildings have <= 10 stories / stack



Thermal strain

Unrestrained objects expand and contract if subjected to temperature increase and decrease, respectively. Thermal strain is defined by a coefficient α for each material. Thermal strain varies linearly with temperature variation.

1 Bar of initial length L
2 Thermal strain ΔL due to temperature increase, computed as:

$$\Delta L = \alpha \, \Delta t \, L$$

where

α = thermal coefficient (in/in/^oF) [/^oC (SI units)]
Δt = temperature increase (+) / decrease (-)
L = initial length

Thermal stress

Thermal stress is caused when thermal strain is prevented by restrains.

3 Bar of initial length L
4 Elongation ΔL due to heat
5 Heated bar reduced to initial length by load P
6 Restrained bar under stress

Thermal stress derivation:

Since $\Delta L = PL / AE$ and $f = P/A$

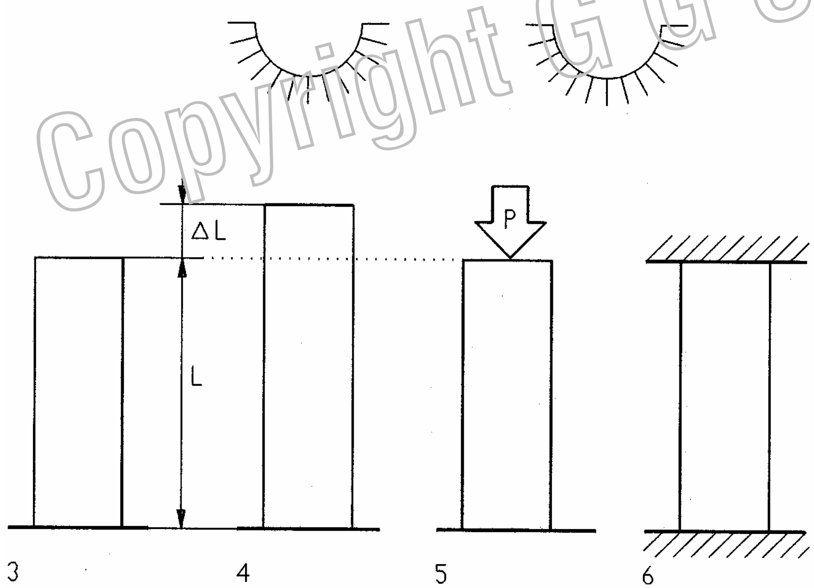
$\Delta L = f L / E \rightarrow f = E \, \Delta L/L$

$\Delta L = \alpha \, \Delta t \, L \rightarrow f = E \, \alpha \, \Delta t \, L/L$

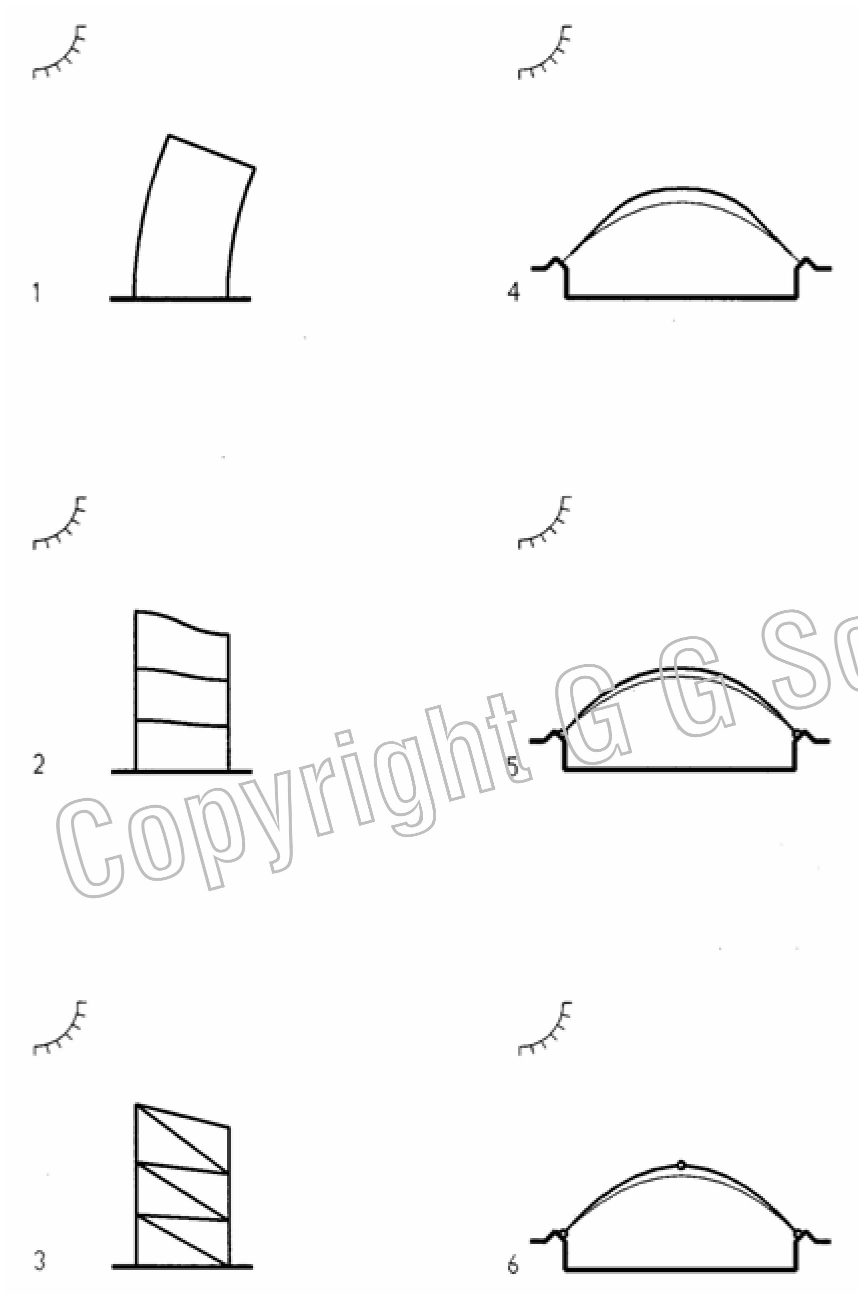
$$f = \alpha \, \Delta t \, E$$

where

f = thermal stress
E = elastic modulus



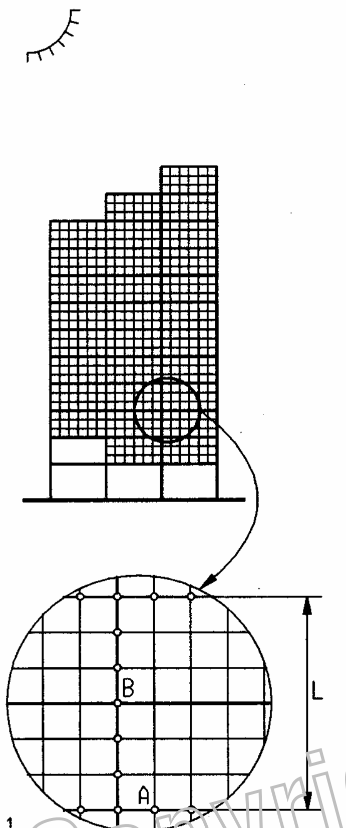
Coefficient of thermal expansion α and elastic modulus E				
Material	US α ($10^{-6}/^oF$)	US Eα (10^6psi)	SI α ($10^{-6}/^oC$)	SI Eα (10^6gPa)
Aluminum	13	10	24	69
Steel	6.5	29	11.7	200
Concrete	6	3 – 4	11	20 – 28
Masonry	4	1 – 3	7	7 – 21
Wood	1.7 – 2.5	1.2 – 2.2	3.5 - 4.5	8 – 15
Glass	44	9.6	80	66
Plastics	68 - 80	0.3 – 0.4	122 - 144	2 – 2.8
Aluminum	13	10	24	70



Thermal effects

Thermal variations cause stress and/or strain in structures. Temperature increase and decrease cause material to expand and contract, in unrestrained objects and stress in restrained objects, respectively, as illustrated in the examples at left.

1 Wall subject to bending stress due to temperature variation
1 Expanding south column causes bending stress in beams
2 Expanding south column causes tensile stress in bracing
3 Expanding fix-end arch subject to reversed bending stress
4 Expanding pin-joined arch subject to bending stress
5 Three-hinge arch, free to expand, without bending stress



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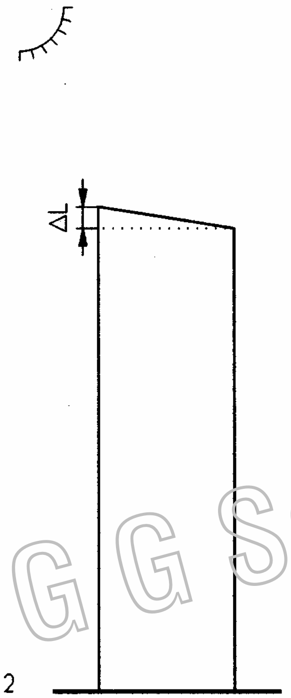

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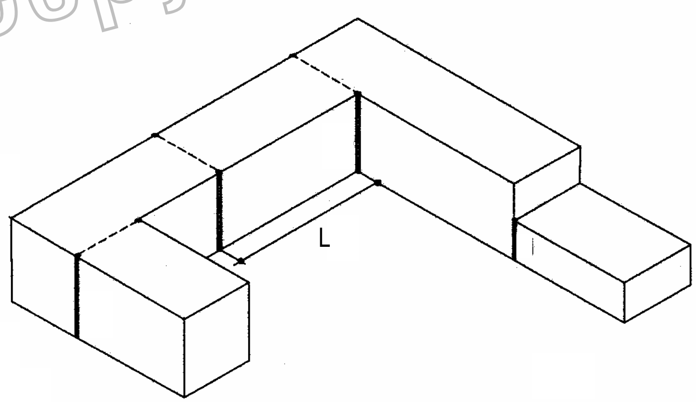

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Thermal examples

1 Curtain wall

Assume:

Aluminum curtain wall, find required expansion joint

$\Delta t = 100\ ^{o}$ F (summer vs. winter temperature)

2 story mullion, L = 30' x 12" = 360"

$\alpha = 13 \times 10^{-6}$ in/in/^{o} F

$E = 10 \times 10^{6}$ psi

Thermal strain

$\Delta L = \alpha\ \Delta t\ L = 13 \times 10^{-6} \times 100\ ^{o} \times 360"$ — $\Delta L = 0.47"$

Use ½" expansion joints — 0.5 > 0.47

Assume ignorant designer forgets expansion joint

Thermal stress:

$f = \alpha\ \Delta t\ E = 13x10^{-6}x100x10x10^{-6}$ psi — f = 13,000 psi

Note: 10^{6} and 10^{-6} cancel out and can be ignored

13,000 psi is too much stress for aluminum

2 High-rise building, differential expansion

Assume:

Steel columns exposed to outside temperature

$\Delta t = 50^{o}$F (south vs. north temperature)

L = 840' (60 stories at 14')

$\alpha = 6.5 \times 10^{-6}$ in/in/^{o} F

Differential expansion

$\Delta L = \alpha\ \Delta t\ L = 6.5 \times 10^{-6} \times 50\ ^{o} \times 840' \times 12"$ — $\Delta L = 3.3"$

Note: the differential expansion would cause bending stress

3 Masonry expansion joints

(masonry expansion joints should be at maximum L = 100')

Assume

Temperature variation $\Delta t = 70^{o}$ F

Joint spacing L=100' x 12" — L = 1200"

Thermal coefficient — $\alpha = 4x10^{-6}/^{o}F$

E-modulus — $E = 1.5x10^{6}$ psi

Required joint width

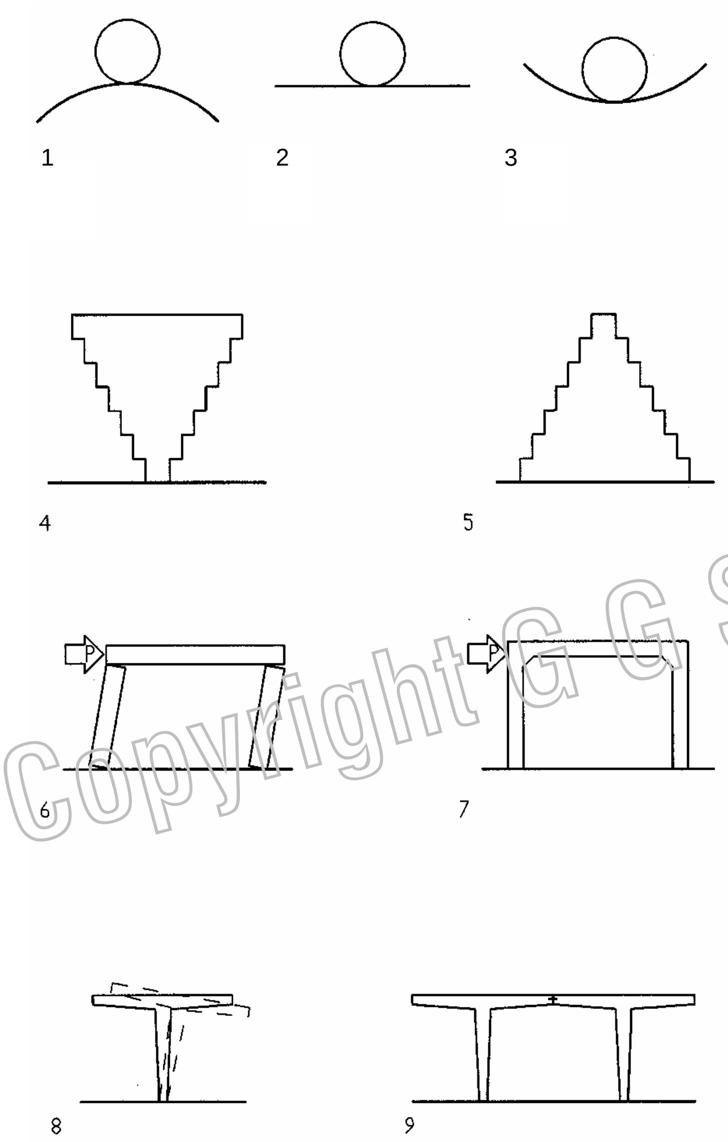
$\Delta L = \alpha\ \Delta t\ L = (4 \times 10^{-6})\ 70^{o}\ (1200")$ — $\Delta L = 0.34"$

Use 3/8" expansion joint — 0.375 > 0.34

Check thermal stress without expansion joint

$f = \alpha\ \Delta t\ E$

$f = 4x10^{-6} \times 70^{o} \times 1.5x10^{6}$ — f = 280psi



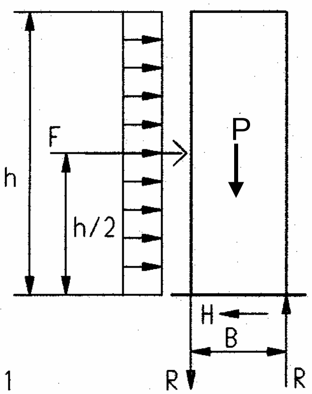
Stability

Stability is more complex and in some manifestations more difficult to measure than strength and stiffness but can be broadly defined as capacity to resist:

- Displacement
- Overturning
- Collapse
- Buckling

Diagrams 1-3 give a theoretical definition; all the other diagrams illustrate stability of conceptual structures.

1 Unstable
2 Neutral
3 Stable
4 Weak stability: high center of gravity, narrow base
5 Strong stability: low center of gravity, broad base
6 Unstable post and beam portal
7 Stable moment frame
8 Unstable T-frame with pin joint at base
9 Stable twin T-frames



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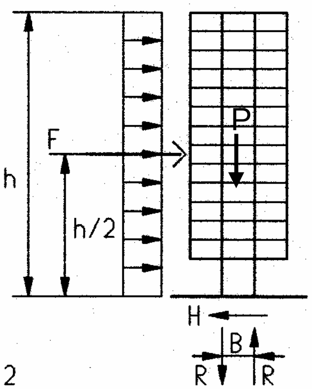

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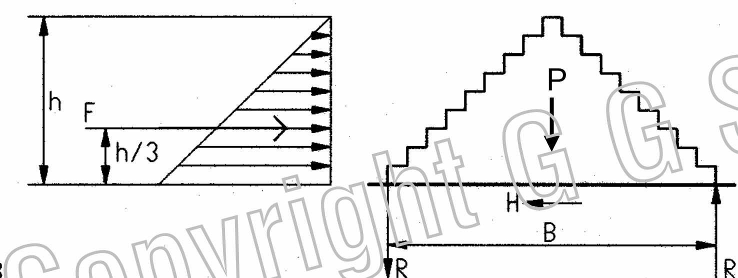

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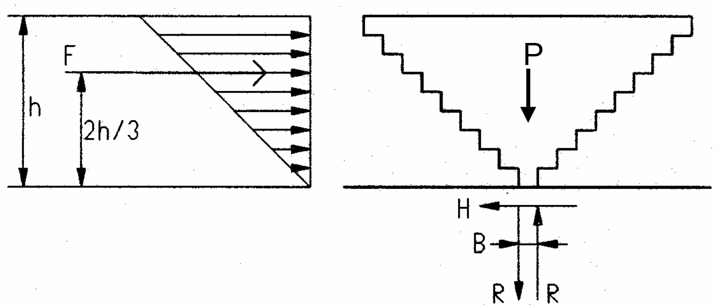


4

Overturn stability

To resist overturning under lateral load requires a stabilizing moment greater than the overturning moment (usually with a safety factor of 1.5). Stabilizing moments are dead weight times lever arm (distance from center of mass to edge of resisting element, assuming a rigid body) as demonstrated on the following examples (assuming uniform wind load for simplicity).

1 Building of vertical extrusion

Assume: 20 stories, 90'x90', B = 90', h = 300	
Wind force F = 70 psf x 90 x 300 / 1000	F = 1,890 k
Overturning moment Mo = F h/2 = 1890x300/2	Mo = 283,500 k'
Dead load P = 50 psf x 90^2 x 20/1000	P = 8.100 k
Stabilizing moment Ms = P B/2 = 8100 x90/2	Ms = 364,500 k'
Check stability (MS > Mo ?)	364,500 > 283,500

1 Building with cantilever core

Assume: 20 stories, 90'x90', B = 300', h = 300	
Wind force F = 70 psf x 90 x 300/1000	F = 1,890 k
Overturning moment Mo = F h/2 = 1890x300/2	Mo = 283,500 k'
Dead load P = 50 psf x 90^2 x 20/1000	P = 8.100 k
Stabilizing moment Ms = P B/2 = 8100x30/2	Ms = 121,500 k'
Check stability (MS > Mo ?)	121,500 < 283,500

Core is unstable without tension piles or large footing

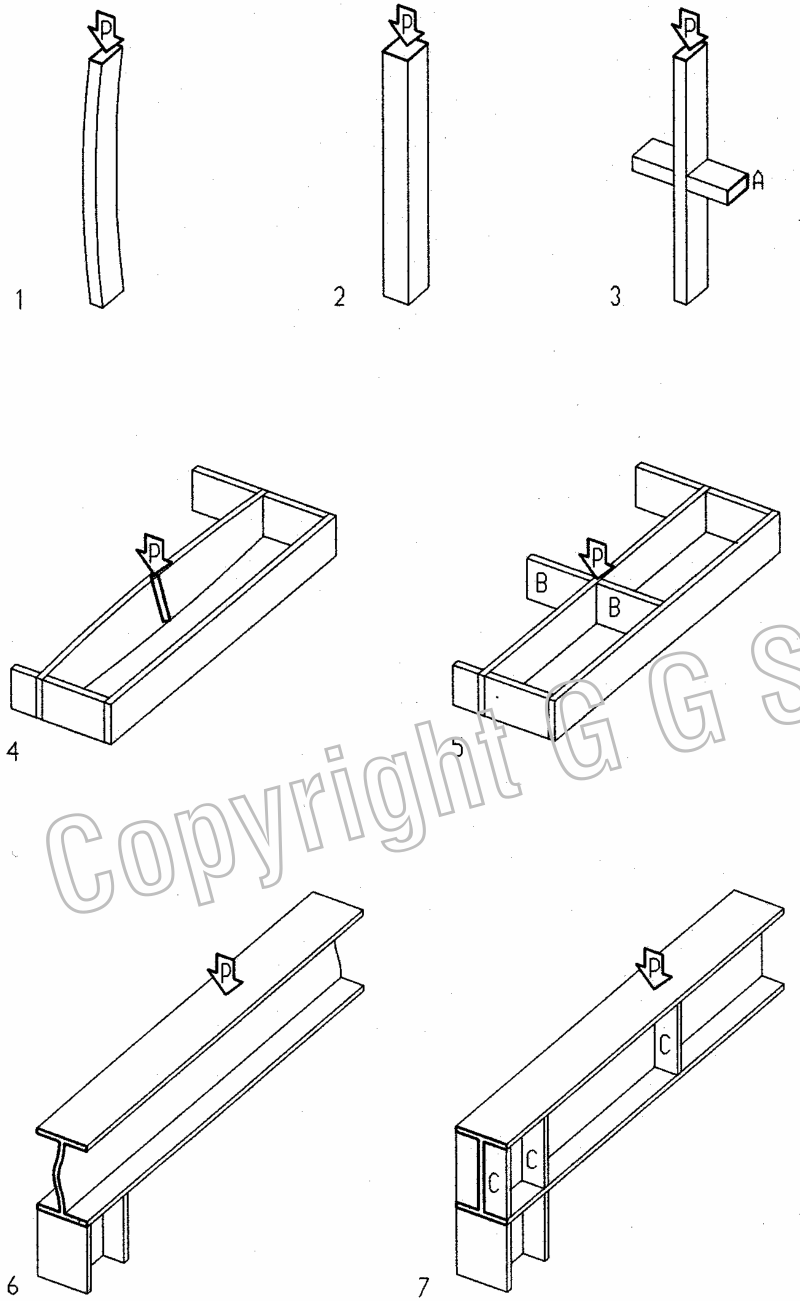
3 Pyramid

Assume: 9 stories, h = 108', B = 204'	F = 750 k
Dead load	P = 4800 k
Overturn moment Mo = F h/3 = 750x108/3	27,000 k'
Stabilizing moment Ms = P B/2 = 4800x204/2	Ms = 489,600 k'
Check stability (MS > Mo ?)	489,600 >> 27,000

4 Inverted Pyramid

Assume: 9 stories, h = 108', B = 12'	F = 750 k
Dead load	P = 4800 k
Overturn moment Mo = F 2/3 h = 750 x 2/3 x 108	54,000 k'
Stabilizing moment Ms = P B / 2 = 4800 x 12 / 2	Ms = 28,800 k'
Check stability (MS > Mo)	28,800 < 54,000

Inverted pyramid is unstable without tension piles or large footing



Buckling stability

Buckling stability is more complex to measure than strength and stiffness and largely based on empirical test data. This introduction of buckling stability is intended to give only a qualitative intuitive understanding.

Column buckling is defined as function of slenderness and beam buckling as function of compactness. A formula for column buckling was first defined in the 18th century by Swiss mathematician Leonhard Euler. Today column buckling is largely based on empirical tests which confirmed Euler's theory for slender columns; though short and stubby columns may crush due to lack of compressive strength.

Beam buckling is based on empirical test defined by compactness, a quality similar to column slenderness.

1 Slender column buckles in direction of least dimension
2 Square column resist buckling equally in both directions
3 Blocking resists buckling about least dimension
4 Long and slender wood joist subject to buckling
5 blocking resists buckling of wood joist
6 Web buckling of steel beam
7 Stiffener plates resist web buckling

A Blocking of wood stud
B Blocking of wood joist
C Stiffener plate welded to web
P Load

6

Bending

Bending elements are very common in structures, most notably as beams. Therefore, the theory of bending is also referred to as beam theory, not only because beams are the most common bending elements but their form is most convenient to derive and describe the theory. For convenience, similar elements, such as joists and girders, are also considered beams. Although they are different in the order or hierarchy of structures, their bending behavior is similar to that of beams, so is that of other bending elements, such as slabs, etc., shown on the next page. Thus, although the following description applies to the other bending elements, the beam analogy is used for convenience.

Beams are subject to load that acts usually perpendicular to the long axis but is carried in bending along the long axis to vertical supports. Under gravity load beams are subject to bending moments that shorten the top in compression and elongate the bottom in tension. Most beams are also subject to shear, a sliding force, that acts both horizontally and vertically. Because beams and other bending elements are very common, the beam theory is important in structural design and analysis.

As for other structural elements, beam investigation may involve analysis or design; analysis, if a given beam is defined by architectural or other factors; design, if beam dimensions must be determined to support applied loads within allowable stress and deflection. Both, analysis and design require finding the tributary load, reactions, shear, and bending moment. In addition, analysis requires to find deflections, shear- and bending stress, and verify if they meet allowable limits; by contrast design requires sizing the beam, usually starting with an estimated size.

The following notations are commonly used for bending and shear stress:

f_b= actual bending stress F_b= allowable bending stress

f_v= actual shear stress F_v= allowable shear stress

Allowable stresses are given in building codes for various materials.
Allowable stresses assumed in this chapter are:

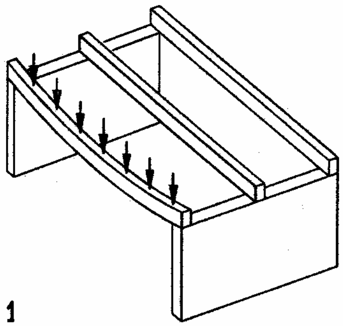
Wood

F_b= 1450 psi (9998 kPa) F_v= 95 psi (655 kPa)

Steel

F_b= 22 ksi (152 MPa) F_v= 14 ksi (97 MPa)

The more complex design and analysis of concrete and masonry will be introduces-later.



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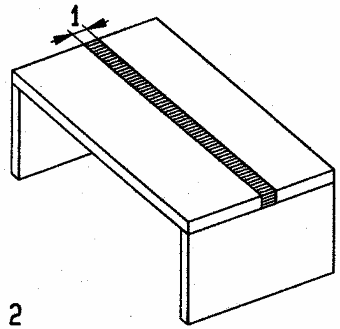

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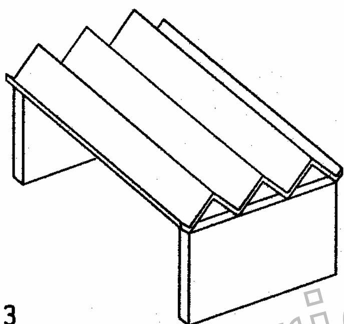

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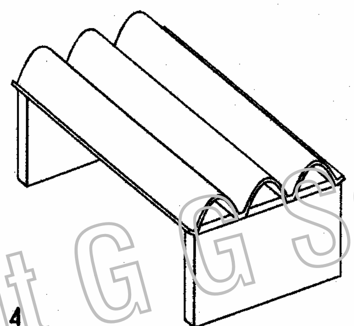

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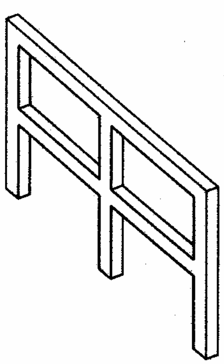

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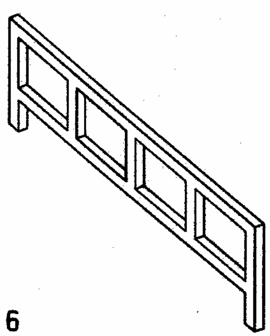
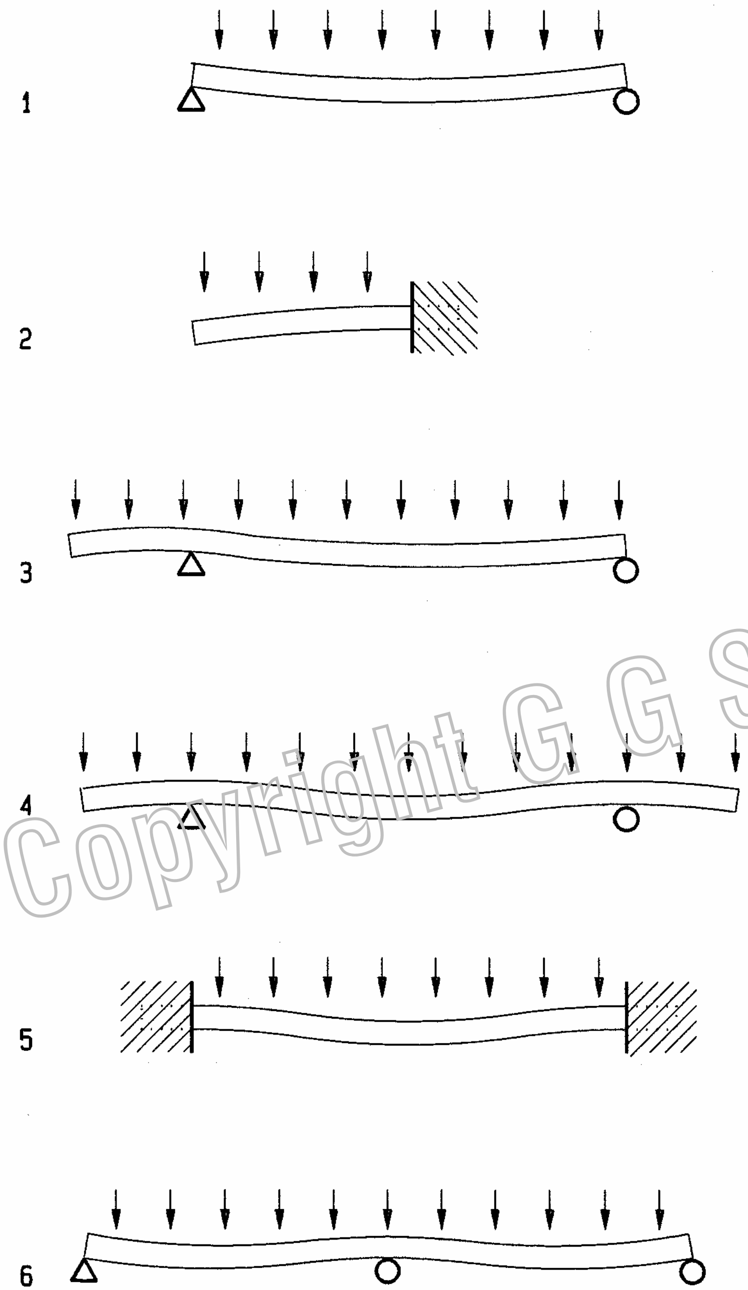


6

Bending elements

As mentioned above, beams are the most common bending elements and their structural behavior described in this chapter applies in general to other bending elements as well. Other bending elements, explored later, include joists, girders, slabs and plates (analyzed as strip of unit width) as well as folded plates, cylindrical shells, moment frames, and *Vierendeel* girders (named after the Belgian inventor of the 19th century). Thus, the theory of bending and shear has broad implications and is very important for structural analysis and design.

1 Beams, one shown deformed under uniform gravity load
2 Slab or deck with a strip of unit width marked for analysis as "beam"
3 Folded plate acts as narrow, inclined beams leaning against one-another
4 Cylindrical shell, acts as beams of semi-circular cross-section
5 Moment frame resists gravity and lateral load in bending
6 *Vierendeel* girder (named after the Belgian inventor of it)



Beam types

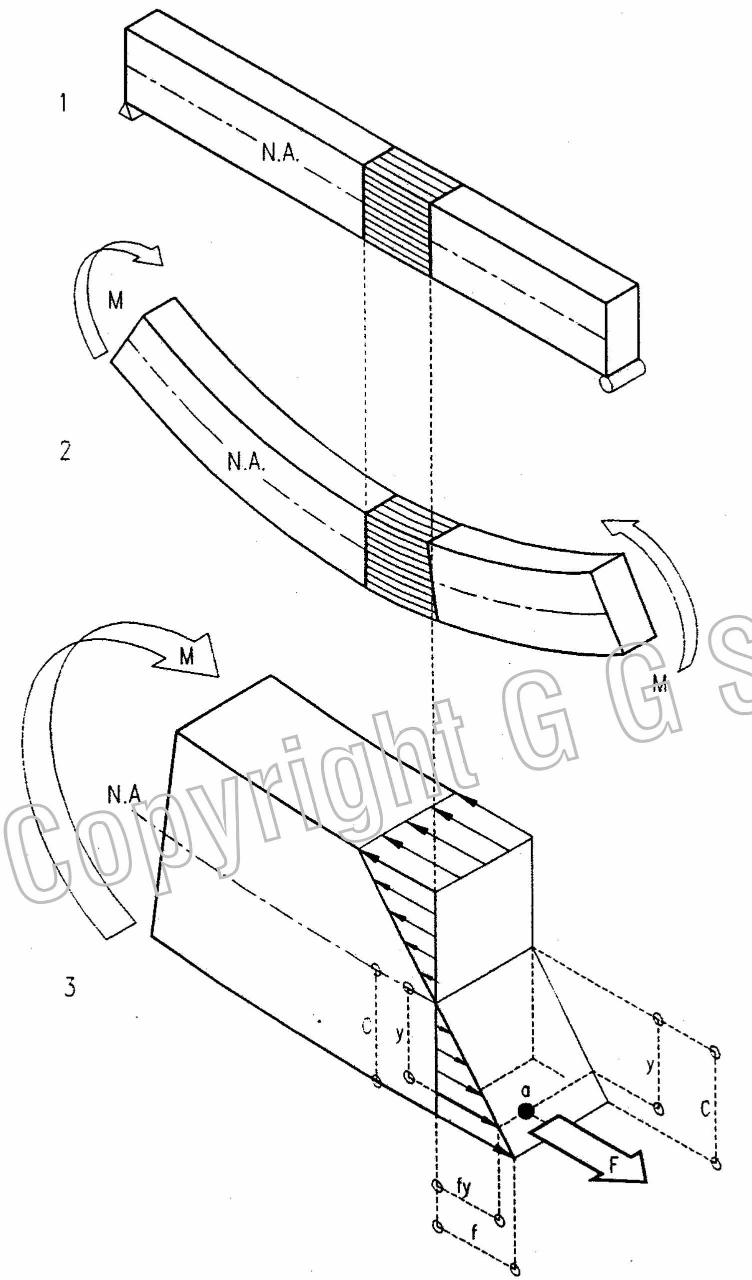
The location, number, and type of supports determine the type of beam.

1 Simple beam
2 Cantilever beam
3 Beam with one overhang
4 Beam with two overhangs
5 Restrained beam
6 Continuous beam

The simple beam is most common in practice. It has two supports, one pin and one roller, and, with three unknown reactions, it is statically determinate. Given their pin and roller supports, overhang beams are also determinate; by contrast restraint and continuous beams are statically indeterminate, since they have more than three unknown reactions.

The simple beam has single concave curvature that results in compression on top and tension at the bottom of the beam. The cantilever beam has single convex curvature, with tension on top and compression at the bottom of the beam. All the other beams change from concave to convex curvatures. Because the cantilever beam has only one support, it must have fixed (moment resistant) support to be stable.

Given equal loads and spans, the cantilever beam has the largest bending moment, followed by the simple beam with half that of the cantilever. Beams with overhang have negative overhang moments that reduces positive field moments. Their reduced bending moment is less than the moment of a simple beam of equal span. Two overhangs yield smaller field moments than a single overhang. Designing a beam with one or two overhangs is a good strategy to greatly reduce bending for better efficiency without extra cost. Given the double curvature of restrained and continuous beams, they, too, have reduced bending moments. In restrained beams this advantage may be in part offset by the fact that the negative end moments must be resisted by supports. The interaction of beam and column provides lateral resistance for moment frames.



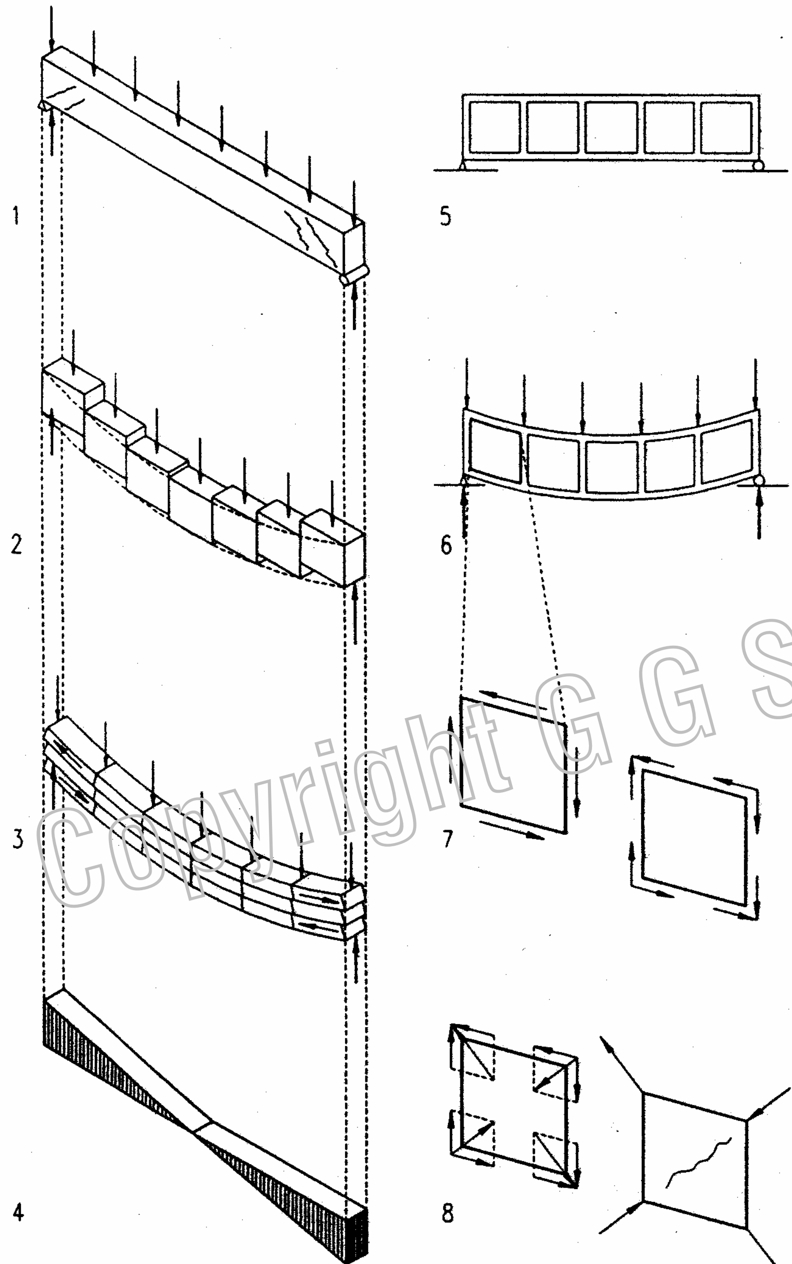
Bending and Shear

Although derivation and numeric examples are required to analyze and design beams, intuitive understanding is an important prerequisite to gain deeper insight into the behavior of beams. The following is an intuitive introduction to beam bending and shear. A simple beam with uniform load is used for convenience.

Bending moment

Gravity load on a simple beam shortens the top and elongates the bottom, causing compressive and tensile stresses at top and bottom, respectively; with zero stress at the neutral axis (N. A.). In beams of symmetrical cross-section, the neutral axis is at the center. The compressive and tensile stress blocks generate an internal force couple that resists the external bending moment caused by load.

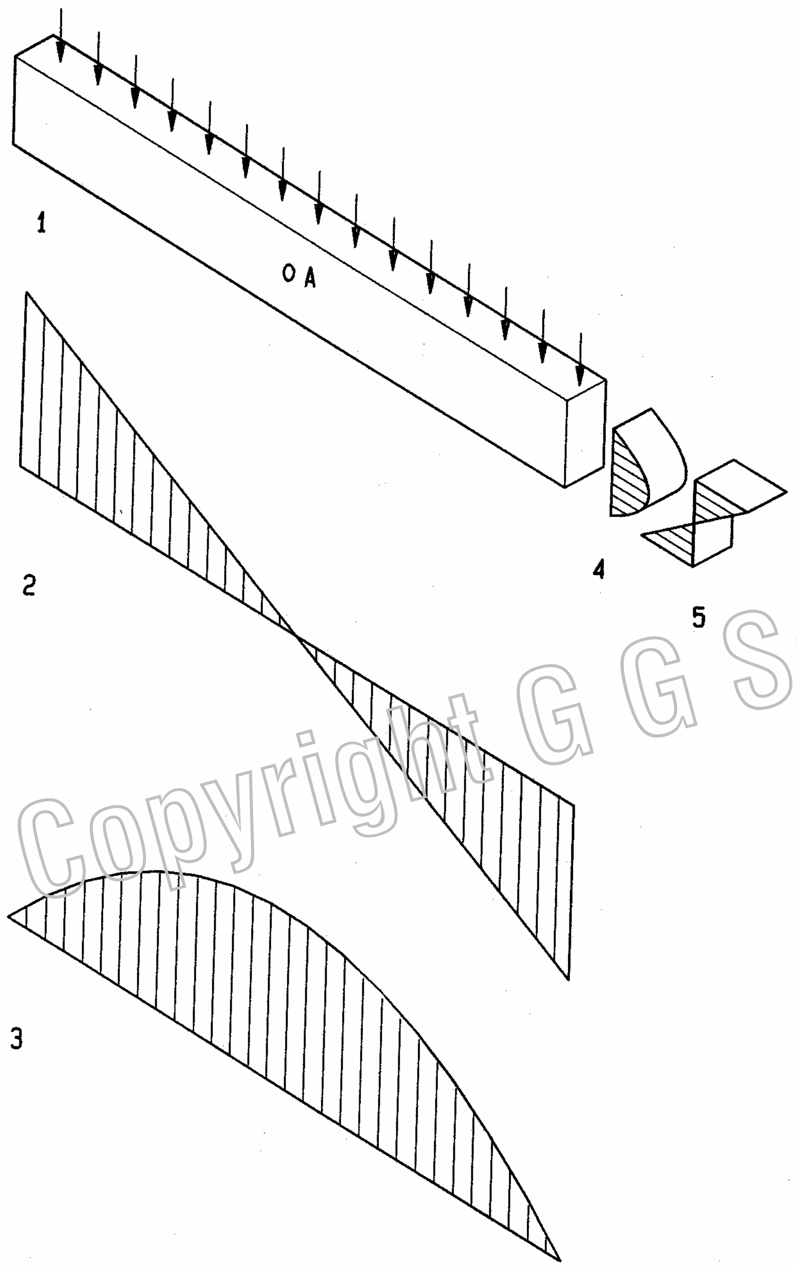
1 Simple beam with pin and roller supports
2 Deformed beam under uniform gravity load
3 Free-body diagram with bending stress block that generates an internal force couple to resist the external bending moment caused by load



Shear force

With few exceptions, described later, shear coexists with bending. When shear is present it acts both horizontally and vertically at equal magnitude. In wood beams horizontal shear is more critical because wood's shear capacity is much smaller parallel than perpendicular to the grain.

1 Beam under uniform load with shear cracks as they occur in some concrete beams near the supports where shear is maximum
2 Tendency of beam parts to slide vertically generates vertical shear stress that is zero at mid-span and increases to maximum at the supports where the vertical shear deformation is greatest
3 Tendency of beam layers to slide horizontally generates horizontal shear that is zero at mid-span and increases toward the supports. This is visualized, assuming a beam composed of several boards
4 Shear diagram reflects shear distribution over beam length
5 Unloaded beam marked with squares to visualize shear
6 Loaded beam with squares deformed into rhomboids due to shear
7 Horizontal and vertical shear couples on a square beam part are equal to balance rotational tendencies ($\Sigma M = 0$). Therefore, horizontal and vertical shear stresses are equal at any point on the beam.
8 Shear vectors generate compression and tension diagonal to the shear. This tends to generate diagonal tension cracks in concrete beams



Bending and shear distribution

Shear and bending diagrams illustrate their respective distribution over the beam's length (simple beam in this case). Similarly, internal shear and bending stresses, caused by shear force and bending moment, respectively, may be drawn to illustrate their distribution over the beam's cross-section.

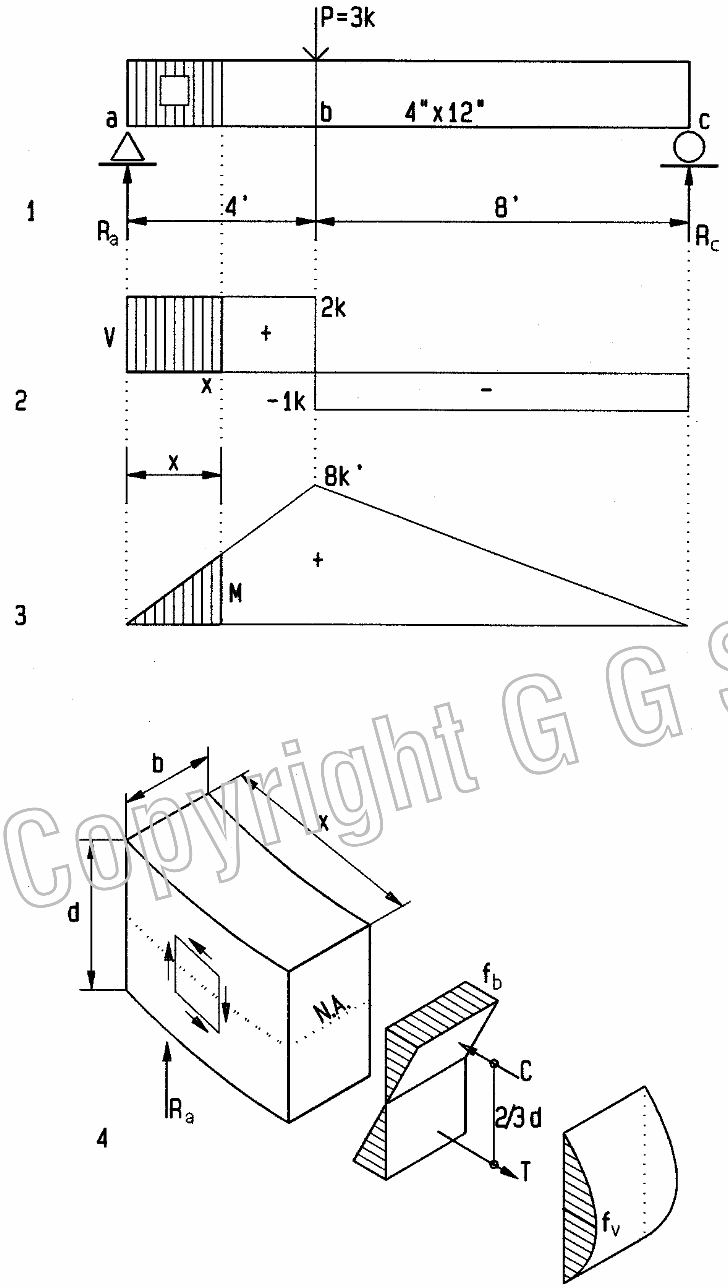
1 Beam diagram
2 Shear diagram shows shear force distribution over beam length
3 Bending diagram shows bending moment distribution over beam length
4 Shear stress diagram shows distribution over beam depth
5 Bending stress diagram shows distribution over beam depth

A Possible location of a pipe hole at beam center and mid-span where both shear and bending stresses are zero

The shear force varies linearly from maximum positive shear at the left support to maximum negative shear at the right support. The left reaction pushing upward and the beam load downward cause a maximum positive (clockwise) shear couple at the left support. Shear reduces to zero at mid-span where it is balanced by gravity load. At the right support shear reaches a negative maximum.

The bending moment varies in parabolic form over the beam's length. It is zero at both supports and maximum at mid-span. Over the depth of the beam, bending stress varies from maximum compression on the top to maximum tension at the bottom.

The variation of bending over the beam length creates differential bending stress that is unbalanced. Thus, the compressive and tensile bending stresses push in opposite directions which causes horizontal shear stress. Shear stress varies from zero on top and bottom to maximum at the neutral axis. The rare case of uniform bending over the beam length, i.e., no differential bending stress, causes zero shear stress. This is called pure bending.



Bending and shear stress

Bending and shear stresses in beams relate to bending moment and shear force similar to the way axial stress relates to axial force (f = P/A). Bending and shear stresses are derived here for a rectangular beam of homogeneous material (beam of constant property). A general derivation follows later with the *Flexure Formula*.

1 Simple wood beam with hatched area and square marked for inquiry
2 Shear diagram with hatched area marked for inquiry
3 Bending moment diagram with hatched area marked for inquiry
4 Partial beam of length x, with stress blocks for bending f_b and shear f_v, where x is assumed a differential (very small) length

Reactions, found by equilibrium $\Sigma M = 0$ (clockwise +)

at c: $+12\,R_a-3(8)=0$; $R_a=3(8)/12$ — $R_a = 2$ k
at a: $-12\,R_c+3(4)=0$; $R_c=3(4)/12$ — $R_c = 1$ k

Shear V, found by vertical equilibrium, $\Sigma V=0$ (upward +).

right of a and left of b — V= 0+2 — V = 2 k
right of b and left of c — V= 2-3 — V = -1 k

Bending moment M, found by equilibrium $\Sigma M=0$ (clockwise +)

at a: — M = +2(0) — M = 0 k'
at b: — M = +2(4) — M = 8 k'
at c: — M = +2(12)-3(8) — M = 0 k'

Bending stress f_b is derived, referring to 4. Bending is resisted by the force couple C-T, with lever arm 2/3 d = distance between centroids of triangular stress blocks. C=T= f_b bd/4, M= C(2d/3) = ($f_b$bd/4)(2d/3) = $f_b bd^2/6$, or

f_b= M/(bd^2/6); where $bd^2/6$ = S= *Section Modulus* for rectangular beam; thus

$$f_b = M/S$$

For our beam: $S= bd^2/6= 4(12)^2/6=$ — $S = 96\ in^3$
f_b= M/S= 8(1000)12/96 * — f_b = 1000 psi

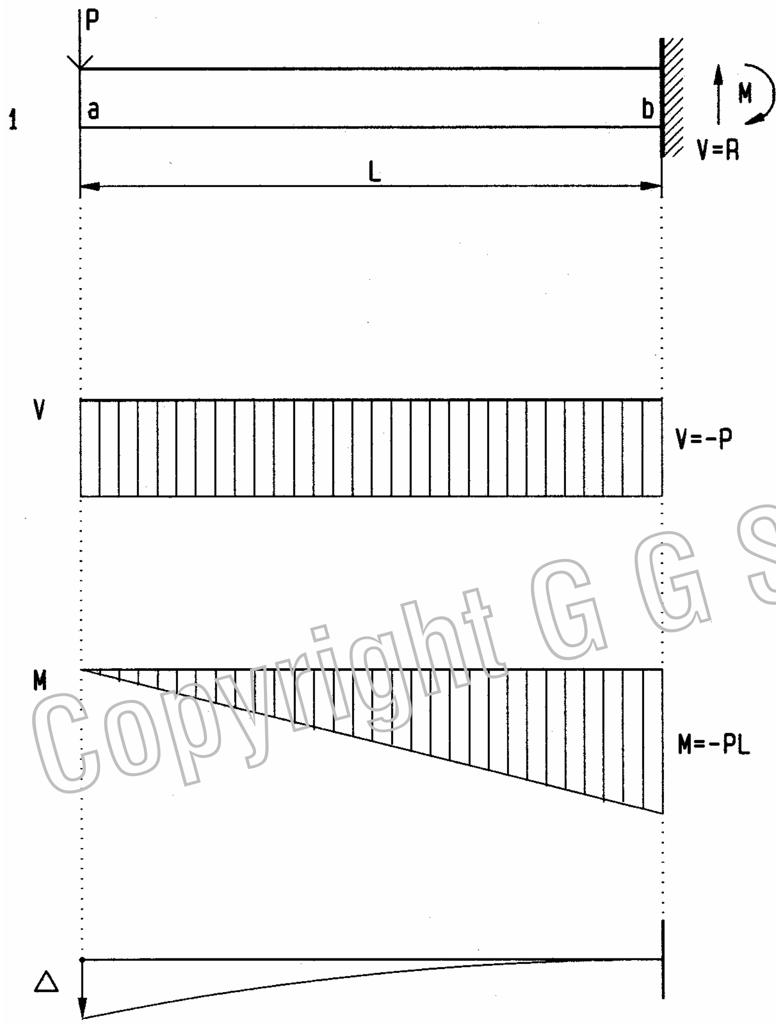
* multiplying by 1000 converts kips to pounds, by 12 converts feet to inches.

Shear stress f_v is derived, referring to 4. Bending stress blocks pushing and pulling in opposite directions create horizontal shear stress. The maximum shear stress is f_v=C/bx, where b = width and x = length of resisting shear plane. Shear at left support is V = R. Let M = bending moment at x, and f_b= bending stress at x, then M = RX = VX, and f_b=M/S=Vx/S. Substituting Vx/S for f_b in C = $f_b$bd/4, the compressive top force, yields C=(Vx/S)(bd/4). Thus f_v=C/(bx) yields f_v=(Vx/S)(bd/4)/(bx). Substituting $bd^2/6$ for S yields

$$f_v = \frac{Vx}{bd^2/6}\frac{bd/4}{bx} = \frac{Vbd/4}{b^2d^2/6} = \frac{6V}{4bd}$$, or

$$f_v = 1.5\ V / bd$$

For the sample beam: f_v = 1.5(2)1000/(4x12) — f_v = 63 psi



Equilibrium Method

Cantilever beam with point load

Assume a beam of length L = 10 ft, supporting a load P = 2 k. The beam bending moment and shear force may be computed, like the external reactions, by equations of equilibrium ΣH=0, ΣV= 0, and ΣM=0. Bending moment and shear force cause bending- and shear stress, similar to axial load yielding axial stress f= P/A. Formulas for bending- and shear stress are given on the next page and derived later in this chapter.

1 Cantilever beam with concentrated load
V Shear diagram (shear force at any point along beam)
M Bending moment diagram (bending moment at any point along beam)
Δ Deflection diagram (exaggerated for clarity)

Reactions, found by equilibrium, ΣV=0 (up +) and ΣM=0 (clockwise +)

at b ΣV= 0 =	R–2= 0	R = 2 k
at b ΣM= 0 =	M–2(10)= 0	M = 20 k'

Shear V, found by vertical equilibrium, ΣV=0 (up +)

right of a = left of b	V= 0-2	V = - 2 k

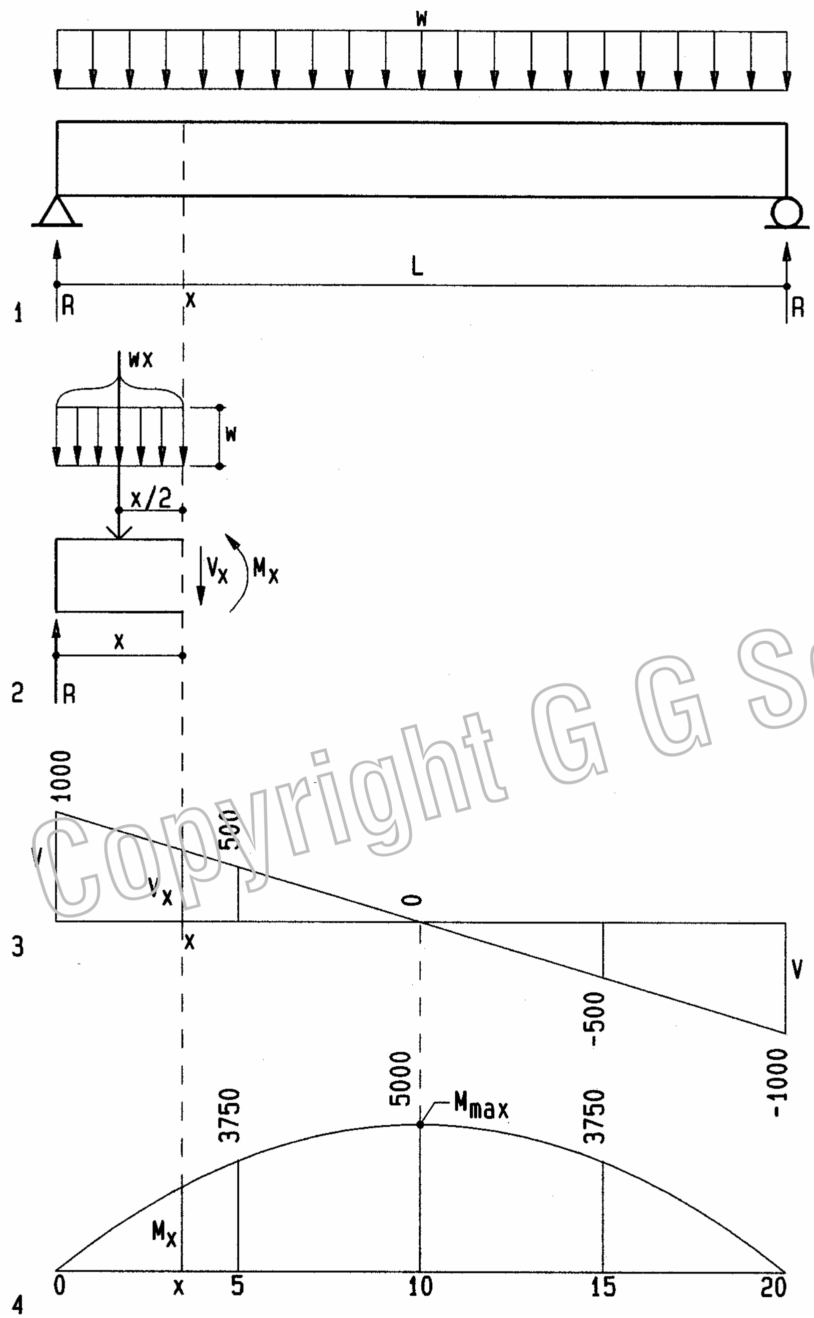
Left of a and right of b, shear is zero because there is no beam to resist it (reaction at b reduces shear to zero). Shear may be checked, considering it starts and stops with zero. Concentrated loads or reactions change shear from left to right of them. Without load between a and b (beam DL assumed negligible) shear is constant.

Bending moment M, found by moment equilibrium, ΣM=0 (clockwise +)

at a M = -2(0)	= 0 k'
at mid-span M = -2(5)	= -10 k'
at b M = -2(10)	= -20 k'

The mid-span moment being half the moment at b implies linear distribution. The support *reaction* moment is equal and opposite to the beam moment.

Deflection Δ is described later. Diagrams visualize positive and negative bending by concave and convex curvature, respectively. They are drawn, visualizing a highly flexible beam, and may be used to verify bending.



Simple beam with uniform load

1 Beam of L= 20 ft span, with uniformly distributed load w = 100 plf
2 Free-body diagram of partial beam x units long
3 Shear diagram
4 Bending moment diagram

To find the distribution of shear and bending along the beam, we investigate the beam at intervals of 5', from left to right. This is not normally required.

Reactions R are half the load on each support due to symmetry

R= w L/2 = 100 (20)/2 R= +1000 lbs

Shear force V_x at any distance x from left is found using $\sum V = 0$

$\sum V = 0$; $R - w\,x - V_x = 0$; solving for V_x

$$V_x = R - w\,x$$

at x = 0'	V = 0 + R_a = 0 +1000	V = +1000 **lbs**
at x = 5'	V = + 1000 - 100 (5)	V = +500 lbs
at x = 10'	V = + 1000 - 100 (10)	V = 0 lbs
at x = 15'	V = + 1000 - 100 (15)	V = -500 lbs
at x = 20'	V = + 1000 - 100 (20)	V = -1000 lbs

Bending moment M_x at any distance x from left is found by $\sum M = 0$.

$\sum M = 0$; $R\,x - w\,x\,(x/2) - M_x = 0$; solving for M_x

$$M_x = R\,x - w\,x^2/2$$

at x = 0	$M = 1000\,(0) - 100\,(0)^2/2$	M = 0 lb-ft
at x = 5'	$M = 1000\,(5) - 100\,(5)^2/2$	M = 3750 lb-ft
at x = 10'	$M = 1000\,(10)-100\,(10)^2/2$	**M = 5000 lb-ft**
at x = 15'	$M = 1000\,(15)-100\,(15)^2/2$	M = 3750 lb-ft
at x = 20'	$M = 1000\,(20)-100\,(20)^2/2$	M = 0 lb-ft

Bending is zero at both supports since pins and rollers have no moment resistance. Since the bending formula $M_x= Rx-wx^2/2$ is quadratic, bending increase is quadratic (parabolic curve) toward maximum at center, and decreases to zero at the right support. For simple beams with uniform load the maximum shear force is at the supports and the maximum bending moment at mid-span (x= L/2) are:

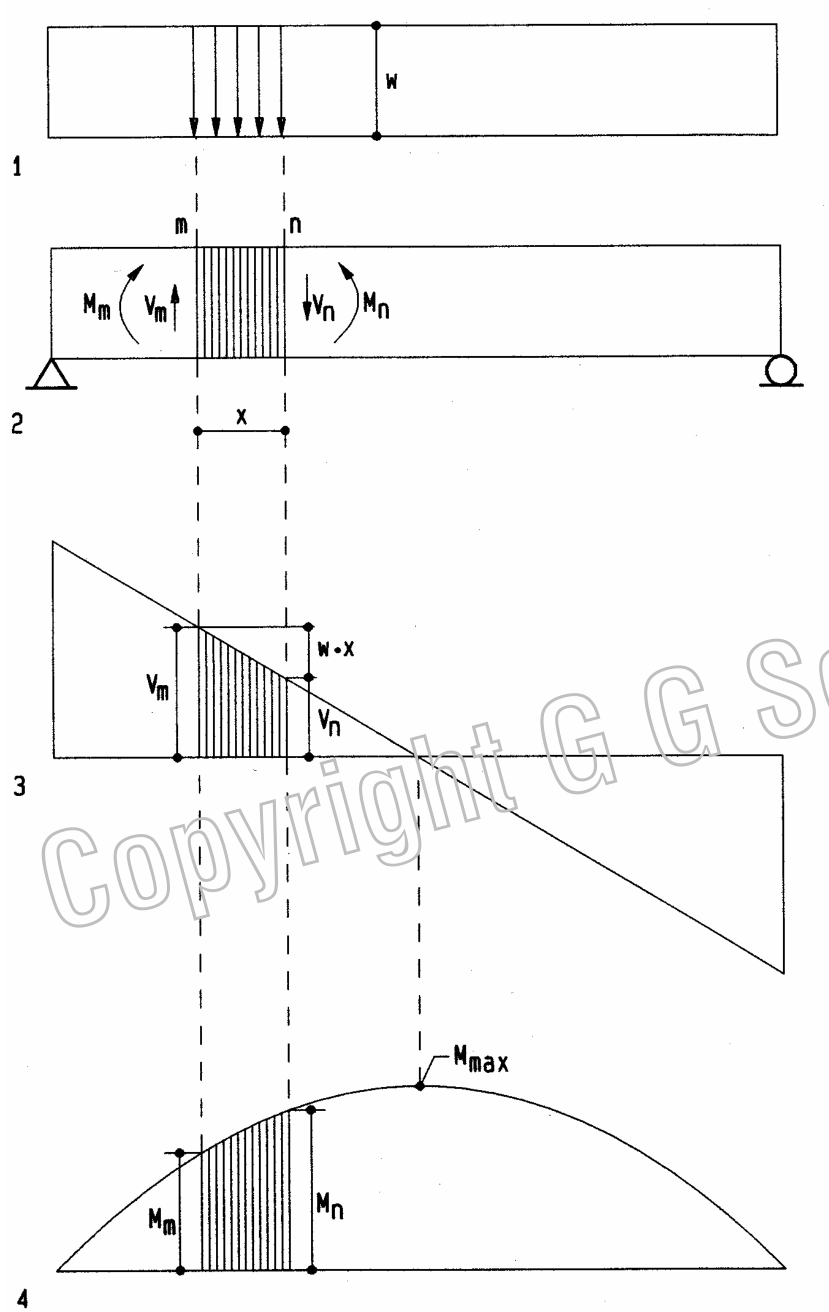
$$V_{max} = R = w\,L / 2$$

$M_{max} = (wL/2)L/2 - (wL/2)L/4 = 2wL^2/8 - wL^2/8$, or

$$M_{max} = wL^2 / 8$$

This formula is only for simple beams with uniform load. Verifying example:

$M_{max}= wL^2/8 = 100(20)^2/8$ M_{max}= +5000 lbs-ft (same as above)



Area Method

The area method for beam design simplifies computation of shear forces and bending moments and is derived, referring to the following diagrams:

1 Load diagram on beam
2 Beam diagram
3 Shear diagram
4 Bending diagram

The area method may be stated:

- The shear at any point n is equal to the shear at point m plus the area of the load diagram between m and n.
- The bending moment at any point n is equal to the moment at point m plus the area of the shear diagram between m and n.

The shear force is derived using vertical equilibrium:

$\sum V = 0$; $V_m - w\,x - V_n = 0$; solving for V_n

$$V_n = V_m - wx$$

where w x is the load area between m and n (downward load w = negative).

The bending moment is derived using moment equilibrium:

$\sum M = 0$; $M_m + V_m x - w\,x\,x/2 - M_n = 0$; solving for M_n

$$M_n = M_m + V_m x - wx^2/2$$

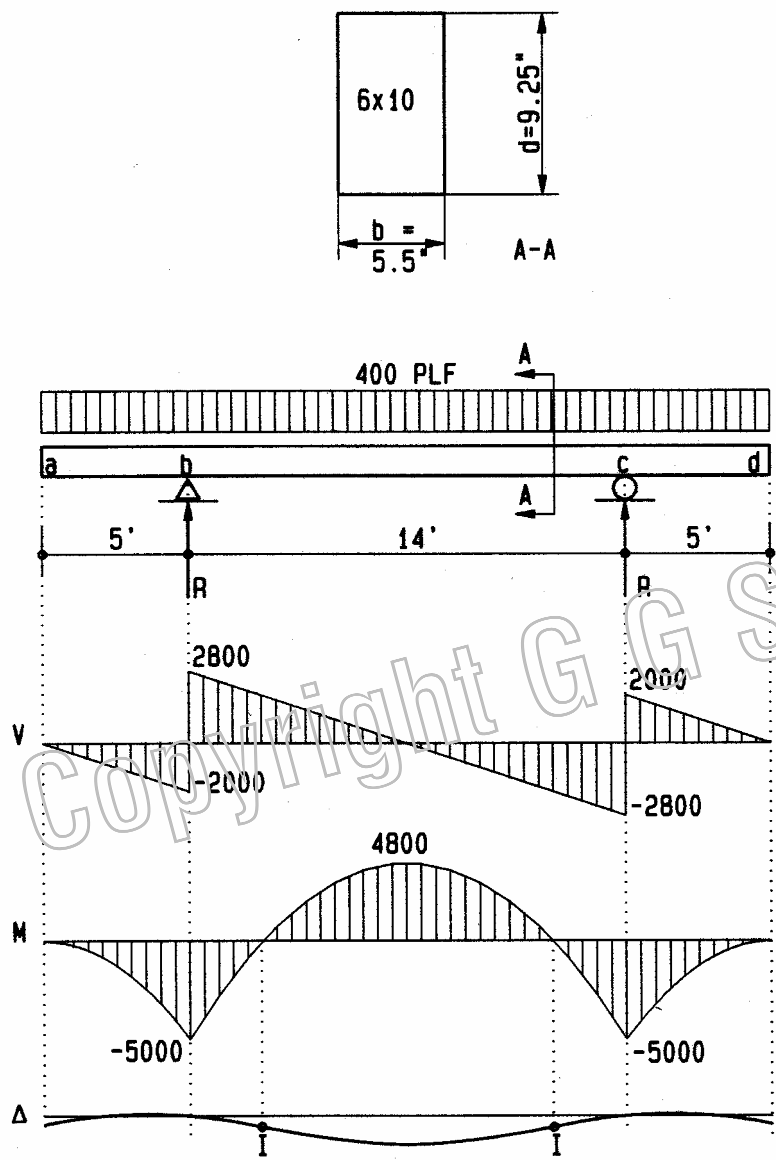
where $V_m x - wx^2/2$ is the shear area between m and n, namely, the rectangle V_m x less the triangle $w\,x^2/2$. This relationship may also be stated as
$M_n = M_m + Vx$, where V is the average shear between m and n.

By the area method moments are usually equal to the area of one or more rectangles and/or triangles. It is best to first compute and draw the shear diagram and then compute the moments as the area of the shear diagram.

From the diagrams and derivation we may conclude:

- Positive shear implies increasing bending moment.
- Zero shear (change from + to -) implies peak bending moment (useful to locate maximum bending moment).
- Negative shear implies decreasing bending moment.

Even though the forgoing is for uniform load, it applies to concentrated load and non-uniform load as well. The derivation for such loads is similar.



Examples

The following wood beams demonstrate the area method for design and analysis. For design, a beam is sized for given loads; for analysis, stresses are checked against allowable limits, or how much load a beam can carry.

Beam design

V Shear diagram.
M Bending diagram.
Δ Deflection diagram.
I Inflection point (change from + to - bending).

Reactions

R= 400plf (24)/2 — R = 4800 lbs

Shear

$V_a = 0$ — $V_a = 0$ lbs
$V_{bl} = 0 - 400(5)$ — $V_{bl} = -2000$ lbs
$V_{br} = -2000 + 4800$ — $V_{br} = +2800$ lbs
$V_{cl} = +2800 - 400(14)$ — $V_{cl} = -2800$ lbs
$V_{cr} = -2800 + 4800$ — $V_{cr} = +2000$ lbs
$V_d = +2000 - 400(5)$ — $V_d = 0$ lbs, ok

Moment

$M_a = 0$
$M_b = 0 - 2000\ (5)/2$ — $M_b = -5000$ lb-ft
$M_{b\text{-}c} = -5000 + 2800\ (7)/2$ — $M_{b\text{-}c} = +4800$ lb-ft
$M_c = +4800 - 2800\ (7)/2$ — $M_c = -5000$ lb-ft
$M_d = -5000 + 2000\ (5)/2$ — $M_d = 0$, ok

Try 4x10 beam

$S = (3.5)\ 9.25^2/6$ — $S = 50\ in^3$

Bending stress

$f_b = M_{max}/S = 5000\ (12)/50$ — $f_b = 1200$ psi
$1200 < 1450$, ok

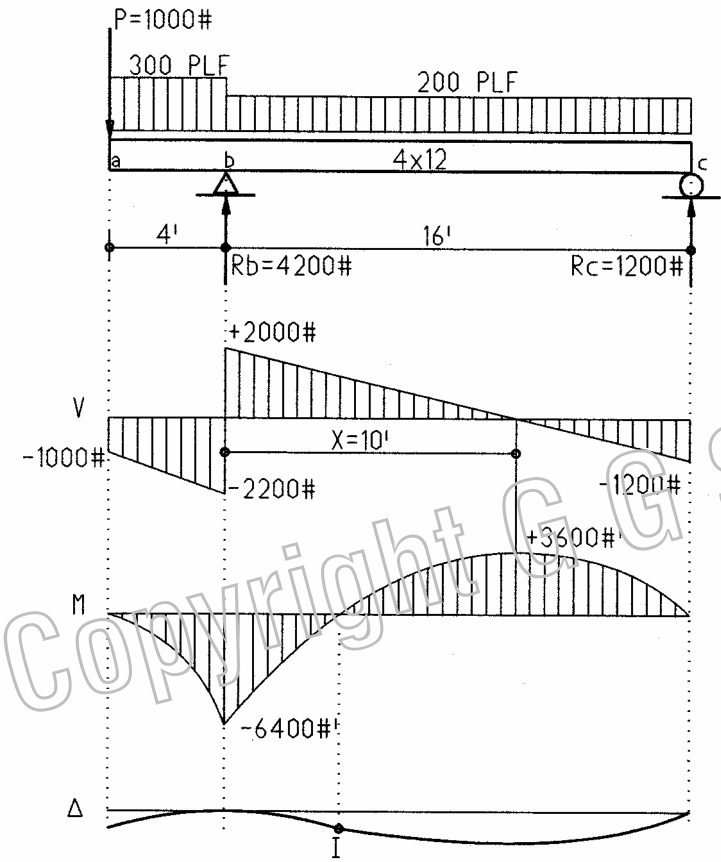
Shear stress

$f_v = 1.5V/bd = 1.5(2800)/[3.5(9.25)]$ — $f_v = 130$ psi
$130 > 95$, not ok

Try 6x10 beam

$f_v = 1.5V/bd = 1.5\ (2800)/[5.5\ (9.25)]$ — $f_v = 83$ psi
$83 < 95$, ok

Note: increased beam width is most effective to reduce shear stress; but increased depth is most effective to reduce bending stress.



Beam analysis

Reactions

$\Sigma M_c = 0 = +16\,R_b - 1000\,(20) - 300\,(4)\,18 - 200\,(16)\,8$

$16\,R_b = 1000(20) + 300\,(4)\,18 + 200\,(16)\,8$

$R_b = (20000 + 21600 + 25600)/16$ — $R_b = +4200$ lbs

$\Sigma M_b = 0 = -16\,R_c - 1000(4) - 300(4)2 + 200(16)8$

$16\,R_c = -1000(4) - 300(4)2 + 200(16)\,8$

$R_c = (-4000 - 2400 + 25600)/16 =$ — $R_c = +1200$ lbs

Shear

$V_{ar} = 0 - 1000$ — $V_{ar} = -1000$ lbs

$V_{bl} = -1000 - 300\,(4)$ — $V_{bl} = -2200$ lbs

$V_{br} = -2200 + 4200$ — $V_{br} = +2000$ lbs

$V_{cl} = +2000 - 200(16) = -R_c$ — $V_{cl} = -1200$ lbs

$V_{cr} = -1200 + 1200$ — $V_{cr} = 0$ lbs

Find x, where shear = 0 and bending = maximum:

$V_{br} - w_2\,x = 0;\ \ x = V_{br}/w_2 = 2000/200$ — $x = 10$ ft

Moment

$M_a = 0$

$M_b = 0 + 4\,(-1000 - 2200)/2$ — $M_b = -6400$ lb-ft

$M_x = -6400 + 10\,(2000)/2$ — $M_x = +3600$ lb-ft

$M_c = +3600 + (16-10)(-1200)/2$ — $M_c = 0$

Section modulus

$S = bd^2/6 = (3.5)11.25^2/6$ — $S = 74\ in^3$

Bending stress

$f_b = M/S = 6400(12)/74$ — $f_b = 1038$ psi

$1038 < 1450$, ok

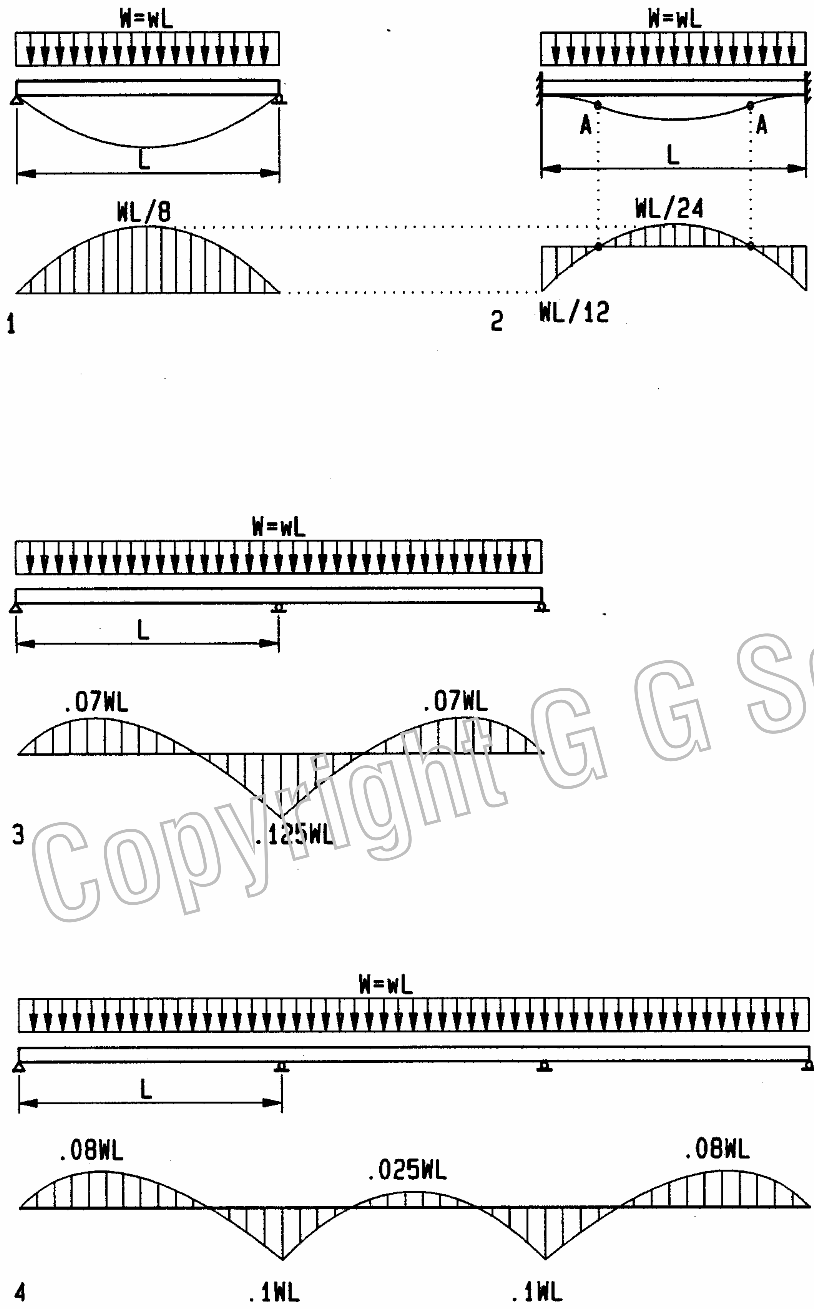
Shear stress

$f_v = 1.5V/(bd) = 1.5(2200)/[3.5(11.25)]$ — $f_v = 84$ psi

$84 < 95 =$ ok

Note: stress is figured, using absolute maximum bending and shear, regardless if positive or negative. Lumber sizes are nominal, yet actual sizes are used for computation. Actual sizes are ½ in. less for lumber up to 6 in. nominal and ¾ in. less for larger sizes: 4x8 nominal is 3½x7¼ in. actual.

Note: in the above two beams shear stress is more critical (closer to the allowable stress) than bending stress, because negative cantilever moments partly reduces positive moments.



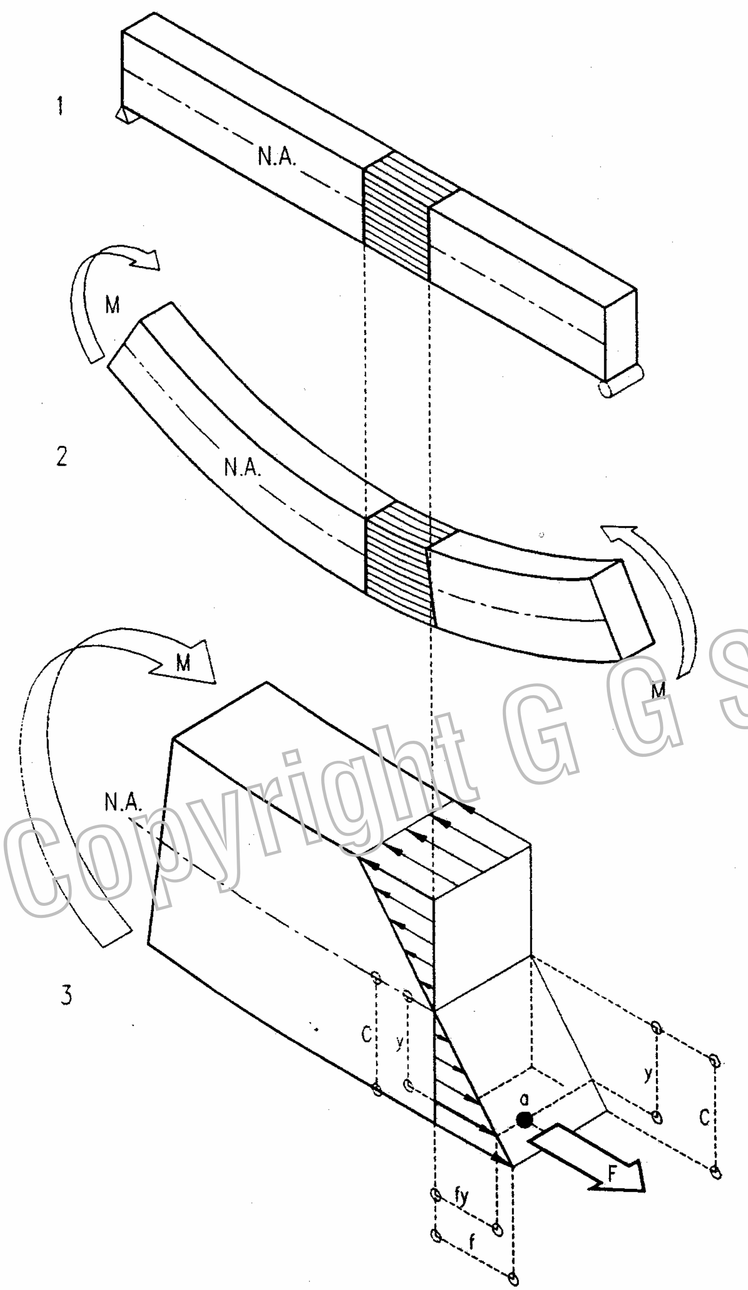
Indeterminate beams

Indeterminate beams include beams with fixed-end (moment resistant) supports and beams of more than two supports, referred to as continuous beams. The design of statically indeterminate beams cannot be done by static equations alone. However, bending coefficients, derived by mechanics, may be used for analysis of typical beams. The bending moment is computed, multiplying the bending coefficients by the total load W and span L between supports. For continuous beams, the method is limited to beams of equal spans for all bays. The coefficients here assume all bays are loaded. Coefficients for alternating live load on some bays and combined dead load plus live load on others, which may result in greater bending moments, are in Appendix A. Appendix A also has coefficients for other load conditions, such as various point loads. The equation for bending moments by bending coefficients is:

$$M = C\ L\ W$$

M = bending moment
C = bending coefficient
L = span between supports
W = w L (total load per bay)
w = uniform load in plf (pounds / linear foot

1 Simple beam
2 Fixed-end beam
(combined positive plus negative moments equal the simple-beam moment)
3 Two-span beam
4 Three-span beam



Flexure Formula

The flexure formula gives the internal bending stress caused by an external moment on a beam or other bending member of homogeneous material. It is derived here for a rectangular beam but is valid for any shape.

1 Unloaded beam with hatched square
2 Beam subject to bending with hatched square deformed
3 Stress diagram of deformed beam subject to bending

Referring to the diagram, a beam subject to positive bending assumes a concave curvature (circular under pure bending). As illustrated by the hatched square, the top shortens and the bottom elongates, causing compressive stress on the top and tensile stress on the bottom. Assuming stress varies linearly with strain, stress distribution over the beam depth is proportional to strain deformation. Thus stress varies linearly over the depth of the beam and is zero at the neutral axis (NA). The bending stress f_y at any distance y from the neutral axis is found, considering similar triangles, namely f_y relates to y as f relates to c; f is the maximum bending stress at top or bottom and c the distance from the Neutral Axis, namely $f_y / y = f / c$. Solving for f_y yields

$f_y = y f / c$

To satisfy equilibrium, the beam requires an internal resisting moment that is equal and opposite to the external bending moment. The internal resisting moment is the sum of all partial forces F rotating around the neutral axis with a lever arm of length *y* to balance the external moment. Each partial force F is the product of stress f_y and the partial area *a* on which it acts, $F = a f_y$. Substituting $f_y = y f / c$, defined above, yields $F = a y f / c$. Since the internal resisting moment M is the sum of all forces F times their lever arm y to the neutral axis, $M = F y = (f/c) \Sigma y y a = (f/c) \Sigma y^2 a$, or $M = I f/c$, where the term $\Sigma y^2 a$ is defined as *moment of inertia* ($I = \Sigma y^2 a$) for convenience. In formal calculus the summation of area *a* is replaced by integration of the differential area *da*, an infinitely small area:

$I = \int y^2 da$ I = *moment of inertia.*

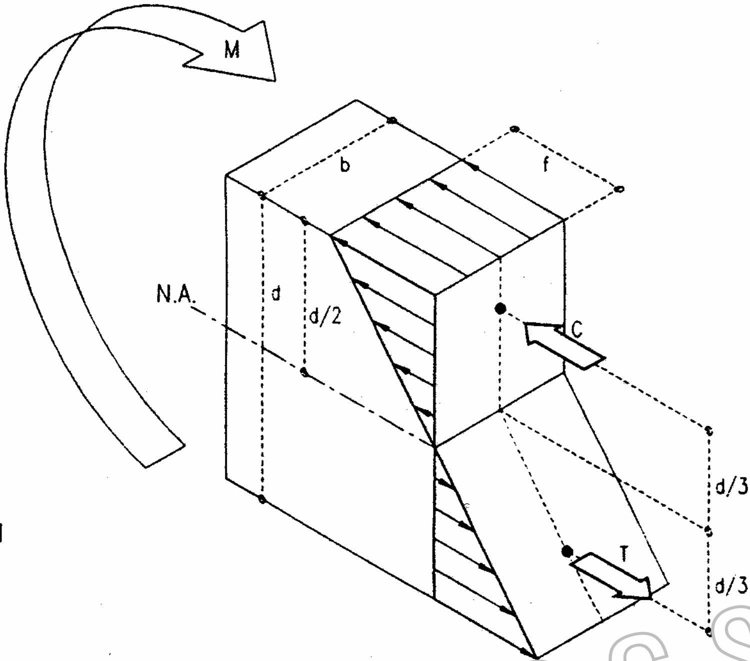
The internal resisting moment equation $M = I f/c$ solved for stress f yields

$f = M c / I$ the *flexure formula*,

which gives the bending stress f at any distance c from the neutral axis. A simpler form is used to compute the *maximum* fiber stress as derived before. Assuming c as maximum fiber distance from the neutral axis yields:

$f = M / S$ *S = I/c = section modulus*

Both the moment of inertia I and section modulus S define the strength of a cross-section regarding its geometric form.



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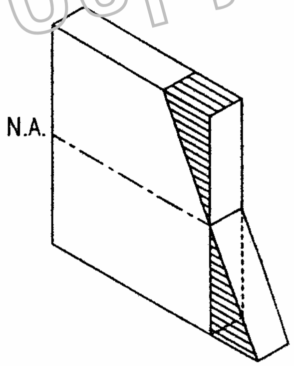

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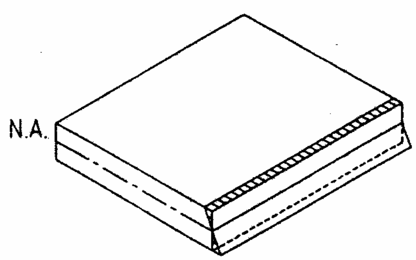


3

Section modulus

Rectangular beams of homogeneous material, such as wood, are common in practice. The section modulus for such beams is derived here.

1 Stress block in rectangular beam under positive bending.
2 Large stress block and lever-arm of a joist in typical upright position.
3 Small, inefficient, stress block and lever-arm of a joist laid flat.

Referring to 1, the section modulus for a rectangular beam of homogeneous material may be derived as follows. The force couple C and T rotates about the neutral axis to provide the internal resisting moment. C and T act at the center of mass of their respective triangular stress block at d/3 from the neutral axis. The magnitude of C and T is the volume of the upper and lower stress block, respectively.

$C = T = (f/2)\ (bd/2) = f\ b\ d/4.$

The internal resisting moment is the sum of C and T times their respective lever arm, d/3, to the neutral axis. Hence

$M = C\ d/3 + T\ d/3$. Substituting $C = T = f\ bd/4$ yields

$M = 2\ (f\ bd/4)\ d/3 = f\ bd^2/6$, or $M = f\ S$,

where $S = bd^2/6$, defined as the section modulus for rectangular beams of homogeneous material.

$S = bd^2/6$

Solving $M = f\ S$ for f yields the maximum bending stress as defined before:

$f = M/S$

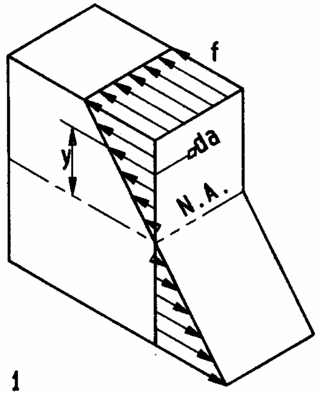
This formula is valid for homogeneous beams of any shape; but the formula $S = bd^2/6$ is valid for rectangular beams only. For other shapes S can be computed as $S = I\ /c$ as defined before for the flexure formula. The moment of inertia I for various common shapes is given in Appendix A.

Comparing a joist of 2”x12” in upright and flat position as illustrated in 2 and 3 yields an interesting observation:

$S = 2\text{x}12^2/6 = 48\ in^3$ for the upright joist

$S = 12\text{x}2^2/6 = 8\ in^3$ for the flat joist.

The upright joist is six times stronger than the flat joist of equal cross-section. This demonstrates the importance of correct orientation of bending members, such as beams or moment frames.



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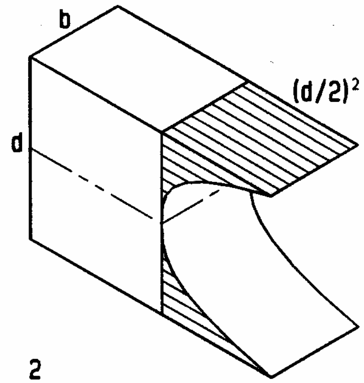

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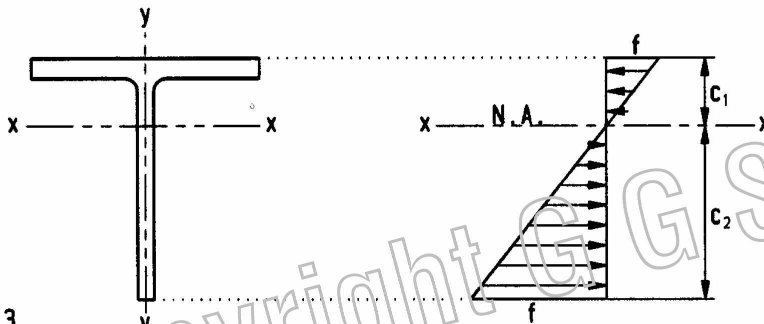

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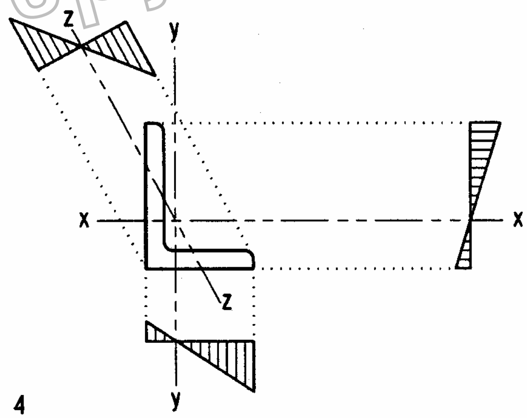


4

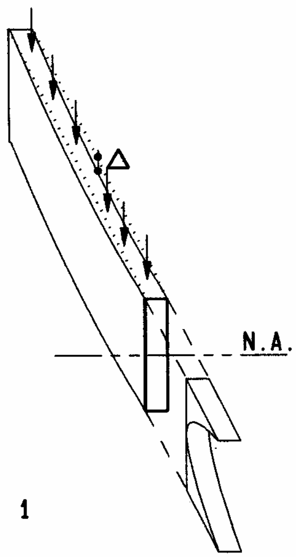
Moment of inertia

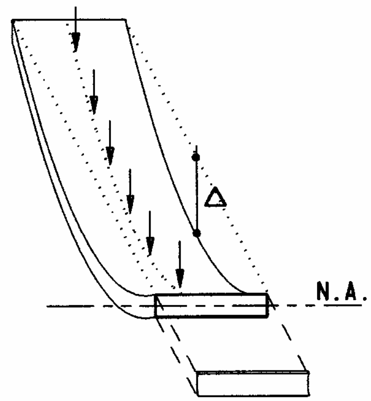
The formula for the moment of inertia $I=\int y^2 da$ reveals that the resistance of any differential area da increases with its distance y from the neutral axis squared, forming a parabolic distribution. For a beam of rectangular cross-section, the resistance of top and bottom fibers with distance $y = d/2$ from the neutral axis is $(d/2)^2$. Thus, the moment of inertia, as geometric resistance, is the volume of all fibers under a parabolic surface, which is 1/3 the volume of a cube of equal dimensions, or $I= bd\,(d/2)^2/3$, or

$I = bd^3 / 12$ (for rectangular beams only)

the moment of inertia of a rectangular beam of homogeneous material. A formal calculus derivation of this formula is given in Appendix A. The section modulus gives only the maximum bending stress, but the moment of inertia gives the stress at any distance c from the neutral axis as $f= Mc/I$. This is useful, for example, for bending elements of asymmetrical cross-section, such as T- and L-shapes.

1 Bending stress distribution over beam cross-section
2 Moment of inertia visualized as volume under parabolic surface
3 T-bar with asymmetrical stress: max. stress at c_2 from the neutral axis
4 Angle bar with asymmetrical stress distribution about x, y, and z-axes: maximum resistance about x-axis and minimum resistance about z-axis





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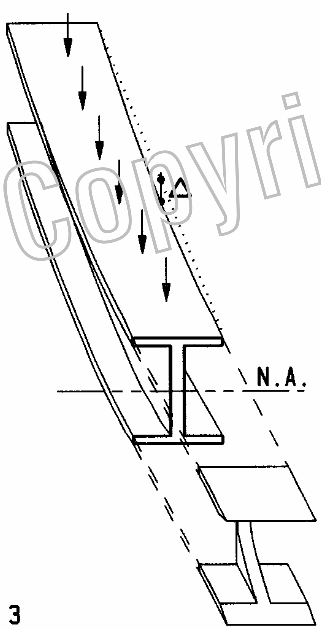
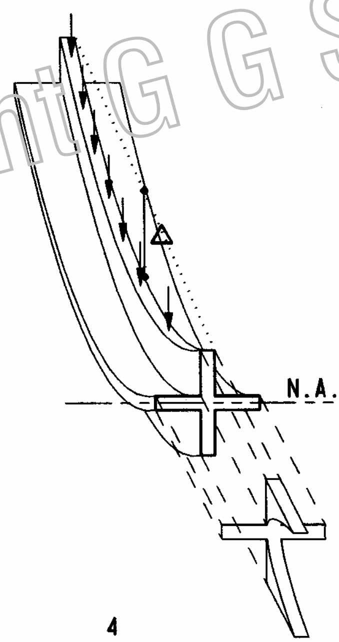




3

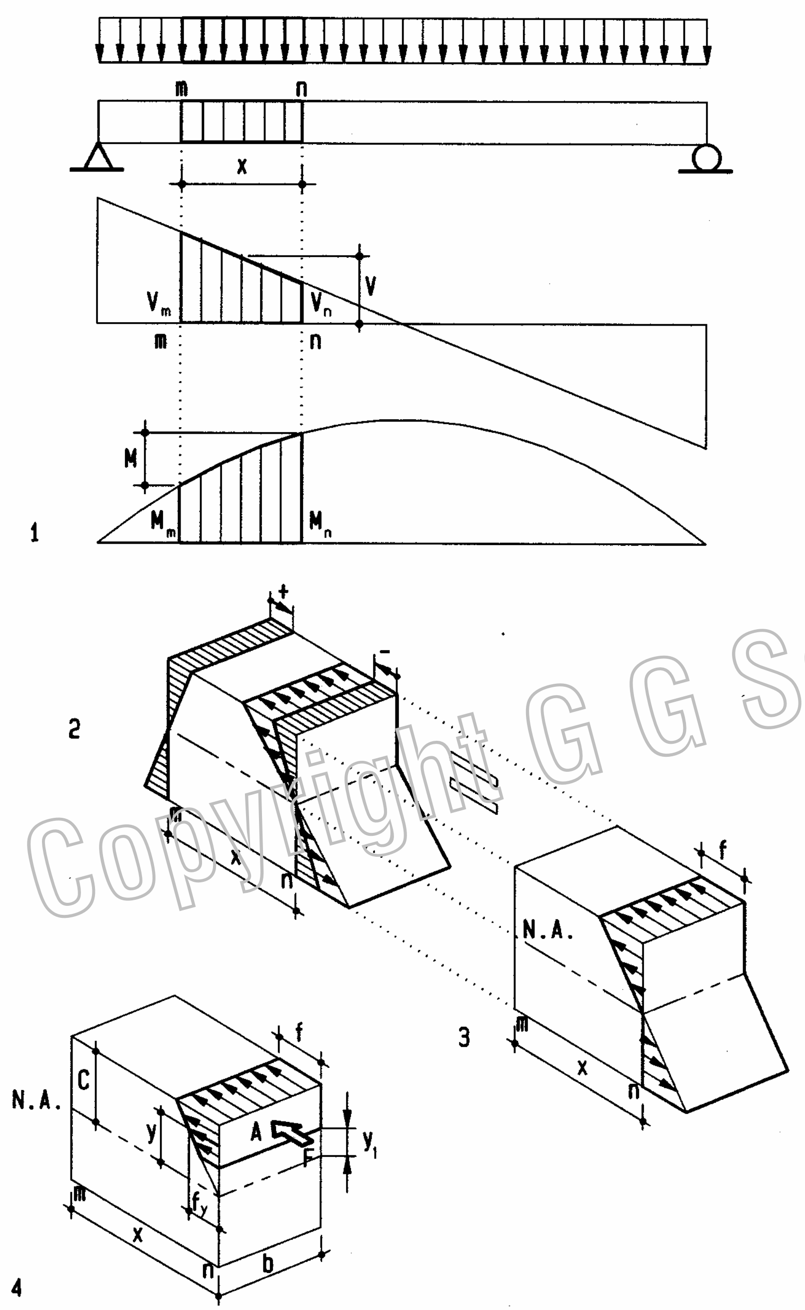
4

Moment of inertia and shapes

The moment of inertia, a measure of geometric strength and stiffness, is greatly effected by a beam's shape. The formula $I= \int y^2 da$ reveals that the resisting capacity of fibers increase is quadratic with vertical distance from the neutral axis. Material far from the neutral axis increases the resistance capacity; by contrast, material located near the neutral axis is relatively ineffective. This is visualized here by the capacity of various beam shapes to resist bending deformation. The moment of inertia is shown here, along with relative deformations under gravity load. This qualitative, intuitive comparison is quantified in *Beam deflection* of this chapter.

1 Efficient upright 2"x12" joist; $I= 2(12)^3/12$ — $I = 288\ in^4$
2 Inefficient flat 12"x2" joist; $I= 12(2)^3/12$ — $I = 8\ in^4$
3 Efficient wide-flange beam
4 Inefficient cross-shaped beam

Given the same cross-section area, the upright joist has a 36 times greater moment of inertia to resist deformation than the flat one. This represents the square of the joist's width-to-depth ratio. A similar contrast can be observed between wide-flange and cross-shaped beams.



Shear stress

The distribution of shear stress over the cross-section of beams is derived, referring to a beam part of length x marked on diagrams. Even though horizontal and vertical shear are equal at any part of a beam, horizontal shear stress is derived here because it is much more critical in wood due to horizontal fiber direction.

1 Beam, shear and bending diagrams with marked part of length x
2 Beam part with bending stress pushing and pulling to cause shear
3 Beam part with bending stress above an arbitrary shear plane

Let M be the differential bending moment between m and n. M is equal to the shear area between m and n (area method), thus M = V x. Substituting V x for M in the flexure formula f= M c /I yields bending stress f= V x c/I in terms of shear. The differential bending stress between m and n pushes top and bottom fibers in opposite directions, causing shear stress. At any shear plane y_1 from the neutral axis of the beam the sum of shear stress above this plane yields a force F that equals average stress f_y times the cross section area A above the shear plane, F = A f_y. The average stress f_y is found from similar triangles; f_y relates to y as f relates to c, i.e., $f_y/y = f/c$; thus f_y = f y/c. Since f= V x c/I, substituting V x c/I for f yields f_y = (V x c/I) y/c = V x y/I. Since F = A f_y, it follows that F = A V x y/I. The horizontal shear stress v equals the force F divided by the area of the shear plane,

V = F/(x b) = A V x y/(I x b) = V A y/(I b)

The term A y is defined as Q, the first static moment of the area above the shear plane times the lever arm from its centroid to the neutral axis of the entire cross-section. Substituting Q for A y yields the working formula

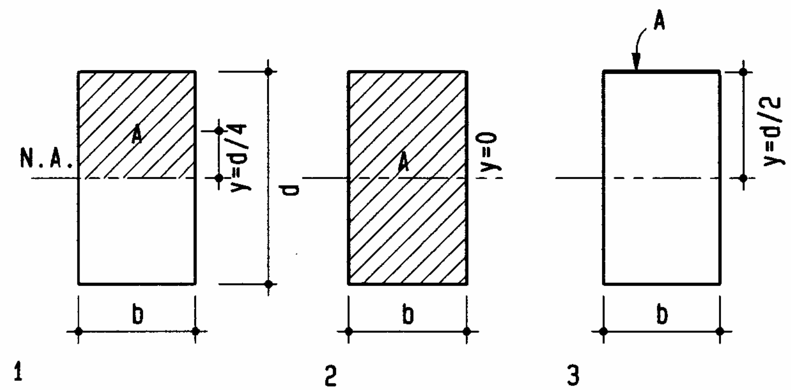
v = V Q / (I b) (shear stress)

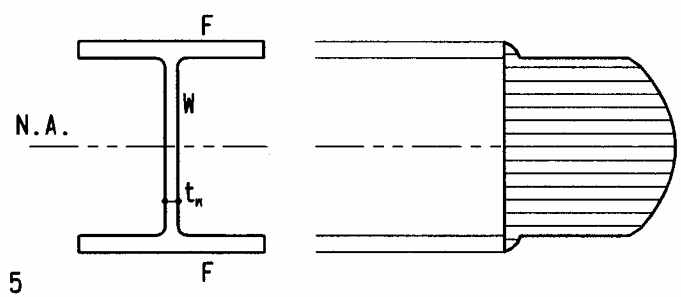
v = horizontal shear stress.
Q = static moment (area above shear plane times distance from centroid of that area to the neutral axis of the entire cross-section
I = moment of Inertia of entire cross section
b = width of shear plane

The formula for shear stress can also be stated as shear flow q, measured in force per unit length (pound per linear inch, kip per linear inch, or similar metric units); hence

q = V Q / I (shear flow)

q = force per unit length





Shear stress in wood and steel beams

Based on the forgoing general derivation of shear stress, the formulas for shear stress in rectangular wood beams and flanged steel beams is derived here. The maximum stress in those beams is customarily defined as f_v instead of v in the general shear formula.

1 Shear at neutral axis of rectangular beam (maximum stress),
$Q = Ay = (bd/2)\ d/4$, or
$Q = bd^2/8$ (Note: d^2 implies parabolic distribution)
$I = bd^3/12$, hence
$v = V\,Q / I\,b = V\,(bd^2/8) / [(bd^3/12)b] = f_v$, or

$$f_v = 1.5\ V / (bd)$$

Note: this is the same formula derived for maximum shear stress before

2 Shear stress at the bottom of rectangular beam. Note that $y = 0$ since the centroid of the area above the shear plane (bottom) coincides with the neutral axis of the entire section. Thus $Q= Ay = (bd/2)\ 0 = 0$, hence
$v = V\ 0/(I\ b) = 0 = f_v$, thus
$f_v = 0$

Note: this confirms an intuitive interpretation that suggests zero stress since no fibers below the beam could resist shear

3 Shear stress at top of rectangular beam. Note $A = 0b = 0$ since the depth of the shear area above the top of the beam is zero. Thus
$Q = Ay = 0\ d/2 = 0$, hence $v = V\ 0/(I\ b) = 0 = f_v$, thus
$f_v = 0$

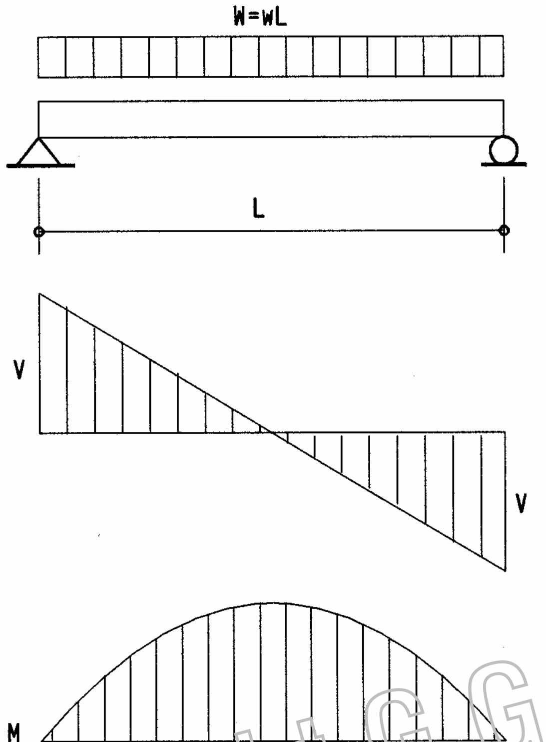
Note: this, too, confirms an intuitive interpretation that suggests zero stress since no fibers above the beam top could resist shear.

4 Shear stress distribution over a rectangular section is parabolic as implied by the formula $Q=bd^2/8$ derived above.

5 Shear stress in a steel beam is minimal in the flanges and parabolic over the web. The formula $v = VQ/(I\ b)$ results in a small stress in the flanges since the width b of flanges is much greater than the web thickness. However, for convenience, shear stress in steel beams is computed as “average” by the simplified formula:

$$f_v = V / A_v$$

f_v = shear stress in steel beam
V = shear force at section investigated
A_v = shear area, defined as web thickness times beam depth



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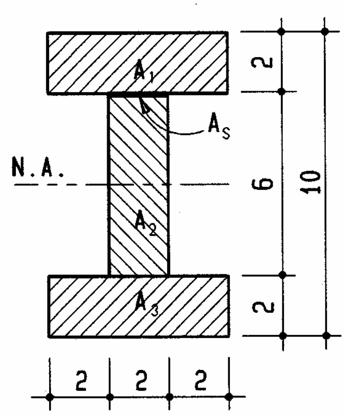


2

Shear stress in wood I-beam

Since this is not a rectangular beam, shear stress must be computed by the general shear formula. The maximum shear stress at the neutral axis as well as shear stress at the intersection between flange and web (shear plane A_s) will be computed. The latter gives the shear stress in the glued connection. To compare shear- and bending stress the latter is also computed.

1 Beam of L= 10 ft length, with uniform load w= 280plf (W = 2800 lbs)
2 Cross-section of wood I-beam

Shear force V= W/2 = 2800/2 — V = 1400 lbs

Bending moment M=WL/8 = 2800(10)/8 — M = 3500 lb'

For the formula v= VQ/(Ib) we must find the moment of inertia of the entire cross-section. We could use the *parallel axis theorem* of Appendix A. However, due to symmetry, a simplified formula is possible, finding the moment of inertia for the overall dimensions as rectangular beam minus that for two rectangles on both sides of the web.

$I = (BD^3 – bd^3)/12 = [6(10)^3 – 2(2)6^3]/12$ — $I = 428\ in^4$

Bending stress f_b= Mc/I = 3500(12)5/428 — f_b = 491 psi

491 < 1450, ok

Note c= 10/2 = 5 (half the beam depth due to symmetry)

Static moment Q of flange about the neutral axis:

Q = Ay = 6(2)4 — $Q = 48\ in^3$

Shear stress at flange/web intersection:

v = VQ/(Ib) = 1400(48)/[428(2)] — v = 79 psi

Static moment Q of flange plus upper half of web about the neutral axis

Q = ΣAy = 6(2)4 + 2(3)1.5 — $Q = 57\ in^3$

Maximum shear stress at neutral axis:

v = VQ/(Ib) = 1400(57)/[428(2)] — v = 93 psi <95, ok

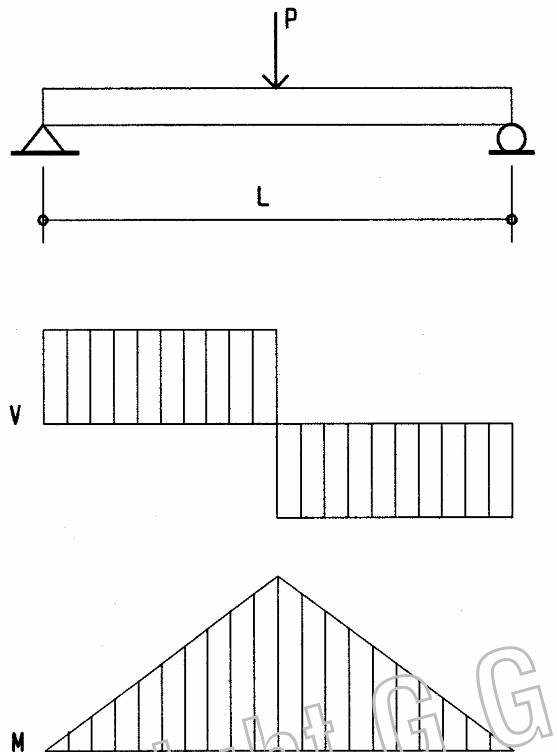
Note: Maximum shear stress reaches almost the allowable stress limit, but bending stress is well below allowable bending stress because the beam is very short. We can try at what span the beam approaches allowable stress, assuming L= 30 ft, using the same total load W = 2800 lbs to keep shear stress constant:

M= WL/8 = 2800(30)/8 — M = 10500 lb-ft

f_b = Mc/I = 10500(12)5/428 — f_b = 1472 psi

1472 >1450, not ok

At 30 ft span bending stress is just over the allowable stress of 1450 psi. This shows that in short beams shear governs, but in long beams bending or deflection governs.



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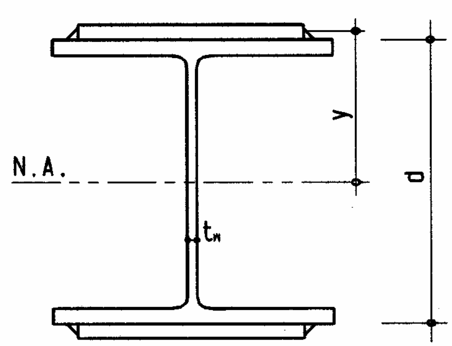


2

Shear stress in steel beam

This beam, supporting a column point load of 96 k over a door, is a composite beam consisting of a wide-flange base beam with 8x½ in plates welded to top and bottom flanges. The beam is analyzed with and without plates. As shown before, for steel beams shear stress is assumed to be resisted by the web only, computed as $f_v = V/A_v$. The base beam is a W10x49 [10 in (254 mm) nominal depth, 49 lbs/ft (6.77 kg/m) DL] with a moment of inertia I_{xx}= 272 in^4 (11322 cm^4) (see Appendix). Shear in the welds connecting the plates to the beam is found using the shear flow formula q = VQ/(I).

1 Beam of L= 6 ft (1.83 m) span with P = 96 k point load
2 Composite wide-flange beam W10x49 with 8x½ inch stiffener plates

Shear force V = P/2 = 96/2 — V = 48 k

Bending moment M = 48(3) — M = 144 k'

Wide-flange beam

Shear area of web A_v = web thickness x beam depth
A_v = 0.34(10) — A_v = 3.4 in^2

Shear stress $f_v = V/A_v$ = 48/3.4 — f_v = 14 ksi
14 < 14.5, ok

Bending stress f_b = Mc/I = 144(12)5.5/272 — f_b = 35 ksi
35 >22, not ok

Since the beam would fail in bending, a composite beam is used.

Composite beam

Moment of inertia $I=\Sigma(I_{oo}+Ay^2)$ (see *parallel axis theorem* in Appendix A)

I= 272+2(8)0.5^3/12+2(8)0.5(5.25)^2 — I = 493 in^4

Bending stress f_b= Mc/I = 144(12)5/493 — f_b = 19 ksi
19 < 22, ok

Since the shear force remains unchanged, the web shear stress is still ok.

Shear flow q in welded plate connection

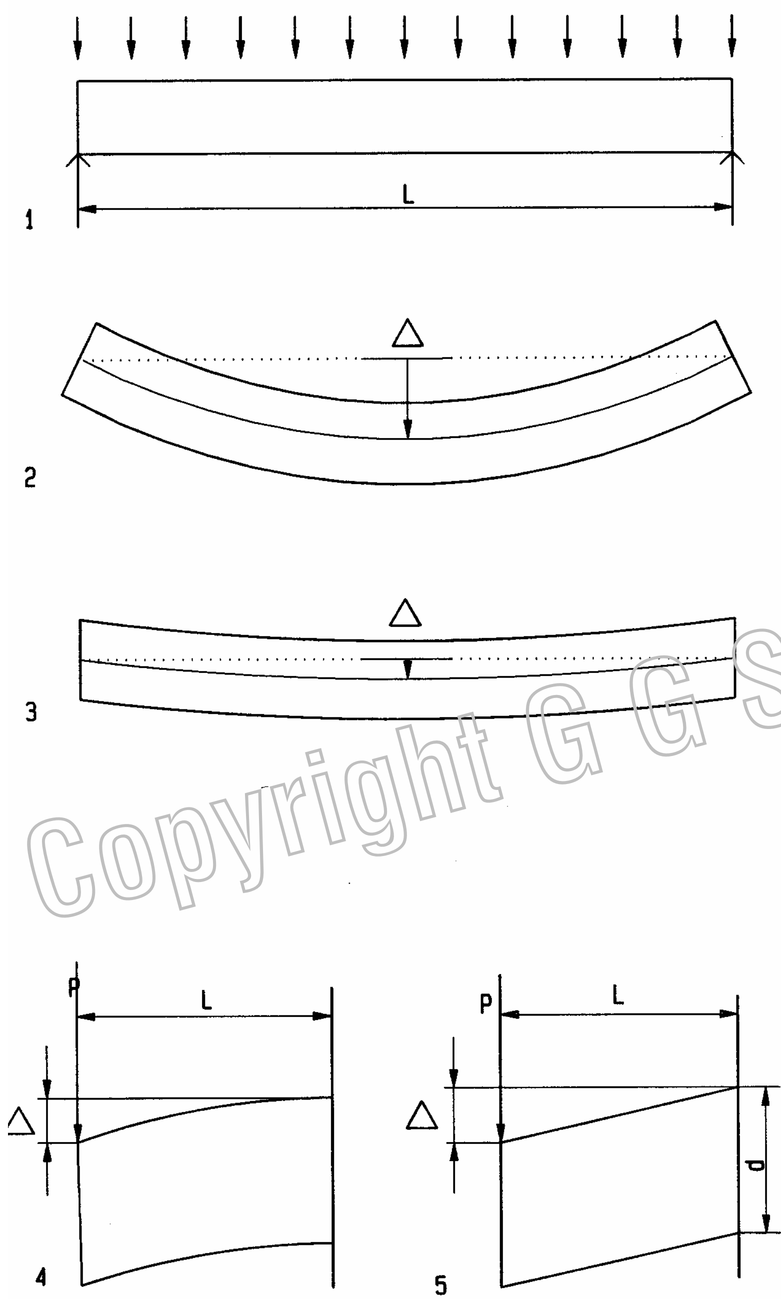
Q= Ay = 8(.5)5.25 = 21 in^3

q_{tot} = VQ/I = 48(21)/493 — q_{tot} = 2 k/in

Since there are two welds, each resists half the total shear flow

$q = q_{tot}/2$ — q = 1k/in
Assume ¼ in weld of 3.2 k/in * strength — 1 < 3.2, ok
* see AISC weld strength table

Note: in this steel beam, bending is stress is more critical than shear stress; this is typical for steel beams, except very short ones.



Deflection

To satisfy stiffness, beam deflection must be limited by code or other factors. For example, to prevent cracks in plaster, codes require deflection to be not more than L/360 for LL or L/240 for combined LL+DL. Excessive deflection may also be unsightly or cause damage to non-load-bearing partitions. Therefore, beams may be oversized for strength to limit deflection.

Beam deflection is caused by both bending and shear, yet, except for very short beams, shear deflection is typically very small and may be ignored.

1 Simple beam under uniform load
2 Bending deflection of simple beam under uniform load
3 Shear deflection of simple beam under uniform load
4 Bending deflection of cantilever beam under point load
5 Shear deflection of cantilever beam under point load

Referring to 4, elastic bending deflection of a cantilever beam under point load is derived on the following pages as:

$\Delta = PL^3 / (3EI)$

E = modulus of elasticity, I = moment of inertia

Referring to 5, shear deflection is defined by the formula:

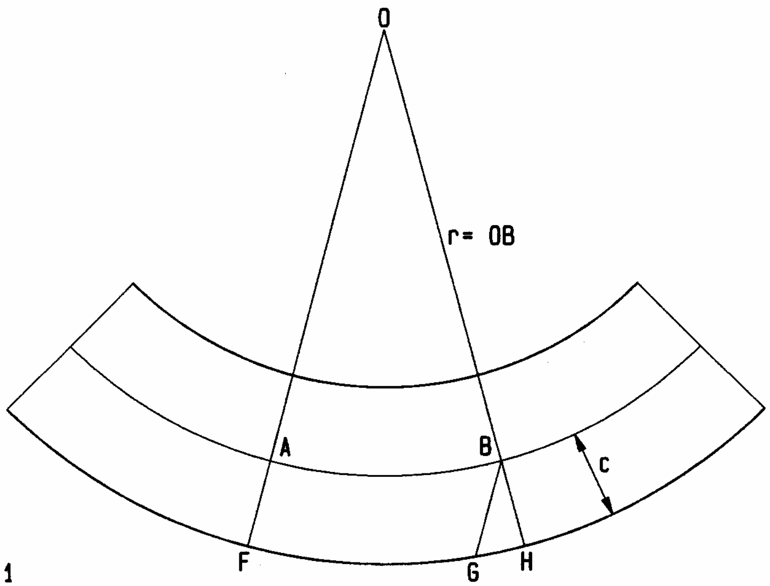
$\Delta = 6PL/(5AG)$

A = cross-section area, G = shear modulus

Shear deflection is not derived, since it is negligible and ignored for most beams. The above formulas show bending deflection increases with the third power of L, but shear deflection increases linearly with L. Shear deflection is equal to shear stress (V/A = P/A) divided by the shear modulus G, modified by 6/5 since shear stress is non-linear over the beam depth.

Referring to 4 and 5, the following highlights the correlation of beam length with shear- and bending deflection. Assuming a 4x6 in (102x152 mm) cantilever steel beam with P=8 k (36 kN), E=30,000 ksi (206,850 MPa), G=12,000 ksi (82,740 MPa), $I=4x6^3/12=72$ in^4 ($29969x10^{-3}$ mm^4), A = 24 in^2 (16 cm^2)

If the beam length is L=60in (152 cm), bending deflection is $\Delta=(8)60^3/[3(30000)72]=.27$in (7mm), but shear deflection is only $\Delta=6(8)60/[5(12000)24]=.002$ in (.05 mm). Thus shear deflection is less than one percent of bending deflection. However, if the beam length equals the beam depth, L=6 in (152 mm), then the bending deflection is reduced to $\Delta=.00027$ in (.007 mm) and shear deflection to $\Delta=.0002$ in (.005 mm); which is about equal to the bending deflection. This confirms, shear deflection is insignificant and may be ignored for beams of typical length, but approaches bending deflection when the beam length is reduced to the beam depth.



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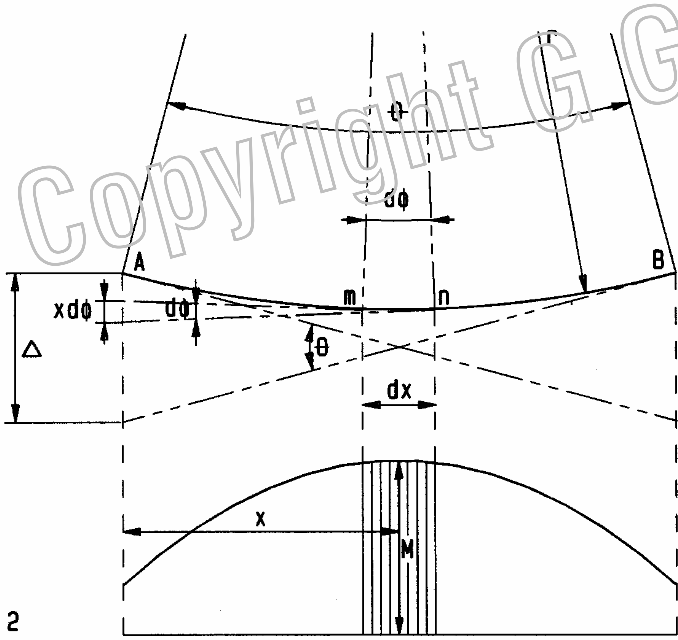


2

Moment-area method

The moment-area method for deflection was developed in 1873 by Charles Green of the USA independent of a similar method developed in 1868 by Otto Mohr of Germany. The derivation of the moment-area theorem is based on fig. 1 and 2, showing part of a deformed beam and its elastic curve AB, respectively; assuming small deformations and constant elastic modulus E and moment of inertia I. Referring to fig. 1, let GB be parallel to FO, then FG=AB, the unstressed length, and $GH/AB=\varepsilon$, the unit strain. Since the elastic modulus is $E=f/\varepsilon$, $f=E\varepsilon$, or $f=E\ GH/AB$; but $GH/AB=c/r$, due to similar triangles. Substituting c/r for GH/AB yields $f=Ec/r$ and, since $f=Mc/I$ (where M= bending moment - see *flexure formula*), $Ec/r=Mc/I$, hence $E/r=M/I$, or $1/r=M(EI)$.

Referring to fig. 2, with angles $d\phi$ and θ measured in radians, $d\phi$ is the angle of the radii at m and n and between the tangents to those radii. The length $dx=r\ d\phi$ and $d\phi/dx=1/r = M(EI)$ (as derived above), or $d\phi=M\ dx\ /(EI)$. The sum of $d\phi$ between A to B is $\theta=\sum d\phi = \sum Mdx/(EI)$, or

$$\theta= A_m/(EI)$$

$A_m = \sum Mdx$, the area of the bending moment diagram between A and B. Hence, the theorem for the beam slope may be stated as follows:

The angle θ between the tangents of points A and B on the elastic curve of a beam is the moment diagram area between A and B, divided by EI.

This theorem can be used to find the elastic slope at any point of a beam. The theorem for deflection (usually of greater interest) is derived next.

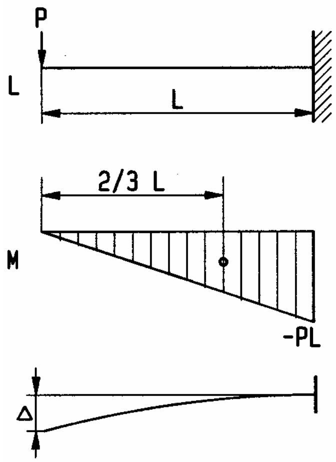
The angle between the tangents at m and n on the elastic curve is $d\phi$ and the vertical displacement between these tangents at A is $xd\phi$. Therefore, the displacement between A and the tangent at B is $\Delta=\sum xd\phi=\sum xMdx/(EI)$, or

$$\Delta= x\ A_m/(EI)$$

where $A_m = \sum Mdx$, area of the bending moment diagram between A and B and times the lever arm from A to the centroid of the bending moment diagram between A and B. Hence, the deflection theorem may be stated as follows:

The vertical displacement Δ of the tangent at B on the elastic curve equals the moment of the area of the bending diagram between A and B times the lever-arm x from its centroid to A, divided by EI.

This theorem can be applied to compute beam deflection as shown on the following pages. The above derivation considers only bending deformation, and ignores shear deformation, which is insignificant as shown before, and can be ignored for most beams.



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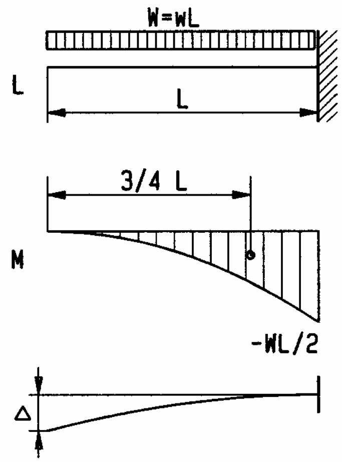

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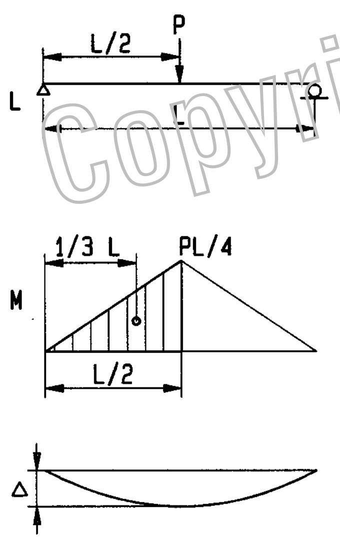

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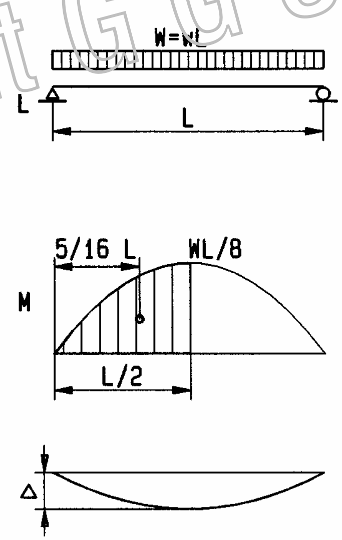


4

Deflection formulas

Based on the moment-area method, the following formulas for slope and deflection are derived for beams of common load and support conditions. Additional formulas are provided in Appendix A. Although downward deflection would theoretically be negative, it is customary to ignore the sign convention and define up- or downward deflection by inspection. The angle θ is the slope of the tangent to the elastic curve at the free end for cantilever beams and at supports for simple beams; Δ is the maximum deflection for all cases. As derived before, θ is the area of the bending moment diagram divided by EI, the elastic modulus and moment of inertia, respectively; Δ equals θ times the lever-arm from the centroid of the bending moment diagram between zero and maximum deflection to the point where θ is maximum.

1 Cantilever beam with point load; $\theta = (PL)(L/2)/(EI)$, $\Delta = \theta\ 2/3\ L$

$\theta = 1/2\ PL^2/(EI)$

$\Delta = 1/3\ PL^3/(EI)$

2 Cantilever beam with uniform load; $\theta = (WL/2)(L/3)/(EI)$, $\Delta = \theta\ 3/4\ L$

$\theta = 1/6\ WL^2/(EI)$

$\Delta = 1/8\ WL^3/(EI)$

3 Simple beam with point load; $\theta = (PL/4)(L/4)/(EI)$, $\Delta = \theta\ 1/3\ L$

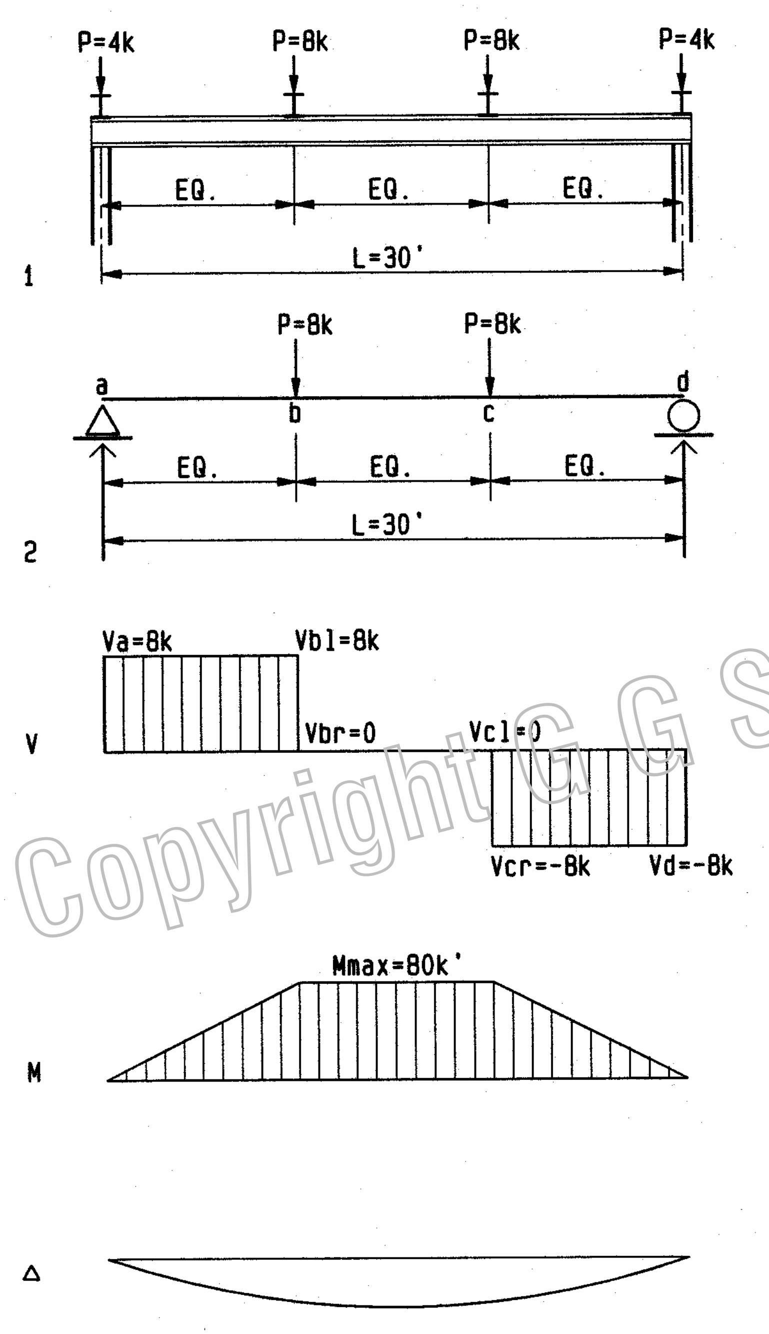
$\theta = 1/16\ PL^2/(EI)$

$\Delta = 1/48\ PL^3/(EI)$

4 Simple beam with uniform load; $\theta = (WL/8)(2/3\ L/2)/(EI)$, $\Delta = \theta\ 5/16\ L$

$\theta = 1/24\ WL^2/(EI)$

$\Delta = 5/384\ WL^3/(EI)$



Steel beam with point loads

This steel beam supports joists that span 32 feet between beams and carry a roof load of 50 psf (30 psf DL and 20 psf LL). The joist reactions generate point loads of P=50 psf(10ft)16ft/1000), P= 8 k beam load. The beam is designed for stress, then verified for deflection and redesigned if necessary.

1 Beam diagram.
2 Load diagram abstraction
Note:
Load at the beam supports is ignored since it is directly supported by columns and hence has no effect on shear, bending moment, or deflection
V Shear diagram.
M Bending moment diagram.
Δ Deflection diagram.

Shear:
$V_a = V_{bl} = R = 2(8)/2$ — $V_a = V_{bl} = 8$ k
$V_{br} = V_{cl} = 8-8$ — $V_{br} = V_{cl} = 0$ k
$V_{br} = V_c = R = 0-8$ — $V_{dr} = V_c = -8$ k

Bending moment:
$M_{max} = 8\ (10)$ — $M_{max} = 80$ k'

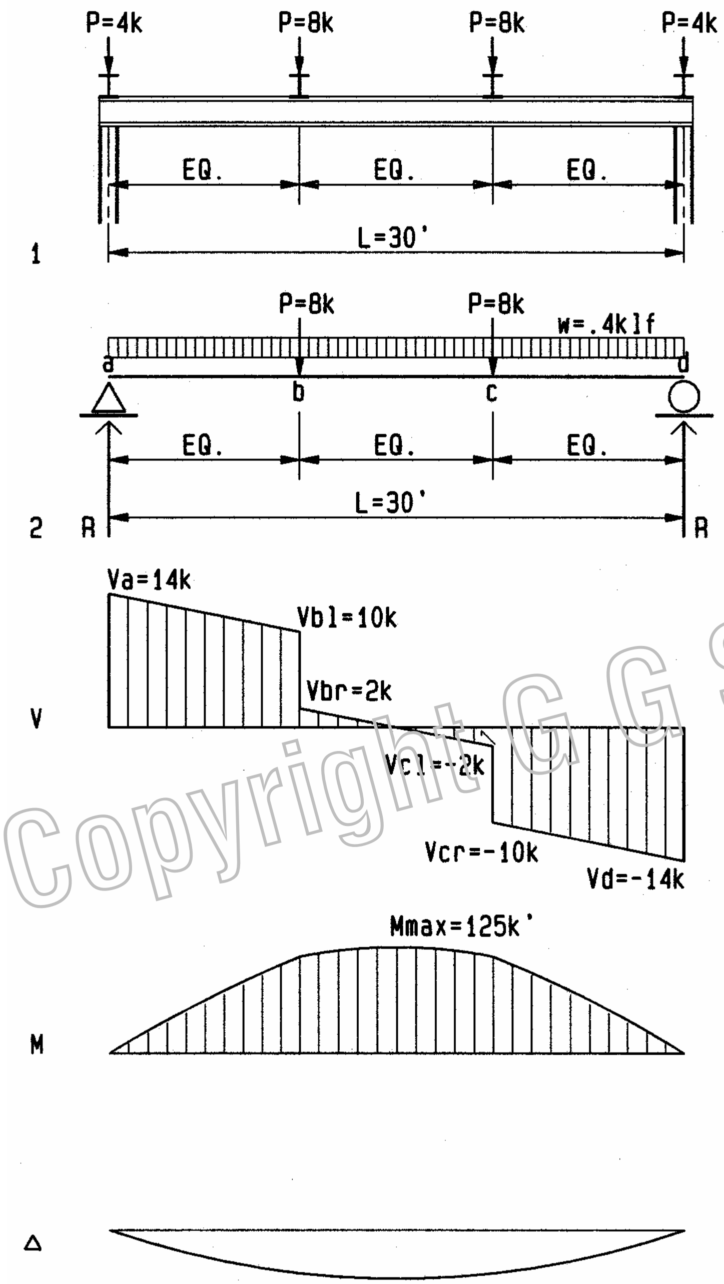
Section modulus S and moment of inertia I (from Appendix D):
$S = M/F_b = 80$ k'(12")/22 ksi — $S = 44\ in^3$
Try W10x45, $S = 49.1\ in^3$ — 49.1 > 44, ok
$I = 248\ in^4$

Deflection (see Appendix A for formula):
L= 30' (12") = 360 in
$\Delta_{max} = (23/684)\ PL^3/(EI) = [(23/684)8(360)^3] / [(30000)248]$
$\Delta_{max} = 1.7$ in
$\Delta_{all} = 360/240 = 1.5$ in — 1.5 < 1.7, not ok

Try W18x35, $S = 57.6\ in^3$ — 57.6 > 44, ok
$I = 510\ in^4$

Deflection:
$\Delta_{max} = (23/684)\ 8\ (360)^3 / [(30000)510]$ — $\Delta_{max} = 0.8$ in
$\Delta_{all} = 2$ in — 1.5 > 0.8, ok

Note: the W10x45 deflects too much, and, with a span/depth ratio of 36:1, is too shallow; but W18x35 has the recommended 20:1 ratio, is lighter and, hence, more economical.



Steel beam with mixed load

This steel beam, too, supports joists that span 32 feet between beams and carry a roof load of 50 psf (30 psf DL and 20 psf LL). The joist reactions generate point loads of P=50 psf(10ft)16ft/1000), P = 8 k beam load. In addition, the beam carries a uniform dead load of 0.4 klf (the beam's own weight plus fire proofing and cladding). The beam is designed for stress, then verified for deflection and redesigned if necessary.

1 Beam diagram

2 Load diagram abstraction
Note: load at the beam supports is ignored since it has no effect on shear, bending moment, or deflection

V Shear diagram

M Bending moment diagram

Δ Deflection diagram

Shear:

V_a = R= [2(8)+0.4(30)]/2 — V_a = 14 k

V_{bl} = 14-0.4(10) — V_{bl} = 10 k

V_{br} = 10-8 — V_{br} = 2 k

V_{cl} = 2-0.4(10) — V_{cl}= -2 k

V_{cr} = -2-8 — V_{cr}= -10 k

V_d = -10-0.4(10) — V_{dr}= -14 k

Bending moment:

M_{max}= 10(14+10)/2 + 5(2)/2 — M_{max} = 125 k'

Section modulus S and moment of inertia I (from Appendix D):

S= M/F_b= 125 k'(12")/22 ksi — S = 68 in^3

Try W18x40, S= 68.4 in^3 — 68.4 > 68, ok

I = 612 in^4

Deflection (see Appendix A for formula):

L= 30' (12") = 360 in

W= wL = 0.4 (30) = 12 k

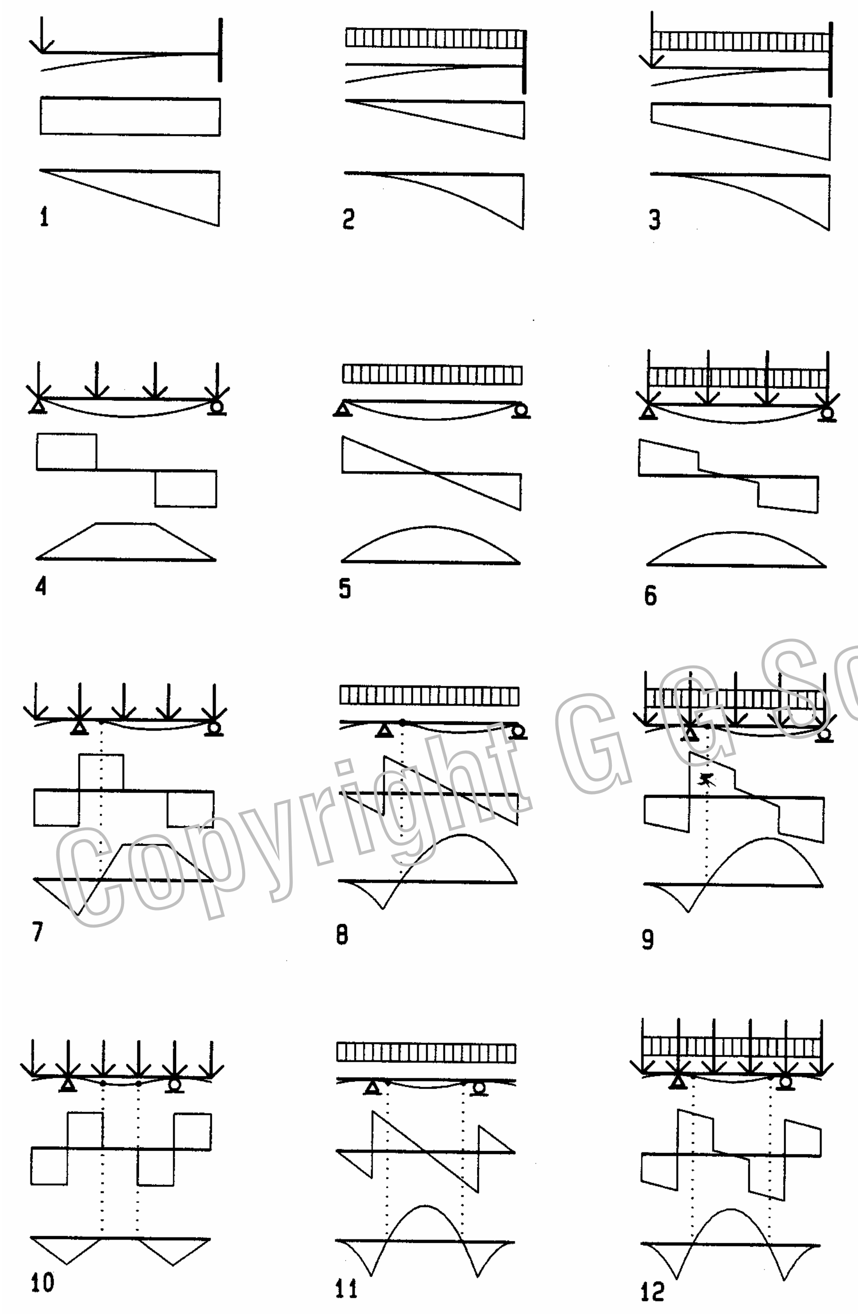
Δ_{max} = (23/684) PL^3/(EI) + (5/384) WL^3/(EI) *

Δ_{max} = [(23/684)8$(360)^3$ + (5/384) 12$(360)^3$] / [(30000)612]

Δ_{max} = 1.1 in

Δ_{all}=360/240 = 1.5 in — 1.5 > 1.1, ok

* Superimposition of equations for point load and uniformly distributed load



Typical beam diagrams

Deflection, shear, and bending diagrams are shown here for typical beams. The beam with deflection and load diagrams are drawn on top with shear and bending diagrams shown below. With experience, these diagrams may be drawn by visual inspection prior to computing. This is useful to verify computations and develop an intuitive sense and visualization regarding shear and bending on beams. The deflection diagram is drawn, visualizing the deflection of a thin board, flexible ruler, or similar device. It is drawn grossly exaggerated to be visible. The shear diagram is drawn at a convenient force scale left to right, starting with zero shear to the left of the beam. Downward uniform load yields downward sloping shear. Downward point loads are drawn as downward offset, and upward reactions yield upward offset. Bending diagrams are drawn, considering the area method; namely, bending at any point is equal to the area of the shear diagram up to that point. Both, shear and bending must be zero to the right of the right beam end. To satisfy this, requires a certain amount of forward thinking and, in complex cases, even working backward from right to left as well as left to right.

1 Cantilever beam with point load
2 Cantilever beam with uniform load
3 Cantilever beam with mixed load
4 Simple beam with point loads
5 Simple beam with uniform load
6 Simple beam with mixed load
7 Beam with one overhang and point load
8 Beam with one overhang and uniform load
9 Beam with one overhang and mixed load
10 Beam with two overhangs and point loads
11 Beam with two overhangs and uniform load
12 Beam with two overhangs and mixed load

8

ASD, LRFD, Masonry, and Concrete Design

ASD (*Allowable Stress Design*) and *LRFD* (*Load and Resistance Factor Design*) are two design and analysis methods currently used for structural design. ASD is the classic method used since the inception of structural design and sometimes referred to as working stress design. LRFD is a new method, increasingly promoted by codes. The difference of the two methods is essentially in the way they consider the issue of safety: ASD uses actual loads to design members for allowable stress that is reduced from ultimate strength or yield stress by a safety factor. By contrast LRFD assigns safety to the load, increasing actual service load by a load factor to design members for stress that is close to the ultimate strength. The load factors provide a more rational safety because dead load is more predictable than live load and therefore has a smaller load factor

LRFD is similar to the *Strength Method* or *Ultimate Strength Method* that has been used for concrete design since about 1960. The two methods are briefly introduced below and demonstrated for masonry design (ASD) and concrete design (LRFD). At present, for masonry design ASD is still more common.

ASD (Allowable Stress Design)

Allowable stress design, also known as *working stress design*, was the traditional method in general use before the advent of the LRFD method. The ASD method is based on service loads as defined by codes. Structural members are designed to resist such loads without exceeding allowable stresses, allowable deflections, and lateral drift. Allowable stresses are based on ultimate strength or yield stress, reduced by safety factors. The safety factors depend on the consistency of a given material and the type of stress. For example, allowable axial tensile stress for steel is 60 % of the yield stress (Fa= 0.6Fy). Allowable deflections for horizontal span members shall not exceed Δ=L/240 for combined dead and live load and
Δ=L/360 for live load only

Δ = maximum deflection
L = span

The lateral drift of vertical structures shall not exceed a fraction of the height (Maximum drift is typically 0.5 % of height).

ASD Load combinations

Based on the 1997 UBC structures and all portions thereof shall resist the most critical effects resulting from the following combinations of loads:

D
D +L + (Lr or S)
D + (W or E/1.4)
0.9D ± E/1.4
D + 0.75[L + (Lr or S) + (W or E/1.4)]

D = Dead load
E = Earthquake load
L = Live load
Lr = Roof live load
S = Snow load
W = Wind load

Most of this book is based on ASD

Allowable stress is defined by a material's *ultimate strength* or *yield strength* and a *factor of safety*. Building codes and trade associations provide allowable stress for various materials and grades of materials, which may also depend on duration of load. Allowable wood stress also depends on temperature, moisture content, size, and if a member is single or repetitive, like closely spaced joists. Relevant factors regarding allowable stress are briefly introduced here and further described later in this chapter.

Wood

Base values for *Douglas Fir-Larch* 2"x5" (5x13 cm) or greater for allowable stress: bending (F_b), tension (F_t), compression (F_c), compression normal to grain ($F_{C\perp}$), horizontal shear (F_v), and elastic modulus (E).

Grade	F_b	F_t	F_c	$F_{c\perp}$	F_v	E	units
Select	1,500	1,000	1,100	625	85	1,600,000	psi
structural:	*10.3*	*6.9*	*7.6*	*4.3*	*0.6*	*1,1032*	*MPa*
No. 1:	1,200	625	1,000	625	85	1,600,000	psi
	8.2	*4.3*	*6.9*	*4.3*	*0.6*	*1,1032*	*MPa*
No. 2:	700	475	1300	625	85	1,300,000	psi
	4.8	*3.3*	*9.0*	*4.3*	*0.6*	*8,964*	*MPa*

Steel

The table gives yield stress (F_y), ultimate strength (F_u), allowable stress for bending (F_b), compression (F_c), tension (F_t), and shear (F_v), elastic modulus (E)

Steel grade	F_y	F_u	F_b	F_c F_t	F_v	E	ksi
ASTM A36	36	58-80	22	22	14.5	29,000	ksi
	248	*400-550*	*150*	*150*	*100*	*200,000*	*MPa*
ASTM A572	50	65	30	30	20	29,000	ksi
	345	*450*	*210*	*210*	*140*	*200,000*	*MPa*

Masonry

Allowable compressive stress F_a, for masonry with special inspection is 25% of specified strength f'_m by the ASD method; reduced for slenderness. **Specified Compressive strength** f'_m is based on compressive strength of masonry units and mortars type M, S, N.

Type	Concrete masonry (ksi)				Clay brick masonry					(ksi)
Unit strength	1.9	2.8	3.75	4.8	4	6	8	10	12	14
f'_m (M or S)	1.5	2	2.5	3	2	2.7	3.35	4	4.7	5.3
f'_m (N)	1.35	1.85	2.35	2.8	1.6	2.2	2.7	3.3	3.8	4.4

Type	Concrete masonry (MPa)				Clay brick masonry					(MPa)
Unit strength	13	19	26	33	28	41	55	69	83	97
f'_m (M or S)	10	14	17	21	14	19	23	28	32	37
f'_m (N)	9	13	16	19	11	15	19	23	26	30

Concrete

By working stress method, allowable stresses are based on compressive strength f'_c. Typical compressive strengths range from 2 to 6 ksi (14 to 41 MPa)

Allowable compressive stress		0.40 f'_c
Allowable compressive bending stress		0.45 f'_c
Allowable shear stress without reinforcing:	beam	1.1 $f'_c{}^{1/2}$
	joist	1.2 $f'_c{}^{1/2}$
	footing & slab on grade	2.0 $f'_c{}^{1/2}$

LRFD (Load and Resistance Factor Design)

LRFD is a new method increasingly promoted by building codes. It is similar to the strength method used for concrete design since the 1960th. LRFD is based on factored loads (amplified service loads) and nominal resistance (reduced ultimate strength). Safety factors are assumed by factored load, rather than allowable stress as in ASD. Typical factored loads are 1.2 dead load and 1.6 live load. The LRFD design method is essentially defined by the equation:

ϕ Design Strength ≥ Required Resistance

ϕ = Resistance factor (ϕ <1)

The resistance factor depends on the material and type of stress, based on reliability and consistency of tests (low reliability = low ϕ)

Resistance factors ϕ					
	Material			Steel	
Stress types	Concrete *	Masonry *	Wood	Limit states	ϕ
Bending	0.9	0.8	0.85	Yielding	0.9
Shear	0.85	0.6	0.75		
Tension				Rupture	0.75
Compression	0.75 spiral 0.70 tied	0.65	0.9	Compression and buckling	0.85
Stability			0.85		

* Strength design (similar to LRFD)

LRFD load combinations

Based on the 1997 UBC structures and all portions thereof shall resist the most critical effects resulting from the following combinations of factored loads:

1.4D
1.2D + 1.6L + 0.5(Lr or S)
1.2D + 1.6(Lr or S) + (f1 L or 0.8W)
1.2D + 1.3W + f1 L + 0.5(Lr or S)
1.2D + 1.0E + (f_1 L + f_2 S)
0.9D ± (1.0E or 1.3W)

D = Dead load
E = Earthquake load
L = Live load
Lr = Roof live load
S = Snow load
W = Wind load
f_1 = 1.0 for floors of public assembly, live load >100 psf and garage live load
0.5 for all other live loads
f_2 = 0.7 for roofs that don't shed snow
0.2 for all other roofs

Since factored loads are based on statistical probability and extensive tests, the LRFD method usually results in smaller members than the ASD method, but the LRFD method is more complex and requires stiffness (deflection and drift) to be computed by actual service loads rather than factored loads. Also, since the LRFD method considers strength rather than stress, the results cannot be verified for allowable stress as in ASD. To verify results requires a second analysis by ASD. This represents a challenge for future refinement of the LRFD method.

Example: roof rafters

Roof steel rafters, sloping 2:12, spaced 10', are subject to the following loads:

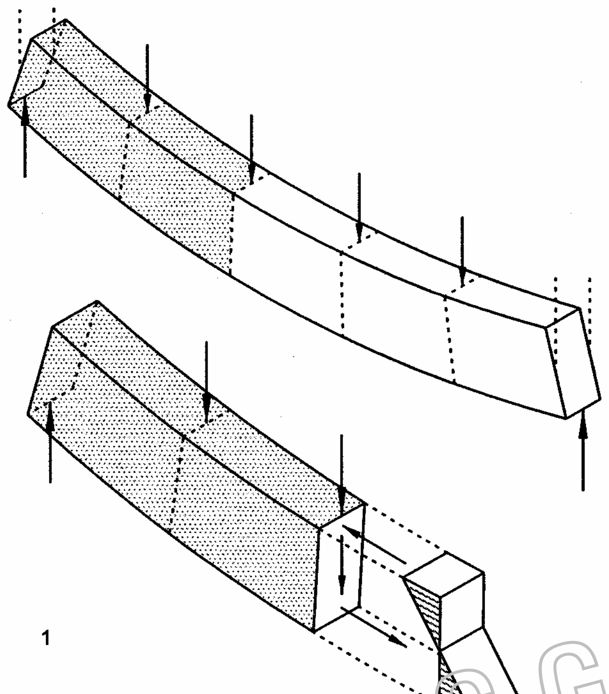
Dead load D = 40 psf
Snow load S = 30 psf
Wind load W = 15 psf (downward)

Find the maximum load effect per liner foot

D = 40 psf (10') D = 400 plf
L = 30 psf (10') L = 300 plf
w = 15 psf (10') w = 150 plf

1.4D
1.4(400 plf) 560 plf
1.2D + 1.6L + 0.5(Lr or S)
1.2(400 plf) + 0.5(300 plf) 630 plf
1.2D + 1.6(Lr or S) + (f1 L or 0.8W)
1.2(400) + 1.6(300 plf) + 0.8(150 plf) 1080 plf
1.2D + 1.3W + f1 L + 0.5(Lr or S)
1.2(4000 plf) + 1.3(150 plf) + 0.5(300 plf) 1905 plf
1.2D + 1.0E + (f_1 L + f_2 S)
1.2 (400 plf) + (0.7(300 plf) 690 plf
0.9D ± (1.0E or 1.3W)
0.9(400 plf) +1.3(150 plf) 555 plf

Governing load effect **w_u = 1905 plf**



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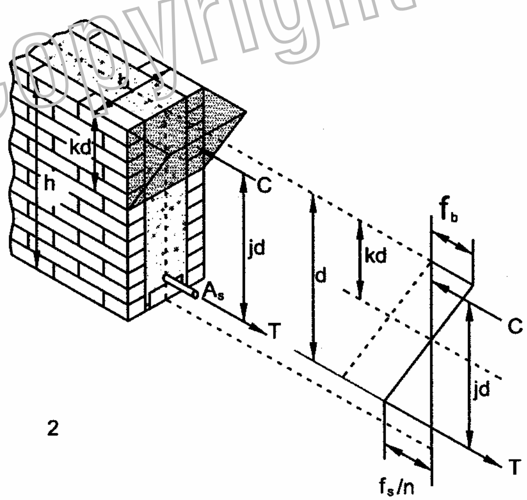


2

Masonry Design (ASD)

Allowable masonry stresses require *special inspection* as defined by bulding codes. **Allowable stresses are one half without special inspection**

1 Beam of homogeneous material resistes gravity bending with maximum top compression, maximum bottom tension and zero stress at the neutral axis

2 Masonry beam resists only compression and steel rebars resist tension. The stiffness difference of masonry and steel are adjusted by the elastic ratio $n = E_s / E_m$

E_s	=	Elastic modulus, steel	E_s = 29,000 ksi
E_m.	=	Elastic modulus. masonry	E_m.= 750 f'_m
n	=	Elastic ratio (steel / masonry)	$n = E_s / E_m$
f'_m	=	Specified masonry compressive strength	f'_m = 1.5 to 5 ksi
F_b	=	Allowable masonry bending stress	$F_b=f'_m/3$, max.2 ksi *
F_s	=	Allowable rebar stress:	$F_s=0.5F_y$, max. 24 ksi *
F_s	=	Allowable stirrup stress:	$F_s=0.4F_y$, max. 24 ksi *
F_v	=	Allowable shear stress if masonry resist all shear	$F_v=(f'_m)^{1/2}$, max.50 psi *
F_v	=	Allowable shear stress if steel resist all shear	$F_v=3(f'_m)^{1/2}$, max.150 psi *
*		**Allowable stresses are one half without special inspection** (Sample tests at start of construction and for every 5000 ft² of masonry)	
b	=	Beam width	
d	=	Effective depth (top of beam to centroid of reinforcing steel)	
h	=	beam depth (top face to bottom face)	
kd	=	Depth of triangular compression stress block	
jd	=	Moment arm, d-kd/3 (distance from tension to compression centroids)	
f_b	=	Maximum compressive bending stress	
f_s	=	Tensile stress in reinforcing steel	
A_s	=	Cross section area of reinforcing steel	
p	=	Ratio of steel area / beam cross section, p = As/bd (0.02% to 2.88%)	

Referring to diagram 2 the following equations are derived:

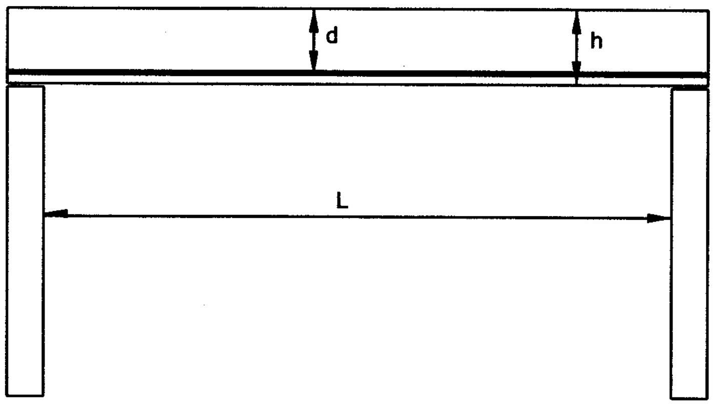
For Balanced beams (with enough reinforcing so that steel and masonry reach their respective limits simultaneously), kd is defined by similar triangles:

$$kd / d = f_b / (f_b + f_s /n)$$

$$k = 1 / [1+ f_s / (n f_b)]$$

Based on the k-factor other factors are deirved:

Resiting lever arm	$jd = d - kd/3$
j-factor	$j = 1 - k/3$
Resisting moment	$M = ½ f_b\, b\, kd\, jd = ½ f_b\, kj\, bd^2$
	$M = R\, bd^2$
Resistance factor	$R = ½ f_b\, kj$
Max. masonry stress	$f_b = 2M / (kj\, bd^2)$
Required steel area	$A_s = M / (F_s\, jd)$
Steel stress	$f_s = M / (A_s\, jd)$



Rebar diameters / **Cross-section areas**

Size	in	in	mm	in²	mm²
#3	3/8	0.375	9.5	0.11	71
#4	4/8	0.500	12.7	0.20	129
#5	5/8	0.625	15.9	0.31	200
#6	6/8	0.750	19.1	0.44	284
#7	7/8	0.875	22.2	0.60	387
#8	8/8	1.000	25.4	0.79	510
#9		1.128	28.7	1.00	645
#10		1.270	32.3	1.27	819
#11		1.410	35.81	1.56	1006
#14		1.693	43.00	2.25	1452
#18		2.257	57.33	4.00	2581

Example: masonry beam design

Design a simply supported brick masonry beam

Assume:

L =16', b = 10", specified compressive strenght f'_m = 1500 psi, with special inspection, F_b = 1500/3 = 500 psi, grade 60 steel, F_s = 24 ksi

Dead load estimate — DL = 300 plf

Live koad estimate — <u>LL = 500 plf</u>

w = 800 plf

Bending moment $M = w L^2/8 = 800(16)^2/8$ — M = 25,600 #'

Elastic modulus

E_m = 750 f'_m = 750(1500)/1000 — E_m = 1,125 ksi

Elastic ratio

$n = E_s/E_m$ = 29,000 ksi / 1875 ksi — n = 25.8

$k = 1/[1+ f_s / (n f_b)]$ = 1/[1+24,000/(25.8 x 500]) — k = 0.35

j = 1 – k/3 = 1 – 0.35/3 — j = 0.88

Resistance factor

R = ½ f_b kj = ½ 500 x0.35 x 0.88 — R = 77

Effective depth required, M = R bd^2

$d = \sqrt{M/bR} = \sqrt{25{,}600x12"/(10x77)}$ — d = 20"

Beam depth h = d+4" = 20+4 (4" for rebar + cover (adjist for modules) — h = 24"

Required steel area

$A_s = M / (F_s jd)$ = 25,600(12") / (24,000 x 0.88 x 20) — A_s = 0.72 in²

Use 1 # 8 bar, $A_s = \pi (0.5)^2$ — A_s = 0.79 in², OK

Example: masonry beam analysis

Assume:

Simple beam, *L=10', b=8", d=32"*, specified compressive strenght f'_m = 1500 psi, without special inspection, F_b =1/2x1500/3 = 250 psi, grade 40 steel, F_s = 20 ksi. 1 # 6 rebar.

Dead load estimate — DL = 600 plf

Live koad estimate — <u>LL = 900 plf</u>

w = 1500 plf

Bending moment $M = w L^2/8 = 1500(10)^2/8$ — M = 18,750 #'

k-factor

$k = 1/[1+ f_s / (n f_b)]$ = 1/[1+20,000/(25.8 x 250]) — k = 0.24

j = 1 – k/3 = 1 – 0.24/3 — j = 0.92

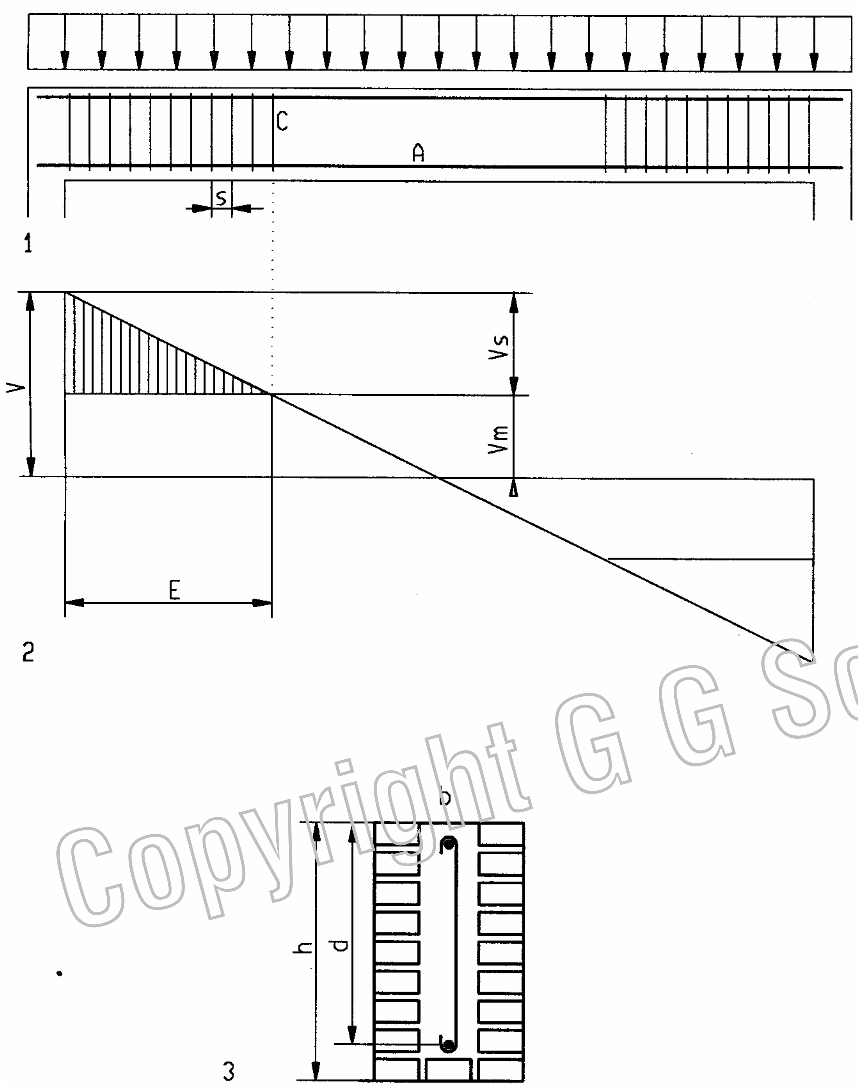
Max. masonry stress

$f_b = 2M / (kj\ bd^2)$ = 2x187503x12" / (0.24x0.92x8x32²) — f_b = 248 psi < 250, OK

Steel cross section ares

Steel stress

$f_s = M / (A_s jd)$ =18750 x 12" / (0.44x0.92x32) — f_s = 17,370 psi < 20,000, OK



Shear reinforcing

Bending members are subject to shear that requires reinforcing to prevent diagainal cracks caused by the tensile components of shear at 45 degrees. Vertical sterrups provide shear reinforcing. Maximum stirrup spacing of d/2 prevents shear cracks. Thus:

$V = V_m + A_v F_s d/s$

V = maximum shear
V_m = shear resisted by masonry
A_v = Cross section area of shear reinforcing
F_s = Allowable steel stress
d = effective beam depth
s = stirrup sacing

Empirical UBC formuals

UBC assumes 2 conditions: 1) all shear reisted by masonry; 2) all shear resisted by steel

Allowable shear stress if masonry resists all shear $F_v = \sqrt{f'_m}$, max 50 psi *

Allowable shear stress if steel resists all shear $F_v = 3\sqrt{f'_m}$, max 150 psi *

* **Allowable stresses are one half without special inspection**

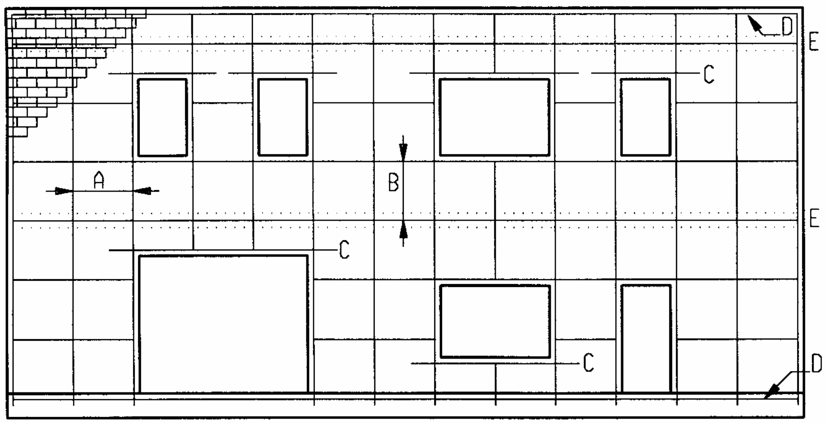
Shear resisted by stirrups (V_m ignored) $V = A_v F_s d / s$
Shear resisted by masonry $V = F_v bjd$
F_v = Allowable masonry shear stress
Computed shear stress (estimate j = 0.9) $f_v = V / bjd$
Stirrup spacing $s = A_v F_s / bf_v$, max. s = d/2

1 Masonry beam with uniform load
2 Shear diagram
3 Beam cross section
A Linear bars resist ensile bending stress
b Beam width
C Stirrups (resist shear stress)
d Effective depth (top of beam to steel centroid
E Zone requiring shear reinforcing
h depth of beam

Example: masonry beam

Assume: simple beam, L = 10', b = 8", d = 32", specified strenght f'_m = 1500 psi, without special inspection, $F_v = ½(1500)^{1/2}$ = 19 psi, grade 40 steel, F_s = 16 ksi

DL+LL w = 1500 plf
Max shear V = wL/2 = 1500 x10/2 V =7500 #
Shear stress $f_v = V / bjd$ = 7500/ 8x0.9x 32 f_v = 33 psi
Stirrup spacing $s = A_v F_s / bf_v$= 0.2x16/8x33 s = 12"
Note: for CMU, s would need to be ajusted a multiple of 8" modules
Shear resisted by masonry $V_m = F_v bjd$ =19x8x0.9x32 V_m = 4378 #
Shear resisted by steel $V_s = V - V_m$ = 7500-4378 V_s = 3122 #
Zone requiring shear reinforcing $E = (L/2)V_s/V$ = 5'x3122/7500 E = 2.1'



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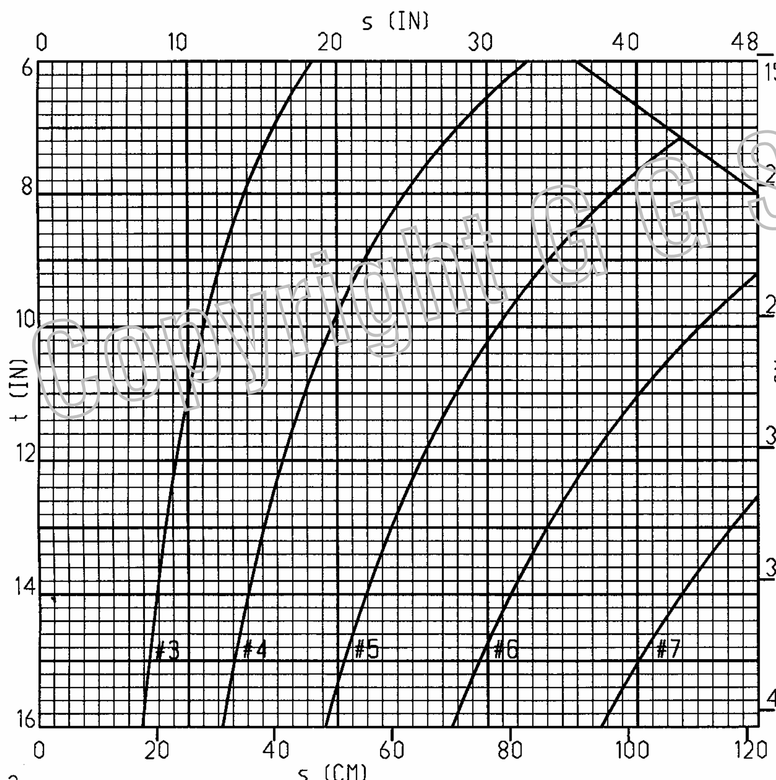


2

Shear walls

Shear wall reinforcement to resist lateral load is required in seismic zones 2 to 4. The reinforcement bars must be provided in both in both vertical and horizontal directions. Horizontal bars must be continuous or spliced at intersections and wall corners. **Minimum reinforcement** is required as follows.

Seismic areas require horizontal and vertical rebars of at least 0.2% of the wall cross section area. Bars in either direction may be 0.1% but shall be at least 0.07% with the remaining 0.13% in perpendicular direction. The greater percentage of bars should run in direction of primary span, normally vertical from floor to floor or roof. Bars shall be arranged as shown in 1: vertical and horizontal bars spaced maximum 4 ft (1.2 m); bars around all openings, the top bar extending at least 24 in (60 cm) or 40 bar diameters beyond openings; on top and bottom of walls; and at structurally connected floors and roofs. Graph 2 gives bar spacing for 0.1 % reinforcing of various wall sizes.

Moderate seismic areas requires rebars with cross-section of min. 0.2 in^2 (129 mm^2), # 4 bars, arranged as follows: vertical bars at 4 ft (1.2 m); horizontal bars spaced 10 ft (3 m); bars around all openings extending at least 24 in (60 cm) or 40 bar diameters beyond openings; bars on top and bottom of walls; and at structurally connected floors and roofs.

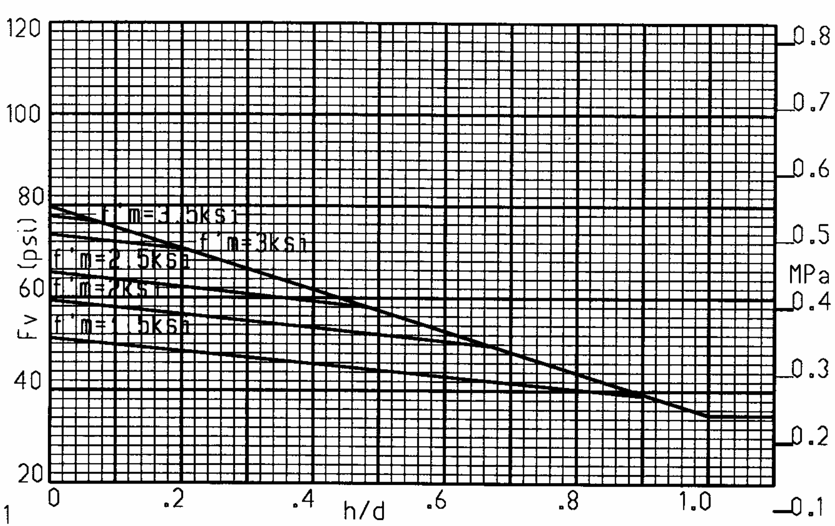
1 Wall elevation with reinforcing bar layout for seismic zones 2 to 4
2 Bar size and spacing for 0.1% reinforcing of wall cross-section area

A Vertical bars, spaced maximum 4 ft or 6 times the wall thickness
B Horizontal bars spaced max. 4 ft in high seismic areas; 10 ft in moderate areas
C Bars at openings, extending min. 2' or 40 bar diameters beyond opening
D Horizontal bars at top and base of wall
E Bars at structurally connected floors and roof
s Spacing of reinforcing bars, sizes #3 to #7 (max. 6 times bar diameter)
t Wall thickness

Rebar diameters				**Cross-section areas**	
Size	in	in	mm	in^2	mm^2
# 3	3/8	0.375	9.5	0.11	71
# 4	4/8	0.500	12.7	0.20	129
# 5	5/8	0.625	15.9	0.31	200
# 6	6/8	0.750	19.1	0.44	284
# 7	7/8	0.875	22.2	0.60	387

Example: 8” CMU wall

# 4 bar spacing (from graph)	s = 24”
# 5 bar spacing, s = 38”, adjust to multiple 8” ≤ 38”	s = 32”
# 6 bar spacing, s = 48” max.	s = 48”



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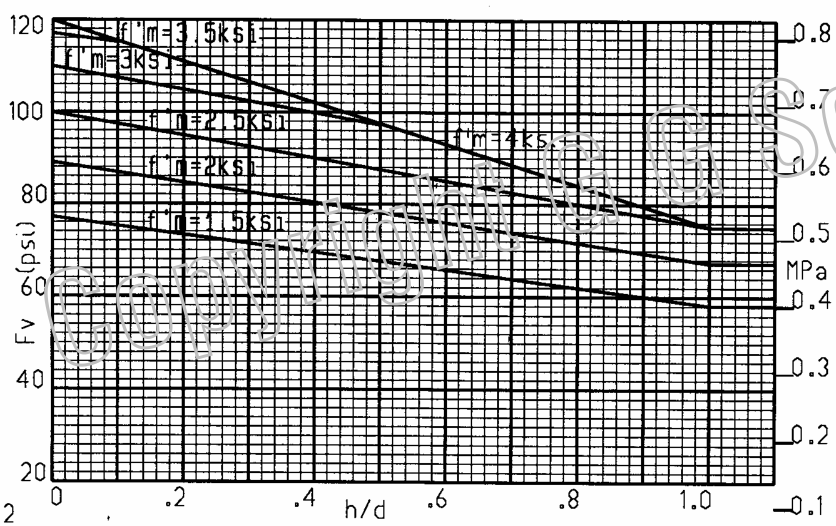

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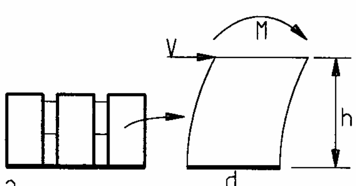

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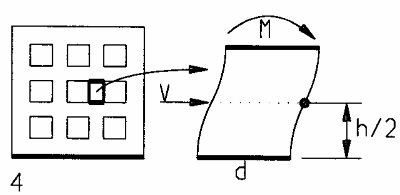


4

Allowable shear stress for reinforced masonry walls is defined two ways masonry to resist all shear or steel to resist all shear. Height-to-width ratio (h/d) also effects shear strength. Narrow walls have less strength than long walls. Allowable shear stress depends on the ratio M/Vd which may be expressed as h/d ratio (M/Vd = Vh/Vd = h/d).

1 Allowable shear stress F_v (assuming masonry resists all shear):

For h/d < 1

$F_v = 1/3\ (4\text{-}h/d)\ (f'_m)^{1/2}$ $\quad F_{v\,(max)}$ = 80 - 45h/d psi *

For h/d => 1

$F_v = 1.0\ (f'_m)^{1/2}$ $\quad F_{v\,(max)}$ = 35 psi *

2 Allowable shear stress F_v (assuming reinforcing resists all shear):

For h/d <1

$F_v = 1/2\ (4\text{-}h/d)\ (f'_m)^{1/2}$ $\quad F_{v\,(max)}$ = 120 - 45h/d psi *

For h/d =>1

$F_v = 1.5\ (f'_m)^{1/2}$ $\quad F_{v\,(max)}$ = 75 psi *

* **Allowable stresses are one half without special inspection**

3 Cantilever wall, free to bend in single curvature

4 Wall with fixed support bends with inflection point at mid-height

d Width of wall or wall element

h Height of wall or wall element

f'_m Specified masonry compressive strength (ksi)

F_v Allowable shear stress (psi and MPa)

M Bending moment M = Vh or M = Vh/2

V Shear force

Bar spacing $\quad s = A_v F_s / bF_v$

A_v= bar area, F_s= allowable bar stress, b= wall width, F_v= allowable masonry shear stress

Example: CMU shear wall design

Assume: h=8', 8" (nominal), f'_m=2 ksi, # 4 bars, F_s=24 ksi, no inspection, design masonry to resist all shear of V = 6,000 #. Try wall length d = 4', h/d=2

Allowable shear stress from graph 1 $\quad F_v$ = 17 psi

Bar spacing $s = A_v F_s / bF_v$ = 0.2x24,000 / (7.625x17) = 37" $\quad$ use s = 32"

Note: bar spacing rounded down to 8" CMU module

Wall shear capacity V = F_v (wall area) = 17x7.625x48" $\quad$ V = 6,222 #

Example: CMU shear wall analysis

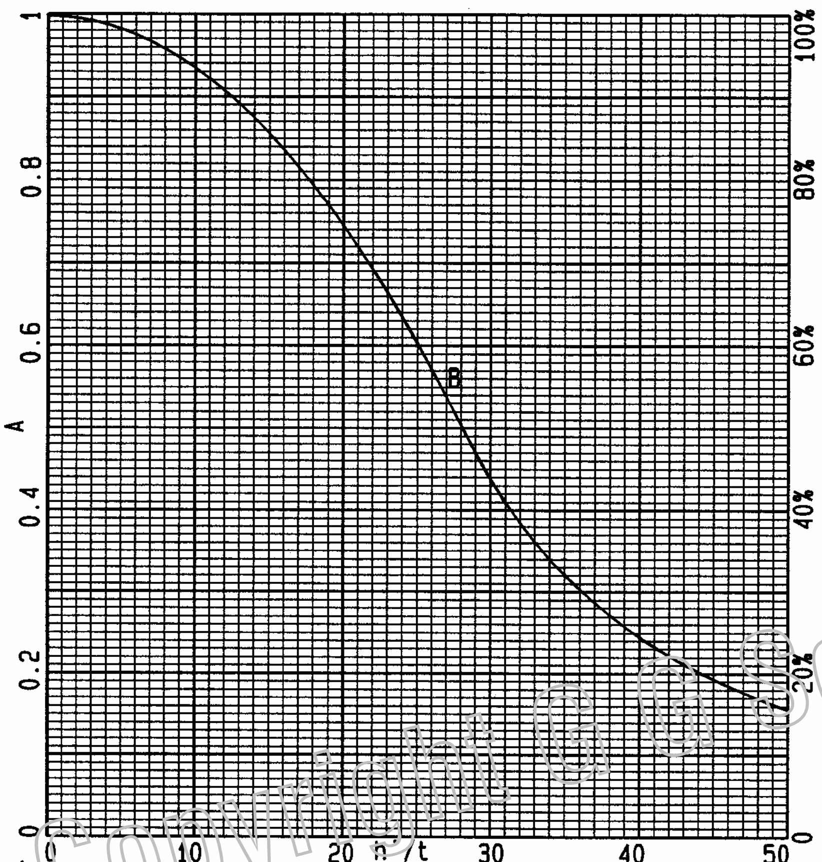
Assume: same wall as above, but inspected and with reinforcing to resist all shear

Allowable shear stress from graph 2 $\quad F_v$ = 58 psi

Bar spacing $s = A_v F_s / bF_v$ = 0.2x24,000 / (7.625x58) = 10.8" $\quad$ use s = 8"

Wall shear capacity V = F_v (wall area) = 58x7.625x48" $\quad$ V = 21,228 #

Note: rebars at 8" vs. 32" and inspection increase capacity from 6 k to 21 k



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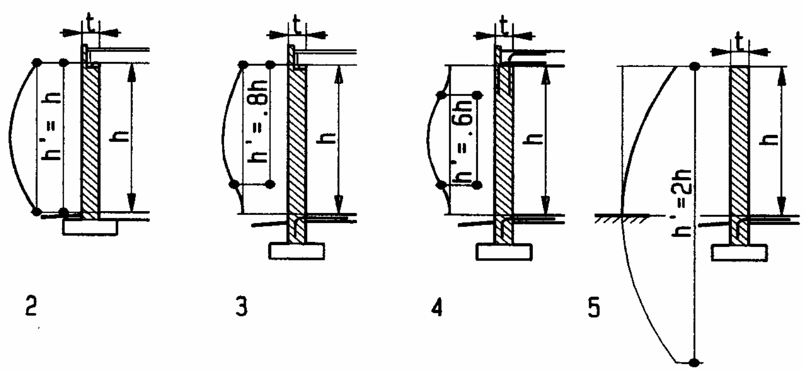


2 3 4 5

Specified Compressive strength *f'*$_m$ for masonry is defined by the strength of masonry units and mortars type M, S, N, with values from ***f'*$_m$ = 1,500 to 4,000 psi.**

Allowable compressive stress F_a for masonry with special inspection with or without grouting is 25% of the specified strength f'_m by the working stress method; reduced for slenderness (shown in graph 1) as followings:

$F_a = 0.25\, f'_m\, [1-(h'/140r)^2]$ * for $h'/r \le 99$ ($h'/t \le 29$)

$F_a = 0.25\, f'_m\, (70r/h')^2]$ * for $h'/r > 99$ ($h'/t > 29$)

For reinforced masonry columns the allowable compressive force P_a is:

$P_a = (0.25 f'_m A_e + 0.65 A_s F_{sc})\, [1-(h'/140\, r)^2]$ * for $h'/r \le 99$ ($h'/t \le 29$)**

$P_a = (0.25 f'_m A_e + 0.65 A_s F_{sc})\, (70\, r/h')^{2}$ * for $h'/r > 99$ ($h'/t > 29$)**

- A_e = Area of masonry (net area for un-grouted masonry)
- A_s = Area of steel reinforcement
- F_{sc} = Allowable compressive stress of steel reinforcement
- h'= effective height of wall
- r = radius of gyration; for convenience, graph 1 substitutes radius of gyration r by thickness t, where $r = (I / A)^{1/2} = 0.289\, t$

*** Allowable stresses and loads are one half without special inspection**

**** For non-square columns the smaller dimension governs slenderness**

1 Slenderness reduction for allowable compressive stress
2 Masonry wall or column with pin support at both ends
3 Masonry wall or column with one fixed support
4 Masonry wall or column with two fixed supports
5 Masonry wall or column freestanding

A Reduction factor for slenderness h'/t
B Slenderness vs. stress reduction curve
h Height of wall or column
h' Effective height, adjusted for support type
t Wall thickness

Example: CMU wall

Assume: h=15', both ends fixed, h'=0.6x15=9', 8" CMU, t = 7.625", f'm = 2000 psi

Find allowable stress F_a

Slenderness h'/t = 9'x12"/7.625 = 9.4

Slenderness reduction (from graph 1) A = 0.94

F_a = 0.25 f'm A = 0.25 x 2000 x 0.94 F_a = 470 psi

Example: brick column

Assume: brick column, 20"x24", h = 30', pin supports, f'm = 2,5 ksi, with 6 #8 steel bars, grade 60, F_a = 60x0.4 = 24 ksi. Find allowable load P

Slenderness h'/t = 30'x12"/20" = 18, slenderness reduction (from graph 1), A = 0.81

$P = (0.25 f'_m A_e + 0.65 A_s F_{sc})(0.81)$

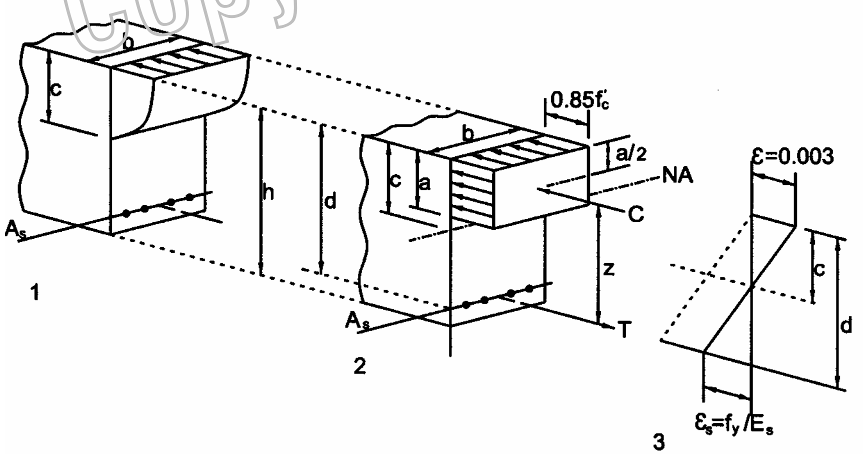
P = (0.25x2.5x20x24+0.65x6x0.44x24)(0.81) P = 276 k

Concrete Strength Design (LRFD)

Concrete *strenth design* is based on ultimatre concrete strenght, reduced by the reduction factor ϕ, similar to LRFD. At ultimte stress, concrete yields, forming a parabolic stress block. But srenght design for rectangular beams assumes a rectangular stress block wich gives similar results demonstrated by tests. Like masonry, concrete is strong in compression but very weak in tension. Hence steel reinforcing is used to resist tenion.

1 = Ultimate bending stress
2 = Bending stress assumed in strength design
3 = Strain of balanced beam

c = distance of neutral axis from top
d = effective depth (from top of beam to centroid of steel)
h = depth of beam
f'_c = specified concrete compressive strength
A_s = cross section area of steel — $A_s = bd\rho$
f_y = steel yield strength
a = depth of concrete stress block — $a = c\,\beta_1$
β_1 = 0.85 for $f'_c \le 4$ ksi, reduced 0.5 per 1 ksi > 4 ksi, min $\beta_1 = 0.65$
Z = resistant moment lever arm — $Z = d - a/2$
C = concrete compression — $C = 0.85\, f'_c\, ab$
T = steel tension — $T = A_s f_y$
ρ = Percentage of reinforcement — $\rho = A_s / bd$



Balanced beam

A convenient reference is the *balanced beam* whith steel reinforcing that reaches yield strength simultaneously with concrete. However, actual reinforcimg should be less to assure ductile bahavior (steel yields before brittle concrete failure). Considring similar triangles, balanced reinforcing ρ_b is derived, assuming E_s = 29,000 ksi:

$c_b / 0.003 = d / (0.003 + f_y/E_s)$

$$c_b = \frac{0.003}{0.003 + f_y/29{,}000}(d) \qquad c_b = \frac{87}{87 + f_y}(d)$$

For equilibrium ($\Sigma H = 0$, C = T) $\quad 0.85 f'_c\, \beta_1\, c_b\, b = A_s f_y$

Thus $\quad c_b = A_s f_y / (0.85\, f'_c\, \beta_1\, b)$

Since $A_s = bd\rho_b$

$$c_b = \frac{\rho_b b d f_y}{85 f'_c \beta_1 b} = \frac{\rho_b d f_y}{85 f'_c \beta_1} = \frac{87}{87 + f_y}(d)$$

Solving for balanced ρ_b

$$\rho_b = \frac{0.85 f'_c \beta_1}{f_y}\left(\frac{87}{87 + f_y}\right)$$

For equilibrium ($\Sigma H = 0$, C = T) $\quad 0.85\, f'_c\, ab = A_s f_y$
Thus $\quad a = A_s f_y / (0.85\, f'_c\, b)$
For moment equilibrium ($\Sigma M = 0$) $\quad M = (C \text{ or } T)(d-a/2) = A_s f_y (d-a/2)$
Substituting a and $A_s = \rho bd$ and (0.59 = (1/0.85)/2) and rearranging yields

$$M = bd^2 \rho f_y \left(1 - 0.59\rho \frac{f_y}{f'_c}\right)$$

Provides the nominal moment **$M = Rbd^2$**

Where **R = Resistance factor**

$$R = \rho f_y \left(1 - 0.59\rho \frac{f_y}{f'_c}\right)$$

The nominal design moment is adjusted by a reduction factor $\phi = 0.9$ **$\phi M_n = M$**

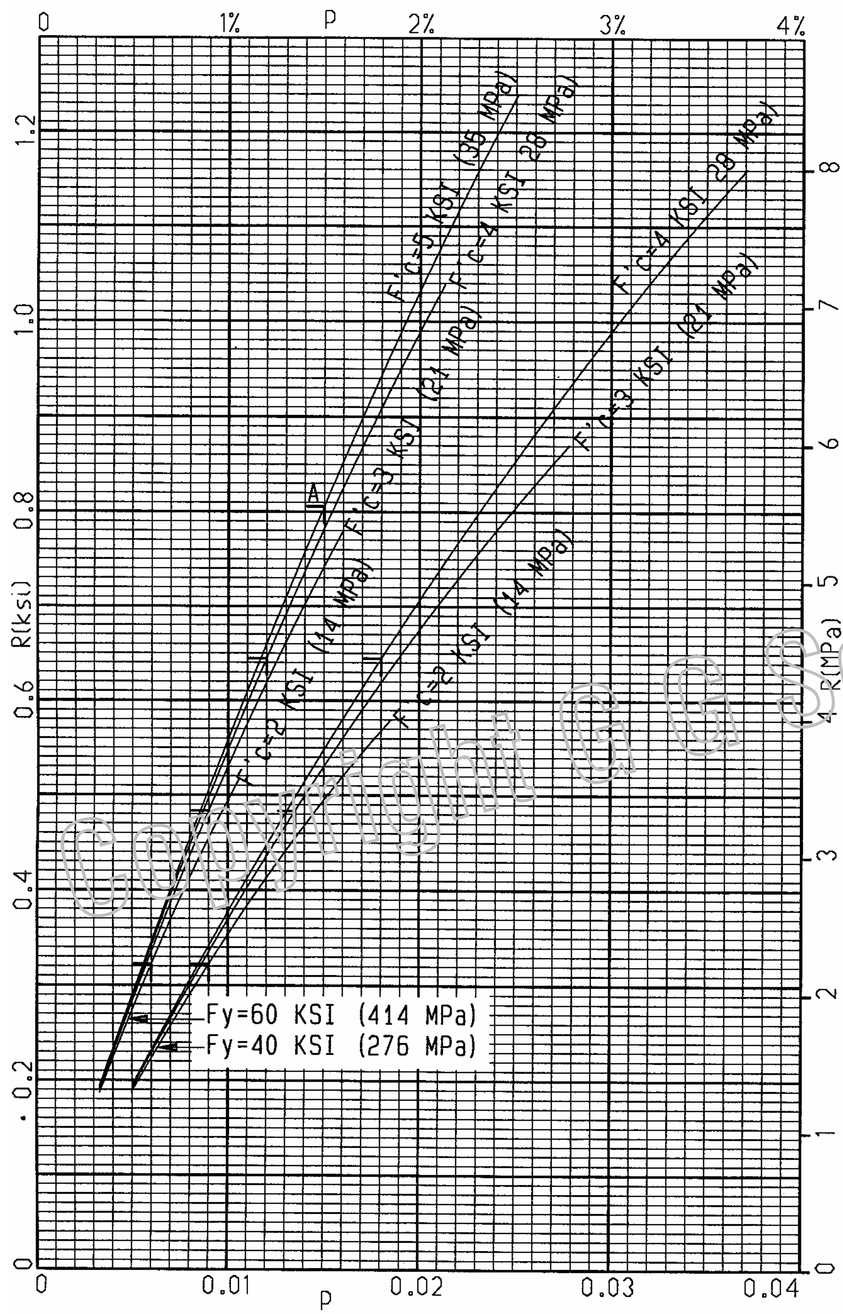
Reduction factors

Bending	$\phi = 0.90$
Shear and torsion	$\phi = 0.85$
Compression (spiral reinforcing)	$\phi = 0.75$
Compression (tied reinforcing) and bearing	$\phi = 0.70$

Reinforcing ratio limits ρ

Minimum	$\rho = 0.2$ ksi / f_y
Recommended	$\rho = 0.18\, f'_c / f_y$
Maximum (75% of balanced reinforcing)	$\rho = 0.75\, \rho_b$
Minimum Resistance factor (at min. $\rho = 0.2$ ksi / f_y)	**R = 0.192**

Note: Balancd reinforcing implies steel and concrete provide equal (balanced) strenght
Less steel provides ductile steel behavior, rathter than britttle concrete failure.



Design graph

The design graph shows ρ-factors on the X-axis and R-factors on the Y-axis. The graph lines extend from minimum to maximum ρ-factors with recommended values marked with an ^. The following examples demonstrate use of the graph:

Example: beam design

Assume: simply supported beam; L = 16'; f'c = 3 ksi; Fy = 60 ksi	
Factored Dead load = 1.4 x 943 plf / 1000	D = 1.32 klf
Factored Live load = 1.7 x 400 plf / 1000	L = 0.68 klf
	w = 2.00 klf
Bending moment $M = w L^2/8 = 2$ klf $(16)^2/8$	M = 64 k'
$M_n = M/\phi$ = 64k'x12"/ 0.9	M_R = 853 k"
Recommended depth (from table below) h = 16'(12")/16	h = 12"
Effective depth (d = h-3" for bar+cover) d = 12-3	d = 9"
Recommended R-factor (from graph)	R = 0.483 ksi
Beam width $b = M_n/(Rd^2) = 853/(0.483x9^2)$	b = 22"
Recommended reinforcement ratio (from graph)	ρ = 0.009
Bar cross section	
$A_s = \rho bd = (0.009)(22)(9) = 1.78$ in²	Use 6 # 5 bars, A_s = 6x0.31 =1.86 in²

Example: alternate beam design

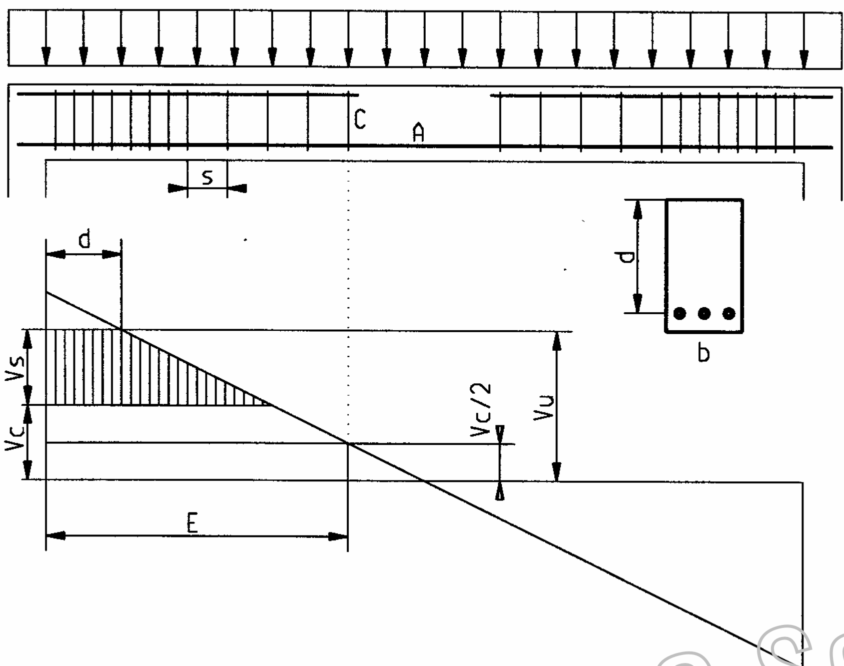
Assume: the same beam to fit an 8" CMU wall, R = 0.483, ρ = 0.009	
Effective depth	
$d = [M_n/(R\ b)]^{1/2} = [853/(0.483x8")]^{1/2}$	d = 15"
Rebar cross section	
$A_s = \rho bd = (0.009)(8x15) = 1.08$ in²	Try 3 # 6 bars, A_s = 3x0.44 = 1.32 in²
Check width: 3 bars+2 spaces+stirrups+cover = 3x6/8+2+1+3 = 8.25 > 8	not OK
Use 2 # 7 bars, A_s = 2x0.6	A_s =1.2 in²
Check width = 2x7/8+1+1+3 = 6.75	6.75 < 8, OK

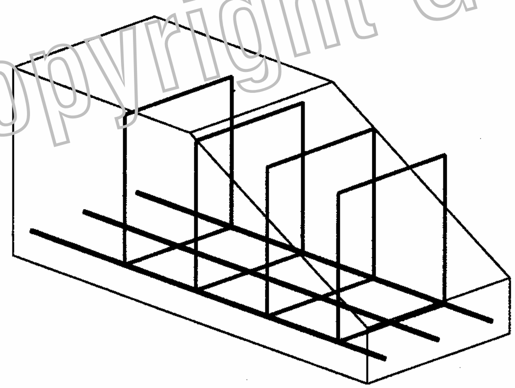
Minimum depths h of beams and slabs unless deflections are computed (L = span)

Support type	Beams & ribs	One-way slabs
1 Simply supported	L/16	L/20
2 One end continues	L/18	L/24
3 Both ends continues	L/21	L/28
4 Cantilever	L/8	L/10

Bar diameters / **Cross-section areas**

Size	in	in	mm	in²	mm²
# 5	5/8	0.625	15.9	0.31	200
# 6	6/8	0.750	19.1	0.44	284
# 7	7/8	0.875	22.2	0.60	387





Shear reinforcement

Shear reinforcement in bending members is required if the factured shear Vu exceeds the shear capacity of concrete Vc,except for:

- Slabs and footings
- Concrete joists
- Beams of <10" depth
- Beams with $V_u < \phi V_c/2$

Concrete shear capacity $V_c = \sqrt{f'_c}\,bd$

Subject to maximum shear stress $F_v = 100$ psi

Shear reinforcing is usually provided by vertical stirrups

Spacing $s = A_v f_v d / V_s$

A_v = total cross section area of stirrups (usually 2 bars per stirrup)

F_y = yield stress of stirrups

d = effective depth (top of beam to steel rebars)

V_s = shear resisted by stirrups

Maximum spacing $s = d/2$

Shear resisted by steel $V_s = V_u/\phi - V_c$

The maximum shear may be taken a distance d from supports

Reinforcing is required where $V > V_c/2$

- A Tensile reinforcement
- b Beam width
- C Shear reinforcing
- d Effective beam depth
- E Distance from support requiring stirrups (at $V_c/2$)
- s Stirrup spacing

Example

Design a simply supported beam, assume: $f'_c = 3$ ksi, $F_y = 60$ ksi, L = 20', b = 10", d = 12"

Factored DL+LL w = 4 klf

Concrete shear capacity $V_c = \phi\sqrt{f'_c}\,bd = 0.85\sqrt{4x10x12}$ $V_c = 18$ k

Maximum factored shear V_u = 4 klf x20'/2 $V_u = 40$ k

Shear at d from support V_u = 40-(4x12/12) $V_u = 36$ k

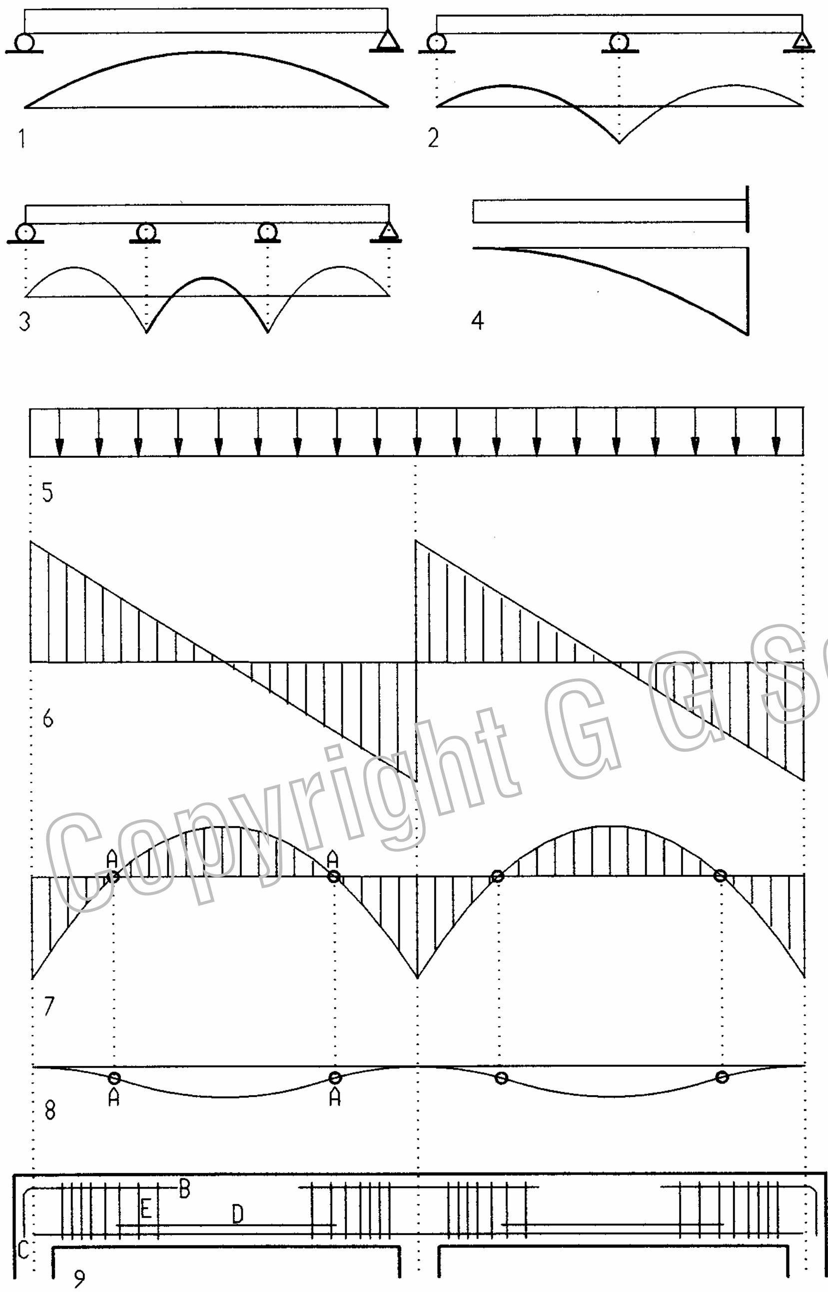
Shear resisted by steel $V_s = V_u/\phi$-VC = 36 / 0.85-22 $V_s = 20$k

Try # 4 srirrups (As = 2x0,2 $A_s = 0.4$ in^2

Spacing $s = A_v f_v d / V_s$ = 0.4x60x12/20 s = 14"

Check max s = d/2 = 12/2 = 6" Use s = 6"

Distance stirrups needed E = $(L/2/V_u)(V_u - V_c/2)$ = (10/40)(40-9) E =7.75'



Continuous and fixed-end beams
Concrete beams may continue over more than two supports, have moment resistant (fixed-end) supports , or both; all of which cause negative support moments that reduce positive mid-span moments and, therefore, require less depth than simply supported beams. Fixed-end supports are usually in beams of moment resisting frames.

1 Simply supported beam (determinate)
2 Continuous beam over 3 supports
3 Continuous beam over 4 supports
4 Cantilever beam
5 Load diagram for beam 9
6 Shear diagram
7 Bending diagram
8 Deflection diagram
9 Beam reinforcement

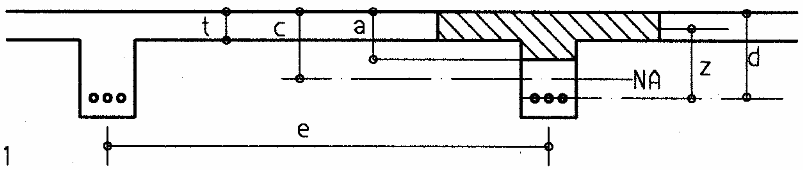
A Inflection points of zero bending moment, change from negative to positive bending
B Top bars at negative bending and convex deflection
C Hook at bar end of maximum stress anchors bar to concrete
D Bottom bars at positive bending and concave deflection
E Stirrups resist shear stress, with increased spacing toward mid-span of zero shear

Minimum depths h of beams and slabs unless deflections are computed (L = span)

Support type	Beams & ribs	One-way slabs
1 Simply supported	L/16	L/20
2 One end continues	L/18	L/24
3 Both ends continues	L/21	L/28
4 Cantilever	L/8	L/10

Reinforcement of continuous and fixed end beams follows shear and bending diagrams. Shear is similar to simply supported beams. Moment distribution varies from positive at mid-span to negative at supports and fixed-ends; causing convex deflections at supports and concave at mid-span with change at inflection points. Reinforcing correlates with the bending diagram: bottom bars at positive bending and top bars at negative bending; both extending somewhat beyond the inflection points to account for variable live loads.

Note:
The reader is referred to books on reinforced concrete design (Spiegel, 1992) for issues beyond the scope of this book, such as design of T-beams, beams with compression reinforcement, bond length of bars, combined axial and compressive stress, etc.



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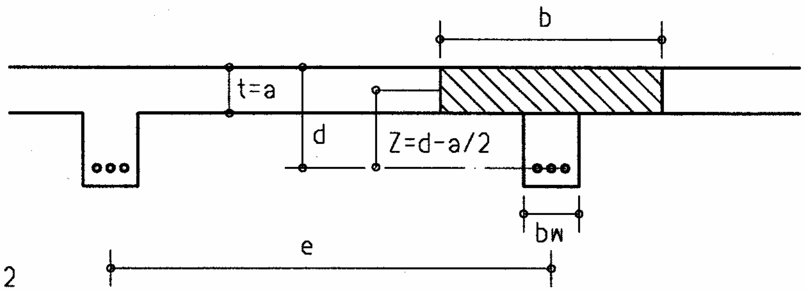

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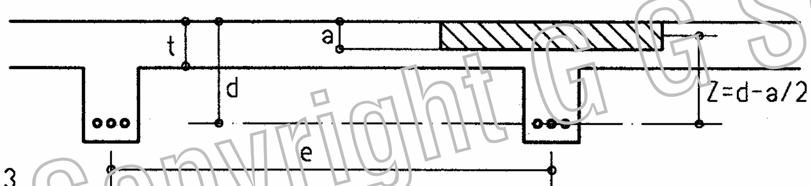

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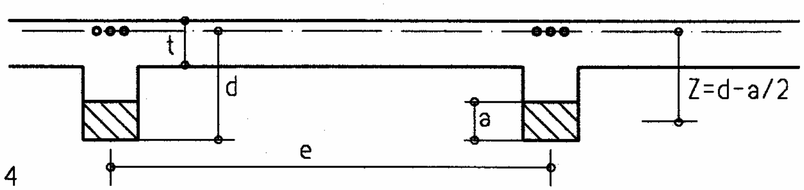


4

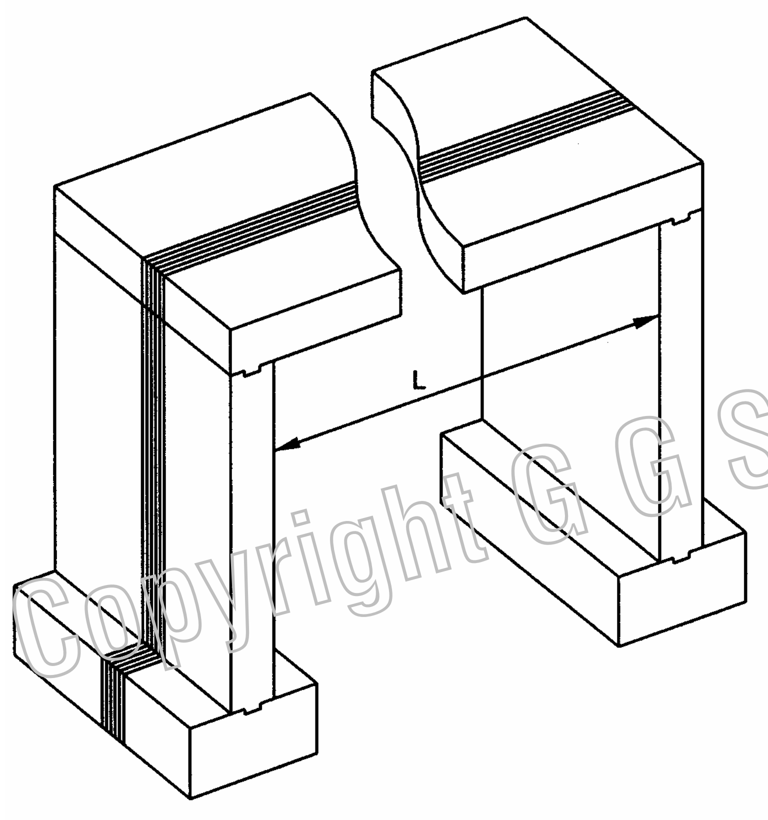
T-Beam

Floor slabs are usually poured together with beams. This provides to combine slab and beam as T-beam, with part of the slab acting as compressive flange and the beam acting as stem or web. Reinforcement at the beam bottom resists tensile stress for positive bending. For negative bending of continuous beams the tensile reinforcement must be on top and the beam resists compression, without benefit of the wider slab. T-beams are, therefore, most efficient as simply supported beams with positive bending only. Shear resistance is limited to the cross section of the web or the area defined by the width of the web and the effective depth of the beam. The flange width provided by the slab is limited to 1/4 of the span, 16 times the slab thickness plus width of the beam, or beam spacing, whichever is less. Depending on the ratio of reinforcement to compressive area, the neutral axis of T-beams may be below or within the slab thickness. For schematic design the resisting lever-arm may be estimated as the distance between center of the slab and center of the reinforcement.

1 T-beam with compression zone depth a > slab thickness t
2 T-beam with compression zone depth a = slab thickness t
3 T-beam with compression zone depth a < slab thickness t
4 T-beam with compression web due to negative bending

a Depth of compression zone
b Width of compression flange, limited by ACI code to the lesser of:
- 1/4 beam span
- 16 times slab thickness plus web width
- beam spacing e from center to center

d Effective depth (distance from reinforcement to compression zone edge)
Z Lever arm of internal resisting moment
(Distance from reinforcement to compression zone center)
NA Neutral Axis
t Slab thickness



One-way slabs

With reinforcing in only one direction, one-way slabs need reinforcing for temperature variation and shrinkage perpendicular to the main reinforcing. As percentage of slab cross section area, temperature reinforcing must be at least:

A_s = 0.20% for grade 40 and 50 steel
A_s = 0.18% for grade 60 steel

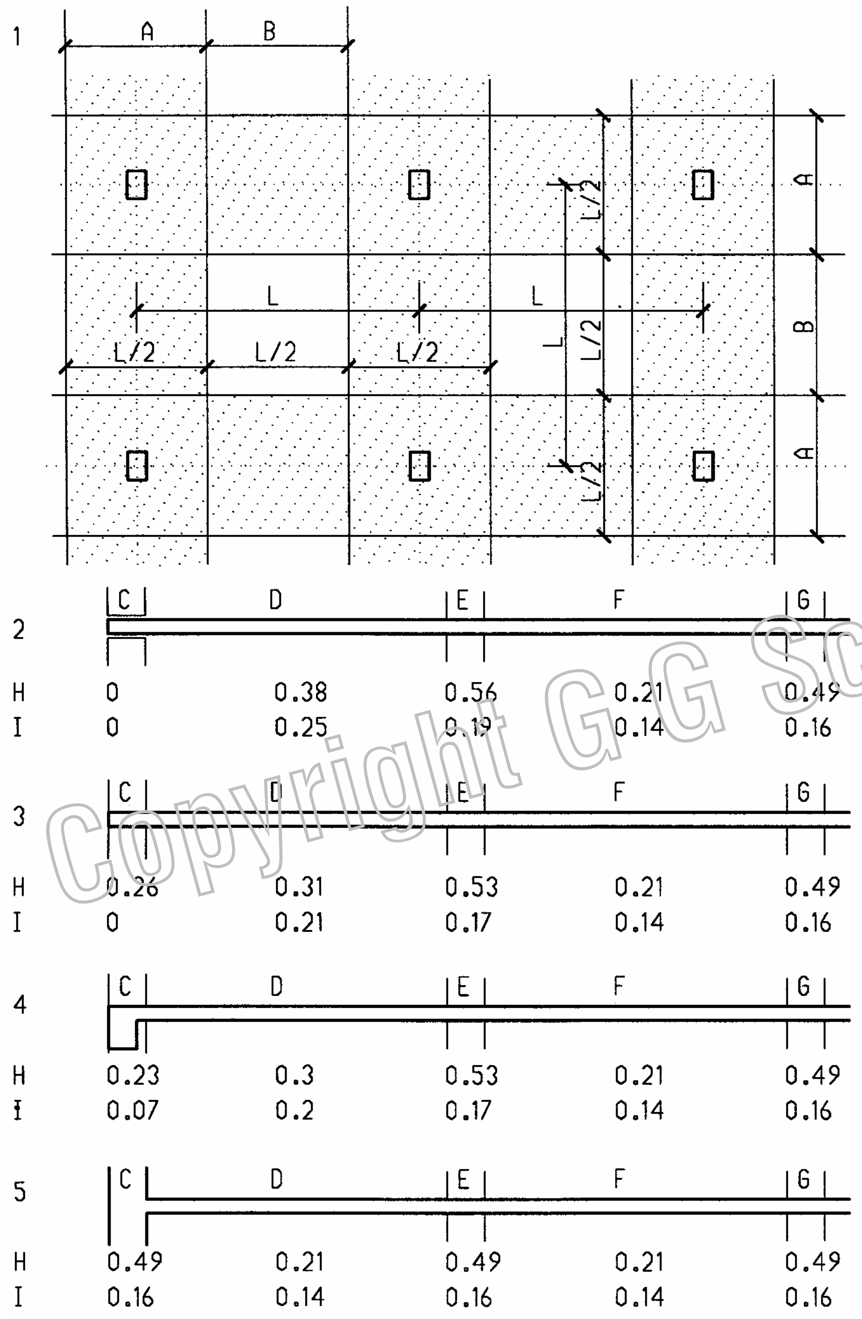
Minimum depths h of beams and slabs unless deflections are computed (L = span)

Support type	Beams & ribs	One-way slabs
Simply supported	L/16	L/20
One end continues	L/18	L/24
Both ends continues	L/21	L/28
Cantilever	L/8	L/10

Bar diameters				**Cross-section areas**	
Size	in	in	mm	in^2	mm^2
# 4	4/8	0.500	12.7	0.20	129
# 5	5/8	0.625	15.9	0.31	200
# 6	6/8	0.750	19.1	0.44	284

Example: One-way slab design

Assume: simply supported slab, L = 16'; f'_c = 3 ksi; F_y = 40 ks, Design a 1' wide strip	
Slab depth h = L / 20 = 16'x12"/20	h = 9.6"
Dead load = 150pcf x9.6"/12" = 120 psf+20 psf partitions+14 psf misc.	DL = 154 psf
Factored loads: (1.4x154 psf DL+1.7x50 psf LL)/1000	w = 0.3 klf
Moment M = $wL_2/8 = 0.3x16^2/8$	M = 9.6 k'
Required resisting moment (k")	
M_n = 12"M/ϕ = 12"x9.6k'/0.9	M_n = 128 k"
Effective depth d = h – bar/2 - cover	
d = 9.6 – 0,75/2 - 0.75	d = 8.5
Resistance factor R = M_n / bd^2 = 128 / (12x8.5^2)	R = 0.148 < 0.192
For min. R = 0.192 min. steel ratio ρ = 0.2 ksi / f_y	ρ = 0.005
Bar area $A_s = \rho bd$ = 0.005x12x8.5	A_s = 0.51 in^2
Use # 6 bars, A_s = 0.44 in^2 per bar	
Bar spacing s = 12"x 0.44 / 0.51	s = 10.3"
Temperature reinforcing	
A_s = 0.002bd = 0.002x12x8.5	A_s = 0.204 in^2
Try #6 bars	
Bar spacing: s = 12"x 0.44 / 0.204	s = 25.9"
Check s vs. ACI spacing limits: 4" < s < 18" < 5h	25.9 > 18, not OK
Use # 4bars	
Bar spacing: s = 12"x0.20 / 0.204	s = 11.8"



Two-way slabs and plates

Two-way systems should have about equal spans both ways. Double spans increase deflection 16 times (4th power of span). They may be thick plates or thin slabs with drop panels at posts to resist shear. They can be designed by *Direct Design Method*, assuming : 1) at least 3 spans; 2) span ratios ≤ 2:1; 3) adjacent spans differ < 1:1.3; 4) post offsets < 1.1L; 5) uniform load L ≤ 2 D. Bending M = M_0 x coefficient (at left), where:

$M_0 = w L^2/8$ L = span; w = 1.4D+1.7L

1 Column strips and middle strips of typical slab
2 Moment coefficients for slab and plate with simply supported end span
3 Moment coefficients for slab and plate supported directly on columns
4 Moment coefficients for slab and plate with edge beam
5 Moment coefficients for slab and plate with end span integral with wall

A Column strip (slab/column moment distribution is not considered)
B Middle strip
C End support (negative moment)
D End span (positive moment)
E First interior support (negative moment)
F Interior span (positive moment)
G Interior support (negative moment)
H Column strip moment coefficients
I Middle strip moment coefficients

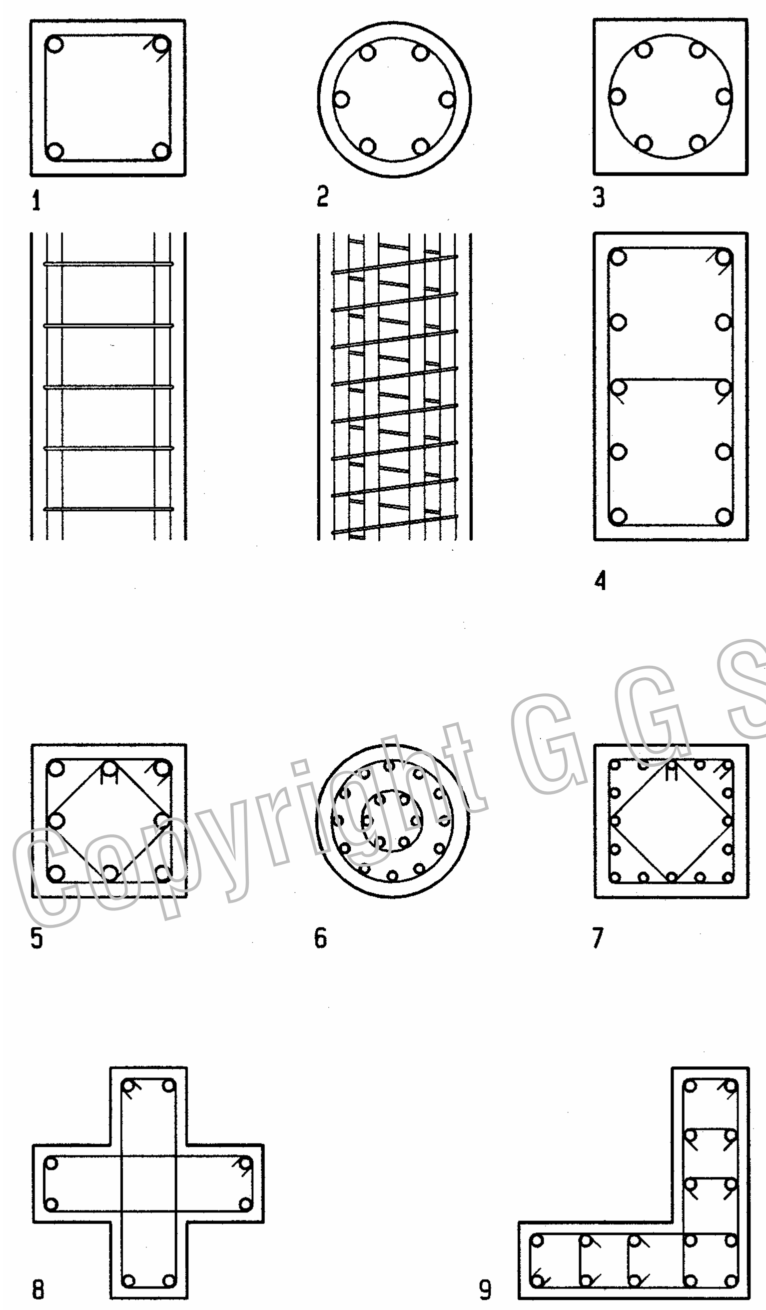
Minimum depths h of two-way slabs unless deflections are computed (L = span)

Support type	f_y = 40 ksi (276 MPa)	f_y = 60 ksi (414 MPa)
Plate without edge beams	L/33	L/30
Plate with edge beams	L/36	L/33
Slab with drop panels without edge beams	L/36	L/33
Slab with drop panels and edge beams	L/40	L/36

Example: Bending moment for slab 2 with simple end span

Assume: Span L = 20', factored dead + live load		w = 0.3 klf
Column strip moments	$M_0 = wL^2/8 = 0.3x20^2/8$	M_o = 15 k'
End span	M = +0.38x15	M = +5.7 k'
First interior post	M = -0.56x15	M = -8.8 k'
Interior span	M = +0.21x15	M = +3.15 k'
2nd interior post	M = -0.49x15	M = -7.35 k'
Middle strip		
End span	M = +0.25x15	M = +3.75 k'
First interior post	M = -0.19x15	M = -2.85 k'
Interior span	M = +0.14x15	M = +2.10 k'
2nd interior post	M = -0.16x15	M = -2.40 k'

Reinforcing is similar to one-way slabs, but two ways, without temperature reinforcing.



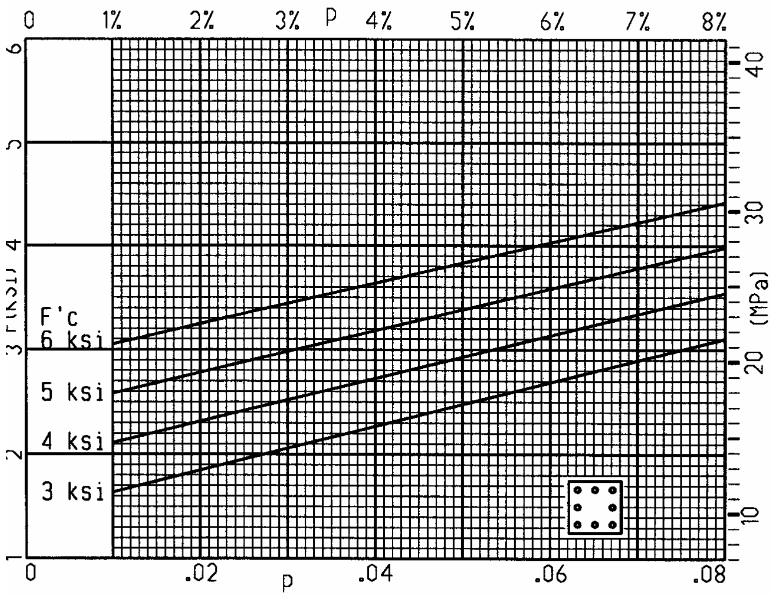
Column

Concrete columns may have square, rectangular, round, or of other cross section with tied or spiral rebars. Compression bars shall be min. No. 5 (16 mm) or greater. The ACI code limits reinforcement to **1% min. and 8% max.** as percentage of column cross section area; but 4% is recommended to prevent rebar crowding. Bars and ties require 1.5 in (38 mm) concrete cover for fire and corrosion protection. Columns of width / height ratios of less than 13 are designed *short columns* without considering buckling.

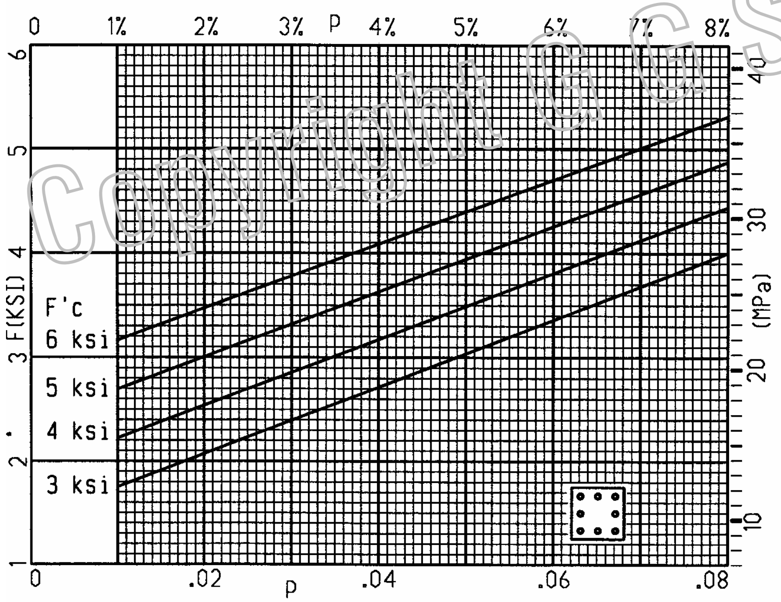
Tied columns must need at least four compression bars held in place by ties: No. 3 ties for compression bars up to No. 10 and No. 4 ties for larger ones. Ties shall secure all corner bars and at least every second bar between corners. Unsecured bars shall be not more than 6 in (15 cm) from a secured bar. Tie hooks shall be 135º. Tied columns are more common, provide somewhat less strength, are less expensive than spiral columns, and adapt easier to cross-, T-, U-, and L-shaped columns. Maximum tie spacing shall be 16 bar diameters, 48 tie diameters, or the least column dimension, whichever is less. For seismic design maximum tie spacing shall be 1/2 the least column dimension near beam intersections.

Spiral columns must have at least five or more vertical compression bars in circular configuration held in place by a continuous circular spiral of about 1/4 in diameter. Spiral columns are usually cylindrical, but spiral reinforcing may also be used for square columns. Spiral columns are about 14% stronger than tied columns of equal cross section area because spirals confine the concrete and rebars better under high stress. Spiral spacing ranges from min. 1 in (25 mm) to max. 3 in (76 mm).

1 Square column with tied reinforcement of minimum 4 bars
2 Round column with spiral reinforcement of minimum 5 bars
3 Square column with spiral reinforcement
4 Rectangular column with tied reinforcement
5 Square column with tied reinforcement of 8 bars
6 Round column with 2-ring spiral reinforcement
7 Square column with tied reinforcement of 16 bars
8 Cross-shaped column with tied reinforcement
9 L-shaped column with tied reinforcement



1 Fy=40 KSI [276 MPa]



2 Fy=60 KSI [414 MPa]

Column design

The strength of concrete columns is defined by concrete strength, grade, amount, and type of steel reinforcing. The theoretical strength without eccentricity is:

$\mathbf{P_o = 0.85\ f'_c(A_g - A_s) + f_yA_s}$

A_g = column cross section area

A_s = area of steel reinforcing

f'_c= specified concrete compressive strength

f_y = yield strength of steel reinforcing

However, concrete columns may be subject to eccentric load or bending moments from beams. Therefore, ACI assumes an implied eccentricity by reduction factors of 0,85 for spiral columns and 0.80 for tied columns. In addition, strength is reduced by Φ = 0.75 for spiral columns and Φ = 0.70 for tied columns. Thus, ACI defines column strength as:

Spiral columns $\Phi P = 0.85\Phi\ [0.85f'_c\ (A_g - A_s) + f_y\ A_s]$

Tied columns $\Phi P = 0.80\Phi\ [0.85f'_c\ (A_g - A_s) + f_y\ A_s]$

For convenient schematic design formulas for stress used in the graphs are:

For spiral columns $F = 0.75x0.85\ [0.85f'_c\ (1 - \rho) + f_y\ \rho]$

For tied columns $F = 0.70x0.80\ [0.85f'_c\ (1 - \rho) + f_y\ \rho]$

Design graphs

The design graphs for tied columns at left and spiral columns on the next page are based on the above equations. Their use is described by examples.

Example: Tied columns, 3-story

Assume: Lateral load resisted by shear walls, design for gravity load only

Tributary area 30'x30', DL = 175 psf, LL 50 psf, f_y = 60 ksi

Factored load w = 1.4x175+ 1.7x50 w = 330 psf

Ground floor (use f'_c = 5 ksi, 4% steel) F = 3.6 ksi

P = 3x30'x30'x330/1000 P = 891k

A = P/F = 891/3.6 A = 248 in^2

Column size, $A^{1/2} = 248^{1/2}$ = 15.7" Use 16"x16"

Steel area $A_s = 0.04x16^2$ = 10.2 in^2 Use 18 # 7 bars, A_s = 10.8 in^2

First floor (use the same column with f'c = 3 ksi)

P = 2x30'x30'x330/2000 P = 594 k

F = P/A = 594/16^2 = 2.32 ksi Use 2.8% steel

Steel area $A_s = 0.028x16^2$ = 7.2 in^2 Use 12 # 7 bars, A_s = 7.2 in^2

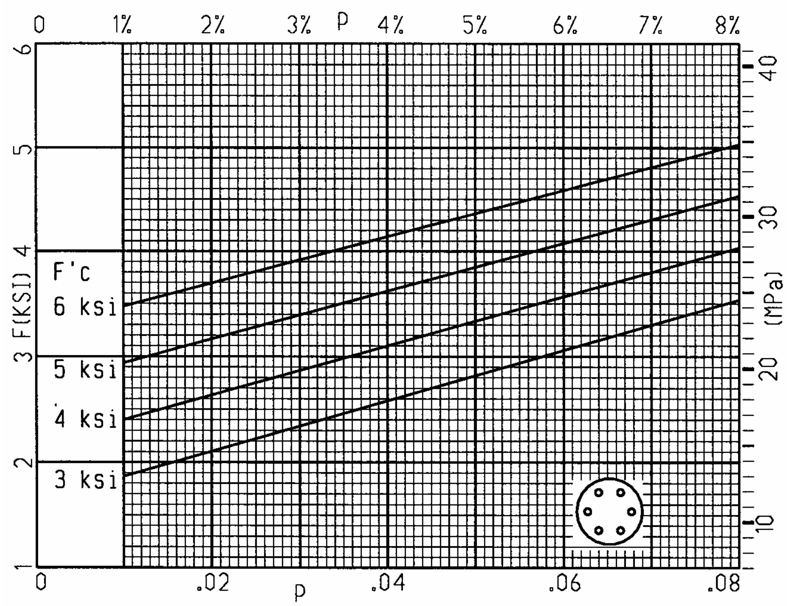
Second floor (use same column with f_y = 40 ksi)

P =1x30x30x330/1000 P = 297 k

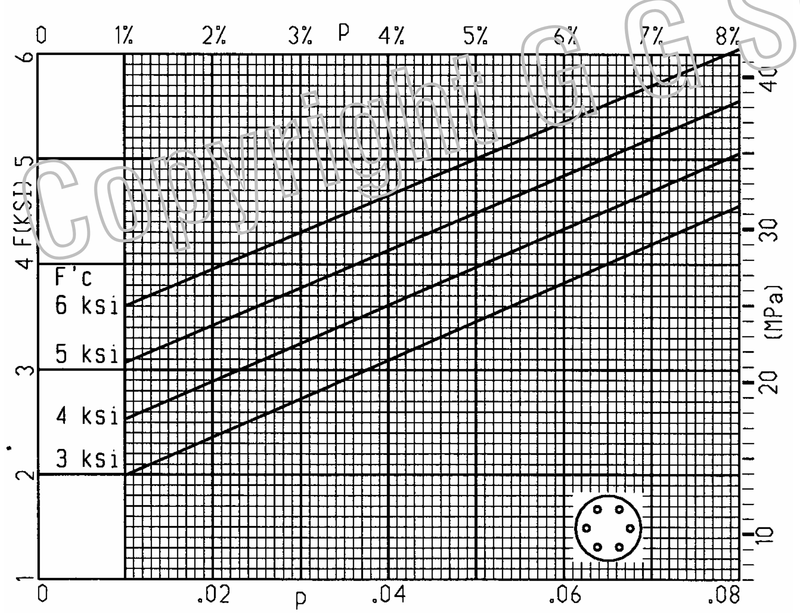
F = P/A = 297/16^2 =1.16 ksi Use 1% steel

Steel area $A_s = 0.01x16^2$ = 2.56 in^2 Use 6 # 7 bars, A_s = 3.6 in^2

Rebar diameters				**Cross-section areas**	
Size	in	in	mm	in^2	mm^2
#7	7/8	0.875	22.2	0.60	387
#8	8/8	1.000	25.4	0.79	510



1 Fy=40 KSI [276 MPa]



2 Fy=60 KSI [414 MPa]

Example: Spiral columns, 3-story

Assume: The same project as above but with spiral columns

Tributary area 30'x30', fy = 60 ksi, DL = 175 psf, LL 50 psf.

Factored load w = 1.4x175+ 1.7x50 — w = 330 psf

Ground floor (use f''_c = 5 ksi, 5% steel) F = 4.5 ksi

P = 3x30'x30'x330/1000 — P = 891k

A = P/F = 891/4.5 — A = 198 in^2

Column size, $2(A/\pi)^{1/2} = 2(198/\pi)^{1/2} = 15.8$" — Use ϕ16"

Column cross section area $A = \pi r^2 = \pi(16/2)^2$ — A = 201 in^2

Steel area A_s = 0.05x201 = 10.1 in^2 — Use 14 # 8 bars, A_s = 11.1 in^2

First floor (use the same column with f'c = 3 ksi)

P = 2x30'x30'x330/2000 — P = 594 k

F = P/A = 594/198 = 2.98 ksi — Use 3.8% steel

Steel area A_s = 0.038x198 = 7.52 in^2 — Use 10 # 8 bars, A_s = 7.9 in^2

Second floor (use the same column with f_y = 40 ksi)

P =1x30x30x330/1000 — P = 297 k

F = P/A = 297/198 = 1.5 ksi — Use 1% steel

Steel area A_s = 0.01x198 = 1.98 in^2 — Use 4 # 7 bars, A_s = 2.4 in^2

9

Lateral Force Design

Lateral loads, acting primarily horizontally, include:

- Wind load
- Seismic load
- Earth pressure on retaining walls (not included in this book)

Wind and earthquakes are the most devastating forces of nature:
Hurricane Andrew 1992, with gusts of 170 mph, devastated 300 square miles, left 300,000 homeless, caused about $ 25 billion damage, and damaged 100,000 homes

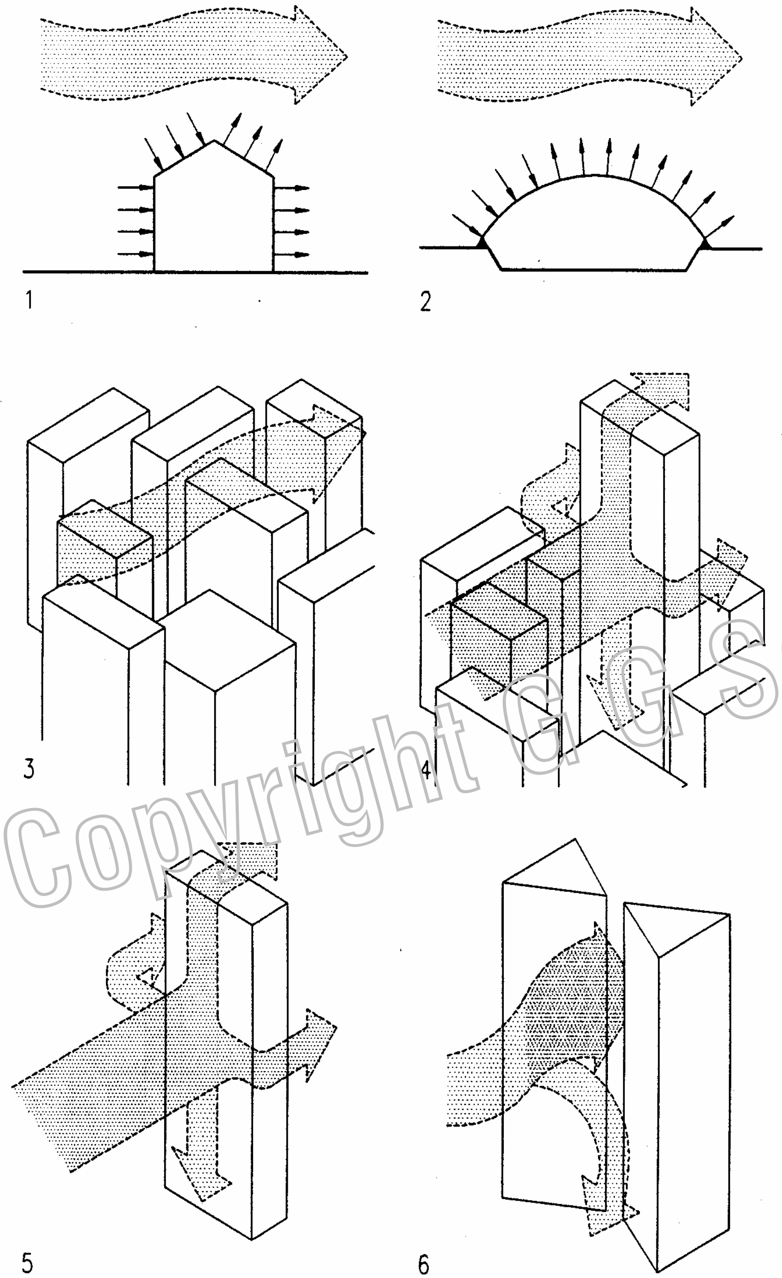
The 1976 Tangshan Earthquake (magnitude 7.8), obliterating the city in northeast China and killing over 240,000 people, was the most devastating earthquake of the 20th century.

Swiss Re reported 2003 world wide losses:

- 60,000 people killed
- Over two thirds earthquake victims
- $70 billion economic losses

IBC table 1604.5. Importance Classification excerpt

Category	Seismic Use Group	Nature of Occupancy	Seismic importance factor	Snow importance factor	Wind importance factor
I	I	Low hazard structures: Agriculture, temporary, minor storage	1	0.8	0.87
II	I	Structures not in categories I, III, IV	1	1	1
III	II	Structures such as: Occupancy >300 people per area Elementary schools >250 students Colleges >500 students Occupancy >5000	1.25	1.1	1.15
IV	III	Essential facilities, such as: Hospitals, polices and fire stations	1.5	1.2	1.15



Wind load

1 Wind load on gabled building
2 Wind load on dome or vault
3 Protected buildings inside a city
4 Exposed tall building inside a city
5 Wind flow around and above exposed building
6 Wind speed amplified by building configuration

Wind channeled between buildings causes a *Venturi* effect of increased wind speed. Air movement through buildings causes internal pressure that affects curtain walls and cladding design. Internal pressure has a balloon-like effect, acting outward if the wind enters primarily on the windward side. Openings on leeward or side walls cause inward pressure. In tall buildings with fixed curtain wall the difference between outside wind pressure and interior pressure causes air movement from high pressure to low pressure. This causes air infiltration on the windward side and outflow on the leeward side. In high-rise buildings, warm air moving from lower to upper levels causes pressures at top levels on the leeward face and negative suction on lower levels. Wind pressure is based on the equation developed by Daniel Bernoulli (1700-1782). For steady air flow of velocity V, the velocity pressure, q, on a rigid body is

$q = \rho V^2 / 2$

ρ = air density (air weight divided by the acceleration of gravity g = 32.2 ft/sec^2)

Air of 15°C at sea level weighs 0.0765 lb/ft^3, which yields:

$q = 0.00256 V^2$ (q in psf)

The American National Standards Institute (ANSI) *Minimum design loads for buildings and other structures* (ANSI A58.1 - 1982), converted dynamic pressure to velocity pressure q_z (psf) at height z as

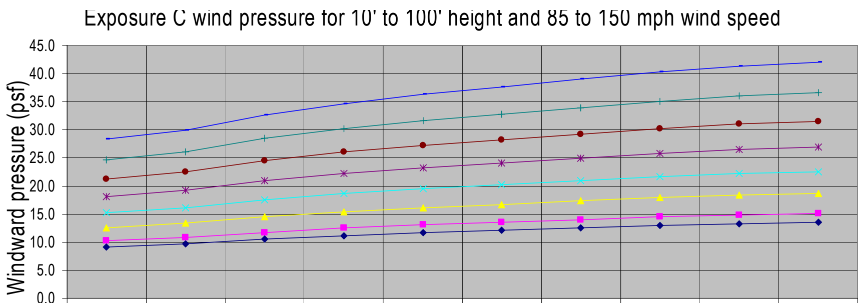
$q_z = 0.00256\, K_z (I\, V)^2$

$K_z = 2.58 (z/z_g)^{2/a}$ (for buildings of 15 ft or higher)

a = Power coefficient (see exposures A – D below)
Z = Height above ground
Z_g = Height at which ground friction no longer effects the wind speed (see exposures A - D below)
I = Importance factor (see IBC table 1604.5)

ANSI A58.1 defined exposures A, B, C, D (IBC uses B, C, D only):

Exposure A	Large city centers	a = 3.0, Z_g = 1500 ft
Exposure B	Urban and suburban areas, wooded areas	a = 4.5, Z_g = 1200 ft
Exposure C	Flat, open country with minimal obstructions	a = 7.0, Z_g = 900 ft
Exposure D	Flat, unobstructed coastal areas	a = 10.0, Z_g = 700 ft



	10	20	30	40	50	60	70	80	90	100
85	9.1	9.6	10.5	11.1	11.7	12.1	12.5	12.9	13.3	13.5
90	10.2	10.8	11.7	12.5	13.1	13.5	14.0	14.5	14.9	15.1
100	12.6	13.3	14.5	15.4	16.1	16.7	17.3	17.9	18.3	18.6
110	15.2	16.1	17.5	18.6	19.5	20.2	20.9	21.7	22.2	22.6
120	18.1	19.2	20.9	22.2	23.2	24.1	24.9	25.8	26.4	26.8
130	21.3	22.5	24.5	26.0	27.3	28.3	29.3	30.3	31.0	31.5
140	24.7	26.1	28.4	30.2	31.6	32.8	33.9	35.1	36.0	36.5
150	28.3	30.0	32.6	34.6	36.3	37.6	39.0	40.3	41.3	41.9

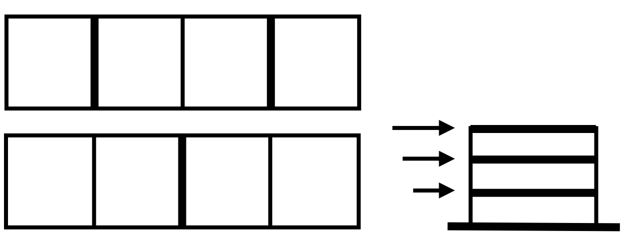
Windward pressure (psf)

85	5.7	6.0	6.5	6.9	7.3	7.6	7.8	8.1	8.3	8.4
90	6.4	6.7	7.3	7.8	8.2	8.5	8.8	9.1	9.3	9.4
100	7.9	8.3	9.1	9.6	10.1	10.5	10.8	11.2	11.5	11.7
110	9.5	10.1	11.0	11.6	12.2	12.6	13.1	13.5	13.9	14.1
120	11.3	12.0	13.1	13.8	14.5	15.0	15.6	16.1	16.5	16.8
130	13.3	14.1	15.3	16.3	17.0	17.7	18.3	18.9	19.4	19.7
140	15.4	16.3	17.8	18.9	19.8	20.5	21.2	21.9	22.5	22.8
150	17.7	18.7	20.4	21.6	22.7	23.5	24.3	25.2	25.8	26.2

Leeward pressure (psf)

85	2.4	2.5	2.8	2.9	3.1	3.2	3.3	3.4	3.5	3.6
90	2.7	2.9	3.1	3.3	3.5	3.6	3.7	3.8	3.9	4.0
100	3.3	3.5	3.8	4.1	4.3	4.4	4.6	4.7	4.9	4.9
110	4.0	4.3	4.6	4.9	5.2	5.4	5.5	5.7	5.9	6.0
120	4.8	5.1	5.5	5.9	6.1	6.4	6.6	6.8	7.0	7.1
130	5.6	6.0	6.5	6.9	7.2	7.5	7.7	8.0	8.2	8.3
140	6.5	6.9	7.5	8.0	8.4	8.7	9.0	9.3	9.5	9.7
150	7.5	7.9	8.6	9.2	9.6	10.0	10.3	10.7	10.9	11.1

Interior pressure (psf)



Example: Wood shear walls

Assume: 66'x120'x27' high, 3 shear walls, L=3x30'=90', wind speed 90 mph, exposure C, Importance factor I = 1, gust factor G = 0.85 (ASCE 7, 6.5.8 for rigid structures > 1 Hz)
For each level in width direction find: wind pressure P, force F, shear V, shear wall type
Interior pressure (from graph for h = 30') p = 3.1 psf
Leeward suction (from graph for h = 30' P= 7.3 psf

Level 3 (h = 29 – use 30' pressure)
Wind pressure (windward + leeward + interior)
p = 11.7+7.3 + 3.1 p = 22.1 psf
Force F = 22.1x120x10'/2 F = 13,260 #
Shear V = F V = 13,260 #
Required wall strength = 13,260 / 90' = 147 plf; use 5/16", 6d at 6" 200 > 147

Level 2 (h = 19' – use 20' pressure)
p = 10.8+7.3+3.1 p = 21.2 psf
Force F = 21.2 x 120 x 10' F = 25,440 #
Shear V = 13,260 + 25,440 V = 38,700 #
Required wall strength = 38,700 / 90' = 430 plf; use 15/32", 8d at 4" 430 = 430

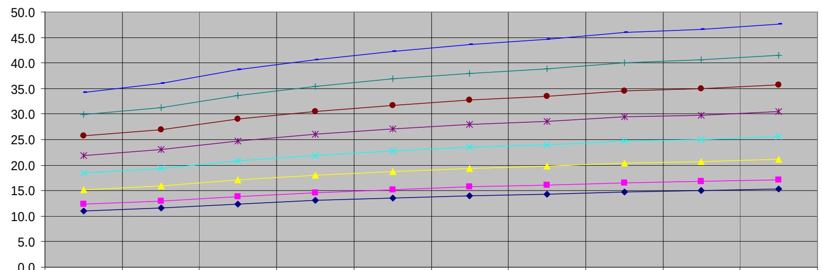
Level 1 (h = 9 – use 10' pressure')
p = 10.2+7.3+3.1 p = 20.6 psf
Force F = 23.7x120x10' F = 24,720 #
Shear V = 38,700 + 24,720 V = 63.420 #
Required strength = 63,420 / 90' = 705 plf; use 15/32", 8d at 2" 730 > 705
Note:
The results are very similar to 9.1 with less computation
See Appendix C for exposure B and D graphs

IBC table 2306.4.1 excerpts
Allowable shear for wood panels with Douglas-Fir-Large or Southern Pine

Panel grade	Panel thickness	Nail penetration	Nail size	Nail spacing at panel edge (inches) 6	4	3	2
				Allowable shear (lbs / foot)			
Structural I sheathing	5/16 in	1 1/4 in	6d	200	300	390	510
	3/8 in	1 3/8 in	8d	230	360	460	610
	7/16 in	1 3/8 in	8d	255	395	505	670
	15/32 in	1 3/8 in	8d	280	430	550	730
		1 1/2 in	10d	340	510	665	870

* Requires 3 x framing and staggered nailing

Exposure D wind pressure for 10' to 100' height and 85 to 150 mph wind speed



	10	20	30	40	50	60	70	80	90	100
85	11.0	11.5	12.4	13.0	13.6	14.0	14.3	14.8	15.0	15.3
90	12.3	12.9	13.9	14.6	15.2	15.7	16.1	16.5	16.8	17.1
100	15.2	16.0	17.2	18.1	18.8	19.4	19.8	20.4	20.7	21.2
110	18.4	19.3	20.8	21.8	22.7	23.5	24.0	24.7	25.1	25.6
120	21.9	23.0	24.7	26.0	27.1	27.9	28.6	29.4	29.8	30.5
130	25.8	27.0	29.0	30.5	31.8	32.8	33.5	34.5	35.0	35.8
140	29.9	31.3	33.6	35.4	36.8	38.0	38.9	40.0	40.6	41.5
150	34.3	36.0	38.6	40.6	42.3	43.6	44.6	45.9	46.6	47.6

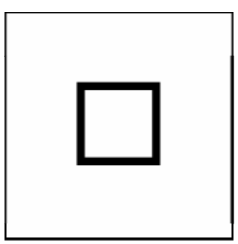
Winward pressure (psf)

85	6.9	7.2	7.8	8.2	8.5	8.8	9.0	9.2	9.4	9.6
90	7.7	8.1	8.7	9.1	9.5	9.8	10.0	10.3	10.5	10.7
100	9.5	10.0	10.7	11.3	11.7	12.1	12.4	12.8	12.9	13.2
110	11.5	12.1	13.0	13.7	14.2	14.7	15.0	15.4	15.7	16.0
120	13.7	14.4	15.4	16.2	16.9	17.4	17.8	18.4	18.6	19.0
130	16.1	16.9	18.1	19.1	19.8	20.5	20.9	21.5	21.9	22.3
140	18.7	19.6	21.0	22.1	23.0	23.7	24.3	25.0	25.4	25.9
150	21.4	22.5	24.1	25.4	26.4	27.3	27.9	28.7	29.1	29.8

Leeward pressure (psf)

85	2.9	3.1	3.3	3.5	3.6	3.7	3.8	3.9	4.0	4.0
90	3.3	3.4	3.7	3.9	4.0	4.2	4.3	4.4	4.4	4.5
100	4.0	4.2	4.5	4.8	5.0	5.1	5.2	5.4	5.5	5.6
110	4.9	5.1	5.5	5.8	6.0	6.2	6.4	6.5	6.6	6.8
120	5.8	6.1	6.5	6.9	7.2	7.4	7.6	7.8	7.9	8.1
130	6.8	7.1	7.7	8.1	8.4	8.7	8.9	9.1	9.3	9.5
140	7.9	8.3	8.9	9.4	9.7	10.1	10.3	10.6	10.7	11.0
150	9.1	9.5	10.2	10.8	11.2	11.5	11.8	12.2	12.3	12.6

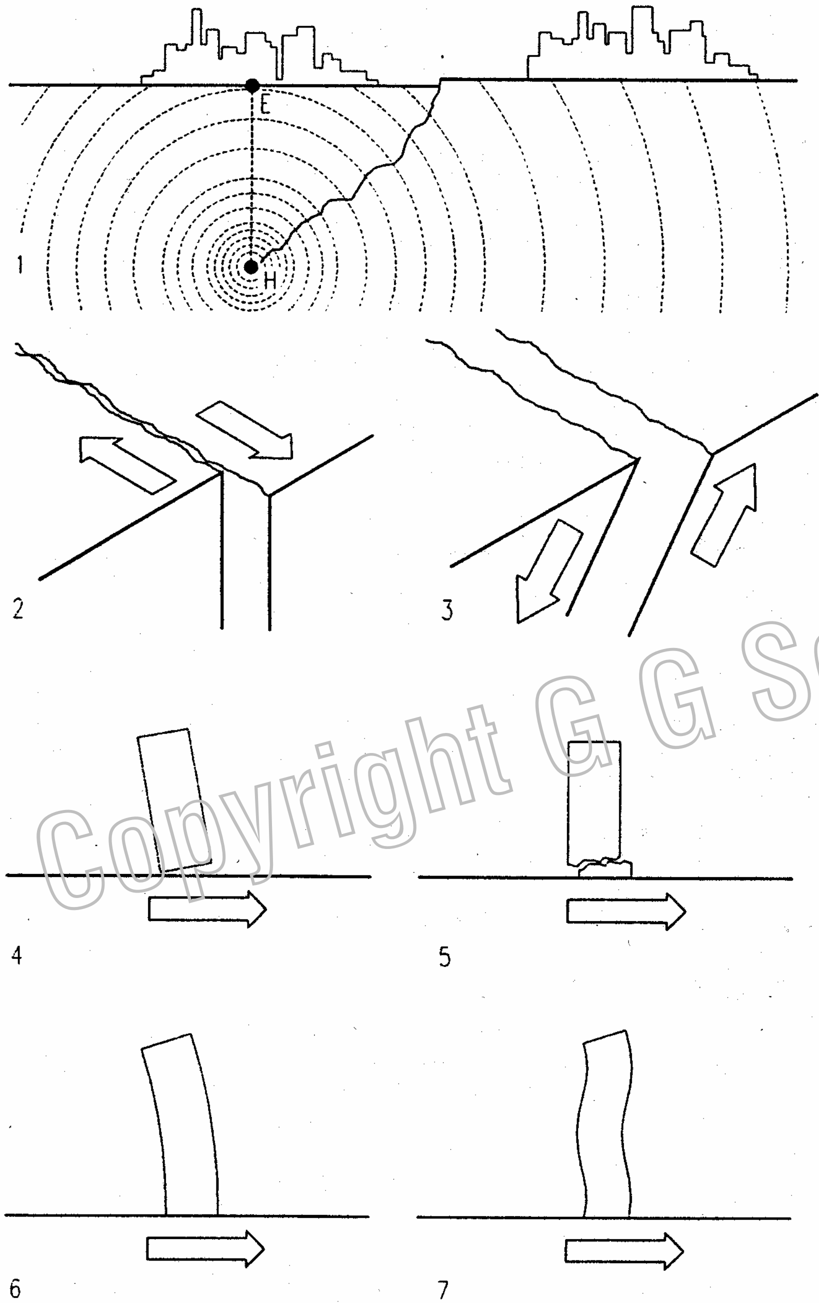
Interior pressure (psf)



Example: CMU shear walls
Assume: Regular flat site
Office building: 6-story, 90'x90'x60', 30'x30' core
30' CMU walls, 8" nominal (7.625")
Shear wall length L = 2 x (30'– 6' doors) = 48'
Shear walls resist all lateral load
Roof fabric canopy, 50'x50'x10', gust factor G = 1.8
Wind speed V = 100 mph
Exposure D
Importance factor I = 1

Interior pressure (assume conservative opening height h = 60') p = 5.1 psf
Leeward pressure (for h = 60')
P = 12.1 psf + 5.1 psf p = 17.2 psf
Average windward pressure (h = 10 to 60')
P = (15.2+16.0+17.2+18.1+18.8+19.4) / 6 p = 17.5 psf
Average combined wind pressure
P = 17.5+17.2+5.1 P = 39.8 psf
Roof canopy pressure P_{canopy} = (12.1)(1.8) P_{canopy} = 21.8 psf
Base shear
V = A P = 90' x (60'-5') x 39.8 + (50'x10'/2)x21.8 V = 202,460 #
Core shear stress
v =V/A = 202,460 / (48'x12"x7.625") v = 46 psi

Note:
Wind on lower half of first floor, resisted by footing, has no effect on shear walls



Seismic Design

Earthquakes are caused primarily by release of shear stress in seismic faults, such as the *San Andreas* Fault, that separates the Pacific plate from the North American plate, two of the plates that make up the earth's crust according to the plate tectonics theory. Plates move with respect to each other at rates of about 2-5 cm per year, building up stress in the process. When stress exceeds the soil's shear capacity, the plates slip and cause earthquakes. The point of slippage is called the *hypocenter* or *focus*, the point on the surface above is called the *epicenter*. Ground waves propagate in radial pattern from the focus. The radial waves cause shaking somewhat more vertical above the focus and more horizontal far away; yet irregular rock formations may deflect the ground waves in random patterns. The Northridge earthquake of January 17, 1994 caused unusually strong vertical acceleration because it occurred under the city.

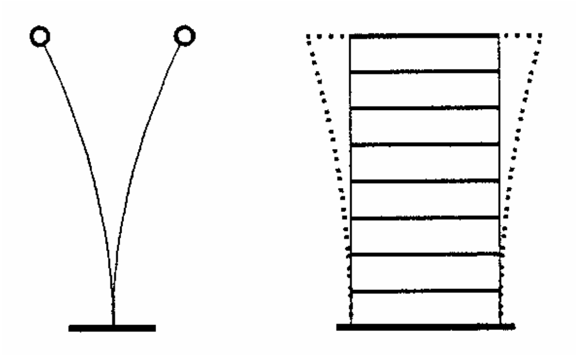
Occasionally earthquakes may occur within plates rather than at the edges. This was the case with a series of strong earthquakes in New Madrid, along the Mississippi River in Missouri in 1811-1812. Earthquakes are also caused by volcanic eruptions, underground explosions, or similar man-made events

Buildings are shaken by ground waves, but their inertia tends to resists the movement which causes lateral forces. The building mass (dead weight) and acceleration affects these forces. In response, structure height and stiffness, as well as soil type affect the response of buildings to the acceleration. For example, seismic forces for concrete shear walls (which are very stiff) are considered twice that of more flexible moment frames. As an analogy, the resilience of grass blades will prevent them from breaking in an earthquake; but when frozen in winter they would break because of increased stiffness.

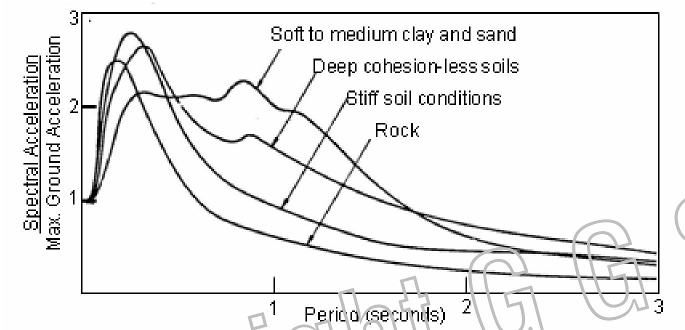
The cyclical nature of earthquakes causes dynamic forces that are best determined by dynamic analysis. However, given the complexity of dynamic analysis, many buildings of regular shape and height limits, as defined by codes, may be analyzed by a *static force* method, adapted from Newton's law F= ma (Force = mass x acceleration).

1 Seismic wave propagation and fault rupture
2 Lateral slip fault
3 Thrust fault
4 Building overturn
5 Building shear
6 Bending of building (first mode)
7 Bending of building (higher mode)

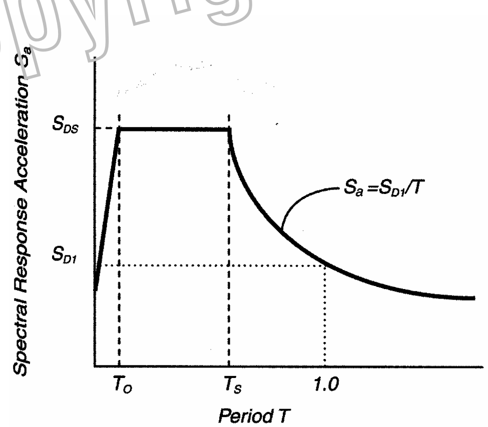
E Epicenter
H Hypocenter



Spectral acceleration



Acceleration spectra for four soil types (by Seed)



IBC Design Response Spectrum

Basic concepts

Earthquake ground shaking generates forces on structures. Though these forces act in all direction, the horizontal (lateral) forces are usually most critical. Seismic forces are

f = m a (Force = mass x acceleration)

m = mass (building dead load)

a = acceleration (Spectral Acceleration)

Note:

Spectral Acceleration approximates the acceleration of a building, as modeled by a particle on a mass-less vertical rod of the same period of vibration as the building.

PGA (Peak Ground Acceleration) is experienced by a particle on the ground

Acceleration Spectra (left)

Based on the 1971 San Fernando and other Earthquakes Seed (1976) developed Acceleration Spectra to correlate time period (X-axis) with acceleration for four soil types. Other studies by Hall, Hayashi, Kuribayashi, and Mohraz demonstrated similar results. *Equivalent Lateral Force Analysis* is based on Acceleration Spectra, abstracted as *Design Response Spectrum*

Design Response Spectrum (left)

The IBC Design Response Spectrum correlate time period T and Spectral Acceleration, defining three zones. Two critical zones are:

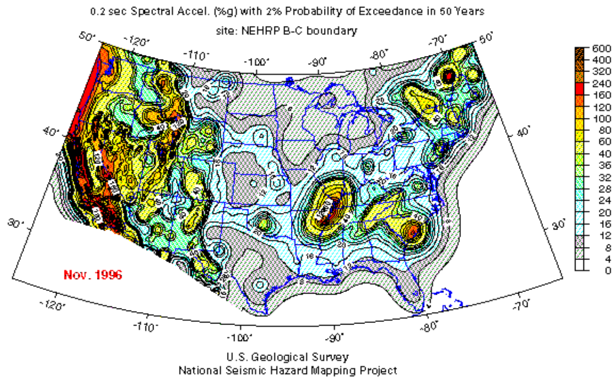
$T < T_S$ governs low-rise structures of short periods

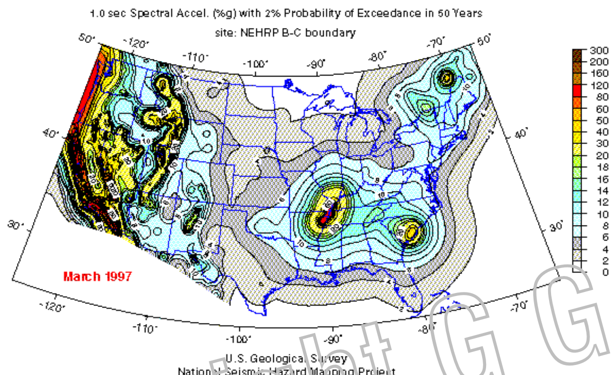
$T > T_S$ governs tall structures of long periods

where

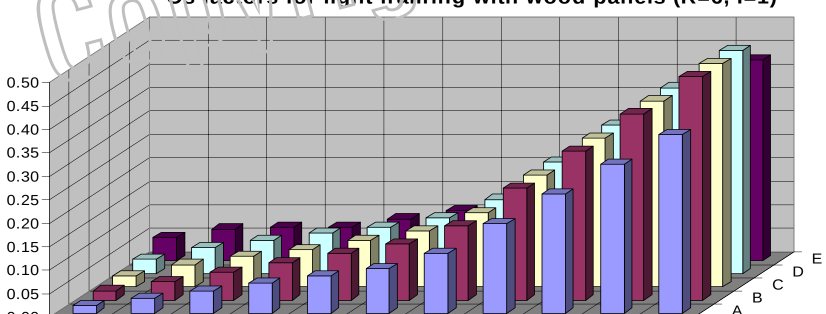
T = time period of structure (T ~ 0.1 sec. per story - or per ASCE 7 table 1615.1.1)

$T_S = S_{DS} / S_{D1}$ (See the following graphs for S_{DS} and S_{D1})





Cs factors for light framing with wood panels (R=6, I=1)



	0.25	0.5	0.75	1	1.25	1.5	2	3	4	5	6
A	0.02	0.03	0.05	0.06	0.08	0.10	0.13	0.19	0.25	0.32	0.38
B	0.02	0.04	0.06	0.08	0.10	0.12	0.16	0.24	0.32	0.40	0.48
C	0.02	0.05	0.07	0.08	0.10	0.12	0.16	0.24	0.32	0.40	0.48
D	0.03	0.06	0.07	0.09	0.10	0.12	0.16	0.24	0.32	0.40	0.48
E	0.05	0.07	0.07	0.07	0.09	0.11	0.14	0.21	0.29	0.36	0.43

Cs for site class A-E for 0.2 sec mapped spectral accelerations Ss (top line)

Analysis steps

Define site class by geologist, or assume default site class D (IBC table 1615.1.1)
Define Mapped Spectral Accelerations S_S and S_1
For overview see USGS maps at left: 0.2 sec low-rise (top) 1 sec high-rise (bottom)
Enter Site coordinates at USGS web site:
http://eqdesign.cr.usgs.gov/html/lookup-2002-interp-D6.html

Enter Latitude:	37.7795	Enter Longitude:	-122.4195
Enter Latitude:		Enter Longitude:	

Enter latitude in the left box in *decimal* degrees (range: 24.6 to 50.0)
Enter negative longitude in the right box (range: -125.0 to -65.0)
Web output:
LOCATION 37.7795 Lat. -122.4195 Long.
Interpolated Probabilistic Ground Motion(Spectral Acceleration SA) in %g, at the site are:

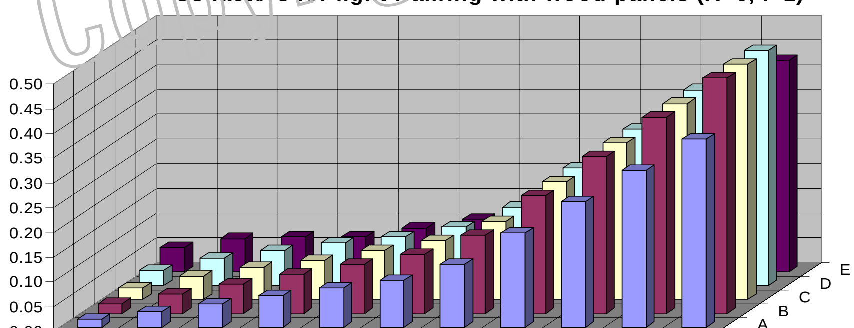
	10%PE in 50 yr.	**2%PE in 50 yr.**	
0.2 sec SA	115.35	**182.76 %**	**→ S_S = 1.83 (for low-rise)**
1.0 sec SA	53.08	**92.41 %**	**→ S_1 = 0.92 (for high-rise)**

Low-rise: $T<T_S$ (structures < 5 stories)
High-rise: $T>T_S$ (structures > 10 stories)
$T_S = S_{DS}/S_{D1}$ (For S_{DS} and S_{D1} see graphs on following pages)
Define base shear V (lateral force at base of structure)
$V = C_S W$
W = Dead load (+ 25% storage live load + 20% flat roof snow load > 30 psf)
C_S= seismic coefficient - see sample graph at left (S_S at top line)
For other structures:
$C_S = I\, S_{DS} / R$ (for $T < T_S$)
Need not exceed
$C_S = I\, S_{D1} / (TR)$ (for $T > T_S$)
I = Importance factor (IBC table 1604.5)
R = R-factor (IBC table 1617.6.2)
S_{DS} and S_{D1} (See graphs on the following pages)
C_S varies with spectral acceleration S_S & S_1 and type of structure
(defined on the following pages)
For example, in seismic areas:
C_S ~ 3 % for tall steel frame structures
C_S ~ 15 % for low-rise wood structures
C_S ~ 30 % for some low-rise masonry structures
W = w A (w = dead load, DL in psf, A = total gross floor area of building)
w varies with type of construction – for example:
w ~ 15 to 25 psf for wood structures
w ~ 70 to 100 psf for steel structures
w ~ 150 to 200 psf for concrete structures

IBC table 1615.1.1 Site class definitions excerpts		
Site class	Soil profile name	Average shear velocity in top 100 ft (30 m)
A	Hard rock	Vs > 5000 ft/s (1500 m/s)
B	Rock	Vs = 2500 to 5000 ft/s (760 to 1500 m/s)
C	Very dense soil & soft rock	Vs = 1200 to 2500 ft/s (370 to 760 m/s)
D	Stiff soil	Vs = 600 to 1200 ft/s (180 to 370 m/s)
E	Soft soil	Vs < 600 ft/s (180 m/s)
F	Soil vulnerable to failure, very organic clay, high plasticity clay, etc.	

IBC table 1617.6.2 excerpt	R-factor	Height limits (ft), categories A-F				
Bearing wall systems		A B	C	D	E	F
Light framed walls with wood panels	6	NL	NL	65	65	65
Light framed walls with other panels	2	NL	NL	35	NP	NP
Ordinary reinforced concrete walls	4	NL	NL	NP	NP	NP
Special reinforced concrete walls	5	NL	NL	160	160	100
Ordinary reinforced masonry walls	2	NL	160	NP	NP	NP
Special reinforced masonry walls	5	NL	NL	160	160	100
Building frame systems						
Ordinary steel concentric braced frames	5	NL	NL	35	35	NP
Special steel concentric braced frames	6	NL	NL	160	160	100
Ordinary steel moment frames	3.5	NL	NL	NP	NP	NP
Special steel moment frames	8	NL	NL	NL	NL	NL

Cs factors for light framing with wood panels (R=6, I=1)



	0.25	0.5	0.75	1	1.25	1.5	2	3	4	5	6
A	0.02	0.03	0.05	0.06	0.08	0.10	0.13	0.19	0.25	0.32	0.38
B	0.02	0.04	0.06	0.08	0.10	0.12	0.16	0.24	0.32	0.40	0.48
C	0.02	0.05	0.07	0.08	0.10	0.12	0.16	0.24	0.32	0.40	0.48
D	0.03	0.06	0.07	0.09	0.10	0.12	0.16	0.24	0.32	0.40	0.48
E	0.05	0.07	0.07	0.07	0.09	0.11	0.14	0.21	0.29	0.36	0.43

Cs for site class A-E for 0.2 sec mapped spectral accelerations Ss (top line)

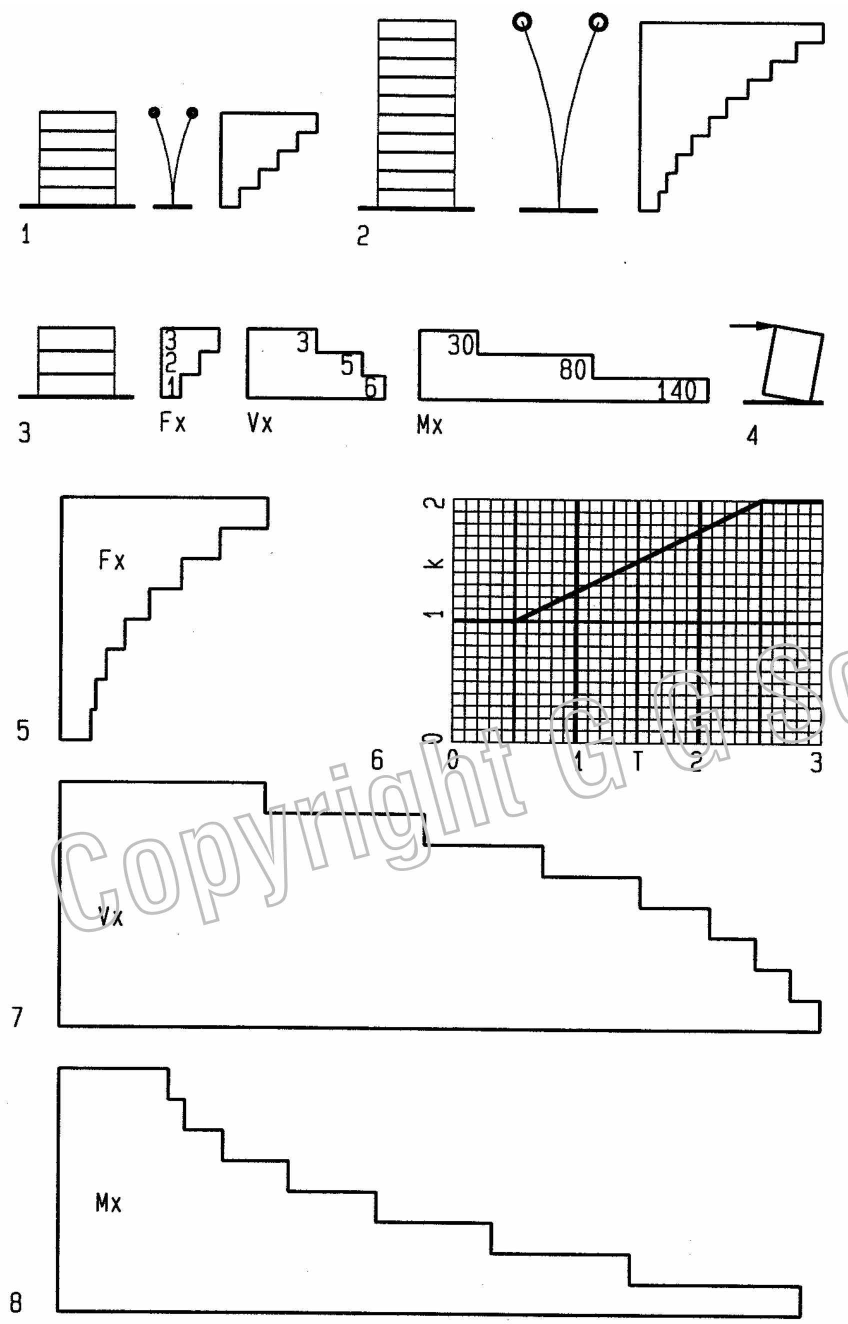
Example: One-story residence, San Francisco
Assume: Light framing with plywood panels
36'x40'x10' high, DL = 25 psf, site class undefined, use default D, I = 1
Enter site coordinates at USGS web site
http://eqdesign.cr.usgs.gov/html/lookup-2002-interp-D6.html
Web site output
0.2 sec Spectral Acceleration **S_s = 1.85**
Design Spectral Accelerations (see graph)
At S_s = 2.0 C_S = 0.16
Interpolate C_S at S_s = 1.85 (C_S /1.85 = 0.16 / 2.0)
C_S = 1.85 x 0.16 / 2.0 C_S =0.15
Building dead weight
W = 25 psf x 36'x40' W = 36,000#
Base Shear
V = C_S W = 0.15 x 36,000 V = 5,400 #

Example: Same residence in San Francisco on site class A
C_S factor (see graph)
Interpolate C_S at S_s = 1.85 (C_S /1.85 = 0.13 / 2.0)
C_S = 0.13x1.85/ 2.0 C_S = 0.12
Base shear V = C_S W = 0.12 x 36,000 V = 4,320 #

Example: Same residence in Tucson
Site class D, S_S = 0.329
C_S factor (see graph)
Interpolate for S_s = 0.329 (C_S/0.329 = 0.06/05)
C_S = 0.329x0.06 / 0.5 C_S = 0.04
Base shear V = C_S W = 0.04 x 36,000 V = 1,440 #

Example: Same residence in Tucson on site class A
C_S factor (see graph)
Interpolate for S_s = 0.329 (at S_s = 0.5 C_S = 0.03)
C_S = 0.329 x 0.03 / 0.5 C_S = 0.02
Base shear V = C_S W = 0.02 x 36,000 V = 720 #

Compare seismic factors
Los Angeles site class D C_s = 0.15
Los Angeles site class A C_s = 0.12
Tucson site class D C_s = 0.04
Tucson site class A C_s = 0.02



Vertical distribution

Seismic forces increase with building height since f = ma (force = mass x acceleration), i.e., increased drift increases acceleration. Thus story forces F_x are story mass times height above ground. For buildings with periods of 0.5 seconds or less the force increase is considered liner. For tall buildings the story-force varies non-linear. Since all story forces are resisted at the ground, each story must resist its own force plus all forces from above. Thus shear per level increases from top to bottom. The overturn moment per level is the sum of all forces above times their distance to the level considered.

1 Linear force increase for T ≤ 0.5 seconds
2 Non-linear force increase for T > 0.5
3 Distribution per level of force
Fx = force per level x
Vx = Shear per level x = sum all forces above
V_2 = 3 k — V_2 = 3 k
V_1 = 3 k + 2 k — V_1 = 5 k
V_0 = 5 k + 1 k — V_0 = 6 k
Mx = overturn moment per level = sum of all forces above times level arm
Assuming 10' story height:
M_2 = 3 k x10' — M_2 = 30 k'
M_1 = 3 k x20' + 2 k x 10' — M_1 = 80 k'
M_0 = 3 k x 30' + 2 k x 20' + 1 k x 10' — M_0 = 140 k'
4 Overturn moment visualized
5 Force per level
$F_x = C_{vx} V$

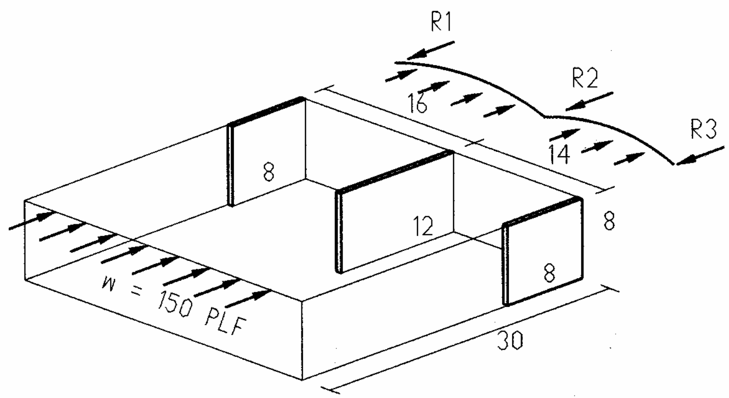
$$C_{vx} = w_x h_x / \sum_{i=1}^{n} w_i h_i^k$$ (vertical distribution factor)

W = total dead weight of level x
h = height of level x above ground
n = total number of stories
k = exponent related to structure period
k = 1 for T ≤ 0.5 seconds
k = 2 for T > 2.5 seconds
k = interpolated between T = 0.5 and 2.5
6 k Interpolation graph
7 Shear per level

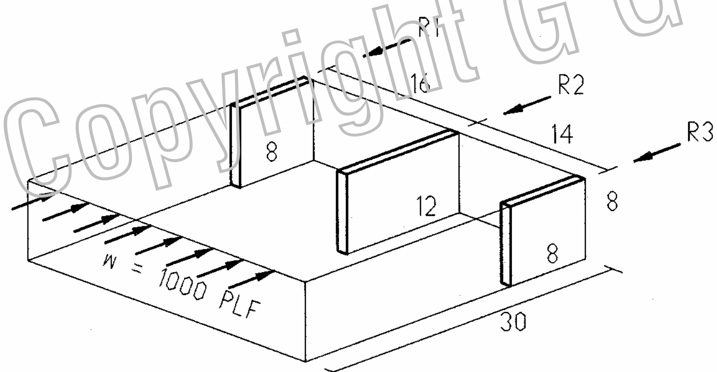
$$V_x = \sum_{i=x}^{n} F_i$$

8 Overturn moment per level

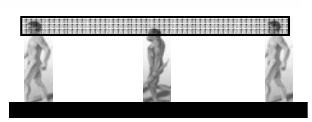
$$M_x = \sum_{i=x}^{n} F_i (h_n - h_i)$$



1



2



Horizontal diaphragms

Horizontal floor and roof diaphragms transfer lateral load to walls and other supporting elements. The amount each wall assumes depends if diaphragms are ***flexible*** or ***rigid***.

1 **Flexible diaphragm**

Floors and roofs with plywood sheathing are usually flexible; they transfer load, similar to simple beams, in proportion to the tributary area of each wall

Wall reactions R are computed based on tributary area of each wall

Required shear flow q (wall capacity)

q = R / L (L = length of shear wall)

R = w (tributary width)	q = R / L (L = shear wall length	
R1 = (150)16/2 = 1200 lbs	q = 1200 / 8'	q = 150 plf
R2 = (150)(16+14)/2 = 2250 lbs	q = 2250 / 12'	q = 188 plf
R3 = (150)14/2 = 1050 lbs	q = 1050 / 8'	q = 131 plf

2 **Rigid diaphragm**

Concrete slabs and some steel decks are rigid; they transfer load in proportion to the relative stiffness of each wall. Since rigid diaphragms experience only minor deflections under load they impose equal drift on walls of equal length and stiffness. For unequal walls reactions are proportional to a resistance factor r.

$r = E\,I / h^3 / \Sigma\,(E\,I / h^3)$

h = wall height

$I = bL^3 / 12$ (moment of inertia of wall)

b = wall thickness

L = wall length

For walls of equal height, thickness and material, the resistance factors are:

$r = L^3 / \Sigma\, L^3$

$L1^3 = L3^3 = 8^3 = 512$

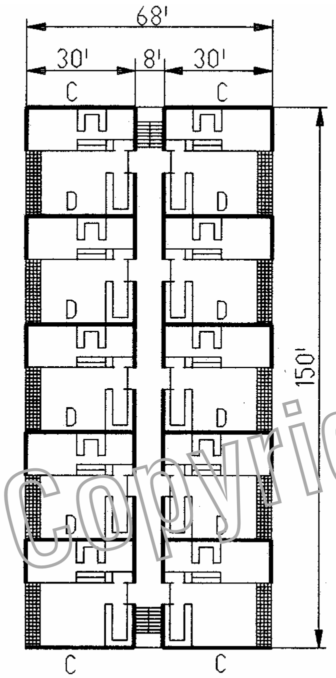
$L2^3 = 12^3 = 1728$

$\sum L^3 = 512 + 1728 + 512$	$\Sigma\, L^3 = 2752$
$r_1 = 512 / 2752$	$r_1 = 0.186$
$r_2 = 1728 / 2752$	$r_2 = 0.628$
$r_3 = 512 / 2752$	$r_3 = 0.186$
Check Σ r	Σ r = 1.000

Total force F

F = 1000 plf x (16'+14') / 1000	F = 30 k
Wall reactions	
$R1 = r_1$ F = 0.186 x 30 k	R1 = 5.58 k
$R2 = r_2$ F = 0.628 x 30 k	R2 = 18.84 k
$R3 = r_3$ F = 0.186 x 30 k	R3 = 5.58 k
Check Σ R	Σ R = 30,00 k

IBC table 2306.4.1 excerpts Allowable shear for wood panels with Douglas-Fir-Large or Southern Pine							
				Nail spacing at panel edge (inches)			
Panel grade	Panel thickness	Nail penetration	Nail size	6	4	3	2 *
				Allowable shear (lbs / foot)			
Structural I sheathing	5/16 in	1 1/4 in	6d	200	300	390	510
	3/8 in	1 3/8 in	8d	230	360	460	610
	7/16 in	1 3/8 in	8d	255	395	505	670
	15/32 in	1 3/8 in	8d	280	430	550	730
		1 1/2 in	10d	340	510	665	870
* Requires 3 x framing and staggered nailing							



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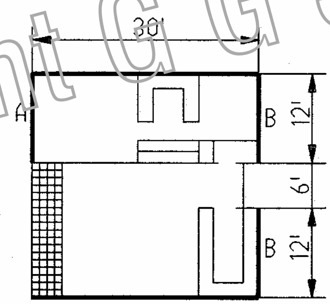

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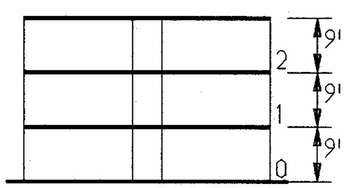


Example: Flexible diaphragm

Assume: plywood diaphragm, plywood shear walls on light wood framing

Dead load — DL= 23 psf

Seismic factor (adjusted for ASD) — $C_s = 0.15$

Dead load per level

W = 23 psf x 68' x 150' / 1000 — W = 235 k

Total DL (3 levels)

$\sum W = 3 \times 245$ k — $\sum W = 705$ k

Base shear

$V = W\,C_s = 705 \times 0.15$ — V = 106 k

Force distribution

Level	W_x	h_x	$W_x h_x$	$w_x h_x/\Sigma w_i h_i$	$F_x = V(w_x h_x/\Sigma w_i h_i)$	$V_x = \Sigma F_x$
2	235 k	27'	6345 k'	0.50	53 k	53 k
1	235 k	18'	4230 k'	0.33	35 k	88 k
0	235 k	9'	2115 k'	0.17	18 k	106 k
		$\Sigma w_i h_i$ = 12,690 k'				V = 106 k

Area per level A = 68 (150) — A= 10200 ft²

Shear per square foot v	v = V / A	v (psi)
V_0 = 106 k = 106000 lbs	v_0 = 106000/10200	v_0=10.4 psf
V_1 = 88 k = 88000 lbs	v_1 = 88000/10200	v_1 = 8.6 psf
V_2 = 53 k = 53000 lbs	v_2 = 53000/10200	v_2 = 5.2 psf

Level 0 shear walls

Wall A = 10.4 psf (15')30'/12' =	390 plf	use 5/16, 6d @ 3" =	390 plf
Wall B = 10.4 psf (19')30'/24' =	247 plf	use 7/16, 8d @ 6" =	255 plf
Wall C = 10.4 psf (34')15'/30' =	177 plf	use 5/16, 6d @ 6" =	200 plf
Wall D = 10.4 psf (34')30'/30' =	354 plf	use 3/8, 8d @ 4" =	360 plf

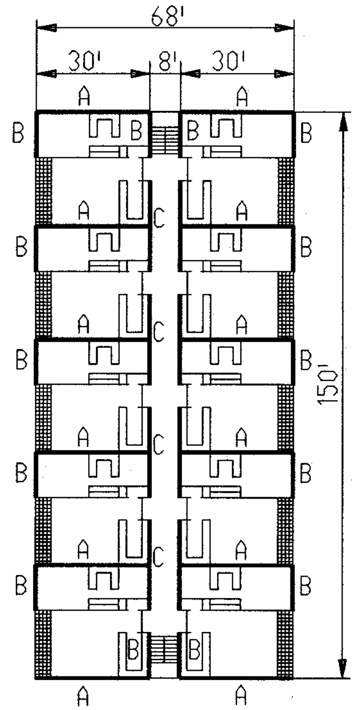
Level 1 shear walls

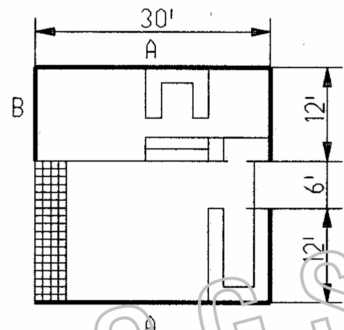
Wall A = 8.6 psf (15')30'/12' =	323 plf	use 15/32, 10d @ 6" =	340 plf
Wall B = 8.6 psf (19')30'/24' =	204 plf	use 3/8, 8d @ 6" =	230 plf
Wall C = 8.6 psf (34')15'/30' =	146 plf	use 5/16, 6d @ 6" =	200 plf
Wall D = 8.6 psf (34')30'/30' =	292 plf	use 5/16, 6d @ 4" =	300 plf

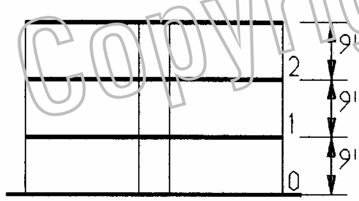
Level 2 shear walls

Wall A = 5.2 psf (15')30'/12' =	195 plf	use 5/16, 6d @ 6" =	200 plf
Wall B = 5.2 psf (19')30'/24' =	124 plf	use 5/16, 6d @ 6" =	200 plf
Wall C = 5.2 psf (34')15'/30' =	89 plf	use 5/16, 6d @ 6" =	200 plf
Wall D = 5.2 psf (34')30'/30' =	177 plf	use 5/16, 6d @ 6" =	200 plf

Note: To simplify construction, fewer wall types could be selected







Example: Rigid diaphragm

Assume: concrete slab on CMU shear walls

Allowable masonry shear stress — Fv = 85 psi

Seismic factor C_s = 0.17x1.5 — C_s = 0.26

Note: increase C_s by 1.5 per IBC 2106.5.1 for ASD method

Dead Load

Wall lengths L = 12 (30')+14 (12)+8 (24) — L= 720'

Wall DL = (720') 8'(7.625"/12") 120 pcf / (68x150) — DL = 43 psf

Floor/roof (12" slab) — 150 psf

Miscellaneous — 7 psf

Σ DL — Σ DL = 200 psf

DL / level: W = 200 psf x 68' x 150'/ 1000 — W = 2,040 k

DL for 3 Levels: W = 3 x 2040 k — W = 6,120 k

Base shear V = Cs W = 0.26 x 6120 — V = 1,591 k

Force distribution

Level	W_x	h_x	W_xh_x	$w_xh_x/\Sigma w_ih_i$	$F_x = V(w_xh_x/\Sigma w_ih_i)$	$V_x = \Sigma F_x$
2	2,040 k	27'	55,080 k'	1591x0.50	796 k	796 k
1	2,040 k	18'	36,720 k'	1591x0.33	525 k	1,321 k
0	2,040 k	9'	18,360 k'	1591x0.17	270 k	1,591 k
			Σw_ih_i = 110,169 k'			V = 1,591 k

Relative wall stiffness:

$R = L^3 / \Sigma L^3$

Wall B: r = $12^3 / [12^3+24^3]$ — r = 0.11

Wall C: r = $24^3 / [12^3+24^3]$ — r = 0.89

Wall cross section areas:

A walls = 12(30')12"(7.625") — A = 32940 in^2

B walls = 14(12')12"(7.625") — B = 15372 in^2

C walls = 8(24')12"(7.625") — C = 17568 in^2

Level 0 (V_0 = 1591 k)

Wall A = (1591) 1000 / 32940 — 48 psi < 85

Wall B = (1591) 1000 (0.11) / 15372 — 11 psi < 85

Wall C = (1591) 1000 (0.89) / 17568 — 81 psi < 85

Level 1 (V_1 = 1321 k)

Wall A = (1321) 1000 / 32940 — 40 psi < 85

Wall B = (1321) 1000 (0.11) / 15372 — 10 psi < 85

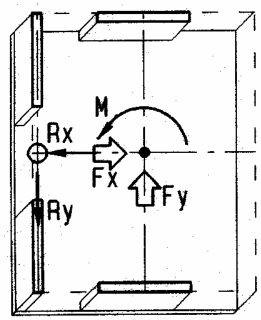
Wall C = (1321) 1000 (0.89) / 17568 — 67 psi < 85

Level 2 (V_2 = 796 k)

Wall A = (796) 1000 / 32940 — 24 psi < 85

Wall B = (796) 1000 (0.11) / 15372 — 6 psi < 85

Wall C = (796) 1000 (0.89) / 17568 — 40 psi < 85



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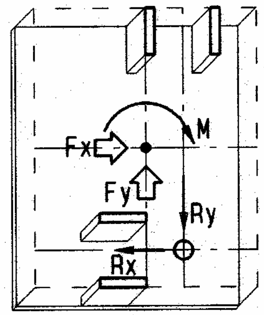

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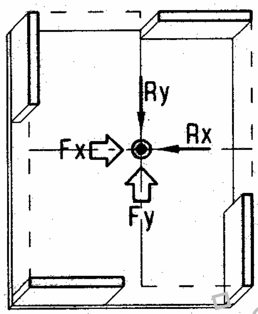

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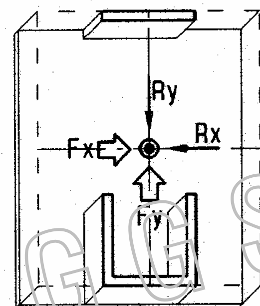

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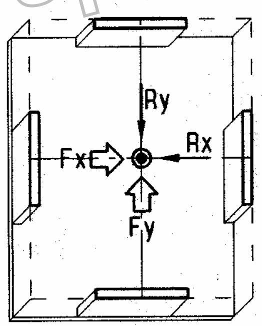

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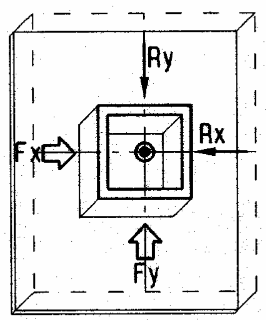


6

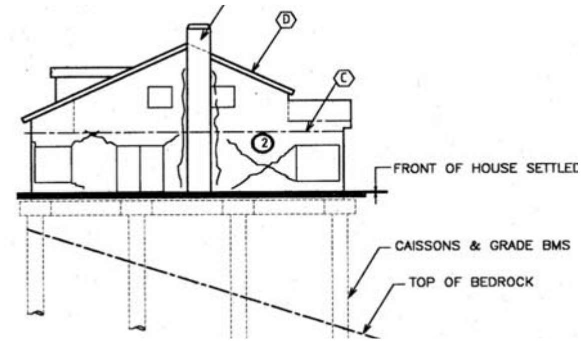
Seismic design issues

Eccentricity

Offset between center of mass and center of resistance causes eccentricity which causes torsion under seismic load. The plans at left identify concentric and eccentric conditions:

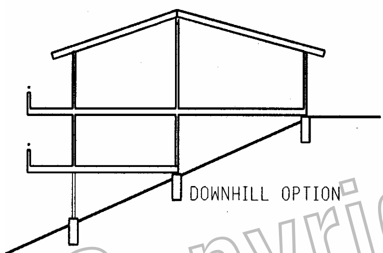
1	X-direction	concentric
	Y-direction	eccentric
2	X-direction	eccentric
	Y-direction	eccentric
3	X-direction	concentric
	Y-direction	concentric
4	X-direction	concentric
	Y-direction	concentric
5	X-direction	concentric
	Y-direction	concentric
6	X-direction	concentric
	Y-direction	concentric

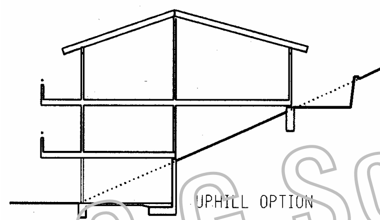
Note:
Plan 5 provides greater resistance against torsion than plan 6 due to wider wall spacing
Plan 6 provides greater bending resistance because walls act together as core and thus provide a greater moment of inertia



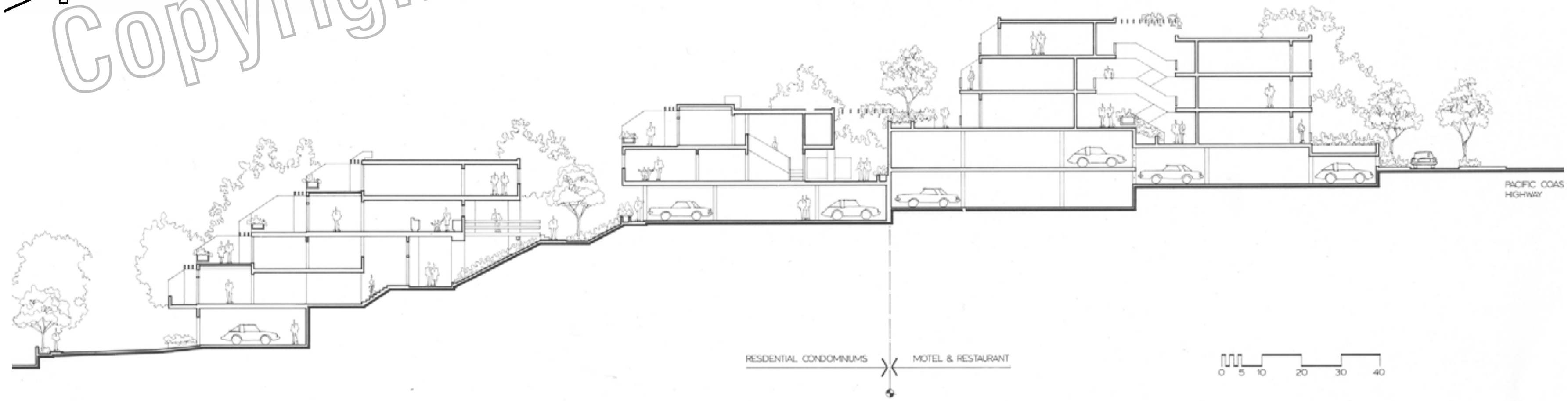
Hillside construction

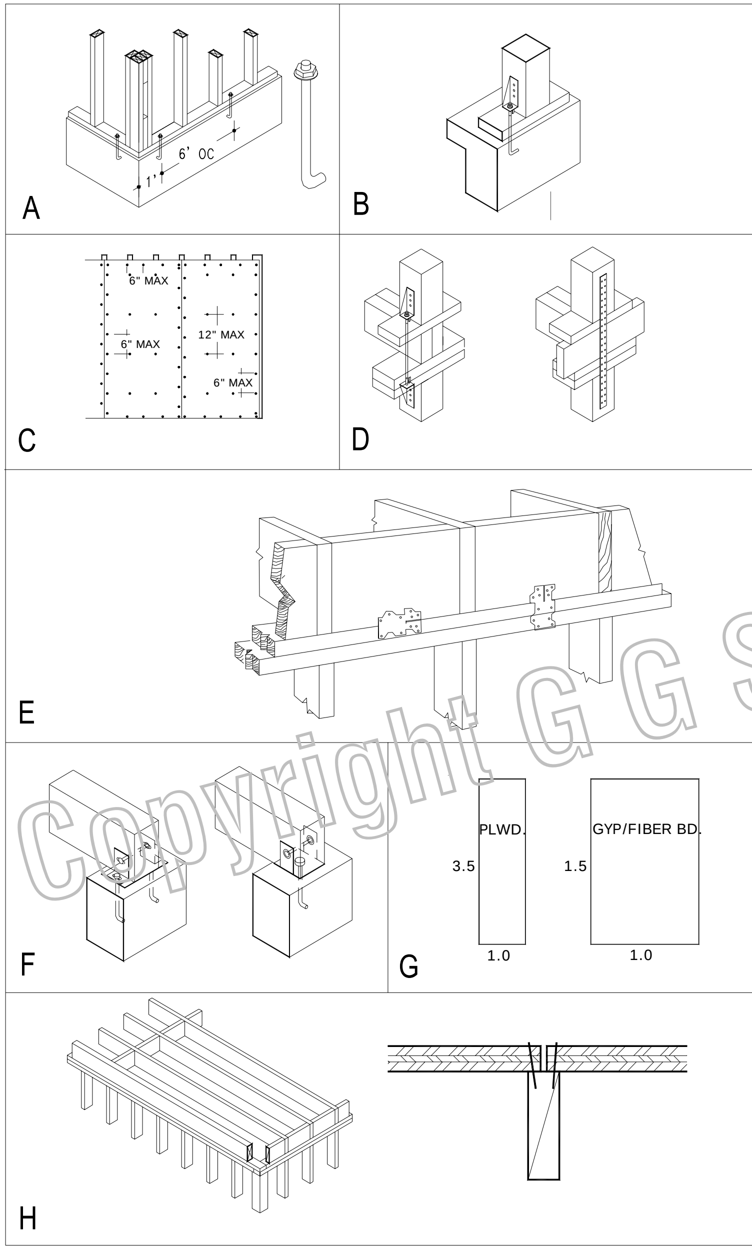
To avoid expensive earthquake settlement repair ……





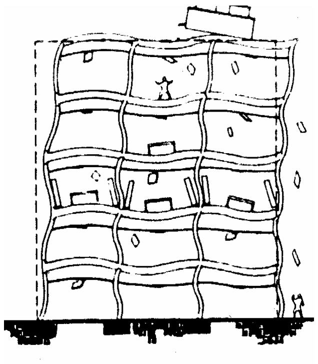
……. …. adapt building to site rather than adapting site to building

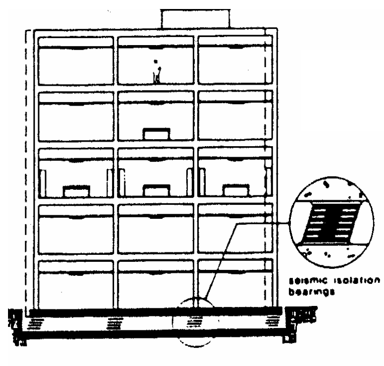




Critical wood-frame items

	Item	Requirements
A	Shear wall anchor bolts	Resist wall slippage
B	Hold-down	Resist shear wall overturning
C	Shear wall nailing	Attach panels to framing
D	Wall-to-wall hold-down	Resist shear wall overturning
E	Framing anchor clips	Transfer shear from floor to floor
F	Beam connection	Resist beam slippage
G	Shear wall width/height ratio Wood panels Gypsum board	Minimum 1 : 3.5 for stability 1:3.5 (Los Angeles, 1:2) 1:2
H	Joist blocking	Transfers shear at panel edges





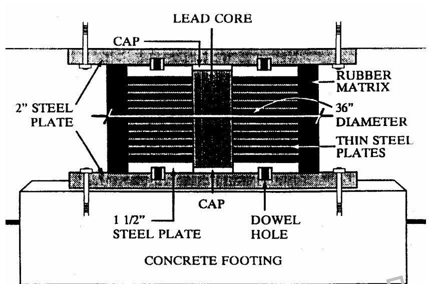
Base Isolator

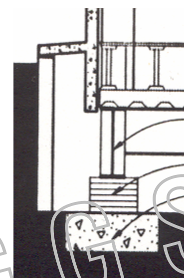
Left: Conventional structure

- Large total and inter-story drift
- Accelerations increase with height
- Potential permanent deformations
- Potential equipment damage

Right: Base isolators:

- Reduce floor accelerations and drift
- Reduce damage to structure and equipment
- Are not good for high-rise structures





Left: Base isolator make-up

- Top and bottom steel plate
- Rubber sheets
- Steel sheets
- Central lead core

Right: Separate building from ground to allow drift

UCLA Kerckhoff Hall base isolator upgrade
Drawings, courtesy Widom Wein Cohen Architects, Santa Monica

10

Conceptual Design

Introduction

Conceptual design usually starts with approximate sizing principle elements of a structure and possible alternatives, followed by thorough analysis during design development. Approximate methods are essential to quickly develop alternate designs. They are also useful to verify final designs and computer analysis. If based on good assumptions, approximate methods can provide results of remarkable accuracy, usually within ten percent of precise results. The following conceptual design examples introduce approximate methods, sometimes referred to as back-of-the-envelop design. They are not meant to replace accurate design but as precursor of accurate design and analysis.

System Selection

Structural design starts with the selection of a system and material; often informed by similar past projects, even if not appropriate. For example, light wood structures are common for residential building where hurricanes cause frequent destruction, though heavy concrete or masonry would resist wind load much better. A rational method is proposed with the objective to select more appropriate systems. However, since design criteria may be conflicting in some cases, selection is both art and science, yet the following criteria make the selection process more objective

- Capacity limit
- Code requirements
- Cost
- Load
- Location
- Resources
- Technology
- Synergy

Capacity limit is based on limits of systems and materials. For example, beams are economical for a given span range. To exceed that range would yield a bad ratio of dead load to live load. A beam's cross section increases with span, resulting in heavier dead load. Eventually, the beam's dead load exceeds its capacity and it would break. Approaching that limit, the beam gets increasingly uneconomical because its dead weight leaves little reserve capacity to carry live load. The span limit can be extended by effective cross section shape. For example, steel beam cross sections are optimized in response to bending and shear stress, to allow greater spans.

Trusses have longer span capacity than beams, due to reduced self weight. They replace the bulk of beams by top and bottom chords to resist global moments, and vertical and diagonal web bars to transfer shear between compression and tension chords. Compared to beams, the greater depth of trusses provides a greater lever arm between compression and tension bars to resist global moments. Similarly, suspension cables use the sag between support and mid-span as moment resisting lever arm. Since cables have higher breaking strength and resist tension only, without buckling, they are optimal for long spans; but the high cost of end fittings makes them expensive solutions for short spans. These examples show, most systems have upper and lower span limits.

Code requirements define structures by type of construction regarding materials and systems; ranging form type I to type V for least and most restrictive, respectively, of the Uniform Building Code (UBC) for example. Each type of construction has requirements for fire resistance, maximum allowable floor area, building height, and occupancy group. Codes also have detailed requirements regarding seismic design; notable structures are categorized by ductility to absorb seismic energy and related height limits. Some code requirements are related to other criteria described in the respective section.

Cost is often an overriding criterion in the selection of structures. In fact, cost is often defined by some of the other selection criteria. However, costs also depend on market conditions and seasonable changes. The availability of material and products, as well as economic conditions and labor strikes may greatly affect the cost of structures. For example, a labor strike in the steel industry may shift the advantage to a concrete structure, or the shortage of lumber, may give a cost advantage to light gauge steel instead of light wood framing. Sometimes, several systems are evaluated, or schematic designs are developed for them, in order to select the most cost effective alternative.

Load imposed on a structure is a major factor in selecting a system. For example, roofs in areas without snow must be designed only for a nominal load, yet roof load in mountain areas may be up to 20 times greater than the nominal load. Structures in earthquake prone areas should be lightweight and ductile, since seismic forces are basically governed by Newton's law, *force equals mass times acceleration* (f=ma). In contrast, structures subject to wind load should be heavy and stiff to resist wind uplift and minimize drift. Structures in areas of daily temperature variations should be designed for thermal load as well, unless the structure is protected behind a thermal insulation skin and subjected to constant indoor temperature only.

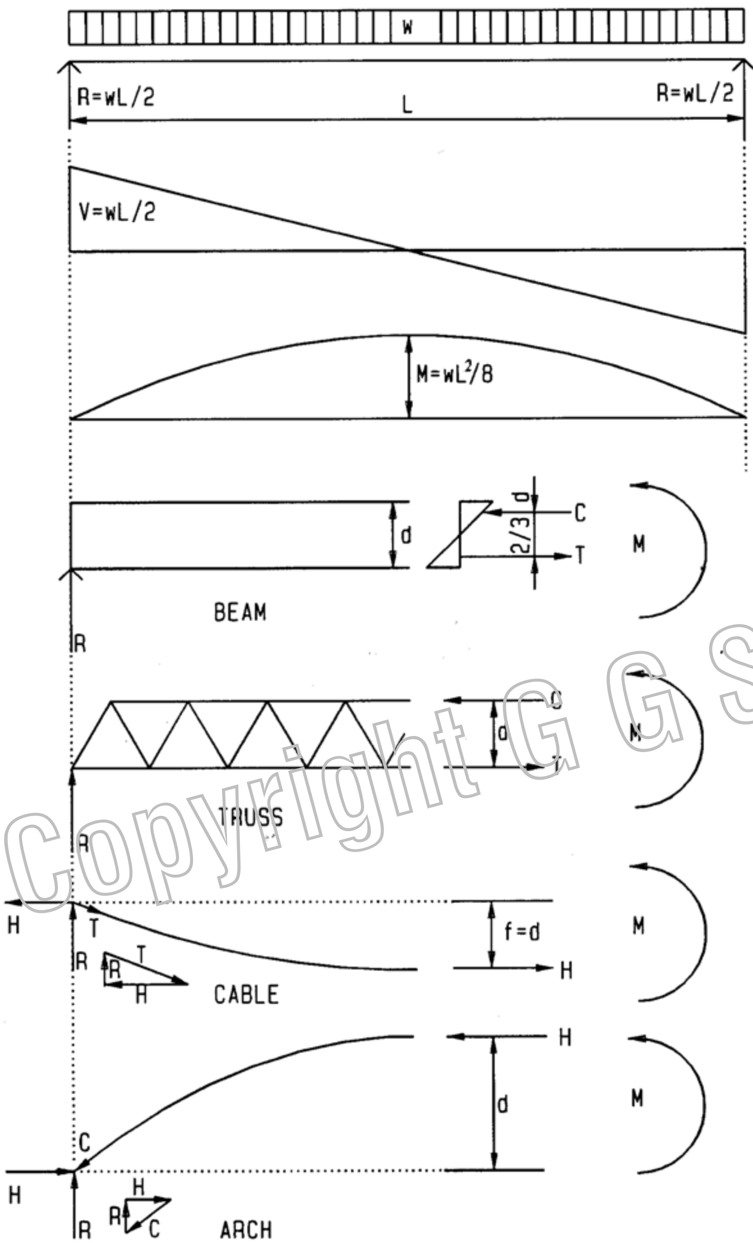
Location may effects structure selection by the type of soil, topography, and ground water level, natural hazards, such as fire, frost, or flood. Local soil conditions affect the foundation and possibly the entire structure. Soft soil may require pile foundations; a mat foundation may be chosen to balance the floating effect of high ground water. Locations with winter frost require deep foundations to prevent damage due to soil expansion in frost (usually a depth of about one meter). Hillside locations may require caisson foundations to prevent sliding, but foundations are more common on flat sites. Locations with fire hazards require non-combustible material. Raising the structure off the ground may be the answer to flooding.

Resources have a strong impact on the selection of structure materials. Availability of material was a deciding factor regarding the choice of material throughout history. The Viking build wood structures, a logical response to the vast forests of Scandinavia, yet stone temples of Egypt and Greece reflect the availability of stone and scarcity of wood. More recently, high-rise structures in the United States are usually steel structures, but the scarcity of steel in some other countries makes concrete structures more common.

Technology available at an area also effects the selection of structures. For example, light wood structures, known as platform framing, is most common for low rise residential structures in the United States, where it is widely available and very well known; but in Europe where this technology is less known, it is more expensive than more common masonry structures. Similarly, in some areas concrete technology is more familiar and available than steel technology. Concrete tends to be more common in areas of low labor cost, because concrete form-wok is labor intensive. On the other hand, prefab concrete technology is less dependent on low labor cost and more affected by market conditions, namely continuity of demand to justify the high investments associated with prefab concrete technology.

Synergy, defined as a system that is greater than the sum of its parts is a powerful concept to enrich architecture, regarding both pragmatic as well as philosophic objectives. Pragmatic example are numerous: Wall system are appropriate for hotel and apartment projects which require spatial and sound separation; but moment frames provide better space planning flexibility as needed for office buildings. However, the core of office buildings, usually housing elevators, stairs, bathrooms, and mechanical ducts, without the need for planning flexibility, often consists of shear walls ore braced frames, effective to reduce drift under lateral loads. Long-span systems provide column-free space required for unobstructed views in auditoriums and other assembly halls; but lower cost short span systems are used for warehouses and similar facilities where columns are usually acceptable.

On a more detailed level, to incorporate mechanical systems within a long-span roof or floor structure, a Vierendeel girder may be selected instead of a truss, since the rectangular panels of a Vierendeel better facilitate ducts to pass through than triangular truss panels. A suspended cable roof may be selected for a sports arena if bleachers can be used to effectively resist the roof's lateral thrust which is very substantial and may require costly foundations otherwise. Synergy is also a powerful concept regarding more philosophical objectives, as demonstrated throughout history, from early post and beam structures, Roman arches, domes and vaults; Gothic cathedrals; to contemporary suspension bridges or roofs. Columns can provide architectural expression as in post and beam systems, or define and organize circulation, as in a Gothic cathedral. The funicular surface of arches, domes and vaults can define a unique and spiritual space. The buttresses to resist their lateral thrust provide the unique vocabulary of Gothic cathedrals. Large retaining walls may use buttressing for rhythmic relieve, as in the great wall of Assisi, or lean backward to express increased stability as the wall of the Dalai Lama palace in Tibet.



Global moment and shear

Global moments help to analyze not only a beam but also truss, cable or arch. They all resist global moments by a couple F times lever arm d:

M = F d; hence F = M / d

The force F is expressed as T (tension) and C (compression) for beam or truss and H (horizontal reaction) for suspension cable or arch, forces are always defined by the global moment and lever arm of resisting couple. For uniform load and simple support, the maximum moment M and maximum shear V are computed as:

$M = w L^2 / 8$

$V = w L / 2$

w = uniform gravity load
L = span

For other load or support conditions use appropriate formulas
Beam
Beams resist the global moment by a force couple, with lever arm of 2/3 the beam depth d; resisted by top compression C and bottom tension T.

Truss
Trusses resist the global moment by a force couple and truss depth d as lever arm; with compression C in top chord and tension T in bottom chord. Global shear is resisted by vertical and / or diagonal web bars. Maximum moment at mid-span causes maximum chord forces. Maximum support shear causes maximum web bar forces.

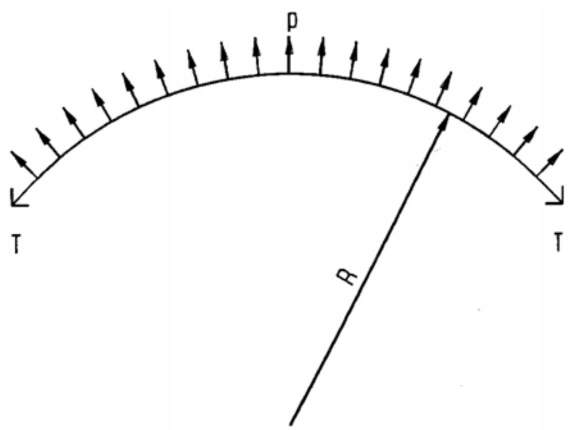
Cable
Suspension cables resist the global moment by horizontal reaction with sag f as lever arm. The horizontal reaction H, vertical reaction R, and maximum cable tension T form an equilibrium vector triangle; hence the maximum cable tension is:

$T = (H^2 + R^2)^{1/2}$

Arch
Arches resist the global moment like a cable, but in compression instead of tension:

$C = (H^2+R^2)^{1/2}$

However, unlike cables, arches don't adjust their form for changing loads; hence, they assume bending under non-uniform load as product of funicular force and lever arm between funicular line and arch form (bending stress is substituted by conservative axial stress for approximate schematic design).



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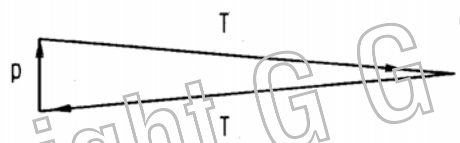

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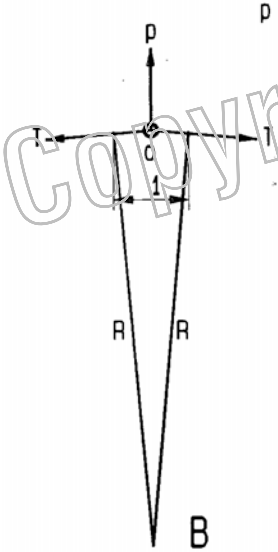


B

Radial pressure

Referring to diagram A, pressure per unit length acting in radial direction on a circular ring yields a ring tension, defined as:

T = R p

T = ring tension
R = radius of ring
p = uniform radial pressure per unit length

Units must be compatible, i.e., if p is force per foot, R must be in feet, if p is force per meter, R must be in meters. Pressure p acting reversed toward the ring center would reverse the ring force from tension to compression.

Proof

Referring to ring segment B:

T acts perpendicular to ring radius R

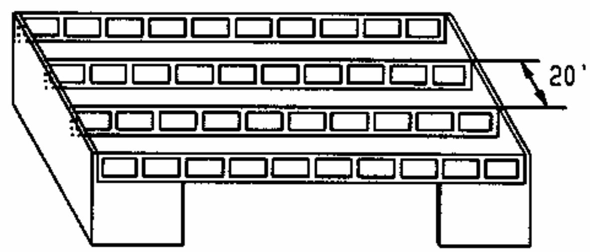
p acts perpendicular to ring segment of unit length

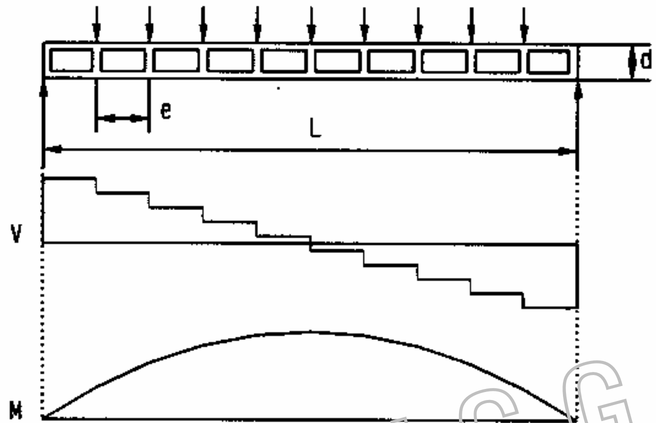
Referring to ring segment B and vector triangle C:

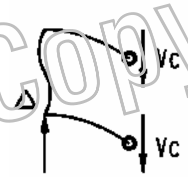
p and T in C represent equilibrium at o in B

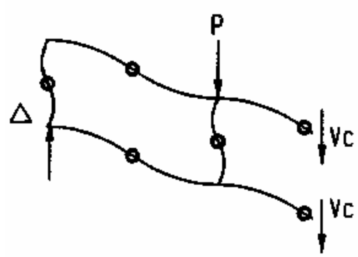
T / p = R / 1 (due to similar triangles), hence

T = R p





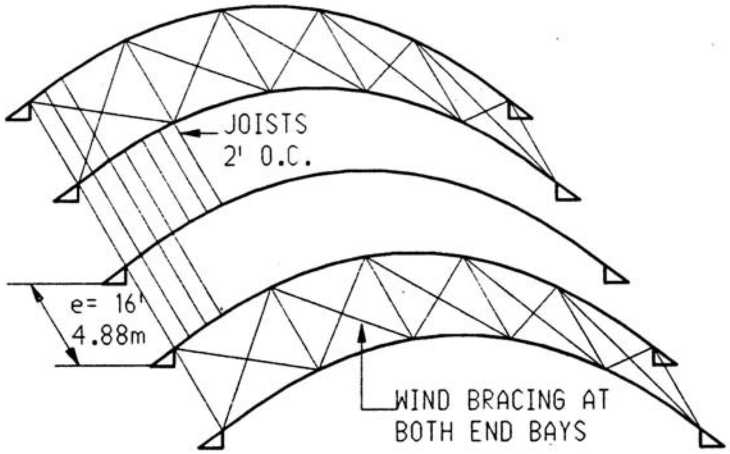


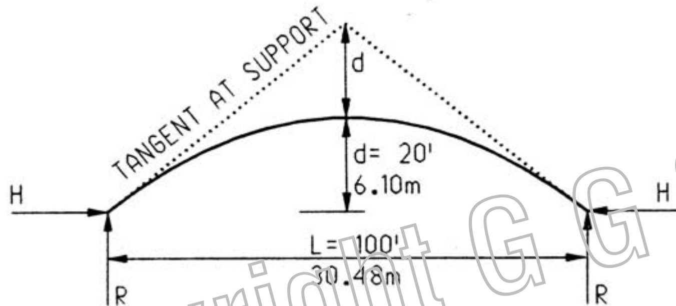


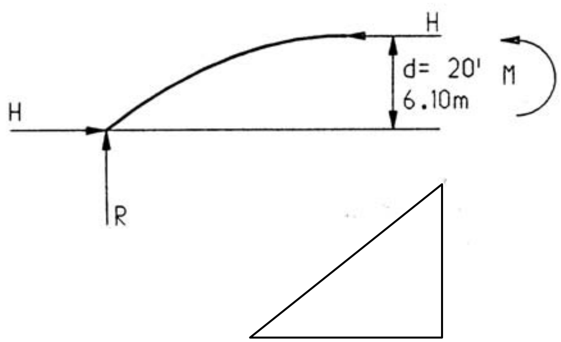
Examples

Vierendeel Girder

Assume	
Steel girders spaced 20'	L = 100'
Allowable stress (60% of F_y = 46 ksi tubing)	F_a = 27.6 ksi
DL= 18 psf	
LL = 12 psf (20 psf reduced to 60% for tributary area > 600 sq. ft.)	
Σ = 30 psf	
Uniform girder load	
w = 30 psf x 20' / 1000	w = 0.6 klf
Joint load	
P= w e = 0.6 x 10'	P = 6 k
Vertical Reaction	
R= w L / 2 = 0.6 x 100' / 2	R = 30 k
END BAY CHORD	
Chord shear	
V_c= (R – P/2) / 2 = (30 – 6/2) / 2	V_c = 13.5 k
Chord bending	
M_c = V_c e/2 = 13.5 x 10'x12" / 2	M_c = 810 k"
Moment of Inertia required	
I= M_c c/ Fa = 810x5"/27.6 ksi	I = 147 in^4
Use ST 10x10x5/16	I = 183 > 147, ok
WEB BAR (2nd web resists bending of 2 adjacent chords)	
2^{nd} bay chord shear	
V_c= (R-1.5 P)/2 = (30–1.5x6)/2	V_c = 10.5 k
2^{nd} bay chord bending	
M_c = V_c e/2 = 10.5x10'x12"/2	M_c = 630 k"
Web bending	
M_w = M_c end bay + M_c 2^{nd} bay = 810 + 630	M_w = 1,440 k"
Moment of Inertia required	
I = M_w c/F_a = 1,440x5/27.6	I = 261 in^4
Use ST 10x10x1/2 web bar	I =271 > 261, ok
MID-SPAN CHORD (small chord bending ignored)	
Mid-span global bending	
M= w L^2/8= 0.6x100^2/8	M = 750 k'
Mid-span chord force	
P= M/d= 750/6	P= 125 k
Use ST 10 x 10 x5/16	297 > 125, ok







Wood Arch

Assume

Three-hinge glue-lam arches spaced 16'.

(Available glue-lam dimensions: ¾" lams; 3 1/8", 5 1/8", 6¾", 8¾" and10 ¾" wide).

Allowable buckling stress (from case studies) $F_c' = 200$ psi

LL = 12 psf (reduced to 60% of 20 psf for tributary area > 600 sq. ft.)

DL = 18 psf (estimate)

Σ = 30 psf

Uniform load

w = 30 psf x 16' / 1000 = w = 0.48 klf

Global moment

$M = w L^2 / 8 = 0.48 \times 100^2 / 8 =$ M = 600 k'

Horizontal reaction

H = M / d = 600 / 20 = H = 30 k

Vertical reaction

R = w L /2 = 0.48 x 100' / 2 = R = 24 k

Arch compression (max.)

$C = (H^2 + R^2)^{1/2} = (30^2 + 24^2)^{1/2}$ C = 38 k

Cross section area required

$A = C / F_c' = 38 / 0.2$ ksi $A = 190$ in^2

Glue-lam depth (try 5 1/8" wide glue-lam)

t = A / width = 190 / 5.125 = 37; use 50 lams of ¾" t = 37.5"

Check slenderness ratio

L / t = 100' x 12" / 37.5" = L/t = 32 ok

Note:

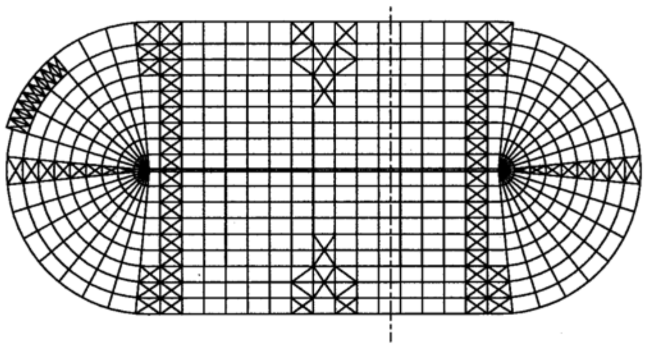
Arch slenderness of L/t = 32 is ok (the 5 1/8" arch width is braced against buckling by the roof diaphragm).

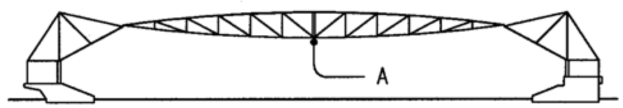
Wind bracing at end bays may consist of diagonal steel rods in combination with compression struts. The lateral thrust of arches may be resisted by concrete piers that may be tied together by grade beams to resist the lateral arch thrust.

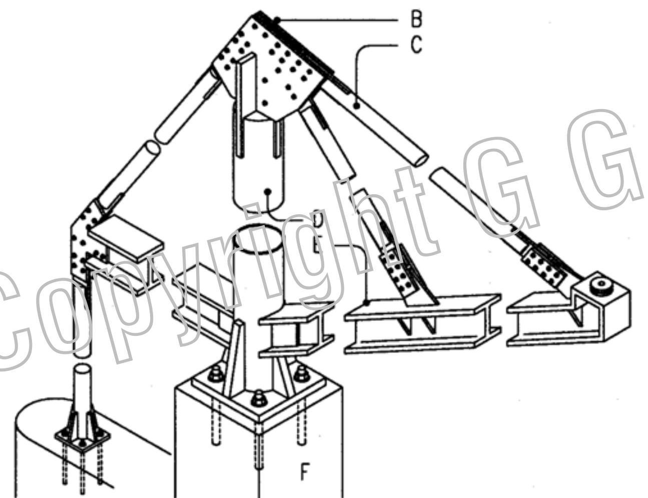
Final design must consider non-uniform load (snow on half the arch) resulting in combined axial and bending stress; the bending moment being axial force times lever arm between funicular pressure line and arch center. The funicular line may be found graphically.

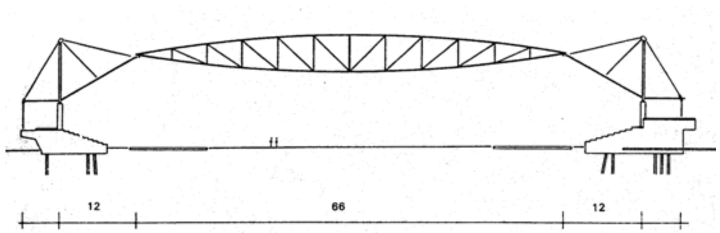
Graphic method

- Draw a vector of the computed vertical reaction
- Draw equilibrium vectors parallel to arch support tangent
- Equilibrium vectors give arch force and horizontal reaction









Case studies

Skating Rink, Heerenveen, Holland

Architect: Van der Zee & Ybema
Engineer: Arie Krijegsman, ABT
Steel trusses

Allowable stress F_y = 36 ksi x 0,6	F_a = 21.6 ksi
Truss span L= 66m/0.3048	L = 217'
Truss spacing e =7.2m/.3048	e = 24'
Truss depth at mid span d = 5.8m/0.3048	d = 19'
DL = o.6 kPa (12.5 psf)	
<u>LL = 0.5 kPa (10.4 psf)</u>	
Σ = 1.1 kPa (22.9 psf)	
Uniform load per truss	
w = 24' x 22.9 psf / 1000	w = 0.55 klf
Mid span point load (center truss, A transfers load of circular end units)	
Tributary area of end units	
$A = \pi r^2/3 = \pi(217'/2)^2/3$	A = 12,278 sq. ft.
Point load per truss	
P = 12,278 x 22.9 psf / 1000 / 16 trusses	P = 18 k
Global moment	
$M = PL/4+wL^2/8$ = 18x217/4 + 0.55 x $217^2/8$	M = 4,214 k'
Chord bar force	
C = T = M/d = 4,214 / 19	C =T = 222 k
Bottom tension chord	
Try wide flange section	W8x35
Try wide flange	
Allowable force P from AISC table (use L = 0' for tension, no buckling)	
P_{all} = 222	222 = 222, ok
Top chord un-braced length L = 217'/12	L = 18'
Top chord bending (negative support bending	
$M = w L^2/12 = 0.55 \times 18^2 / 12$	M = 15 k'
Try W12x50	
A = 14.7in^2, I_x= 394 in^4, r_x= 5.17" (y-axis is braced by roof deck)	
Bending stress	
f_b = M c / I = 15k'x12"x 6"/394	f_b = 2.74 ksi
Axial stress f_a = C / A = 222 k /14.7 in^2	f_a = 15.1 ksi
Slenderness KL/r_x = 1x18'x12" / 5.17"	kL/r = 42
Allowable buckling stress (from AISC table)	F_a =19 ksi
Check combined stress $f_a/F_a + f_b/F_b$ <=1	
$f_a/F_a + f_b/F_b$ = 15.1/19 + 2.74 / 21.6 = 0.92	0.92 < 1, ok
Use	W12x50

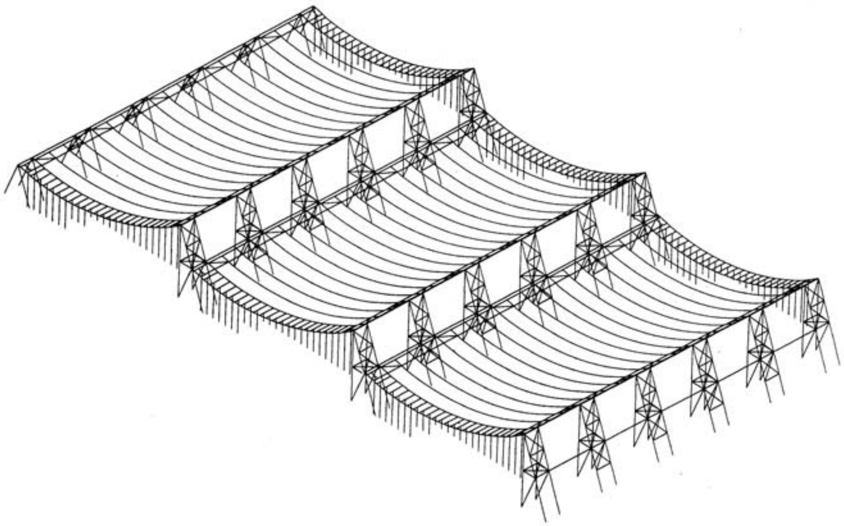


Exhibit Hall 26 Hanover

Architect: Thomas Herzog
Engineer: Schlaich Bergermann

Given

Steel suspender bands 30x400 mm (1.2x16”), spaced 5.5 m (18’)

LL =	0.5 kN/m^2	(10 psf)
DL =	1.2 kN/m^2	(25 psf)
Σ =	1.7 kN/m^2	(35 psf)

Uniform load

w = 1.7 kN/m^2 x 5.5 m = w = 9.35 kN/m

Global moment

M = w L^2 / 8 = 9.35 x 64^2 / 8 M = 4787 kN-m

Horizontal reaction

H = M / f = 4787 / 7 H = 684 kN

Vertical reaction R (max.)

Reactions are unequal; use R/H ratio (similar triangles) to compute max. R

R/H = (2f+h/2) / (L/2), hence

R = H (2f+h/2) / (L/2) = 684 (2x7+13/2) / (64/2) 438 kN

Suspender tension (max.)

T = $(H^2 + R^2)^{1/2}$ = $(684^2 + 438^2)^{1/2}$ T = 812 kN

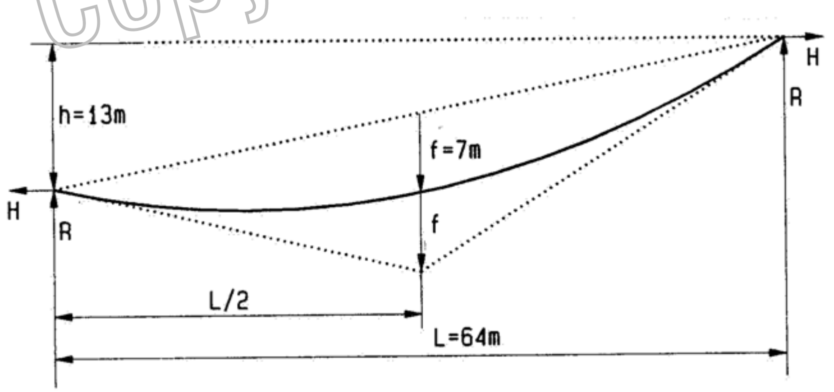
Suspender stress (A = 30x400 mm)

f = T / A = 1000x812 / (30x400) f = 67.7 MPa

US unit equivalent

67.7 kPa x 0.145 f = 9.8 ksi

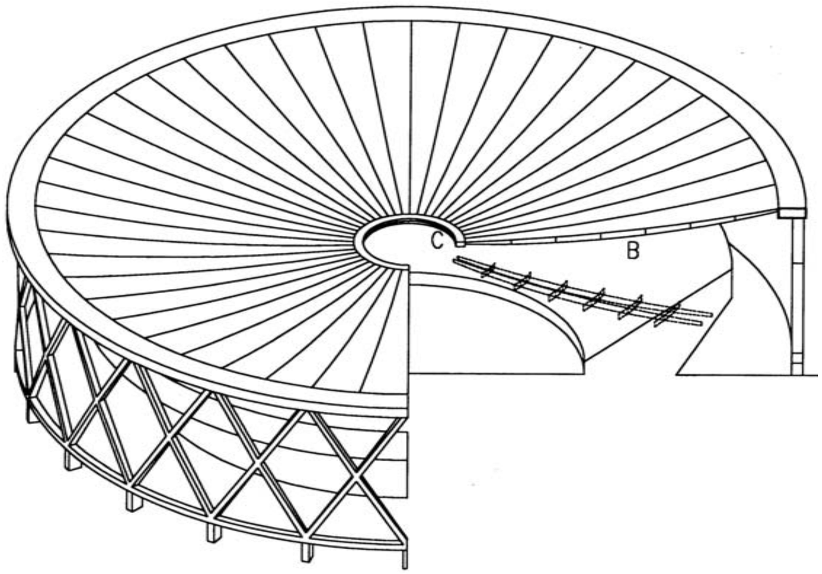
9.8 < 22 ksi, ok

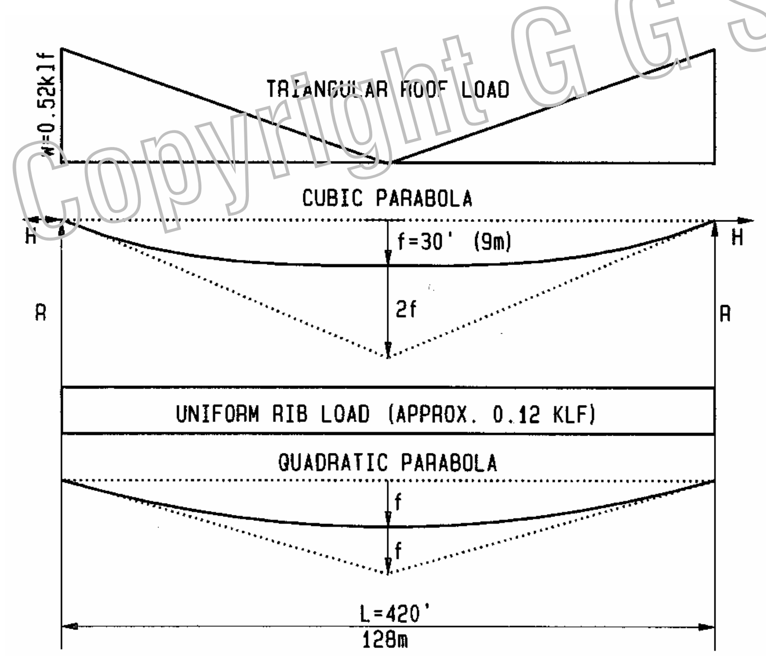


Graphic method

- Draw a vector of the total vertical load
- Equilibrium vectors parallel to support tangents give cable forces
- Equilibrium vectors at supports give H and R reactions.

Note: The unequal support height is a structural disadvantage since the horizontal reactions of adjacent bays don’t balance, but it provides lighting and ventilation, a major objective for sustainability. The roof consists of prefab wood panels, filled with gravel to resist wind uplift. Curtain wall mullions at the roof edge are prestressed between roof and footing to prevent buckling under roof deflection. In width direction the roof is slightly convex for drainage; which also gives the interior roof line a pleasing spatial form.





Oakland Coliseum

Architect/Engineer: Skidmore Owings and Merrill

Assume

Allowable cable stress (210 ksi breaking strength / 3) — F_a = 70 ksi

Radial suspension cables, spaced 13' along outer compression ring

LL=	12 psf	(60% of 20 psf for tributary area > 600 sq. ft.)
DL=	46 psf	(estimate)
Σ =	58 psf	

Uniform load

w = 58 psf x 13' / 1000 — w = 0 to 0.75 klf

Global moment [cubic parabola with origin at mid-span]

$M_x = w\,L^2/24\,(1 - 8\,X^3 / L^3)$ for max. M at mid-span, X=0, hence

$M = w\,L^2 / 24 = 0.75 \times 420^2 / 24$ — M = 5,513 k

Horizontal reaction

H = M / f = 5,513 / 30 — H = 184 k

Vertical reaction

R = w L / 2 = (0.75/2) x 420' / 2 — R = 79 k

Cable tension (max.)

$T = (H^2 + R^2)^{1/2} = (184^2 + 79^2)^{1/2}$ — T = 200 k

Metallic cross section required

Am = T / Fa = 200 / 70 ksi — Am= 2.86 in^2

Gross cross section (70% metallic)

Ag = Am / 0.70 = 2.86 / 0,70 — Ag = 4.09 in^2

Cable size

$ø = 2(Am / \pi)^{1/2} = 2(4.09 / 3.14)^{1/2} = 2.28$" — use ø 2 3/8"

Steel tension ring (inner ring radius r= 15', cable spacing = 0.94')

T= H r / 0.94 = 184 x 15 / 0.94 — T = 2,972 k

Cross-section area (assume high-strength steel Fa = 30 ksi)

A= T/F_a = 2,936/30= 98 in^2

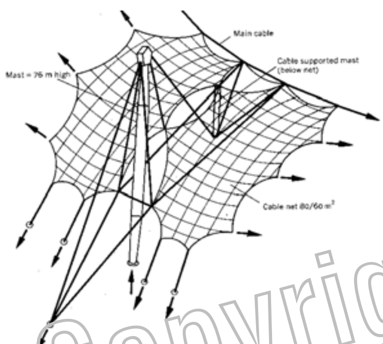
Try W24x335, A= 98.2 in^2 > 98 — use W24x335

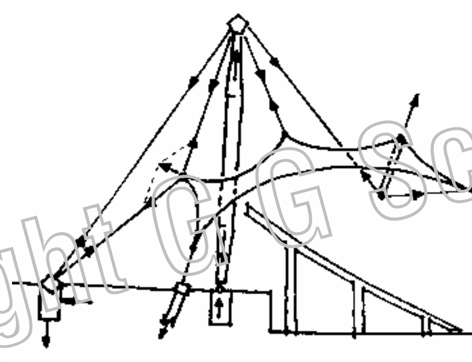
Concrete compression ring (r = 210', e = 13')

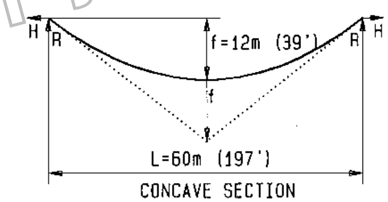
C = H r / e = 184 x 210 / 13 = — C = 2,972 k

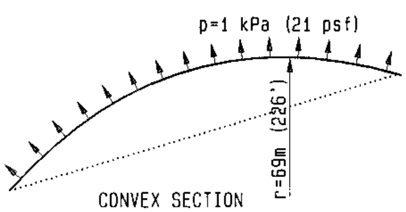
Cross-section area (assume allowable buckling stress F_c'= 1.2 ksi)

A = C / F_c' = 2,972 / 1.2 = 2,477 in^2 ~72"x34" — Use 6x3'









Olympic Stadium Munich

Architect: Guenter Behnisch
Engineer: Leonhardt und Andrae

Assume

Allowable cable stress (210 ksi breaking strength / 3) F_a = 70 ksi

DL =	5psf	5 psf
LL =	20 psf	wind uplift 21 psf
Σ =	25 psf	net uplift 16 psf

Uniform load (cable spacing 75 cm = 2.5')

Gravity w = 25 psf x 2.5' w = 62.5 plf

Wind uplift 16 psf x 2.5' p = 40 plf

Global moment

$M = w L^2 / 8 = 62.5 \times 197^2 / 8$ M = 303,195 #

Horizontal reaction

H = M / f = 303,195 #" / 39' H = 7,774 #

Vertical reaction

R = w L / 2 = 62.5 x 197' / 2 R = 6,156 #

Gravity tension (add 10% residual prestress)

$T = 1.1 (H^2 + R^2)^{1/2} = 1.1 (7,774^2 + 6,156^2)^{1/2}$ Gravity T = 10,908 #

Wind uplift tension (add 10% residual prestress)

Wind suction is normal to surface, hence T = p r [r = curvature radius)

T = 1.1 p r = 1.1 x 40 x 226' Wind T = 9,944 #

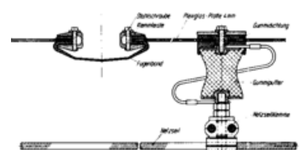
10,907 > 9,944 gravity governs

Metallic cross section area (assume twin ½" net cables, 70% metallic)

$A_m = 0.7 \times 2 \pi r^2 = 2 \times 3.14 (0.5/2)^2$ A_m= 0.28 in²

Net cable stress

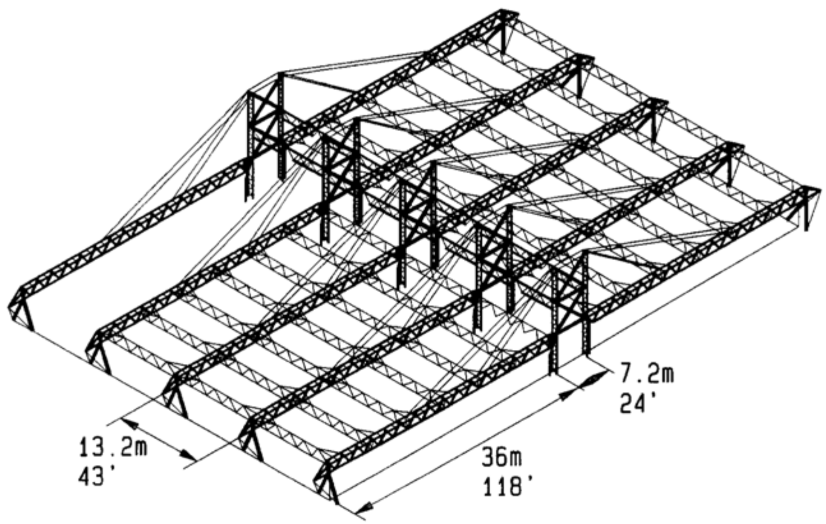
$f = T / A_m = 10,908 / (0.28 \times 1000)$ f = 39 ksi < 70 ok



5.19
Detail showing attachment of the plexiglass cladding panels to the net cables

Plexiglas joint / support

Net cable joint



Imos Factory Newport UK
Architect: Richard Rogers
Engineer: Anthony Hunt

This microprocessor factory is a cable stayed structure. Stay rods, instead of stay cables, are suspended from trussed steel pylons along a central circulation spine. The rods support prismatic roof trusses at third points to reduce truss depth and weight for a column-free floor area of maximum flexibility. Truss joists, spaced 6m (20'), support the roof deck. Mechanical equipment, located over the central spine, also allows for optimal flexibility, as required for this facility.

Assume

Allowable rod stress (high strength steel) — F_a = 207,000 kPa (30 ksi)

DL	=	0.7 kPa	(14.7 psf)
LL	=	0.5 kPa	(10.4 psf)
Σ	=	1.2 kPa	(25.1 psf)

Uniform load
w = 1.2 kPa x 13.2m — w = 15.8 kN/m

Tributary load per rod P
P = w L' = 15.8 kN/m x 12m — P = 190 kN

Rod tensions
$T_{A\text{-}B}$ (from vector triangle) — $T_{A\text{-}B}$ = 370 kN

$T_{A\text{-}C}$ (from vector triangle) — $T_{A\text{-}C}$ = 248 kN

Rods A-B cross section (2 rods)
A = $T_{A\text{-}B}$ / (2 F_a) = 370 / (2 x 207,000) = 0.000894 m^2 — A = 894 mm^2

Rod diameter
$\varnothing = 2\,(A/\pi)^{1/2} = 2\,(894/3.14)^{1/2}$ — 2 $\varnothing$ 34 mm

Convert to inches
34 mm/25.4 — 2 $\varnothing$ 1.33"

Rods A-C cross section (4 rods)
A = $T_{A\text{-}C}$ / (4 Fa) = 246 / (4 x 207,000) = 0.000297 m^2 — A = 297 mm^2

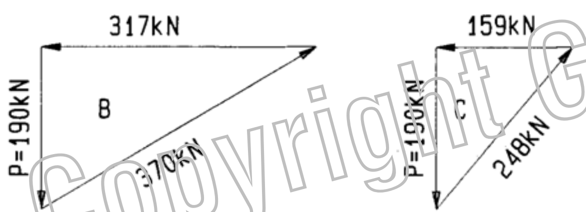
Rod diameter $\varnothing$
$\varnothing = 2\,(A/\pi)^{1/2} = 2\,(297/3.14)^{1/2}$ — 4 $\varnothing$ 20 mm

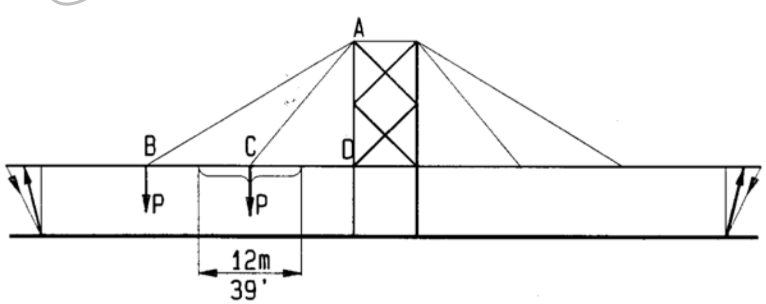
Covert to inches
20 mm/25.4 — 4 $\varnothing$ 0.79"

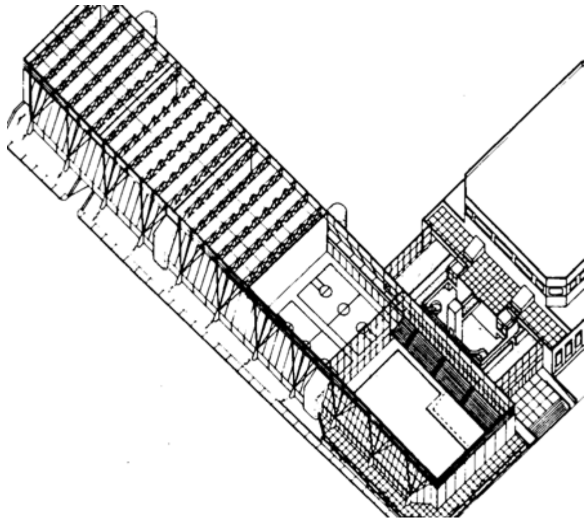
Note: design the truss for added compression, from vector triangles

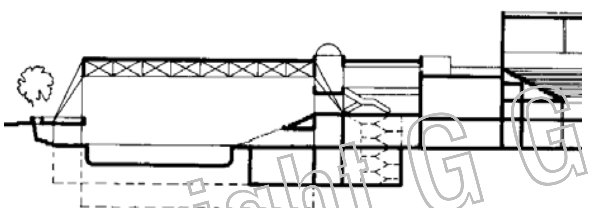
Between B-C add 317 kN — B-C = 317 kN

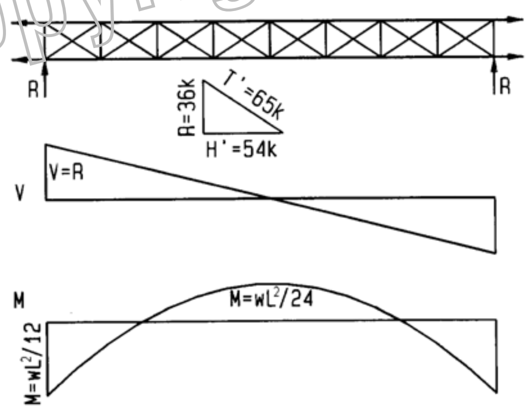
Between C-D add 317 + 159 kN — C-D = 476 kN











Sports Center UC Berkeley
Architect: G G Schierle
Engineer: T Y Lin

Assume:
Cable truss with vertical compression struts

Span	L= 120'
Depth	d = 10'
Spacing	e = 20'
Allowable cable stress (210 ksi / 3)	F_a= 70 ksi

Prestress 60% of F_a (50% + temperature change reserve)

DL = 18 psf
LL = 12 psf (60% of 20 psf for tributary area > 600 sq. ft.)
Σ = 30 psf

Uniform load per truss
w = 30 psf x 20'/1000 — w = 0.6 klf

Global moment (fixed support)
$M = wL^2/12 = 0.6 \times 120^2/12$ — M = 720 k'

Chord force (assume 10% residual prestress)
T = 1.1 M/d = 1.1 x 720/10' — T = 79 k

Chord cross section area (70% metallic)
A= T/(0.7 F_a) = 79/(0.7 x 70ksi) — $A = 1.61\ in^2$

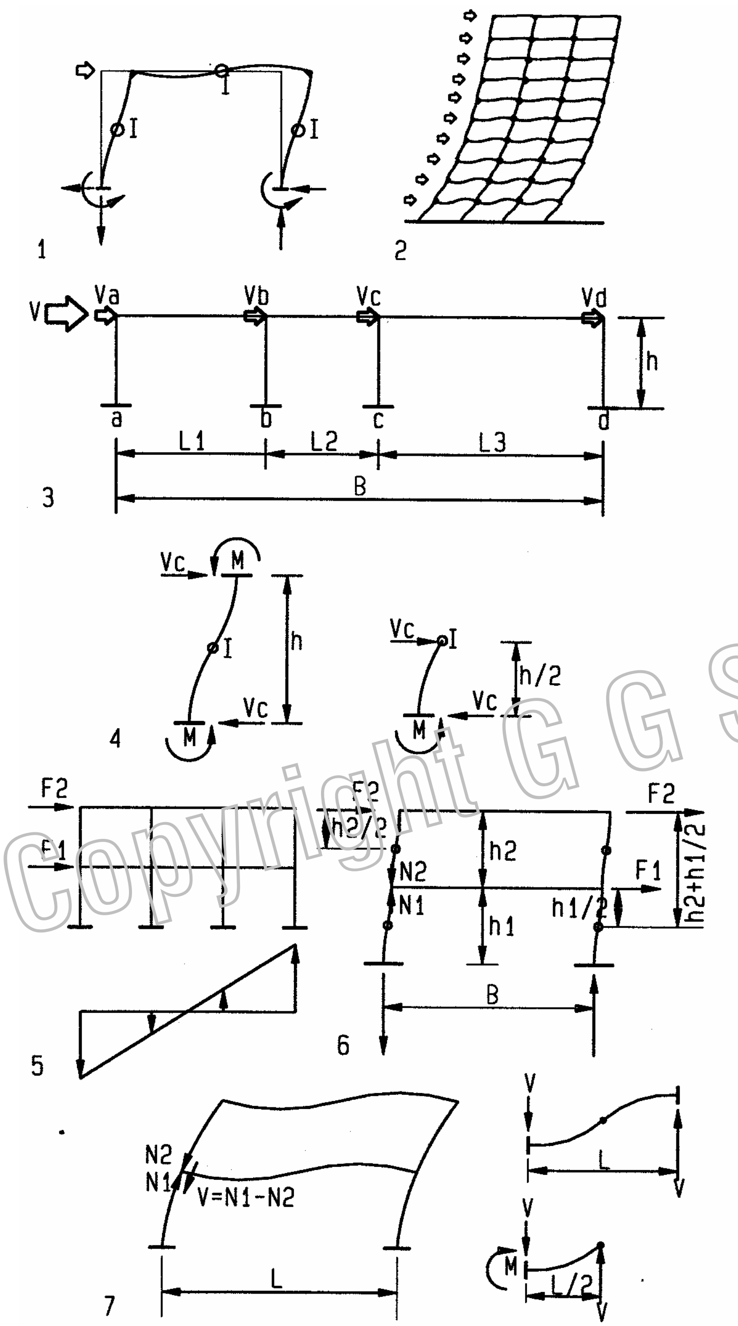
Chord cable size
$\phi = 2(A/\pi)^{1/2} = 2(1.61/\pi)^{1/2} = 1.43$" — Use ϕ1.5"

Vertical reaction (without guy cable force)
R = wL/2 = 0.6 x 120'/2 — R = 36 k

Diagonal cable force (assume 10% residual prestress)
T = 1.1 T' = 1.1 x 65 k (from vector triangle) — T = 72 k

Diagonal cable cross section (twin cables, 70% metallic)
A = T'/(2 x 70 x 0.7) = 72/(2 x 70 x 0.7) — $A = 0,73\ in^2$

Diagonal cable size
$\phi = 2(A/\pi)^{1/2} = 2(0.73/\pi)^{1/2} = 0.96$ in — Use ϕ1 in



Portal method

The Portal Method for rough moment frame design is based on these assumptions:

- Lateral forces resisted by frame action
- Inflection points at mid-height of columns
- Inflection points at mid-span of beams
- Column shear is based on tributary area
- Overturn is resisted by exterior columns only

1 Single moment frame (portal)

2 Multistory moment frame

3 Column shear is total shear V distributed proportional to tributary area:
Va = (V/B) L1 / 2
Vb = (V/B) (L1+L2) / 2
Vc = (V/B) (L2+L3) / 2
Vd = (V/B) L3 / 2

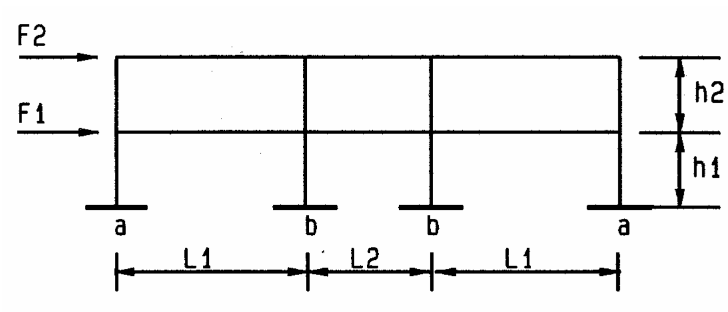
4 Column moment = column shear x height to inflection point
Ma = Va h / 2
Mb = Vb h / 2
Mc = Vc h / 2
Md = Vd h / 2

5 Exterior columns resist most overturn; the portal method assumes they resist all

6 Overturn moments per level are the sum of forces above the level times lever arm of each force to the column inflection point at the respective level:
M2 = F2 h2 / 2 (level 2)
M1 = F2 (h2+h1 / 2) + F1 h1 / 2 (level 1)
Column axial force = overturn moment divided by width B
N = M / B
Column axial force per level:
N2 = M2 / B (level 2)
N1 = M1 / B (level 1)

7 Beam shear = column axial force below beam minus column axial force above beam
Level 1 beam shear:
V = N1 - N2
Roof beam:
V = N2 - 0 = N2
Beam bending = beam shear times distance to inflection point at beam center
M = V L / 2
Beam axial force is negligible and assumed 0

Example: 2-story building
Assume:
L1 = 30’
L2 = 20’
B = 30+20+30 = 80’
h = h1 = h2 = h = 14’
F1 = 8 k
F2 = 12 k



1st floor

Base shear
V = F1+F2 = 8+12 V = 20 k

Column shear
Va = (L1/2) (V/B) = 15’x20/80 Va = 3.75 k
Vb = (L1+L2)/2 (V/B) = (20+30)/2 (20/80) Vb = 6.25 k

Column bending
Ma = Va h/2 = 3.75 x 14/2 Ma = 26 k’
Mb = Vb h/2 = 6.25 x 14/2 Mb = 44 k’

Overturn moments
M1 = F2 (h2+h1/2)+F1 h1/2 = 12 (14+7)+8x7 M1 = 308 k
M2 = F2 h2/2 = 12x7 M2 = 84 k’

Column axial load 1st floor
N1 = M1/B = 308/80 N1 = 3.9 k

Column axial load 2nd floor
N2 = M2/B = 84/80 N2 = 1.1 k

Beam shear
V1 = N1–N2 = 3.9-1.1 V1 = 2.8 k

Beam bending
M1 = V1 L1/2 = 2.8x30/2 M1 = 42 k’
M2 = V1 L2/2 = 2.8x20/2 M2 = 28 k’

2nd floor

2nd floor shear
V = F2 V = 12 k

Column a shear
Va = (L1/2) (V/B)=15’x12/80 Va = 2.25 k

Column b shear
Vb = (L1+L2)/2 (V/B) = (30+20)/2x12/80 Vb = 3.75 k

Column a bending
Ma = Va h/2 = 2.25 x 14/2 Ma = 15.75 k’

Column b bending
Mb = Vb h/2 = 3.75 x 14/2 Mb = 26.25 k’

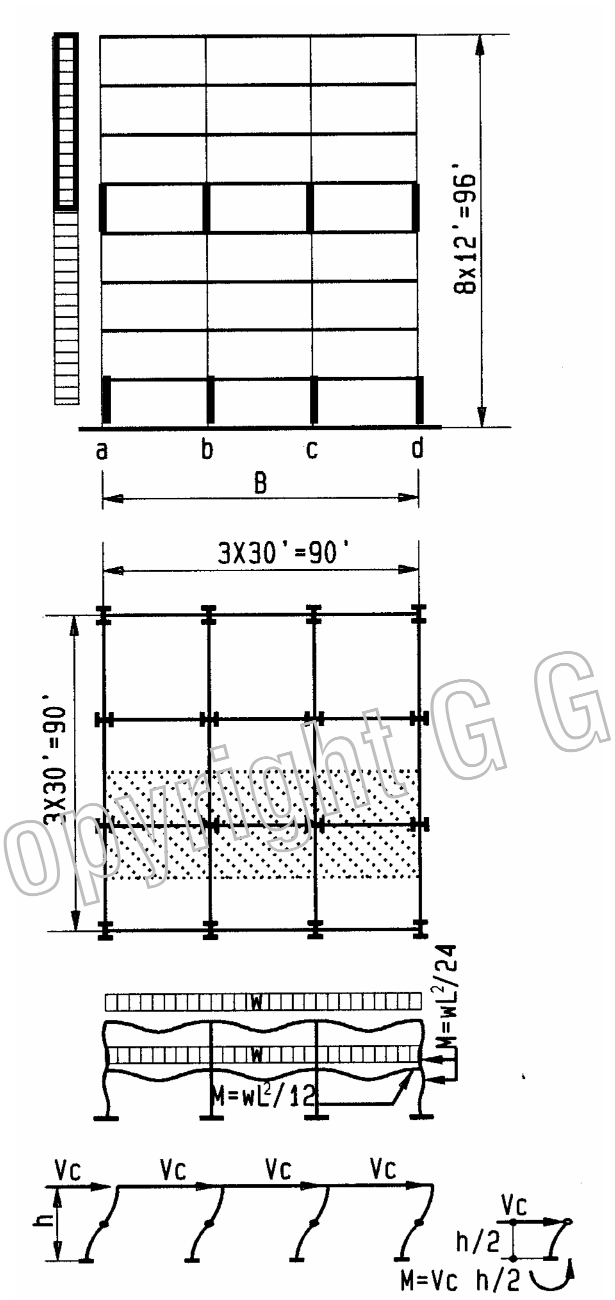
Overturn moment
M2 = F2xh/2 = 12x14/2 M2 = 84 k’

Column axial load
N2 = M2/B = 84/80 N2 = 1.0 k

Beam shear
V2 = N2 V2 = 1.0 k

Beam 1 bending
M1 = V2 L1/2=1.0x30/2 M1 = 15 k’

Beam 2 bending
M2 = V2 L2/2=1.0x20/2 M2 = 10 k’



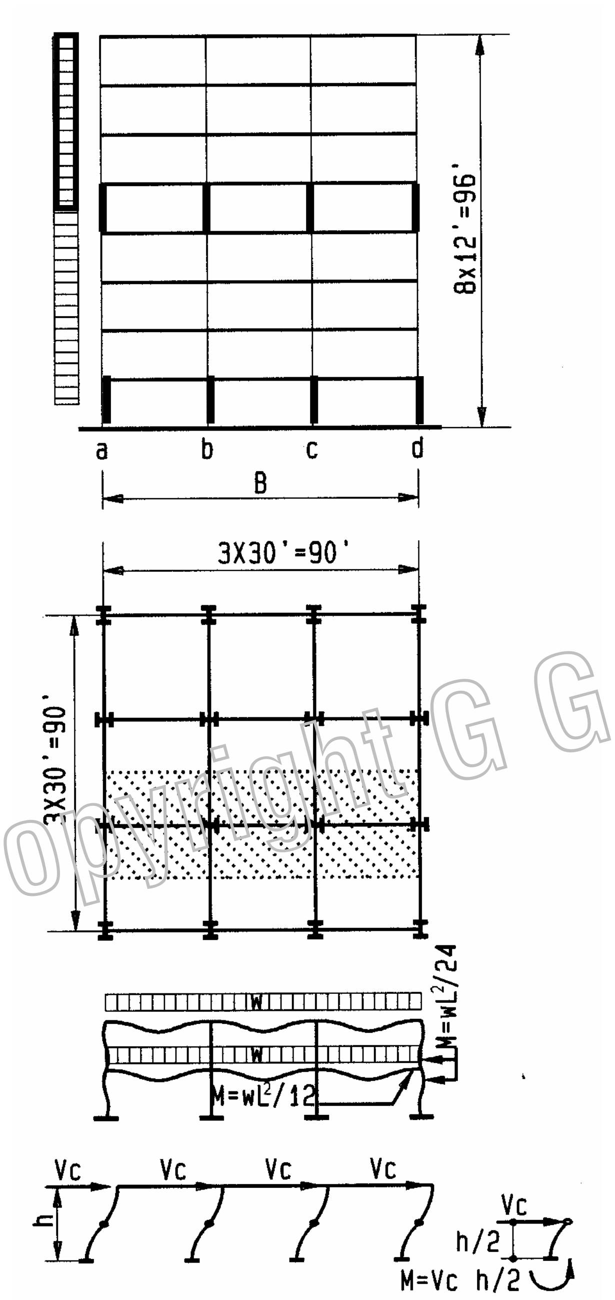
Moment frame

Eight story steel moment frame, high strength steel, F_y = 50 ksi F_a = F_b = 30 ksi
Live load 50 psf, load reduction R in percent per IBC
R = 0.08 (A-150)
A = tributary area
Max. reduction: 40% for members supporting a single level, 60% for other members

Gravity load	Beam (psf)	Column (psf)
Framing	10	10
Concrete slab	37	37
Partitions	20	20
Floor / ceiling	3	3
DL	70	70
LL	50 x 0.6 = 30	50 X 0.4 = 20
Total DL + LL	100	90
Average wind pressure		P = 33 psf

Design ground floor and 4th floor

Uniform beam load (shaded tributary are)
W = 100 psf x 30'/1000 — w = 3 klf
Uniform column load (distributed on beam)
w = 90 psf x 30'/1000 — w = 2.7 klf
Base shear
V = 33 psf x 30' x 7.5 x 12'/1000 — V = 89 k
Level 4 shear
V = 33 psf x 30' x 3.5 x 12'/1000 — V = 42 k
Overturn moments
Ground floor M_0 = 33 psf x 30' x $(7.5x12)^2$ /2 / 1000 — M_0 = 4,010 k'
First floor M_1 = 33 psf x 30'$(6.5 x12)^2$ /2 / 1000 — M_1 =3,012 k'
Fourth floor M_4 = 33 psf x 30'$(3.5 x12)^2$ /2 / 1000 — M_4 = 873 k'
Beam design
Column a & d axial load
N_0 = M_0 / B = 4,010 / 90 — N_0 = 45 k
N_1 = M1 / B = 3,012 / 90 — N_1 = 34 k
Beam design
Beam shear
V = N_0 – N_1 = 45-34 — V = 11 k
Beam bending
$M_{lateral}$ = V L/2 = 11x30/2 — $M_{lateral}$ = 165 k'
$M_{gravity}$ = $wL^2/12$ = $3x30^2/12$ — $M_{gravity}$ = 225 k'
ΣM = 165 + 225 — Σ M = 390 k'
Required S_x = M / F_b = 12"x390 k'/ 30 ksi — S_x = 156 in^3
Use W18x86 — S_x = 166 > 156
Note: W18 beam has optimal ratio L/d = 20

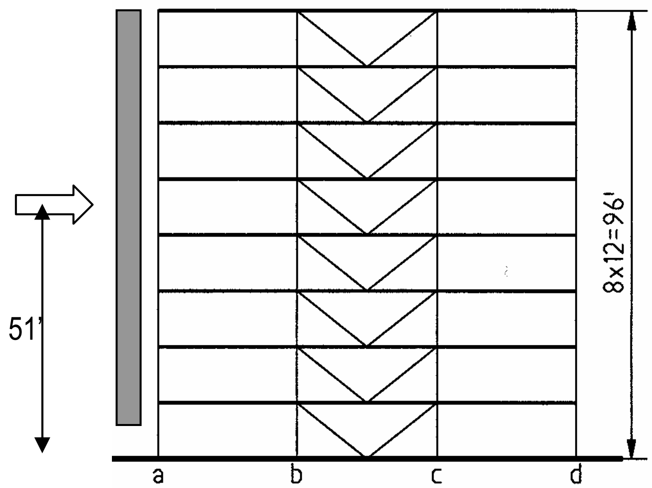


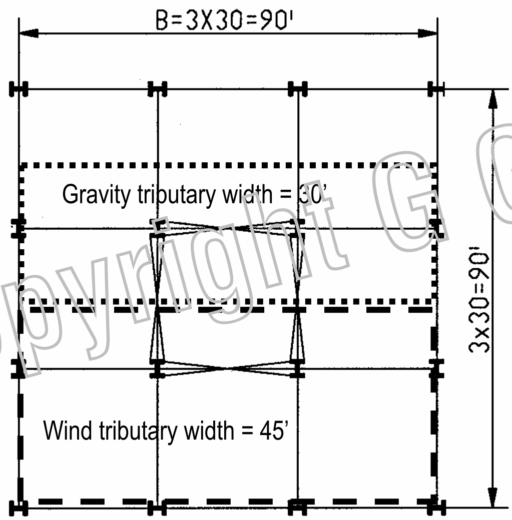
Ground floor

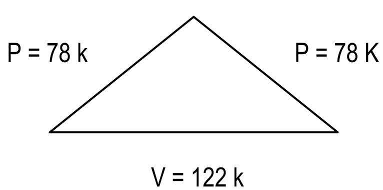
Column shear			
Column	$V_c = L_{tributary} V / B$		V_c
a & d	15x89/90		14.8 k
b & c	30x89/90		29.7 k
Column bending			
Column	$M_{lateral} = V_c h/2$	$M_{gravity} = wL^2/24$	ΣM
a & d	14.8 x12/2 = 89 k'	$2.7x30^2/24$ = 101 k'	190 k'
b & c	29.7x12/2 = 178 k'	0	178 k'
Column axial force (n = # of stories)			
Column	$P_{lateral} = M_o/B$	$P_{gravity} = n\,w\,L_{tributary}$	Σ P
a & d	4,010 / 90 = 45 k	8x2.7x15 = 324 k	369 k
b & c	0	8x2.7x30 = 648 k	648 k
Column axial force + bending (ΣP = P+M B_x, estimate B_x than verify)			
Column	P	M B_x (convert M to k")	Σ P
a & d	365 k	12"x190 k'x0.185 = 422 k	787 k
b & c	648 k	12"x178 k'x0.185 = 395 k	1043 k
Design column (assume KL = 1.2 x12 = 14')			
Column	Use	Check $P_{allowable}$ vs. P	Check $B_{x\ estimate}$ vs. B_x
a & d	W14x109	803 > 785, OK	0.185 = 0.185, OK
b & c	W14x145	1090 > 1043, OK	0.185 > 0.184, OK

4th floor

Column shear			
Column	$V_c = L_{tributary} V / B$		V_c
a & d	15x42/90		7 k
b & c	30x42/90		14 k
Column bending			
Column	$M_{lateral} = V_c h/2$	$M_{gravity} = wL^2/24$	Σ M
a & d	7 x12/2 = 42 k'	$2.7x30^2/24$ = 101 k'	143 k'
b & c	14x12/2 = 84 k'	0	84 k'
Column axial force (n = # of stories)			
Column	$P_{lateral} = M_o/B$	$P_{gravity} = nwL_{tributary}$	Σ P
a & d	873/90 = 10 k	4x2.7x15 = 162 k	172 k
b & c	0	4x2.7x30 = 324 k	324 k
Column axial force + bending (ΣP = P+M B_x, estimate B_x than verify)			
Column	P	M B_x (convert M to k")	Σ P
a & d	172 k	12"x143 k'x0.196 = 336 k	508 k
b & c	324 k	12"x84 k'x0.196 = 198 k	522 k
Design column (assume KL = 1.2 x12 = 14')			
Column	Use	Check $P_{allowable}$ vs. P	Check $B_{x\ estimate}$ vs. B_x
a & d	W14x82	515 > 508, OK	0.196 = 0.196, OK
b & c	W14x90	664 > 522, OK	0.196 > 0.185, OK







Braced frame

Eight story braced frame: high strength steel, F_y = 50 ksi $F_a = F_b$ = 30 ksi

Loads		
Gravity load	Column	Beam
DL	70 psf	70 psf
LL	50x0.4 = 20 psf	50x0.6 = 30 psf
Total DL+LL	90 psf	100 psf
Average wind pressure		P = 30 psf

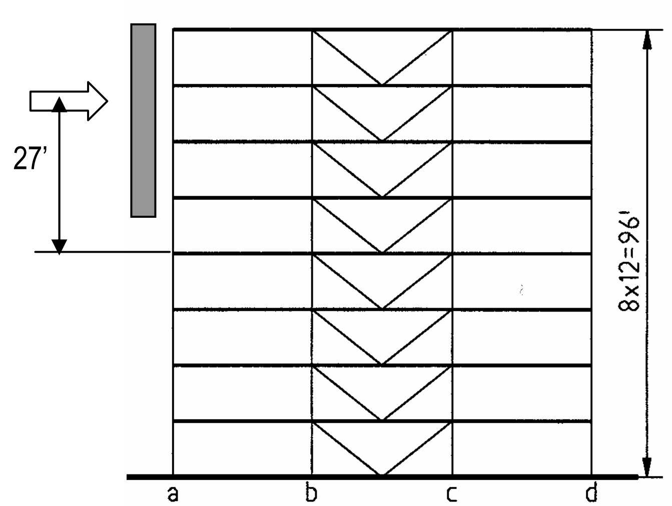
Beam load
w = 100 psf x 30'/1000 w = 3 klf
Column load (per foot on beam)
w = 90 psf x 30'/1000 w = 2.7 klf
Base shear
V = 30 psf x 45 x 90'/1000 V = 122 k
Overturn moment
Lever arm (to floor level for braced frames)
L = (8x12'-6')/2+6' L = 51'
M_0 = V L = 122 k x51' M_0 = 6,222 k'

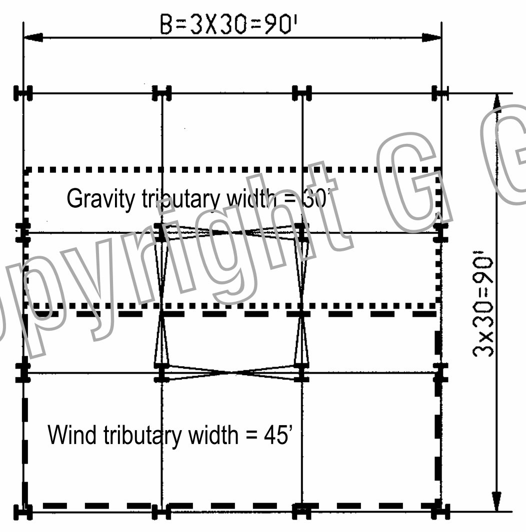
Column and brace axial forces			
Column	$P_{lateral} = M_0/30'$	$P_{gravity} = n\ w\ A_{tributary}$	Σ P
a & d	P = 0	P = 8x2.7x15 = 324 k	324 k
b & c	P = 6,222 k'/30' = 207 k	P = 8x2.7x30 = 648 k	855 k
Brace	See vectors (tension & compression, design for compression)		78 k

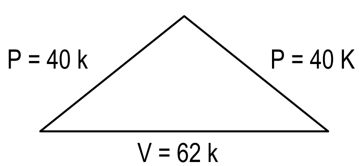
Column and brace design (K = 1 for pin joints)				
Column	Force P	KL length (K=1)	Use	$P_{allowable}$ vs. P
a & d	324 k	12'	W14x61	410 > 324
b & c	855 k	12'	W14x120	919 > 855
Brace	78 k	$L = (12^2+15^2)^{1/2} = 20'$	TS6x6x5/16	93 > 78

Beam *aldesign
Bending moment
$M = wL^2/8 = 3\text{ klf} \times 30^2/8$ M = 338 k'
Section Modulus
$S = M/F_b$ = 12" x 338 k' / 30 ksi $S = 135\text{ in}^3$
Use W18x76 146 > 135
Deflection $\Delta = (5/384)\ W\ L^3/(E\ I)$
$\Delta = (5/384)(3x30)(30x12)^3/(30{,}000x1330)$ Δ = 1.37"
Allowable Δ = L/240 = (30'x12")/240 = 1.5 1.5 > 1.37, ok

Note: ignore brace beam support of inner beam since lateral load may act in addition to gravity load







4^{th} floor design

4^{th} floor shear

V = 33 psf x 45 x 3.5x12 / 1000 V = 62 k

Overturn moment

Lever arm (to floor level for braced frames)

L = 3.5x12'/2+ 6' L = 27'

M_4 = V L = 62 k x27' M_4 = 1,674 k'

Column and brace axial forces			
Column	$P_{lateral} = M_0/30'$	$P_{gravity}$ = n w $A_{tributary}$	Σ P
a & d	P = 0	P = 4x2.7x15 = 162 k	162 k
b & c	P = 1,674 k'/30' = 56 k	P = 4x2.7x30 = 324 k	380 k
Brace	See vectors (tension & compression, design for compression)		40 k

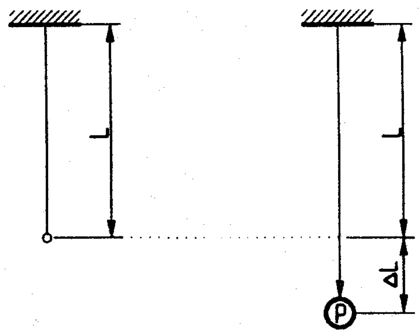
Column and brace design (K = 1 for pin joints)				
Column	Force P	KL length (K=1)	Use	$P_{allowable}$ vs. P
a & d	162 k	12'	W8x31	189 > 162
b & c	380 k	12'	W14x61	410 > 380
Brace	40 k	L = $(12^2+15^2)^{1/2}$ = 20'	TS6x6x3/16	60 > 40

Compare material

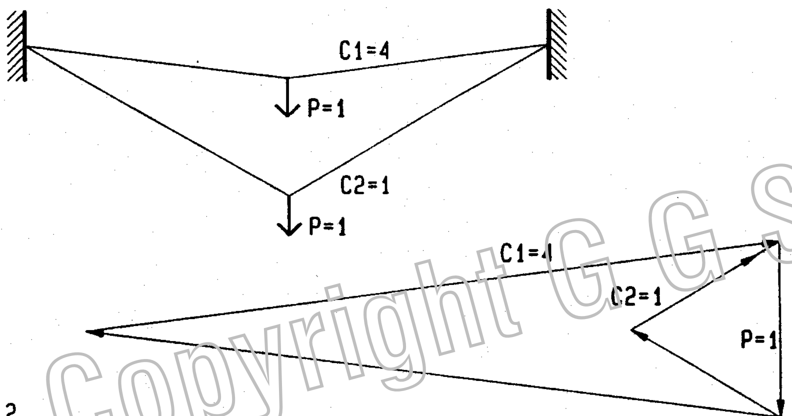
The amount of steel required per square foot (m^2) is used to compare framing systems. The steel at mid height provides a quick average weight to compare, assuming all bays of approximately the same size provides the following comparative results.

Moment frame				
Member	Weight / ft	Length each	Total length	Weight
8 columns W14x82	82 plf	12'	96'	7,872 #
8 columns W14x90	90 plf	12'	96'	8,640 #
24 beams W18x86	86 plf	30'	720'	61,920 #
18 joists W18x35	35 plf	30'	540'	18,900 #
Total				97,332 #
Total per square foot	97,332 / (90x90)			12 psf

Braced frame				
Member	Weight / ft	Length each	Total length	Weight
8 columns W8x31	31 plf	12'	96'	2,976 #
8 columns W14x61	61 plf	12'	96'	5,856 #
24 beams W18x76	76 plf	30'	720'	54,720 #
18 joists W18x35	35 plf	30'	540'	18,900 #
4 braces TS6x6x3/16	14.53 plf	20'	80'	1,162 #
Total				83,614 #
Total per square foot	83,614 / (90x90)			10.3 psf



1



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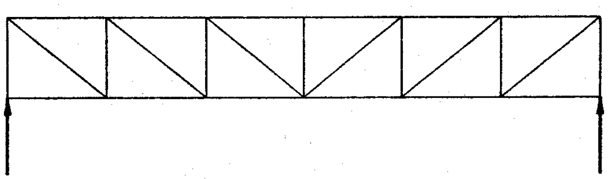


3

Test models

Static models are useful to test structures for strength, stiffness, and stability. They may have axial resistance (truss), bending resistance (beam), or both axial and bending resistance (moment frame). Static models have three scales: geometric scale, force scale, and strain scale. The geometric scale relates model dimensions to original dimensions, such as 1:100. The force scale relates model forces to the original structure. For a force scale of 1:100, one pound in the model implies 100 pounds in the original structure. The force scale should be chosen to keep model forces reasonable (usually less than 50 pounds). The strain scale relates model strain (deformation) to strain in the original structure. A strain scale of 1:1 implies model strain relates to original strain in the geometric scale; given a geometric scale of 1:10 a model strain of 1 inch implies 10 inch original strain. For structures with small deformations may a strain scale of 5:1, for example, helps to visualize strain. However, structures with large strain like membranes require a strain scale of 1:1 to avoid errors (see 2). Scales are defined as:

Geometric Scale: $S_G = L_m/L_o$ = model dimension / original dimension

Force Scale: $S_F = P_m/P_o$ = model force / original force

Strain Scale: $S_S = \varepsilon_m/\varepsilon_m$ = model strain / original strain

The derivation for axial and bending resistance models assumes:

- A = Cross-section area
- E = Modulus of elasticity
- I = Moment of inertia
- k = Constant of integration for deflection; for cantilever beams with point load $\Delta = kPL^3 / (EI)$ where k = 1/3
- m = Subscript for model
- o = Subscript for original structure

Axial resistance

Unit Strain $\varepsilon = \Delta L/L$ $\Delta L = P\,L / (AE)$ hence

Force $P = A\,E\,\Delta L/L = A\,E\,\varepsilon$ hence

Force Scale = $S_F = P_m/P_o = A_mE_m / (A_oE_o)\ \varepsilon_m/\varepsilon_o$ since $\varepsilon_m/\varepsilon_o = S_S$

$S_F = A_mE_m/(A_oE_o)\ S_S$

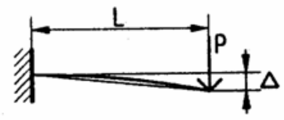
$S_F = A_mE_m/(A_oE_o)$ if $S_S = 1$

$S_F = A_m/A_o = S_G^2$ if $E_m=E_o$

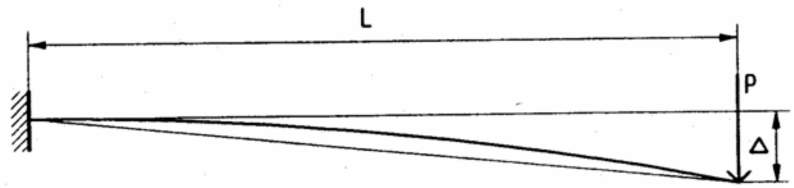
1 Axial strain $\Delta L = P\,L/(AE)$; unit strain $\varepsilon = \Delta L/L$

2 Structures with large deformations, such as membranes, yield errors if the strain scale S_S is not 1:1; as demonstrated in the force polygon

3 Structures like trusses, with small deformations, may require a strain scale S_S >1 to better visualize deformations.

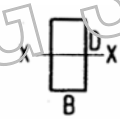


1

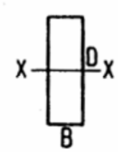


2

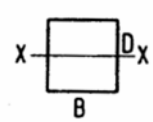
Case A: Assuming B = 1 and D = 2
A = 1X2 = 2
I = 1x2³/12 = 0.66



Case B: Assuming B = 1 and D = 4
A = 1X4 = 4 = 2 x case A
I = 1x4³/12 = 5.33 (=8x0.66) = 8 x case A



Case C: Assuming B = 2 and D = 2
A = 2X2 = 4 = 2 x case A
I = 2x2³/12 = 1.33 (=2x0.66) = 2 x case A



3

Bending resistance

Unit Strain $\varepsilon = \Delta/L$ $\Delta = kPL^3 / (EI)$ hence

Force $P = EI\Delta / (kL^3) = EI / (kL^2)\ \Delta/L$ hence

Force Scale $S_F = P_m/P_o = [E_mI_m / (E_oI_o)]\ k_o/k_m\ L_o^2/L_m^2\ \varepsilon_m/\varepsilon_o$

Since the model and original have the same load and support conditions the constants of integration $k_m = k_o$, hence the term k_o/k_m may be eliminated. The term $L_o^2/L_m^2 = 1/S_G^2$ = 1/geometric scale squared, and $\varepsilon_m/\varepsilon_o = S_S$ = strain scale. Therefore the force scale is:

$S_F = E_mI_m/(E_oI_o)\ 1/S_G^2\ S_S$

$S_F = E_mI_m/(E_oI_o)\ 1/S_G^2$ If $S_S = 1$

$S_F = I_m/I_o\ 1/S_G^2$ If $E_m = E_o$, or simplified

$S_F = S_G^2$ assuming all model dimensions, including details, relate to the original in the geometric scale

In the simplest form the force scale is equal to the geometric scale squared for both axial and bending resistant models. Thus a model with a geometric scale of 1:100 has a force scale of 1:10,000 if it is made of the same material or modulus of elasticity as the original structure.

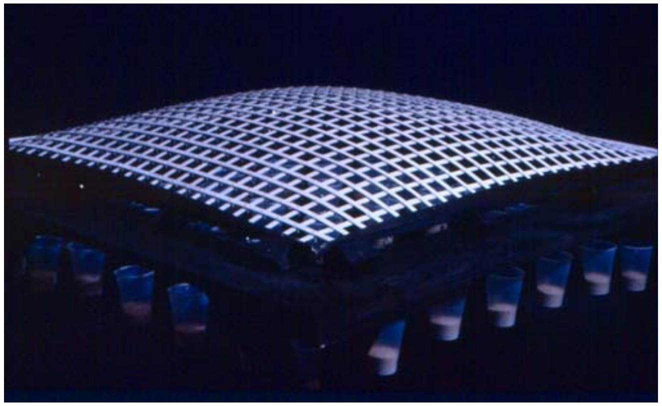
Combined axial and bending resistance

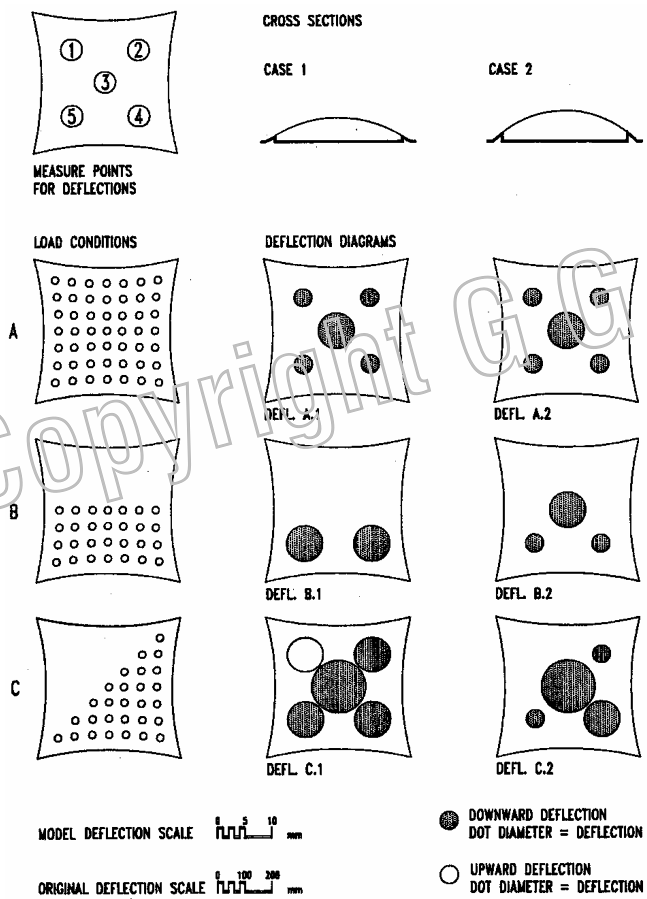
Models with both axial and bending resistance, such as moment frames, should be of the same material or elastic modulus as the original in order to avoid errors. Referring to diagrams 3, if, for example, the elastic modulus of a model is half as much as in the original structure and the cross-section area is doubled to compensate for it, then the moment of inertia is four times greater, assuming area increase is perpendicular to the bending axis. For small adjustments this can be avoided by increasing the area parallel to the bending axis. Large differences in stiffness, such as wood simulating steel, with an elastic modulus about 20 times greater, are not possible. In such a case the strain scale could be 20:1 to amplify deflection rather than adjusting the cross-section area.

1 Model strain $\varepsilon = \Delta/L$ must be equal to the original strain.
$\Delta = k\ PL^3/EI$ where $k = 1/3$

2 Original strain $\varepsilon = \Delta/L$
$\Delta = k\ PL^3/EI$ where $k=1/3$
Since k is the same in the model as in the original, for equal load and support conditions, it may be eliminated from the force scale equation

3 Correlation between cross-section area A and moment of inertia I demonstrates incompatibility between A and I since they increase at different rates, unless the increase is only in width direction





Test stand

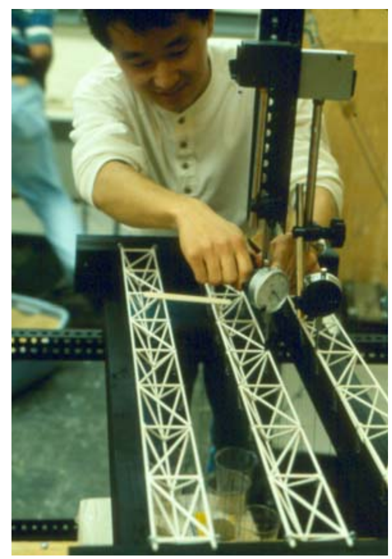
- Light gauge steel frame 3'x5'
- Frame to support test models
- Adjustable platform for loads below the test model (crank mechanism lowers platform to apply load)
- Blocking deice holds load platform at any position
- Support frame for measure gauges above test model

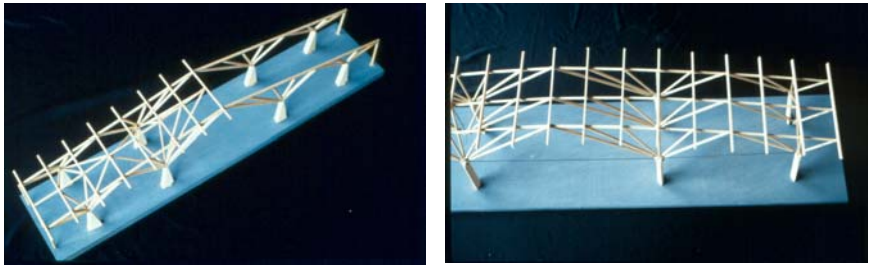
Test procedure

- Position model with open base to allow loads below
- Suspend load cups filled with lead or sand from model (support loads on load platform before loading)
- Attach measure gauges above model
- Lower load platform with crank to apply load
- Measure deformations and stress
- Apply alternate loads (half load, etc.)
- Record deformations and stresses for all load conditions

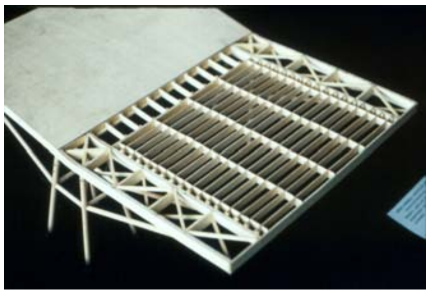
Note

- Apply loads briefly to avoid creep deformation
- Apply loads gradually to avoid rupture
- Test all load conditions that may cause critical deformation or stress
- Adjust design if deformation or stress exceeds acceptable limits

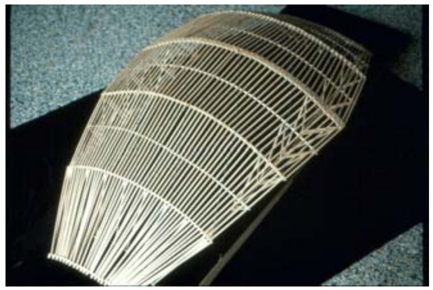




Tree structure wood design model

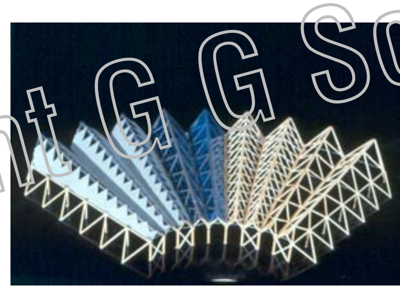


Wood cantilever roof



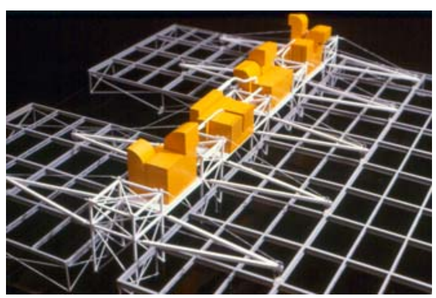
Wood grid shell

Arch / grid shell



Folded truss

Prismatic truss



Cable stayed roof

For most material the E modulus may be defined, applying load on a cantilever and computing E as:

$E = PL^3/(I\Delta)$

For fabric the E modulus is defined, applying axial load on a 5 inch fabric strip:

$E = PL/(A\Delta)$

- E = Elastic modulus in lbs/in^2 (lbs/linear in for fabric)
- Δ = deformation
- A = width of fabric strip
- P = point load
- L = length of cantilever or fabric strip
- I = moment of inertia

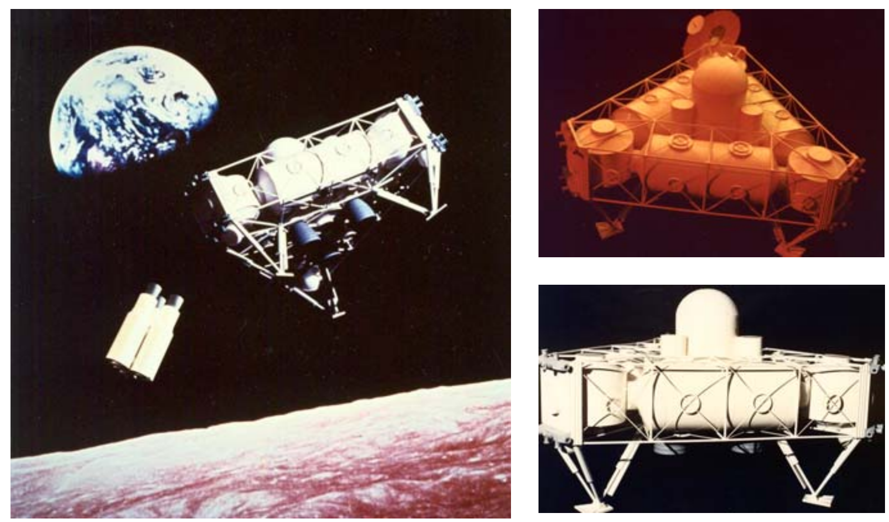
Note:

L should be as long as possible for accuracy

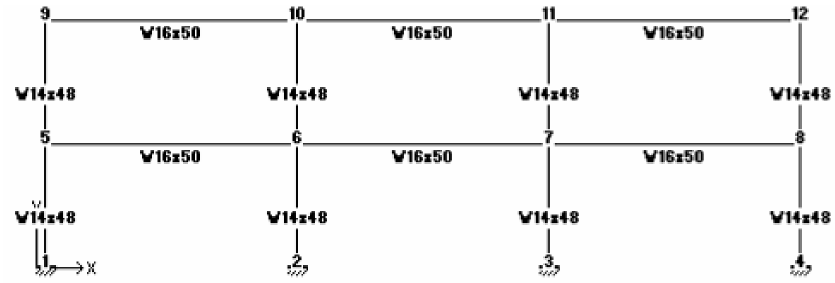
Use average E of several tests

Master thesis by Madhu Thangavelu

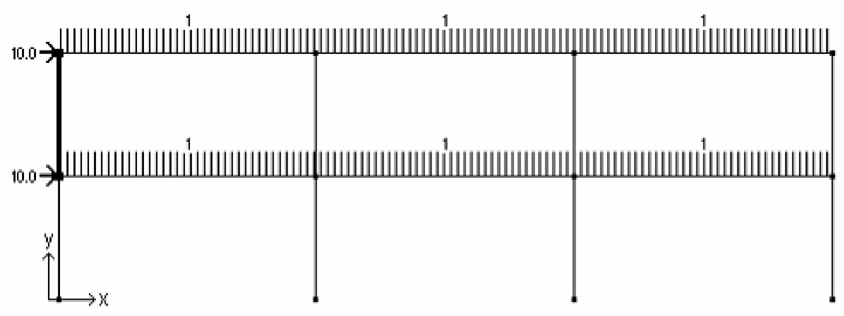
MALEO: Modular Assembly in Low Earth Orbit, to avoid assembly of lunar station by costly remote control robotics. Light-weight cable truss of stable triangular configuration supports three fuel tank modules for habitation, research, power and control.



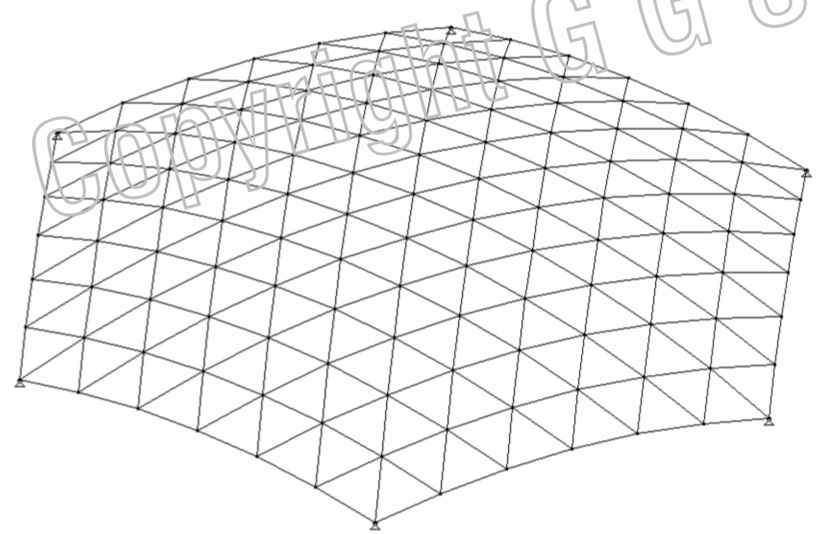
MALEO: Model



Moment frame (with joint numbers and member names)



Load diagram (uniform beam load, lateral point load)



Hexagonal grid shell dome

Computer aided design

Advance in computer technology made structural design and analysis widely available. The theory and algorithm of structural design programs is beyond the scope of this book. However, a brief introduction clarifies their potential and use.

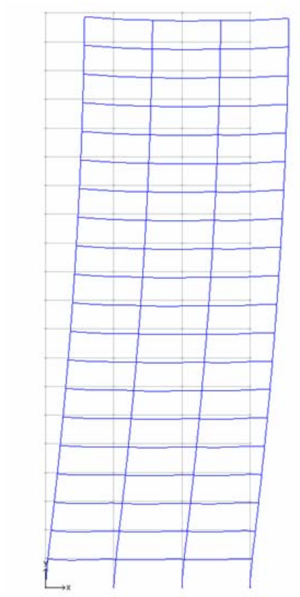
Structure programs generate and solve a stiffness matrix of the structure. Based on the degree of freedom of joints, the output provides stress and strain. A two-dimensional truss with pin joints has two degrees of freedom and thus two unknowns per joint, X and Y-displacement. A three-d truss has three unknowns per joint. Two-D frames have four unknowns, X, Y-displacement and X, Y-rotation, but three-D frames have six unknowns per joint, X, Y, Z-displacement and X, Y, Z-rotation.

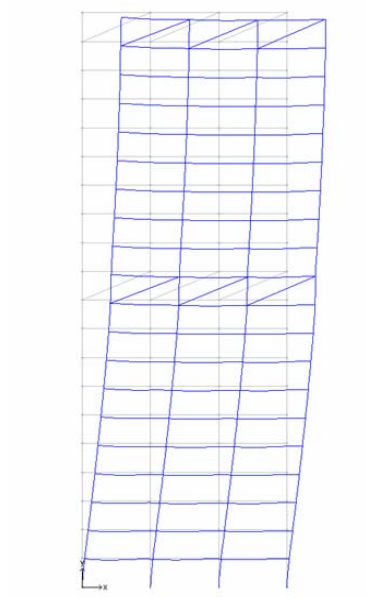
The structure input is defined by joints, members connecting the joints and loads. Joints of three-d structures are defined by X, Y, Z-coordinates, joint type (pin or moment joint), and degree of freedom, regarding X, Y Z-displacement and X, Y, Z-rotation (joints attached to the ground are fixed with pin or moment joints). Members are defined by properties, cross section area, moment of inertia, and modulus of elasticity. Some members may have *end release* at one or both ends, to allow pin joints of braces to connect to moment joints of beam to column, for example. End releases are simulated by a dummy pin adjacent to the moment joint. The geometry of a structure may be defined in the analysis program or imported as DFX file from a CAD program. Loads are defined as distributed or point load. Gravity load is usually assigned as uniform beam load, yet lateral wind or seismic loads are usually assigned as point loads at each level.

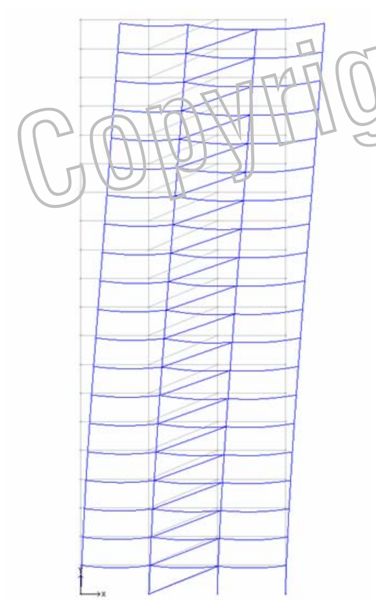
Output includes force, stress, and deformation for members, joint displacement and rotation, as well as support reactions. Output may be in tables and / or graphic display. Graphic display provides better intuitive understanding and is more convenient to use.

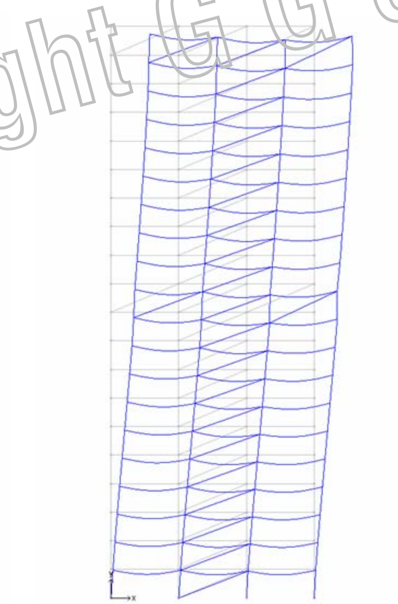
Some programs simulate non-linear material behavior and / or non-linear geometric behavior. For example, non-linear material may include plastic design of steel with non-linear stress/strain relation in the plastic range. Non-linear geometric analysis is for structures with large displacements, such as cable or membrane structures. Non-linear analysis usually involves an iterative algorithm that converges after several iterations to a desired level of accuracy. Some programs include a *prestress* element to provide form-finding for membranes structures. Some programs provide dynamic analysis, sometimes referred too as 4-D analysis. Programs with advanced features provide greater versatility and accuracy, but they are usually more complex to use.

Multiframe-4D used for the demonstrations features 2-d and 3-d static and 4-d dynamic analysis. For static analysis Multiframe is very user friendly, intuitive, and thus good for architecture students. The 4-d dynamic feature is beyond the scope of this book. The examples presented demonstrate 2-d and 3-d design/analysis. A very convenient feature are tables of steel sections with pre-defined properties for US sections and for several other countries. The program features US and SI units.









Belt truss effect

CAD-analysis provides efficient means to compare framing systems. For convenience the following example was done with constant W18 beams and W14 columns, 30' beam spans and 12' story heights. The results, comparing the effect of belt and top trusses on a moment frame and a braced frame are very reveling:

20-story moment frame

Gravity load	w = 3 klf
Wind load	P = 10 k / level

Frame:	Drift
Frame only	15.1"
Top truss	14.9"
Belt truss	14.2"
Top and belt truss	14.0"

20-story braced frame

Gravity load	w = 3 klf
Wind load	P = 10 k / level

Frame	Drift
Frame only	17.6"
Top truss	11.4"
Belt truss	11.1"
Top and belt truss	8.6"

Note:

Belt and top trusses are much more effective to reduce drift at the braced frame than at the moment frame. The combined belt and top trusses reduce drift:

- 7 % at moment frame
- 49 % at braced frame

Interpreting the results clarifies the stark difference and fosters intuitive understanding of different deformation modes of moment and braced frames.

PART IV

HORIZONTAL SYSTEMS

Part III presents structure systems, divided into two categories: horizontal, and vertical/lateral. Horizontal systems include floor- and roof framing systems that support gravity dead- and live load and transfer it to vertical supports, such as walls and columns. As the name implies, vertical/lateral systems include walls, columns and various other framing systems that resist gravity load as well as lateral wind- and seismic load.

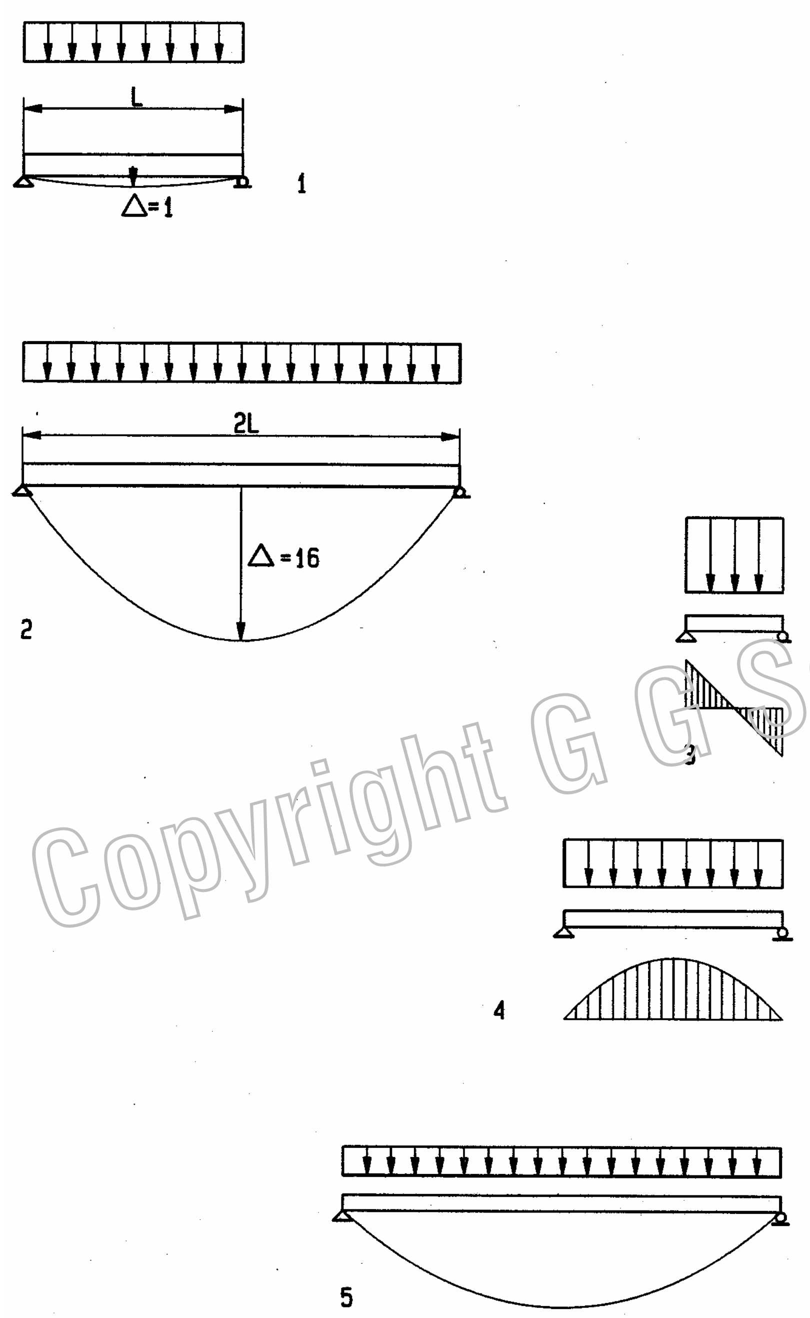
In the interest of a structured presentation, both, horizontal and vertical/lateral systems are further classified by type of resistance controlling the design. This also helps to structure the creative design process. Though many actual systems may include several modes of resistance, the controlling resistance is assumed for the classification. For example, cable stayed systems usually include bending elements like beams, in addition to cables or other tension members. However, at least at the conceptual level, their designed is controlled more by tension members than by bending. Therefore they are classified as tensile structures. Horizontal systems are presented in four chapters for structures controlled by bending, axial, form and tensile resistance. Vertical/lateral systems are presented in three chapters for structures controlled by shear-, bending-, and axial resistance.

11

HORIZONTAL SYSTEMS

Bending Resistant

Bending resistant systems include joist, beam, girder, as well as Vierendeel frame and girder, folded plate and cylindrical shell. They carry gravity load primarily in bending to a support structure. Shear is typically concurrent with bending; yet bending usually controls the design. Though bending resistant elements and systems are very common, they tend to be less efficient than other systems, because bending varies from maximum compression to maximum tension on opposite faces, with zero stress at the neutral axis. Hence only half the cross-section is actually used to full capacity. Yet, this disadvantage is often compensated by the fact that most bending members are simple extrusions, but trusses are assembled from many parts with costly connections. Like any structure system, bending elements are cost effective within a certain span range, usually up to a maximum of 120ft (40m). For longer spans the extra cost of more complex systems is justified by greater efficiency.



Bending Concepts

Some concepts are important for an intuitive understanding of bending members and their efficient design. They include the effects of span and overhang, presented in this section. Other concepts, such as optimization and the *Gerber* beam, are included in the following section.

Effect of span

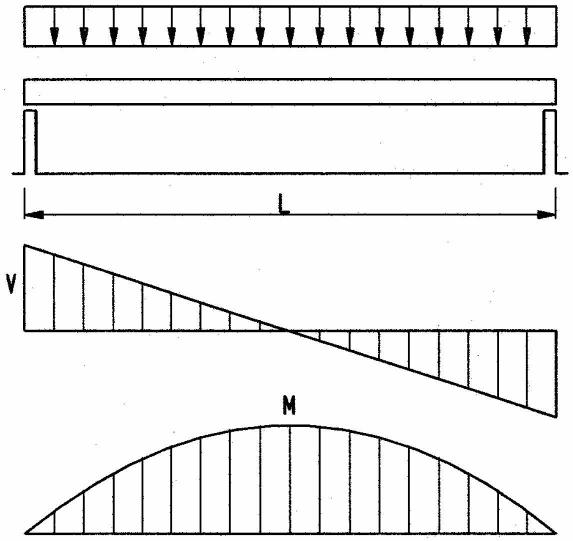
The effect of the span L for bending members may be demonstrated in the formulas for deflection, bending moment and shear for the example of a simple beam under uniform load.

$\Delta = (5/384)\ wL^4/\ (EI)$
$M = wL^2/8$
$V = wL/2$
where

Δ= Maximum deflection
E= Elastic modulus
I = Moment of Inertia
L= Length of span
M= maximum bending moment
V= maximum shear force
w= Uniform load per unit length

The formulas demonstrate deflection increases with the 4th power of span, the bending moment increases with the second power, and shear increases linearly. Although this example is for a simple beam, the same principle applies to other bending members as well. For a beam of constant cross-section, if the span is doubled deflection increases 16 times, the bending four times, but shear would only double. Thus, for long bending members deflection usually governs; for medium span bending governs, yet for very short ones, shear governs

1 Beam with deflection $\Delta = 1$
2 Beam of double span with deflection $\Delta = 16$
3 Short beam: shear governs
4 Medium-span beam: bending governs
5 Long-span beam: deflection governs



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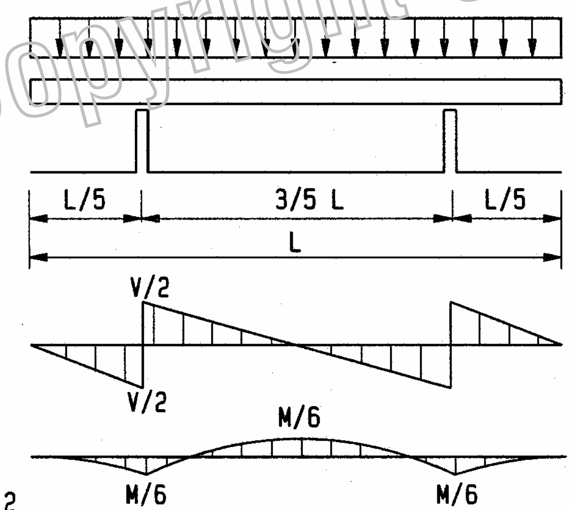

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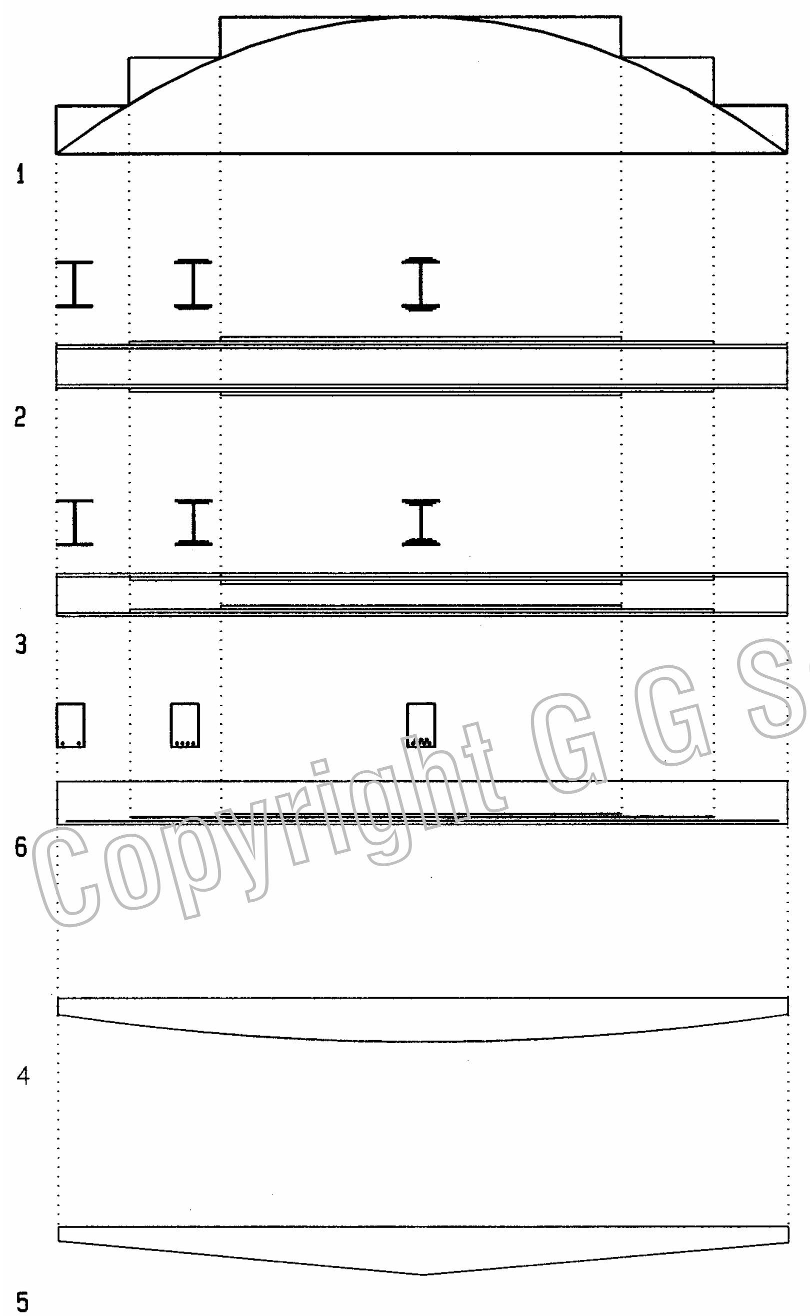
Effect of overhang

Bending moments can be greatly reduced, using the effect of overhangs. This can be describe on the example of a beam but applies also to other bending members of horizontal, span subject to gravity load as well. For a beam subject to uniform load with two overhangs, a ratio of overhangs to mid-span of 1:2.8 (or about 1/3) is optimal, with equal positive and negative bending moments. This implies an efficient use of material because if the beam has a constant size – which is most common – the beam is used to full capacity on both, overhang and span. Compared to the same beam with supports at both ends, the bending moment in a beam with two overhangs is about one sixth ! To a lesser degree, a single overhang has a similar effect. Thus, taking advantage of overhangs in a design may result in great savings and economy of resources.

1 Simple beam with end supports and uniform load

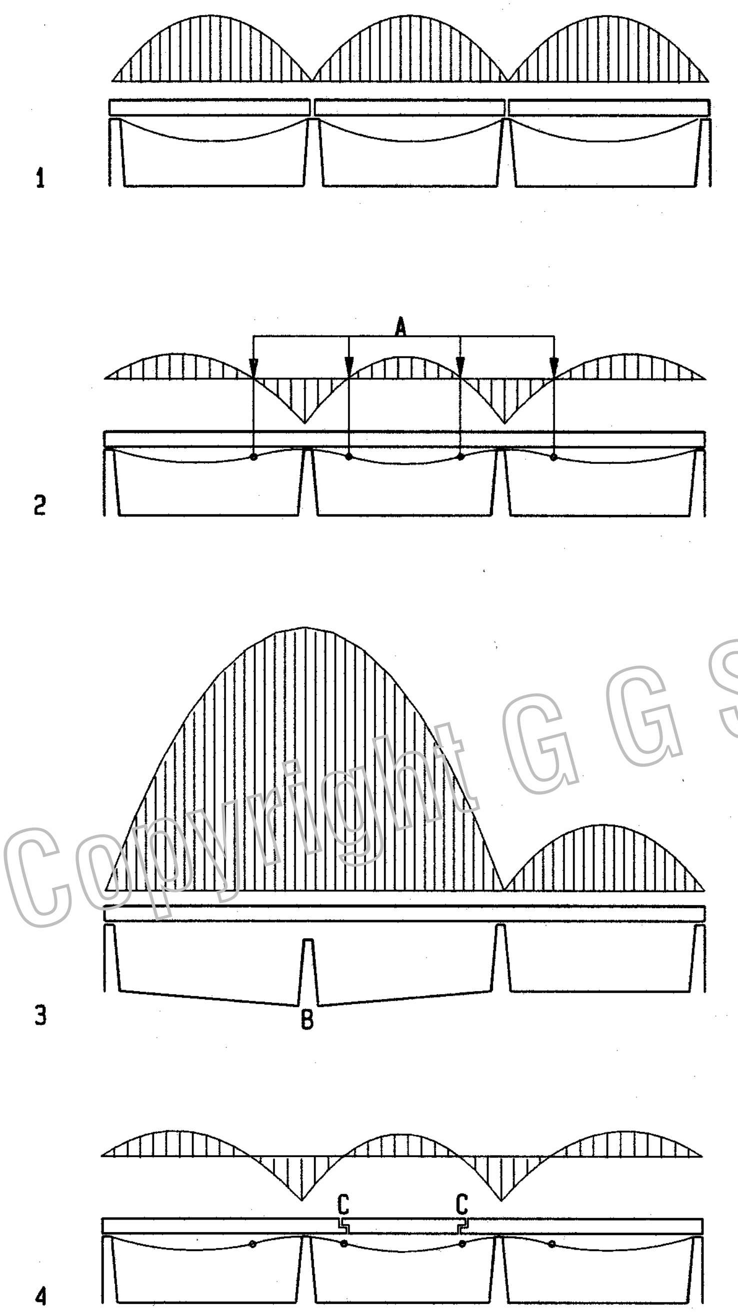
2 Cantilevers of about 1/3 the span equalize positive and negative bending moments and reduces them to about one sixth, compared to a beam of equal length and load with but with simple end support



Bam optimization

Optimizing long-span girders can save scares resources. The following are a few conceptual options to optimize girders. Optimization for a real project requires careful evaluation of alternate options, considering interdisciplinary aspects along with purely structural ones.

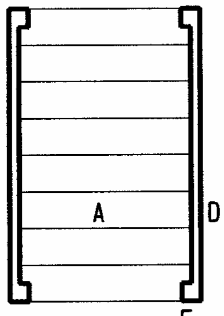
1 Moment diagram, stepped to reflect required resistance along girder
2 Steel girder with plates welded on top of flanges for increased resistance
3 Steel girder with plates welded below flanges for increased resistance
4 Reinforced concrete girder with reinforcing bars staggered as required
5 Girder of parabolic shape, following the bending moment distribution
1 Girder of tapered shape, approximating bending moment distribution



Gerber beam

The *Gerber* beam is named after its inventor, Gerber, a German engineering professor at Munich. The Gerber beam has hinges at inflection points to reduce bending moments, takes advantage of continuity, and allows settlements without secondary stresses. The Gerber beam was developed in response to failures, caused by unequal foundation settlements in 19th century railroad bridges.

1 Simple beams over three spans
2 Reduced bending moment in continuous beam
3 Failure of continuous beam due to unequal foundation settlement, causing the span to double and the moment to increase four times
4 Gerber beam with hinges at inflection points minimizes bending moments and avoids failure due to unequal settlement



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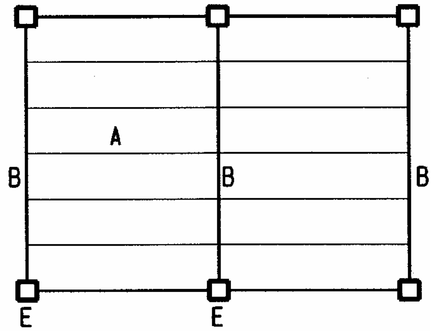

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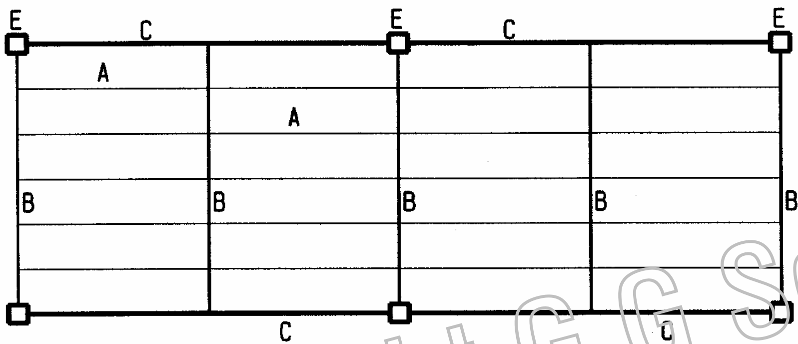

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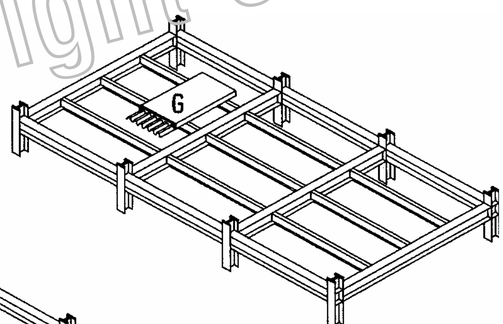

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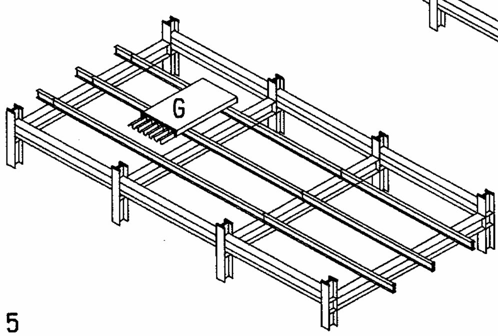


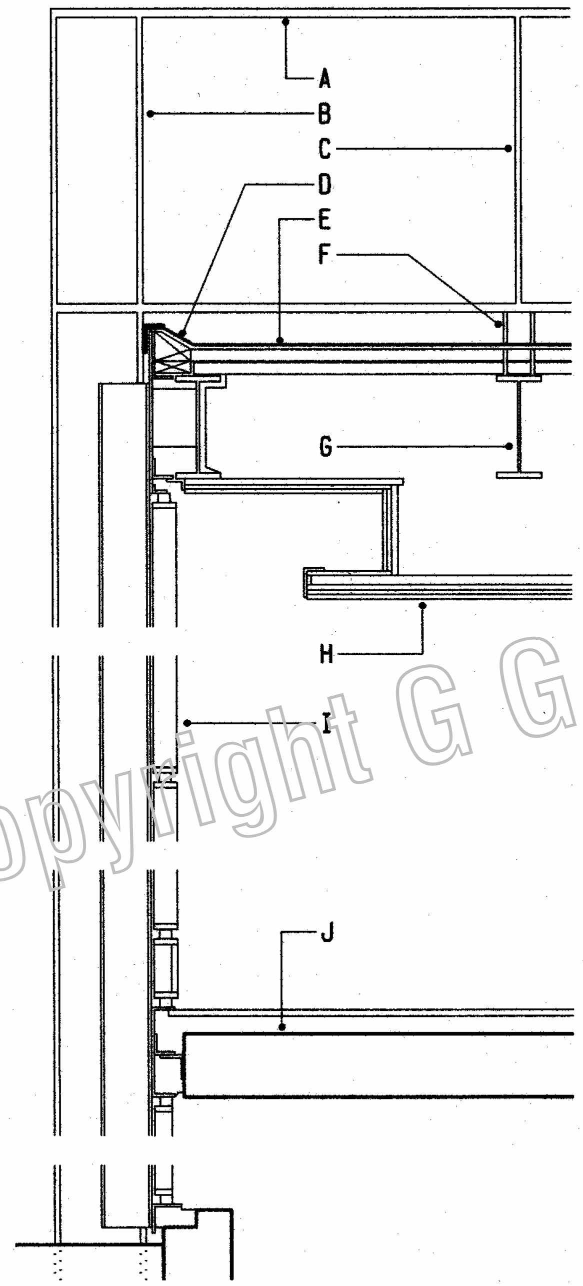
5

Joist, Beam, Girder

Joists, beams, and girders can be arranged in three different configurations: joists supported by columns or walls[1]; joists supported by beams that are supported by columns[2]; and joists supported by beams that are supported by girders that are supported by columns[3]. The relationship between joist, beam, and girder can be either flush or layered framing. Flush framing, with top of joists, beams, and girders flush with each other, requires less structural depth but may require additional depth for mechanical systems. Layered framing allows the integration of mechanical systems; with main ducts running between beams and secondary ducts between joists. Further, flush framing for steel requires more complex joining, with joists welded or bolted into the side of beams to support gravity load. Layered framing with joists on top of beams with simple connection to prevent displacement only

2 Single layer framing: joists supported directly by walls
3 Double layer framing: joists supported by beams and beams by columns
4 Triple layer framing: joists supported by beams, beams by girders, and girders by columns
5 Flush framing: top of joists and beams line up
May require additional depth for mechanical ducts
6 Layered framing: joists rest on top of beams
Simpler and less costly framing
May have main ducts between beams, secondary ducts between joists

A Joists
B Beam
C Girders
D Wall
E Column
F Pilaster
G Concrete slab on corrugated steel deck



1

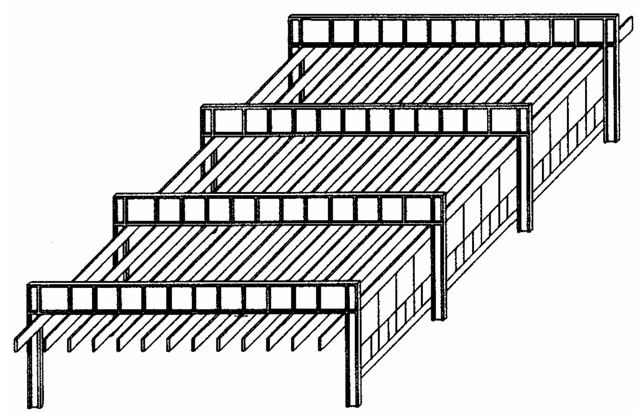
Crown Hall, IIT, Chicago (1956)
Architect: Mies van der Rohe

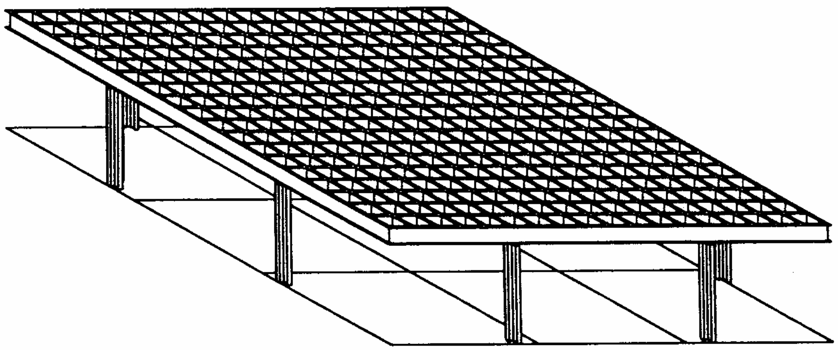
Crown Hall for architecture at the Illinois Institute of Technology, exemplifies Mies' architecture of universal space and structural expression, exposing girders and columns on the outside. His tectonic objectives of exposed girders above the roof reduces air conditioned due to less interior volume but also implies penalties: the girder top is not braced against buckling, and the roof is punctured at many suspension points for potential leaks. A column-free space of 120x220ft (37x67m) is spanned by four moment resistant steel portals of 14in (360mm) wide-flange columns and four ft (1.2m) deep plate girders that span the entire width. The portals, spaced 60ft (18m), support steel joists suspended from the girders on bracket hangers. The joists, spaced 10ft (3m) on center, overhang 20ft (6m) at end portals. To resist buckling, the girders have stiffener plates welded to the web at intervals of the joists. Besides stability, they give the girders a tectonic articulation.

1 Wall cross section
2 Structural diagram

A Top flange of girder
B Column
C Stiffener plate welded to girder web
D Cant-strip at roof edge
E Roofing membrane
F Suspension brackets
G Roof joist
H Ceiling
I Glass wall
J Concrete floor

2





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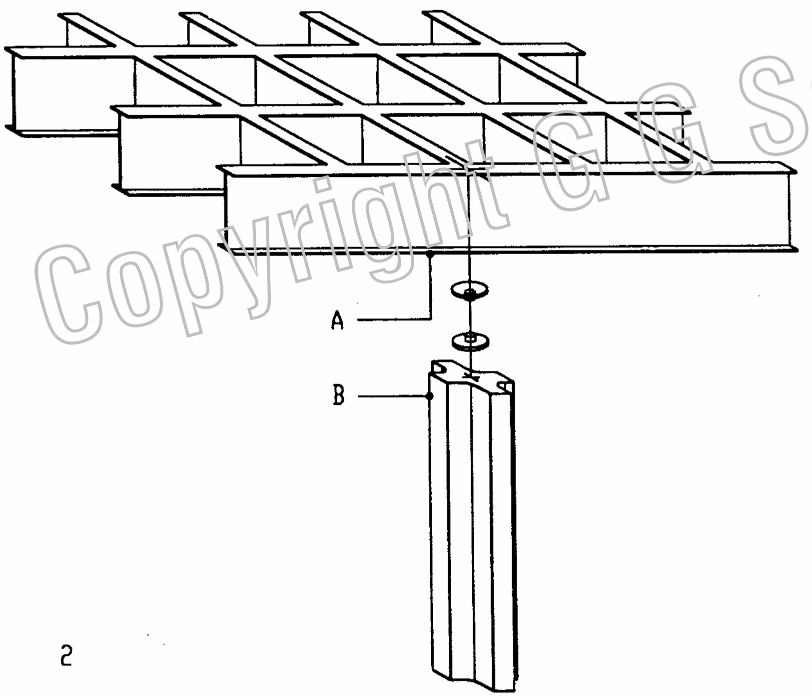


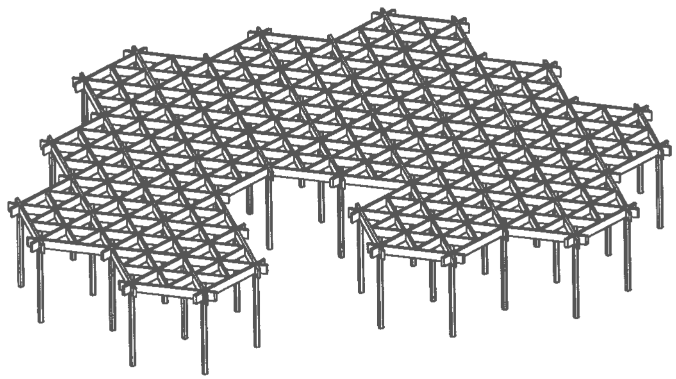
2

National Gallery, Berlin (1968)
Architect: Mies van der Rohe

The National Gallery was initially commissioned in 1962 for Berlin's twentieth century art collection. In 1965 it was merged with the National-galerie and renamed accordingly. A semi-subterranean podium structure of granite-paved concrete is the base for the main structure; a steel space-frame of 64.8m (212ft) square has a clear interior height 8.4m (28ft). At the roof edge eight cross-shaped steel columns with pin joint at the roof cantilever from the podium for lateral resistance. Based on a planning module of 1.2m, the unique space-frame consists of two-way steel shapes, 1.8m deep, spaced 3.6m on center in both directions. The shallow span/depth ratio oh 33 required the roof to have a camper to cancel deformation under gravity load. The entire roof was assembled on ground from factory pre-welded units and hydraulically lifted in place on a single day.

1 Steel roof framing concept
2 Steel roof framing detail

A Steel edge beam
B Cross-shaped steel column



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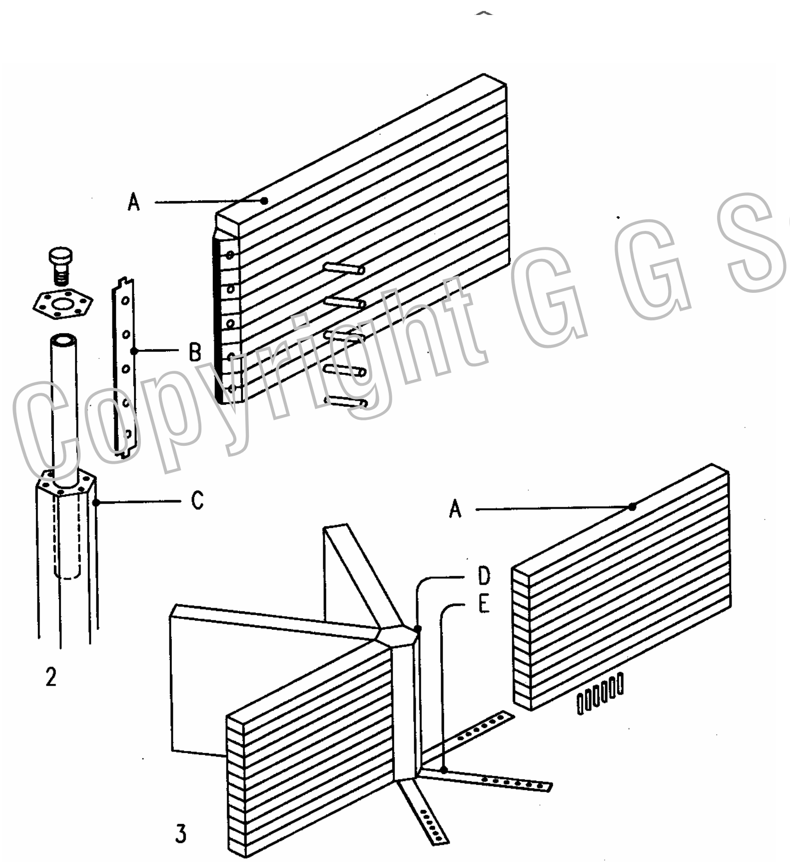


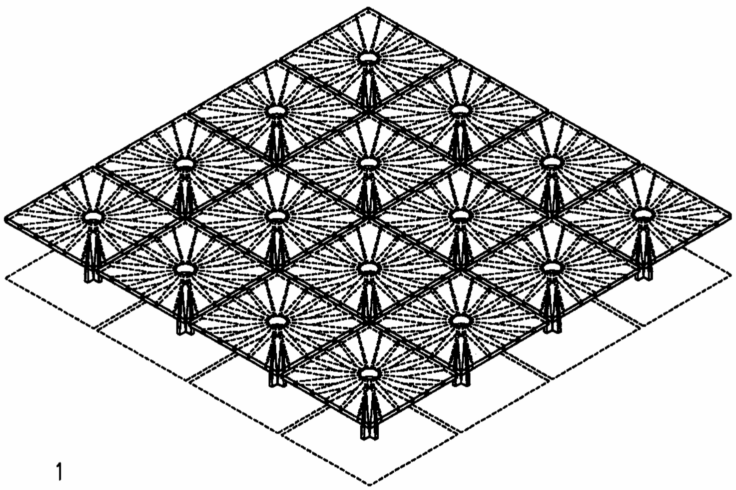
School in Gurtweil, Germany (1972)
Architect: H. Schaudt
Engineer: Ingenieurbüro für Holzbau

A grid of equilateral triangles is the base of this honeycomb of ten hexagonal classrooms. The side length of each regular hexagon is 5.4 m (18 ft). The composition of classrooms defines a free-form hall with entry from a central court. The sloping site provides space for a partial ground floor for auxiliary spaces below the classroom level.

Laminated beams spanning three ways presented a challenge to minimize the number of beam intersections. The continuous beams need moment connections. To this end, the main roof and floor structures have identical configurations but different support conditions. Six columns support each hexagonal classroom at the vertices. The classroom floors have an additional column at the center of each hexagon to support the cross point of three girders that span the six hexagon vertices. Those columns do not interfere with the auxiliary spaces below classrooms. Three beams span between the girders to form four triangular panels. Floor joists rest on the beams and support a particle board sub-floor with acoustical and thermal insulation. To provide uninterrupted classrooms, the roof structure has no column support within each hexagon. The column-free spaces required beams with moment connections. The roof deck consists of planks with tongue-and-groove. Diagonal steel rods, 24 mm (1 in) ϕ, with turnbuckles, brace some peripheral columns to resist lateral wind load.

1 Floor structure (roof is similar but without column at hexagon centers)
2 Column supporting center of floor hexagon
3 Moment resistant joint of roof beam at hexagon center without column

A Laminated girder, 12x60 cm (5x24")
B Steel insert bar with dowels ties beams to column
C Hexagonal laminated column, ϕ 21 cm (8")
D High-strength concrete core resists compression at top of roof beam
E Steel strap, 10x80 mm (3/8x3"), resists tension at bottom of roof beam
E Tension straps, 10x80mm, at bottom of beams



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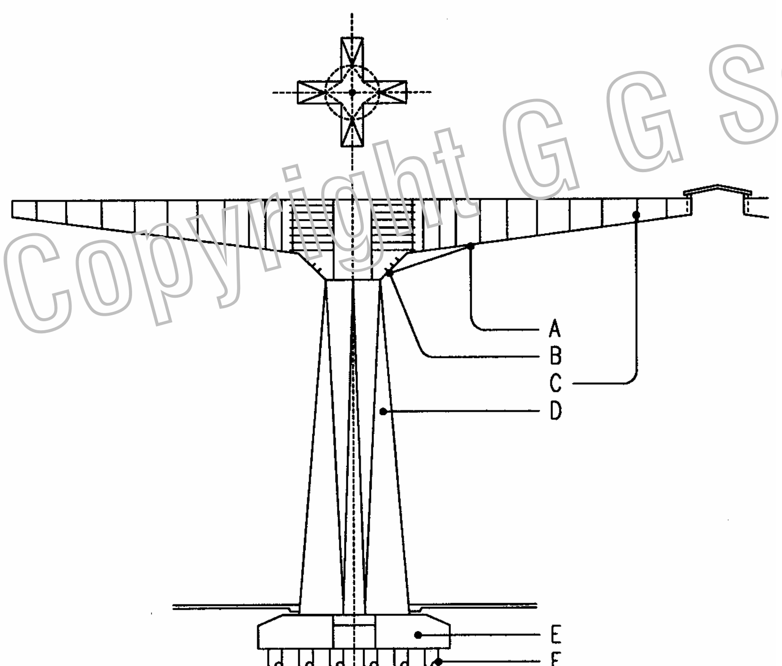
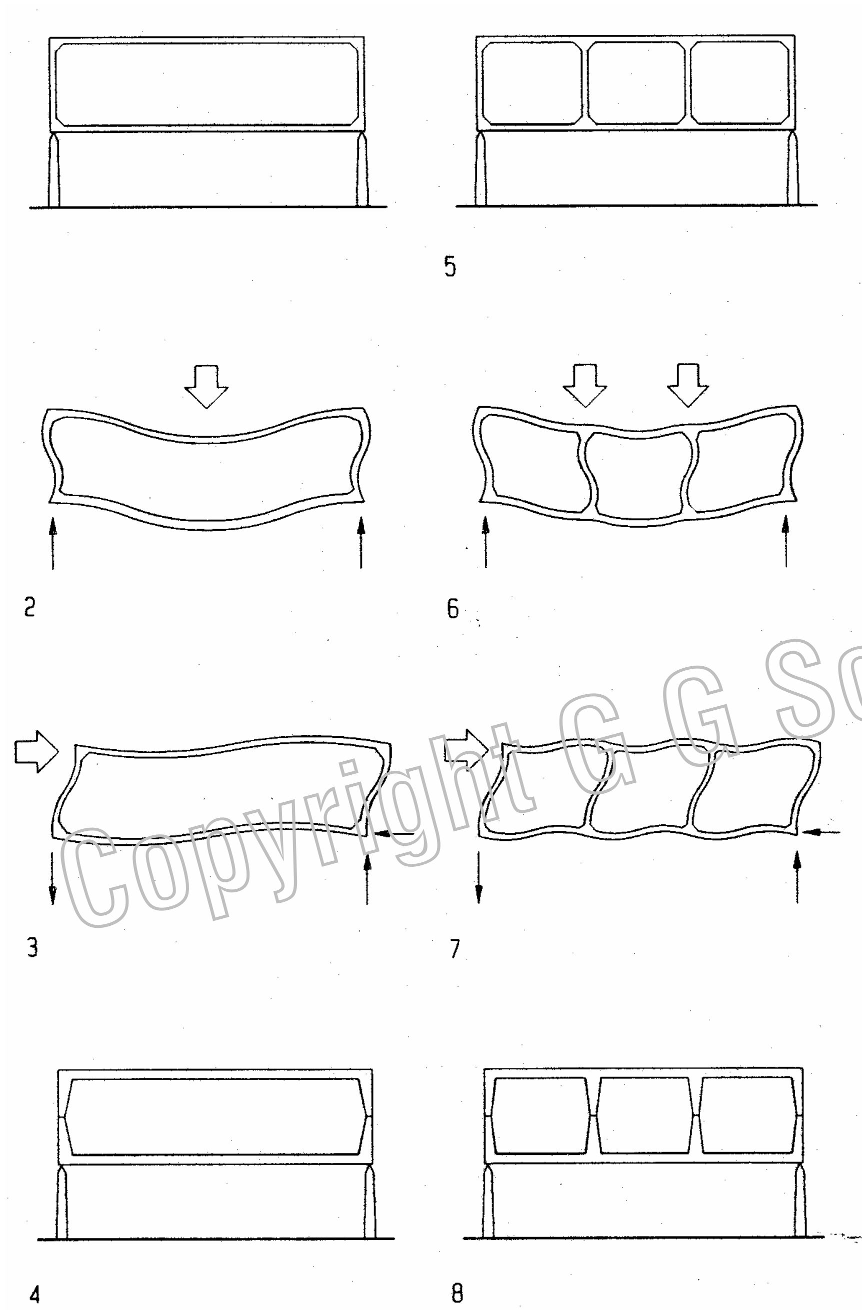


2

Labor Palace Turin (1961)

Architect/Engineer: Pierre Luigi Nervi

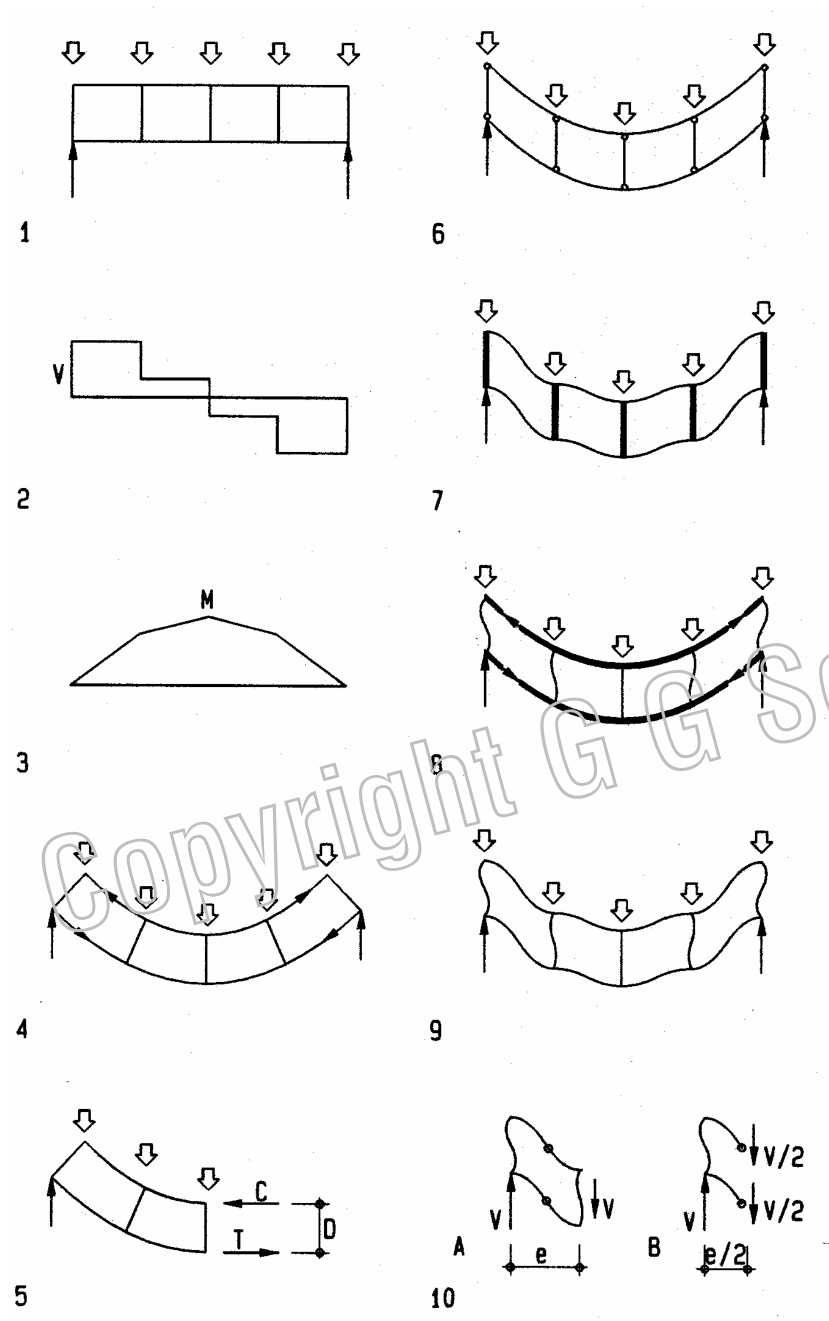
This project, first price of a design competition, was build for the centenary of Italy's unification in 1981 to house an international labor exhibition. The classic order of this structure is a departure from Nervi's funicular oeuvre. Due to a short time from design to completion, one of the design objectives was fast construction. The solution of 16 freestanding mushroom structures allowed for sequential manufacture and erection, a critical factor for speedy completion. The facility measures 158x158m and has a height of 23m. Each of the 16 units measures 38x38m. The mushrooms are separated by gabled skylights of 2m width that help to accentuate individual units visually, provide natural lighting and structural separation. Each mushroom consists of concrete pylons that taper from 2.4m at the top to 5.5m at the base in response to the increasing bending moment toward the base. The pylons are rounded at the top and cross-formed at the base. Twenty tapered steel plate girders cantilever from the pylons in radial patterns; with increasing depth toward the support in response to greater bending. Triangular brackets strengthen the transition from girder to pylon. Stiffener plates welded to girder webs stabilizes them against buckling and provide a visual pattern in response to the structural imperatives.



Vierendeel

Named after the Belgian builder *Vierendeel* who is credited with its invention, the Vierendeel girder has rectangular panels, composed of struts, which are connected by moment resistant joints. Since load is resisted in frame action rather than truss action, the terms Vierendeel *girder or frame* (depending on configuration) are more suitable than Vierendeel *truss* which implies triangular panels and axial stress rather than bending stress. Compared to trusses with triangular panels, rectangular Vierendeel panels allow ducts or pipes to pass through with greater ease. Rectangular panels also have a different visual appearance. Vierendeel girders resist load in combined axial and bending stress and, thus, tend to be less efficient than trusses of triangular panels, which resist load in axial tension and compression. Bending stress in members varies from zero at the neutral axis to maximum at the outer fibers, but axial stress is uniform and thus more efficient. For convenience, we refer to horizontal and vertical struts as ***chord*** and ***web***, respectively. The load bearing of Vierendeel girders and frames can be visualized by magnifying their deformation under load. A single-bay Vierendeel[1] deforms under gravity load similar to a moment resisting portal frame[2]: top and bottom chords develop positive moments at mid span and negative moments at both ends, with two inflection points at the transition. Chord rotation is transmitted to webs and deforms them into S-shapes. The resulting web moments are inverse on top and bottom, with inflection points of zero moment at mid height. Under lateral load[3] both chord and web struts are deformed with single inflection points in the middle. In multi-bay girders[5], too, webs deform under both gravity- and lateral loads similar to frames[6-7], with inflection points that may be hinged. However, the chords develop single inflection points for both lateral and gravity loads, except the center bay which has two inflection points under gravity load. In girders with even number of bays and a center web strut, all chords have single inflection points. Since all web struts, assume inflection points under both gravity and lateral load, they could have hinges at those points, provided those hinges can resist out-of-plane deformation to avoid instability.

1 Single-bay Vierendeel girder
2 Deformation under gravity load
3 Deformation under lateral load
4 Web struts with hinges at inflection points
5 3i-bay Vierendeel girder
6 Deformation under gravity load
7 Deformation under lateral load
8 3-bay web struts with hinges at inflection points



Vierendeel girders resist load in combed beam action and frame action as shown on the left and right diagrams, respectively. Load causes *global* shear and bending which elongates the bottom in tension and shortens the top in compression. The internal reaction to global shear and bending is different in a Vierendeel compared to a beam. A beam's bending stress varies gradually over the cross-section, but global bending in a Vierendeel causes concentrated tension and compression forces in the chords. By visual inspection we can derive simple formulas for approximate axial and shear forces and bending moments. Respective stresses are found using formulas for axial, shear and bending stress and superimposing them. Chord tension T and compression C are computed, dividing the respective global moment M by frame depth D (distance between centers of chords).

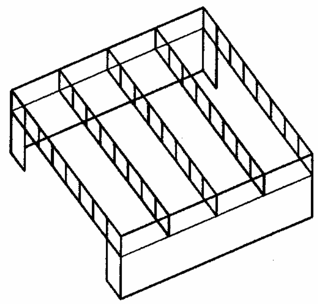
$$C = T = M / D$$

Bending of individual struts can be visualized too. In a structure where moment resistant strut/chord connections are replaced with hinges, chords would deflect as independent beams[6]. Assuming flexible chords and stiff webs, vertical shear would deform chords to S-shapes with inflection point. Assuming flexible webs and stiff chords, horizontal shear, caused by a compressive force pushing outward on top and a tensile force pulling inward on the bottom, would deform webs to S-shapes with inflection point. The combined effect of these two idealized cases imparts S-shaped deformation and inflection points in both chord and web struts. The deformation yields strut bending moments which vary from positive to negative along each strut. Top and bottom chords carry each about half the total shear V. Assuming inflection points at midpoints of chords, the local chord moment M is half the shear V multiplied by half the chord length.

$$M = (V / 2) (e / 2)$$

The moment M is maximum at supports where shear is greatest and equal to support reactions. For equilibrium, webs have to balance chord moments at each joint. Their moment equals the difference of adjacent chord moments.

1 Gravity load on a Vierendeel
2 Global shear (in overall system rather than individual members)
3 Global bending (in overall system rather than individual members)
4 Compression and tension in top and bottom chord, respectively
5 Free-body visualizes derivation of chord tension T and compression C
6 Global shear deformation
7 Chord bending, assuming flexible chords and stiff webs
8 Web bending, assuming flexible webs and stiff chords
9 Combined chord and web bending under actual condition
10 Free-bodies visualize derivation of chord bending moment M



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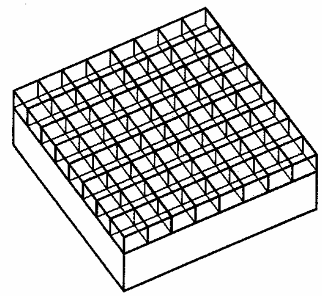

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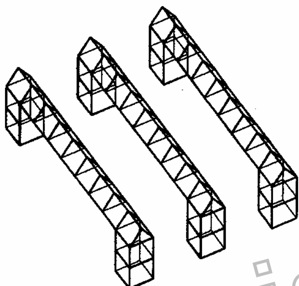

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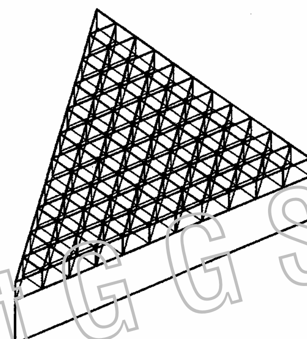

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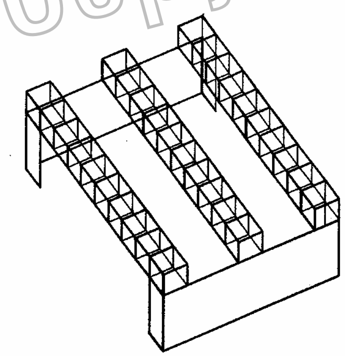

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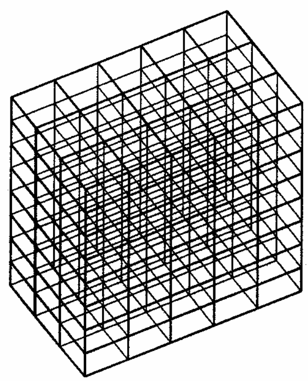


6

Configurations

Vierendeels may have various configurations, including one-way and two-way spans.

One-way girders may be simply supported or continuous over more than two supports. They may be planar or prismatic with triangular or square profile for improved lateral load resistance. Some highway pedestrian bridges are of the latter type. A triangular cross-section has added stability, inherent in triangular geometry. It could be integrated with bands of skylights on top of girders.

When supports are provided on all sides, Vierendeel frames of two-way or three-way spans are possible options. They require less depth, can carry more load, have less deflection, and resist lateral load as well as gravity load. The two-way option is well suited for orthogonal plans; the three-way option adapts better to plans based on triangles, hexagons, or free-form variations thereof.

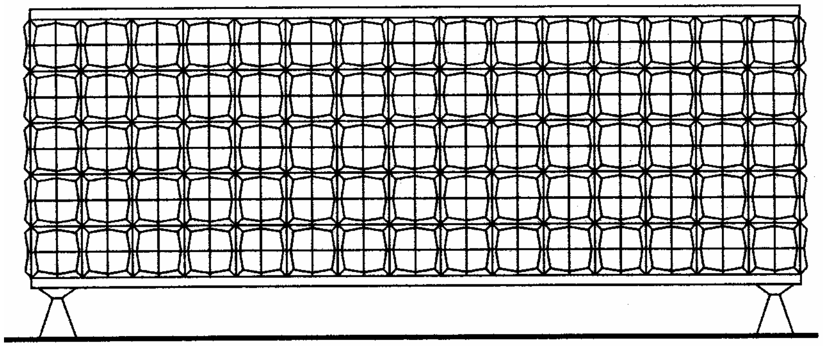
Moment resistant space frames for multi-story or high-rise buildings may be considered a special case of the Vierendeel concept.

1 One-way planar Vierendeel girder
2 One-way prismatic Vierendeel girder of triangular cross-section
3 One-way prismatic Vierendeel girder of square cross-section
4 Two-way Vierendeel space frame
5 Three-way Vierendeel space frame
6 Multi-story Vierendeel space frame

Beinecke library, Yale University, New Haven, Connecticut (1963)
Architect and Engineer: Skidmore, Owings and Merrill

The Beinecke library of Yale University for rare books has a 5-level central book tower, freestanding within a single story donut-shaped hall that extends over the full height of the tower. The tower holds 180,000 books and is climatically separated from the surrounding hall by a glass curtain wall.

The library's five-story open space is framed by a unique structural concept. Four Vierendeel steel frames, 50 ft (15 m) high, support the roof and wall load and span 131 and 88 ft (40 and 27m) in length and width, respectively. The frames are supported by a reinforced concrete plate that transfers the load via steel pin joints to four reinforced concrete pylons. The Vierendeel frames consist of 8'-8" (2.6m) prefabricated steel crosses, welded together during erection. The crosses express pin joints at mid-points of chord and web struts, where inflection points of zero bending occur.



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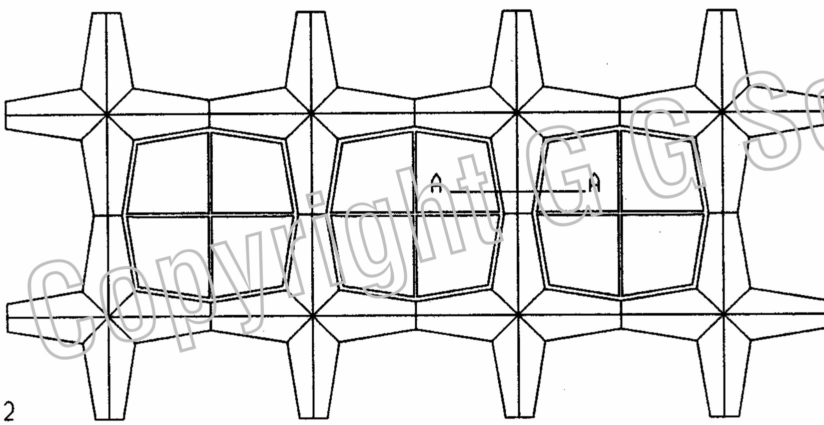

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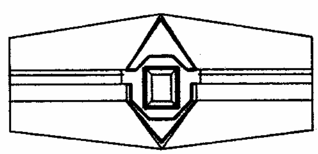

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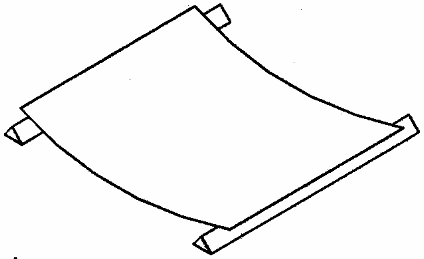

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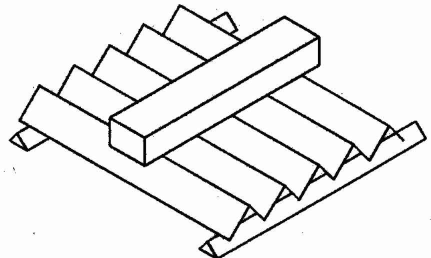

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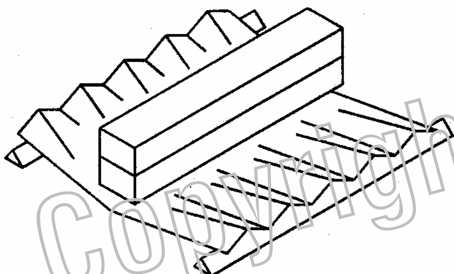

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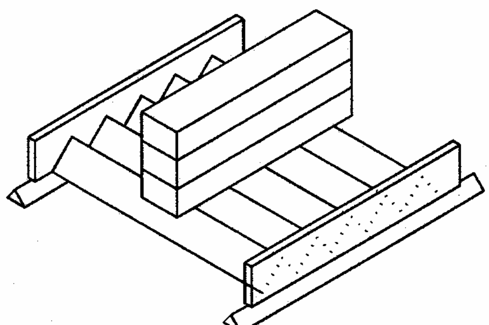


4

Folded Plate

The effect of folding on folded plates can be visualized with a sheet of paper. A flat paper deforms even under its own weight. Folding the paper adds strength and stiffness; yet under heavy load the folds may buckle. To secure the folds at both ends increases stability against buckling

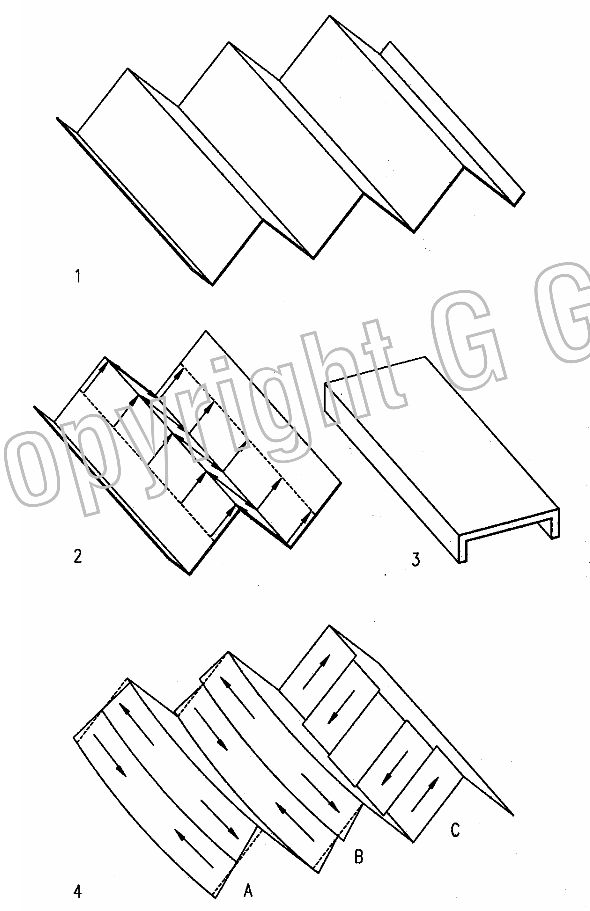
1 Flat paper deforms under its own weight
2 Folding paper increases strength and stiffness
3 Paper buckling under heavy load
4 Secured ends help resist buckling

Folded plate behavior

Folded plates combine slab action with beam action. In length direction they act like thin inclined beams of great depth, stabilized against buckling at top and bottom by adjoining plates. In width direction they are one-way slabs that span between adjacent plates.

1 Folded plate concept
2 Slab action in width direction
3 Slab-and-beam equivalent
4 Beam action in length direction

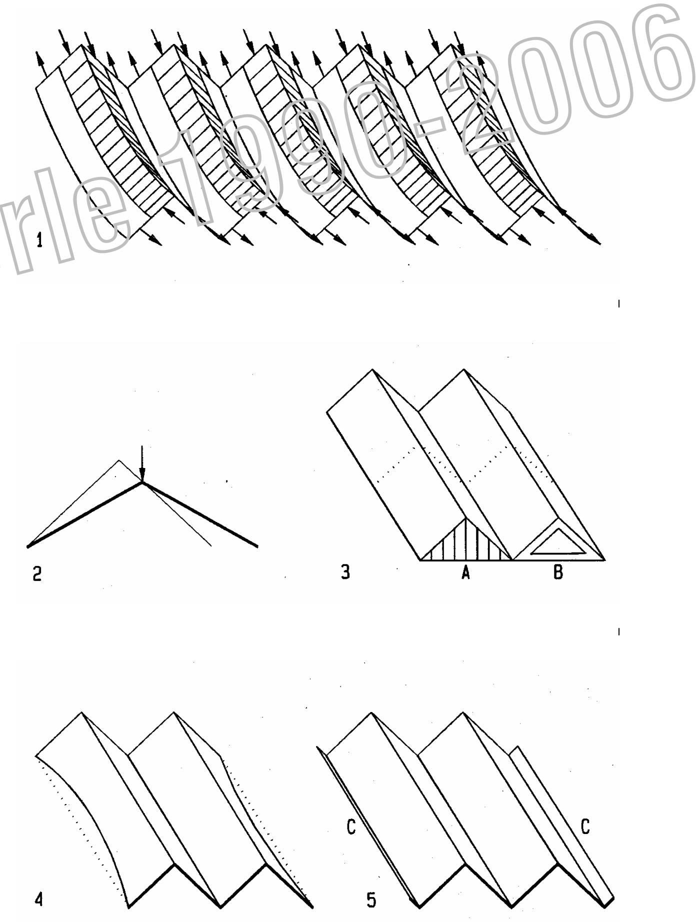
A Bending deformation causes top compression and bottom tension
B Horizontal shear caused by compression and tension
C Vertical shear is maximum at supports and zero at mid span



Bending in folded plates causes top compression and bottom tension. Folded plates also tend to flatten out under gravity load, which may be prevented by walls or frames at end supports. Tendency of end panel buckling can be resisted by edge beams.

1 Bending visualized as external compression and tension forces
2 Flattened folded plate under gravity load
3 Folded plate with walls and frames to resist flattening
4 Buckled end panels
5 End panels stabilized by edge beams

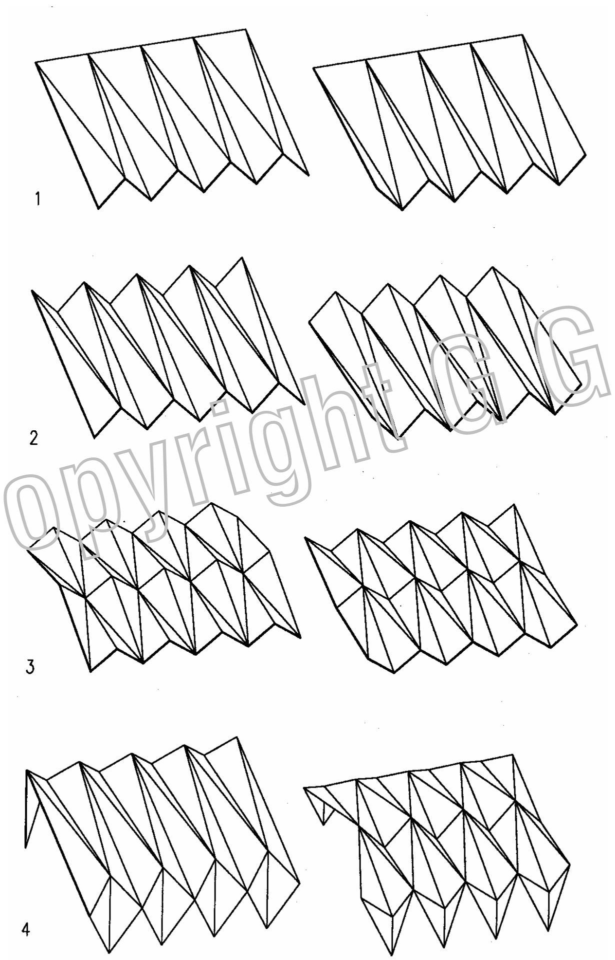
A Stabilizing wall at support and, for long systems, at mid-span
B Stabilizing frame at support and, for long systems, at mid-span
C Edge beam to stabilize end panel against buckling



Folded plate forms

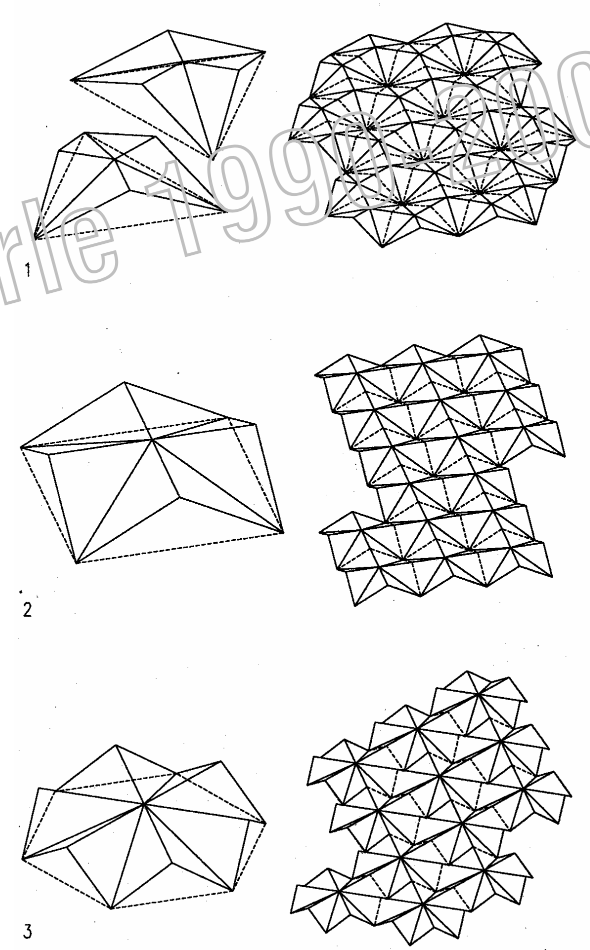
Folded plates may have many one-way, two-, or three-way spans. They may be motivated by aesthetic or spatial objectives, or to add strength and stability to a system. In areas with snow, flat folded plates are problematic since snow can accumulate in the valleys. One-way systems are shown below; Two and three-way systems are right.

1 Folded plate with one straight and one gabled edge
2 Folded plate with offset gabled edges
3 Folded plate with gabled edges offset at mid-span
4 Folded plate with vertical support folding and gables offset at mid-span



Folded plates may be two or three-way systems.

1 Three-way folded plate unit and assembly on triangular base plan
2 Two-way folded plate unit and assembly on square base plan
3 Three-way folded plate unit and assembly on hexagonal base plan

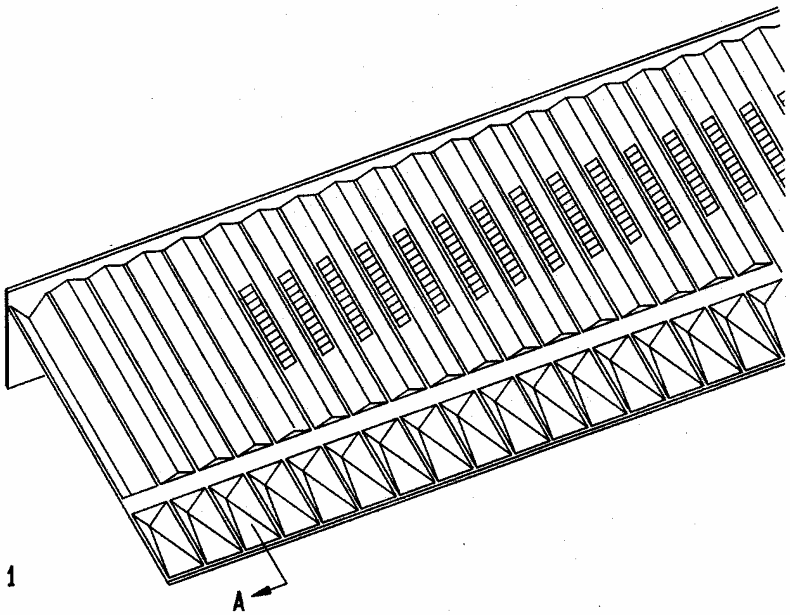


Railroad Station Savona, Italy (1958-61)
Architect: Antonio Nervi
Engineer: Pier Luigi Nervi

This first prize of a design competition consists of site-cast concrete folded plates, supported by a folded plate concrete wall on the rear which also provides lateral stability. Ten concrete pylons support the public entry front. The pylons are cantilevered from a grade beam for lateral stability in length direction. The pylons transform from rectangular cross-section in length direction at the ground to rectangular cross-section in width direction at the roof. They support a u-shaped roof girder that is integral with and supports the folded plate roof. One-third of the roof overhangs in front, beyond the girder. The overhang is tapered, transforming from the folded plate profile to a flat roof edge. The taper makes an elegant edge in logic response to the diminished negative bending moment requiring less depth at the edge. Light-weight gabled roof elements cover the folded plates over interior space for waterproofing. Over the central area skylights, integrated in the roof, provide natural lighting.

1 Folded plate concrete roof layout
2 Typical folded plate concrete unit

A Cross-section through roof overhang with tapered folded plates and u-shape girder
B Length-section through folded plates



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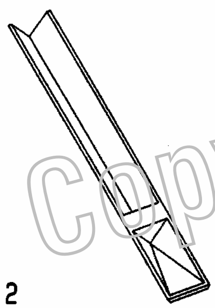

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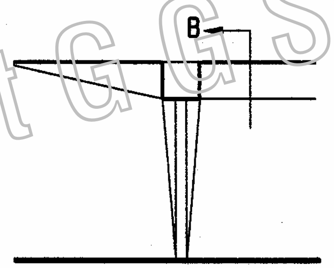

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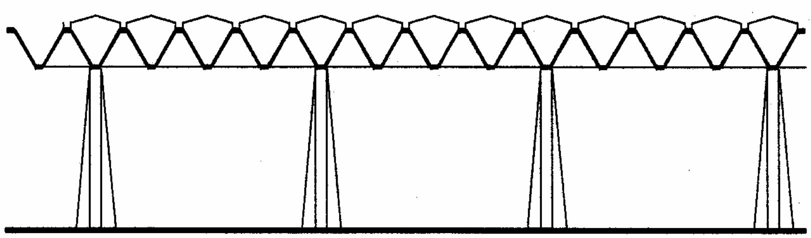


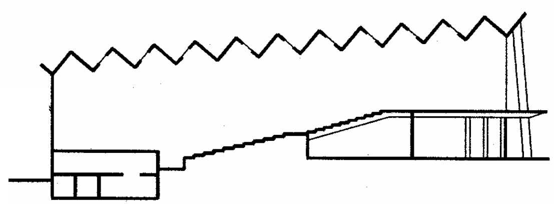
B

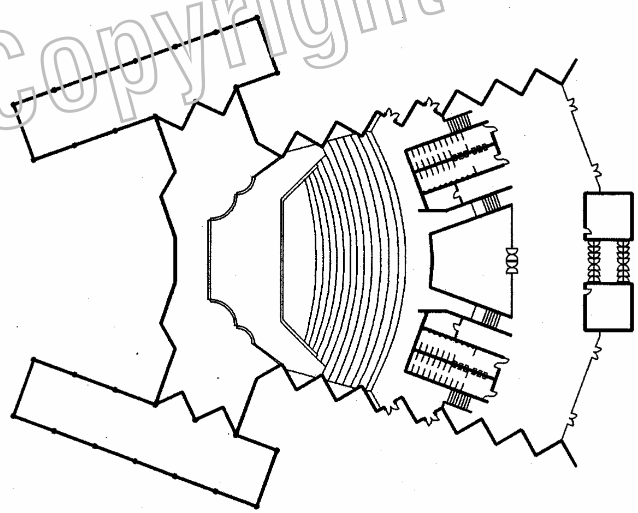
Gunma Music Center, Takasaki, Japan (1961)
Architect: Antonin Raymond
Engineer: Tsuyashi Okamoto

This Gunma music center for the Gunma Philharmonic Orchestra, consist of a folded plate concrete roof of 60 m span and folded plate walls, that form frames to resist gravity and lateral loads. The architect, a former student of Frank Lloyd Wright at Taliesin took the challenge to design the center for the following requirements:

- The center had to be fire and earthquake proof
- Good acoustics for the music center
- Provide for Kabuki performances that required a revolving stage

The folded plate roof is 3.3 m deep for a span/depth ratio of 1:18. Two wings flanking the stage for meeting and green rooms, also have folded plate roofs.



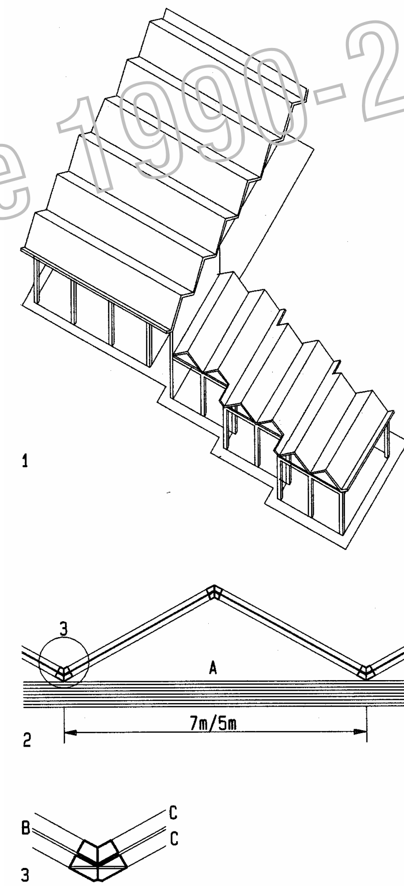


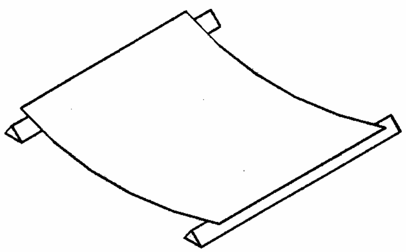
Shopping Center, Würzburg, Germany
Architect: Schönewolff and Geisendörfer
Engineer: Julius Natterer

The folded plate wood roof modules are 7 m wide and span 16.25 m for the large space; 5 m wide and span 12.5 m for small spaces.

1 View of folded plate wood roof
2 Cross-section of typical folded plate module
3 Detail of valley joint

A Tie strut 135x520 mm
B Folded plate cross planking 4 cm
C Transverse ribs, 8 x 16 cm, spaced 1.9 m





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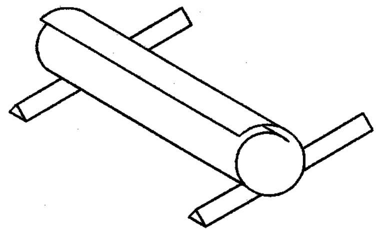

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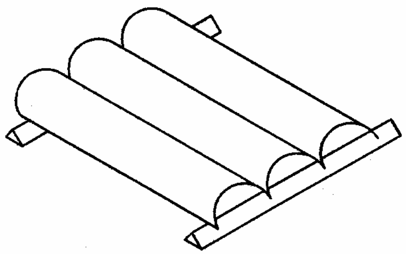

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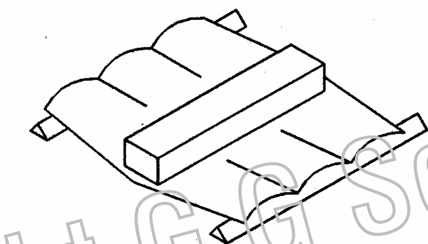

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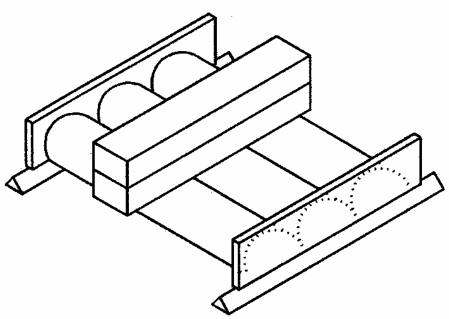
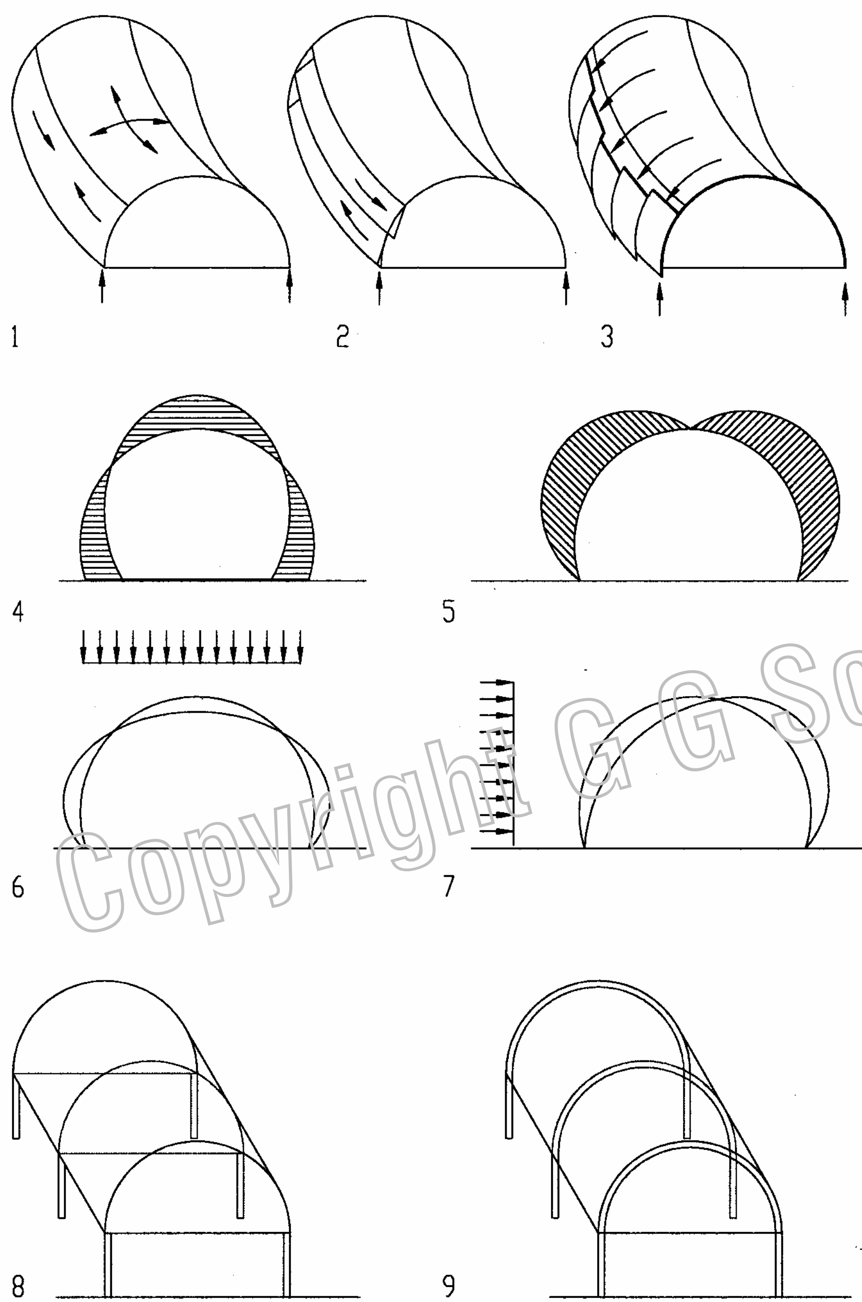


5

Cylindrical Shell

The shape effect of cylindrical shells can be visualized with paper. A flat paper deforms even under its own weight. To roll or bend paper into cylindrical shapes adds strength and stiffness; yet heavy load may flatten and buckle the paper. Securing both ends prevents buckling

1 Flat paper deforms under its own weight
2 Rolling paper increases strength and stiffness
3 Cylindrical form also increase strength and stiffness
4 Paper buckling under heavy load
5 Secured ends help resist buckling



Cylindrical shell behavior

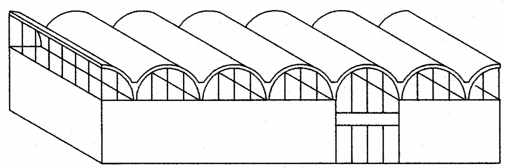
Considering their name, cylindrical shells could be part of shells; but they are included here because they resist load primarily in bending, unlike shells which act primarily in tension and compression. Most cylindrical shells have semi-cylindrical cross-sections and act much like beams of such cross-section, spanning horizontally to transfer gravity load to supports. Similar to beams under gravity load bending in cylindrical shells cause compressive stress on top and tensile stress at the bottom; unlike vaults with primary span in width direction. Differential bending stress, pushing and pulling on top and bottom generates horizontal shear stress in cylindrical shells. To satisfy equilibrium, horizontal shear causes also vertical shear which can be visualized as tendency of individual parts to slide vertically with respect to one-another. Stress distribution over the cross-section is also similar to beams. Bending stress varies from maximum compression on top to maximum tension on the bottom, with zero stress at the neutral axis. In contrast shear stress is maximum at the neutral axis and zero on top and bottom. Compressive stress in cylindrical shells causes buckling which can be resisted by cross-walls or ribs.

1 Compressive stress on top, tensile stress at bottom, with some arch action
2 Horizontal shear generated by differential compressive and tensile stress
3 Vertical shear visualized
4 Bending stress distribution
5 Shear stress distribution
6 Buckling under gravity load
7 Buckling under lateral load
8 Wall panels to resist buckling
9 Ribs to resist buckling

Configurations

Cylindrical shells can have various configurations: cross-sections of half or quarter cylinders, or other curved forms; they may have closed ends or be open at one or both ends; they may be simply supported, cantilevered, or span two supports with one or two overhangs. The end units may be open or closed. Butterfly cross-sections are also possible if designed to resist bending in width direction. The intersection between adjacent shells must incorporate a gutter to drain rainwater. In snow areas, horizontal cylindrical shells are problematic, since snow would accumulate in the valleys.

1 Semi-circular cylindrical shells, simply supported, with glass ends
2 Shallow units cantilever from a beam, designed to resist rotation
3 Butterfly units, cantilevered from pylons



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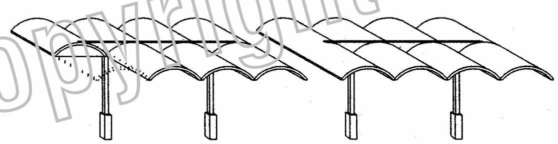

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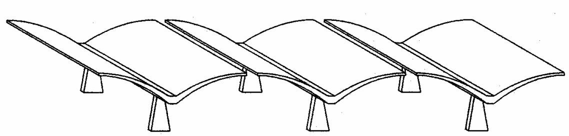
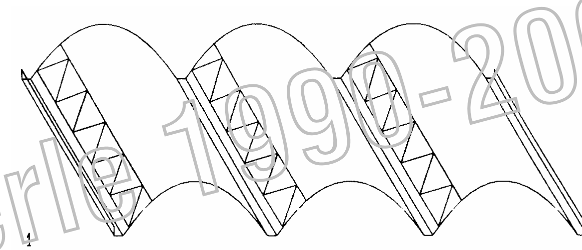


3

Skylights

Various skylight forms maybe integrated with cylindrical shells. This has been a popular solution for natural lighting of industrial buildings. Combining the inherent strength, stiffness, and stability of cylindrical shell forms with natural lighting is a logical design strategy. The skylights may be inclined in the shell form, or flat on top, or in the vertical plane of a quarter-cylindrical shell. Skylights could be incorporated with a truss as part of the cylindrical shell. An important factor in integrated skylights is waterproofing to prevent leaks, and to incorporate some form of gutter for drainage.

1 Cylindrical shell with truss skylight
2 Skylight on top of cylindrical shell
3 Vertical skylight with cylindrical shell of quarter cross-section



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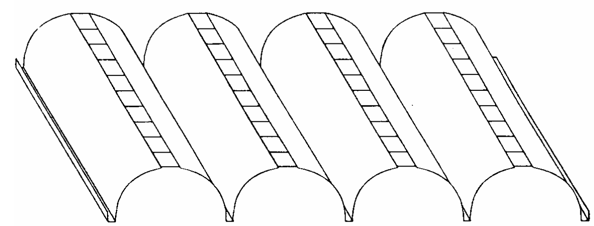

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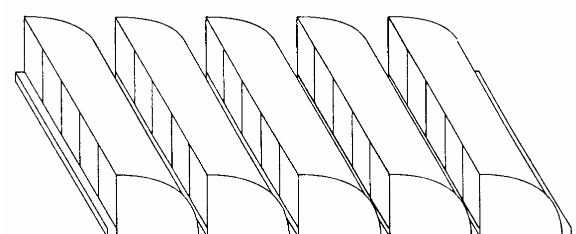

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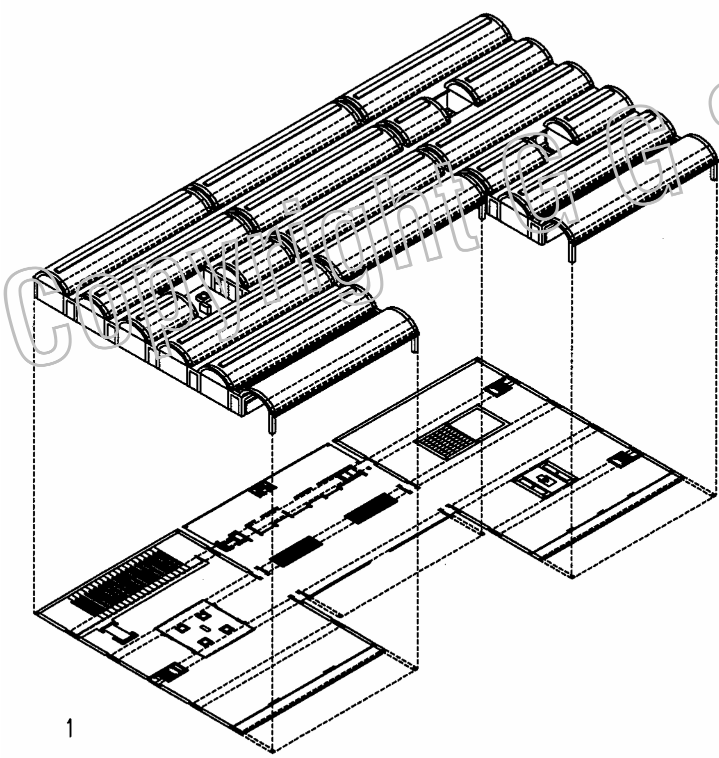
Kimbell Art Museum
Architect: Louis Kahn
Engineer: Kommendant

The Kimbell Art Museum is composed of three parts: the central main entrance, facing bookstore and library is flanked by two gallery wings, one on each side. The gallery wings include atrium courtyards. The entire facility is composed of 16 modules of about 30x100ft (9x31 m). The modules consist of cycloid shells, 24ft (7.3m) wide with a flat part of 6ft (1.8m) between them (the cycloid cross-section is formed by a point on a moving wheel). A 30in (76 cm) wide skylight extends on top of each shell unit. A metal deflector below each skylight reflects the daylight against the interior surface of the cycloid shells for indirect natural lighting. The cycloid shells consist of post-tensioned cast-in-place concrete. They where cast by using a movable form-work used repetitively. The flat roof between cycloid shells forms an inverted U to house mechanical ducts and pipes as required.

1 Exploded isometric view
2 Cycloid, formed by a point on a cycle that moves horizontally
3 Cross-section of cycloid shells

A Point on the cycle that
B Cycloid traced by the point on a cycle
C Linear skylight
D Reflectors of polished metal
E Mechanical ducts
F Duct cover



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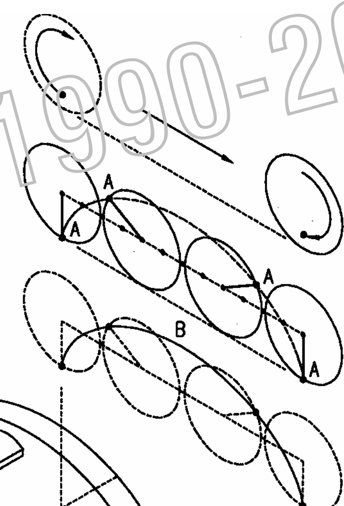

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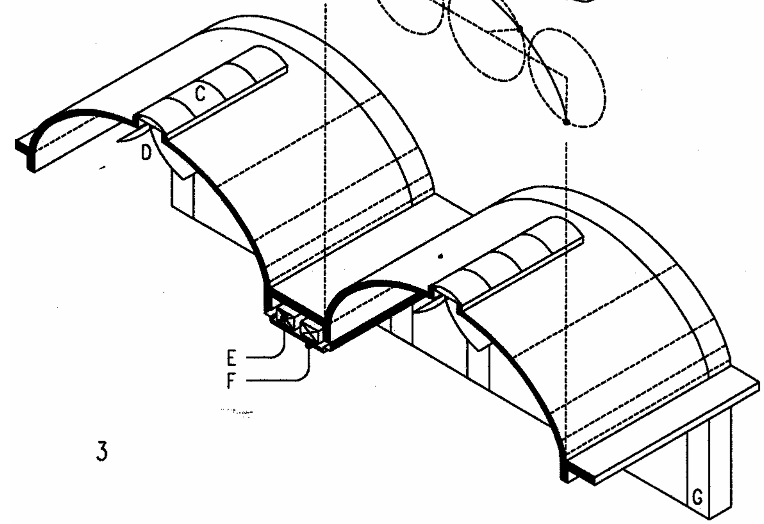

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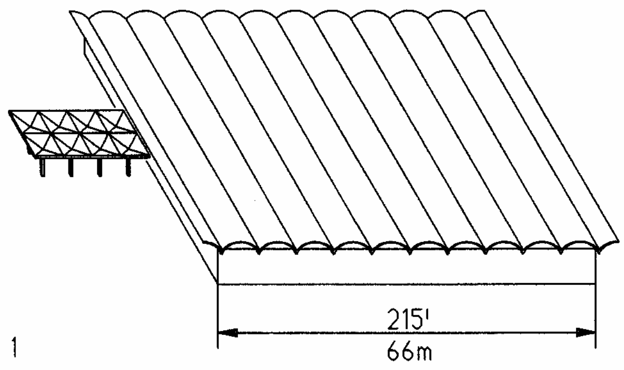

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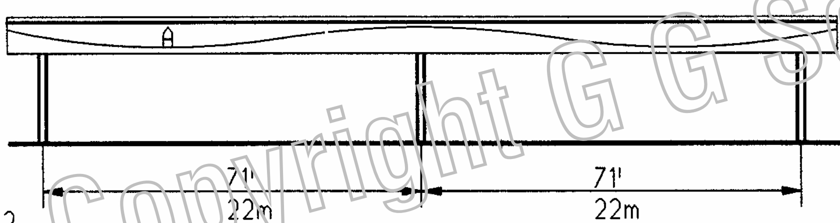

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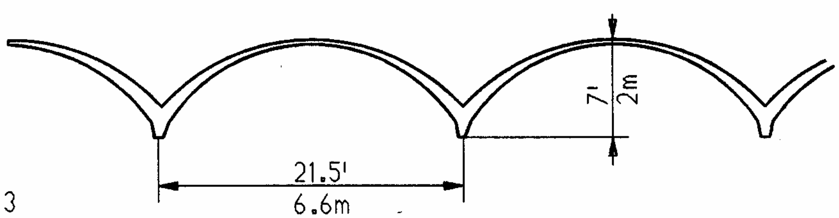


3

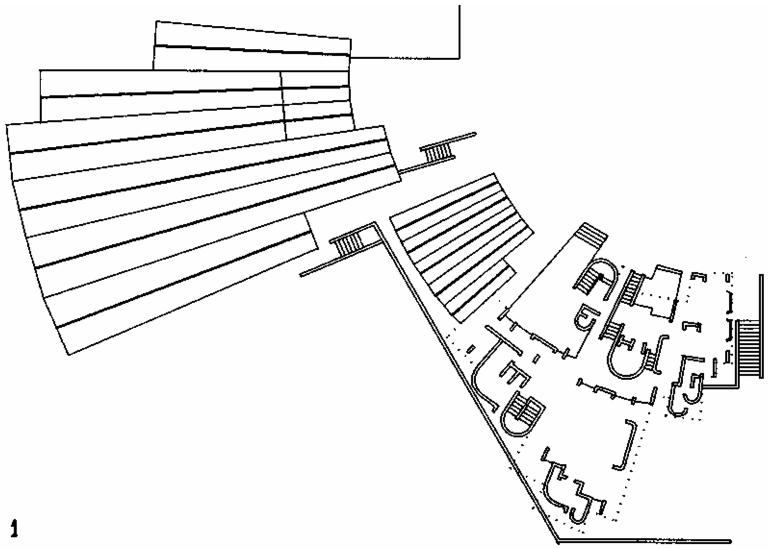
California Museum of Science and Industry

Architect: California State Architect Office
Engineer: T. Y. Lin and Associates

The roof of this rectangular museum consists of ten cylindrical shells and two half shells as curved overhangs on the north and south sides. A group of eight inverted conical shells provides a canopy for the main south side entry. The cylindrical shells provide spatial relief and articulation for this stark rectangular plan. They continuo over two bays and have span/depth ratios of 10. Post-tensioned tendons are draped to approximate a parabola in space. Reflecting the bending deformation of the shells, the parabolic form has an uplifting effect to counteract and minimize deflection. The tendons are prestressed to produce a camber, designed to offset deflection due to dead load and partial live load. The cylindrical shells where site-cast, using lightweight concrete (80% of normal weight concrete) to minimize dead load. This is important in areas of seismic activity, like Los Angeles, since seismic forces are proportional to mass, which corresponds to deadweight. The shell thickness increases toward the base where they form beams between adjacent units.

1 Isometric roof plan
2 Length section in east-west direction
3 Typical shell cross-section

A Post-tensioned prestress tendons, draped to offset deflection



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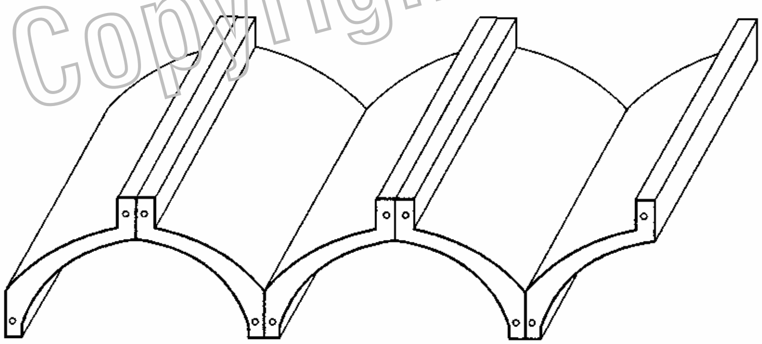

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Kindergarten Yukari, Tokyo
Architect: Kenzo Tange

The fan-shaped plan of the Yukari Kindergarten for 280 children is designed in response to a conic site of mild slope. The director of the facility, an artist, wanted an environment of artistic inspiration for children of this kindergarten. The plan and space are strongly defined by prefabricated cylindrical concrete shells, consisting of twin quarter-circular elements with top stems for assembly and to hold the prestress tendons. Fan-shaped shells accommodate the plan layout: each twin unit covers a modular space; large spaces are covered by several units. Glass end walls emphasize the cylindrical shells and extend them visually to the outside. Unit lengths vary with the spatial requirements. The plan shows at left the roof and at right the floor plan with shells as dotted lines.

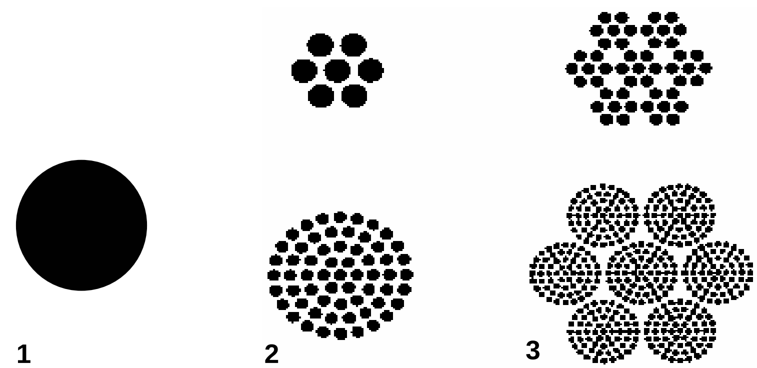
14

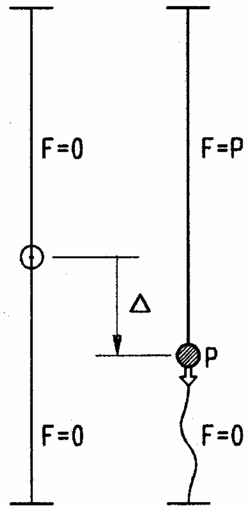
HORIZONTAL SYSTEMS
Tensile Resistant

Tensile-resistant systems include stayed, suspended, cable truss, anticlastic, and pneumatic structures. Although compression, bending and shear may be present in some tensile structures, tensile stress is most prominent. For example, cable-stayed systems may include bending resistant beams and joists, yet they are secondary to primary stay cables or rods. Compared to bending and compression, tensile elements are most efficient, using material to full capacity. Bending elements use only half the material effectively, since bending stress varies from compression to tension, with zero stress at the neutral axis. Compression elements are subject to buckling of reduced capacity as slenderness increases. Furthermore, some tension elements, such as steel cables, have much greater strength than columns or beams of mild steel, because they are *drawn* (stretched) during manufacturing to increase strength. However, the overall efficiency of tensile structures depends greatly on supports, such as ground anchors. If poorly integrated, they may require a large share of the budget. Therefore effective anchorage is an important design factor. For example, the use of self-stabilizing compression rings or infrastructures, such as grandstands, to resist tensile forces can be an effective means of reducing support costs.

Tension members

1 Steel rod
E = 30,000 ksi, F_a = 30 ksi, 100 % metallic
2 Strand consists of 7 or more wires (provides good stiffness, low flexibility)
E = 22,000 to 24,000 ksi; F_a = 70 ksi, 70% metallic
3 Wire rope consists of 7 strands (provides good flexibility, low stiffness)
E = 12,000 to 20,000 ksi, F_a = 70 ksi, 60% metallic





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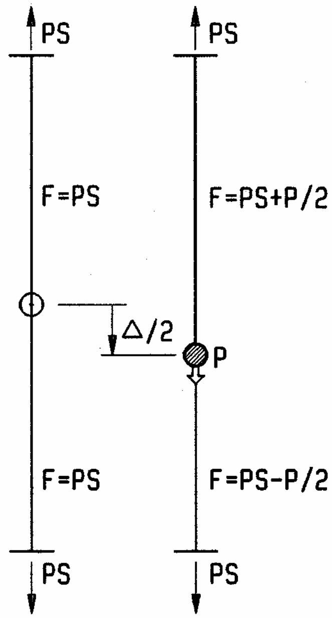

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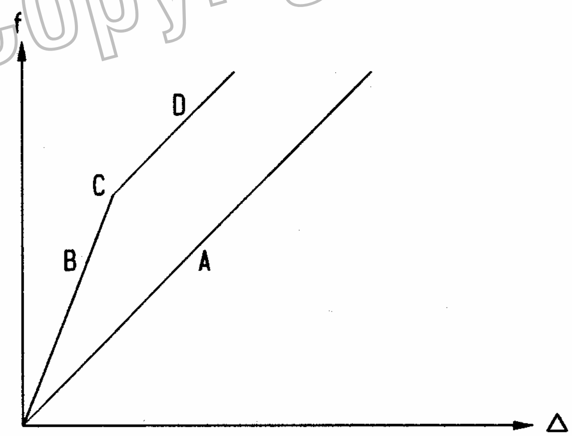

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Prestress

Tensile structures usually include flexible membranes and cables that effectively resist tensile forces but get slack under compression. Yet, under some load conditions, compressive forces may be induced in flexible tensile members. Prestress allows flexible members to absorb compressive stress without getting slack which would cause instability. Prestress also reduces deformation to half. These phenomena may be observed on a simple string.

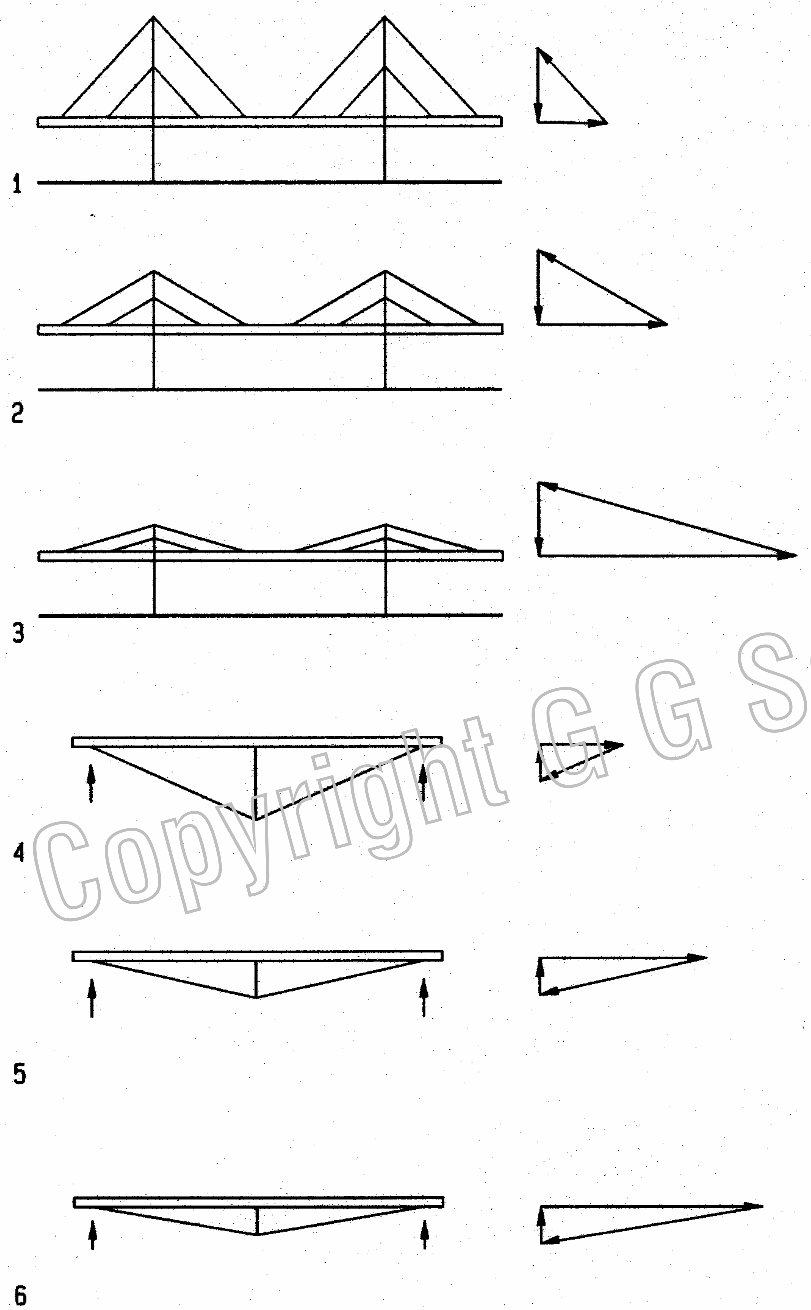
Consider a vertical string fastened on top and bottom. If a load is applied at mid-height, the top link absorbs the entire load, and the lower link will get slack and unstable.

Now consider the same string prestressed (with turnbuckles for example). The same load applied at mid-height will be carried half by the top link (through increase of prestress) and half by the lower link (through decrease of prestress). Since both links are active, each will absorb only half the load, reducing the deformation to half and avoiding the lower link from getting slack and unstable. Since half the load is absorbed by each link, when the applied load reaches twice the prestress or more, the lower link will get slack, just as the string with no prestress. Given similar conditions in a structure, prestress should be at least half of the design load to prevent slack members and instability. Also, loss of prestress due to creep and temperature variation should be considered.

The correlation between prestress, load, and deformation, described above, is visualized in the stress/strain diagram below.

1 String without prestress
2 String with prestress
3 Stress/strain diagram of both strings

A Stress/strain line of un-prestressed string
B Stress/strain line of prestressed string
C Point where prestress is reduced to zero under load
D Stress/strain line of string after loss of prestress
F Force
f Stress
P applied load
PS Prestress
Δ Deformation



Stayed Structures

Stayed structures consist of beams or trusses that are intermittently supported by strands or rods (strands and rods have greater stiffness than wire ropes and hence reduce deflection). Although stays usually support structures, pulling from above, they may also push from below by means of compression struts. The latter is also referred to as cable-propped or just propped. Given the slope of stays, they generate not only a vertical uplift but also a horizontal reaction in the supported members and masts. In beams the horizontal reactions yield compression; in masts they introduce bending and overturn moments, unless stays on both sides balance the horizontal reactions.

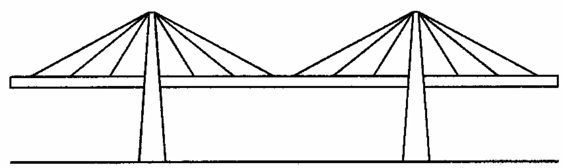
The span/depth ratio of stayed and propped structures is an important design factor. A shallow depth results in great tension and compression in stays and beam respectively, a steep slope has the opposite effect. The relationship of cable slope and resulting forces is illustrated in the diagrams, showing various slopes and resulting forces for an assumed gravity load as vertical vector. Optimal span/depth ratios depend on both, architectural and structural factors. Architectural factors include appearance and spatial considerations. Structural factors include the impact on deflection, overall cost of stays, beams, masts, and compression struts. As a rule of thumb, the optimal slope for stays is about 30 degrees. Optimum span/depth ratio for propped systems is about 10 to 15.

1 Steep stay slope causes small forces but high masts
2 Stay slope of 25° to30° is usually optimal
3 Shallow stay slope causes high forces but low masts
4 Steep props cause small forces but great depth
5 Span/depth ratio of about 10 to 15 is optimal
6 Shallow props cause great forces but small depth

Configurations

Stayed structures may have radial or parallel strands, called radial and harp systems, respectively. Combinations of both systems are also possible. Harp systems have constant stay forces; the force of radial systems varies with the stay slope. The tributary length between radial stays may be adjusted to keep forces constant, i.e., strands with shallow slope support small tributary lengths.

1 Radial system (stay forces vary with slope)
2 Harp system (constant stay forces)
3 Mixed system, combining radial and harp patterns
4 System with variable distance between stay supports to equalize stay forces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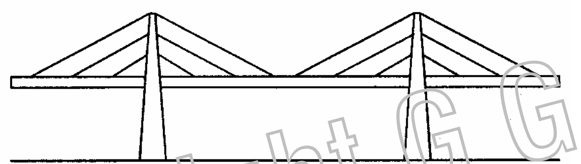
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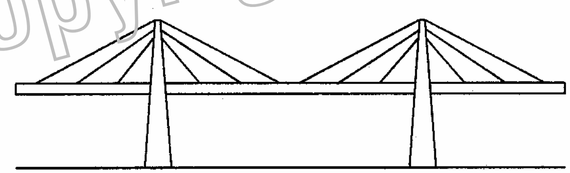
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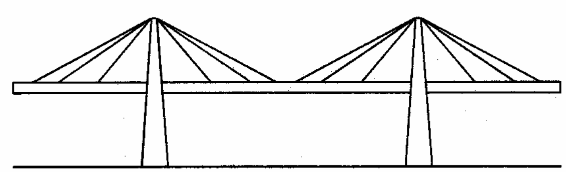
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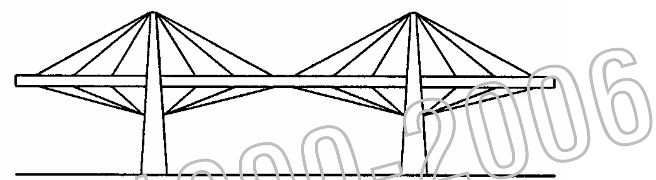
For light-weight roofs, with wind uplift greater than the roof dead weight, stays could by added roof to resist wind uplift. Stays can also branch out like trees to reduce length. Single masts must be designed to resist overturning under unbalanced load. One-sided load causes unbalanced conditions that require guy cables. The dead weight of an inclined mast may help to balance loads.

1 Stay cables below the roof resist wind uplift
2 Inverse tree stays reduce length but require more joints
3 Single tower with tie-downs at both beam ends to resist overturning
4 One-sided support with guy cable are unbalanced and less efficient; the inclined mast can help to balance the one-sided load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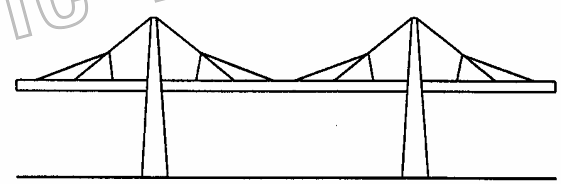
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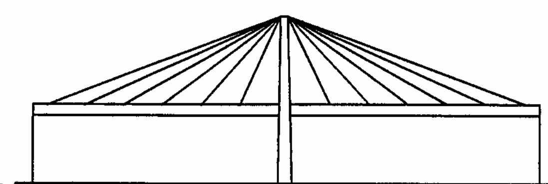
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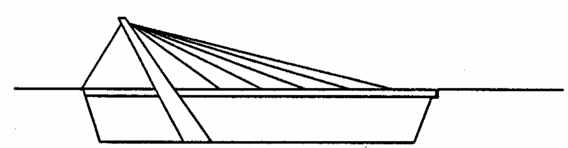
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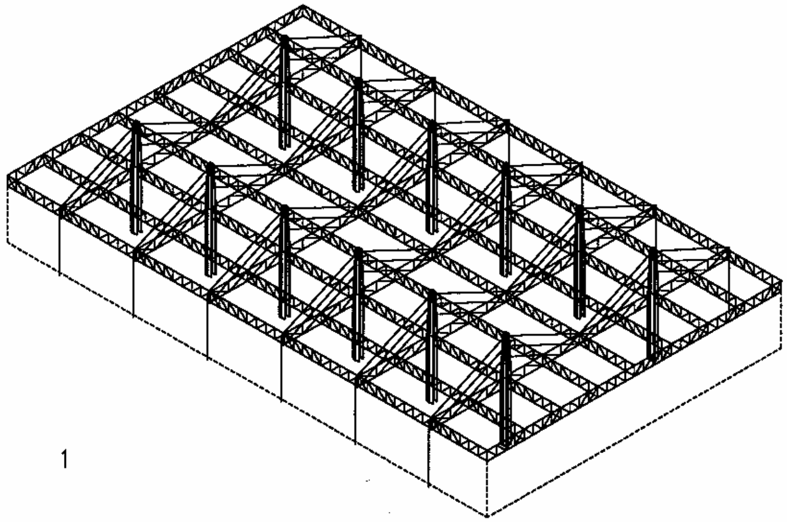

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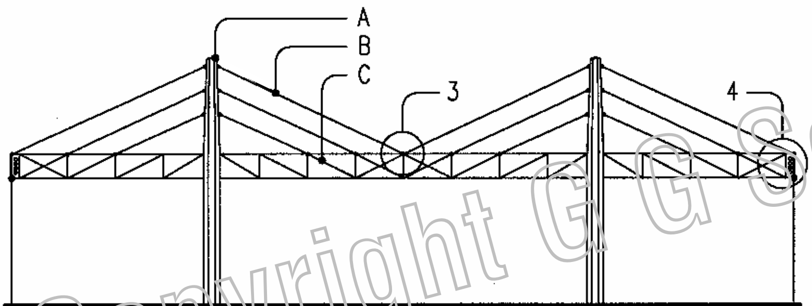

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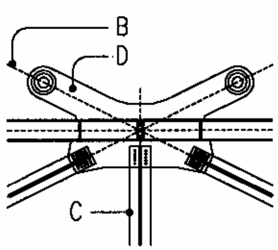

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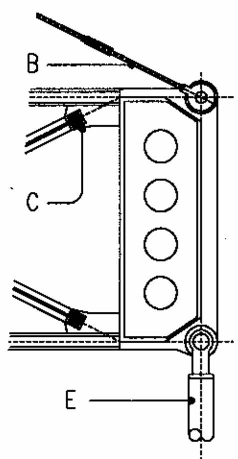

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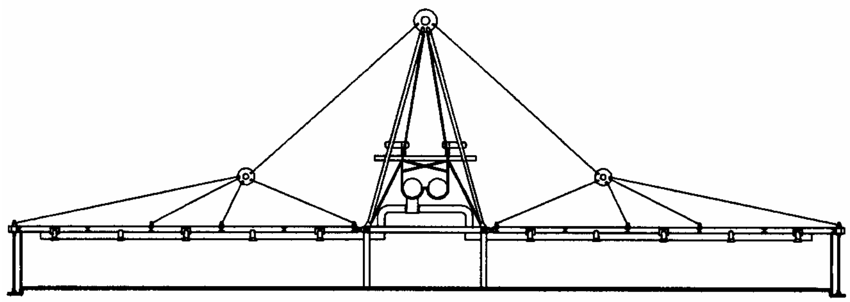
McCormick Place, Chicago (1987)
Architect: Skidmore, Owings and Merrill
Engineer: Knight and Associates

The expansion of McCormick Place exhibit hall, located over existing railroad tracks, required a long-span roof to provide column-free exhibit space without interfering with the tracks. Several structure systems had been investigated before selecting a stayed roof. The roof is suspended from 12 concrete pylons, spaced 120x240ft, with 120ft overhangs on both long sides. The pylons project 60 feet above the roof. The clear interior height is 40 feet. Stay cables consist of 3.75in galvanized steel strands, coated with corrosion resistant PVC, arranged in parallel harp form at an angle of 25 degrees. The stays support steel trusses which support secondary trusses, both 15ft deep and exposed at the interior. The concrete pylons are shaped to incorporate mechanical ducts which bring conditioned air from a mezzanine below the main floor and exhaust it over the roof, without mechanical equipment exposed on the roof. The roof truss edges are tied to the podium of the main hall to provide stability for unbalanced load. The podium is supported by steel columns, spaced to accommodate the rail tracks. Combined with the deep trusses, the stays have enough redundancy that they can be removed and replaced without affecting the structure's integrity. A glass band along the entire façade under the roof trusses and roof skylights, provide natural lighting.

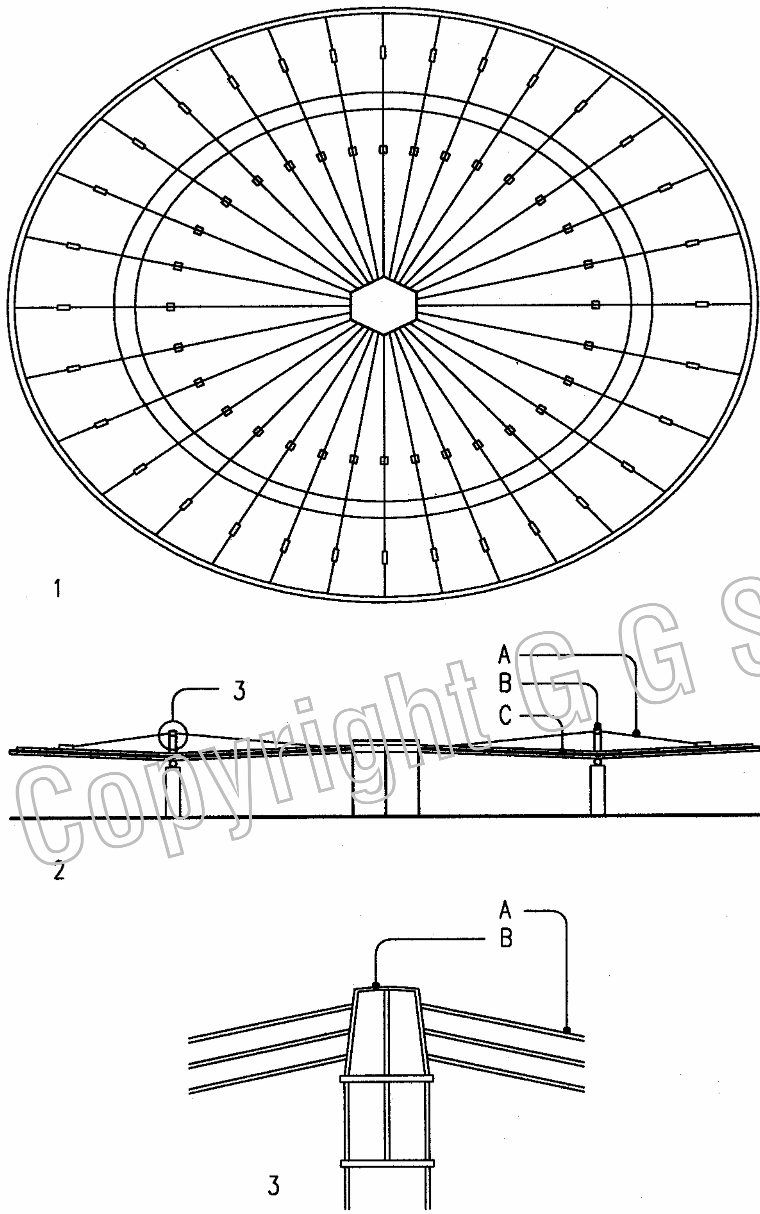
1 Isometric roof structure
2 Cross-section of upper level with stayed roof
3 Mid-span stay support detail
4 Roof edge detail

A Concrete pylons, shaped to accommodate mechanical ducts
B Stays, 3.75in galvanized steel strands, PVC coated
C Truss web bar
D Stay connection bracket
E Steel tie secures roof to [odium



Patscenter, Princeton, USA (1986)
Architect: Richard Rogers
Engineer: Ove Arup and Robert Silman

Patscenter is a research facility for PA Technology. The stayed roof structure was chosen to provide column-free work space and to express technology as architectural language desired by the client. One design criteria was to resist wind uplift load without added dead weight on the roof. Based on a module of 4.5x9m, the single-story facility measures 54x72m. The plan layout, as well as the structure and service technology are all arranged along a central spine, 9m wide and flanked by two 22.5m wide work areas. The entire roof is suspended from 9 triangular pylons, spaced 9m along the central spine and supported by moment resistant portal frames. Steel stay rods on each side of the pylons branch out to support the roof. Intersecting joints for the branches consist of circular steel plates to which the rods are attached by means of standard fittings. Steel rods where chosen over stay cables for greater stiffness and to facilitate painting. The two inner stays are compression struts to resist wind uplift, with both outer stay secured to columns that are tied to foundations. Using graphic vector analysis, the engineers studied the geometry of the inverted tree branches to determine branch forces and overall stability. The roof rests on joists, spaced at 4.5m, spanning 9m between beams which are suspended form the pylons by stay rods. The beams continue over the full width of each wing. A platform for mechanical equipment is suspended by rods of triangular configuration to provide lateral stability for the pylons in length direction. In width direction, lateral stability is provided by the triangular pylons and moment frames along the central spine.

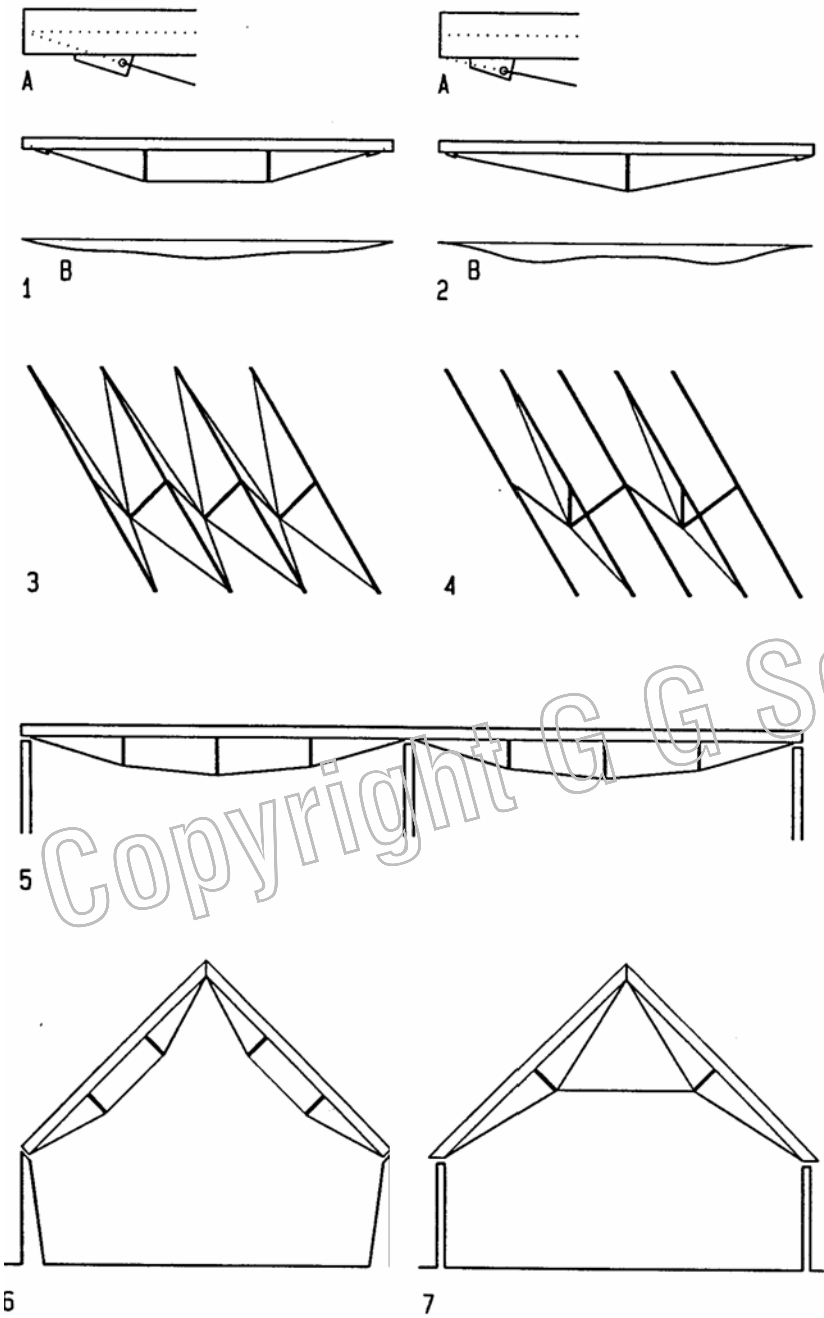


Pan Am Terminal, J F K airport, New York (1959)
Architect/Engineer: Tippetts, Abbet, McCarthy, Stratton

This air terminal was designed with a large overhanging roof to protect boarding passengers. Passenger circulation is straight forward. Departing passengers arrive at the center and fan out to the peripheral departing gates. Arriving passengers proceed in reversed direction. The structure, completed in 1959, was designed with an elliptical roof of 422/528ft, with overhang that projects 114 ft beyond the building enclosure. Thirty-two radial steel girders are supported by stays, each consisting of six 2.5in zinc-coated strand bundles. The stays, attached near the edge of the steel girders, run over saddles of a mid ring of concrete columns and are anchored to an inner ring of columns. The position of columns is such that the girder load on both side is approximately balanced, a strategy which made this giant overhang economically feasible. Steel joists, spaced 19ft span between the radial girders to support the roof metal deck.

1 Roof plan
2 Section
4 Mast top detail

A Stays, 6 - 2.5 in ϕ strands at each
B Stay saddle, rests on concrete columns
C 32 radial steel girders, 4.5ft to 7ft deep

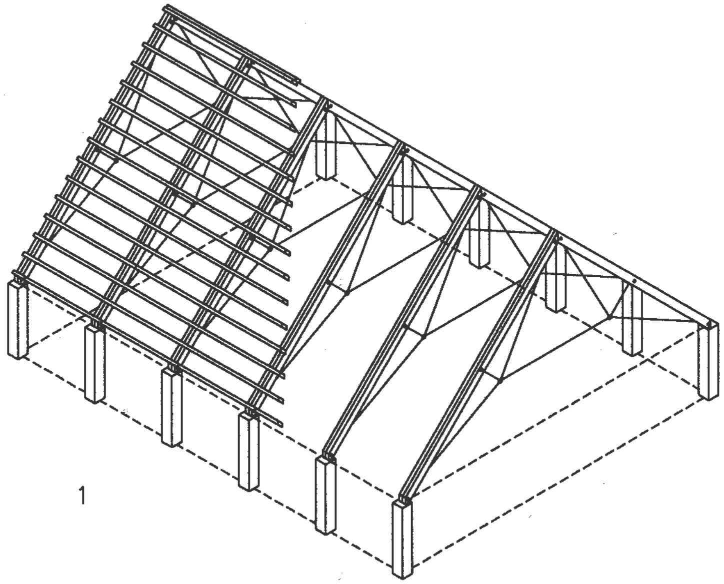


Propped Structures

Propped structures are supported from below rather than from above the horizontal span members. They may consist of one or more struts which, propped by a strand or rod, give elastic support to a girder. Struts require fixed connections to the girder to prevent rotation; or they may form triangular configurations. The connection of tension members may be concentric or eccentric. Concentric connections exert uniform compressive stress on beams. Eccentric connections may be designed to cause negative support moments that reduce the positive span moment and deflection.

A Concentric tie joint
B Eccentric joint (may reduce beam bending)

1 Twin struts with concentric tie connection
2 Single strut with eccentric tie connection to reduce beam bending
3 V-struts supporting two adjacent beams provide lateral bracing
4 Vertical and V-struts supporting three adjacent beams provide lateral bracing
5 Continuous propped beam
6 Gable with propped rafters supported by buttress to resist lateral thrust
7 Gable with propped rafters and tie rod to resist lateral thrust



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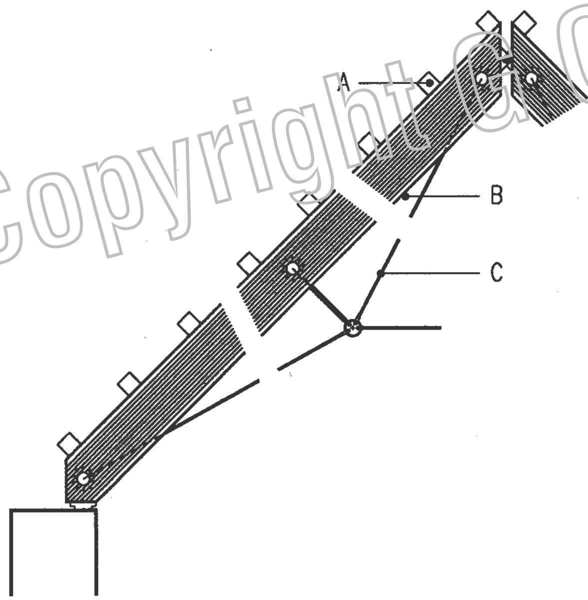

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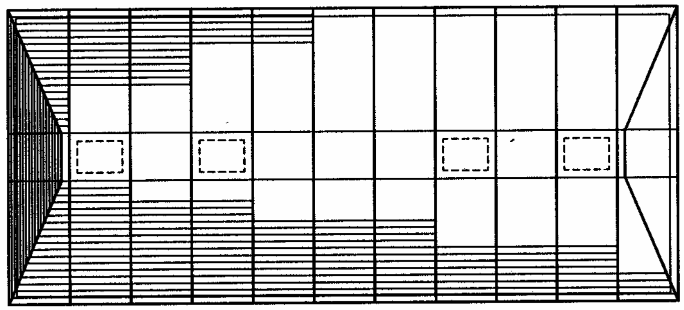
St. Martin Church, Ingolstadt, Germany (1981)
Architect: Hempel and Brand
Engineer: Sailer + Stepan

A simple gable roof over a rectangular plan defines the space of this church. The exposed wood structure adds natural warmth and a sense of balance. Six laminated three-hinge twin-girder assemblies, span 20m across the full width of the nave. The twin girders rest on concrete piers that cantilever from footings to resist gravity load, lateral wind load and part of the outward roof thrust. Roof purlins that span between girders support tongue-and-groove boards of diagonal patterns. The diagonal patterns stabilize the roof for lateral wind load.

1 Structure system
2 Half section of three-hinge twin girders with prop cable and strut

A Roof purlins; 20x20cm, spaced 1m
B Twin girders; 2-20x50cm, spaced 5m, span 20m
C Steel rods support twin girders and resists outward thrust



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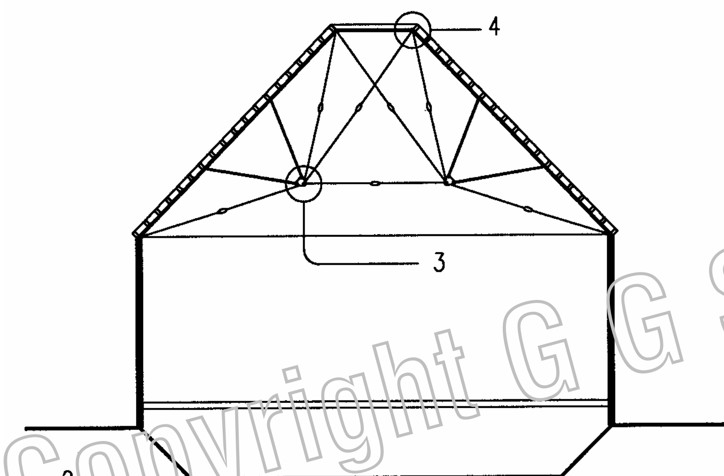

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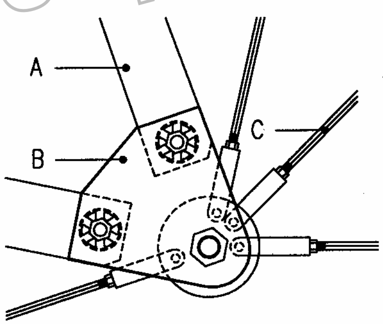

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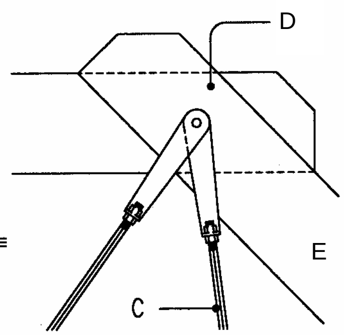

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Concert Hall, Snape, UK (1967)
Architect/Engineer: Ove Arup

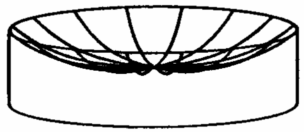
Remodeling this former malt house into a concert hall had to be done with care to preserve the original character of the malt house, including the roof shape with four large wood ventilation shafts. The concert hall with 840 seats measures 18.3x42m and has a height of 15.5m to the flat part of the trapezoidal roof. The roof trusses are spaced 3.8m, span 18.6 m and consist of:

- Twin rafters
- Compression struts
- Tension rods

The trusses are supported by peripheral walls. The twin rafters are propped by two diagonal compression struts which are supported by tie-rods that are part of the lattice truss. A compression strut links the rafters on top. Longitudinal joists support two layers of planks that make up the roof diaphragm.

1 Roof plan
2 Cross-section
3 Tension rod joint
4 Tension rod to rafter connection

A Diagonal compression struts, 9.5x11cm lumber
B Steel plates
C Tie-rods, 19mm diameter
D Top compression strut, 9.5x23cm lumber
E Rafters, 2 – 4.5x23cm lumber



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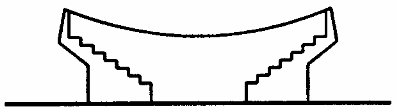

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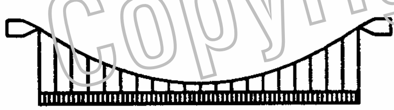

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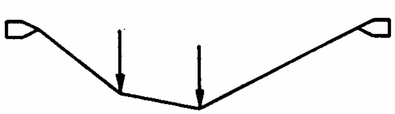

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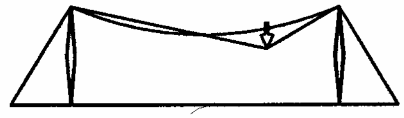

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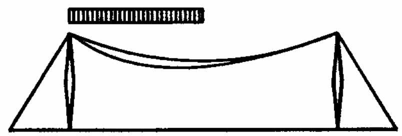

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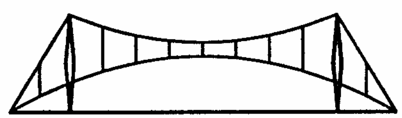

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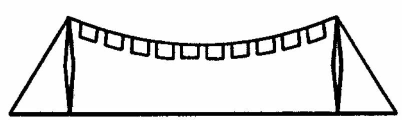


10

Suspended Structures

Suspended structures are used for long-span roofs. They are most effective if the curvature is compatible with spatial design objectives, and the horizontal thrust is resisted by a compression ring or by infrastructures, such as grandstands. Suspended cables effectively resist gravity load in tension, but are unstable under wind uplift and uneven loads. Under its own weight a cable assumes the funicular shape of a catenary (Latin for chain line). Under load uniformly distributed horizontally, the funicular will be parabolic; under point load the funicular is a polygon. Thus, without some means of stabilizing, cables assume different shapes for each load. Furthermore, under wind uplift suspended cables tend to flutter. Several means can be used to stabilize cables for variable loads and wind uplift. Among them are stabilizing cables, anticlastic (saddle-shaped) curvature, described later, and ballast weight. However, in seismic areas ballast weight would increase the mass and thus lateral loads.

1 Suspended roof with compression ring to absorb lateral thrust

2 Suspended roof with grandstands to resist lateral thrust

3 Catenary funicular under cable self weight

4 Parabolic funicular under horizontally distributed load

5 Polygon funicular under point load

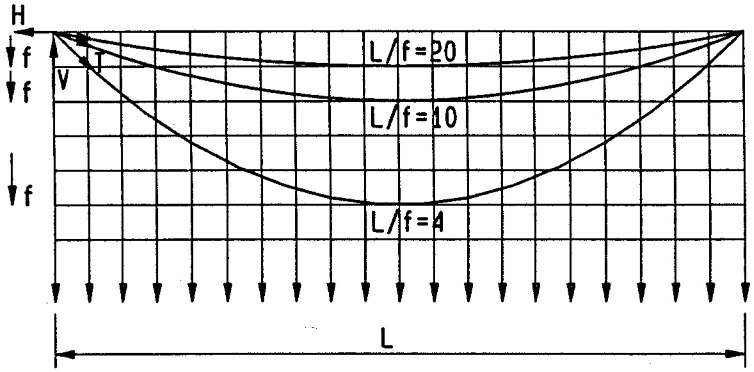
6 Deformed roof under point load

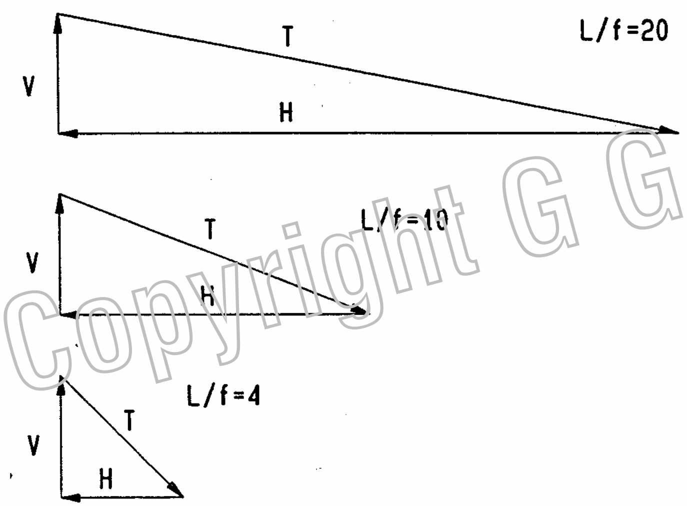
7 Deformed roof under uneven load (snow at one side, for example)

8 Roof subject to wind uplift

9 Roof with convex stabilizing cables to resist uplift and uneven loads

10 Dead load to resist uplift and reduce deformation under uneven load





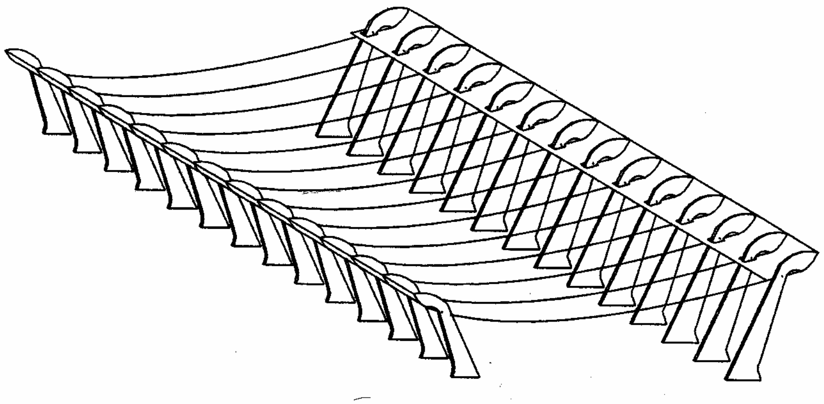
Span/sag ratio

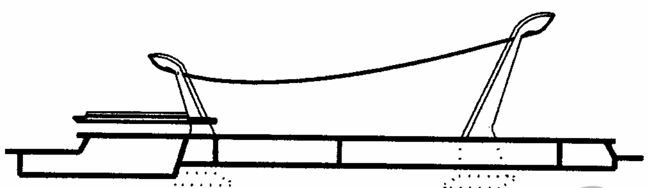
The span/sag ratio of suspended roofs is an important design factor (span is the horizontal distance between supports and sag the vertical distance between supports and cable low-point at mid-span). Considering constant gravity load, the effect of various span/sag ratios can be seen by equilibrium vector polygons at the supports. Constant gravity load causes approximately constant vertical reaction V for all sags, but horizontal reaction H and cable tension T vary with the span/sag ratio. Consider the cable at left under uniform load. The three equilibrium vector triangles below the cable clearly show:

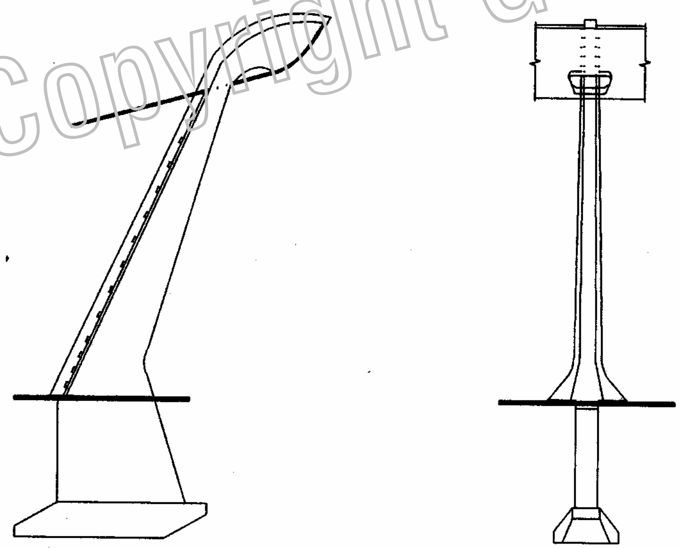
- A small sag (shallow roof) causes a large cable force and horizontal thrust
- A big sag has the opposite effect but requires tal and more costly supports

The optimal span/sag ratio is usually about 10, depending on space requirements

f Sag: distance between supports and cable low point at mid-span
L Length of span between supports
H Horizontal support reaction
T Maximum cable tension
V Vertical support reaction

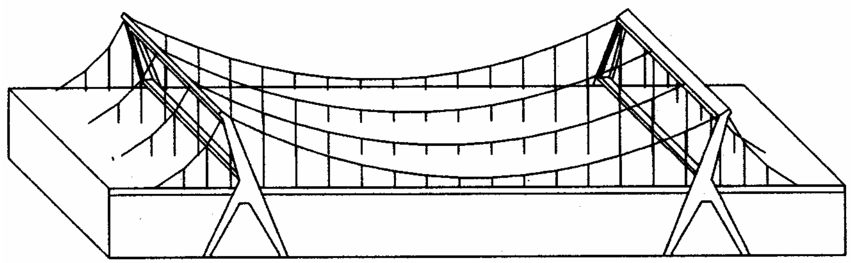






Dulles airport terminal, Washington, DC (1958-62)
Architect: Ero Saarinen
Engineer: Ammann and Whitney

The Dulles international airport terminal near Washington, DC, has a cable roof supported by concrete pylons. The outward leaning pylons partly resist the cable thrust. Based on the dimensions of movable loading docks, designed by Saarinen, the pylons are spaced at 40 ft (12m) for a column-free concourse space of 150x600ft (46x183m), recently expanded, extruding the same structure. Given the slanted pylons, the suspension cables actually span 161 ft (49 m). Concrete edge beams span the pylons at heights that vary from 65ft (20m) along the entry to 40ft (12m) facing the runways. Suspended from the edge beams are 128 bridge strands of ∅ 1in (25mm) which support site-cast concrete roof panels. The concrete dead weight resists wind uplift and minimizes roof deformations under unbalanced roof loads. In Saarinen's own words the Dulles roof is "a strong form between earth and sky that seems both to rise from the plain and hover over it." It presents functional integrity and synergy of form and structure.



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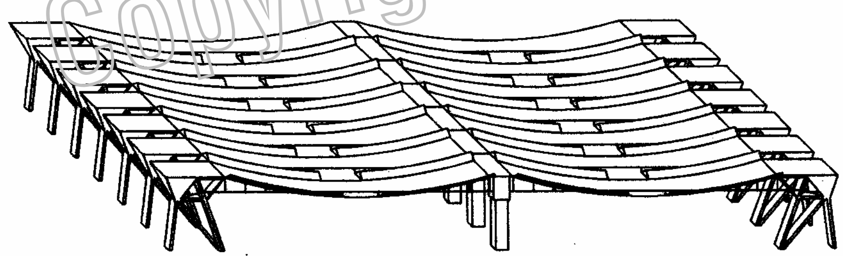

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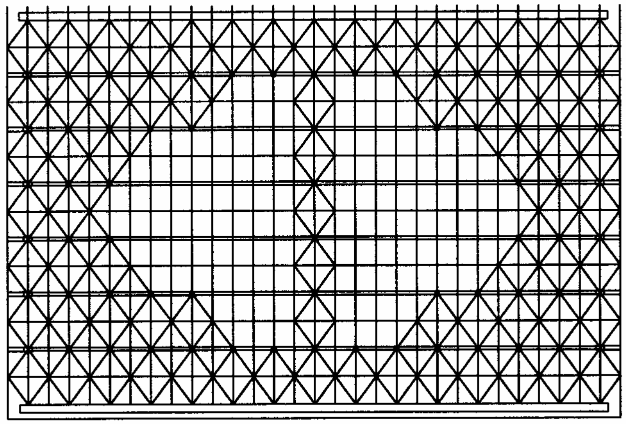
1 Burgo factory, Mantua, Italy (1961-63)
Engineer: Pier Luigi Nervi

The large production machines of this paper factory required a column-free interior space of 30x250m and a 140m opening between exterior supports. Nervi's solution was a roof structure like a suspension bridge. Two concrete frames support four parabolic cables from which a flat concrete roof is suspended by hangers. The frames are braced as inverted Y's for lateral stability in length direction. The suspension cables' lateral thrust is resisted in the concrete roof slab. A glass wall provides the non-structural enclosure.

2 Lufthansa aircraft hanger, Frankfurt (1968-72)
Architect: Beckert & Beckert
Engineer: Helmut Bomhard, Dyckerhoff & Witmann

This aircraft maintenance hanger of 100x270m accommodates up to six 747 jets. Large hanger doors required the roof to span the long way, with a concrete girder on two columns supports at mid-span. Recessed columns create overhangs to reduce the girder bending moment. The roof consists ten bands of pre-stressed, suspended concrete slabs, separated by linear gable skylights. Given an overall height limit of 34m for air traffic safety, and an interior height of 24m, the roof structure was limited to 10m depth for a span/depth ratio of 13.5 between supports. At both ends the suspension roof rests on inclined supports with ballast weight to resist lateral trust. Prismatic steel containers filled with concrete provide the ballast. Straight horizontal tension strands resist outward support displacement under wind uplift, restrain the ballast gravity load, and contribute to overall stability. Perpendicular struts tie the suspended slabs together for rotational stability. The curvilinear roof, flooded with natural light, creates a floating interior space, in contrast to the normally heavy material of concrete.



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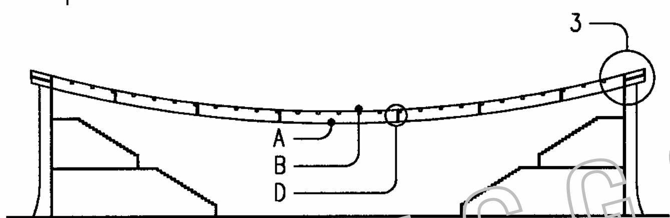

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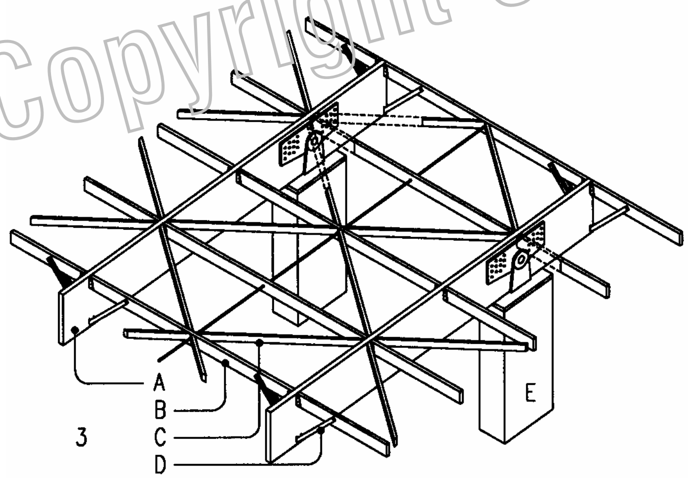

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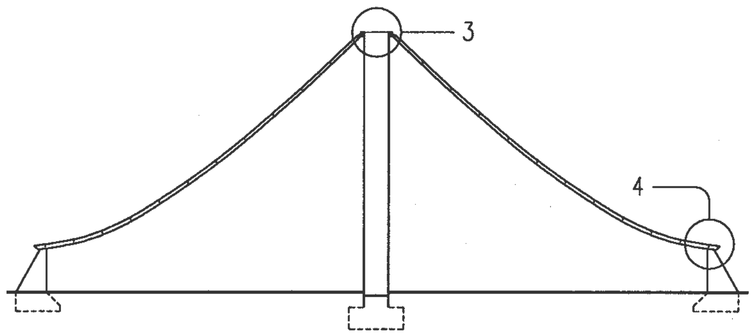
Sports hall, Dijon, France (1976)
Architect: J. F. Devaliere
Engineer: R. Weisrock, SA

With floor plan dimensions of 47.25x70m, this sports hall has a suspension roof spanning the length in response to an interior profile of spectator seating for 4,000. Glue laminated tension girders; spaced 6.75m are suspended from concrete piers with pin joints. They act primarily in tension, but have sufficient bending stiffness to resist deformation under unbalanced gravity load and wind uplift. To facilitate transportation, they are spliced at center. Glue laminated joists, spaced at 2.53m; support a metal roof with thermal insulation. Wood struts brace the girders to the joists against rotation. Roof heating is provided to remove snow. A grid of diagonal wood slats provides lateral wind bracing in the roof plane.

1 Roof plan
2 Cross-section
3 Isometric roof framing detail

A Glue laminated tension girders, 16x150cm
B Glue laminate joists, 11x33cm
C Diagonal wood bracing slats
D Girder bracing, 5x15cm



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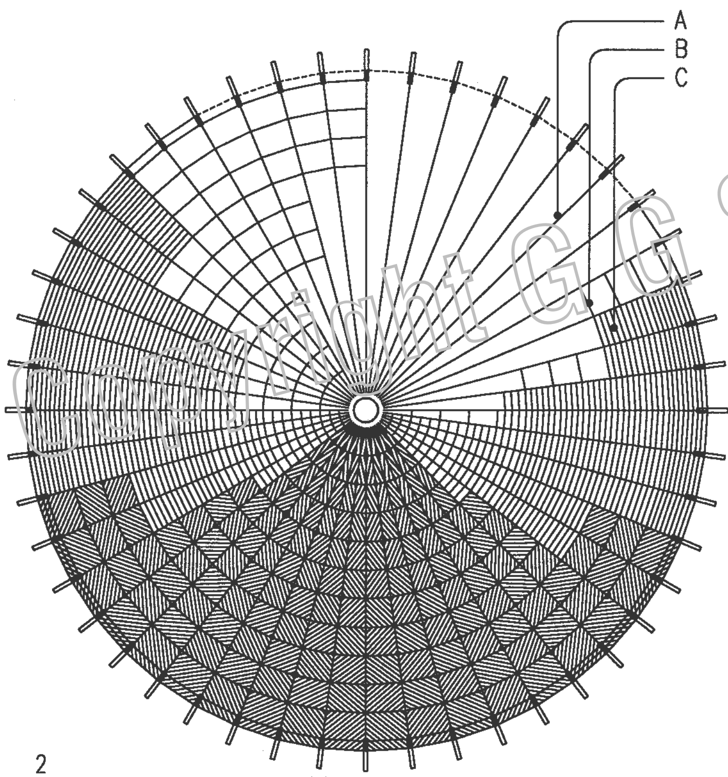

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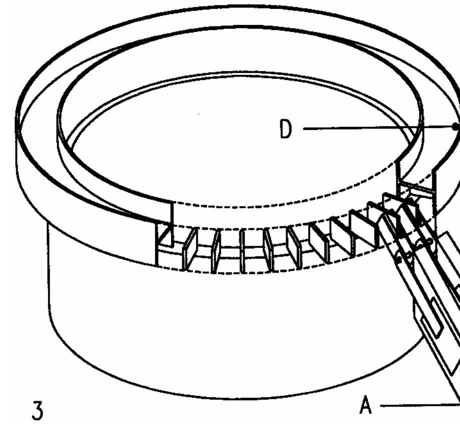
Recycling hall, Vienna (1981)
Architect: L. M. Lang
Engineer: Natterer and Diettrich

This recycling center features a tent-like wood structure of 560 feet (170m) diameter that soars to a height of 220 feet (67m) above ground, supported by a central concrete mast. The suspended wood roof consists of 48 radial laminated ribs that rise from outer concrete pylons with wood compression ring to the mast top. The ribs follow the funicular tension line to carry uniform roof load in pure tension, but asymmetrical loads may cause bending stress in the radial ribs that are designed as semi-rigid tension bands with some bending resistance capacity. Diagonal boards form the roofing membrane and add shear resistance to the assembly of ribs and ring beams. The cylindrical concrete support mast cantilevers from a central foundation. It was designed to resist asymmetrical erection loads and to contribute to lateral wind load resistance. The peripheral pylons are triangular concrete walls with metal brackets on top to secure the radial ribs.

1 Cross section
2 Roof plan
3 Top of central support mast
4 Typical roof assembly

A Radial laminated wood tension rib, 7.8x31-43 (20x80-110cm)
B Laminated wood ring beams, 5x15in (12x39cm)
C Laminated wood compression ring
D Steel tension ring
E Steel anchor bracket



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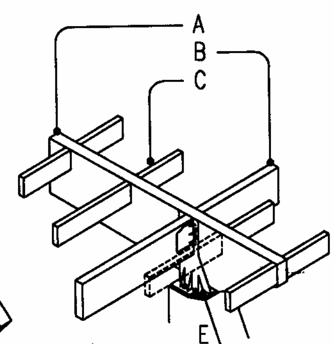

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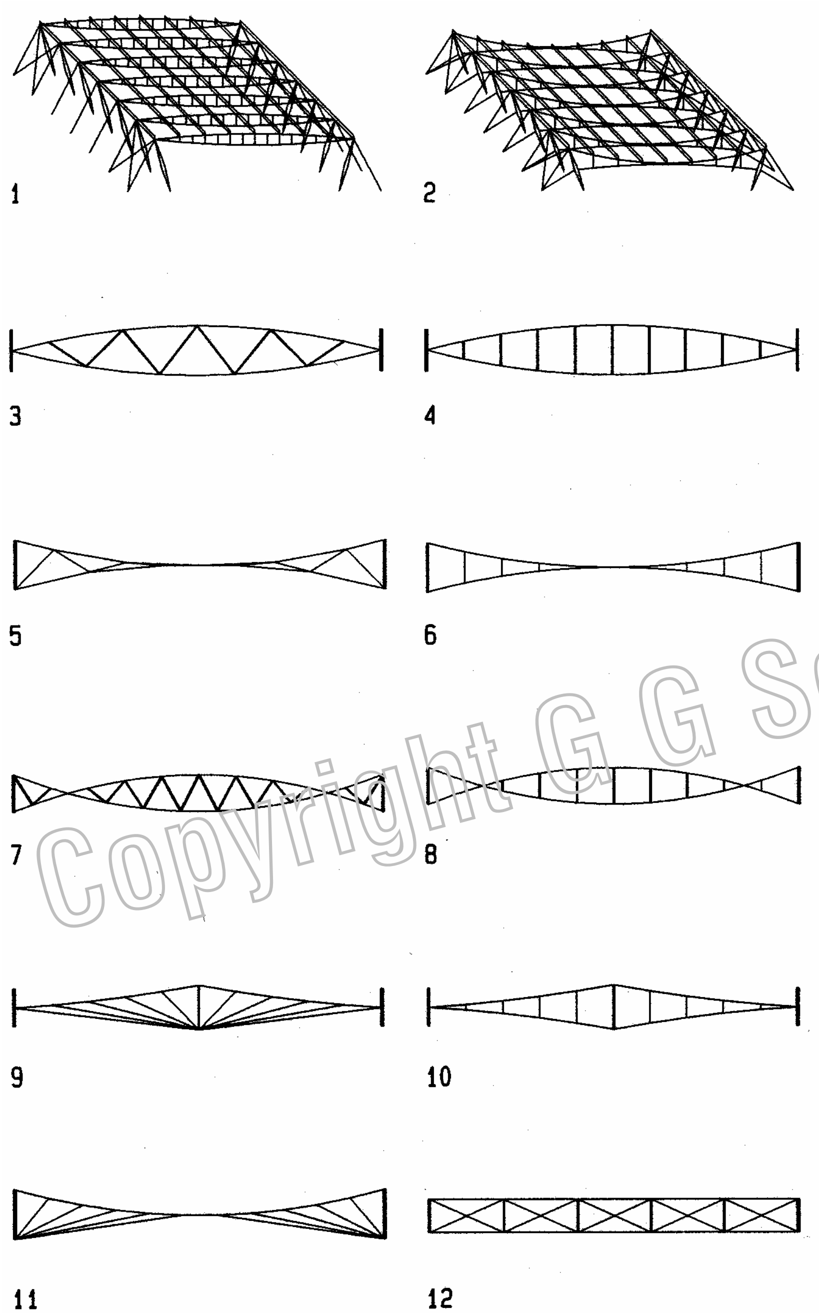
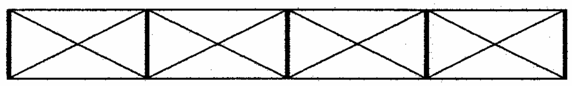


Cable Truss

Cable trusses evolved from needs to stabilize suspension structures against wind uplift and unbalanced gravity loads, using a second set of cables with opposing curvature. The Swedish engineer Jawerth developed a cable truss with diagonal brace cables separating top support- and bottom stabilizing cables that resist wind uplift. This system was widely used in the 1960's. Lev Zetlin and other US engineers designed cable trusses with various other configurations, including lintel shapes with compression struts separating bottom support- and top stabilizing cables. In 1969 the author and his students at UC Berkeley developed trusses with flat chord cables separated by compression struts and diagonal truss cables. Model tests, a full scale prototype, and extensive computer analysis demonstrated great stiffness of these trusses in one-way, two-way, and three-way layouts.

1 Isometric of lintel trusses with bottom support- and top stabilizing cables separated by vertical compression struts
2 Isometric of concave trusses with top supporting- and bottom stabilizing cables separated by vertical tension struts
3 Lintel truss with diagonal compression braces
4 Lintel truss with vertical compression struts
5 Concave truss with diagonal tension braces
6 Concave truss with vertical tension struts
7 Concave/lintel truss with diagonal compression braces
8 Concave/lintel truss with vertical compression struts
9 Concave gable truss with fan support and stabilizing cables and central compression strut
10 Concave gable truss with tension struts and central compression strut
11 Concave support cable and fan stabilizing cables
12 Parallel chord truss, vertical compression struts and diagonal tension braces



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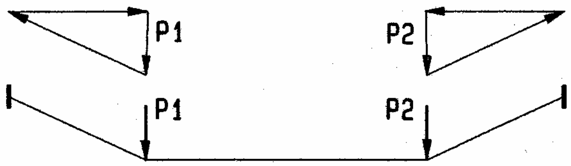

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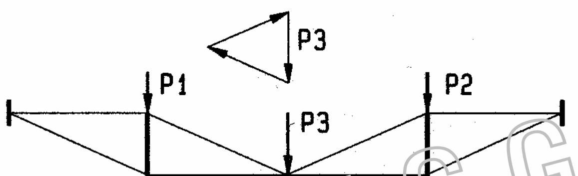

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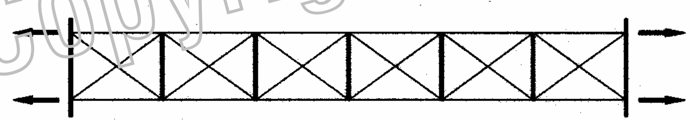

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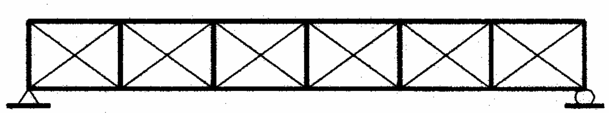

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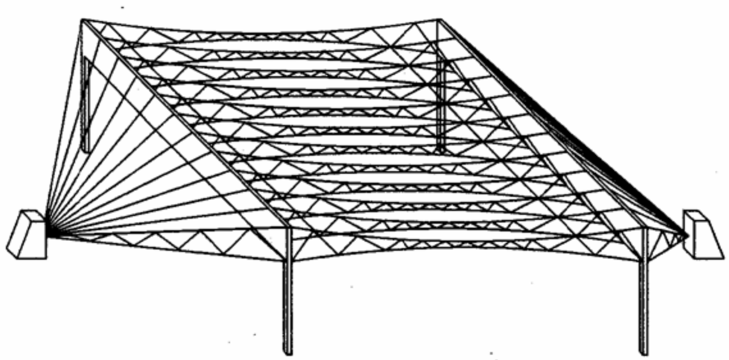
Parallel chord cable truss

The load bearing mode of parallel chord cable trusses is more complex than that of concave or lintel type trusses since they have no funicular cable but it may be explained as follows.

Consider a four-bay truss with loads P1 and P2. They are transferred to the supports be a polygon formed by the center bay bottom chords and end-bay diagonal braces. A third load applied at the center strut is transferred by a second polygon in conjunction with the latter one. Thus, half the bars resist the load in active tension and the other (passive) bars resist the load by reducing prestress. For uplift wind load the load bearing is reversed with active bars becoming passive and vice versa. This load bearing mode applies also to trusses with more bays, as long as they are prestressed in order for passive bars to resist load by reducing prestress. In these trusses the prestress must be externally stabilized. Trusses with compression chord bars may be internally stabilized and simply supported. In order to avoid slack cables, prestress must be at least half of the design load stress as described at the beginning of this chapter. When prestress approaches zero under load, the bar forces are about equal to those in a conventional truss under equal load and proportions and can be found by graphic vectors or other static methods.

1 Externally prestressed cable truss with four bays
2 Load bearing polygon formed to resist two loads
3 Load bearing polygons to resist three loads
4 Externally stabilized truss with six bays
5 Internally stabilized truss with six bays



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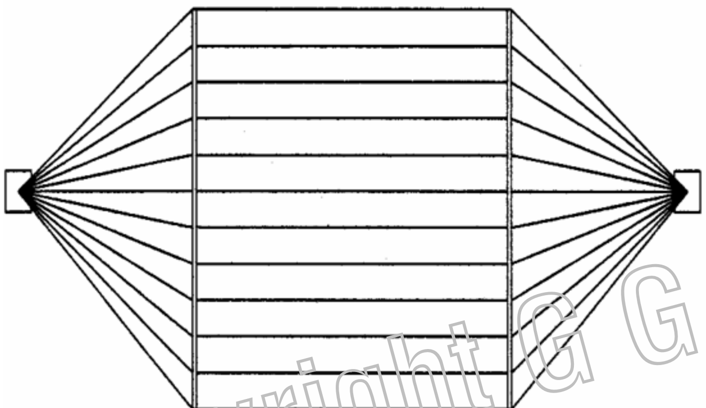

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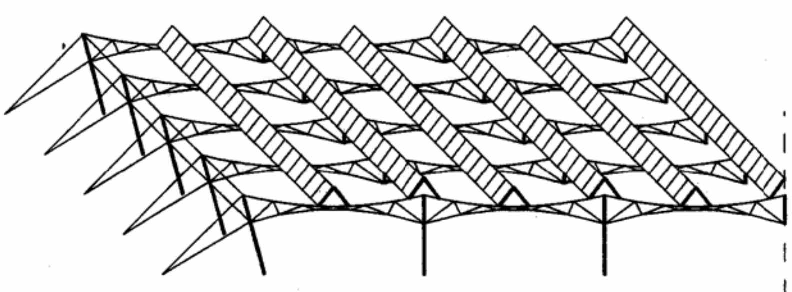

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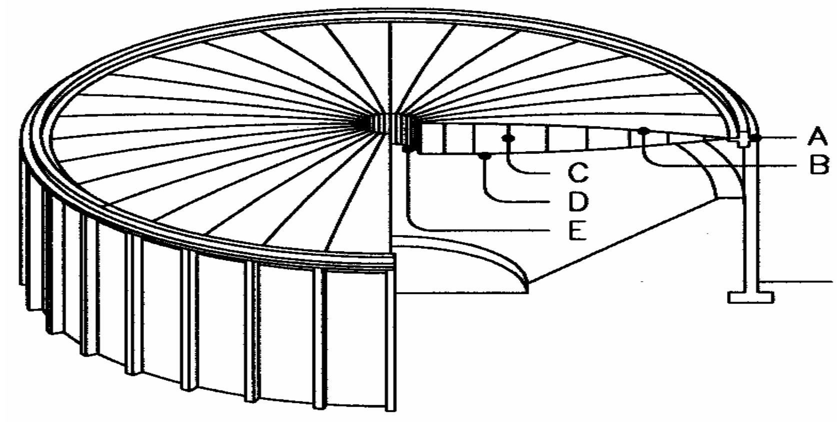
1 Open air theater Ötigheim, Germany (1961)
Architect: E. Heid
Engineer: Jawerth

The roof structure for this largest German outdoor theater resembles a hammock with cable trusses that span between two girders that are supported by two columns each. The cable trusses span 37 m (121 feet) between the girders and converge to two ground anchors. The truss depth of 3.6 m (12 feet) results in a span/sag ratio of about 20 for concave and convex cables that support gravity load and wind uplift, respectively. Crescent-shaped seating layout, with two columns near the front edge, provides unobstructed views for most seats. The prestressed cable trusses stabilize the girders against buckling and rotation. The roof consists of metal deck and two membranes over rigid insulation. The rigid insulation dampens rain water pounding, rather than providing thermal insulation which is not needed for the outdoor theater.

2 Plan, open air theater Ötigheim
The plan shows the area between side girders represents as square of 37x37 m (121x121 feet). The radial convergence of cable trusses toward ground anchors induces compressive stress in addition to bending stress in the girders. The girder bending moments could have been greatly reduced by recessing the columns to provide overhangs of about 1/3 the span between columns. The column recess would have also improved unobstructed views from most seats.

3 Factory at Lesjöers, Sweden
Architect: Lennart Bergström
Engineer: Jawerth

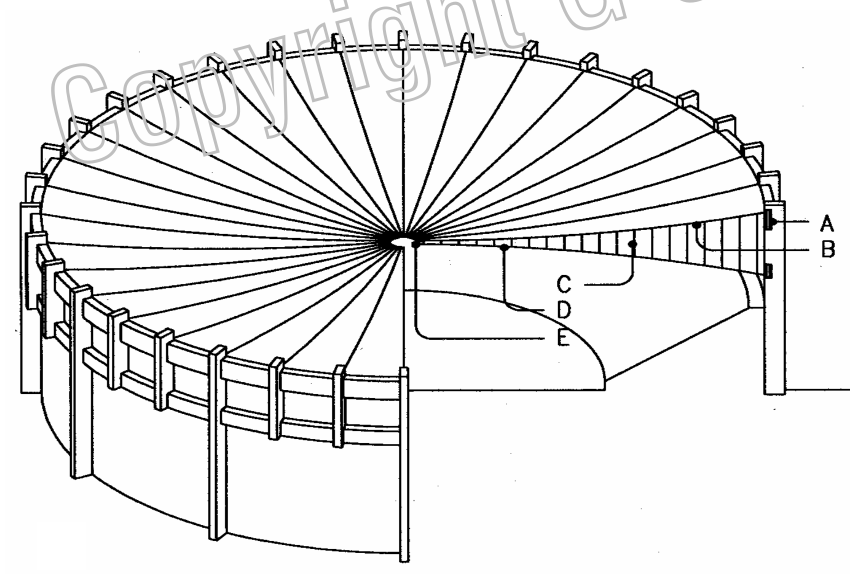
The factory features cable trusses, spaced 4 m (13 feet) with intermediate supports. Five bays of 16 m (53 feet) and two end bays of 6 m (20 feet) provide a total length of 92 m (302 feet). Continued arrangement balances lateral trust of adjacent trusses. Linear skylights over the supports and at truss mid-spans provide natural lighting. The inclined end supports equalize forces in guy cables and truss cables. The angle of inclination can be determined by graphic vectors: equal angles between mast and cables causes imply equal cable forces.



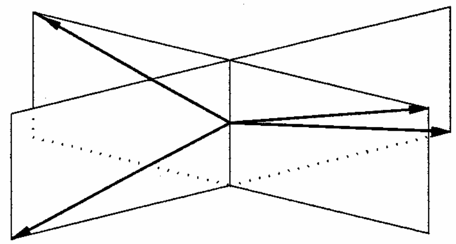
Auditorium Utica, USA (1958)
Architect: Gehron and Seltzer
Engineer: Lev Zetlin

For a seating capacity of 6,500, the Utica auditorium has a circular plan of 240 ft (73m) diameter. Radial cable trusses of concave lintel profile provide the roof structure. Compression struts separate bottom and top strands. The cable trusses are supported by a circular exterior concrete compression ring and connected to two steel tension rings at the center. The circular compression ring is a highly efficient method to support cable roofs, eliminating the need for costly lateral supports. The cable trusses are prestressed by jacking the central tension rings apart. Different lengths of top and bottom chords induced different prestress and natural frequencies to reduce vibration due to wind gusts. The bottom support cables are 2in (50mm) zinc coated strands with 175kip prestress. The stabilizing top cables are 15/8in (41mm) strands with 135kip prestress. The roof was erected in three weeks, with temporary support of the central tension ring only.

A Circular concrete compression ring
B Top stabilizing cable, 15/8" (42mm) strands
C Steel compression struts
D Bottom cable, 2" (50mm) strands
E Steel tension ring



Convex alternate
It is of interest to consider the implications of an alternate convex roof. Cable trusses of convex profile require two outer compression rings but only one central tension ring. The vertical compression struts can be replaced by tension rods. Two compression rings would likely cost more than a single compression ring since compression rings have to be designed to resist buckling under unbalanced load... The inward sloping roof would require rain water to be removed by pumps, but the concave Utica roof is self-draining.



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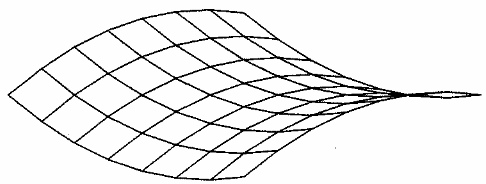

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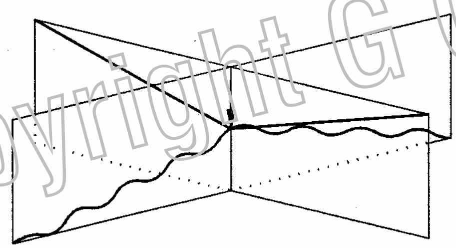

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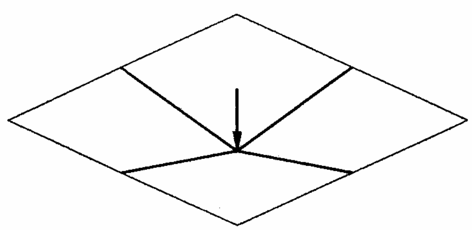


4

Anticlastic Structures

Anticlastic tensile structures are flexible membranes or cable nets of saddle-shaped curvature. The term membrane is used here to imply membranes and cable nets. Given the nature of flexible membranes, double curvature and prestress are essential for stability. This can be observed with simple string models. Two strings pulled in opposite directions stabilize a point at their intersection. If the strings are in non-parallel planes the stability will be three-dimensional. Similarly, if a series of strings cross in opposite directions they stabilize a series of points at their intersection. The cross points form a surface, stabilized by anticlastic curvature. The surface may be a membrane of fabric or other material or a cable net. Although anticlastic curvature provides stability, some elastic deformation is possible due to material elasticity. Thus, steel cable nets with high elastic modulus deform less than fabric membranes with lower stiffness.

In addition to curvature, prestress is also required to stabilize anticlastic membranes. This too can be observed on a string model. Applied load elongates one string in tension and shortens the other in compression. Without prestress, the compressed string will get slack and unstable; but prestressed strings absorb compressive stress by reduction of prestress. Since prestress renders both strings active to resist load, the resulting deflection is reduced to half compared to non-prestressed condition where only one string is active. This observation is also described under *Prestress* at the beginning of this chapter.

Flat membranes are unstable. This, too, can be observed on a string model. Two strings in a flat surface must deform into a polygon to resist load (a straight string would assume infinite forces). Therefore, flat membranes are unstable under load. Similarly, synclastic (dome-shape) membranes would deform excessively under gravity load and flutter in wind.

1 Two strings crossing in non-parallel planes stabilize a point in space
2 A series if strings (or a membrane) form a stable surface
3 Without prestress, one string (or series of strings) would get slack under load, causing instability
4 Strings in a flat surface deform excessively under load, causing instability

Minimal surface

As the name implies, a minimal surface covers any boundary with a minimum of surface area. The minimal surface is defined by three criteria:

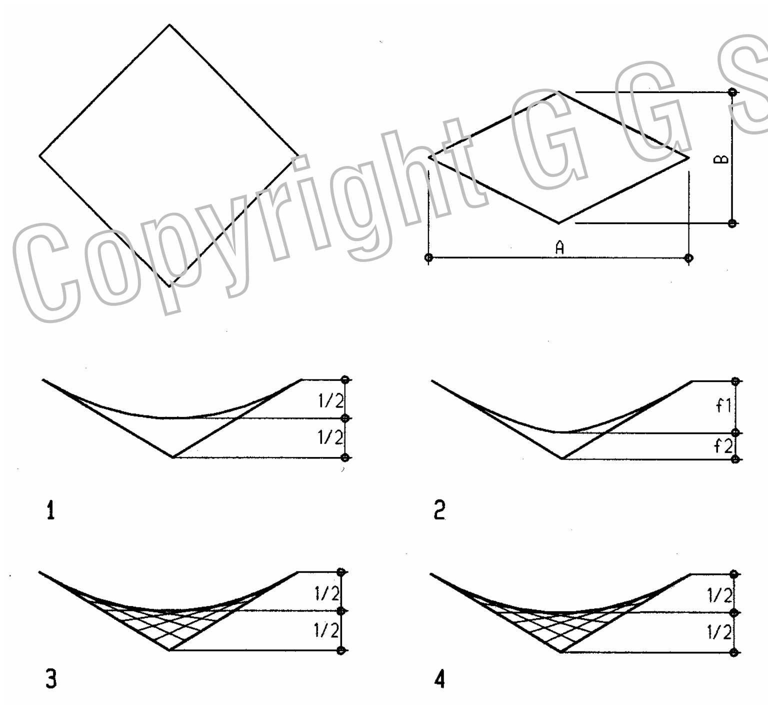
- Minimum surface are between any boundary
- Equal and opposite curvature at any point
- Uniform stress throughout the surface

A minimal surface may be anticlastic or flat. A surface of flat or triangular boundaries is always flat. Flat membranes are unstable structures. Increased curvature increases stability. The minimal surface can be studied on soap film models; but they disappear quickly. The author studied quadrilateral plastic models that keep a minimal surface after drying. The models revealed:

$$f1/f2 = A/B$$

This is contrary to Hyperbolic Paraboloid shells. The surface of HP shells passes at mid-height between low and high points regardless of boundary conditions.

1 Minimal surface of square plan
2 Minimal surface of rhomboid plan
3 Hyperbolic Paraboloid of square plan
4 Hyperbolic Paraboloid of rhomboid plan



Minimal surface equations

The minimal surface models also revealed equations that define the principle curvature of equilateral minimal surfaces (Schierle, 1977):

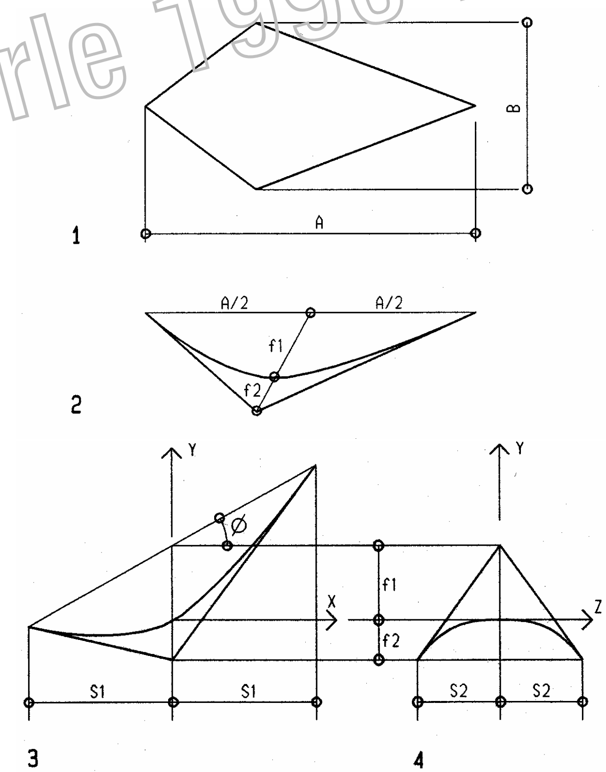
$$Y = f1(X/S1)^{(f1+f2)/f1} + X \tan \phi$$

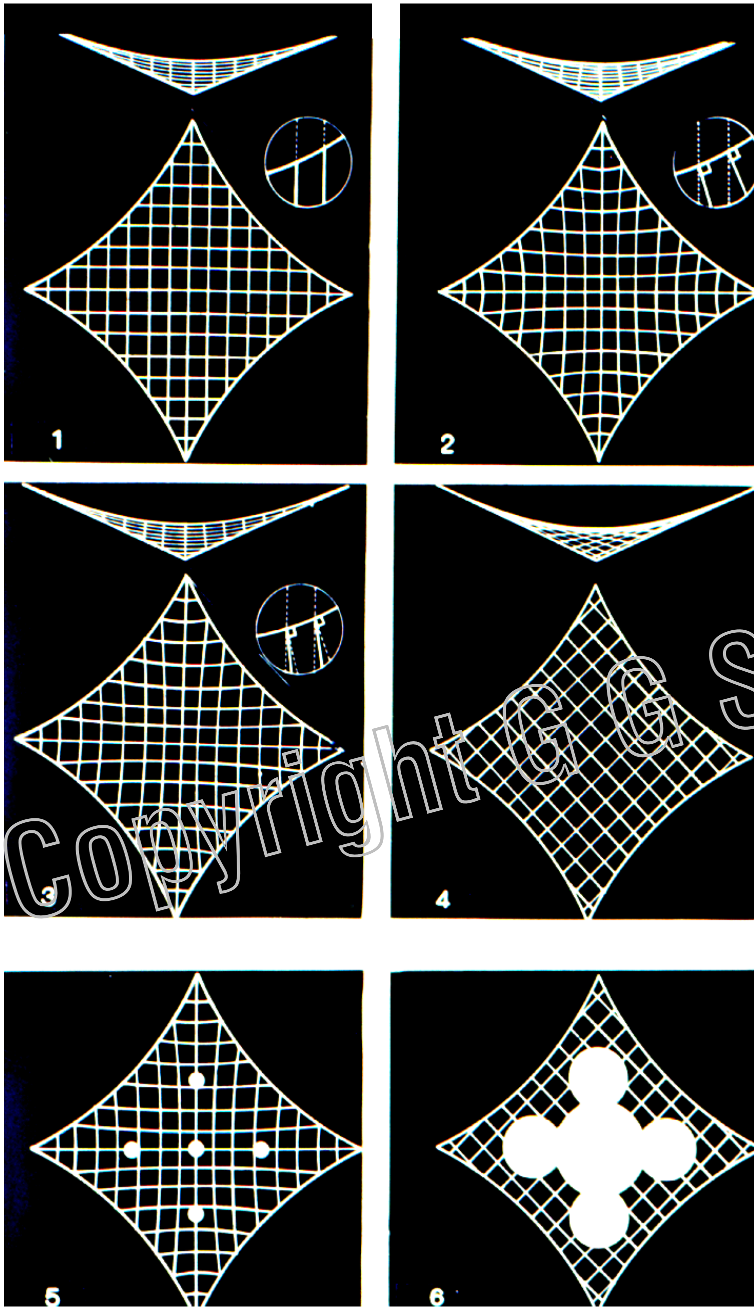
$$Y = f2\,(Z/S2)^{(f1+f2)/f2}$$

The equations are based on empirical studies of minimal surface models of plastic film, measured by means of a projected light grid with an accuracy of only 1.26% standard deviation. The findings were first published in the *Journal for Optimization Theory and Application* (Schierle, 1977)

Although the equations are for minimal surface of quadrilateral plans, they provide reasonable accuracy for other boundaries as well. This should be further studied.

1 Plan view of quadrilateral minimal surface
2 Length section of quadrilateral minimal surface
3 Length section with Y-axis vertical
4 Cross section





Cable nets

Anticlastic cable nets may approximate a minimal surface. However, even if mesh cables have equal prestress, minor mesh distance variations cause uneven density and stress distribution. Various cable net configurations are possible. The method of manufacture is a primary factor in defining a cable net. If assembled on-site, support cables may be strung from edges to hang in vertical planes. Stabilizing cables may also be placed in a vertical plane position. Such a net could have perfectly square meshes in plan projection. Cables could also be arranged following the principle curvatures with the least distance across the surface. Such a net is more complex to manufacture. A cable net can also be prefabricated as an orthogonal grid of equal meshes flat on the ground. The meshes deform from squares into rhomboids to assume the anticlastic curvature. This method was used for the German pavilion of the 1968 Montreal Worlds fair and for sports facilities of the 1972 Olympic Games in Munich. Cables parallel to the edges are nearly straight, like generating lines of a Hyperbolic Paraboloid shell. Such flat cables will deform much more under applied load than curved cables. This was first demonstrated by tests the author conducted with students at the University of California, Berkeley in 1967. One cable nets was tested with curved cables and the other with flat cables. Under equal load the net with flat cables deformed about six times more than the one with curved cables. The test results, drawn as dots on both nets, show the great difference in stiffness. It is clear from this test, later confirmed by computer, that nets with flat cables are not a viable solution as structures. The test results have been widely published.

1 Net with cables arranged in vertical planes, with square grid projection

2 Net with cables in direction of principle curvature

3 Net of square meshes, prefabricated flat, deform into rhomboids in space

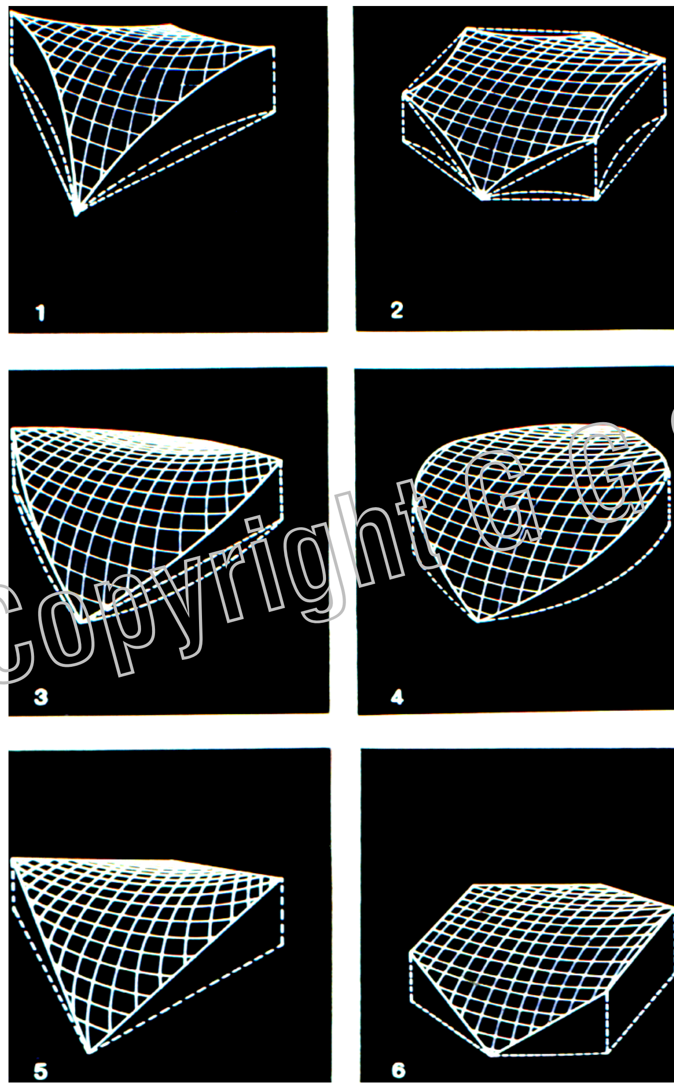
4 Net of cables running nearly straight in direction of generating lines

5 Net 3 under gravity load (dots show small deformations)

6 Net 4 under gravity load (dots show 6 times greater deflection than net 3)

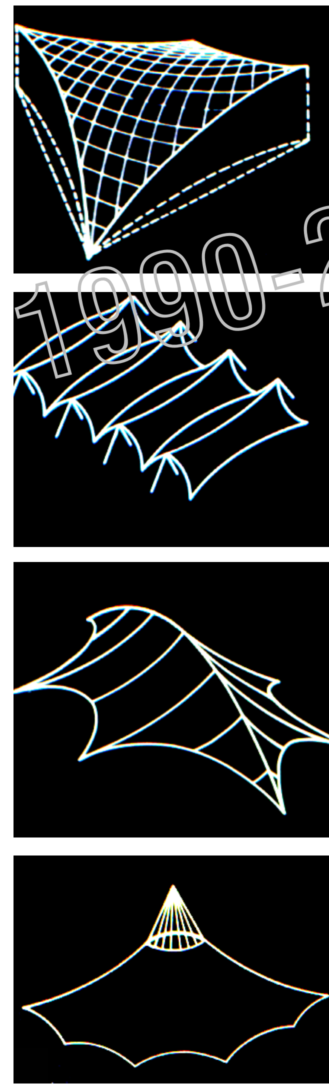
Edge conditions

Anticlastic structures may have three edge types: **cable, arch, beam/truss.** Cables resist membrane stress in tension, forming a curve of equal radius in space under uniform prestress. Arches resist primarily in compression, with bending under some loads. Edge beams resist in bending and trusses axial mode. Regarding architectural objectives, straight beams or trusses are easy to connect walls, but edge cables require more complex enclosures.

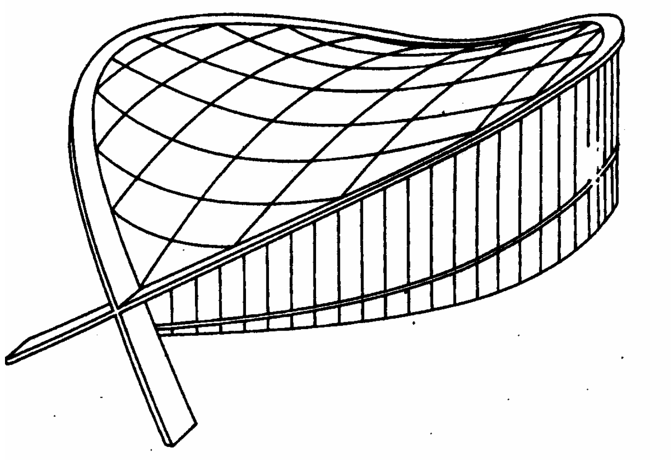


Surface conditions

Anticlastic membranes may have four surface conditions with many variations: **saddle shape, wave shape, arch shape, point shape**. The most appropriate type for a given situation is determined by architectural and structural considerations.



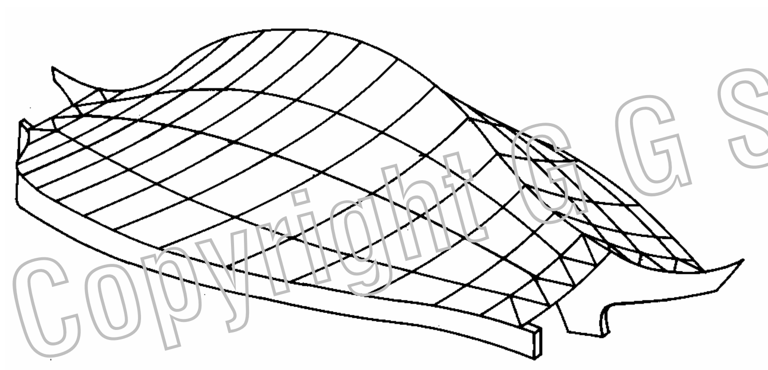
Pioneering structures (1953)



State Fair Building, Raleigh, North Carolina

Architect: Novicki and Deitrick
Engineer: Severud, Elstad, Krueger

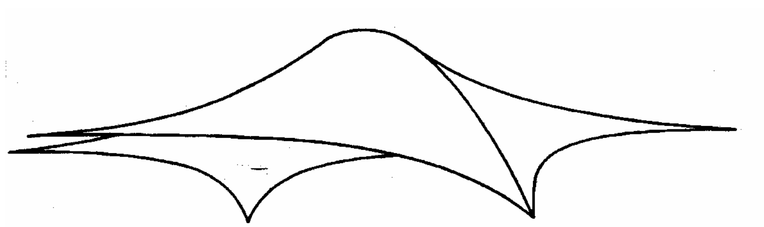
Built in 1953, the Raleigh arena was the first saddle shape roof of curved cables (Schuchow's 1889 Nischni Nowgorod exhibit anticlastic cable roof had straight cables. The support cables span 300 ft with 31 ft sag, for a span/sag ratio of 9.7. The concave support cables, spaced about 6 ft vary from ¾ to 15/16 inch, the convex cables vary from ½ to ¾ inch. The concrete compression arches are inclined about 22 degree to the horizontal and supported by concrete covered steel columns. The roof consists of corrugated metal with thermal insulation.



Hockey rink, Yale University (1958)

Architect: Ero Saarinen
Engineer: Severud, Elstad, Krueger

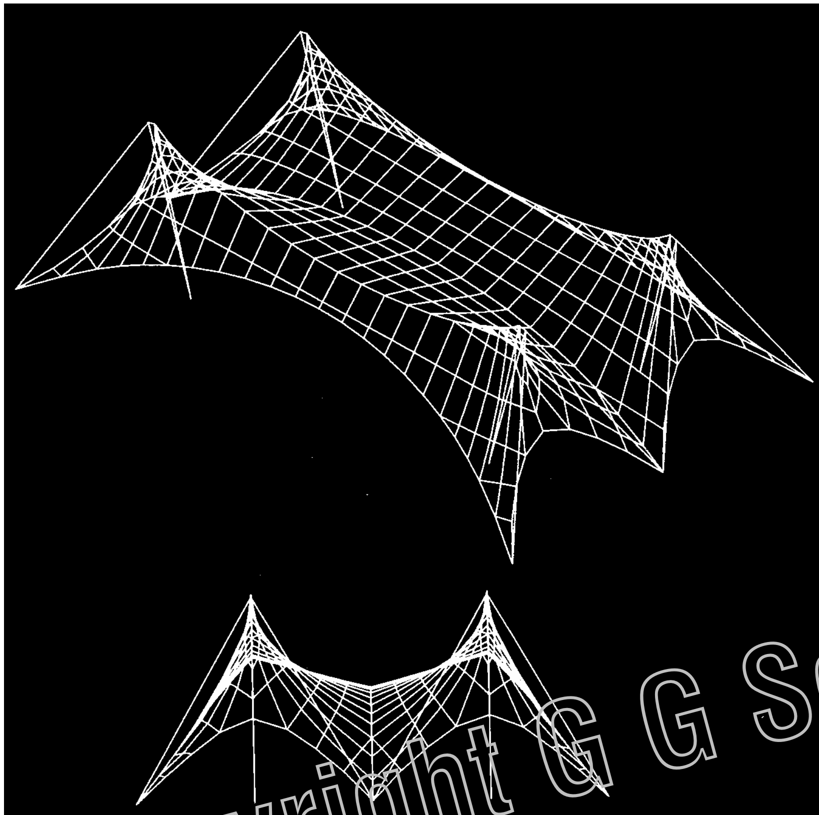
The 200/85 feet rink, built 1958, has an arch supported anticlastic cable net. The central concrete arch is designed to resists unbalanced load in bending, rather than using the cable net for stability. Both support and stabilizing cables are 1¾ inch diameter, spaced 6 feet. The roof consists of 2x8 inch wood planking, nailed to 2x6 inch wood strips. The oval plan provides most spectators seating at the preferred location near the rink center.



Entry arch Kőln (1957)

Architect: Frei Otto
Engineer: Fritz Leonhard

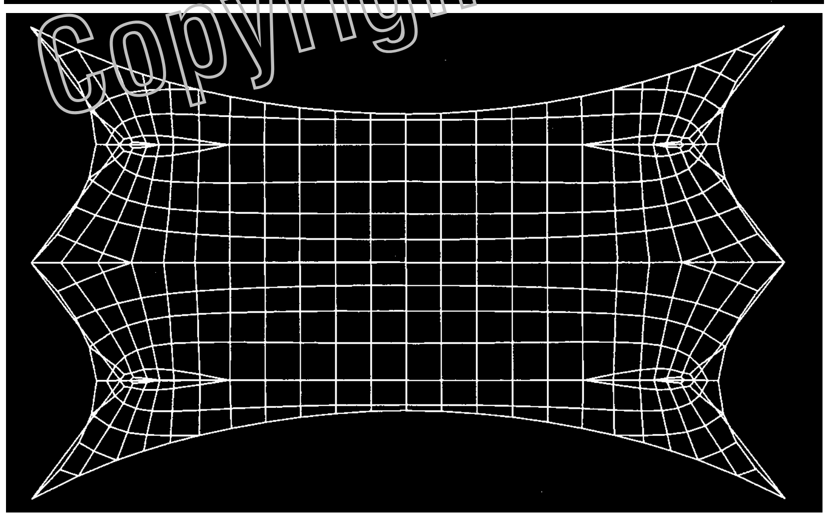
This arch marked the entry of the 1957 Kőln garden show. The steel arch of 112 feet span and only 7.5 inch diameter is stabilized by the membrane to resist unbalanced loads and wind uplift. The membrane, projecting 39 feet on both sides of the arch is prestressed by edge cables which are supported by four steel masts.

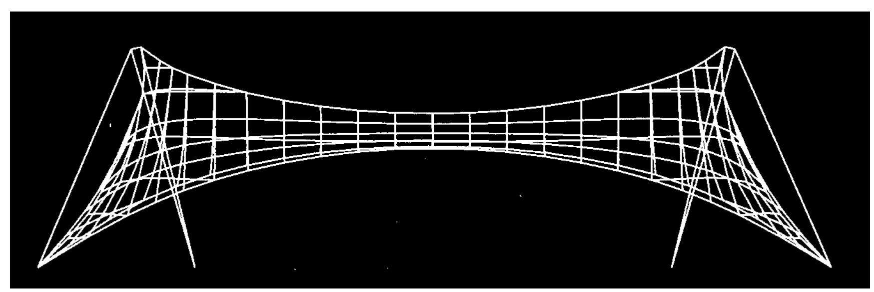


Wave shapes

Recycling center project, Mil Valley, California (1971)
Architect: Neil Smith and G G Schierle
Engineer: G G Schierle

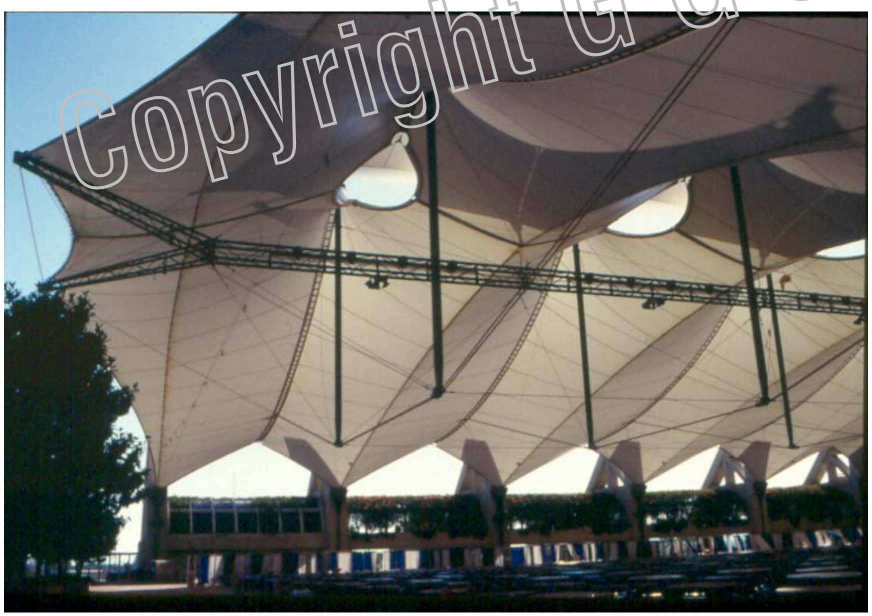
This structure was designed to expand easily in response to needed growth, using a modular system. A base module of 25x80ft (7.6x24m) is supported by 22ft (6.7m) high masts. Two half end modules provide enclosure at both ends. Only one base module with both end enclosures is shown here. For sustainable energy efficiency the membrane was designed of translucent natural canvass allowing natural daylight. Edge, ridge, and valley cables where designed as bridge rope for flexibility in adjusting to the curvatures. Membrane prestress was introduced by turnbuckle adjustment at cable ends anchored to helix ground anchors. Variable prestress was required in order for the ridge cables to remain in vertical plains as required for repeatability of the modules. The prestress levels were determined by computer analysis. Mats were designed as standard steel pipes with pin joint attachment to the foundation. The pin joints avoided bending stress for optimal efficiency; moment resistant joints would introduce bending stress in the masts under any movement.

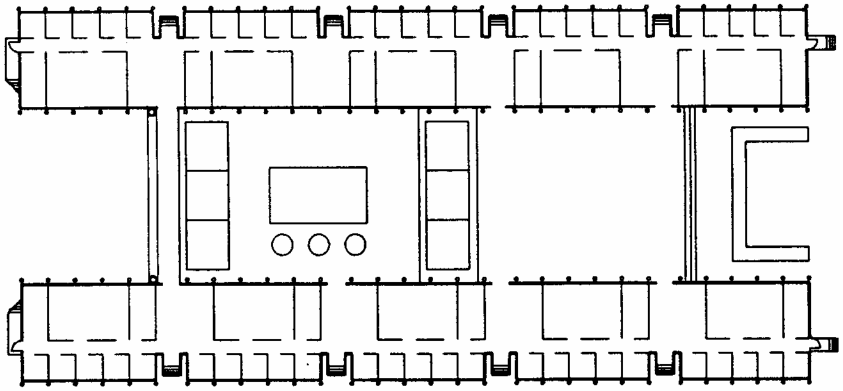




San Diego Convention Center (1987-89)
Architect: Arthur Erickson and a joint venture team
Engineer/membrane designer: Horst Berger

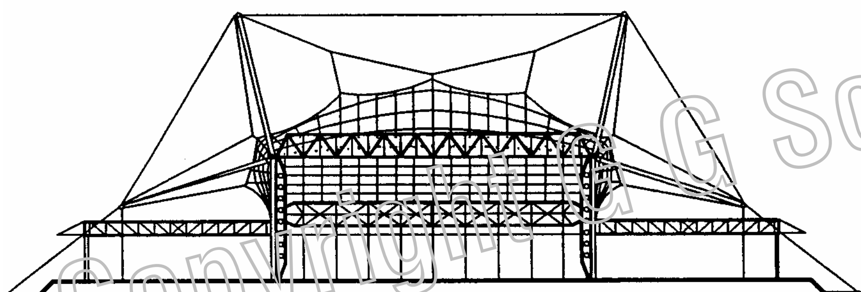
The San Diego Convention Center features a linear plan of 1.7 million square feet (157,935 m^2). Part of the top level is designed as semi-outdoor exhibit space, covered with a wave-shape membrane roof covering an area of 300 x 300 feet. The membrane undulates between ridge and valley cables that are suspended from triangular concrete pylons spaced 60 x 300 feet. Openings at membrane ridges provide natural ventilation. The openings are protected by secondary membranes hovering above. Flying buttress masts, supported by guy cables, hold up the ridge cables to provide a column-free space. The guy cables are also suspended from the triangular concrete pylons. This creative support system makes the translucent roof appear hovering seemingly weightless above the space which is flooded in filtered natural light. The lightweight roof provides a stark contrast to the conventional concrete infrastructure. The concrete pylons reinforce this contrast with compelling elegance. The Teflon-coated glass fiber membrane provides a fireproof enclosure as required for permanent structures.

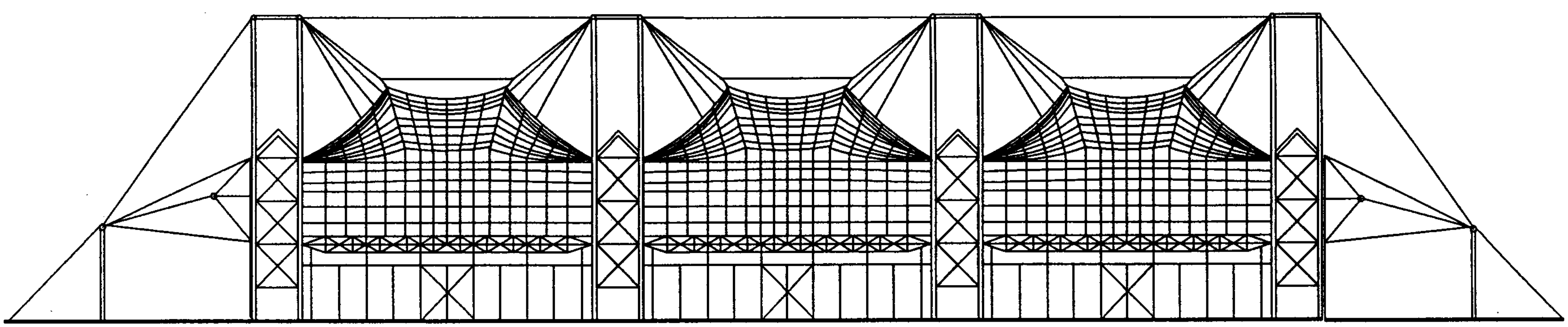


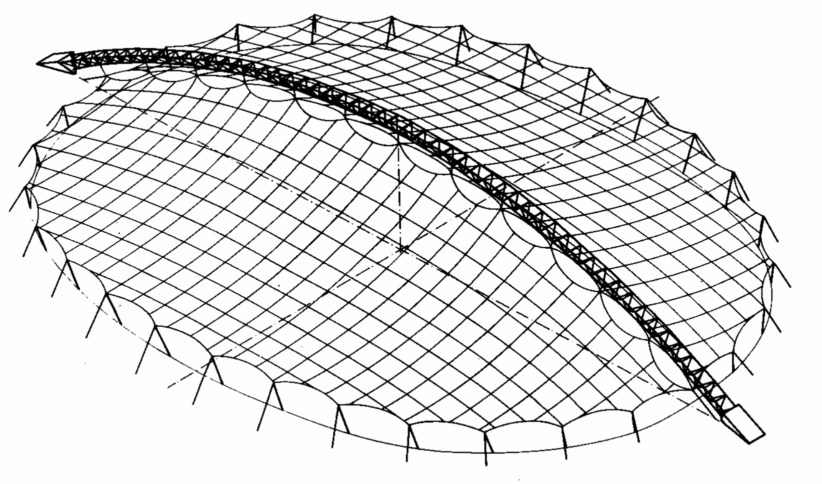


Schlumberger Research Center, Cambridge, UK (1984)
Architect: Michael Hopkins
Engineer: Ove Arup

The Schlumberger Center conducts basic research for oil exploration which includes drilling, fluid mechanics, etc. that work in close cooperation. Thus a major objective was to facilitate contacts among theoretical and experimental researchers as well as administrators. The client also required an option for future expansion. The design features a 24m wide central space for drilling equipment and a recreation area between two single story research wings with private offices, discussion rooms and laboratories, all separated from the central space by 21mm thick sound insulation glass. The central space is covered by a removable fabric, suspended from a network of cables that are supported by braced steel frames. Three wave-shape membranes have two ridge cables each. The ridge cables are suspended from overhead guy cables. The translucent fabric provides natural lighting and is removable to provide access for replacing the drilling equipment.

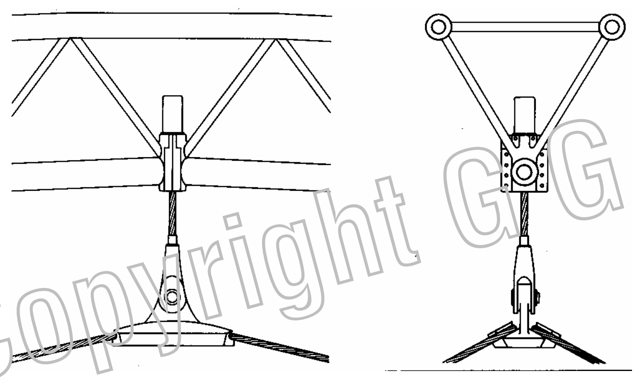


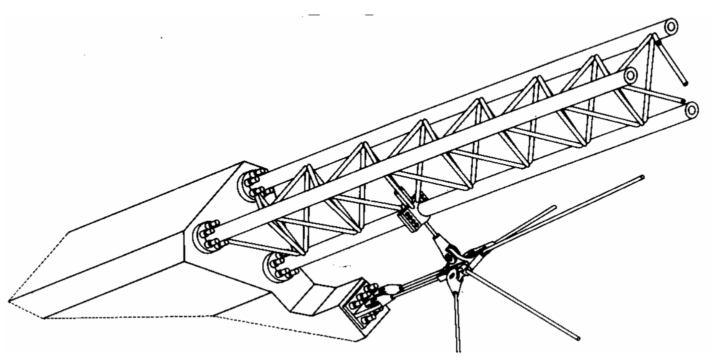




Skating rink, Munich (1983)
Architect: Kurt Ackermann
Engineer: Sclaich Bergermann

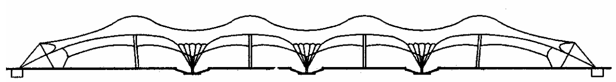
This facility, initially designed as ice skating rink was recently converted into an inline-skating facility due to the increasing popularity of this new sport. The elliptical rink of 88x67m is covered by a cable net roof, suspended from a central arch and supported along the edges by a series if steel masts with guy cables. A prismatic trussed steel arch spans 104m between concrete abutments. The arch supports the cable net and is itself stabilized by it. The cable net is suspended to the arch by means of looping edge cables along the central spine. The space between the edge cables is designed as a skylight that exposes the arch from the inside and provides natural lighting in addition to a translucent roofing membrane. The cable net of double strands has 75x75cm meshes to which a lattice grid of wood slats is attached at the joints. A translucent PVC membrane is nailed to the lattice grid. This unusual combination of materials creates a unique interior spatial quality of quite elegance, contrasting the lightness of the translucent fabric membrane with the warms of the wood lattice grid.

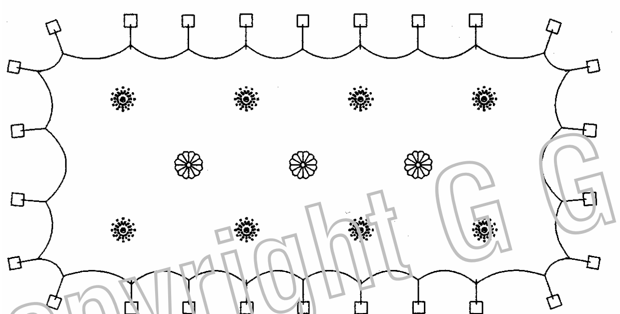


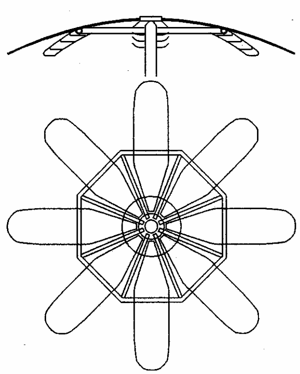


Garden show pavilion Hamburg (1963)
Architect: Frei Otto

This pavilion for the international Garden Show 1961 covers an area of 29/64m and has 5.5m high masts. The point shape roof was fabricated as flat fabric without patterns. The canvass stretched enough to assume the curvature between high and low points. The high points are supported by steel masts with laminated wood springs over octagonal steel ring to avoid stress peaks. Low points are anchored to the ground to resist wind uplift and act as drainage points with rain water collector basins. Membrane edge cables are anchored to the ground by guy cables.

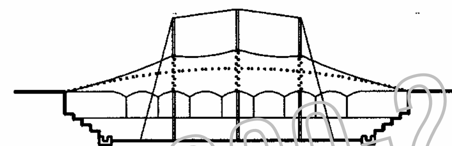


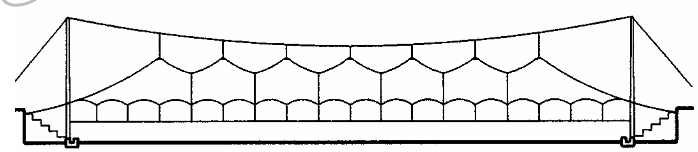


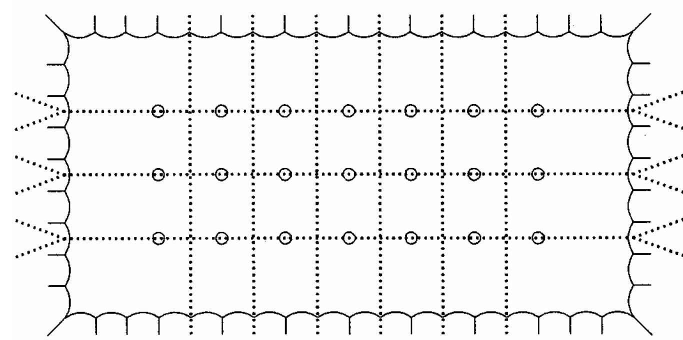


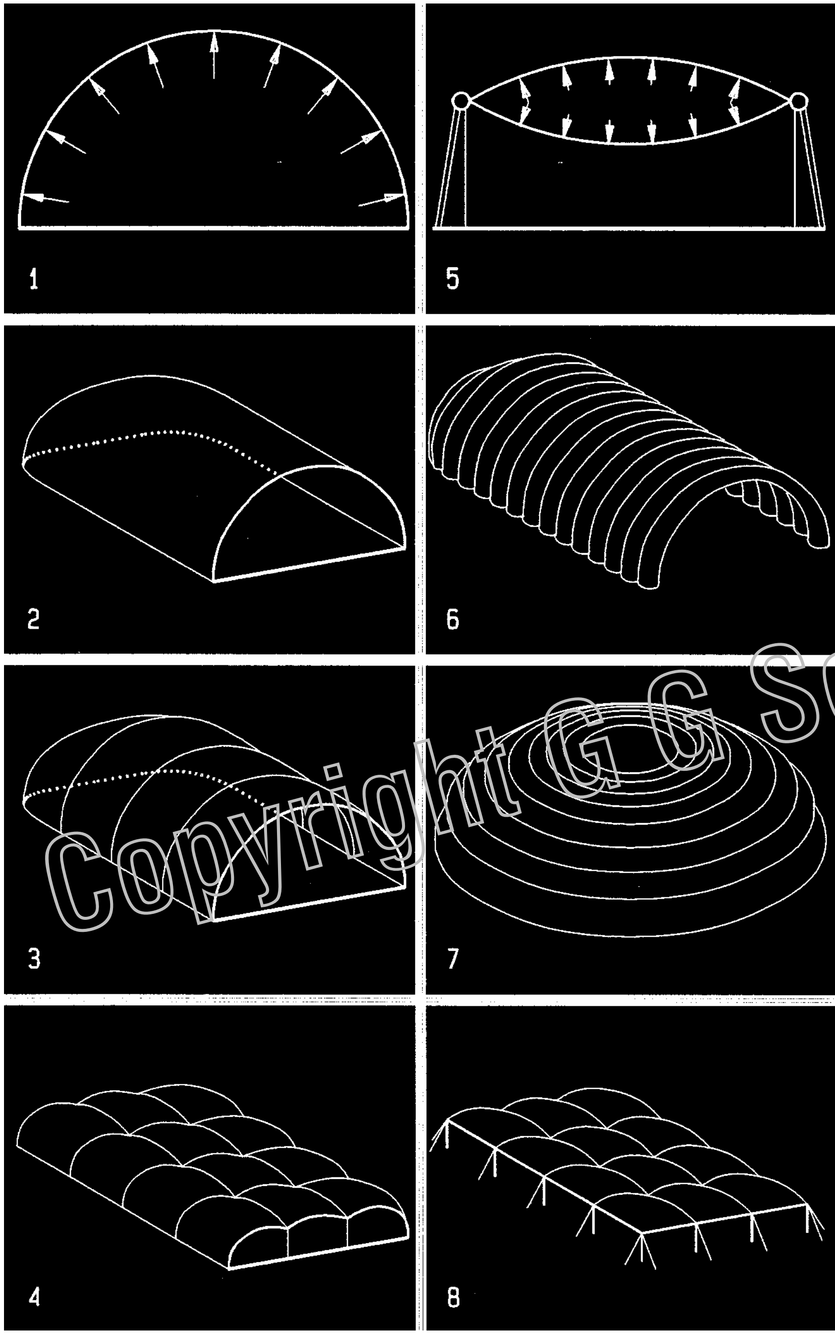
Ice skating rink Villars, Switzerland (1959)
Architect: Frei Otto

The sunken skating area is surrounded by spectator seating and covered by a point shape canvass membrane roof of 32x64m. The roof membrane hangs from three suspension cables that span the length of the rink with steel masts at both ends. Metal dishes distribute the membrane stress at support points. Light fixtures are suspended from the same support points. Guy cables anchor membrane edge cables to the ground.









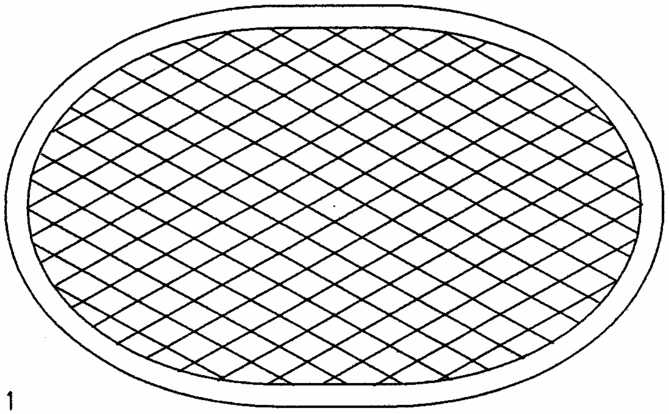
Pneumatic Structures

Pneumatic structures are flexible membranes that derive their stability from air pressure. They usually have synclastic curvature like domes, but anticlastic curvatures are possible as well. Two generic types of pneumatic structure are *air supported* (low pressure) and *air inflated* (high pressure) systems. The air pressure in inflated high pressure structures is 100 to 1000 times greater than in air supported low pressure structures.

Air supported structures typically have a single fabric layer enclosing a space in form of domes or similar shapes. The fabric is supported by inside air pressure. However, considering human comfort, air pressure can be only slightly higher than outside atmospheric pressure. The low air pressure makes air supported structures more vulnerable to flutter under wind load. Since the usable space is under air pressure, door openings must have air locks, usually in form of revolving doors to minimize loss of air pressure. Air supported structures require continuous air supply, usually with standby electric power generator to retain air pressure in case of power outage.

Air inflated structures are hermetically enclosed volumes that are inflated under high pressure much like a football to provide stability. They can have various tubular or cushion forms with high air pressure between two layers of fabric that provide usable space under normal air pressure. The air pressure ranges from 2 to 70 meters of water, yielding 2.8 to 100 pounds per square inch pressure, enough to resist gravity and lateral load. Without air pressure they would have no stability. Air inflated structures also require some continuous air supply to make up for pressure loss due to membrane leaks.

1 Air supported dome or vault
2 Air supported vault
3 Air supported vault with support cables
4 Air supported dome repetitions
5 Air inflated cushion
6 Air inflated tubular vault
7 Air inflated tubular dome
8 Air inflated cushion repetitions



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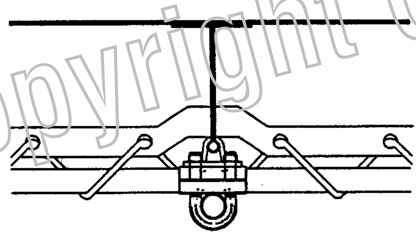

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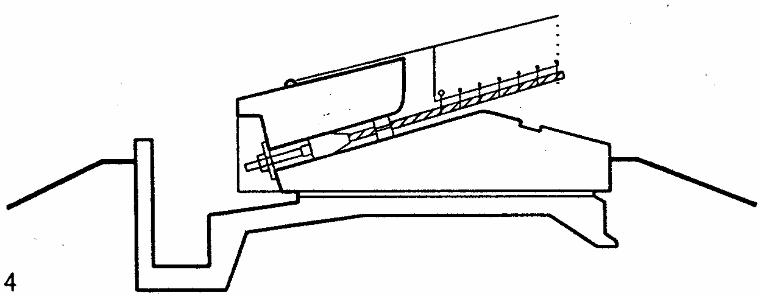

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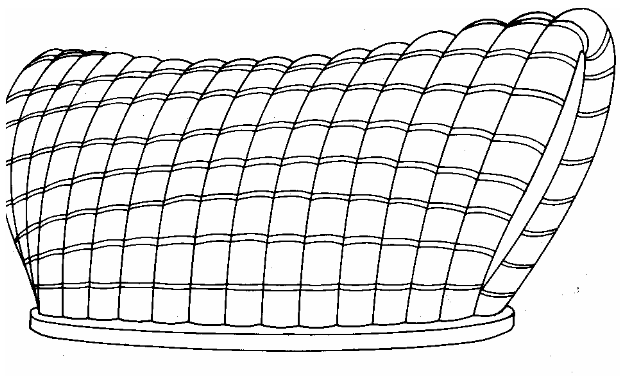
US Pavilion, Expo 70, Osaka (1970)
Architect: Davis, Brody, Chermayeff, Geismar, De Harak
Engineer: David Geiger

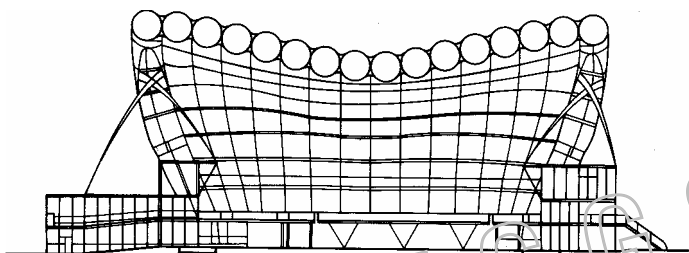
The US Pavilion was the first large-scale pneumatic structure in 1970 with an elliptical plan of 466x272 feet (142x83 meters); yet rising only 20 feet (6 meters) from a peripheral earth berm, the structure had a very low profile. This shallow curvature was possible because the translucent roof membrane was laced to a grid of diagonal cables, spaced 20x20 feet (6x6 meter) that provided the primary support. The tension cables were supported by a concrete compression ring on top of the earth berm by means of adjustable anchor bolts. The compression ring formed a gutter to collect rain water along the periphery. Bending moments that could have been generated in the compression ring resulting of asymmetrical loads, were transferred to and resisted by the earth berm. The pavilion impressed not only by its great size but by its combination of understatement and technical innovation and refined sophistication.

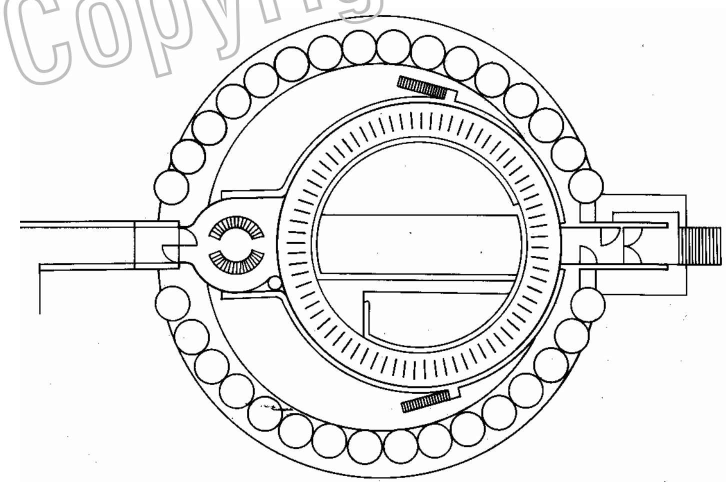
1 Elliptical roof plan
2 Length section
3 Laced membrane to cable attachment
4 Concrete compression wing with gutter and adjustable cable anchors

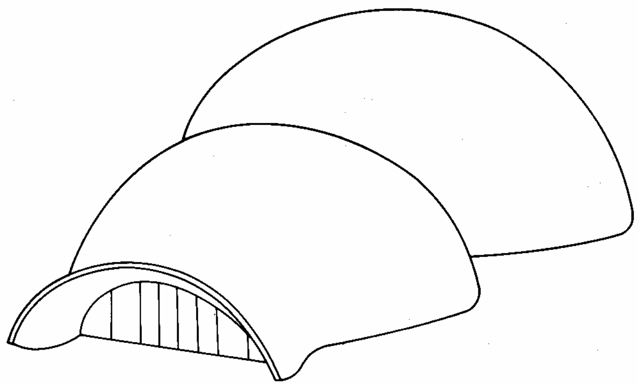
Fuji Pavilion, Expo 70, Osaka (1970)
Architect: Yutaka Murata
Engineer: Mamoru Kawaguchi

The Fuji Pavilion housed an exhibit and light show of the Fuji Corporation in a unique, organic form. Over a circular floor plan of 164 feet (50 meter) diameter, the pavilion featured a vaulted fabric structure composed of 16 pneumatic arched tubes. The tubes of 13 feet (4 meter) diameter were tied together by horizontal belts at 13 feet (4 meter) intervals. The tubes consisted of two vinyl fabric layers that were glued together for improved tear resistance. Given the circular floor plane, the arching tubes of equal length form cross sections that vary from semi-circular at the center to semi-elliptical at the entries on both opposite ends. To adjust the structure's stiffness in response to various wind pressures, the tubes were connected by pipes to a multi-stage turbo blower that provided 1,000 to 2,500 mm water pressure.



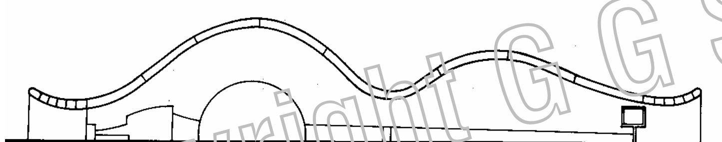


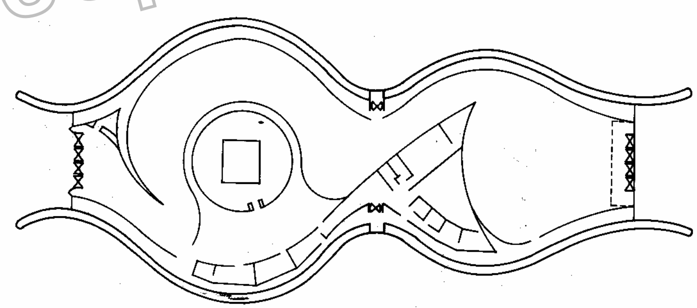




Atoms for Peace pavilion (1960)
Architect: Victor Lundy
Engineer: Severud, Elstad, Krueger

This pavilion housed a traveling exhibit of the United States Atomic Energy Commission that was sent throughout Central and South America in 1960 when the adverse effect of atomic energy were not yet fully understood. The pavilion included a cinema with seating for 300 and a demonstration reactor under a double skin air supported structure. The structure had air pressure between the two fabric layers as well as the inside usable space; seemingly combining air supported and air inflated technologies; but it is indeed air supported. The double skin fabric of vinyl coated polyamide improved the thermal performance and added structural rigidity. The space between the double membrane was divided into air chambers, separated by fabric ribs that provided additional strength. The stout outside form reflected the interior space, given the constant spacing of 4 feet (1.2m) between the two membranes. The portable pavilion of 131/328 feet (40x100 meters) was erected at each new exhibit site in 12 days by a crew of 12 workers.





PART V

VERTICAL SYSTEMS

Vertical structures are presented in four categories, considering primary resistance to load: shear, bending, axial, and suspended (tensile). Although most structures combine several categories, one usually dominates. For example, axially stressed braced frames may also have moment resistant joints, yet the bracing provided most strength and stiffness.

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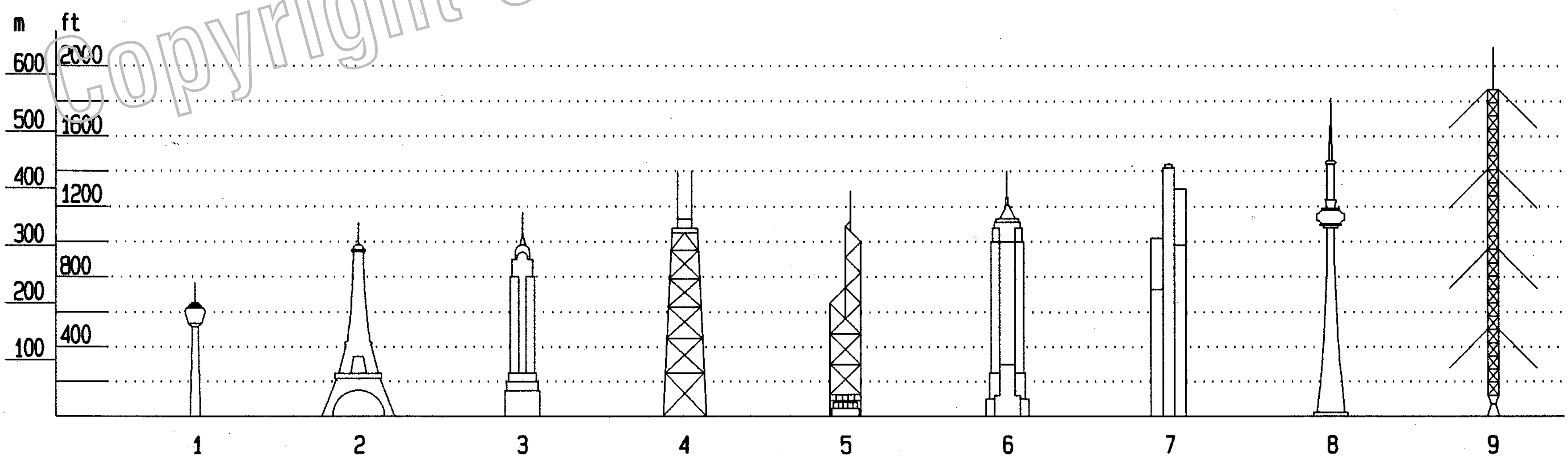
VERTICAL SYSTEMS

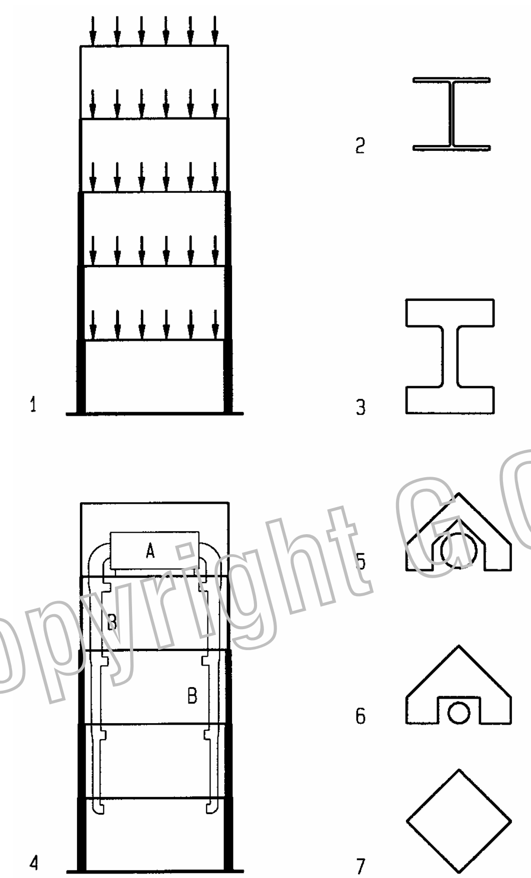
General Background

Vertical structures have been a challenge since the famed tower of Babylon. Motivations to build tall structures include: a desire to reach toward heaven; to see the world from above; the prestige of being tallest, and high land costs. The tallest church tower in Ulm, Germany exemplifies the spiritual motivation. The Eiffel tower allows seeing Paris from above. The towers of the Italian hill-town San Gimignano, and contemporary corporate office buildings express power and wealth; the latter are also motivated by high land cost. Traditional building materials like wood and masonry imposed height limitations overcome by new materials like steel and prestressed concrete. The Eiffel tower in Paris marks the beginning of tall steel towers. Prestressed concrete towers were pioneered 1955 by Fritz Leonhard with a television tower in Stuttgart.

Tall Structures

1	SDR television tower Stuttgart	1955	217 m	712 feet
2	Eiffel tower Paris	1889	300 m	984 feet
3	Chrysler building New York,	1930	319 m	1047 feet
4	John Hancock tower Chicago	1968	344 m	1127 feet
5	Bank of China tower Hong Kong	1988	369 m	1211 feet
6	Empire State building New York	1933	381 m	1250 feet
7	Sears tower Chicago	1974	443 m	1453 feet
8	CN tower Toronto	1976	553 m	1814 feet
9	Transmission tower Warsaw	1974	643 m	2110 feet

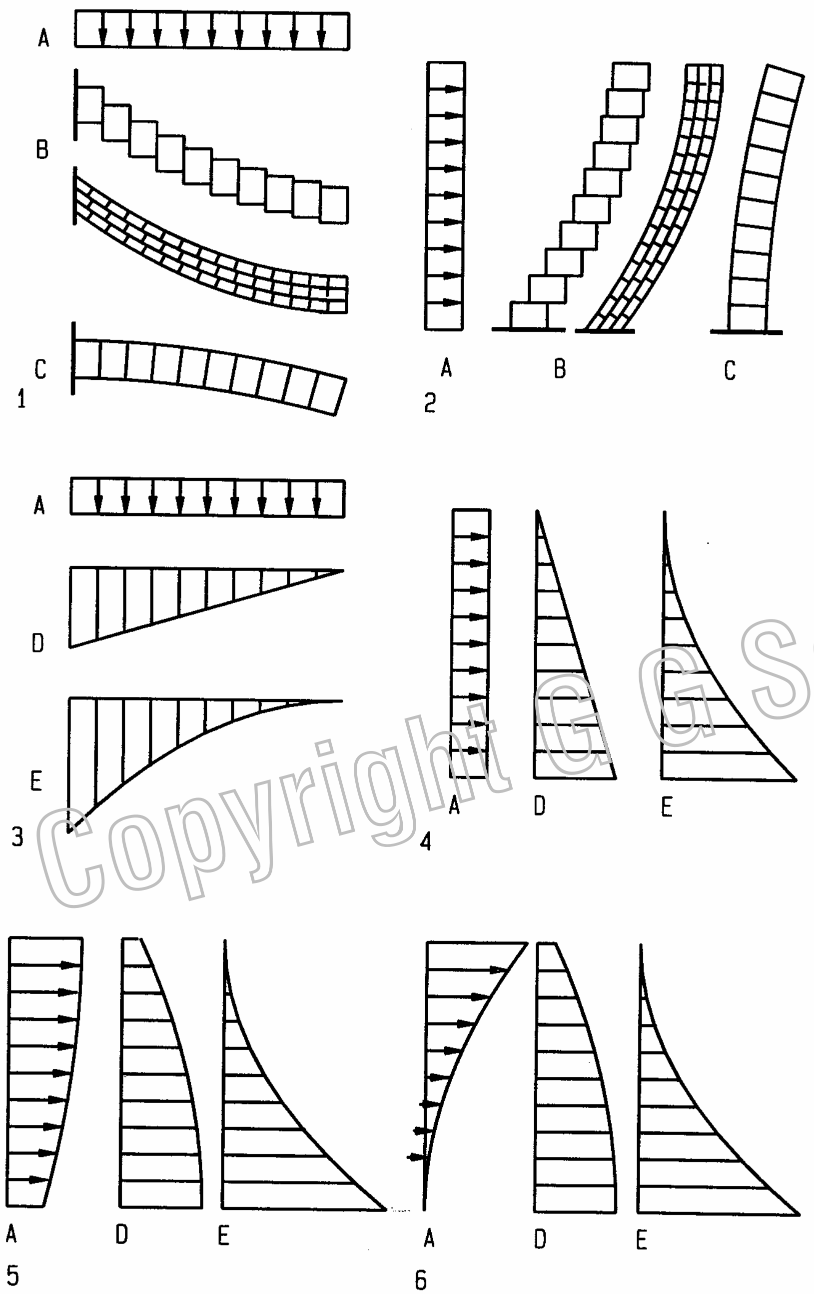




Gravity load

Gravity load is the combined live and dead load, acting vertically to generate compressive stress in supporting columns or walls. At every level they carry the combined loads from above. Since load accumulates from top down, members at the top carry the least; those at the bottom carry the most. Steel structures usually have the same nominal column size but of increasing unit weight, resulting in thin columns at top and bulky ones at ground level. For example W14 wide flange columns come in many weights from 43 to 730 plf (64 to 1,086 kg/m) with capacities of 272 to 4,644k (1,210 to 20,656 kN). It is also possible to use higher strength steel at lower floors. However, increase in steel strength does not yield higher stiffness since the modulus of elasticity of steel is constant regardless of strength. For concrete structures it is possible to increase concrete strength and stiffness, or to increase the cross sectional area. If a mechanical room is on the top floor it is possible to balance the decreasing need for duct sizes from top down, with need for increasing column sizes from top to bottom. Eero Saarinen designed the CBS tower New York with such a strategy but was only partly consistent since the lower floors are served from a mechanical room on the second floor.

1 structure with increasing column size as load increases from top down
2 Light-weight wide-flange column
3 Heavy weight wide-flange column of equal nominal size as in 2 above
4 Increasing column size dovetails with reducing duct size from top down
5 Small column cross section and large duct size on top column
6 Large column cross section and small duct size on lower floor column
7 Large column on ground level where no duct space is needed

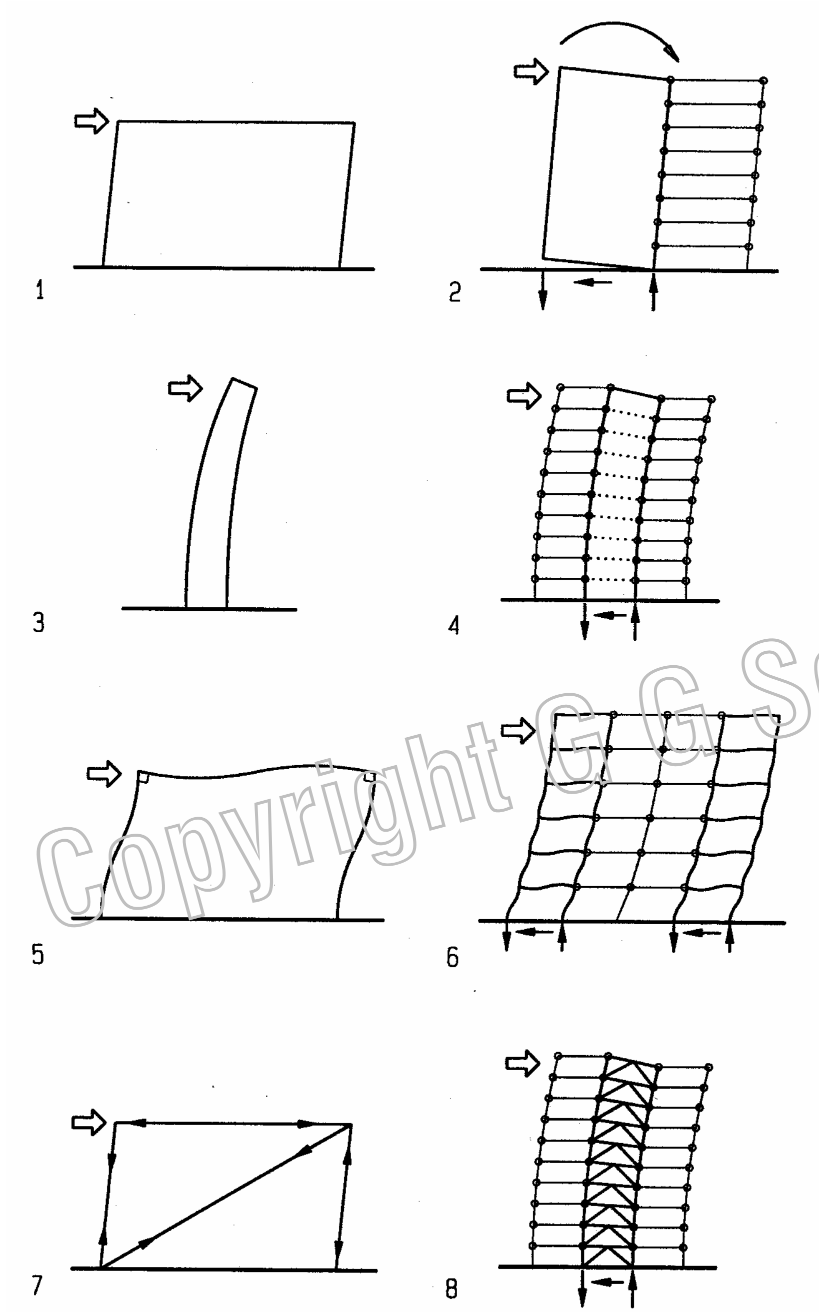


Lateral load

The effect of lateral load on tall structures is similar to gravity load on cantilevers, such as balconies. Tall structures act like cantilevers projecting from the ground. Lateral load generates shear and bending that may be presented in respective shear and bending diagrams as in a cantilever beam. Yet there are important differences. The shear and bending diagrams for buildings are usually *global*, for the entire system rather than for individual elements like beams. For example, global bending (overturn moment) causes axial tension and compression in columns, and local shear and bending in beams. Further, lateral wind and seismic loads are non-uniform. Wind force increases with height due to higher wind speed and reduced friction. Seismic forces increase with height in proportion to increasing acceleration (acceleration increases with height due to increased drift). However, shear increases from top to bottom since the structure at each floor must resist not only the force at that floor but the forces from all floors above as well. The non-uniform wind and seismic loads cause nonlinear shear distribution.

1 Cantilever beam with shear and bending deformation
2 Tall building with shear and bending deformation
3 Shear and bending diagrams for uniform load on a cantilever beam
4 Shear and bending diagrams for idealized uniform load on a building
5 Vertical distribution of wind force, shear, and bending diagrams
6 Vertical distribution of seismic force, shear, and bending diagrams

A Load/force diagram
B Shear deformation
C Bending deformation
D Shear diagram
E Moment diagram



Lateral resistance

Lateral loads may be resisted by shear walls, cantilevers, moment frames, braced frames, or combinations thereof. The choice of a suitable system depends on structural and architectural objectives. Shear walls and braced frames are strong and stiff, cantilevers and moment frames are more ductile, to dissipate seismic energy. Shear walls are good for apartments or hotels that require party-walls between units. Moment frames offer better planning flexibility required for office buildings with changing tenant needs.

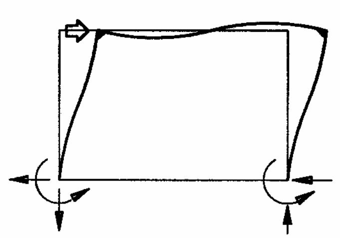
Shear walls resist lateral load primarily through in-plane shear. They may be of reinforced concrete or masonry, or, for low-rise, of wood studs with plywood or particle board sheathing. Short shear walls tend to overturn and must be stabilized by dead load or tie-downs.

Cantilevers are slender elements that resist load primarily in bending. Pole houses are cantilevers; but more commonly, cantilevers are of reinforced concrete or masonry, anchored to foundations, wide enough to resist overturn. Overturn cause compression on one side and tension on the other. Compression acts in addition to gravity, tension may be partly offset by gravity compression. In tall cantilevers, tension due to lateral load may be greater than gravity compression, resulting in net tension.

Moment frames consistent of posts and beams connected by moment resistant joints. They may be of steel or reinforced concrete. To resist seismic load, concrete should have ductile reinforcement that yields before brittle concrete failure. Ductile design results in greater concrete members with less reinforcing steel.

Braced frames may have diagonal-, A-, X-, or V-braces. The best bracing scheme depends on structural and architectural considerations. K-bracing tends to buckle columns and must not be used. X-bracing allows no doors and requires more joints for greater cost; but X-bracing can be of tension rods to eliminate buckling. A- and V-braces are shorter than single diagonals and result in reduced buckling. (However, beams must be designed for the full span since bracing may adversely affect the beam load). Braced frames are usually of steel but may be of reinforced concrete or wood (for low-rise).

1 Long shear wall resists in-plane load in shear primarily
2 Shear wall supports adjacent bays (slender walls tend to overturn)
3 Cantilever resists lateral load primarily in bending
4 Cantilever supporting adjacent bays
5 Moment frame requires moment resisting beam-column joints to resist lateral load by beam-column interaction
6 Moment frames at both ends supports intermittent bays
7 Braced frame with diagonal bracing
8 braced center core supports adjacent bays



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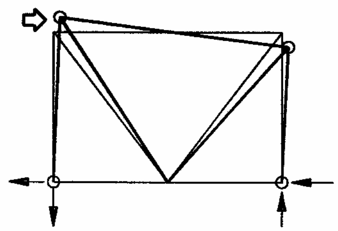

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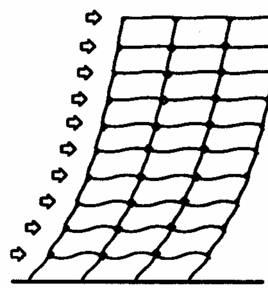

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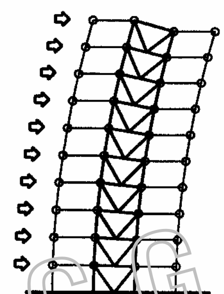

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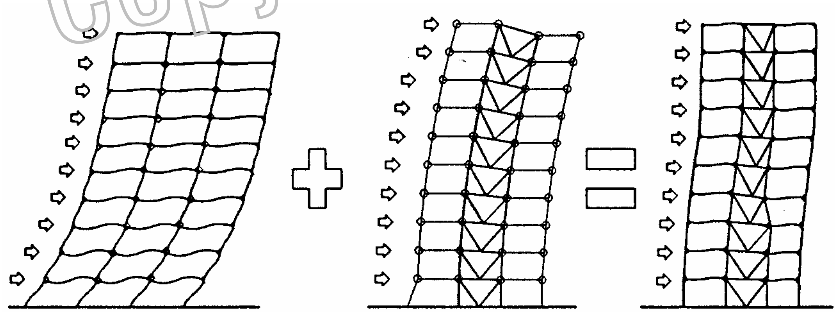

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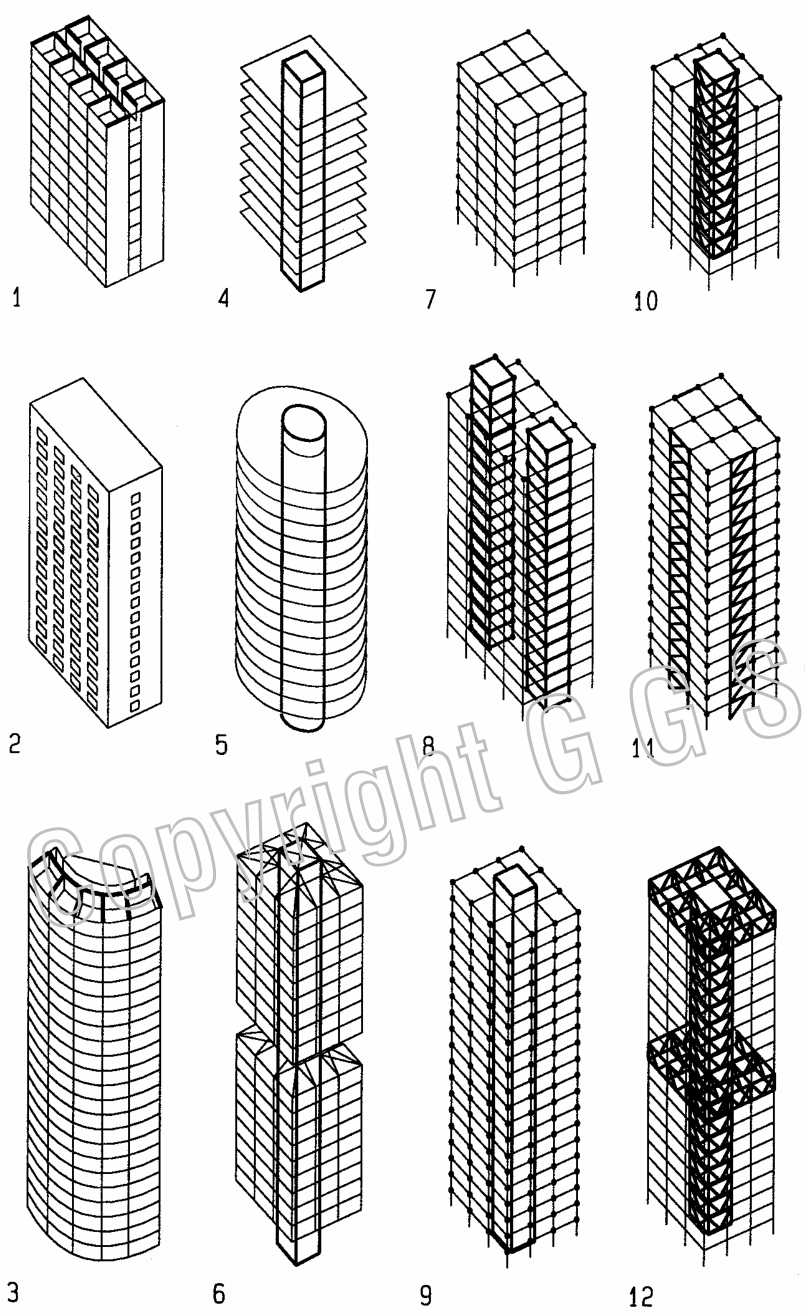
Braced /moment frame

Combined braced/moment frames are used to reduce drift under lateral load. Moment frames have the greatest drift at the building base, but braced frames have the greatest drift on top. Combining the two systems reduces drift at both base and top. The objectives to reduce drift are:

- To prevent occupant discomfort
- To reduce the risk of failure of cladding and curtain walls
- To reduce secondary stress caused by *P-Δ effects*
  (the P-Δ effect generates bending moments caused by column gravity load P and the lateral drift Δ as lever arm)

1 Bending resistance of moment frame portal under lateral load
2 Axial resistance of braced frame portal under lateral load
3 Lateral drift of moment frame is maximum at base
4 Lateral drift of braced frame is maximum on top
5 Reduced drift of combined braced/moment frame

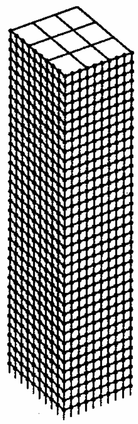


Structure systems

The vertical-lateral framing systems of wall, cantilever, braced frame, and moment resisting frame, shown from left to right, may be optimized for height and use, including combinations of systems. The importance to select an efficient system increases with building height in order to achieve a low weight per floor area ratio for the structure. The late engineer Fazlur Kahn of Skidmore Owings and Merrill recommended the following systems for various heights:

Concrete moment resisting frame	20 stories
Steel moment resisting frame	30 stories
Concrete shear wall	35 stories
Braced moment resisting frame	40 stories
Belt truss	60 stories
Framed concrete tube	60 stories
Framed steel tube	80 stories
Braced tube	100 stories
Bundled tube	110 stories
Truss tube without interior columns	120 stories

1 Cellular shear walls
2 Exterior shear walls
3 Curved shear walls
4 Cantilever core with cantilever floors
5 Cantilever round core with cantilever floors
6 Cantilever core with suspended floors
7 Moment resistant frame
8 Moment frame with two shear cores
9 Moment frame with single shear core
10 Braced core
11 Braced exterior bays
12 Braced core with outrigger trusses



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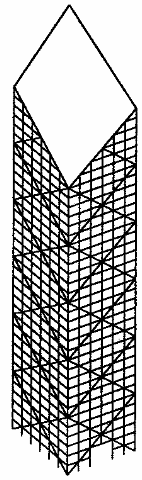

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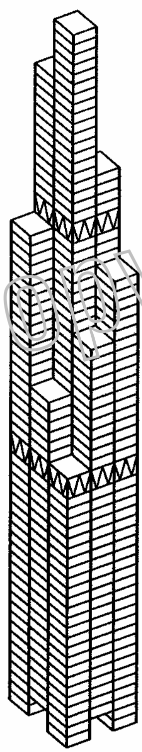

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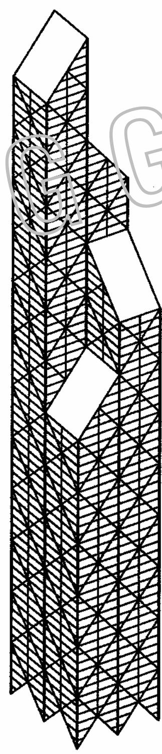

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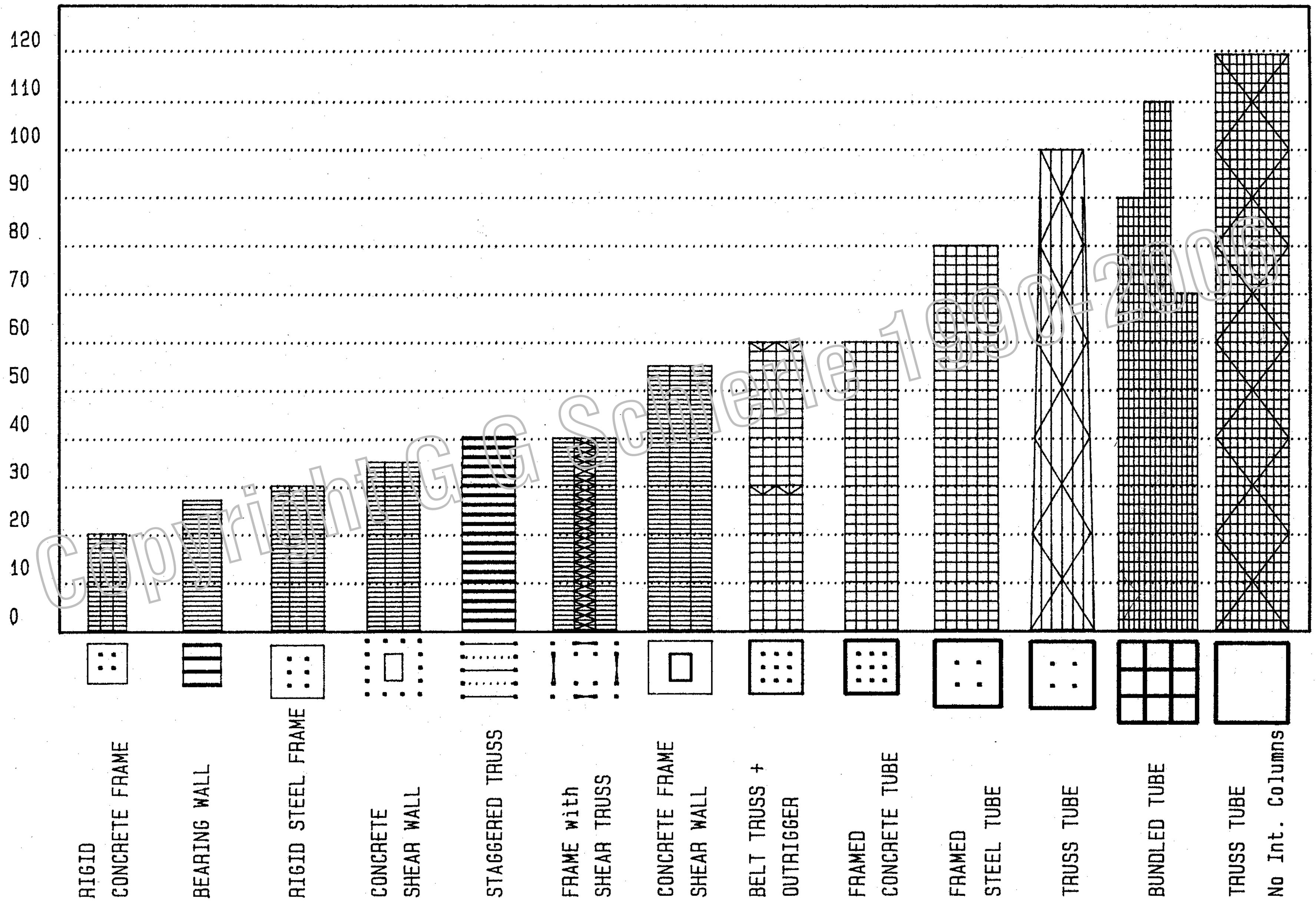
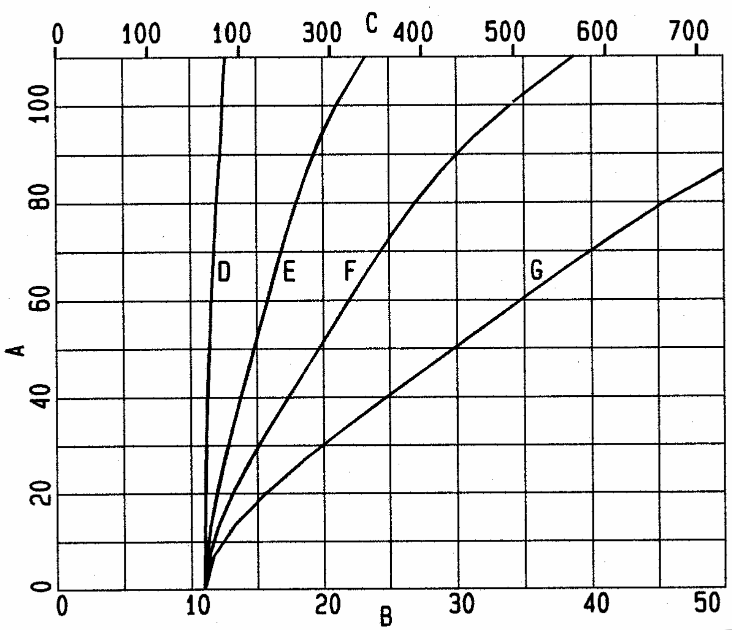
Tubular systems incorporate lateral resistance into the skin by either some form of bracing or narrowly spaced columns with moment resisting beam-column connections. For very tall buildings, bundled tubes increase lateral resistance with interior cell “walls” to reduce shear lag between exterior skin bracing. The Sears tower in Chicago has a framed bundled tube structure.

1 Framed tube
2 Braced tube
3 Bundled tube, framed
4 Bundled tube, braced

Structure systems vs. height

The diagram is based on a study by the late Fazlur Kahn regarding optimal structure system for buildings of various heights, defined by number of stories.





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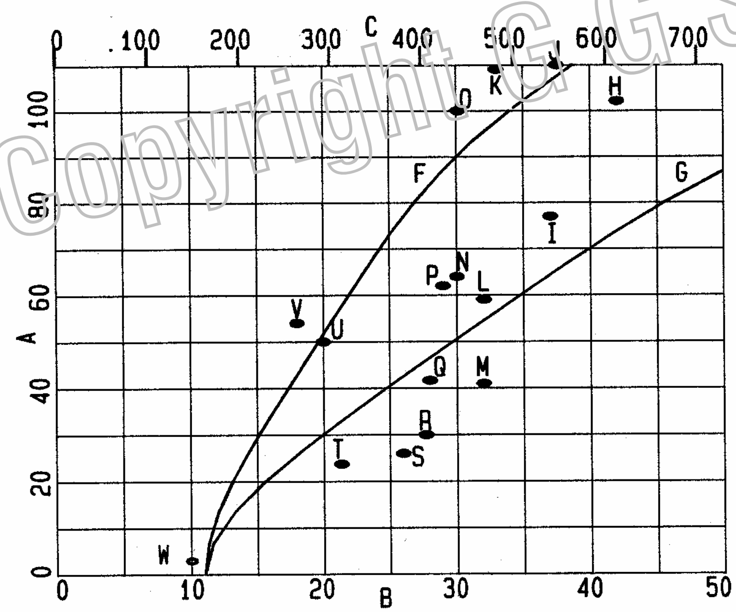

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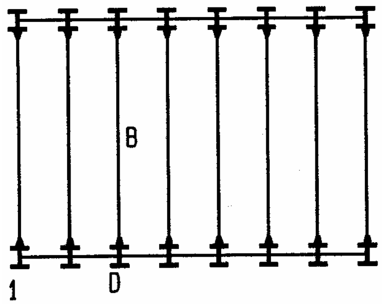
Structure weights

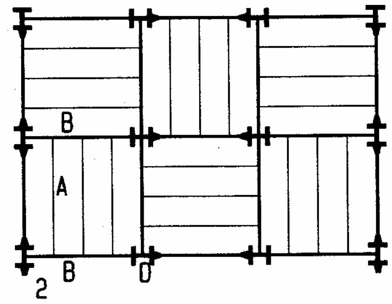
The amount of structural steel required per floor area is a common measure of efficiency for steel structures. Comparing various systems demonstrates the importance of selecting a suitable system. As shown in the diagram, considering gravity load alone, the structural weight would increase only slightly with height. The effect of lateral load, however, accelerates the increase dramatically and at a non-linear rate.

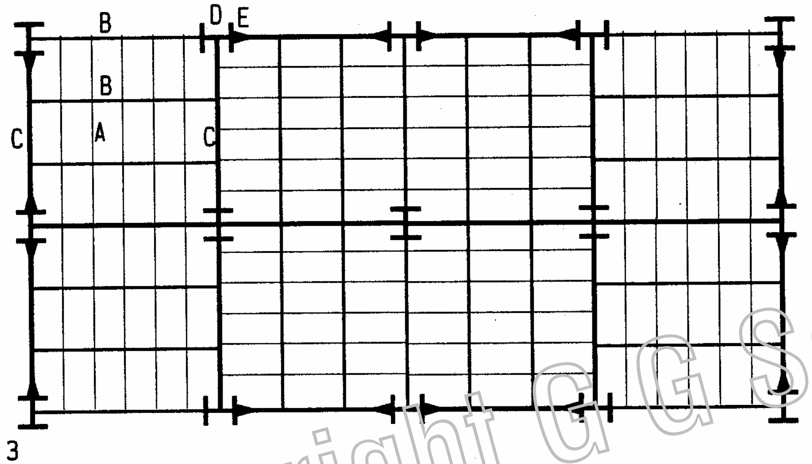
1 Structural steel weight related to building height (by Fazlur Kahn)
2 Weight of structural steel per floor area of actual buildings

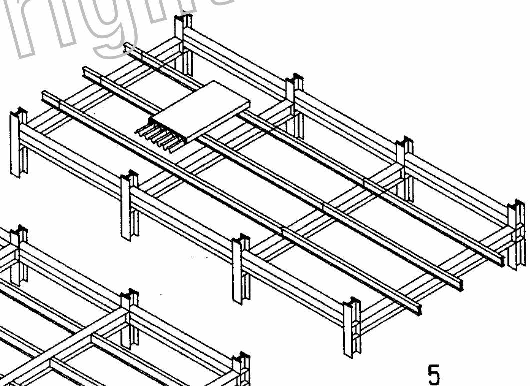
A Number of stories
B Weight of structural steel in psf (pounds per square foot)
C Weight of structural steel in N/m²
D Weight of structural steel considering floor framing only
E Weight of structural steel considering gravity load only
F Weight of structural steel for total structure optimized
G Weight of structural steel for total structure not optimized

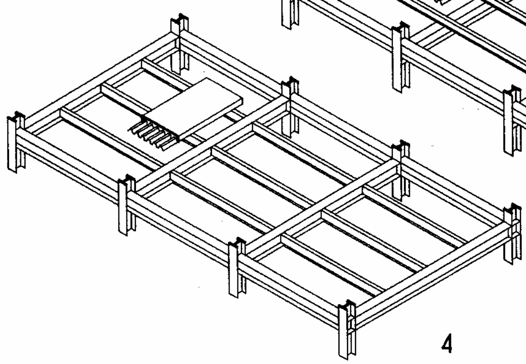
H Empire State building New York
I Chrysler building New York
J World Trade center New York
K Sears tower Chicago
L Pan Am building New York
M United Nations building New York
N US Steel building Pittsburgh
O John Hangkock building Chicago
P First Interstate building Los Angeles
Q Seagram building New York
R Alcoa building Pittsburgh
S Alcoa building San Francisco
T Bechtel building San Francisco
U Burlington House New York
V IDS Center Minneapolis
W Koenig residence Los Angeles









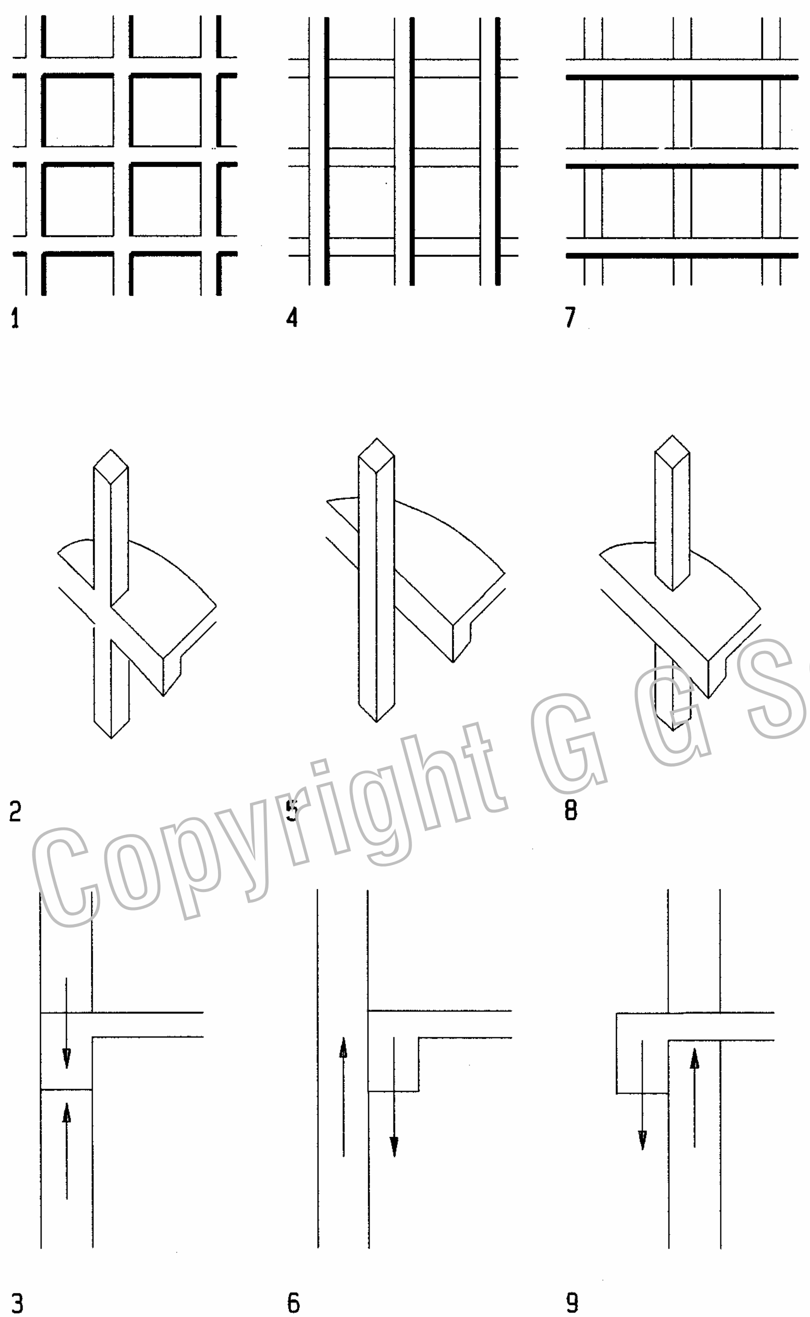


Floor and roof framing

The layout of horizontal framing is important not only for the transfer of gravity load to columns but also to resist lateral load. For example, beam framing should transfer gravity load to columns subject to uplift forces caused by overturn moments. Gravity load may cancel uplift at least in part to avoid the need for foundation anchorage. It is also advantageous to design beams to distribute gravity load to girders evenly, rather than all load to some and none to others.

1 Single-layer system: beams rest on columns
2 Two-layer system: joists rest on beams that rest on columns
3 Three-layer system: joists rest on beams that rest on girders that rest on columns
4 Flush joist and beam
Requires joist connection into side of beam
Joists resist rotational buckling of beam
Mechanical ducts must run below framing
5 Layered framing
Provides easy connections
Main ducts between beam, feeder ducts between joists reduces height
Joists don't resist rotational buckling of beams

A Joist
B Beam supports joist
C Girders supports beam
D Column supports beam or girder
E Symbol for moment resisting connection common in framing plans



Beam-column interface

The type of interface between spandrel beam and column on a facade is important considering architectural and structural implications. Assuming moment resisting connections, the best structural solution is to frame the beam directly into the column for effective moment transfer without torsion. If shown on the facade, this expresses most clearly a moment resisting frame. Beams may run behind the column to express verticality, or in front of the column to express horizontally; yet both cases generate torsion in the beam and bending in the column due to eccentricity.

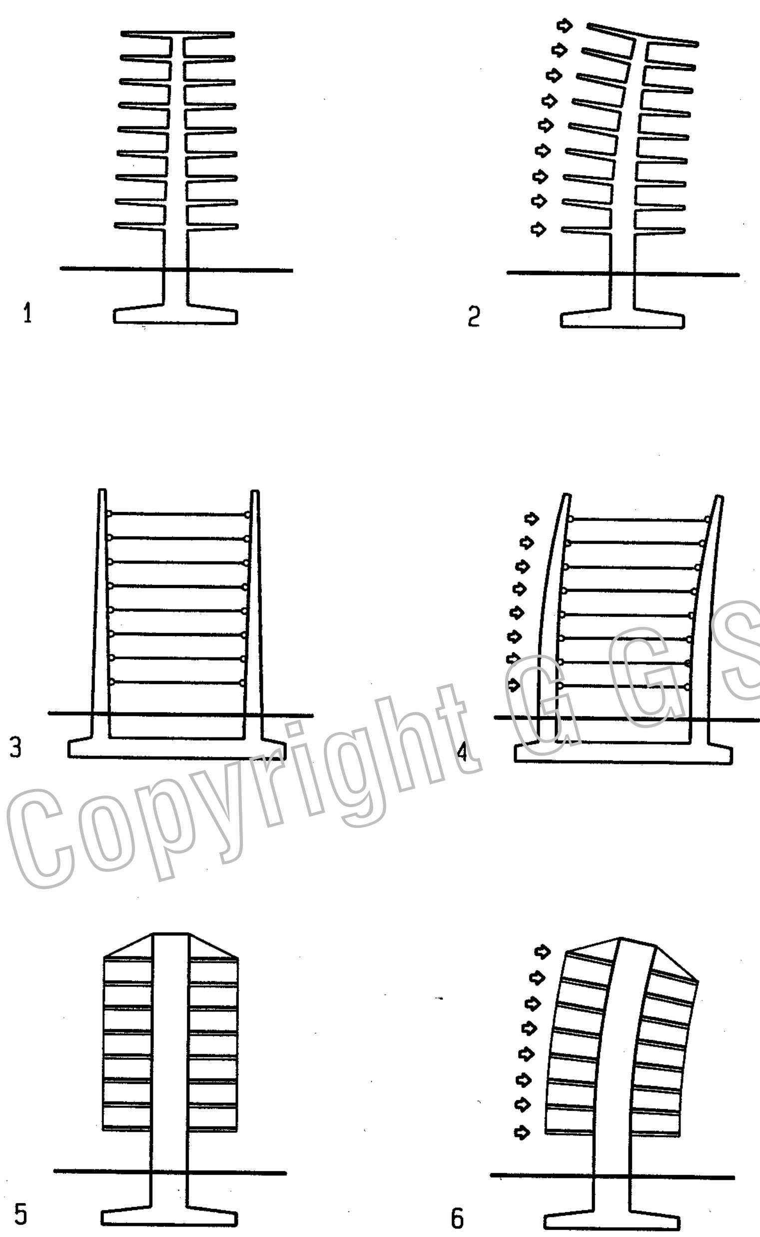
1 Visual expression of frame
2 Axon of beam framed directly into column
3 Section of beam framed directly into column
4 Visual expression of columns
5 Axon of beam running behind column
6 Section of beam running behind column yielding a moment couple
7 Visual expression of beam
8 Axon of beam running in front of column
9 Section of beam running in front of column yielding a moment couple

17

VERTICAL SYSTEMS

Bending Resistant

Bending resistant structures include cantilever, moment frame, framed tube, and bundled tube structures. They resist lateral load by combed axial and bending stresses. Since bending stress varies from tension to compression with zero stress at the neutral axis, only half the cross section is effectively engaged. This makes them less stiff than shear walls or braced frames, but it provides greater ductility to absorb seismic energy in the elastic range, much like a flower in the wind. On the other hand, bending resistance implies large deformations that may cause damage to non-structural items. Bending resistant structures are sometimes combined with other systems, such as braced frames or shear walls, for greater stiffness under moderate load; but moment frames provide ductility under severe load, after the bracing or shear walls may fail.

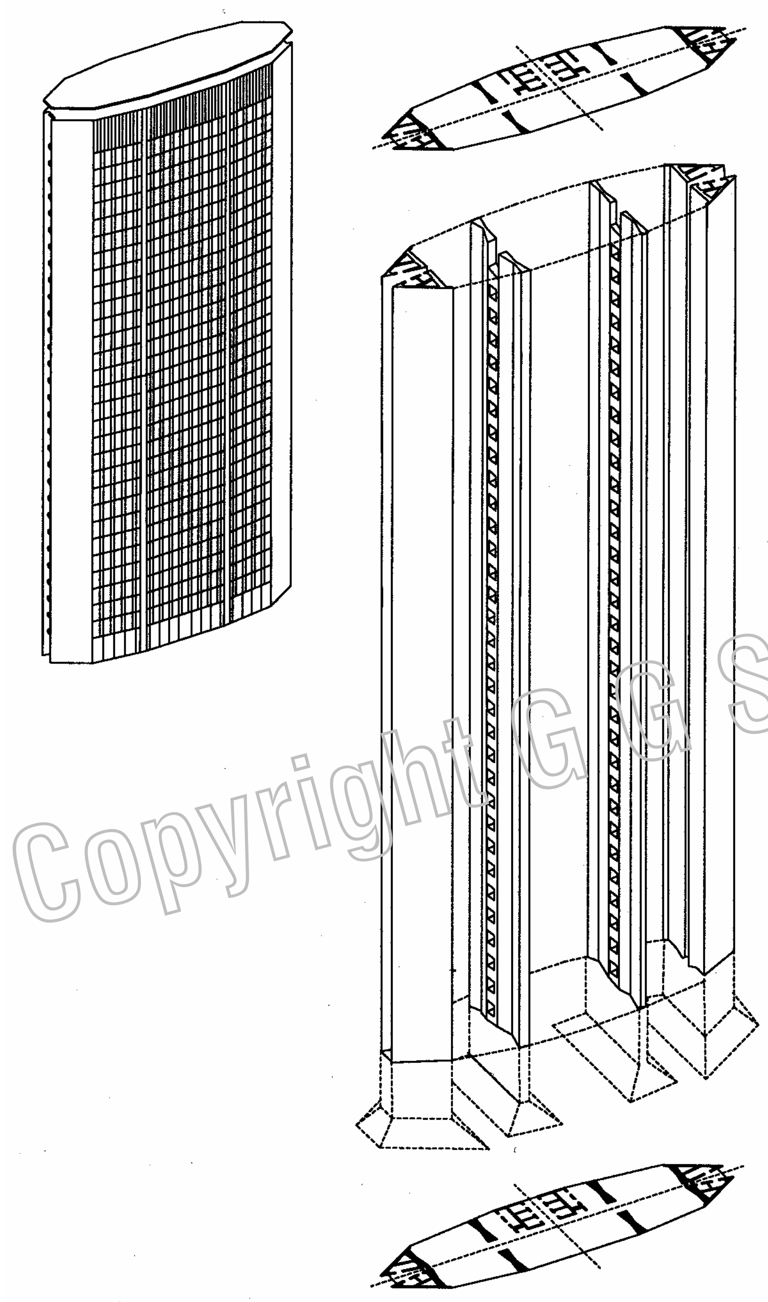


Cantilever

Cantilever structures consist of towers that rise from the foundation. They resist gravity load in compression and lateral load in bending and shear, similar to moment frames. Cantilevers are subject to global bending of the entire tower, whereas moment frames are subject to localized bending of columns and beams joint by moment resistant joints. The global bending of cantilever towers increases from minimum on top to maximum at the base; whereas the local bending of beams and columns in moment frames varies at each level from positive to negative.

Cantilever towers may be very slender walls, hollow boxes, or solid columns. Compared to shear walls, which resist lateral load in shear, cantilevers resist primarily in bending. The most common materials are reinforced concrete and wood poles of pole houses. Floors may also cantilever from the towers. Cantilevers need large foundations to resist overturn moments. Cantilever systems of multiple towers may have joint foundations that tie the towers together for better stability.

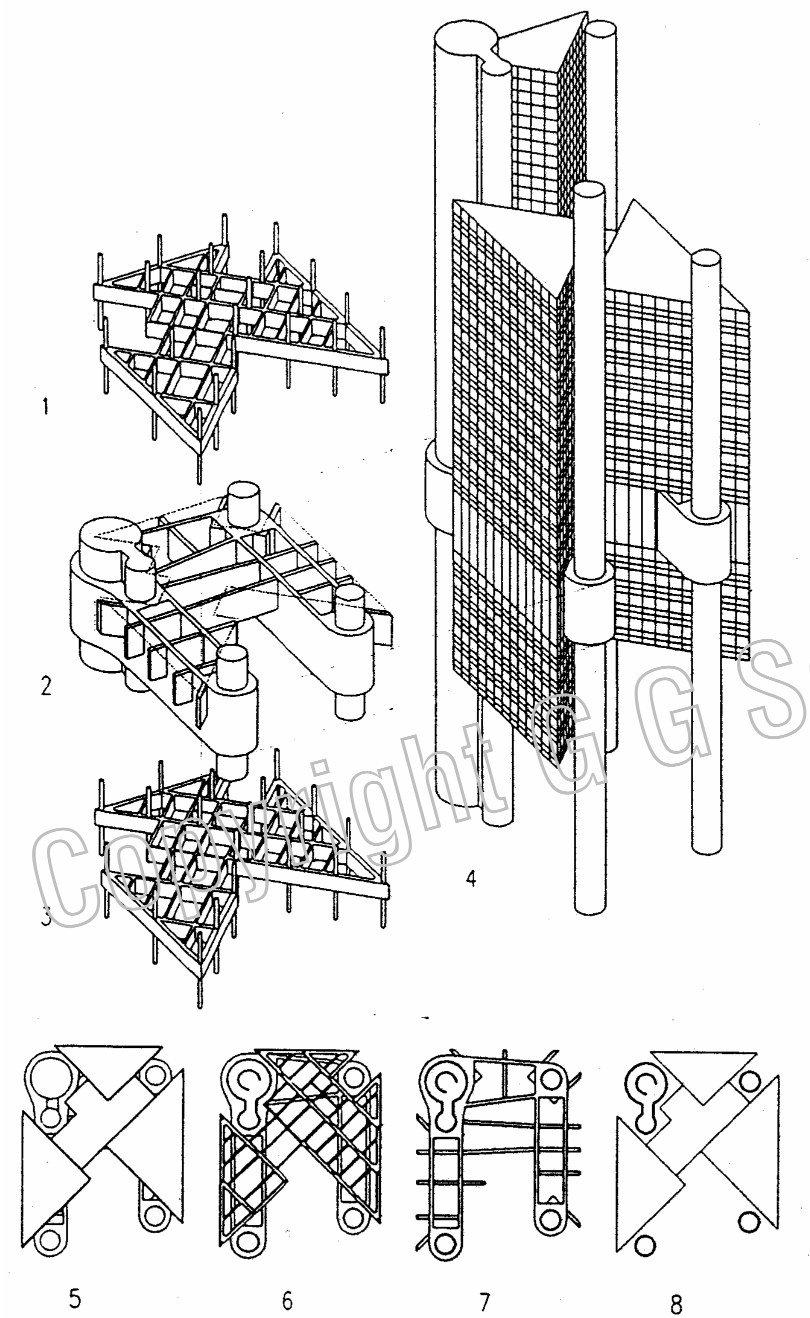
1 Single tower cantilever with cantilever floors
2 Single tower cantilever under lateral load
3 Twin tower cantilever with joined footing for improved stability
4 Twin tower cantilever under lateral load
5 Single tower cantilever with suspended floors
6 Single tower suspension cantilever under lateral load



Pirelli tower, Milan (1956-58)
Architect: Ponti, Fornaroli, Rosselli, Valtolina, Dell Orto
Engineer: Arturo Danusso, Pier Luigi Nervi

Facing Milan's central station across a major urban plaza, the 32-story Pirelli tower rises prominently above the surrounding cityscape. A central corridor, giving access to offices, narrows toward both ends in response to reduced traffic. The reinforced concrete structure features two twin towers in the midsection for lateral resistance in width direction and triangular tubes at both ends for bilateral resistance. The towers and tubes also support gravity load. The gravity load of the towers improves their lateral stability against overturning. The central towers are tapered from top to base, reflecting the increasing global moment and gravity load. The towers are connected across the central corridors at each level by strong beams that tie them together for increased stability. In plan, the central towers are fan-shaped to improve buckling and bending resistance. The tubular end towers of triangular plan house exit stairs, service elevators, and ducts. Concrete rib slabs supported by beams that span between the towers provide column-free office space of 79 and 43 ft (24 and 13 m). The plan and structure give the tower its unique appearance, a powerful synergy of form and structure.

Floor plan:	18 x 68 m (59 x 223 ft)
Height:	127 m (417 ft))
Typical story height:	3.9 m (12.8 feet)
Height/width ratio	7



Hypo Bank, Munich (1980)

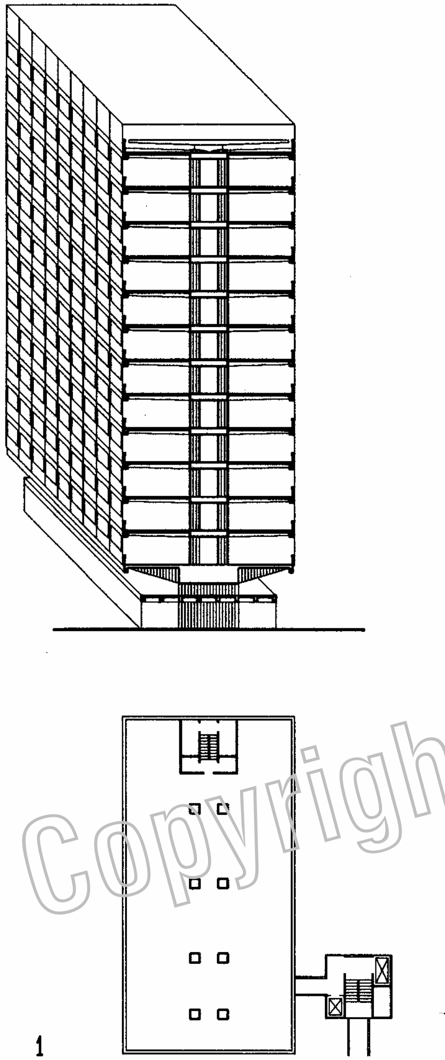
Architect: Bea and Walter Betz

The design objective for the Hypo Bank headquarters was to create a landmark for Munich and a unique architectural statement for the bank. Built 1980, the 22 story bank has 114m height. The structure consist of four tubular concrete towers that support a platform which supports 15 floors above and 6 floors suspended below. The suspended floors had been built from top down simultaneous with upper floors being built upwards. Four towers combined with a platform form a mega-frame to resist gravity and lateral loads. The four towers include exit stairs in prestressed concrete tubes of 7m diameter and 50 to 60 cm wall thickness. A fifth tower of 12.5m diameter, houses eight elevators and mechanical equipment. The support platform consist of prestressed site-cast concrete of 50cm thick concrete slabs on top and bottom, joined by 1.5m rib walls that are tied around the towers. The formwork for the platform was assembled on ground and lifted 45 m by 12 hydraulic jacks.

The office space consists of three triangular units, joined by a T-shaped center. Two-way beams for office floors are supported by columns above the platform and suspended below. Three sub-grade levels include parking, security control, and loading stations.

Floor plan:	7 m (23 ft) diameter towers
Height:	114 m (374 ft)
Height/width ratio	16 per tower

1 Typical upper floor supported by columns above the platform
2 Story-high platform forms a mega frame with four towers
3 Typical lower floor suspended from the platform
4 Isometric view of building
5 Roof plan
6 Typical office floor framing
7 Support platform framing
8 Typical floor plan layout



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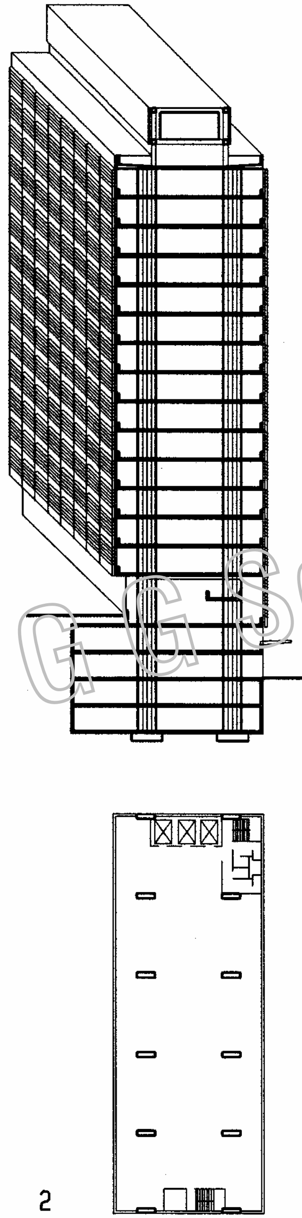

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1 Commerzbank Düsseldorf (1965)
Architect: Paul Schneider-Esleben

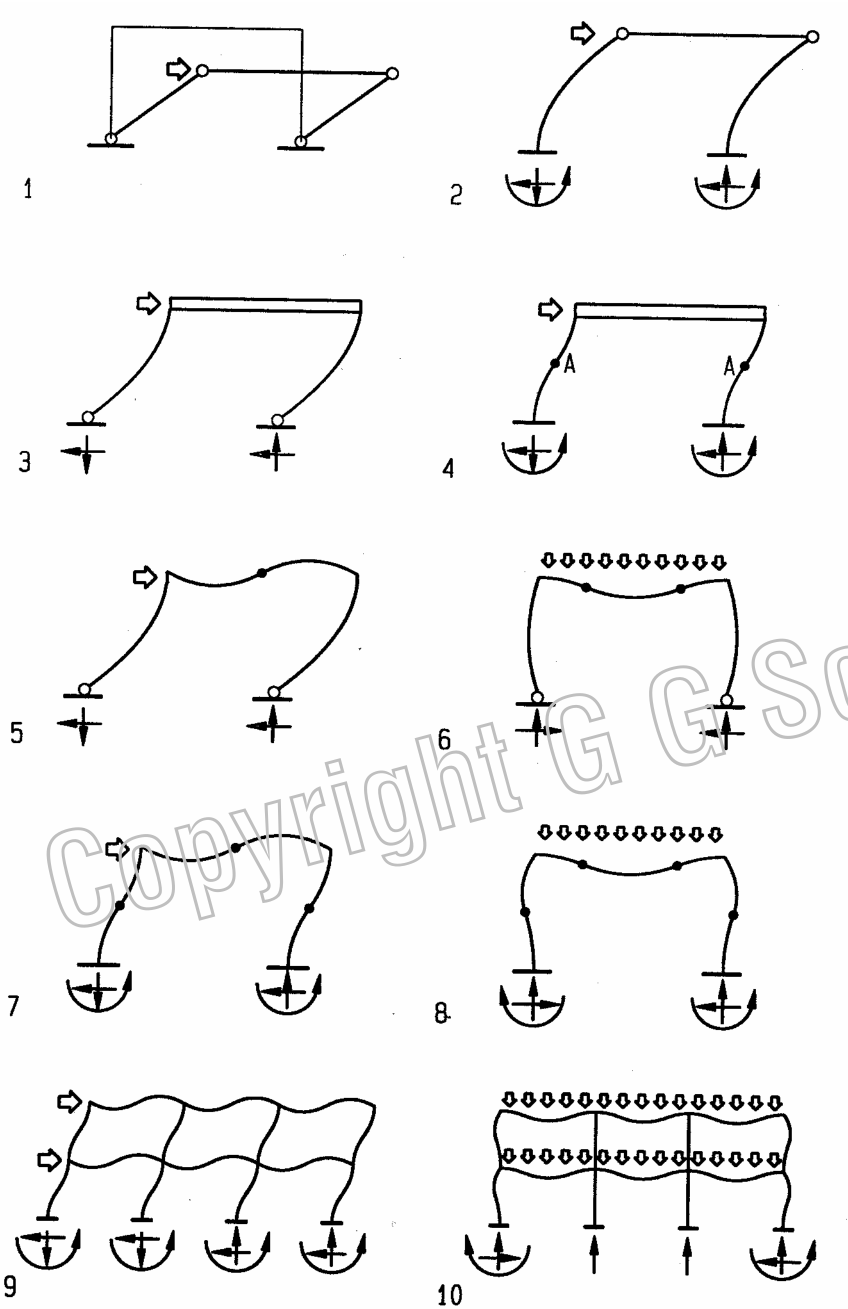
This 12-story bank building is located at the boundaries between the old and new banking district of Düsseldorf, linked by a pedestrian footbridge to an older building of the bank. The 12-story building above a 2-story podium was initially designed to allow a drive-in bank at street level. A free-standing service core supports the pedestrian bridge and makes the link to the office floors. A second stair and bathroom core is located at the far end of the building, providing undivided and flexible office space. The curtain wall façade is designed and manufactured using vehicular technology of insulating sandwich panels. The structure consists of reinforced concrete. Two rows of square cantilever columns support cantilever beams and concrete floor slabs. The interior core helps to resist lateral load in length and width directions, but the exterior core at the other end of the building resist lateral load in width direction only.

Floor plan:	16 x 32 m (52 x 104 ft)
Height:	44 m (144 ft)
Typical story height	3 m (9.8 ft)
Height/width ratio	10 per cantilever

2 Lend Lease House Sydney (1961)
Architect: Harry Seidler

This 15-story office tower with north-south orientation of its length axis has movable exterior blinds for sun control. They give the facade an ever-changing appearance. On sunny mid-days, they are horizontal for optimal sun protection. On cloudy days, in lowered position, they tend to darken the inside rooms. The orientation provides inspiring views to the Sydney harbor and a nearby botanical garden. A two-story showroom with mezzanine floor is located on the ground floor, above a four-story underground parking garage. The office floors feature elevators, stair and bathrooms on one end and an exit stair at the opposite end, providing flexible office floors. Mechanical equipment is in a roof penthouse. The structure consists of reinforced concrete. Two rows of wall-shape cantilever columns support cantilever slabs. The cantilever columns resist both gravity and lateral loads.

Floor plan:	12 x 30 m (39 x 98 ft)
Height:	38 m (125 ft)
Height/width ratio	4.7 per twin cantilever



Moment frame

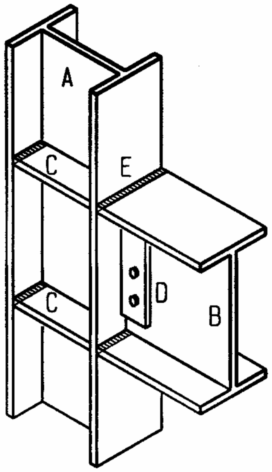
Moment frames consist of one or more portals with columns joint to beams by moment resistant connections that transmit bending deformation from columns to beam and vice-versa. Beams and columns act together to resist gravity and lateral loads in synergy and redundancy. Bending resistance makes moment frames more ductile and flexible than braced frames or shear walls. The ductile behavior is good to absorb seismic energy, but increases lateral drift, a challenge for safety and comfort of occupants, and possible equipment damage.

Moment frames provide optimal planning freedom, with minimal interference of structure. Office buildings that require adaptable space for changing tenant needs usually use moment frames. To reduce lateral drift in tall buildings, dual systems may include bracing or shear walls, usually at an interior core where planning flexibility is not required. Given the high cost of moment-resistant joints, low-rise buildings may provide only some bays with moment resistant frames. The remaining bays, with pin joints only, carry gravity load and are laterally supported by adjacent moment frames.

Moment frame behavior can be visualized by amplified deformations. The connection of column to beam is usually perpendicular and assumed to remain so after deformation. Under lateral load columns with moment joints at both ends assume positive and negative bending at opposite ends, causing S-shapes with inflection points of zero bending at mid-span and end rotation that rotates the ends of a connected beam. By resisting rotation, beams help to resist lateral load. Similarly, a beam subject to bending under gravity load will rotate the columns connected to it and thus engage them in resisting the gravity load. Columns with moment-resistant joints at both ends deform less than columns with only one moment joint. Deformations under gravity and lateral loads are visualized in the diagrams, with dots showing inflection points of zero bending stress.

1 portal with hinged joints unable to resist lateral load
2 Moment joints at base, hinge joints at beam, large drift
3 Moment joints at strong beam, hinge joints at base, large drift
4 Moment joints at base and strong beam, drift reduced to half
5 Hinged base, moment joints at beam, beam forms inflection point
6 Gravity load, hinged base, beam moment joints, 2 beam inflection points
7 Lateral load, all moment joints, inflection points at beam and columns
8 Gravity load, all moment joints, inflection points at beam and columns
9 Multi-bay frame deformation under lateral load
10 Multi-bay frame deformation under gravity load

A Inflection point of zero bending stress



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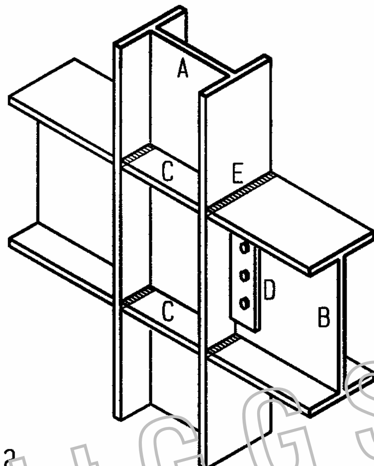

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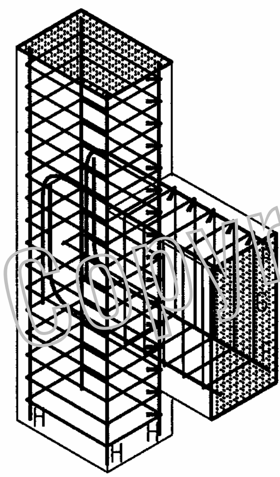

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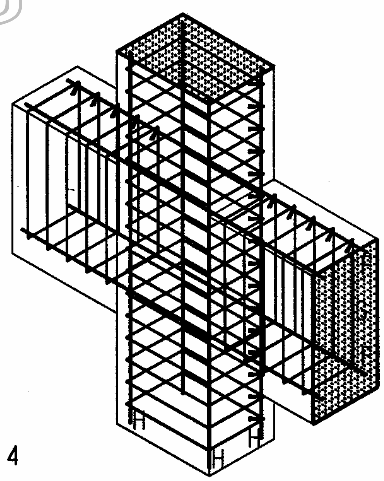

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Moment-resistant Joints

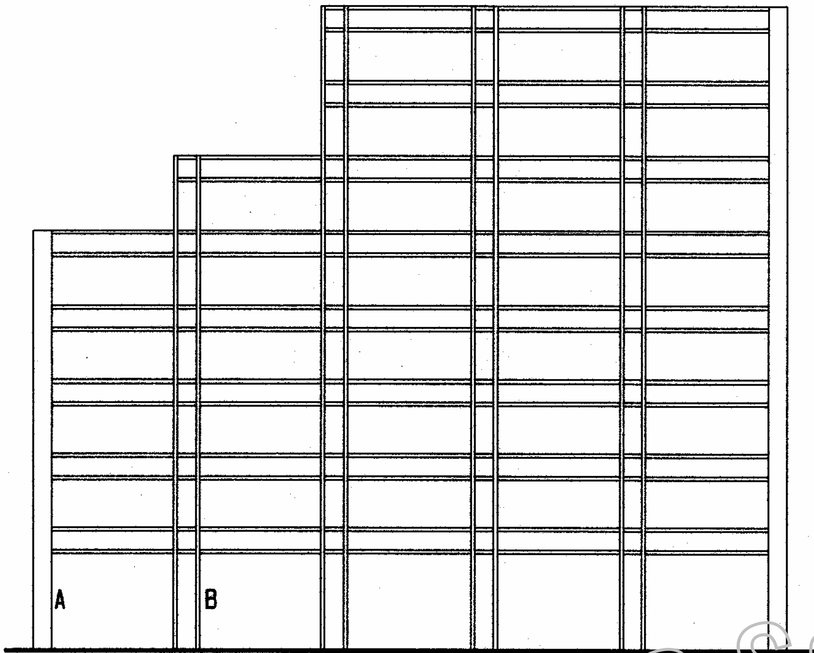
Moment-resisting joints usually consist of steel or concrete. They join members (usually column to beam) to transfer bending moments and rotations of one member to the other. The moment resistant connection makes post and beam act in unison to resist both gravity and lateral loads. In seismic regions, moment frames must be ductile to absorb seismic energy without breaking.

Steel moment joints are usually wide-flange beams connected to wide-flange columns. Generally, post and beam are connected about their strong axis. Semi-rigid joints connect the strong beam axis to the weak column axis. Moment resistant joints require stiffener plates welded between column flanges. They resist bending stress of beam flanges that tend to bend column flanges without stiffener plates. Compact columns with very thick flanges do not require such stiffener plates. Steel is a ductile material which is good to absorb seismic energy in the elastic range. Yet the seismic performance of steel joints was challenged by failures during the 1994 Northridge Earthquake. The failure resulted primarily from joint welds. Research developed solutions for moment-resisting steel joints, notably *dog-bone* beam ends to form plastic hinges to reduce stress at the joints.

Concrete frames achieve ductile joints by proper steel reinforcing, designed to yield before the concrete crushes in brittle mode. Usually that implies 25% to 50% less steel and more concrete than used for balanced design (balanced design has just enough reinforcing to balance the concrete strength). Ductile design also requires: closely spaced tie bars near beam/column joints; column rebars to extend through beams; beam rebars to extend through columns; and column ties to continue through beams.

1 Moment-resisting steel joint at end column
2 Moment-resisting steel joint at interior column
3 Moment-resisting concrete joint at end column
4 Moment-resisting concrete joint at interior column

A Steel wide-flange column
B Steel wide-flange beam
C Stiffener plates resist bending stress of beam flanges
D Steel bar welded to column in shop and bolted to beam in field
E Weld, joining beam flange to column
F Steel reinforcing bars in concrete beam
G Steel ties to restrain reinforcing bars from buckling
H Column reinforcing bars to resist compression and bending



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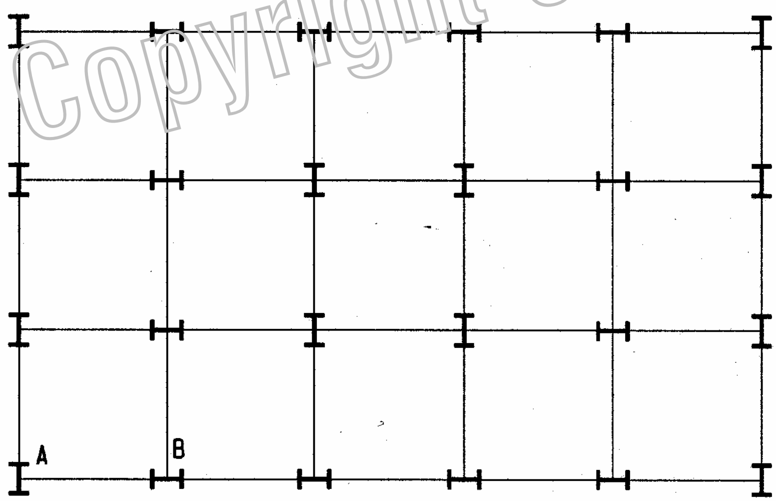


2

Steel framing

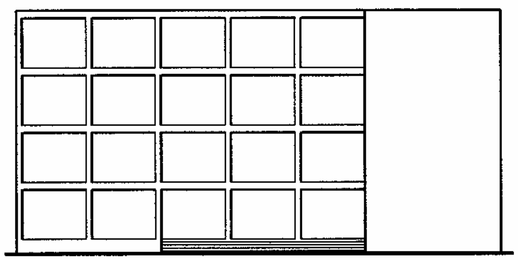
Steel framing with wide-flange profiles requires careful orientation of columns in order to achieve proper strength and stiffness to resist lateral load in both orthogonal directions. Measured by the moment of inertia, typical wide-flange columns have a stiffness ratio of about a 3:1 about the x and y-axis, respectively, yet some deep sections have stiffness ratios up to 50:1, about strong to weak axes. Therefore, column orientation for lateral resistance is an important design consideration for moment frames. Assuming equal lateral load and column size, half of the columns should be oriented in either direction. For unequal loads, column orientations should provide strength proportional to loads. For example a rectangular building has more wind load on the long than on the short facade. If wind governs lateral design, this should be considered in column orientation. Further, column orientation should provide symmetry of stiffness in both directions to prevent torsion. Torsion would occur for example if one end of a building has columns with greater stiffness than the other end. Also to better resist possible torsion from asymmetric mass distribution, columns should be placed near or at the building edge, rather than near the center of mass where they have no effective lever arm to resist torsion. Column size should also account for setbacks on upper floors, to account for asymmetric wind or seismic load resulting from such setbacks.

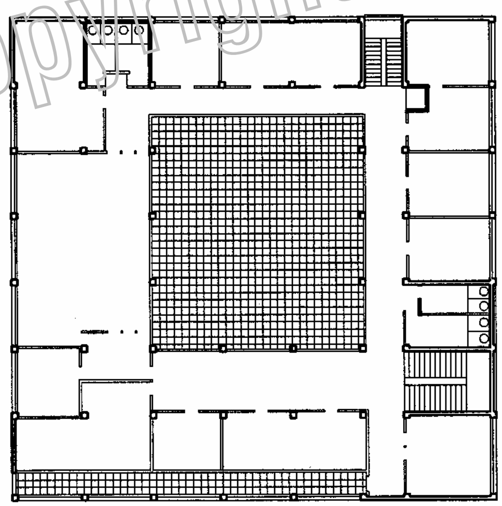
1 Front view of moment resisting frame with setback floors on top
2 Column layout in plan for moment resistance in both directions

A Column oriented for lateral support in width direction
B Column oriented for lateral support in length direction

Casa Terragni, Como, Italy (1936)
Architect: Guiseppe Terragni

With 33.2x33.2x16.6m height, the building is a perfect half cube. The plan is organized around a central atrium, surrounded by circulation. Terragni used the concrete moment frame as organizing grid in a liberal manner, modified as required to meet planning needs: the 4.75m grid is reduced for circulation and increased for large spaces. Beams of variable depth express the respective spans. The front facade is recessed behind a veranda to emphasize the frame. Moment frames with shear walls have proven a failsafe solution in earthquakes prone areas: shear walls provide good stiffness under moderate load, and the moment frame provides ductility if shear walls fail in sever earthquakes.





Commonwealth (formerly Equitable) Building (1944-48)
Architect: Pietro Belluschi

The Equitable Building 1948 pioneered the clear expression of a steel moment frame, a model for many subsequent buildings. With this building Beluschi also pioneered the first double glazed aluminum curtain wall of simple elegance... The building is a National Historic Landmark of mechanical engineering because it was the first building using heat pumps for efficient air conditioning... It was the first skyscraper to use double-paned glass. The first building with air conditioning completely sealed and the first to use a flush curtain wall design. The first building completely clad in aluminum. 1982 the American Institute of Architects awarded the building the 25-year award. The building is a compelling testimony of Beluschi's philosophy of simplicity.

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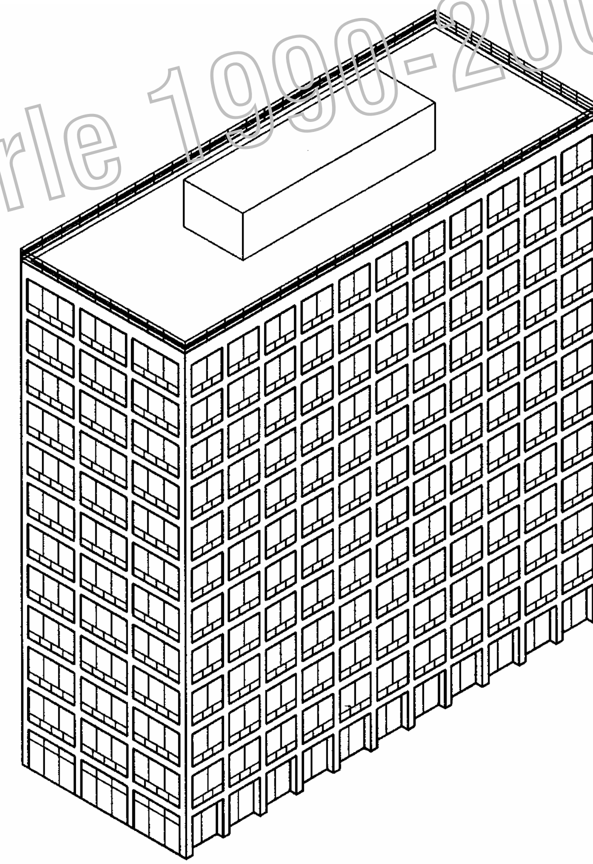
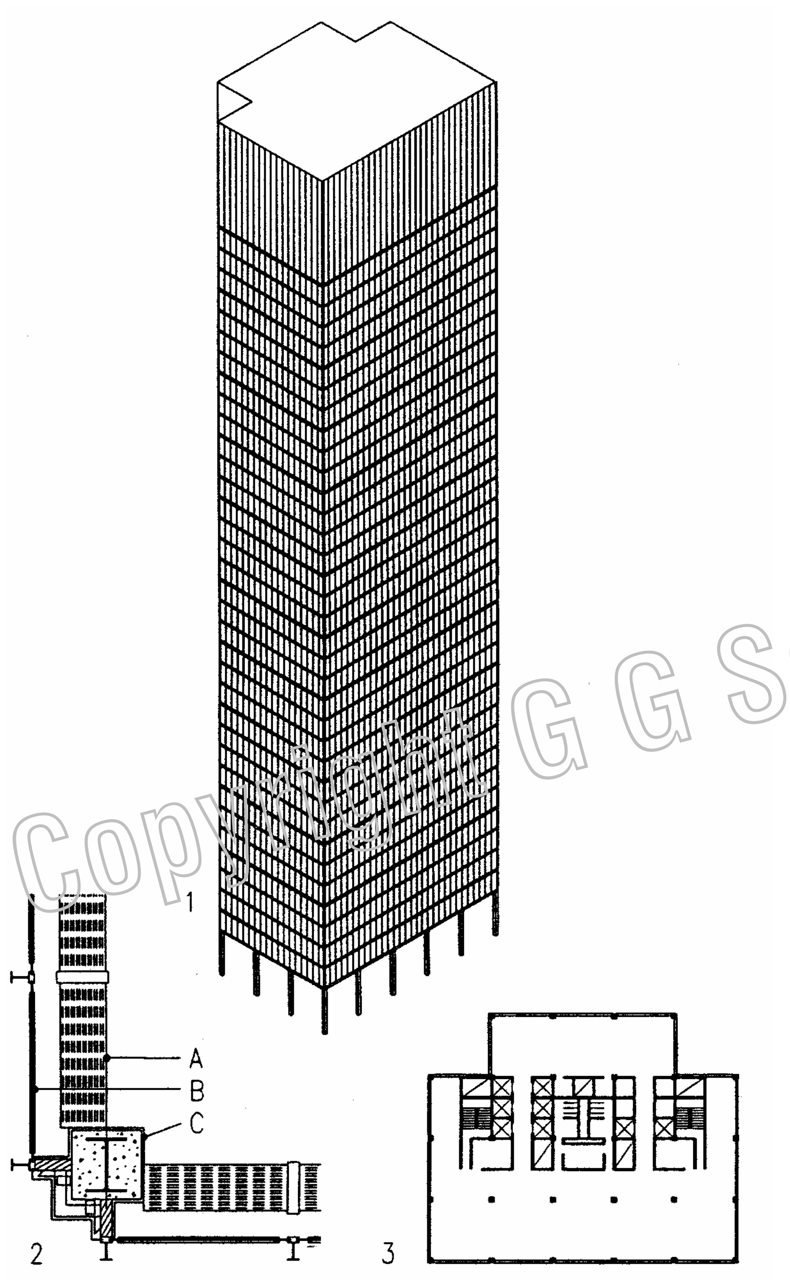




Seagram building, New York (1954-58)

Architect: Mies van der Rohe, Philip Johnson, Kahn and Jakobs

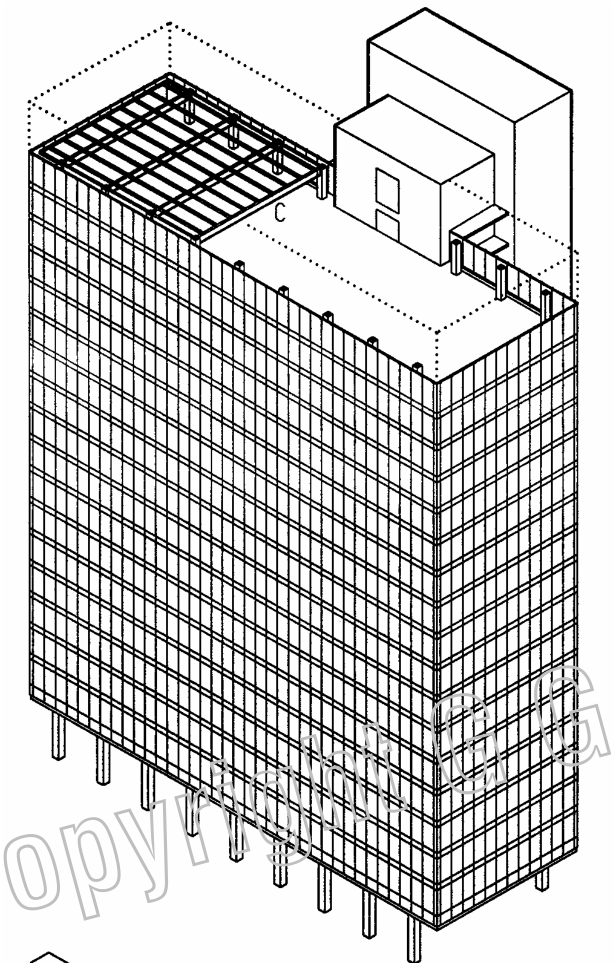
Engineer: Severud, Elstad, Krueger

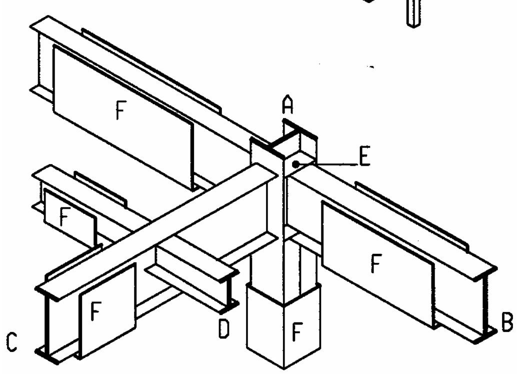
The 38-story Seagram building is a classic icon of modem architecture. It was the result of unique cooperation between the client, Samuel Bronford, his daughter, Phyllis Lambert as planning director, and the architects. The building exemplifies Mies' philosophy of *Baukunst* (art and craft of building), with great attention to detail and proportion. The structure, based on a 28 ft (8.5 m) module, is expressed as colonnade at the base to signal the entrance. The skin of the mechanical floor on top provides a visual cap. Most of the structure is concealed behind the curtain wall which eliminates thermal stress and strain due to outside temperature variations, an important factor in tall structures. The recessed rear gives the tower its classic proportions of five to three for front and side, respectively. The steel moment frame structure is embedded in concrete for fire protection and added stiffness. The core walls have diagonal bracing up to the 29th floor for additional wind bracing. Concrete shear walls up to the 17th floor provide additional stiffness.

Floor plan:	84 x 140 feet (26 x 43 m) without extrusion
Height:	525 feet (160m)
Typical story height:	13.6 feet (4.15m)
Height/width ratio	6.3 without extrusion

1 Axon view of tower
2 Comer detail of structure and skin
3 Typical plan with recessed comers to express 3 to 5 proportion

A Air conditioning duct as parapet
B Glare reducing pink glass appears without color from inside
C Bronze cover of steel column embedded in concrete





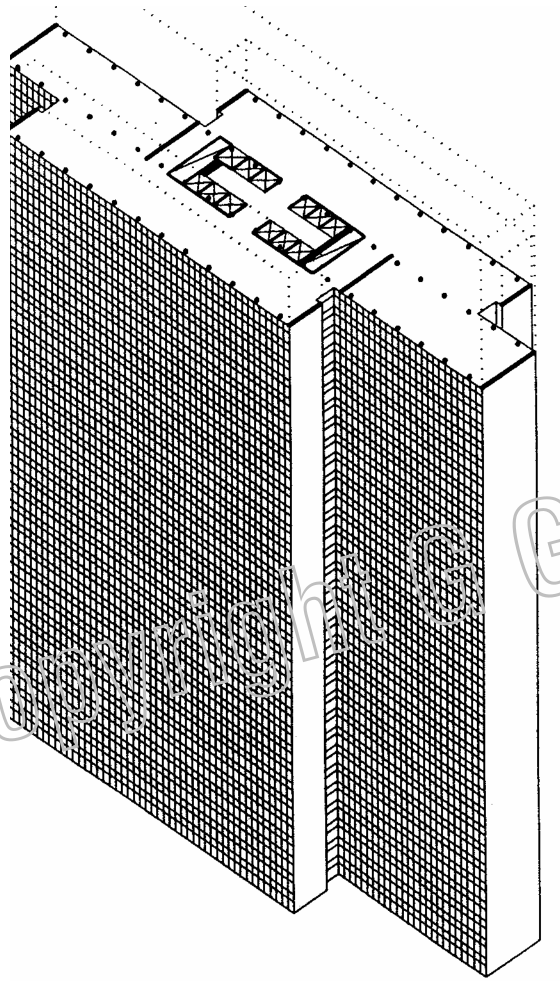
Crown Zellerbach building, San Francisco (1959)
Architect: SOM and Hertzka and Knowles
Engineer: H. J. Brunnier

The 20-story Crown Zellerbach headquarters building covers about one third of a triangular site on Market Street, the main street of San Francisco. The building features a large office wing flanked by an external core for stairs, elevators, bathrooms, and mechanical ducts. The exterior core gives the office wing a column-free floor area for optimal space planning flexibility. A planning module of 5.5 feet (1.6 m) provided for good size office spaces.

The structure is a moment resistant steel frame with wide-flange girders spanning 63 feet (19 m) across the width of the building, supported by wide-flange columns, spaced 22 feet (6.7 m) on center. Spandrel beams connect the columns in the longitudinal direction. Steel joists, spaced 7 feet (2.1 m), support concrete slabs on cellular metal decks. The joists cantilever at each end of the building. All columns are oriented with their strong axis to provide moment resistance in the width direction, giving the building much greater strength and stiffness in width than in length direction. Since the building is much longer than wide, the column orientation is good for wind load which is greater on the long faced; but it is less effective for seismic load which is greater in length direction. Also the eccentric service tower causes seismic torsion. The fire exits on both side of the service tower are too close together for fire safety and would not be allowed by current code. The building is supported by a mat foundation, 8 feet (2.4 m) deep, extending the full width and length of the building. The foundation rests on firm soil 45 feet (13.7 m) below grade under a 2-story parking garage. The steel structure is protected by fire proofing that consists of stucco applied to metal lath wrapped around beams and columns.

Floor plan:	201x69 ft (61x21m) without exterior core
Height:	320 ft (96m)
Typical story height:	13.67 ft (4m)
Height/width ratio	4.6 without exterior core

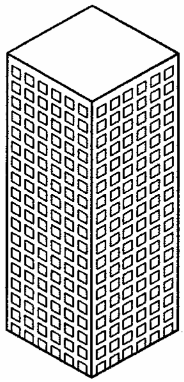
A Column spaced 22ft (6.7 m)
B Spandrel beam
C Girder spanning the full width of the building
D Joist spaced 7ft (2.1 m)
E Stiffener plate for moment connection
F Fire proofing on metal lath



Thyssen tower, Düsseldorf (1957-60)
Architect: Hentrich and Petchnigg
Engineer: Kuno Boll

The Thyssen tower's unique plan of three slabs is a composition with efficient circulation and good delighting for all offices that are never more than 7m (23 ft) from a window. The floor area of offices is 62.7% of the gross floor area. Located at the center of town, the long axis is oriented north-south with a park to the North. The central block includes the service core and, as tallest block, houses mechanical and elevator equipment in the top floors of this 25-story tower. Parking for 280 cars is in the underground garage, rapped around the building. The long facades feature glass curtain walls; the narrow end facades are glad in stainless steel. The steel frame structure is embedded in concrete for fire protection and to provide additional stiffness. The columns consist of steel pipes produced by the building owner. The structural module is 7x4.2 m (23x14 ft), with some variation between central and outer slabs. Braced end walls provide some additional stiffness to resist wind load on the long building sides. The exterior composition of the building, expressing the internal organization, has earned the nickname "Drei-Scheiben Haus" (Three-slab-house). The pristine design, combining American know-how with European sophistication stands as an icon of the modern movement in Europe.

Floor plan:	21x80 m (70x226 feet)
Height:	94 m (308 feet)
Typical story height:	12.2 feet (3.7 m)
Height/width ratio	4.48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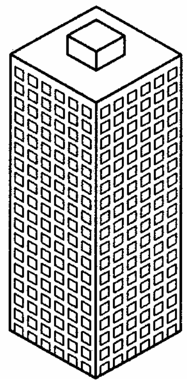
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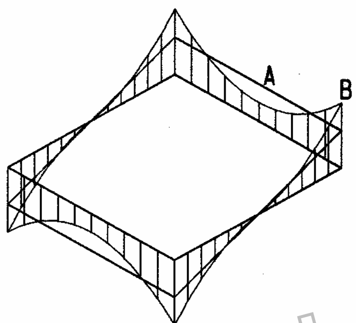

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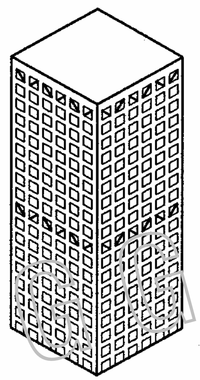
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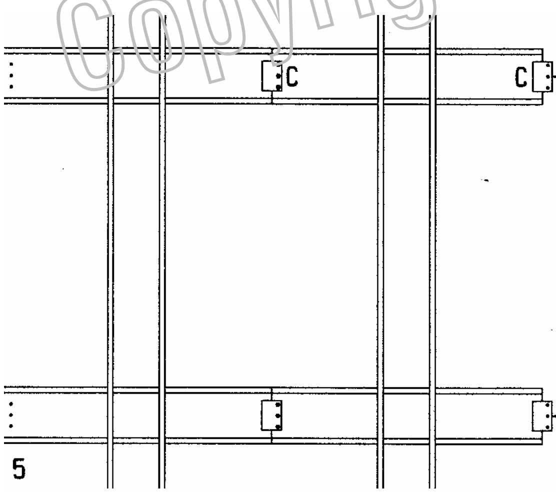

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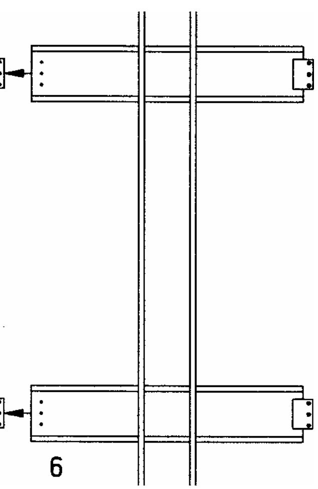
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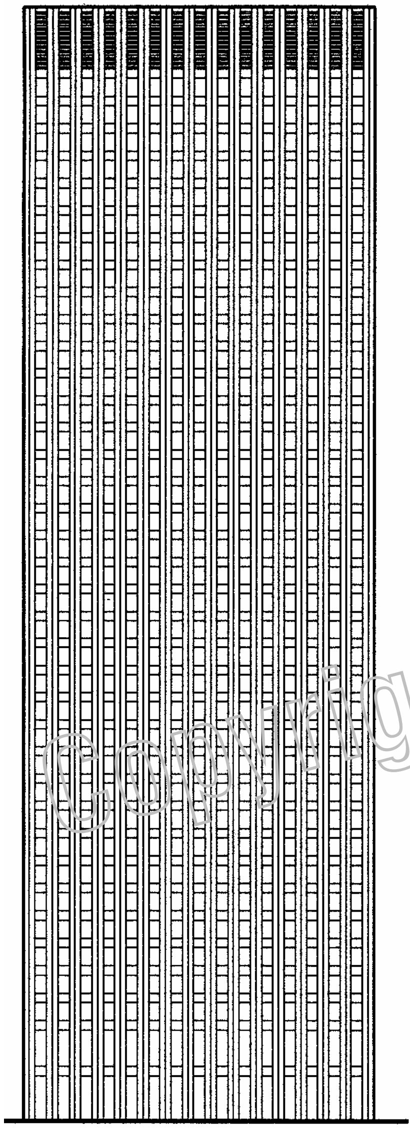
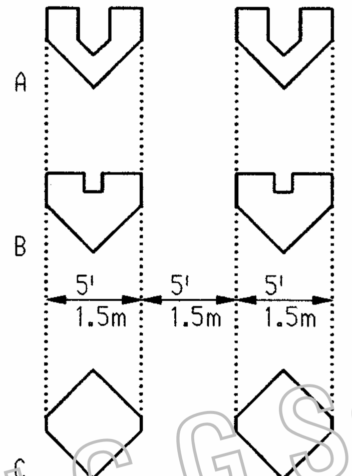
Framed Tube

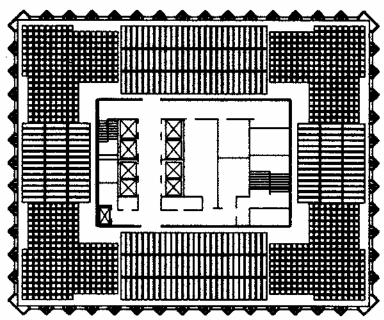
Framed tubes are a variation of moment frames, wrapping the building with a “wall” of closely spaced columns and short spandrel beams. To place the lateral resistance system on the façade rather then at the interior gives it a broader base for greater stability as well as improved rotational resistance. In addition, the lateral resisting system on the façade allows smaller columns on the interior to carry gravity load only. Further, designing floors and roof to span the full width of a building can make the interior completely column free for optimal flexibility. A major challenge of framed tubes is the high cost of numerous moment resistant joints between closely spaced columns and beams. To minimize this adverse cost factor, designers often use prefab methods to weld the joints in the fabrication hop rather than on the job site. This process also improves quality control and reliability.

1 Framed tube without interior core
2 Framed tube with interior core
3 Global stress diagram of framed tube
4 Framed tube with belt and top truss for additional stiffness
5 Prefab frame with joints located at beam inflection point of zero bending
6 Prefab element ready for assembly

A Reduced shear resistance (shear lag) at hollow interior
B Peak axial force from overturn moment
C Pin joint at inflection point of zero beam bending stress







CBS Tower New York (1961-650
Architect: Eero Saarinnen
Engineer: Paul Weidlinger

The 38-story CBS tower is a stark vertical extrusion of the rectangular floor plan. Columns forming a framed tube are expressed as triangular extrusions on the upper floors and diamond shaped on the ground floor. The triangular columns include niches for mechanical ducts and pipes. The niches decrease from top to bottom with the decreasing duct sizes that run down from the mechanical room on the top floor. The decreasing niches result in increasing net column size that coincides with increasing load as it accumulates from top down. Concrete floors span between the walls of a central core and the framed tube, providing a column-free donut-shape floor space for flexible use. The four sides facing the core feature one-way rib slabs, but the four corners have two-way waffle slabs, designed to make the transition from one direction to the other. Glad in black granite the closely spaced triangular columns express a stark verticality, perforated with regular windows on all but the top and ground floors. The top mechanical floor has ventilation louvers instead of windows; the ground floors have taller windows and doors. The articulation of top and bottom of the façade emphasizes the most prominent part of the building, a strategy often use for the design of tall buildings.

Floor plan:	155x125 feet (47x38m)
Height:	494 feet (151m)
Typical story height:	12 feet (3.66m)
Floor-to-ceiling height:	8.75 feet (2.67m)
Height/width ratio	3.9

A Column profile at top floor
B Column profile at lower floors
C Column profile at ground floor

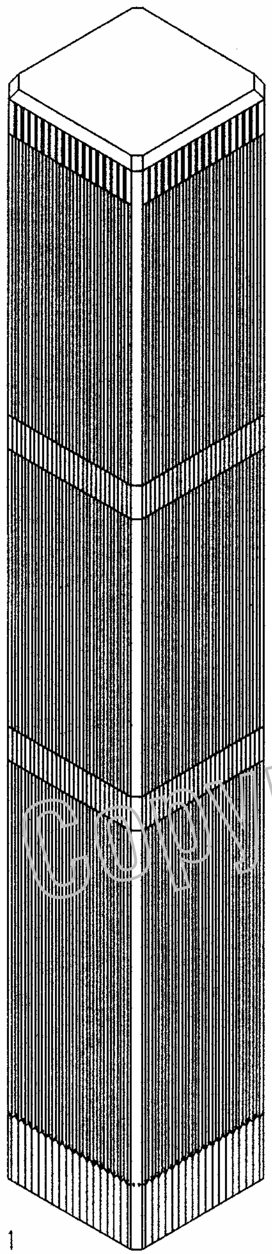
World Trade Center, New York (1977
(demolished by terrorists 9-11-2001)

Architect: Minoru Yamasaki and E. Roth
Engineer: Skilling, Helle, Jackson, Robertson

The World Trade center housed 50,000 employees and up to 80,000 visitors daily in two 110-story towers. Both towers, in diagonal juxtaposition, were vertical extrusions of square plans, with very closely spaced steel columns. Each tower had two-story mechanical spaces on top, near the bottom, and two distributed at 1/3 intervals, with elevator sky-lobbies two floors above each. Each tower had 100 passenger and four service elevators. Each sky-lobby was reached by 11 or 12 elevators from ground floor; with five express elevators non-stop to the 107th and 110th floors. Since elevators are stacked, 56 shafts needed, take 13% floor area on each floor. The framed tube structure consisted of 56 box steel columns on each façade, joint at each floor by spandrel beams with moment resistant connections. This giant Vierendeel frame was assembled from prefab elements of three two-story columns with beam and column joints at mid-span and mid-height where inflection points of zero bending occur under lateral load. Combined with rigid floor diaphragms, the towers formed torsion-resistant framed tubes that cantilever from a five-story underground structure that houses train and subway stations as well as parking for 2,000 cars. Although the framed tube columns overall dimensions are constant, their wall thickness increases from top to bottom in response to increasing loads. Floor truss joists span from the framed tube to columns around the central core. Mechanical ducts run between truss joists for reduced story height. The core columns are designed to carry gravity load only. The framed tube resisted both lateral and gravity load.

Floor plan (square):	208x208 feet (63.4x63.4 m)
Height:	1361 feet (415m)
Typical story height:	12 feet (3.66m)
Floor-to-ceiling height	8.6 feet (2.62m)
Height/width ratio	6.5

1 Axon of tower
2 Typical floor framing plan
3 Typical prefab two-story façade assembly
4 Typical framed tube column size and spacing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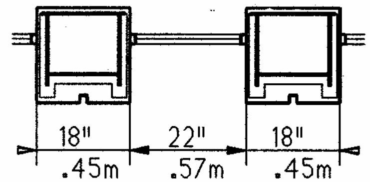

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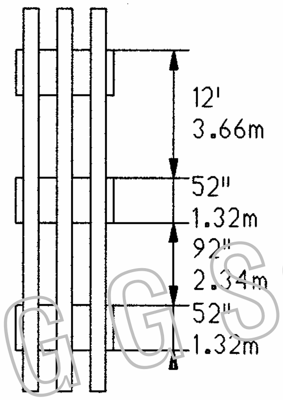

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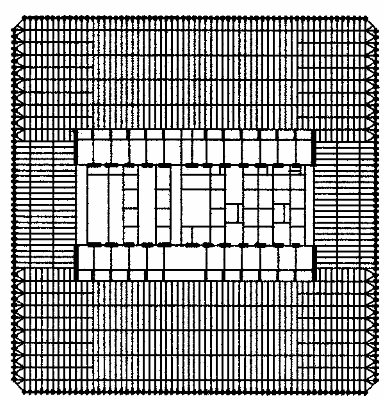
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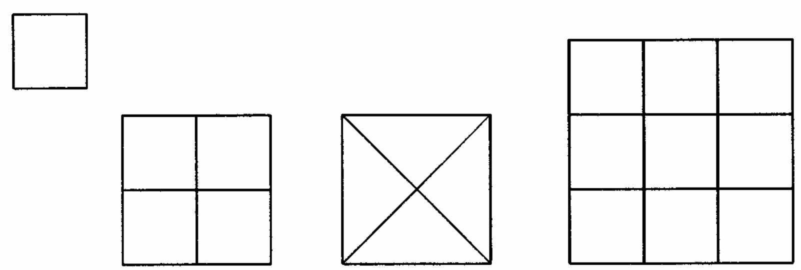

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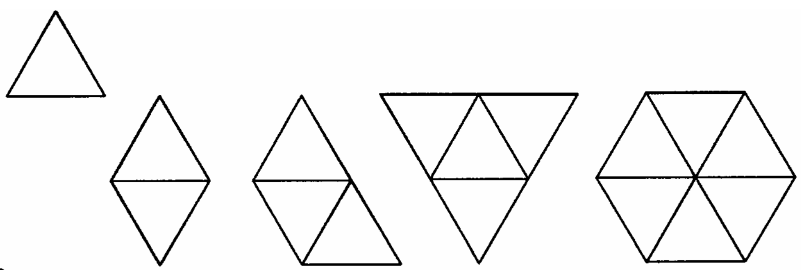

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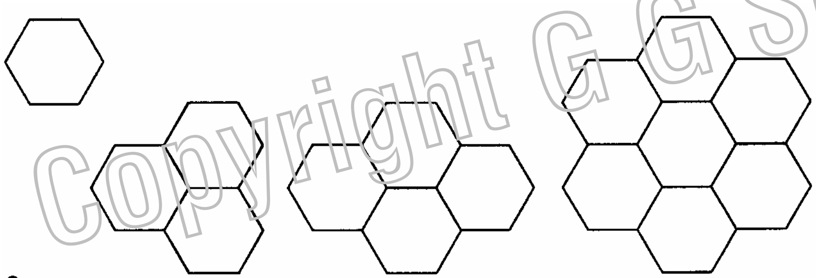

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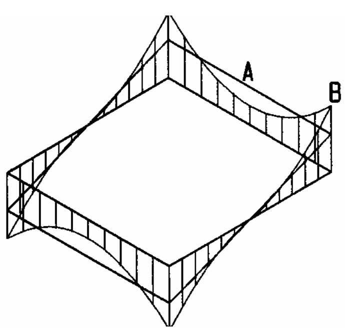

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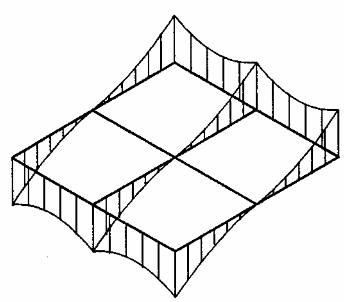

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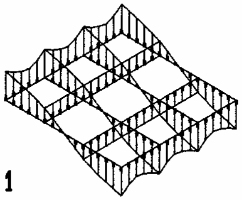
Bundled Tube

Bundled tube structures are composed of tubes framed by closely spaced columns joined to beams to form moment frames. The bundled tubes resulting from the rows of columns add lateral resistance to the structure, transferring shear between exterior columns subject to tension and compression under lateral load. This shear transfer makes it possible for the exterior columns to act in synergetic unison, whereas independent columns would act alone to provide much less lateral resistance. Bundled tubes transfer shear not only through exterior frame "walls" but also through interior cell "walls" thereby reducing shear lag.

An alternative to framed bundled tube are braced bundled tube systems. However, though they provide greater stiffness, the braces disrupt spatial flow between interior columns. Regarding plan geometry, bundled tubes may have bundles of square, rectangular, or triangular polygons that are repeatable. However, hexagonal polygons would be less efficient

1 Square tube modules
2 Triangular tube modules
3 Hexagonal tubes would be less effective to reduce shear lag
4 Framed tube shear lag
5 Bundled tube with reduced shear lag

A Shear lag between connecting shear walls
B Peak resistance at shear wall



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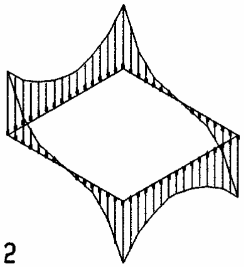

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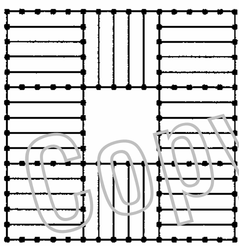

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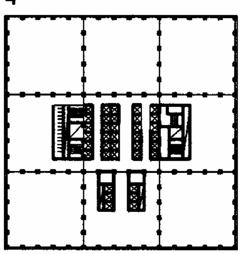

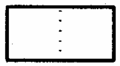

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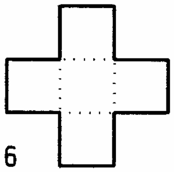

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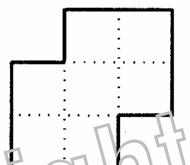

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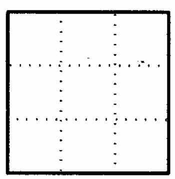

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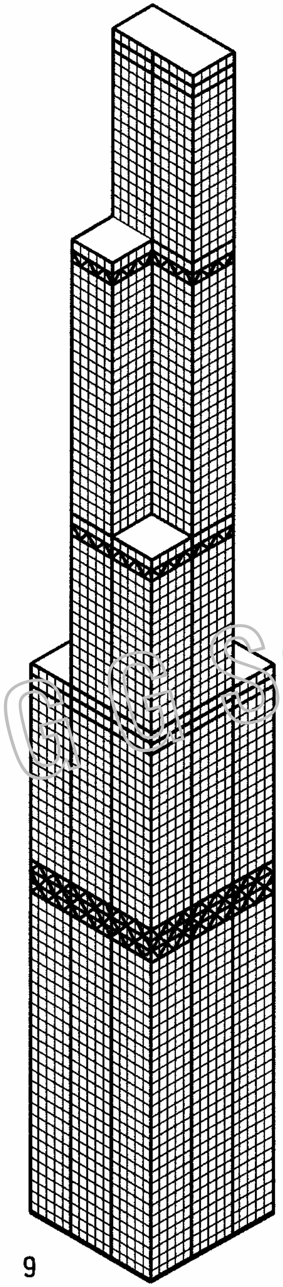

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Sears tower, Chicago (1973)
Architect/ Engineer: Skidmore, Owings and Merrill

With 110 stories, the Sears Tower was the tallest building in the world for many years and occupies an entire city block on the southwest of Chicago's loop. The tower starts at ground level with nine square modules of 75 feet (22.9m) each. The nine modules gradually reduce to a twin module on top in response to needed office space and also to reduce wind resistance and overturn moments. The large areas of the lower floors are occupied by Sears; the smaller floors at higher levels serve smaller rental needs. Elevators serve the building in three zones of 30 to 40 stories separated by sky-lobbies that are reached by double deck elevators express elevators. The building façade is glad in black aluminum and tinted glass. The structure is anchored to a five-story underground structure. Nine bundled tubes are separated by rows of columns, spaced 15 feet (4.6m) on center. The columns, welded to beams, form moment resisting portals to transfer global shear under lateral load from compressed to tensed side of the structure, to reduce lateral drift. This shear transfer between exterior walls reduces "shear lag" and gives the bundled tube greater strength and stiffness to resist lateral loads. The bundled tube concept conceived facilitates the setbacks as the floors get smaller toward the top. Belt trusses at three levels in conjunction with mechanical floors reduce lateral deflection by about 15 percent and help distribute uneven gravity load caused by floor setbacks. The horizontal floor framing consists of trusses that span 75 feet (23m) between columns and support concrete slabs on metal deck. The one-way floor trusses of 40 inch (1m) depth change direction every 6th floor to redistribute the gravity load to all columns. Trusses consist of top and bottom T-bars, connected by twin angle web bars. They allow mechanical ducts between top and bottom chords. The small truss depth was possible, using composite action; shear studs engage the concrete slab in compression for increased resistance.

Floor plan at ground:	225 x 225 feet (69 x 69m)
Height:	1,450 feet (442m)
Typical story height:	13 feet (3.96m)
Height/width ratio	6.4

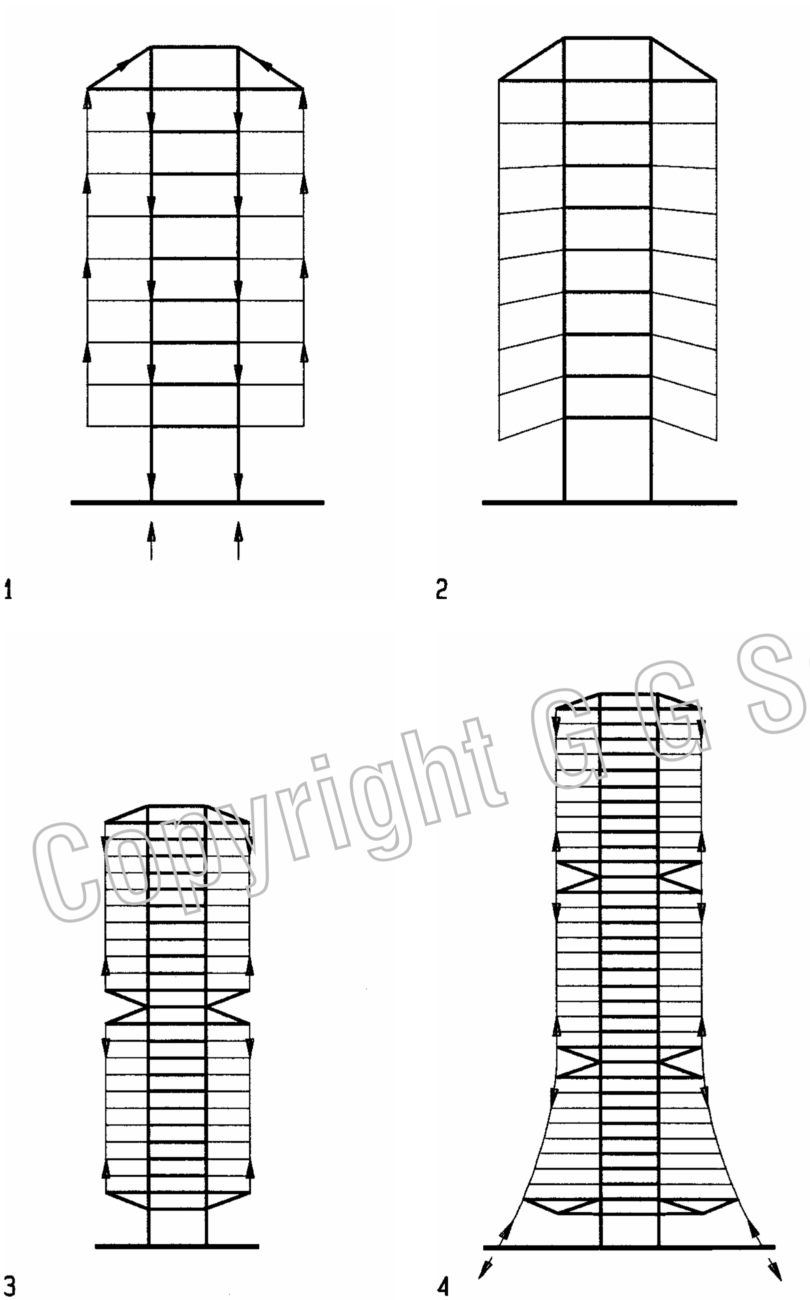
1 Tower axon
2 Base floor plan
3 Floor framing
4 Stress diagram of single framed tube with shear lag between walls
5 Stress diagram of bundled tube with reduced shear lag
6 Floor plan at ground floor
7 Floor plan starting at 51st floor
8 Floor plan starting at 66th floor
9 Floor plan of top floors

19

Vertical Systems Suspended

Vertical systems, suspended, also referred to as **suspended high-rise** structures, are different from suspension structures like suspension bridges, which are draped from two suspension points; suspended high-rise structures hang usually about vertically from top. A rational for suspended high-rise structures is to free the ground floor from obstructions. Other architectural and structural reasons are described on the next page.

Regarding Lateral load, the challenge of suspended high-rise is usually a narrow footprint and slender aspect ratio. Thus their behavior is comparable to a tree, where the drunk resists load primarily in bending and large roots are required to resist overturning. Properly designed, the narrow aspect ratio can enhance ductility to make the structure behave like a flower in the wind to reduce seismic forces.



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Suspension rational

At first glance suspended high-rise structures seem irrational, given the load-path detour: gravity load travels to the top and then down to the foundation. However, as described below, there are advantages, both architectural and structural, that justify this detour. Understanding the pros and cons and their careful evaluation are essential for design.

Challenges

- Load path detour: load travels up to the top, then down to foundation
- Combined hanger / column deflection yields large differential deflection

Architectural rational

- Less columns at ground floor provides planning flexibility and unobstructed view
- Facilitates top down future expansion with less operation interference
- Small hangers instead of large columns improve flexibility and view

Structural rational

- Eliminates buckling in hangers, replacing compression with tension
- High-strength hangers replace large compression columns
- Floors may be built on ground and raised after completion
- Concentration of compression to a few large columns minimizes buckling

Design options

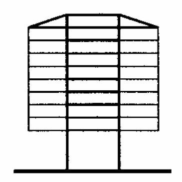
- Multiple towers with joint footing to improve overturning resistance
- Multiple stacks to limit differential deflection
- Adjust hangers for DL and partial LL to reduce deflection
- Prestress hangers to reduce deflection to half

1 Gravity load path
Load travels to top, then down to foundation

2 Differential deflection is cumulative
Shortening of columns and elongation of hangers are additive

3 Prestress can reduce deflection to half
Top resists half the load through increase of prestress
Bottom resists half the load through decrease of prestress

4 Ground anchors for improved stability
(assuming hangers as ground anchors are ok)



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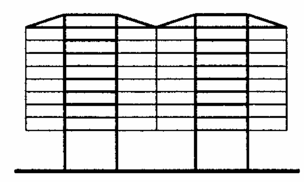

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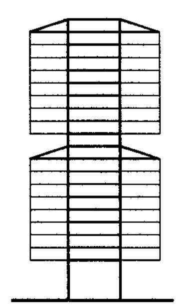

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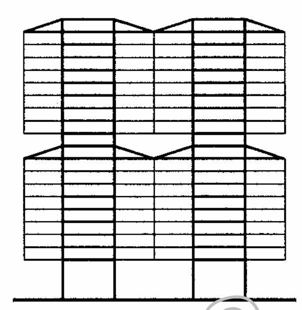

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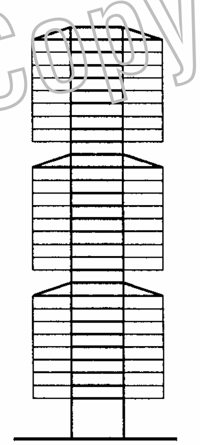

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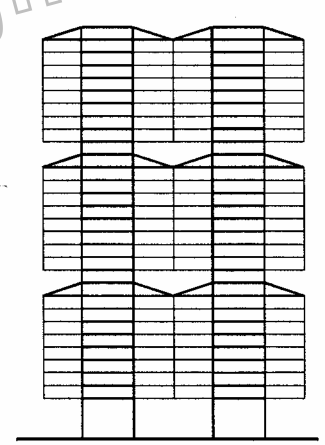

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Design options

Suspended high-rise structures may be designed in various configurations with distinct limitations and implications regarding behavior. The following description provides guidelines for rational design, starting with the introduction of a terminology, followed by implications of various design options:

- Single towers (one vertical support)
- Multiple towers (several vertical supports)
- Single stacks (one set of floors)
- Multiple stacks (several sets of floors)

The effect of these options are described and illustrated as follows:

1 Single tower / single stack
Single towers require large footing like a tree to resist overturning

2 Multi towers
Multiple towers with joint footing increase stability

3 Twin stacks
Twin stacks reduce the length of hangers and thus differential deflection
(ten stories per stack limits differential deflection to < 2 inch (50 mm)

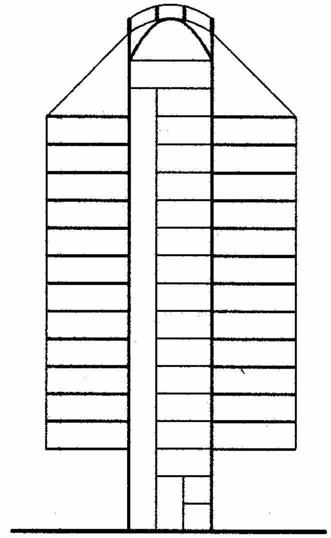
4 Twin stacks / towers
Twin stacks reduce the length of hangers and thus differential deflection
Twin towers with joint footing increase stability

5 Triple stacks
Three or more stacks limit hanger length and thus differential deflection

6 Triple stacks / twin towers
Three or more stacks limit hanger length and thus differential deflection
Two or more towers with joint footing increase stability

Limits

An important limit for suspended high-rise structures is the limited number of floors per stack. More than ten floors per stack would cause unacceptable differential deflections. Conventional columns in compression are subject to about equal strain under load. Suspended high-rise structures are subject to greater differential deflection since hangers elongate but columns shorten under gravity load. Without buckling, the high tensile stress of hangers causes greater strain which further increases differential deflection.



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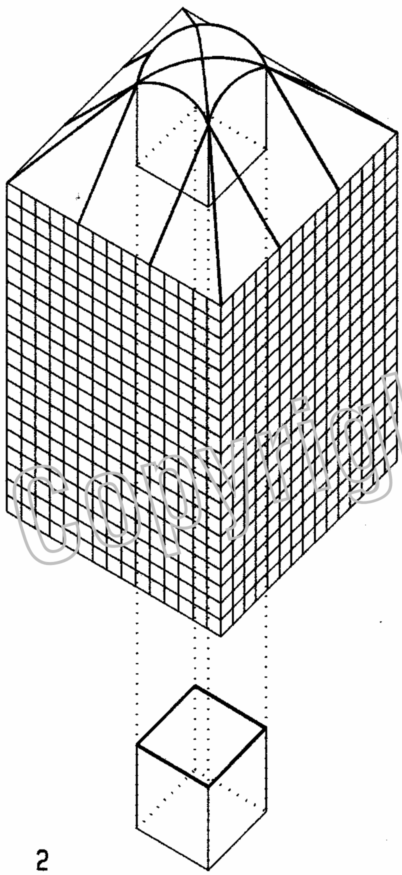

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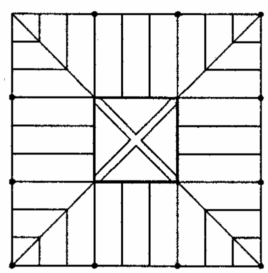

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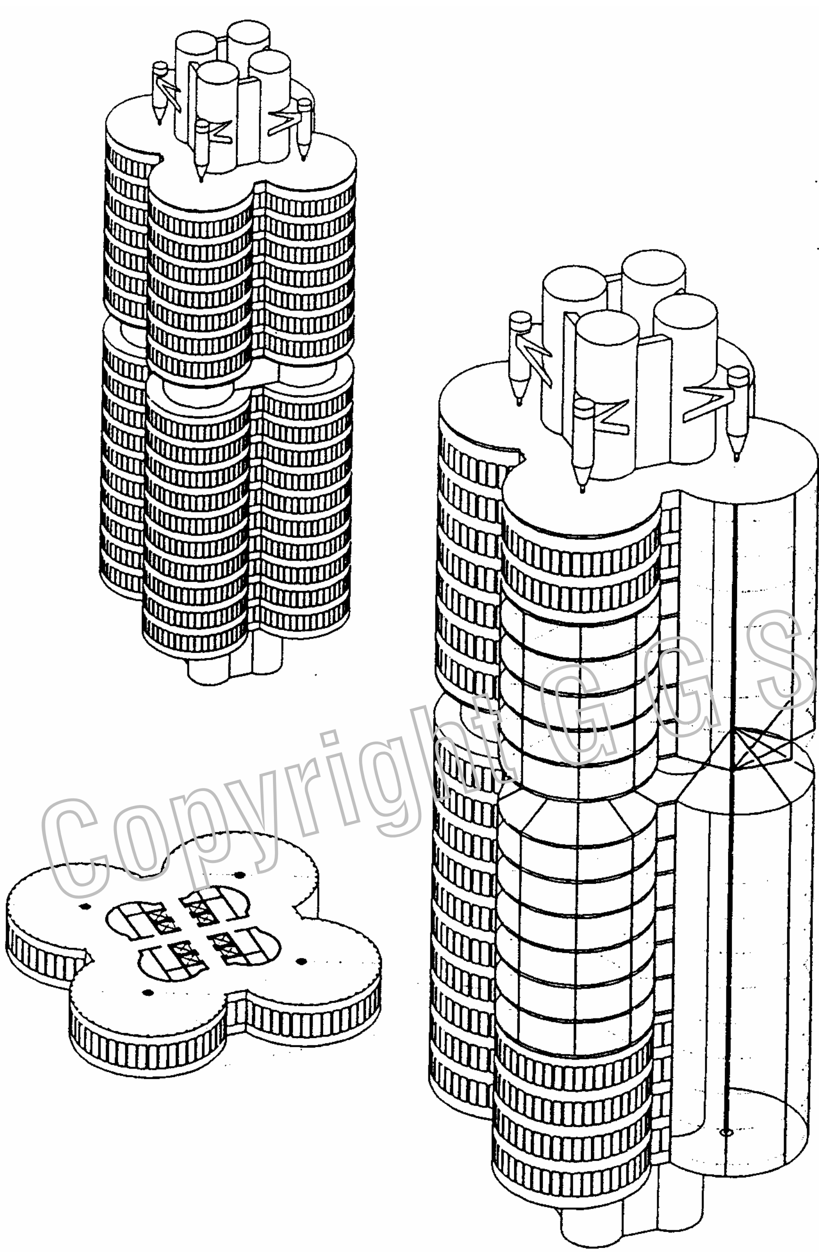
Case studies

Westcoast Transmission Tower, Vancouver (1969)
Architect: Rhone and Iredale
Engineer: Bogue Babicki

The 12-story tower, initially designed and built as Westcoast Transmission headquarters, has become an architectural icon of Vancouver. With support of the City of Vancouver, the historically significant building was converted in 2005 to 180 unique residential suites in studio, one and two bedroom configurations. The suspension concept was selected to provide an unobstructed view to the beautiful bay of Vancouver. According to the Bogue Babicki, the suspension option was also more economical than a conventional alternative they had considered. The suspended structure, stating 30 feet (9 m) above grade provided unobstructed views at ground level to the beautiful bay of Vancouver. The tower is supported by a site-cast concrete core, 36 feet (11 m) square. The floors are suspended by 12 cables. Each cable consists of two 2 7/8" (73 mm) diameter strands. The sloping guy cables have two additional 2 ½" (64 mm) diameter strands (the 45 degree slope increased their vector force by 1.414).

Size:	108x108 feet (33 x33 m)
Core size:	36x 36 feet (11x11 m))
Height	12 stories, 224' (68 m)
Typical story height:	12 feet (3.65 m)
Core height/width ratio:	6.2

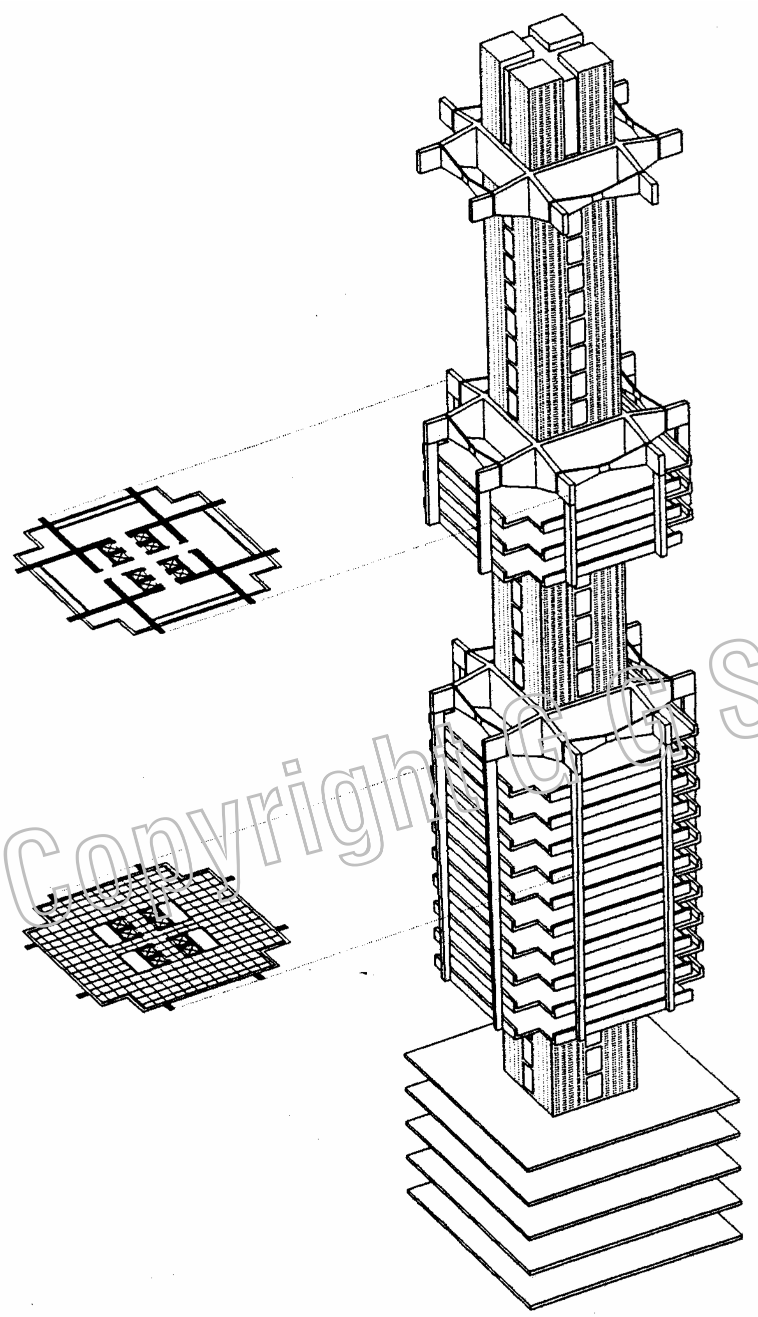
1 Section
2 Exploded axon
3 Floor framing



BMW Headquarters Munich (1972)
Architect: Karl Schwanzer
Engineer: Helmut Bomhard

The Viennese architect Karl Schwanzer won the international design competition for the BMW tower with his idea to represent the automobile company in form of a four-cylinder engine. Four cylinders are suspended from an assembly of four semi-cylindrical concrete cores by means of hangers, suspended from concrete cores of stairs, elevators, etc. The core extends as four cylinders on top of the floor stacks. Each floor is supported by a hanger at its center and stabilized by the core. To keep differential deflection within acceptable limits, the tower is partitioned into two stacks of eleven and seven office floors of the lower and upper stacks, respectively. Eight elevators, stairs and services are located in the core. Except for the four central hangers, the office space around the core is free from columns to provide highly flexible office areas. Construction of the tower started with the central core in conventional method; but then proceeded from top down. Post-tensioned concrete floor plates, cast on the ground, where lifted up by hydraulic means; starting with the top floor, followed by successive floors downward. Silver gladding exterior conveys a sophisticated high-tech image, true to the BMW philosophy.

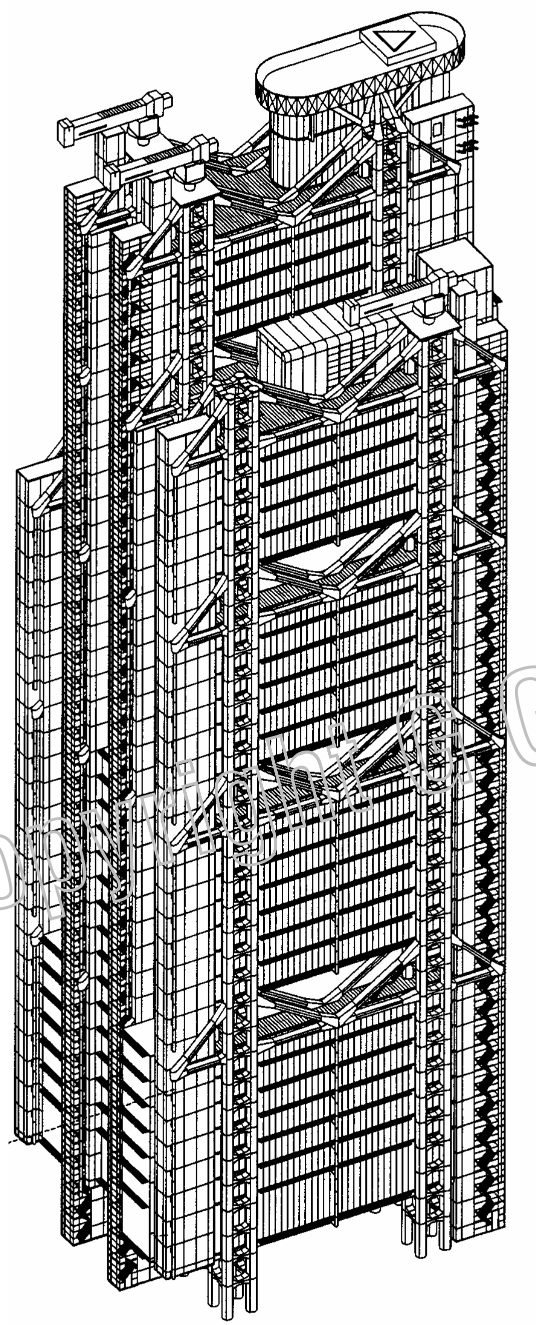
Size:	52, 30 m (172 feet) diameter
Core size	24.4 m (80 feet)
Height:	18 suspended stories, 101 m (331 feet)
Typical Story height:	3.82 m (10.8 feet)
Core height/width ratio:	4.1



Standard Bank Center, Johannesburg (1968)
Architect: Hentrich and Petschnigg
Engineer: Ove Arup and Partners

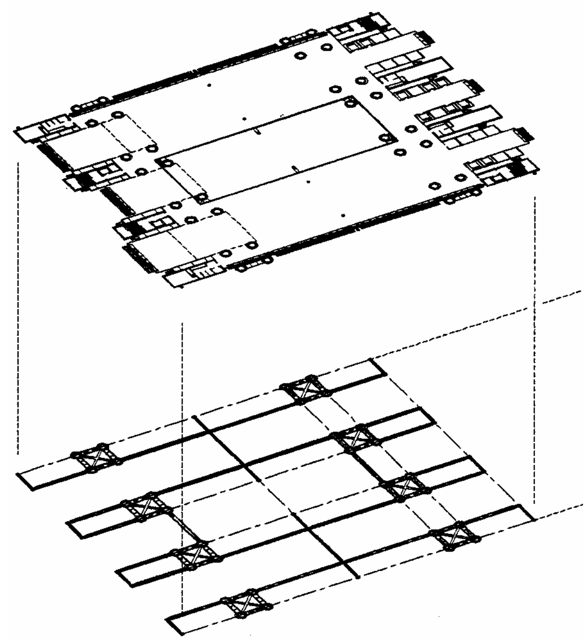
The Standard Bank Center is located in the financial center of Johannesburg. Given the dense surroundings, the design objective was to access the center via an open plaza with the least amount of bulk and obstructions. The response to this objective was a suspended structure. The central support core only keeps the plaza level open for free and spacious access. The suspension system also facilitated construction at the dense urban surrounding. After the central core was built, floors were suspended from three cantilevers. To limit differential deflection, the building is organized into three stacks of nine office floors each, suspended from concrete cantilever beams of 18 feet (5.4 m) depth. The cantilever beams are attached to the outside face of the concrete core by shear connection. The cantilever floors house the mechanical equipment and transformer stations. Basement floors for computer rooms and parking provide stability for the central core.

Floor size:	112x112 feet (34.29x34.28 m)
Core size:	48x48 feet (14.63x14.63 m)
Building height:	27 stories, 456 feet (139 m)
Core height:	520 feet (158.5 m)
Core height/width ratio:	10.8



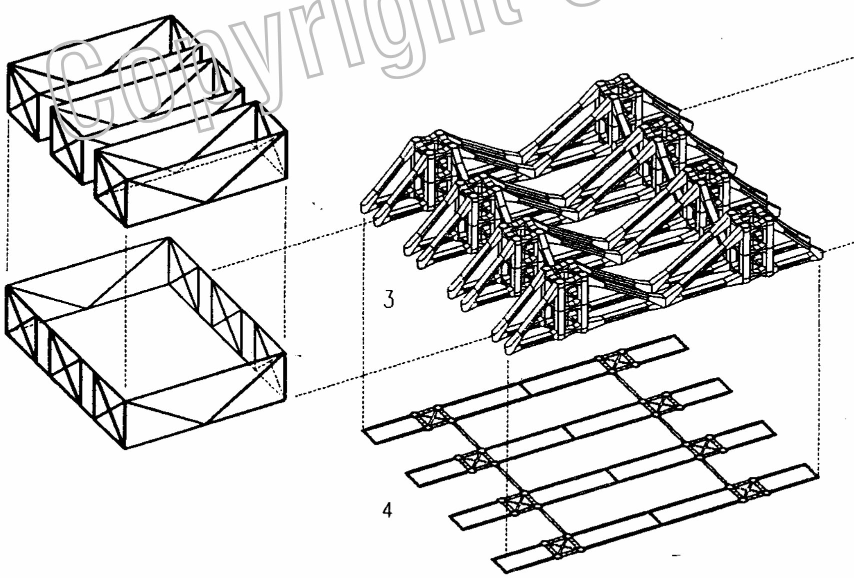
Hon Kong and Shanghai Bank (1985)
Architect: Norman Foster
Engineer: Ove Arup

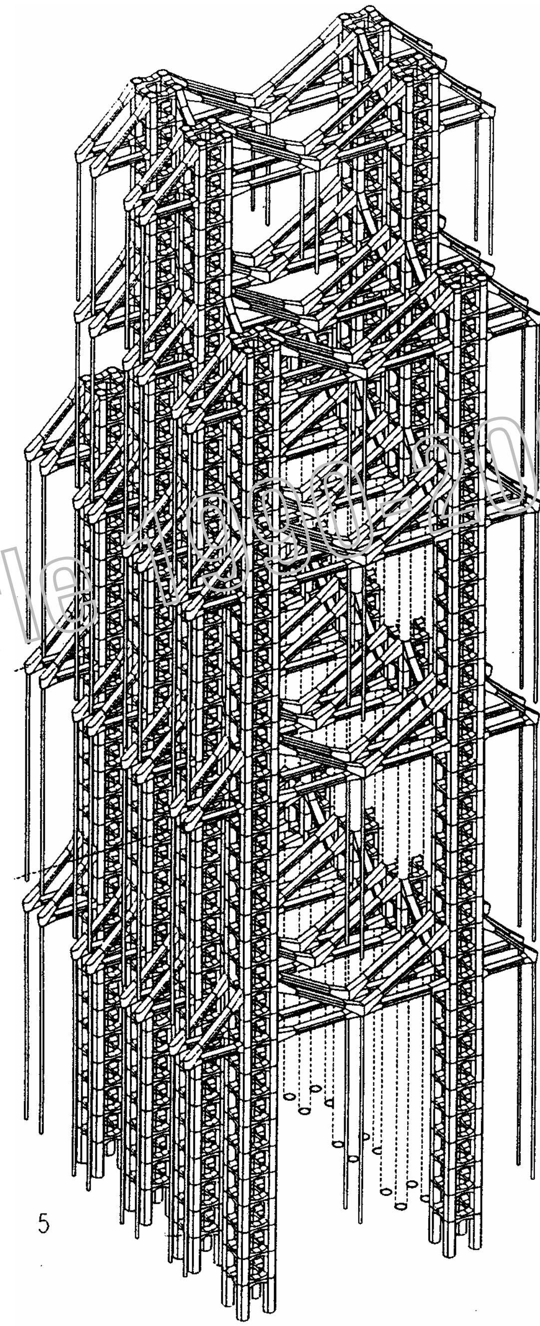
The design of the Hong Kong and Shanghai Bank emerged from a competition among seven invited architects. Foster's winning scheme is a suspension system intended to provide large public space at ground level without interior columns. The large floor area of 55x70m (180'x230') is supported by 8 Vierendeel towers, each consisting of four round columns spaced 5.1x4.8 m (17'x16') and connected at each level with tapered beams. The floors are suspended from twin suspension trusses which span the towers and cantilever from them on both sides to support service modules and exit stairs. A large floor area of 33.6x55 m (110'x180') between the towers are disrupted by only eight hangers, an additional benefit of the suspension scheme, besides the open ground floor. The space between two-story high suspension trusses serves as focal point of each stack of floors, as reception, conference and dining areas and lead to open recreation terraces with dramatic views of Hong Kong.
The maximum mast pipe diameter is 1400 mm (55"0 and 100 mm (3.9") thick
The maximum hanger pipe diameter is 400 mm (16") and 60 mm (2.4") thick

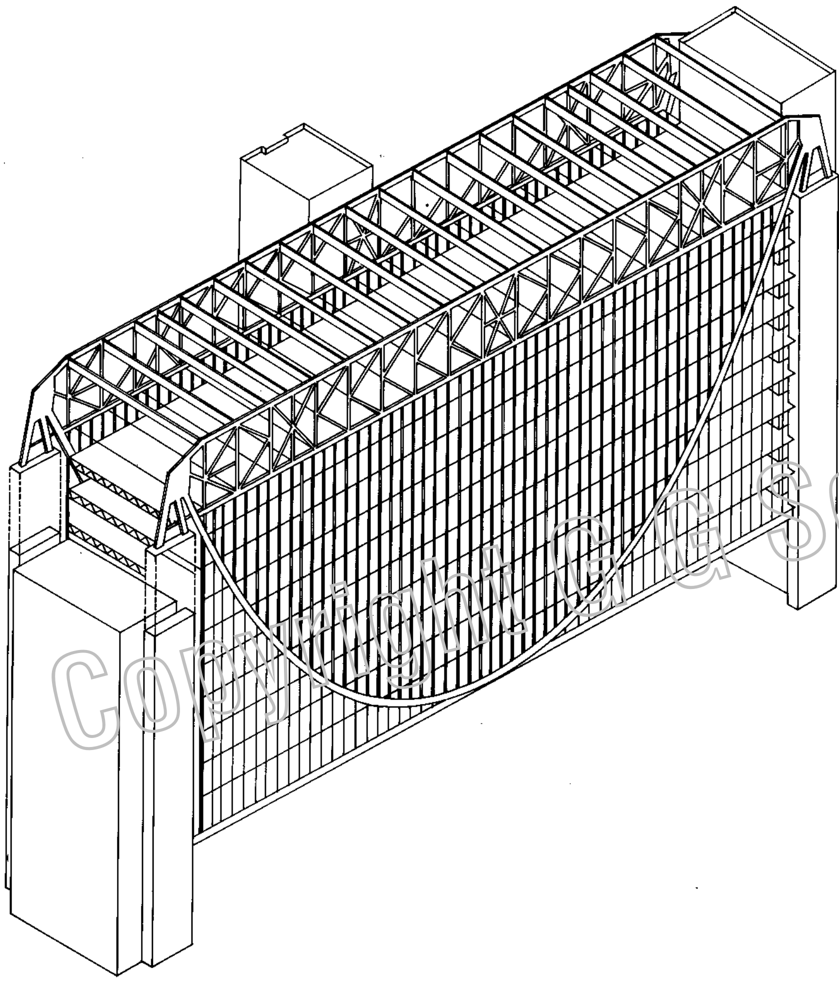


The suspension trusses and X-bracing perpendicular to them are also intended as belt trusses to reduce drift under lateral load. However, since the Vierendeel towers are moment resistant, the belt trusses are less effective than they would in conjunction with truss towers.

Size:	55x70m (180'x230')
Tower axis distance	38 m (126’)
Height	35 floors, 180 m (590’)
Typical story height:	3.9 m (12.8’)
Height/width ratio:	4.7







Federal Reserve Bank, Minneapolis (1971-73)
Architect: Gunnar Birkerts
Engineer: Skilling, Helle, Christiansen, Robertson

The Federal Reserve Bank features a structure similar to suspension bridges. The floors are suspended from parabolic "cables". However, the "cables" are actually wide-flange steel sections of parabolic curvature to balance the distributed floor loads. A major reason to suspend the building from two towers was to keep the bank vaults located below grade free of columns. Wide flange parabolic suspenders of 37 inch (940mm) span 328 feet (100m) between two concrete towers. Trusses on top of the towers resist the lateral trust of the parabolic suspenders. Floors above the suspenders are supported by compression columns, whereas those below are suspended by tension hangers. The façade treatment reflects the compressive and tensile support zones by different recess of the glass line with respect to curtain wall mullions. Floor construction of concrete slabs rests on steel trusses that span the 60 feet (18m) width without interior columns.

Size:	335x60 feet (102x18 m)
Span between towers	275 feet (84 m)
Height	220 feet (67 m)
Typical story height	12.5 feet (3.8 m)
Height/width ratio:	3.7

23

Concrete

Material

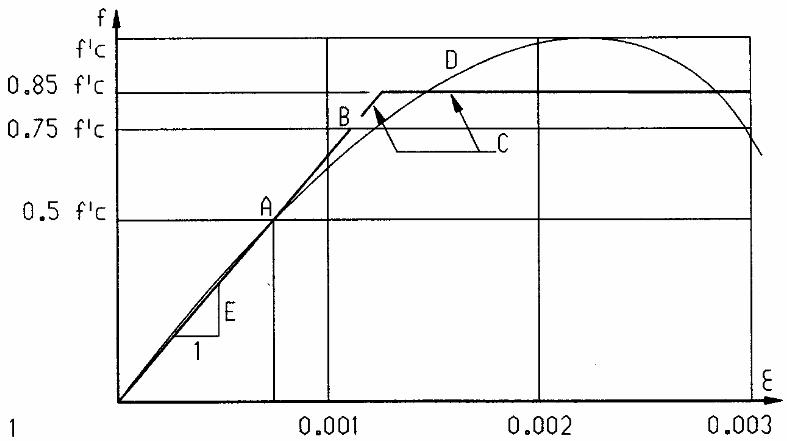
Concrete is a versatile material that can be molded into many forms. It was first known in ancient Rome. Quarrying limestone for mortar, the Romans discovered that burning the limestone mixed with silica and alumina yields cement stronger and more adhesive than ordinary lime mortar. The new material also could be used for underwater construction. Mixing the cement with sand and other materials, the Roman's invented the first concrete and used it widely in their construction, often filled between masonry.

The technology of concrete construction was lost with the fall of the Roman Empire. Only toward the end of the eighteenth century did British inventors experiment to develop concrete again. In 1825, Joseph Aspdin patented *Portland cement* which he named after Portland limestone of similar color. The material was soon in wide use and the name Portland cement is still common today. It consists of lime, silica, and alumina, burned to clinkers in a furnace at about 3000º F (1650º C), and then crushed to a fine powder.

Concrete, consisting of cement, sand, and gravel mixed with water, is strong in compression, but very weak in tension and shear. Thus, concrete by itself is limited to applications subject to compressive stress only. This limit was soon recognized and by 1850 several inventors experimented with adding reinforcing steel to concrete. In 1867 the French gardener, Joseph Monier, obtained a patent for flower pots made of reinforced concrete. He went on to build water tanks and even bridges of reinforced concrete. Monier is credited to invent reinforced concrete...

As any material, concrete has advantages and disadvantages. Concrete ingredients are widely available and rather inexpensive. Concrete combines high compressive strength with good corrosion and abrasion resistance. It is incombustible and can be molded in many forms and shapes. Concrete's main disadvantage is its weakness in resisting tension and shear. Steel reinforcing needed to absorb tensile stress can be expensive. Concrete has no form by itself and requires formwork that also adds much to its cost. The heavy weight of concrete yields high seismic forces but is good to resist wind uplift. Concrete is inherently brittle with little capacity to dissipate seismic energy. However, concrete frames with ductile reinforcing can dissipate seismic energy. The inherent fire resistance of concrete is an obvious advantage in some applications.

Today, concrete serves many applications usually with reinforcing. In buildings, concrete is used for items like footings and retaining walls, paving, walls, floors, and roofs. Concrete is also used for moment resistant frames, arches, folded plates and shells. Apart from buildings, many civil engineering structures such as dams, bridges, highways, tunnels, and power plants are of concrete.



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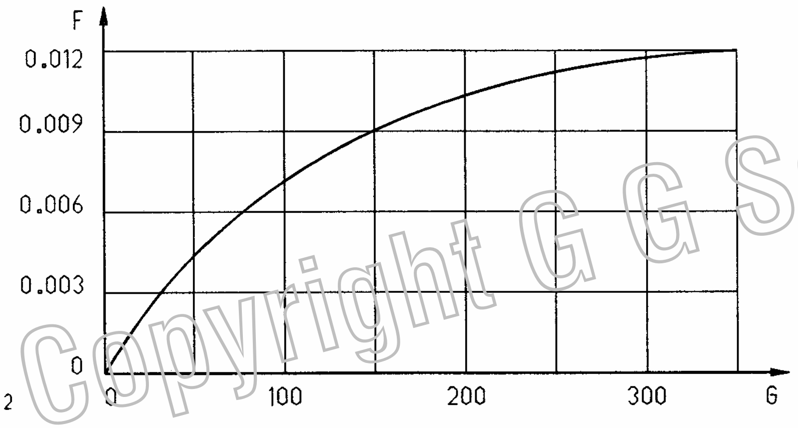

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Concrete properties

Normal concrete has compressive strengths of 2 to 6 ksi (14 to 41 MPa) and high strength concrete up to 19 ksi (131 MPa). Low-strength concrete is used for foundations. Concrete strength is determined by the water-cement ratio (usually 0.6) and the cement-sand-gravel ratio (usually 1-2-3). Specified compressive strength of concrete f'_c, usually reached after 28 days, defines concrete strength. By the *strength method* (ultimate strength method) a structure is designed to 85% of the specified compressive strength f'_c, with factored loads as safety factor. By the *working stress method*, a structure is designed to allowable stress, i.e., a fraction of the specified compressive strength f'_c.

Allowable concrete stress for working stress method		
Compressive bending stress		$0.45\ f'_c$
Bearing stress (full area):		$0.25\ f'_c$
Bearing stress (1/3 area):		$0.375\ f'_c$
Shear stress without reinforcing:	beam	$1.1\ f'_c{}^{1/2}$
	joist	$1.2\ f'_c{}^{1/2}$
	footing and slab	$2.0\ f'_c{}^{1/2}$
Shear stress, with reinforcing:		$5\ f'_c{}^{1/2}$
Elastic modulus (w = concrete density in pcf):		$w^{1.5}\ 33\ f'_c{}^{1/2}$

Temperature increase causes expansion of concrete defined by the thermal coefficient α=5.5x10^{-6} in/in/°F (3.1x10^{-6} m/m/°C). Hence, concrete slabs need temperature reinforcing to prevent cracks due to uneven expansion. Concrete also has creep deformation over time, mostly during the first year. Concrete shrinks about 1.3% due to loss of moisture, notably during curing. The temperature reinforcing helps to reduce shrinkage cracks as well. Density of concrete is determined by the type of aggregate. Light-weight concrete weighs about 100 pcf (1602 kg/m^3). Normal concrete 145 pcf (2323 kg/m^3) without reinforcing and 150 pcf (2403 kg/m^3) with reinforcing.
Concrete has good fire resistance if reinforcing steel is covered sufficiently.
An 8 in (20 cm) wall provides 4 hours and a 4 in (10 cm) wall 2 hours fire resistance.

1 Stress-strain curves for concrete
2 Concrete creep (deflection with time)
3 Concrete strength increase with time as percentage of 28-day strength

A Point defining line of E-module on curve
B Elastic limit of idealized line for working stress method
C Idealized line for strength method
D Actual stress-strain curve
E Elastic modulus, defined as the slope from 0 to 0.5 f'_c
ε Unit strain, in/in (m/m)
F Unit creep strain, in/in (m/m)
G Days after pouring concrete

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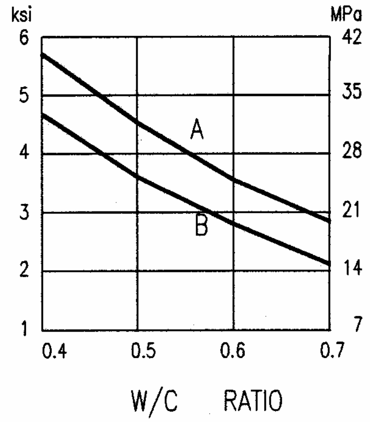

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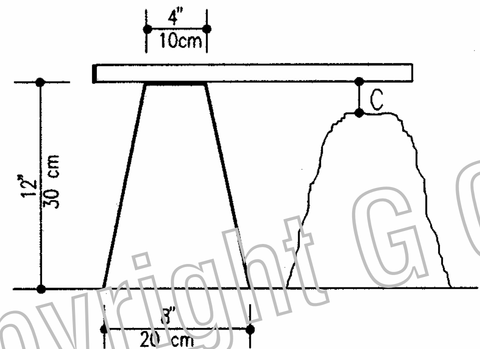

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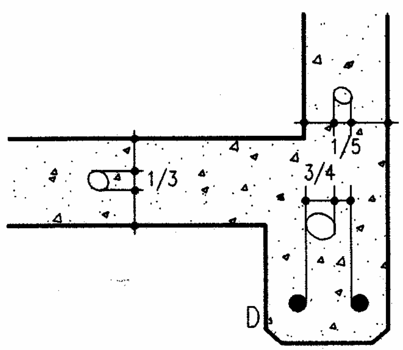


Cement comes in bags of 1 ft^3 (.028 m^3), classified by ASTM-C150 as:

Type I	Normal cement (for most general concrete)
Type II	Moderate resistance to sulfate attack
Type III	High early strength
Type IV	Low heat (minimizes heat in mass concrete, like dams)
Type V	High resistance to sulfate attack

Types IA, IIA, IIIA correspond to I, II, III, but include air-entraining additives for improved workability and frost resistance.

Water must be clean, free of organic material, alkali, oil, and sulfate. The water-cement ratio defines the strength and workability of concrete. Low water content yields high strength, but is difficult to work. Typical water ratios are 0.4 to 0.6, verified by a *slump test*. For this test, a metal cone is filled with concrete and tamped. Lifting the cone slumps the concrete to under 3 in (7 cm) for foundations and walls, and 4 in (10 cm) for columns and beams.

Aggregate should be clean and free of organic material. Fine aggregate (sand) is less than 1/4 in (6 mm). Coarse aggregate (gravel or crushed rock) is used in normal concrete. Lightweight concrete has aggregate of shale, slate, or slag. Perlite and Vermiculite are aggregates for insulating concrete.

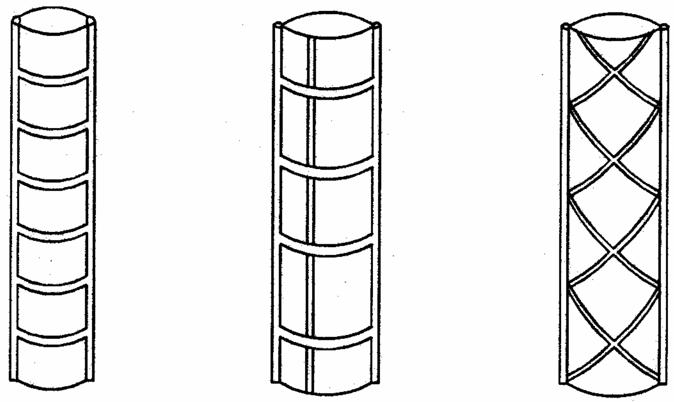
Admixtures are substances added to concrete to modify its properties:

- Air entrained agents improve workability and frost resistance
- Accelerators reduce the curing time and increase early strength
- Retarders slow the curing and allow more time to work the concrete
- Plasticizers improve the workability of concrete
- Colors and pigments add colors to concrete

Curing of concrete is a process of hydration until it reaches its full strength. Although this process may take several months, the design strength is reached after 28 days. During the curing process the concrete should remain moist. Premature drying results in reduced strength. Exposed concrete surfaces should be repeatedly sprayed with water or covered with a protective membrane during curing. This is most important in hot or windy climates. The curing process accelerates in hot temperatures and slows down in cold temperatures. Concrete shrinks about 2 % during curing. This may cause cracks. Synthetic fibers of 1/8 to 3/4 in (3 to 20 mm) are increasingly added to improve tensile strength and reduce cracking of concrete.

1 Concrete compressive strength defined by water-cement ratio
2 Slump test: sheet metal cone and slumped concrete, C = slump
3 Maximum aggregate sizes: 1/3 of slab, 1/5 of wall, 3/4 of bar spacing

A Compressive strength of normal concrete
B Compressive strength of air entrained concrete
C Slump is the amount the wet concrete settles



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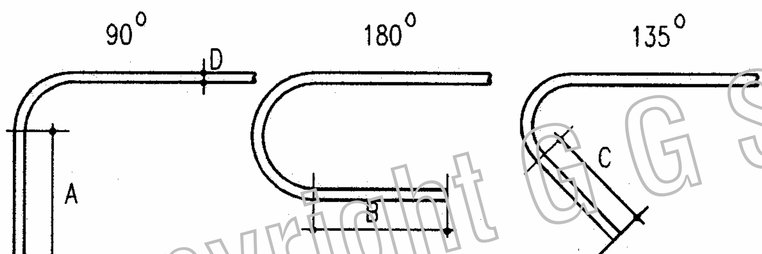

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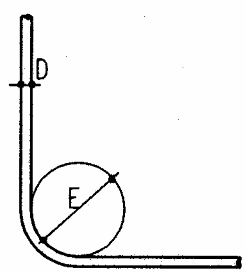

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Reinforced concrete

Concrete is strong in compression, but weak in tension and, when cracked, has zero tensile strength. Under tensile stress, concrete requires reinforcement with *deformed bars* or *welded wire fabric*. Concrete and steel are compatible, with thermal coefficients of α = 6x10^{-6}/F° and 6.5x10^{-6}/F° for concrete and steel, respectively. With different thermal expansions, major thermal stress would result. Some temperature reinforcement is required to prevent cracks. Concrete protects the embedded steel from fire and corrosion, but cracks cause steel corrosion by exposing it to humidity.

Deformed bars have round cross sections with ribs to bond with concrete. Under certain conditions bars need a hook at the end to resist slippage. Bar sizes are designated by numbers 3 to 18. Up to size 8, bar numbers correspond to the bar diameter in eighth of an inch (No. 7 = 7/8 in. Bars are available in the following grades and corresponding yield strengths f_y

Grade	40		50		60		90*	
f_y ksi (MPa)	40	(275)	50	(345)	60	(414)	90*	(620*)

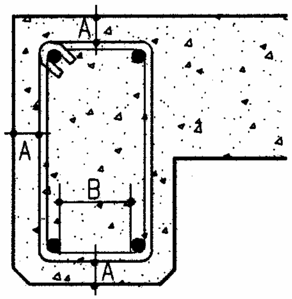
∗ Available for bars No. 14 and 18 only (for compression reinforcement).

Properties of reinforcing bars

	Diameter		Area		Weight	
Size	in	mm	in²	mm²	lb. /ft	kg/m
3	0.375	9.50	0.11	71	0.376	0.560
4	0.500	12.70	0.20	129	0.668	0.994
5	0.625	15.88	0.31	200	1.043	1.552
6	0.750	19.05	0.44	284	1.502	2.235
7	0.875	22.22	0.60	387	2.044	3.042
8	1.00	25.40	0.79	510	2.670	3.973
9	1.128	28.65	1.00	6.45	3.400	5.060
10	1.270	32.26	1.27	819	4.303	6.404
11	1.410	35.81	1.56	1006	5.313	7.907
14	1.693	43.00	2.25	1452	7.650	11.380
18	2.257	57.33	4.00	2581	13.600	20.240

1 Deformed bars with stamp for mill, bar #, steel type, and grade
3 Bar hooks of 90°, 180°, and 135°; the latter for stirrups and ties only
2 Minimum bar bend defined by bar diameter

A Hook length: 6D, stirrups and ties; 12D all others; or min. 2.5 in (6 cm)
B Hook length: 4D; or min. 2.5 in (6 cm)
C Hook length: 6D (10D for seismic regions); or min. 2.5 in (6 cm)
D Bar diameter
E Bend diameter: 4D, No. 5 bars and smaller for stirrups and ties only;
Other bars: 6D, No. 3 to 8; 8.5D, No. 9 to 11; 10.5D, No.14 and 18



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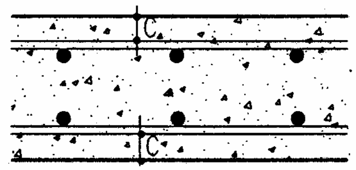

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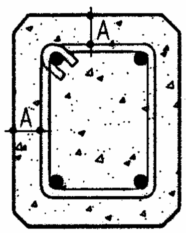

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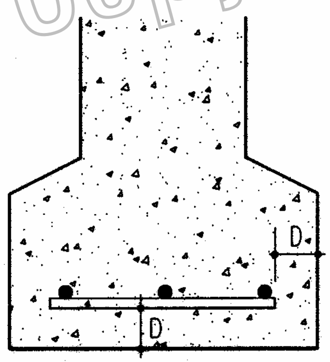

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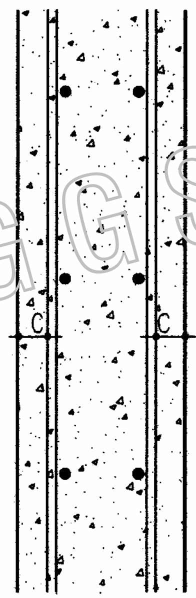

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Bar cover is the distance between bar edges and the outside surface of concrete. Structurally, reinforcing is most effective near the surface to resist cracking and bending. The distance from the neutral axis near the center increases the resisting lever arm for steel. This makes it more effective to resist bending. However, bars placed too close to the surface are more susceptible to corrosion and are poorly protected against fire. The ACI code defines the minimum cover for various members and exposure conditions for the purpose of fire and corrosion resistance. This is particularly important for members in contact with soil, such as foundations, or basement walls.

Bar spacing must be wide enough to allow wet concrete to flow freely and to transfer stress between spliced bars. But spacing should be close enough to provide effective reinforcing. The ACI code defines upper and lower limits for bar spacing.

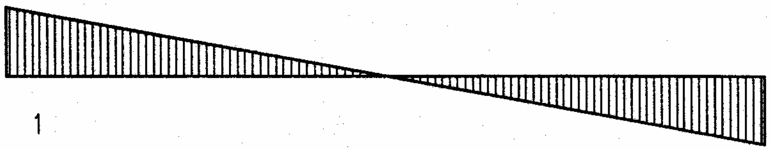
The diagrams show minimum bar cover and spacing for typical concrete structures, including beam, post, foundation, and slab and plate.

1 Beam
2 Slab or plate
3 Column
4 Foundation
5 Wall

A Minimum bar covers: 1.5" for beam and post; 1" for joist
B Minimum bar spacing for beam: 3/4" or 1.33 max. bar ϕ
C Minimum bar cover for slab and plate: 3/4" for #5 bar and smaller (1.5" when exposed to weather); 2" for #6 bars and larger
D Minimum bar cover for foundation: 3"

Welded Wire Fabric is common as reinforcement for slabs on grade and thin slabs. It consists of orthogonal welded wire mesh. Wires are smooth or deformed for better bonding and come in yield strengths from 56 to 70 ksi (386 to 483 MPa). The largest wire has 0.2 sq in area and 1/2 in (13 mm) diameter. A typical welded wire fabric designation is 4x6-W10xW20, implying:

4x6 Wire spacing (in)
W10xW20 wire size and type (W for smooth, D for deformed wires)



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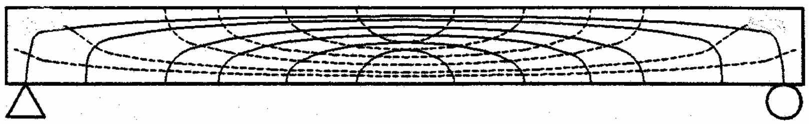

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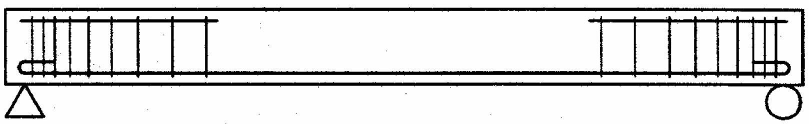

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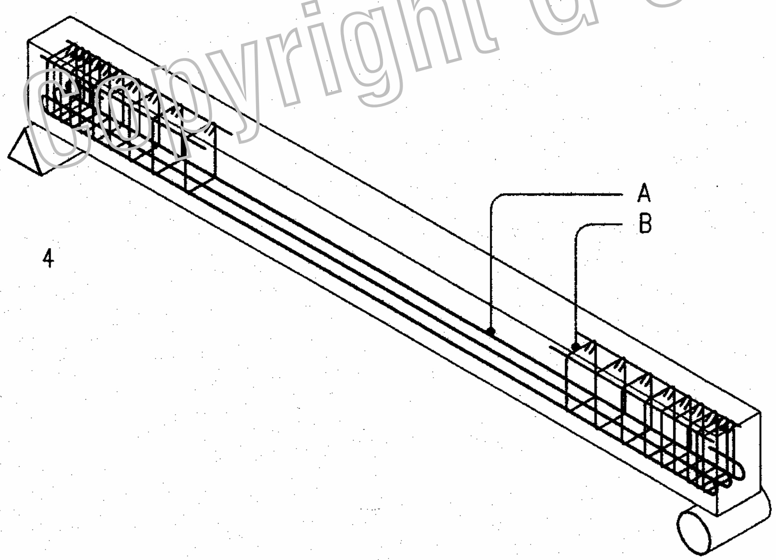


4

Beam Reinforcement

Concrete beams require reinforcement for bending and shear in correlation with the respective stress patterns. This is illustrated for a simply supported beam under uniform load and for other beams on the next page.

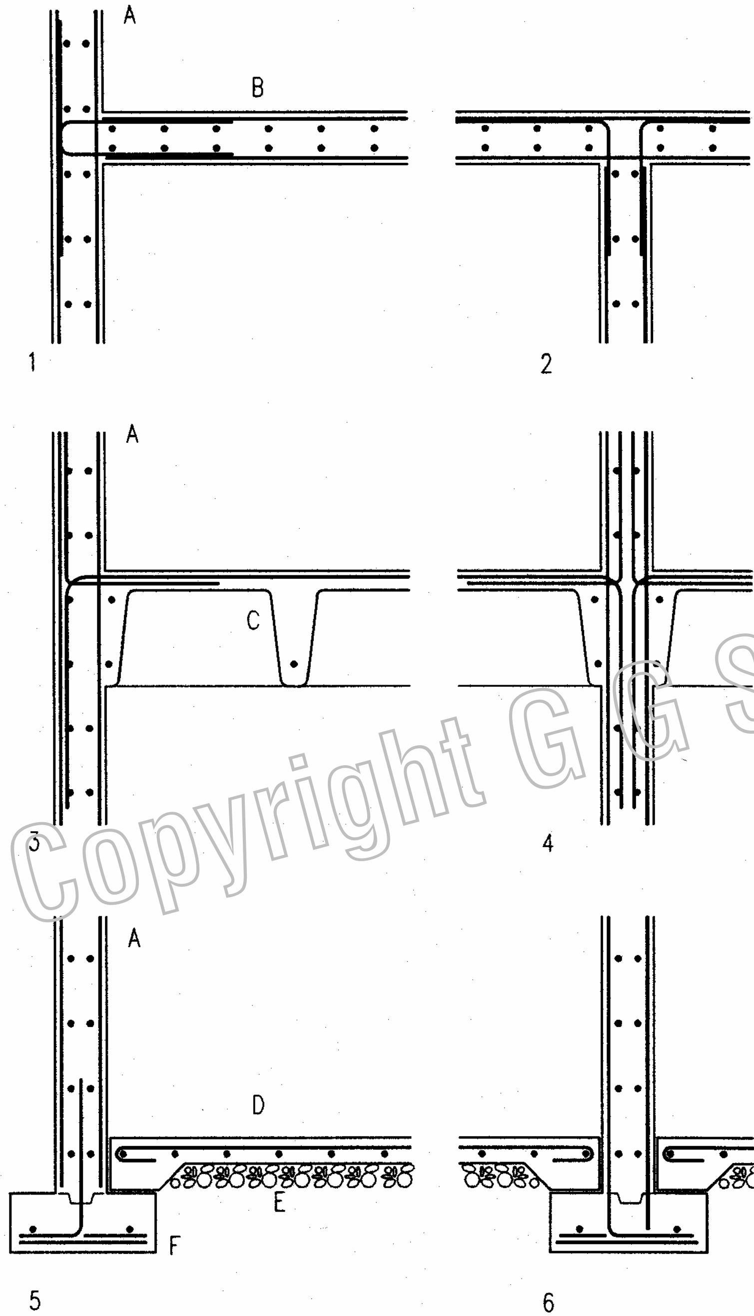
Bending reinforcement is placed where the bending moment causes tensile stress. A simply supported beam under uniform gravity load deforms downward to generate compression on top and tension at the bottom. Thus, bending reinforcement is placed at the bottom. Beams with negative bending require tensile reinforcement on top. This is the case in beams with moment resistant supports, cantilever beams, and beams continuing over three or more supports. Some beams may require additional bars at mid-span or over supports to resist increased bending moment. Beams of limited depth also require compressive reinforcement to make up for insufficient concrete. Some reinforcement bars have hooks at both ends to anchor them to the concrete if the bond length between steel and concrete is insufficient. Deformed bars usually don't need hooks, given sufficient bond length. Temperature reinforcement resists stress caused by temperature variation and shrinkage during curing.

Shear reinforcement is placed where the shear stress exceeds the shear strength of concrete which is very small compared to compressive strength. Beams under uniform gravity load have maximum shear at supports which decreases to zero at mid-span. Thus, shear reinforcement in form of *stirrups* is closely space near the supports and spacing increases toward mid-span. Stirrups are usually vertical for convenience, though combined horizontal and vertical shear stresses generate diagonal tension which may cause diagonal cracks near the supports. Small longitudinal bars on top of a beam tie the stirrups together.

1 Shear diagram: maximum shear at supports and zero at mid-span
2 Isostatic or principal stress lines: diagonal tension, dotted, near support
3 Side view of beam with reinforcement
4 Axon view of beam with reinforcement

A Bottom steel bars resist tensile stress

B Stirrups resist shear stress which, for uniform load, is maximum at the supports and zero at mid-span



Wall

The unsupported height to width ratio of bearing walls should not exceed 25 with 6 in (15 cm) minimum thickness - 8 in (20 cm) for basement walls. Non-bearing walls may be 4 in (10 cm) thick. Walls of 10 in (25 cm) or thicker should have 2 layers of reinforcement. Unless reinforcement is determined to be greater for a given condition, the following minimum reinforcement shall be provided as a percentage of the wall cross-section area.

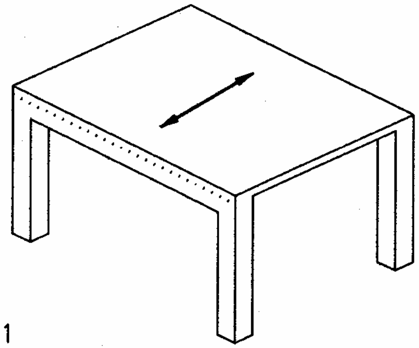
Horizontal reinforcement:	0.25% min.
Vertical reinforcement:	0.15% min.

Additional reinforcement is required at wall tops, corners, around all openings, as well as foundations.

Dowel bars connect walls to foundations, floor and roof slabs. They should overlap with rebars at least 40 bar diameters or the n=minimum computed bond length.

1 Exterior wall with flat slab
2 Interior wall with flat slab
3 Exterior wall with rib or waffle slab
4 Interior wall with rib or waffle slab
5 Exterior wall with foundation and slab on grade
6 Interior wall with foundation and slab on grade

A Wall with 2 layers of reinforcement
B Flat slab with dowel bars connected to wall
C Rib or waffle slab with dowel bars connected to wall
D Slab on grade with construction joint at wall
E Gravel bed under slab on grade
F Foundation with dowel bars and key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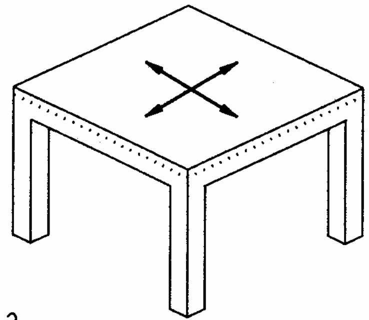
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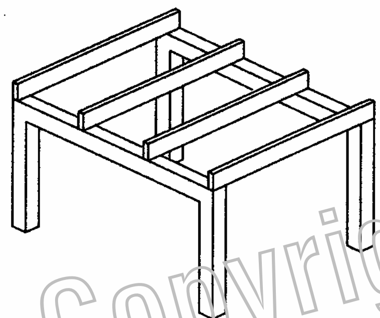
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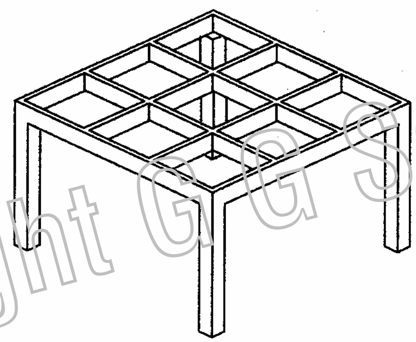
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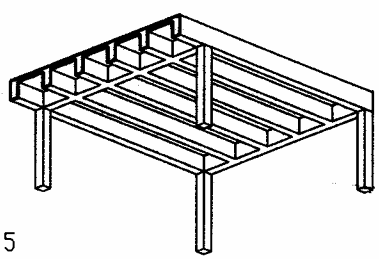
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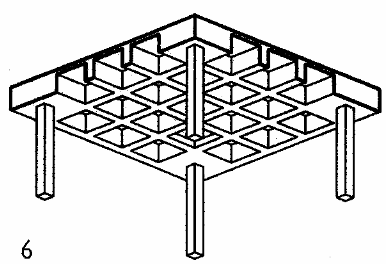
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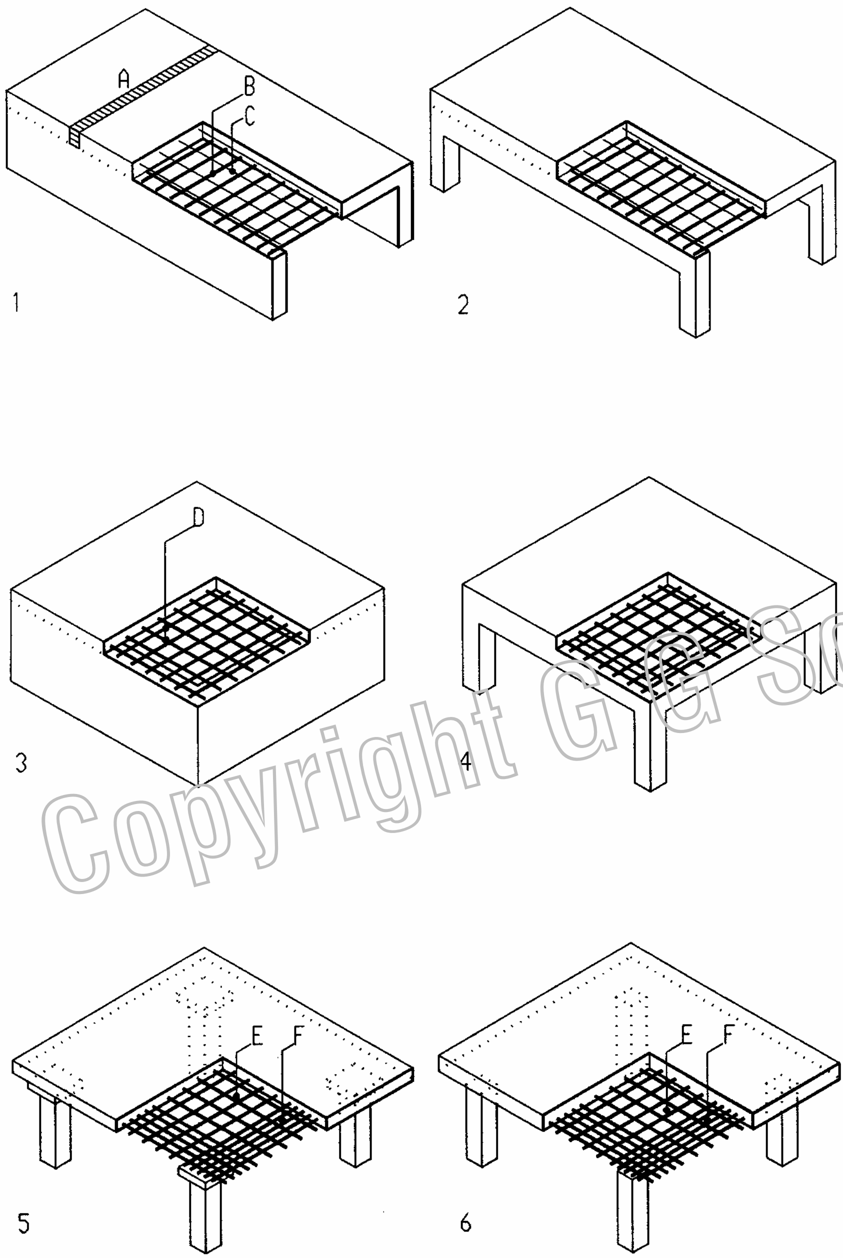
Slab

Depending on the support conditions, concrete slabs may span one-way or two-ways. If supports are on two opposite sides, a one-way slab is the only option. For a slab supported along all edges and of approximately equal span in both directions, a two-way slab is preferred and more efficient. However, if spans in the two directions are different, a one-way slab is better since deflection increases with the fourth power of span, causing 16 times greater deflection for double spans. In two-way slabs of unequal spans, the rebars spanning the short direction carry most of the load and bars spanning the long direction are ineffective. The ratio between short and long span should not exceed 1:2, but is most effective at 1:1.

The span capacity for slabs is about 20 or 30 feet for one-way and two-way slabs, respectively. Slabs exceeding those limits require intermediate beams or joists. One-way and two-way slabs are shown on the left and right, respectively.

1 One-way slab supported by two edge beams
2 Two-way slab supported by four edge beams
3 One-way beams (supporting slab) supported by two edge beams
4 Two-way beams (supporting slab) supported by four edge beams
5 One-way Rib-slab (pan joist) supported by beams
   Slab depth 2.5" to 4.5" (6 to 10 cm)
   Total depth 10" to 24" (25 to 60 cm); L/d= 20-28
   Rib width 5" to 10" (13 to 25 cm)
   Rib spacing 2' to 3' (0.6 to 1 m)
6 Two-way waffle slab with solid panels over columns to resist shear:
   Slab depth 2.5" to 4.5" (6 to 10 cm)
   Total depth 10" to 24" (25 to 60 cm); L/d = 33
   Rib width 5" to 6" (13 to 15 cm)
   Waffle size 2' to 5' (0.6 to 1.5 m)



Slab and plate

Depending on span and support type concrete slabs may span one-way or two-way. For supports on two sides one-way span is the only option. For supports on all sides and about equal span in both directions, two-way slabs are better, but for unequal spans one-way slabs are better. Deflection increases with the fourth power of span or 16 times greater deflection for a double span. Therefore, rebars spanning the long way are ineffective since the shorter span deflects less and carries most load. Slabs and plates have reinforcement at the bottom of mid-span and on top of multi-span supports. The diagrams show only one layer of reinforcement for clarity. Sections on the next page show both top and bottom reinforcement. The following slab span/depth ratios L/d give minimum slab depths if deflection is not checked.

One-way slabs have rebars in one direction but require some rebars to resist stress due to temperature variation and shrinkage. The temperature reinforcing runs perpendicular to main reinforcing and must be a minimum percentage of the concrete cross section area as follows:

Grade 40	0.20% reinforcement
Grade 60	0.18% reinforcement

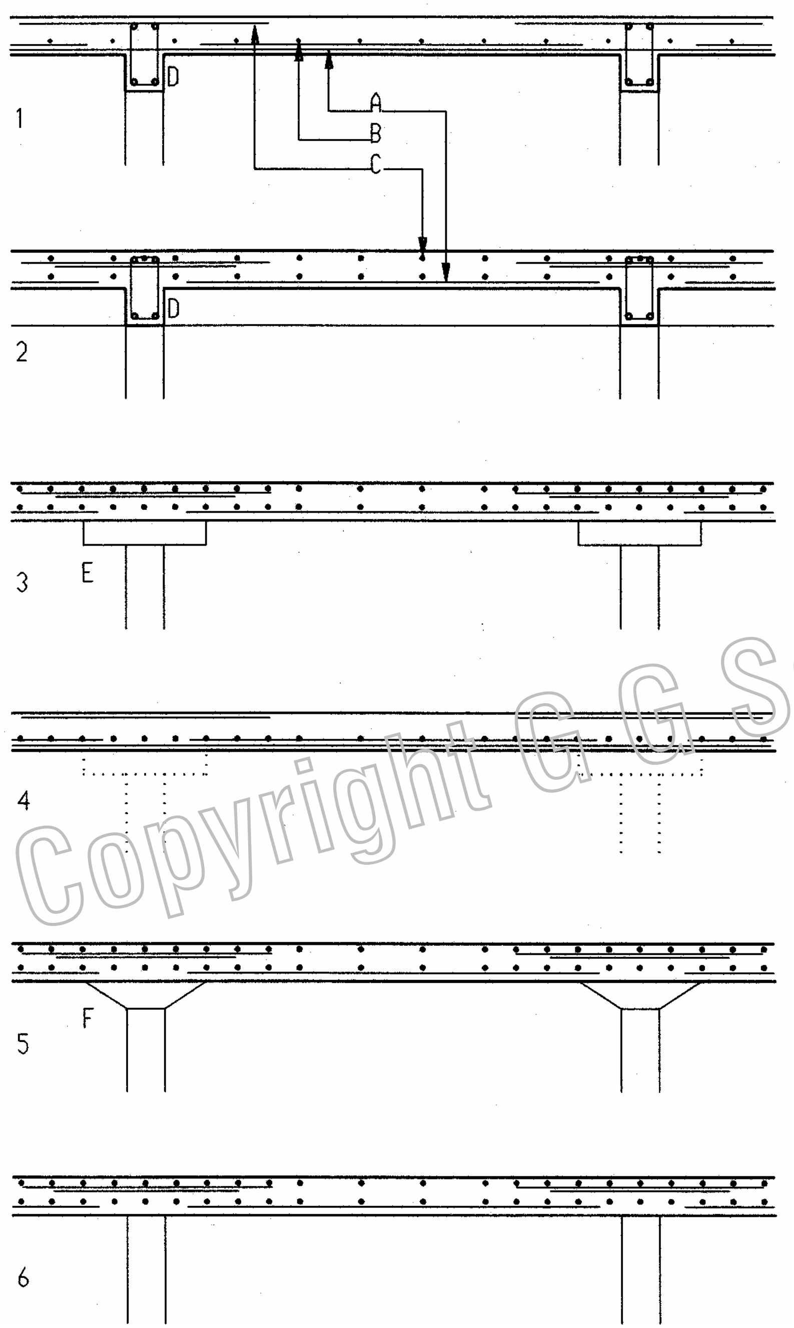
Slab span/depth ratios L/d

Grade	Cantilever	Simple support	one end cont...	Both ends cont.
40	13	25	30	35
60	10	20	24	28

Two-way slabs need no temperature rebars. Slabs without beams require different reinforcement for *middle strips* and *column strips.* Column strips need more rebars since they carry a greater load share. Two-way slabs need about 20 % less depth, but require support on all sides.

1 One-way slab on walls
2 One-way slab on beams
3 Two-way slab on walls; L/d = 36 for multiple bays
4 Two-way slab on beams; L/d = 36 for multiple bays
5 Two-way slab on columns with drop panels to resist shear at columns; L/d = 33 for multiple bays
6 Two-way plate on columns without drop panels; L/d = 30 for multiple bays (for moderateload; low formwork cost)

A Concrete strip of 1' (1 m) wide assumed for slab analysis like a beam.
B Reinforcement of one-way slab
C Temperature reinforcement of one-way slab
D Reinforcement of two-way slab running both ways
E Middle strip reinforcement
F Column strip reinforcement carries a greater share of the load



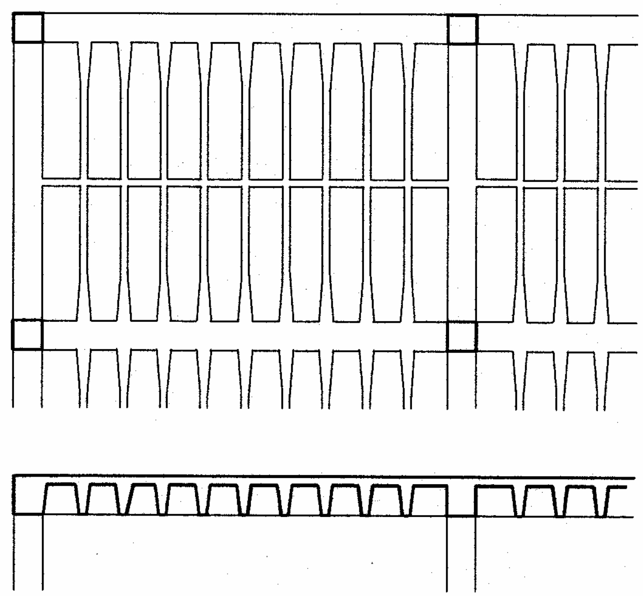
Slab and plate reinforcement

Slabs and plates require bending reinforcement at zones of tensile stress, at bottom of mid-span, top of fixed-end support, and of multi-span supports. One-way slabs require some perpendicular reinforcement due to temperature variation and shrinkage. Thin slabs are sometimes reinforced with welded wire fabric rather than individual bars. Shear stress is usually resisted by concrete alone without shear reinforcement. Flat slabs without beams have drop panels, or column caps, to resist high shear on top of columns. Plates are similar to slabs without drop panels or column caps. They are commonly used for relatively light loads, such as roofs.

Depth of slabs and plates, depending on span, range from
4 to 12 in (10 to 30 cm) for slabs on beams
6 to 12 in (15 to 30 cm) for flat slabs on columns; and
6 to 14 in (15 to 36 cm) for plates

1 One-way slab on beams
2 Two-way slab on beams
3 Two-way flat slab on columns with drop panels, column strip
4 Two-way flat slab on columns with drop panels, mid-strip
5 Two-way flat slab with mushroom columns, column-strip
6 Two-way plate without drop panels, column strip

A Top reinforcement at zone of negative bending
B Temperature reinforcement of one-way slab
C Bottom reinforcement in zone of positive bending
D Beam
E Drop panel on top of column
F Mushroom panel on top of column



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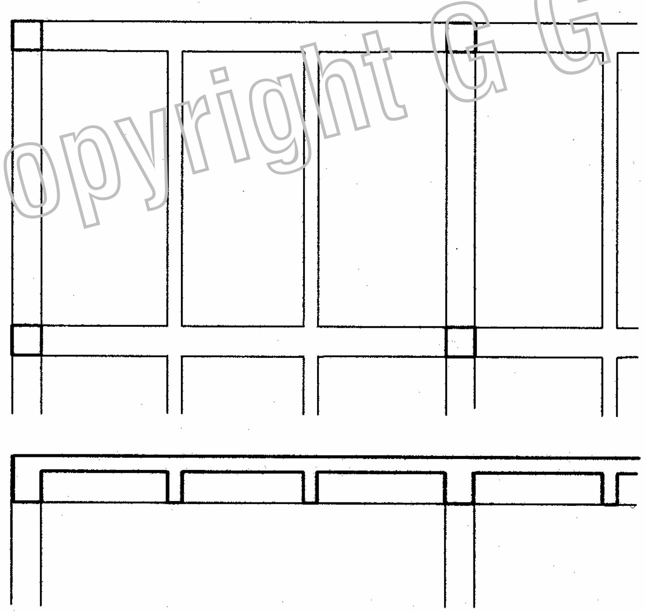

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Rib slab

Rib slabs, also called pan joists, are one-way systems for medium spans where flat slabs or plates would be too deep and heavy. Rib slabs reduce dead weight by eliminating concrete between ribs, providing structural depth without bulk. The tensile steel for positive bending is placed at the bottom of ribs and rib top and slab, resist compressive stress like a T-beam. Given narrow spacing of ribs, the minimum slab depth is determined not by depth/span ratio but by rebar size plus concrete cover. Long ribs may need bracing by cross-ribs to prevent buckling. Because of one-way span, rib slabs are not limited to square plans like two-way slabs. Rib slabs are formed placing reusable, prefabricated pans of plastic or steel on wood boards. Ribs may be tapered near the supports to resist the maximum support shear. Design and analysis of rib slabs is similar to T-beams.

Rib slab dimensions:
Span/depth ratio L/d = 20-24 (d = total depth)
Maximum span = 50' (15 m)
Slab depth 2.5" to 4.5" (6 to 11 cm)
Total depth 12" to 24" (30 to 60 cm)
Rib width 5" to 9" (13 to 23 cm)
Rib spacing 2' to 3' (60 to 90 cm)

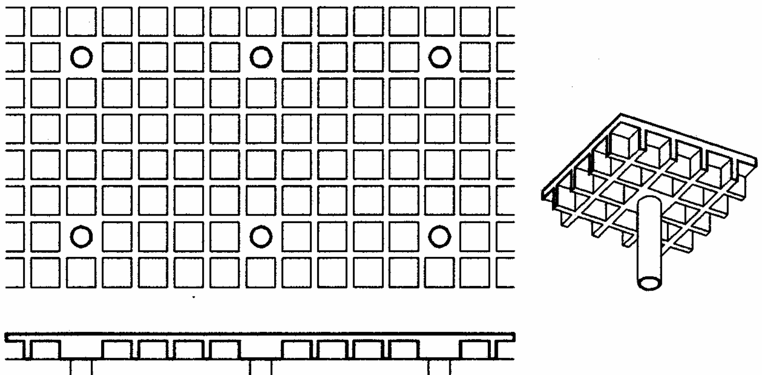
Slab on beam

Slabs on beams may be one- or two-way systems depending on proportions of spans between the boundary beams. For equal spans, two-way slabs are appropriate. For unequal spans, one-way slabs are better and should span the shorter direction. Slabs on beams are an intermediate solution between flat slab or plate, and rib or waffle slabs. The formwork for slab on beam is more complex and costly than for rib slabs.

The slab on beam is designed by the *Direct Design Method* as two-way system and as beam-like strip as one-way system.

Slab on beam dimensions:
Span/depth ratio L/d = 30-36
Maximum span = 30' (9 m)
Slab depth 4" to 12" (10 to 30 cm)

1 Rib slab braced by intermediary cross rib
2 Slab on beam may be one-way or two-way span



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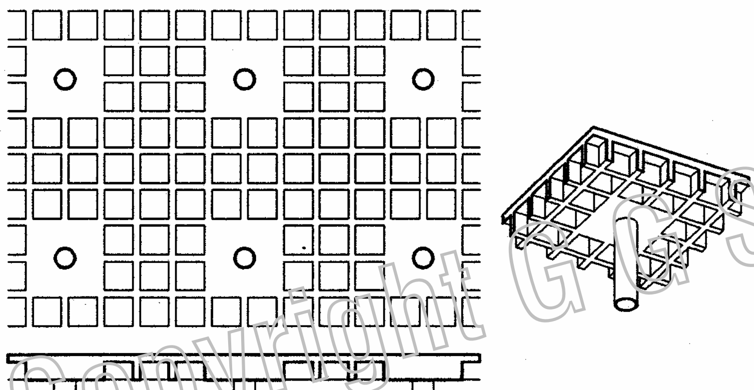

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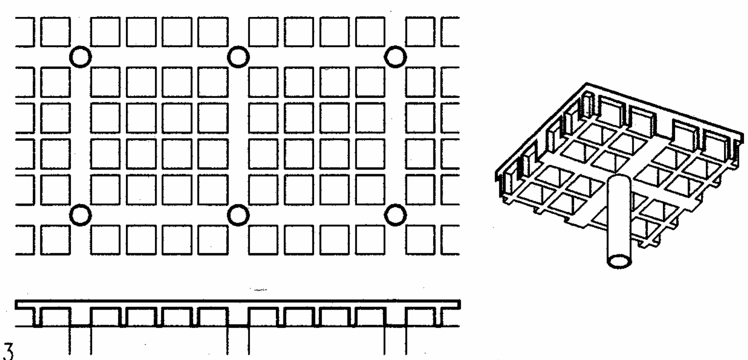

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Waffle slab

Waffle slabs are two-way systems for medium spans where flat slabs or plates would be too deep and too heavy. They reduce dead weight by two-way ribs to eliminate excess weight between the ribs. The tensile steel for positive bending is placed at the bottom of ribs and the rib top and slab resist compressive stress. Since negative bending reverses the stress, waffle slabs are not efficient as cantilever with negative bending. Waffle slabs need either solid panels on top of columns to resist shear stress or two-way beams. Waffle slabs are formed placing re-usable prefabricated pans of plastic or steel over a grid of wood boards.

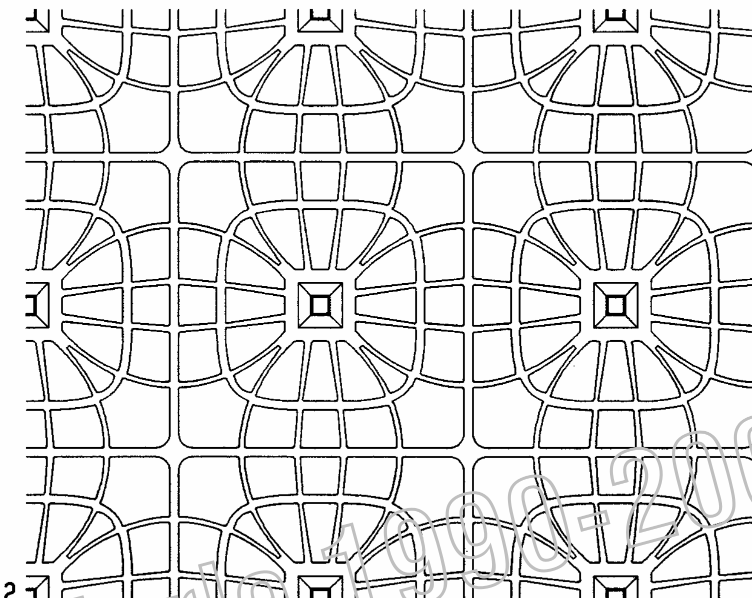
Waffle slabs may be designed similar to the previously described *Direct Design Method* like a flat slab on columns. Typical waffle dimensions: 2' to 5' (60 to 150 cm)

1 Waffle slab with a single solid panel over columns
2 Waffle slab with four solid panels over columns
3 Waffle slab supported by beams

Special slabs

The Italian engineer Arcangeli proposed a waffle slab with curvilinear ribs that follow isostatic lines for optimum stress distribution and a more elegant appearance. Pierre Luigi Nervy built such a slab for a wool factory in Rome. For a tobacco factory in Bologna, Nervi built a waffle slab with ribs wedged toward supporting beams for increased shear capacity.

1 Slab with isostatic ribs, proposed by Arcangeli
2 Slab with isostatic ribs; for a wool factory in Rome, by Nervi
3 Waffle slab with ribs wedged to increase shear capacity



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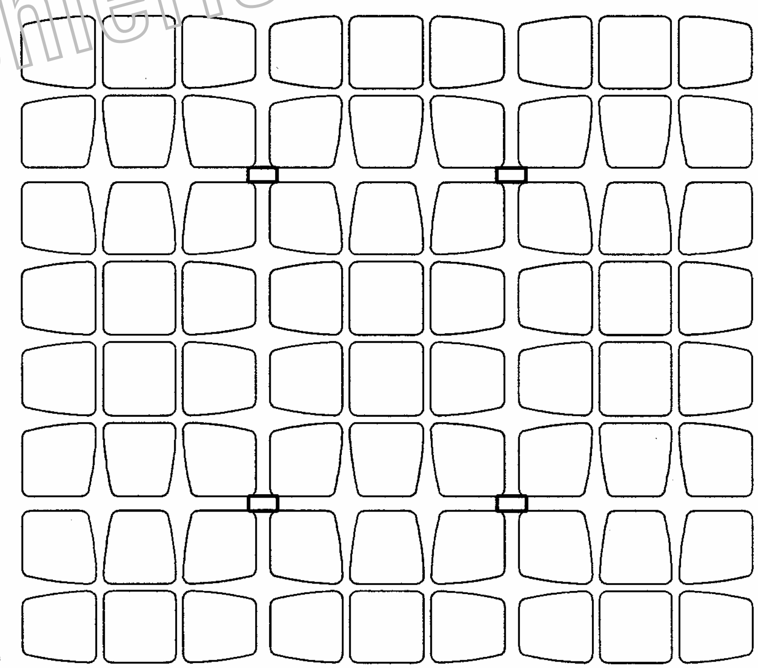

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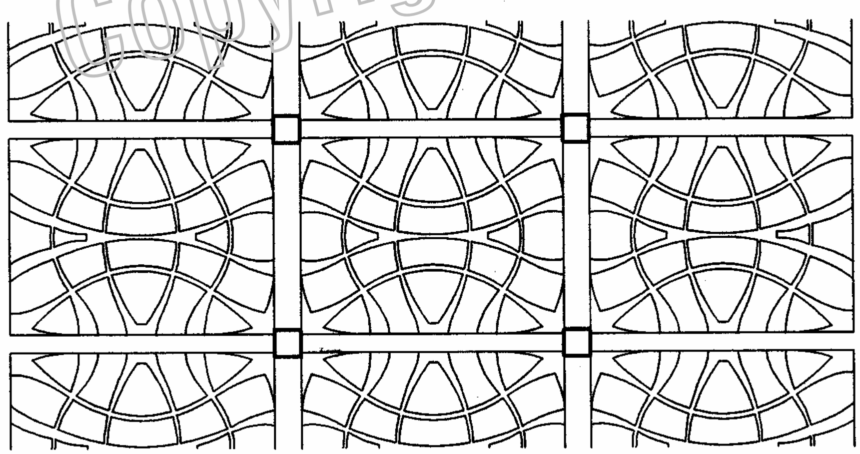
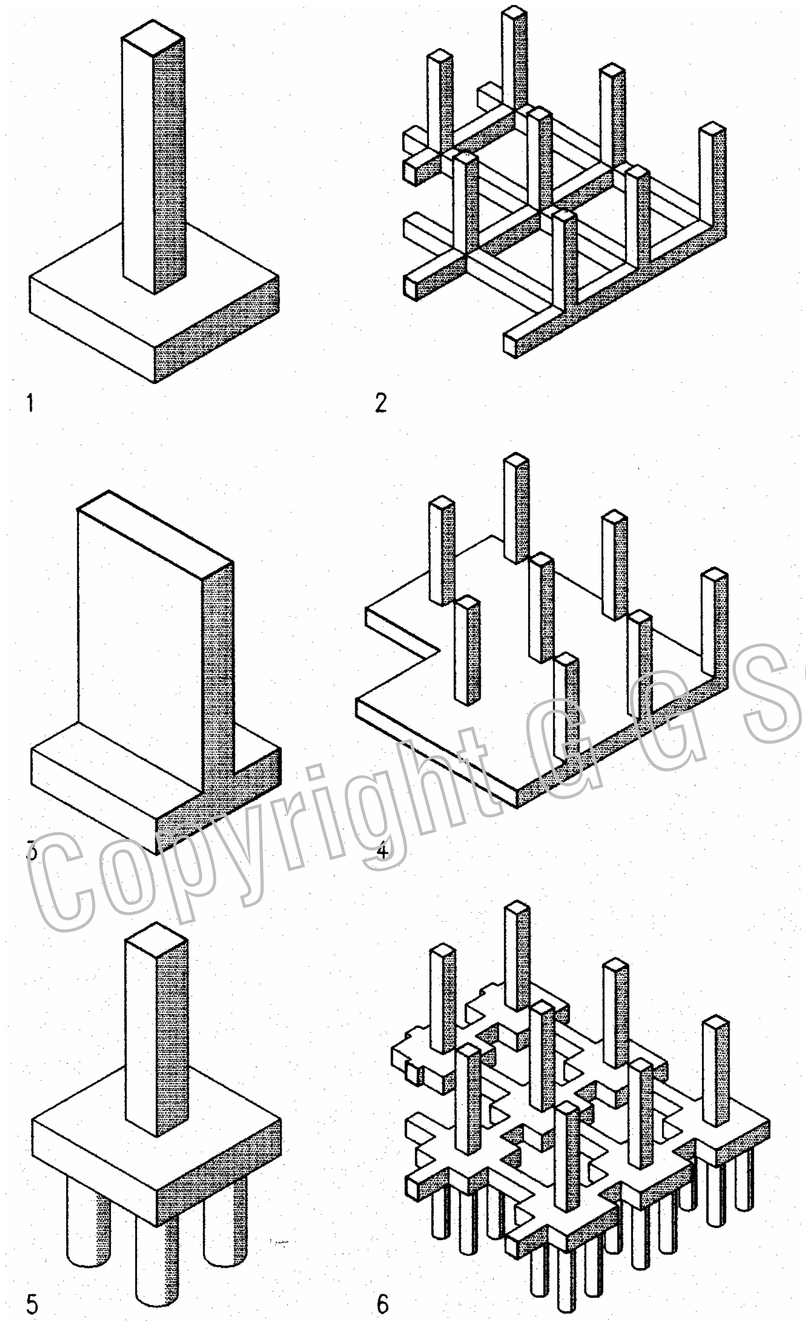




Foundation

Foundations include building parts connected with the ground, namely slabs on grade, basement walls, and footings supporting walls and columns. Foundations are often exposed to groundwater and thus corrosion. Given their corrosion resistance, concrete foundations are very common. Foundations must support various load combinations:

- Gravity live and dead load
- Lateral loads caused by earthquake, wind, and earth pressure
- Lateral thrust of arches, suspension cables, domes, shells, etc.
- Uplift caused by wind on light-weight structures, overturning of tall buildings due to wind and seismic load; or groundwater buoyancy.

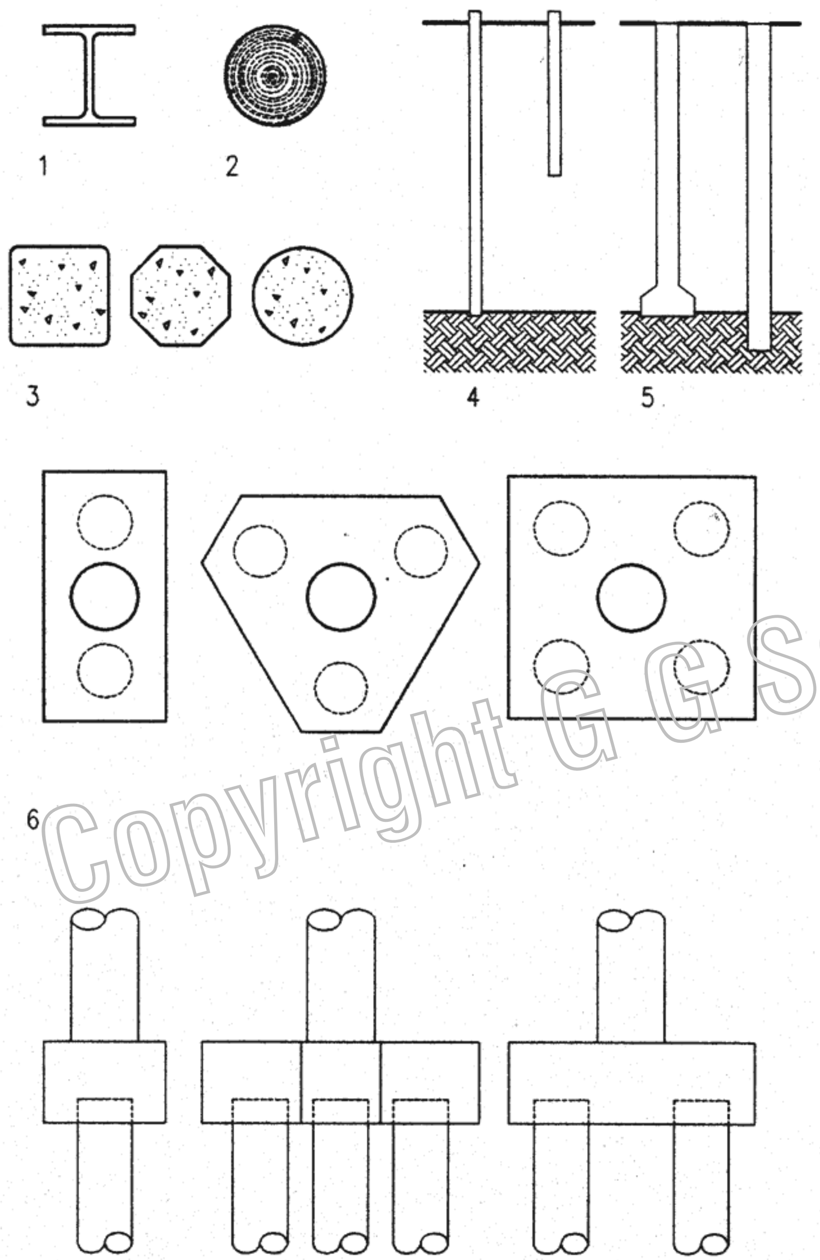
Foundation design is based on the following considerations:

- Imposed loads described above
- Allowable soil pressure of various soil types
- Allowable soil pressure of various soil types
- Soil settlement (critical when soil of uneven stiffness exists on a site)
- Allowable bending and shear stress of reinforced concrete foundations
- Depth of frost (ground water expansion due to frost causes cracks)
- Hot climate ~ 0, moderate climate ~ 3 ft (1 m); cold climate ~ 7 ft (2 m)

Soil Capacity, approximate allowable pressures (rounded for kPa)

Soil type	Approximate allowable soil pressures	
Soft clay	2 ksf	100 kPa
Stiff clay	4 ksf	200 kPa
Sand, compacted	6 ksf	300 kPa
Gravel	15 ksf	700 kPa
Sedimentary rock	50 ksf	2400 kPa
Hard rock (basalt, granite)	200 ksf	9600 kPa

1 Column footings are usually square, from 1 to 2 ft (30 to 60 cm) thick with two-way rebars at bottom and dowel bars extending into column
2 Grade beams carry uniform load (of walls) to caissons, piers, orpiles; or distribute column loads and tie them together as shown
3 Wall footings are linear and usually about twice the width of the wall, with bottom rebars in length direction and cross bars for wide footings
4 Mat foundations can bridge uneven settlements in variable soil and may counteract groundwater buoyancy with appropriate mat depth
5 Pile caps distribute column load to piles and tie them together
6 Grade beams may connect pile caps for increased lateral stability



Pile and pier

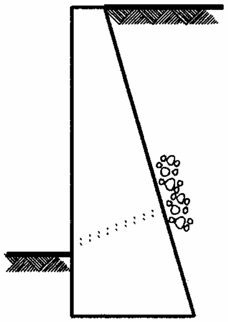
Piles and piers, or caissons, are used in poor soil conditions. They have been used since antiquity. Vitrusius wrote of wood piles driven by machinery. Piles are driven into soil but piers are poured into excavated shafts.

Piles are driven into soft soil by a *pile driver* either as friction piles relying on soil friction, or end-bearing piles to rest on firm soil or rock. Uplift loads are resisted by friction with coefficients ranging from 0.35 to 0.6. Pile capacities are verified by resistance to the last blows or by test loads. Piles can be vertical or inclined for lateral load. Due to capacity limits and difficulty in precise placement, piles are in clusters of usually three or more, spaced about 30 to 36 in (75 to 90 cm) and supporting columns at cluster centroid. Pile caps distribute column load to piles and tie them together. Piles come in steel, wood and concrete. Wood piles are peeled and pressure treated trunks. They need dry soil or should be fully under water to prevent rotting. Steel piles of round pipes or H-shapes can be weld-spliced to great length as they are driven. H-piles displace the least soil for easy penetration. Concrete piles may be plain, reinforced, prestressed, precast, or site-cast. They are the most common because of inherent corrosion resistance

Pile type	Diameter	Max. length	Capacity range
Wood	12 - 24 in	80 ft	60 - 100 k
	30 - 60 cm	24 m	270 - 450 kN
Steel pipe	10 - 36 in	200 ft	100 -400 k
	25 - 90 cm	60 m	450 - 1800 kN
Steel H-pile	8 - 14 in	300 ft	80 - 400 k
	20 - 36 cm	91 m	360 - 1800 kN
Concrete	8 - 36 in	200 ft	60 - 200 k
	20 - 90 cm	60 m	270 - 900 kN

Piers are usually shorter than piles and are used in firmer soil where pile driving is difficult. They are cast in place against excavated soil or a steel form that is gradually removed as the concrete is poured. Piers resting on soft soil may need a *bell* at the bottom to enlarge the bearing area but those on rock have straight shafts. The bell is formed by partially removing the form and compacting the concrete to push it outward. Placing of piers is more precise than piles. Thus, only a single pier is needed to support a column. Pier diameters range from 1.5 to 7 ft (0.5 - 2.1 m) with bells 2 to 3 times wider. Capacities range from 70 to 10,000 k (300 to 45,000 kN).

1 Steel H-pile cross section
2 Wood pile cross section
3 Concrete pile cross sections
4 Piles: end-bearing pile at left; friction pile at right
5 Piers: bell pier at left; straight pier at right
6 Pile caps: common plans and cross sections



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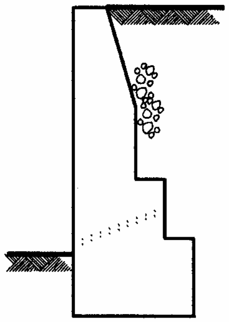

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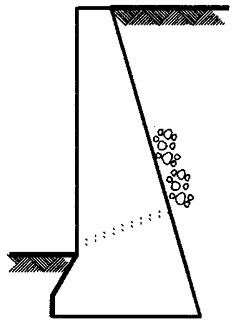

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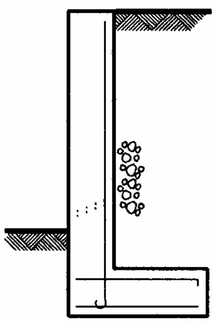

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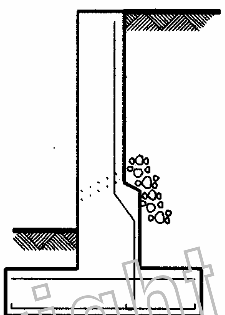

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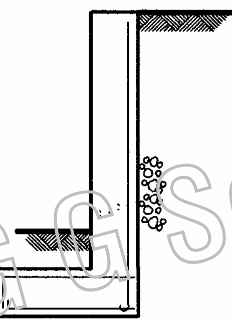

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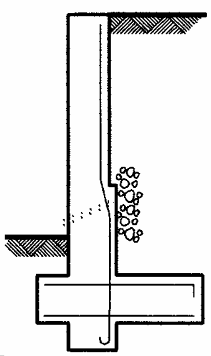

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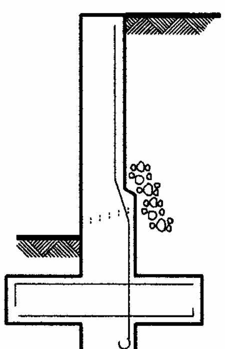

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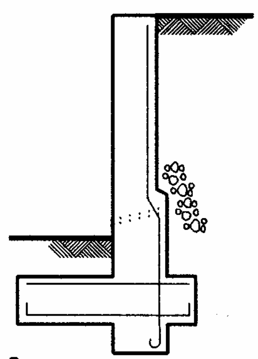

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Retaining wall

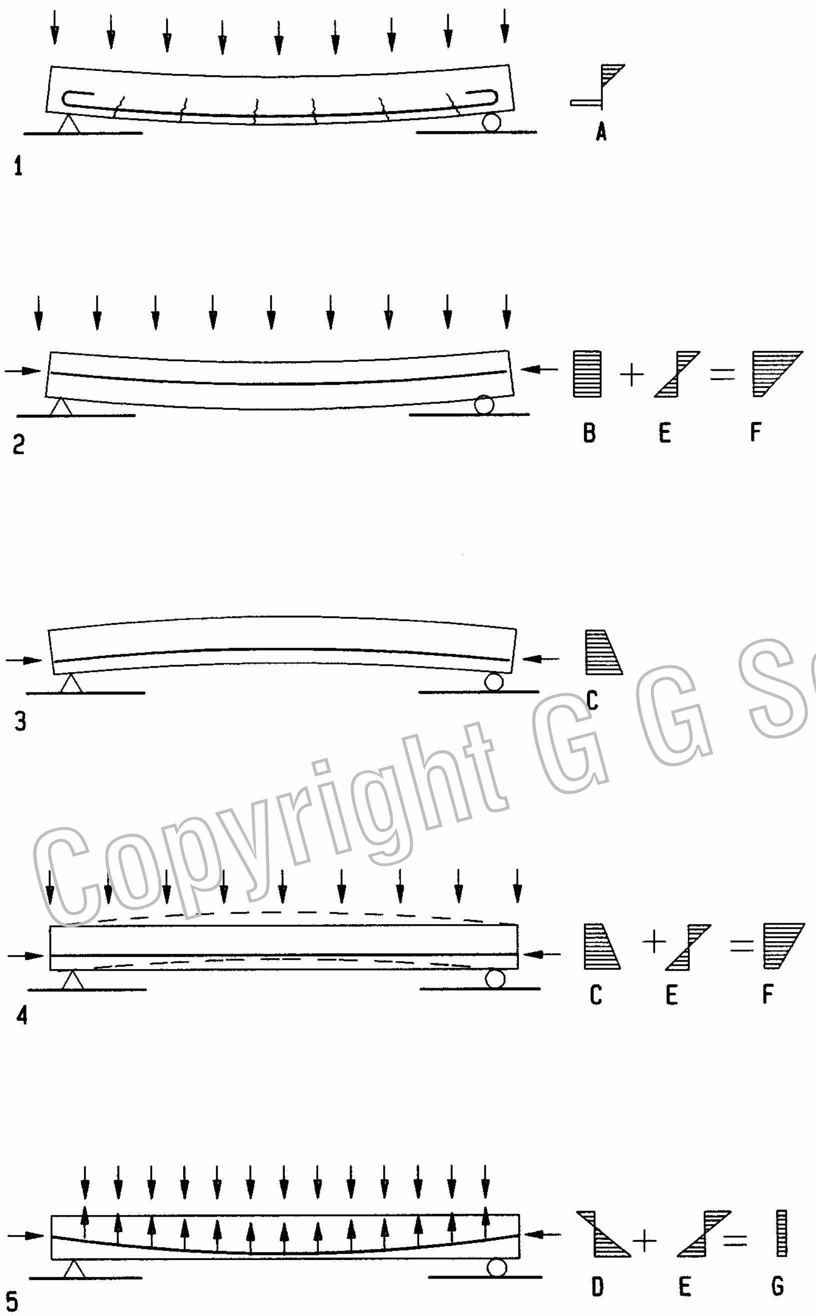
Retaining walls facilitate abrupt changes in topography. They resist lateral soil pressure and possible surcharges, such as buildings. Three types of retaining walls are mass walls (rare today), cantilever walls of concrete and masonry. Retaining walls should have weep holes spaced about 10 ft (3 m) with gravel backfill. They also require expansion joints spaced about 30 ft (10 m) to prevent cracking. Retaining wall design depends on location. At property lines the footing must point away from the property line, otherwise footings are located to best resist overturning, using soil as ballast. Cantilever retaining walls require footing widths of about 2/3 the height from top of footing to top of wall. Walls with sloped backfill need footings of about 1-1/4 their height and also need a *key* below the footing to help resist lateral sliding. Walls up to 6 ft (1.8 m) height require 8 in (20 cm) width. Higher walls up to 9 ft (3 m) height require 12 in (30 cm) at their lower portion. Depending on height, vertical rebars range from # 3 to # 8, spaced 8 to 32 in (20 to 81 cm) and horizontal bars are spaced about 16 in (40 cm). Retaining walls are usually designed using equivalent fluid pressure as lateral load.

Mass walls resist lateral pressure by their mass or dead weight and are thus very bulky. They are usually of plain concrete, but may be reinforced to reduce cracking. Mass walls are about 12 in (30 cm) wide on top and increase in width about 1/3 of the distance from the top.

Concrete walls are cantilever retaining walls that resist lateral pressure by being cantilevered from the ground. They balance overturn moments by their own weight combined with soil surcharge imposed on their footing and resist sliding by lateral soil pressure and friction at the base. They are more expensive than concrete masonry walls due to the expense of formwork.

Concrete masonry walls are also cantilever retaining walls that resist lateral pressure by cantilever action. They resist overturn moments by their own weight and soil imposed on the footing and resist sliding by lateral soil pressure and friction at the base. The footing usually requires a key below the footing to help resist lateral sliding. Concrete masonry retaining walls are most common due to a balance of strength and economy.

1-3 Mass retaining walls
4 Concrete /CMU wall at property line with adjacent land lower
5 Concrete / CMU wall not at property line
6 Concrete / CMU wall at property line with adjacent land higher
7 Concrete / CMU wall at property line with adjacent land lower
8 Concrete / CMU wall not at property line
9 Concrete / CMU wall at property line with adjacent land higher



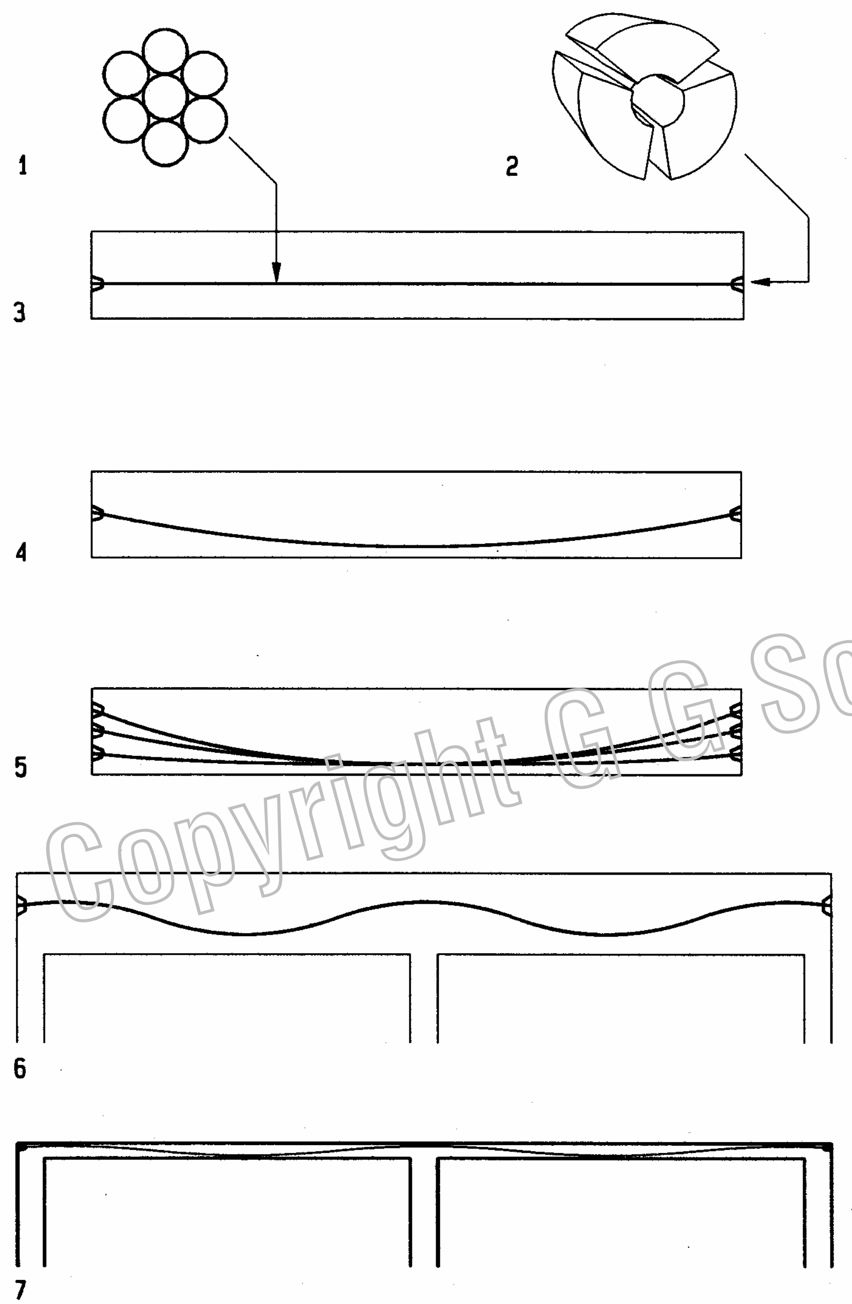
Prestressed concrete

The effect of prestress on concrete is to minimize cracks, reduce depth and dead weight, or increase the span. Analogy with a non-prestressed beam clarifies this effect. In a simply supported non-prestressed beam the bottom rebars elongate in tension and concrete cracks due to tensile weakness. In prestressed concrete *tendons* (high-strength steel strands) replace rebars. The tendons are pulled against concrete to compress it before service load is applied. Service load increases the tension in tendons and reduces the initial concrete compression. Avoiding tensile stress in concrete avoids cracks that may cause corrosion in rebars due to moisture. Further, prestress tendons can take the form of bending moments that balances the service load to minimize deflection. This, combined with higher strength concrete of about 6000 psi (40 MPa), allows for longer span or reduced depth in beams.

Pre-tensioning and *post-tensioning* are two methods to prestress concrete. They are based on patents by Doehring (1886) and Jackson (1888); yet both were unsuccessful due to insufficient stress that dissipated by creep. Doehring stressed wires before casting the concrete, and Jackson used turnbuckles to stress iron rods after the concrete had cured. Subsequent experiments by others led to the first successful empirical work by Wettstein in 1921 and the first theoretical study by French engineer Eugene Freyssinet during 1928, followed by his practical development. In 1961 the US engineer T Y Lin pioneered prestress tendons that follow the bending diagram to balance bending due to load. Lin's method controls deflections for any desired load, usually dead load and about half the live load. By his method, before live load is applied a beam (or slab) bows upward. Under full load, they deflect and under partial live load they remain flat. Diagram 5 illustrates this for a simply supported beam.

1 Simply supported beam without prestress, cracked at tensile zone
2 Prestress beam with concentric tendon deflects under service load
3 Prestress beam with eccentric tendon pushes up without service load
4 Same beam as 3 above with service load balanced at max. mid-span moment but not elsewhere since tendon eccentricity is constant
5 Prestress beam with parabolic tendon to balance bending moment

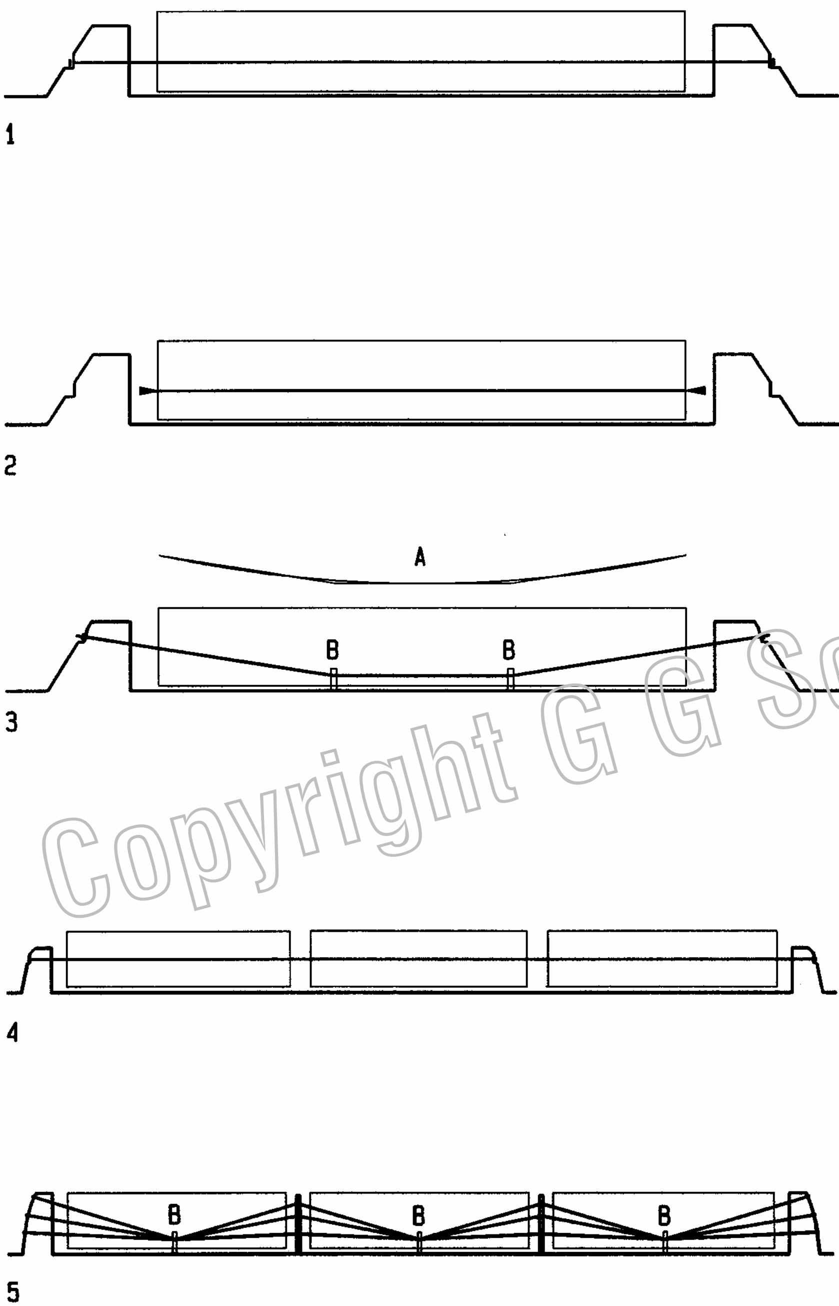
A Bending stress: top concrete compression and bottom steel tension
B Prestress uniform due to concentric tendon
C Prestress with greater compression near tendon at bottom
D Prestress for eccentric tendon: bottom compression and top tension (tension where tendon is outside beam's inner third (*Kern*). Simply supported beam of zero end moments has concentric tendon at ends
E Service load stress: top compression and bottom tension
F Combined stress from prestress and service load
G Combined stress with uniform distribution due to balanced moments



Tendons are high strength steel strands used in prestressed concrete. They have a breaking strength of 270 ksi (1860 MPa) and are composed of 7 wires, six of them laid helically around a central wire. Tendons come in sizes of 0.5 and 0.6 in (13 and 15 mm) diameter. Both sizes are used in post-tensioned concrete but only small tendons are used in pre-tensioned concrete which requires no bond length. The great strength of tendons allows initial stress levels high enough to make up for loss of stress due to creep, most notably during initial curing.

Post-tensioning begins by placing of metal or plastic tubes that house the prestress tendons prior to the pouring of concrete. Some tendons come enclosed in the tubes, but most are inserted after concrete has cured. Tubes prevent tendons to bond with the concrete to allow free movement for subsequent prestress operation. Once concrete has reached sufficient strength, the tendons are prestressed using hydraulic jacks that press against concrete to transfer prestress into it. Short members have tension applied at one end only but long members may require tension at both ends to overcome friction. Several devices are available to anchor the ends of post-tensioned tendons into concrete. One such device is a conical wedge that holds tendons by mechanical friction between the tendon and a rough surface of the device. Post-tensioning is usually done at the building site. It allows tendons to take any form desired to balance bending moments induced by service load.

1 Tendon cross section.
2 Conical tendon lock rests against steel sleeve (not shown) inserted into concrete and squeezes the tendon and hold it by friction
3 Beam with straight eccentric tendon below neutral axis
4 Parabolic tendon emulate bending moment to reduce deflection
5 Multiple parabolic tendons with offset anchors
6 Continuous beam with tendons emulating bending moment distribution
7 Continuous slab with tendons emulating bending moment distribution

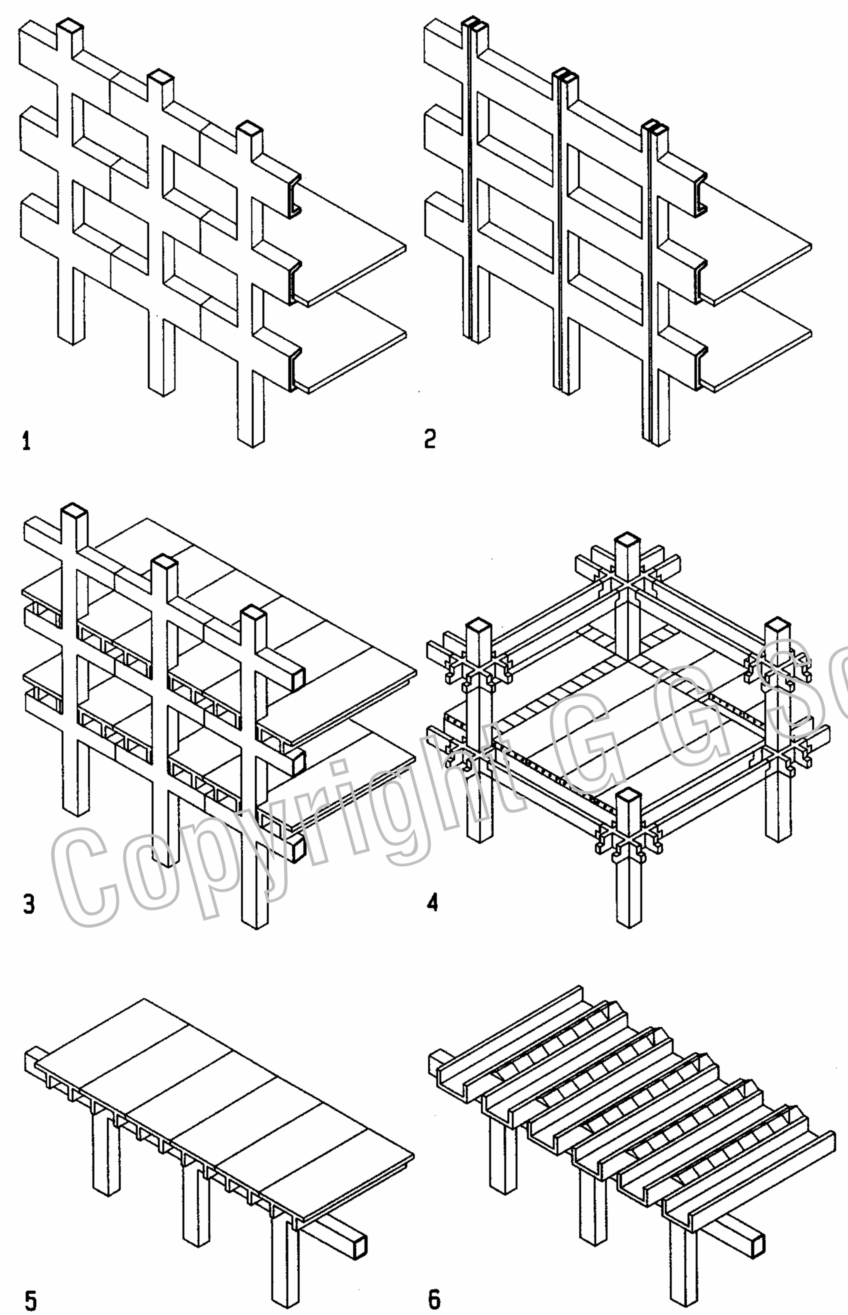


Pre-tensioning stresses tendons between abutments in a precast concrete plant. Once concrete has sufficient strength (with the help of steam in about 24 hours) tendons are cut off at the abutments to transfer prestress into the concrete. After cut off, tendons are anchored by friction with the concrete at both ends. Since abutments are difficult to secure at construction sites, pre-tensioning is usually done in a precast plant. Similar pieces may be laid in a row and cast together requiring only two abutments per row. Pre-tensioning is simplest with straight tendons, but some approximate curves that emulate bending moments are possible. They require temporary tie-downs to be cut off along with tendons after curing is complete.

Pre-tensioned members must be carefully handled during transportation to avoid damage or breakage. Since the reinforcement is designed for a given load direction, any reversed load may result in overstress and possible breakage. To avoid this they must be placed on the truck in the same position as in their final installation. Also, to avoid breakage of corners, it is advisable to provide corners with chamfers.

1 Beam with tendons anchored to abutments
2 Beam with tendons cut off to transfer prestress into concrete after it has reached sufficient strength
3 Beam with tendon tie-down to approximate bending moment curves
4 Row of pre-tensioned members with tendon anchors at ends only
For members like walls and columns with possible bending in any direction, tendons may be placed at the center
5 Row of pre-tensioned beams with tendon tie-down to approximate bending moments of service loads

A Close approximation of parabolic bending moment by tendon shape
B Temporary tendon tie-downs to approximate bending moments are cut off after tendons are cut from abutments

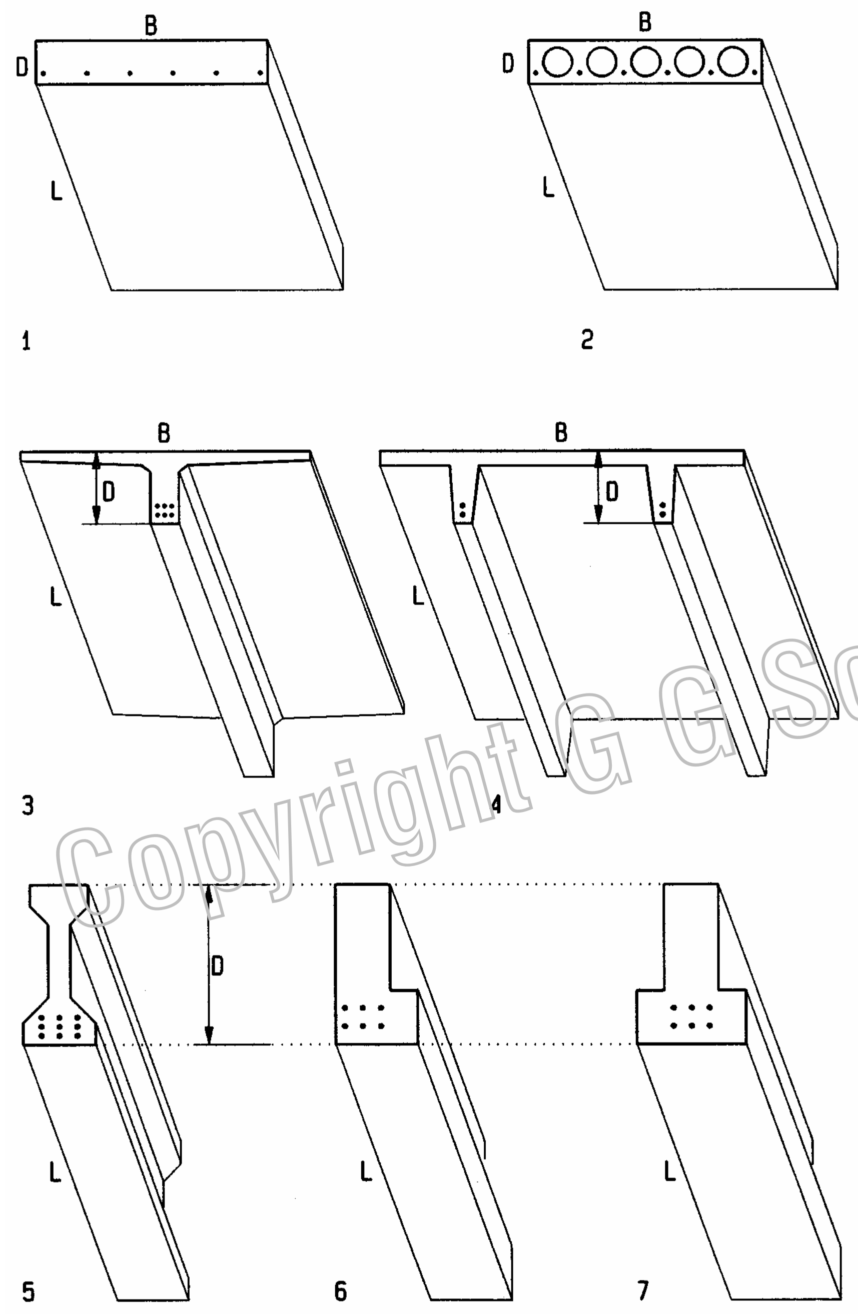


Precast concrete

Precast concrete comes in a wide variety of shapes for both structural and architectural applications. Presented are structural systems and members: floor and roof members, columns, and walls. Though precast members may be of ordinary concrete, structural precast concrete is usually prestressed. The primary reinforcement of prestressed concrete is with tendons, yet normal rebars are often used as stirrups to resist shear. Rebars are also added for different loads during transportation and erection. Compared to site-cast concrete, precast concrete provides better quality control, repeated use of formwork, faster curing with steam, and concurrent operations while other site work proceeds. The advantages must offset the cost of transportation to a construction site. Precast concrete is similar to steel framing by allowing preparatory site work to be concurrent, yet it has the advantage to provide inherent fire resistance. Steel on the other hand, has lower dead weight, an advantage for seismic load that is proportional to dead weight. To reduce high costs of formwork the number of different precast members should also be reduced; yet this objective must be balanced by other considerations. For example, fewer parts may result in a monotone and uninspired design. Combining precast with site-cast concrete may satisfy economy as well as aesthetic objectives.

Precast framing allows many variations, both with and without site-cast concrete. A few typical examples are presented. They are possible with columns of several stories, limited primarily by transportation restrictions. The capacity of available cranes could also impose limitations. In such cases, columns should be spliced near mid-height between floors where bending moments from both gravity and lateral loads are zero.

1 T-columns with deep spandrel beams support floor and roof slabs. Shear connections between adjacent beams combine them to moment frames to resist lateral as well as gravity loads
2 Frames of split columns and deep spandrel beams support floor and roof slabs for gravity and lateral loads. Shear connections at adjacent split columns tie the frames together for unified action
3 T-columns with normal spandrel beams support floor and roof rib slabs Shear connections between adjacent beams combine them to moment frames to resist lateral as well as gravity loads
4 Tree-columns with beam supports allow flexible expansion. Twin beams allow passage of services between them. Lateral load resistance must be provided by shear walls or other bracing
5 Rib slab or double T's supported on site-cast frame
6 U-channels with intermittent skylights supported on site-cast frame

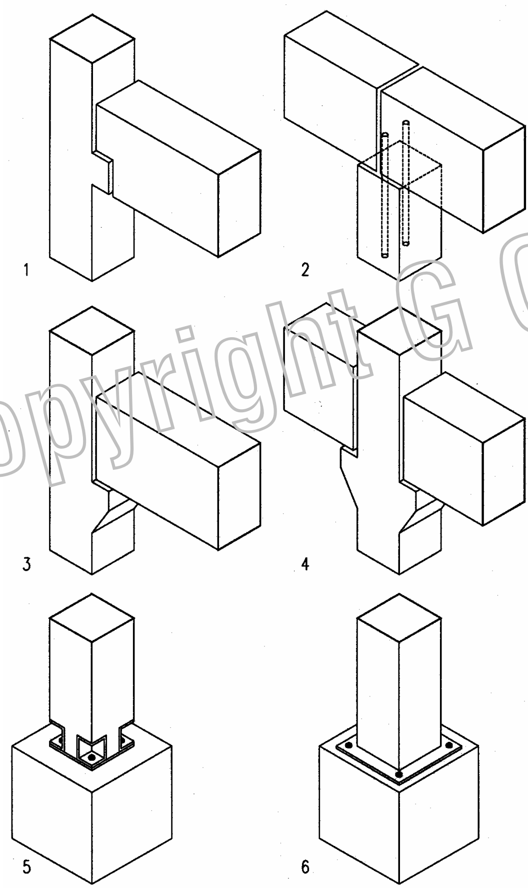


Floor and roof members span primarily horizontally to carry load in bending to supporting columns or walls. Some are designed for bridge structures but also used in buildings. A primary objective is to keep the dead weight low yet providing relatively long spans. To this end long-span members have ribs or hollow cores to reduce weight yet seeking optimal synergy for the tensile and compressive properties of steel and concrete, respectively. Precast members are usually covered with site-cast concrete of about two inch (5 cm) to provide a smooth surface and bond individual panels together. The following dimensions are approximate and vary by precast plant.

	Item	B	D	L	L/D
1	Solid plank	2-4 ft	4-8 in	10-30 ft	25-40
		0.6-1.2 m	10-20 cm	3-10 m	
2	Hollow plank	2-8 ft	6-12 in	20-50 ft	30-40
		0.6-2.4 m	15-30 cm	7-15 m	
3	Single T	4-8 ft	12-48 in	20-120 ft	20-40
		1.2-2.4 m	30-120 cm	7-36 m	
4	Double T	4-8 ft	12-48 in	20-120 ft	20-40
		1.2-2.4 m	30-120 cm	6-36 m	
5	I-beam		20-48 in	20-120 ft	15-30
			50-120 cm	6-36 m	
6	L-beam		20-48 in	20-60 ft	10-20
			50-120 cm	6-24 m	
7	Inverted T		20-48 in	20-60 in	10-20
			50-120 cm	6-24 m	

Precast columns must support beam gravity load and prevent sliding under lateral load. This is possible by steel dowels, grouting, bolting, welding, or tie-rods. Dowel bars are joint by grouting after erection. Bolting and welding connect metal plates welded to rebars of joining members. Bearing pads prevent overstress due to uneven surface of beam or column. Precast columns several stories high, limited by transportation constraints are spliced at the mid-height of a story where moments are zero.

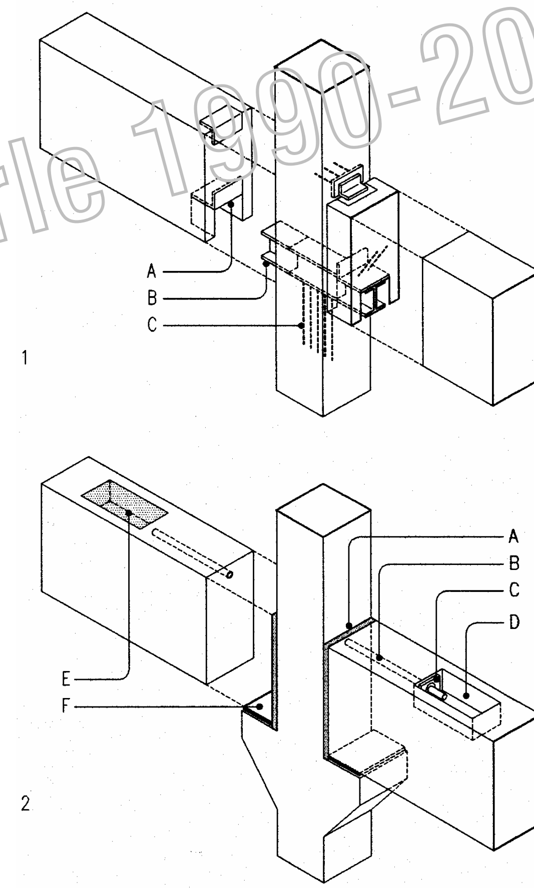
1 Column with semi-corbel recessed into beam for flush bottom face
2 Column with two beams connected by steel dowels
3 Column with one-sided corbel to support beam
4 Column with two-sided corbels to support beams
5 Column splice with recessed metal brackets
6 Column to footing connection with steel plate and anchor bolts

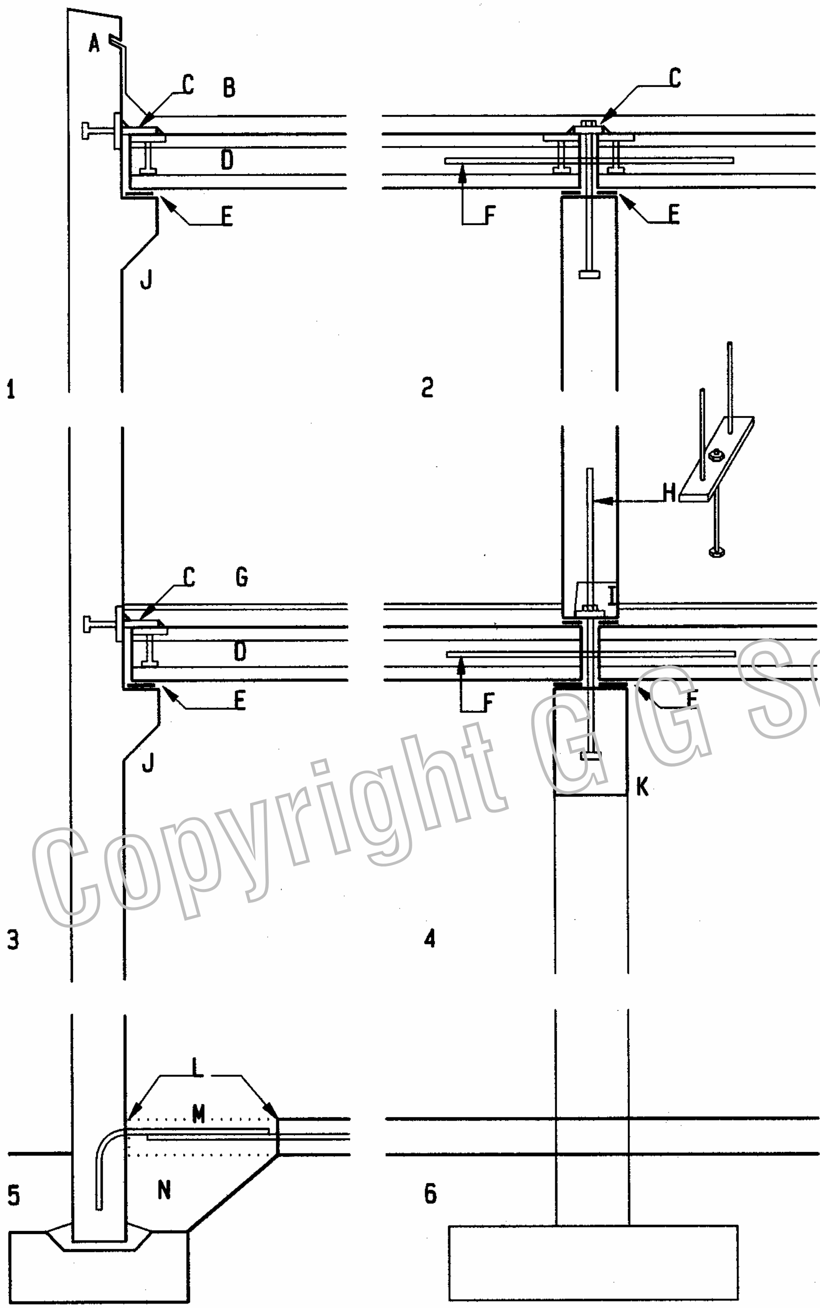


Concealed moment resistant joints

1 Concealed beam/column joint
A Steel angle cast into beam end
B Wide-flange steel haunch cast into column
C Rebars welded to steel haunch

2 Moment resistant beam/beam joint (or similar beam/column joints)
A Dry-pack grout
B Post tensioned rod extending through column
C Anchor plate
D Pocket for tensioning jack
E Grouted pocket after completion of post tensioning
F Bearing pad



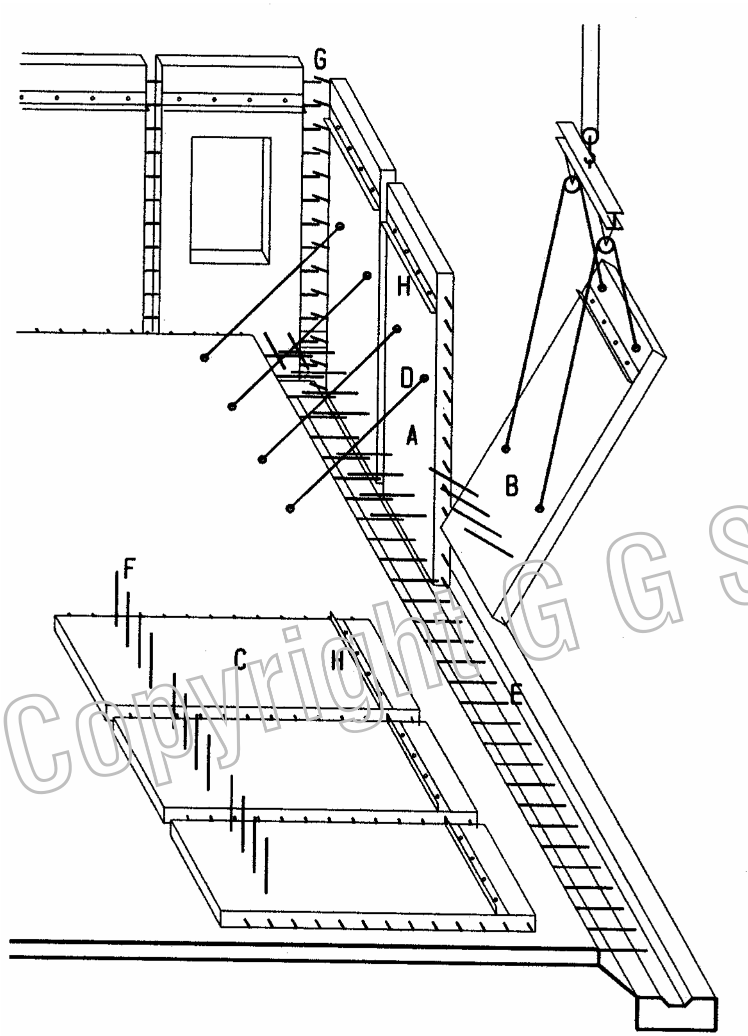


Precast wall

Walls can be precast in many sizes and configurations: small modular panels; walls extending entire spaces; complete walled boxes (hotel rooms); single story walls; and multi-story walls. The choice of size and configuration depends on means of construction and limitations imposed by transportation requirements. Exhaustive coverage is beyond the scope of this book. Only some basic conditions are presented. Precast walls may be prestressed or with normal reinforcement that would be similar to site-cast concrete walls presented before. For thick walls two layers of tendons are placed near the wall faces. For most walls single layers are placed at the center of walls to resist buckling and bending in any direction. For walls that continue over several stories concrete floors and roofs are supported by concrete corbels, steel and wood by steel and wood ledgers, respectively. Walls extending from floor to ceiling are anchored to floors or roofs and adjacent walls by welded or bolted steel brackets, or by dowel bars inserted in grout or site-cast concrete. Exterior walls may be temporarily stabilized by dowel bars that are tied or welded to dowel bars extending from floor slabs. A slot in the slab is left open to that end and is filled after the wall is done.

1 Roof support by exterior wall corbel with welded brackets
2 Roof support by interior wall with bolted and welded brackets
3 Floor support by exterior wall corbel with welded brackets
4 Floor support by interior beam with bolted bracket
5 Exterior footing with wall tied to floor by overlapping dowels. A slot in floor slab is filled after dowels are tied or welded together
6 Interior column footing

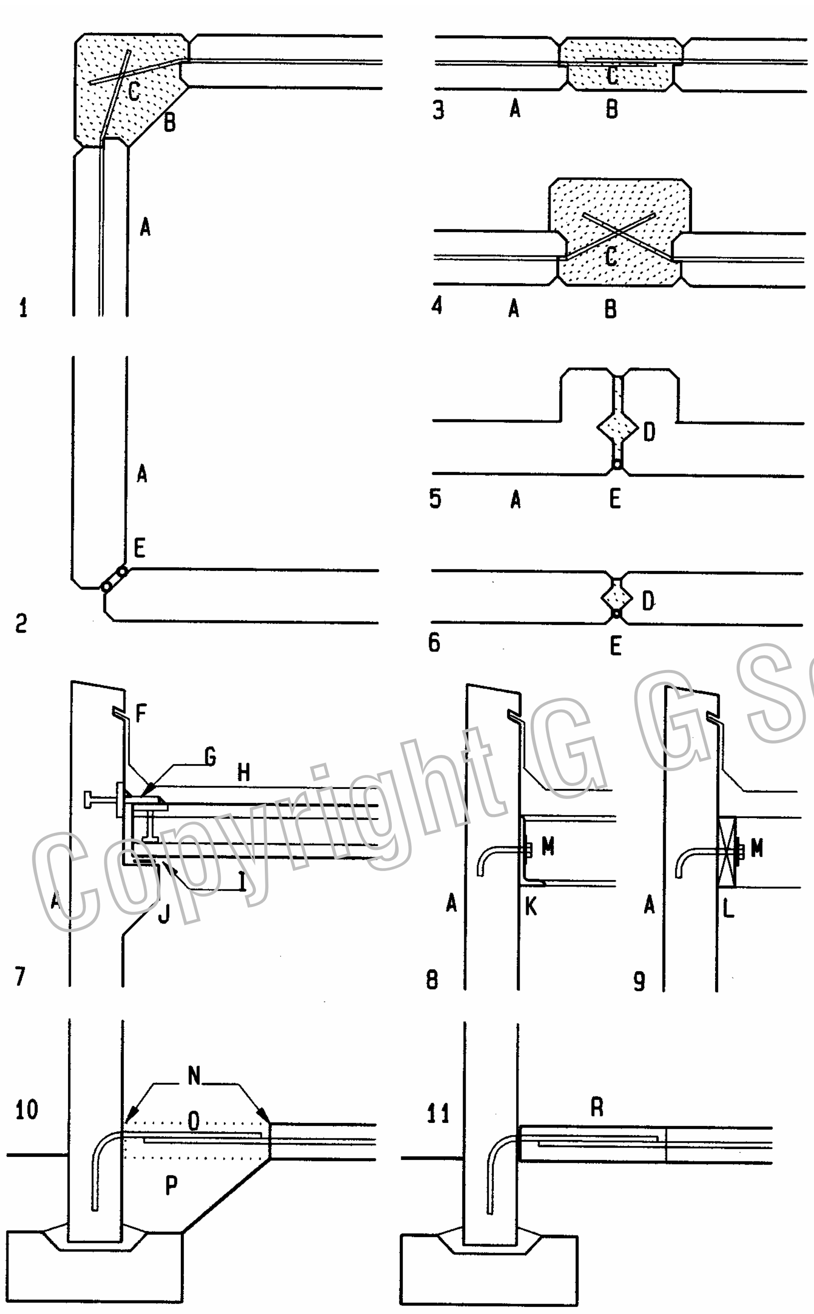
A Metal reglet for flashing, inserted in concrete wall
B Roofing membrane on rigid thermal insulation or site-cast concrete
C Metal plate welded to connect brackets in wall and floor or roof
D Precast roof slab (hollow core or other)
E Bearing pads of neoprene or rubber to distribute load evenly
F Grouted dowel bars to connect adjacent floor slabs
G Site-cast concrete top for smooth floor and to join precast slab panels
H Steel bracket to tie walls of adjacent floors together
I Pocket for bolting bracket (grouted after completion)
J Corbel for floor and roof (or ledger for wood or steel floors and roofs)
K Concrete beam, precast or site-cast
L Slot in floor slab to tie or weld wall to floor dowels (filled subsequently)
M Wall and floor dowels tied or welded together to anchor wall to floor
N Gravel to support floor slab in connecting slot



Tilt-up concrete

Tilt-up concrete is a precast technique performed on the building site. Due to relatively low cost, tilt-up construction is popular for industrial buildings and warehouses, but is adaptable to other building types. Wall panels are cast on top of the floor slab under construction to eliminate most formwork. The floor slab must be smooth and flat and carefully treated with a bond-breaking compound to prevent wall panels to bond with the slab. Depending on the wall finish desired, panels may be cast with the outside face facing up or down. The latter is more common and is the only logical option when corbels or other projections for floors or roofs are needed on the inside. Electrical wiring and similar items needed, including lifting and temporarily bracing inserts, are installed prior to concrete. Panels may be up to 30 ft (9 m) wide and 60 ft (18 m) high, but they are usually much smaller. The lifting capacity of available cranes must be considered in selecting panel sizes. The crane capacity should be about twice the panel weight to account for initial inertia and panel bonding with the floor slab. Once the panel has reached sufficient strength, after about seven days, the lifting process begins. Panels must be designed not only for service load but for any possible load during lifting and erection. After erection, the wall panels are braced with telescopic steel braces that allow proper alignment. The panels are connected to the floor slab by overlapping dowel bars that extend from wall panels and floor slab. Dowels are tied or welded together to anchor panels to the floor for initial stability until roof or floor diaphragms are in place. Those diaphragms transfer lateral loads to wall panels that act as shear walls parallel the load.

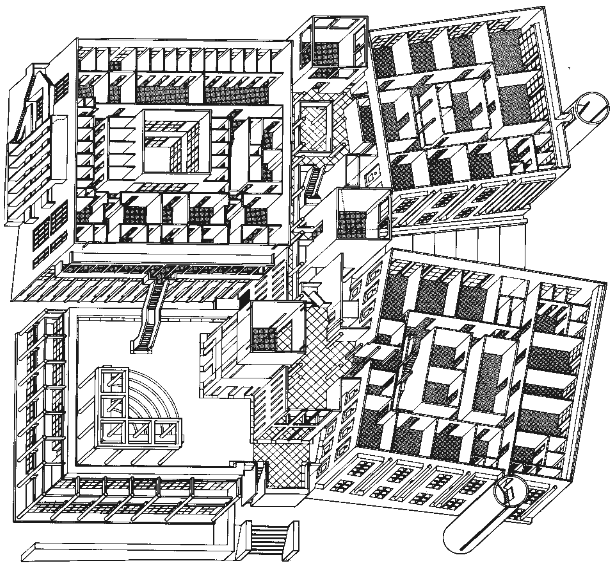
A Wall panel installed with temporary bracing
B Wall panel being tilt-up and lifted into the footing
C Wall panel on the floor slab ready for tilt-up
D Steel braces with telescopic spindle for panel alignment
E Floor slab dowel bars to be tied or welded to wall dowels
F Wall panel dowel bars to be tied or welded to floor dowels
G Possible dowel bars at panel edges to tie panels together
H Steel ledger supports steel roof (wood ledgers or concrete corbels are used for wood or concrete roofs)



Tilt-up details are similar to other precast wall details but some are unique to tilt-up construction. Wall panels may be connected by various butt joints or cast-in-place concrete splices or pilasters or by precast columns. Only some common details are presented. Walls may have concrete corbels, steel or wood ledgers, for concrete, steel and wood floors or roofs, respectively. Reinforcement, except for connecting dowels, is not shown for clarity.

1 Wall corner with cast-in-place concrete splice
2 Wall corner with mitered butt joint and sealant
3 Wall joint with cast-in-place concrete splice
4 Wall joint with cast-in-place concrete pilaster
5 Wall joint with grouted key in precast double pilaster
6 Wall joint with grouted key in butt joint
7 Roof supported by exterior wall corbel with welded brackets
8 Roof supported by exterior wall with steel ledger
9 Roof supported by exterior wall with wood ledger
10 Exterior footing with wall tied to floor by overlapping dowels. A floor slab slot is filled after dowels are tied or welded together
11 Exterior footing after floor slab is completed

A Tilt-up wall panel
B Cast-in-place concrete splice
C Dowel bars extending from wall panels into concrete splice
D Grouted concrete shear key ties panels together
E Silicone sealant with backer rod
F Metal reglet for flashing, inserted in concrete wall
G Metal plate welded to connect wall and roof brackets
H Roofing membrane on rigid insulation or concrete topping
I Bearing pads of neoprene or rubber to distribute load evenly
J Corbel to support roof or floor
K Steel angle ledger to support metal roof or floor
L Wood ledger to support wood roof or floor
M Anchor bolt to connect steel or wood ledger to wall panel
N Slot in floor slab to tie or weld wall to floor dowels
O Wall and floor dowels tied or welded together to anchor wall to floor
P Gravel to support floor slab in connecting slot



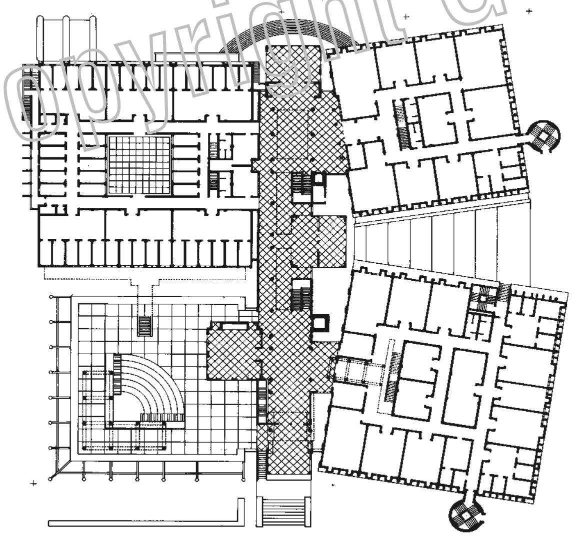
Projects

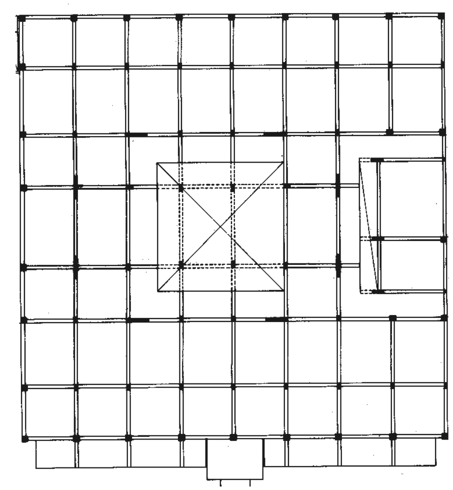
Hellas research foundation, Crete

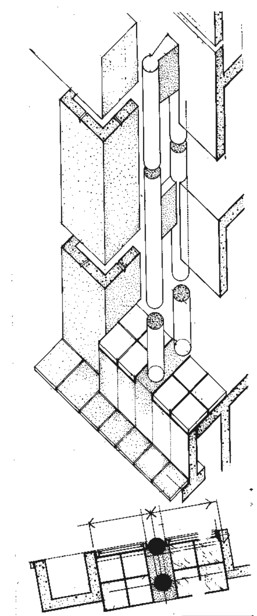
Architect: Panos Koulermos

Engineer:Technical offices of the Hellas Research Foundation and the Greek Ministry of Industry, Research and Technology

This research foundation has four building blocks, clockwise from top left: administration and mathematics; physics and laser research; molecular biology; and a plaza over research offices. A vaulted gallery, reminiscent of the vaults in nearby Heracleon, is supported by columns that flank a gallery to link the various elements, evoking memories of classical Greek architecture. Moment resisting reinforced concrete frames supports two-way concrete slabs between its beams. The frames are based on a module that varies from 4.8 to 6m (16 to 20 ft) in response to program needs and is combined with shear walls for increased resistance to lateral load. Concrete masonry partitions provide further resistance. Koulermos explored the plastic qualities of concrete to articulate facades in response to program needs. The oblique research wings integrate in a 1.2 m (4 ft) deep envelope zone exposed columns and u-shaped walls for sun control and as mechanical chases that facilitate changing needs. The plaza block features freestanding columns to frame dramatic views and support sun-shading devices. Articulation of the administration wing facade reflects interior functions. Facade elements of research wings, next to typical floor framing, are shown below.





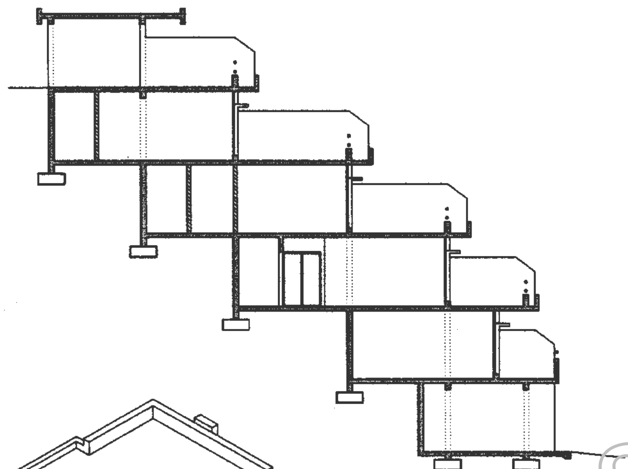


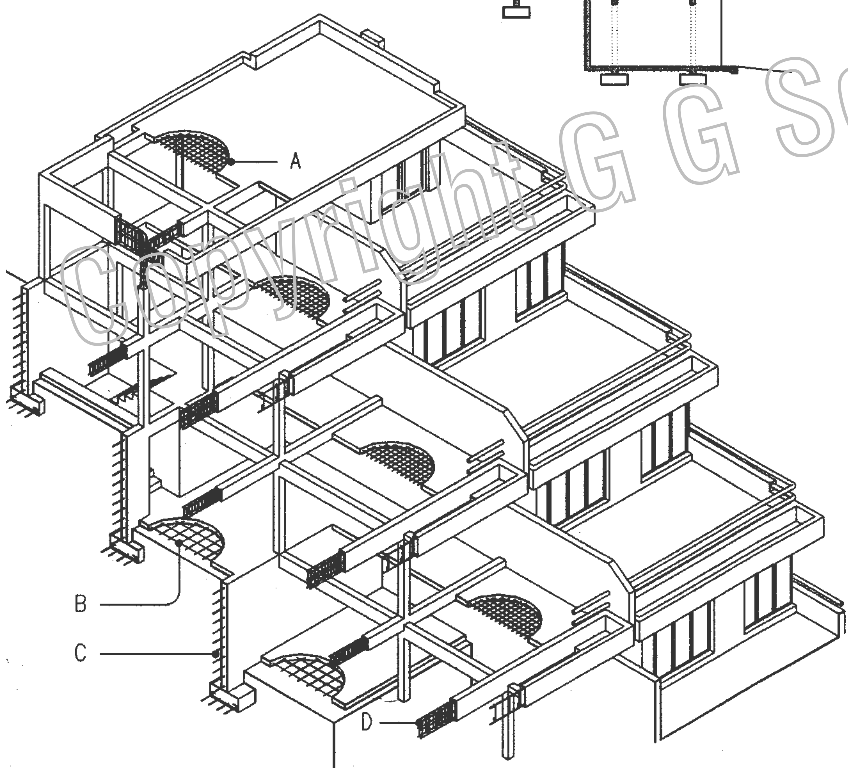
Terrace Homes Taipei, China (1980)
Architect: G.G. Schierle
Engineer: Kuan Wai Yen

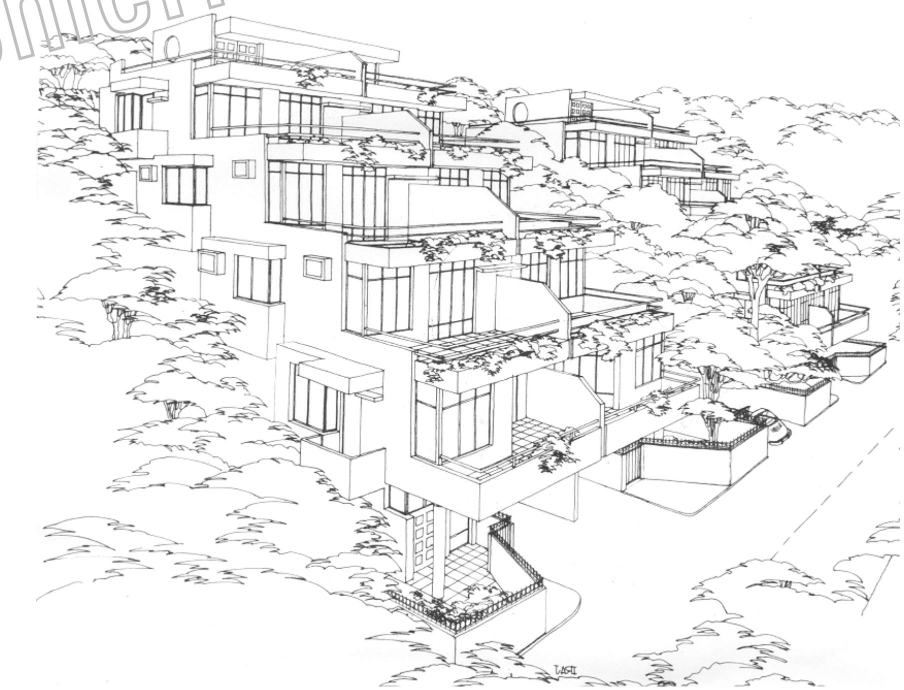
This 200 unit community on a hillside with dramatic valley view includes eight types of terrace homes for various needs and local topographies. Most units are in groups of four; two of them with access from below with street level parking, living above and bedrooms on the third floor. The other two units, with access from above, are in reversed order. Both types have two large terraces with party walls and planters for privacy.

Moment resistant concrete frames, based on a grid in the range of 3 to 4 m (10 to 13 ft) in response to space needs, support two-way concrete slabs and are vertically aligned for straight load paths. Masonry walls add stiffness to minimize movement in wind or moderate earthquakes, whereas frames provide ductility for fail-safe performance should brick walls fail in severe earthquakes, a proven effective combination in seismic regions.

A Two-way reinforced concrete slab on beams, all site-cast
B Concrete slab on grade
C Retaining wall with waterproofing
D Concrete beam





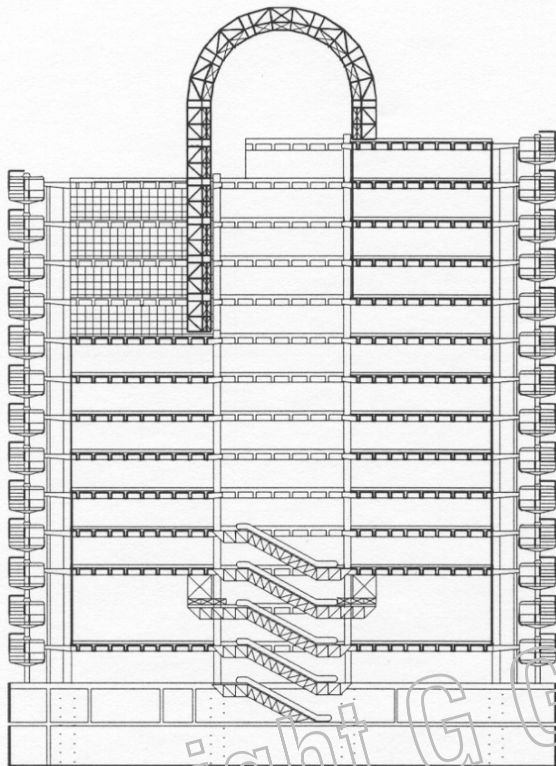


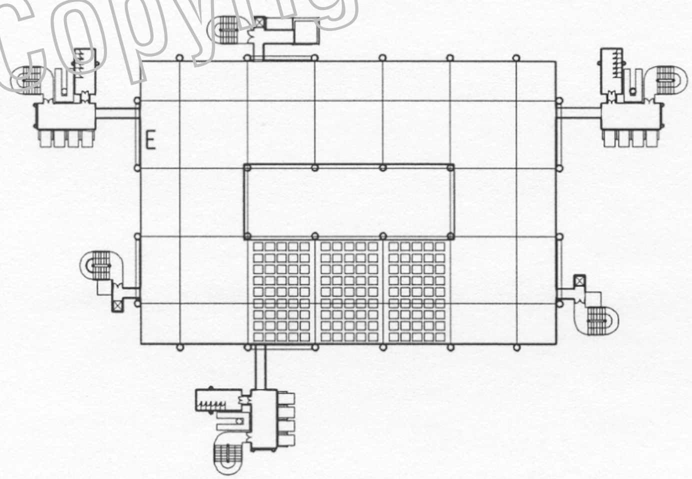
Lloyd's of London (1986)

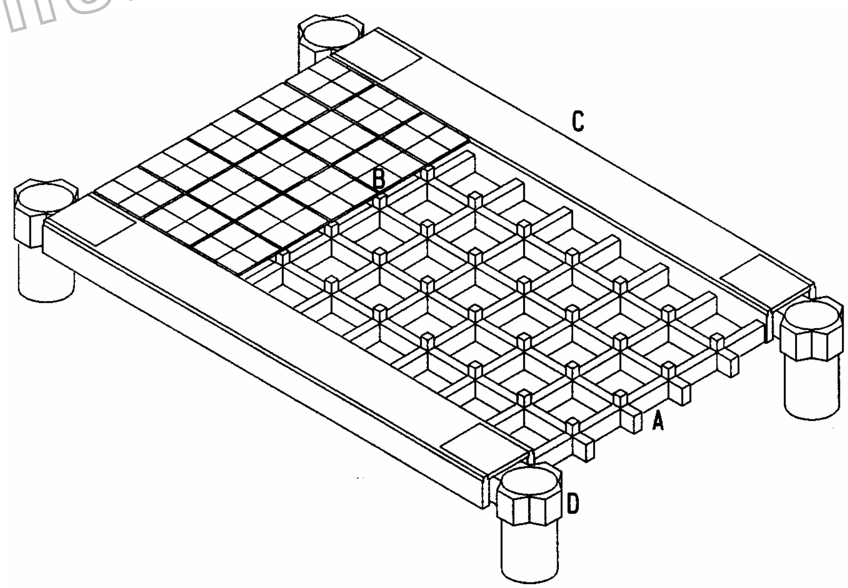
Architect: Richard Rogers
Engineer: Ove Arup

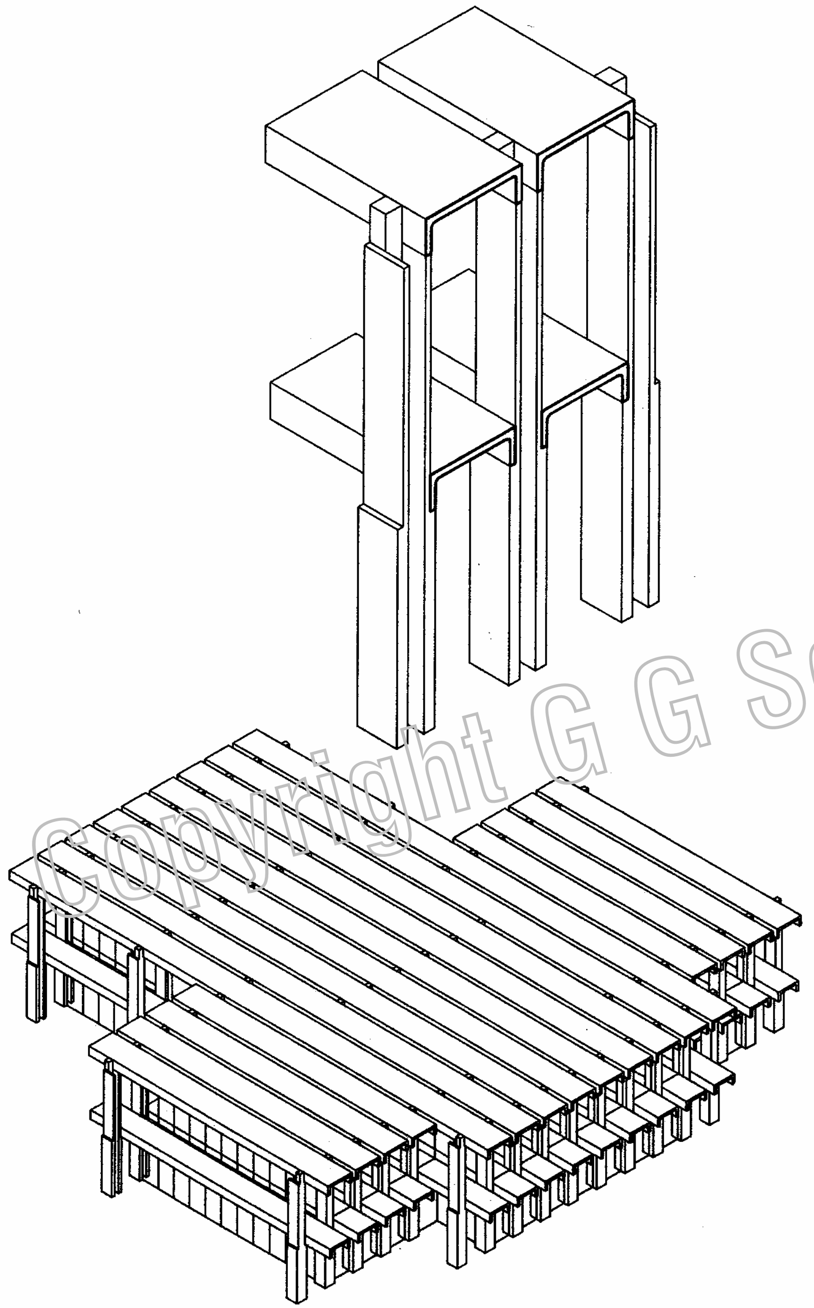
Loyd's of London, a society of syndicated underwriters, needs a market place known as "The room". The program required a facility for the twenty-first century, including an underwriter room three times larger than the old one. Given the location in London's financial district, an area of mostly small and winding streets, the design is in response to this urban context as well as the program. A permanent structure for the central space, expected to last, is flanked by service towers, expected to adapt with changing technology. The central space is an efficient box with an atrium covered by vaulted truss. Roof terraces step from 12 stories facing a high-rise on the North to six stories on the South. The service towers with stairs and elevators respond to small scale urban context. Located on the outside they allow continued operations during future changes. Reinforced concrete columns, spaced 35x59 ft (11x18 m), support the main structure.

A One-way ribs at 1.8 m (6 ft) and cross-ribs act as diaphragm to carry lateral load to vertical bracing. Cross ribs distribute rib load to adjacent ribs
B Access floors on stubs provide space for service distribution
C Precast, prestressed concrete channel beams
D Cantilever brackets support beams
E Diagonal bracing on six façade bays resists lateral loads







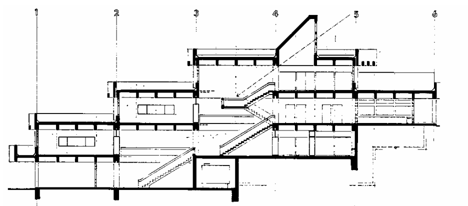


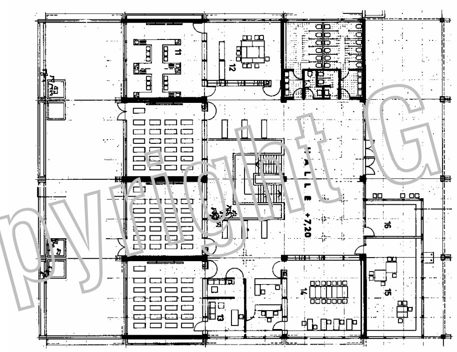
Cafeteria, state college Hayward, California (1967)
Architect: Worley K Wong and Associates
Engineer: Eric Elsesser

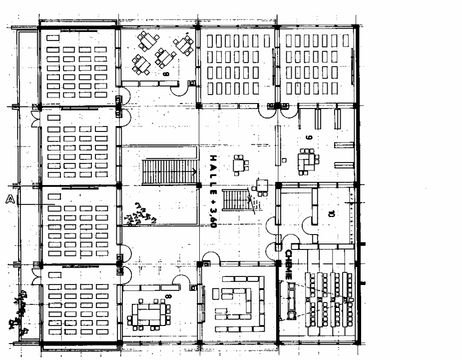
This cafeteria was built with two structural precast concrete components: Channel beams supported by channel columns; though the beams vary in lengths and width. A large interior atrium links the two levels and provides visual relieve to the strictly modular structure. The columns cantilever from underground foundations to resist lateral load in bending as well as gravity axial load. Columns step back at floor and roof levels to provide supporting shoulders for beams. Thus, the columns are largest at the base where moments from lateral loads are greatest. Roof beams are slightly wider than floor beams due to column setbacks. The void of channel columns, and the aligned space between beams, house electrical and mechanical services and recessed folding partitions. On-site concrete provides the finish over and bridges the gap between beams. Beams are connected to columns by inserts, welded together after erection. Neoprene pads provide smooth load transfer from beams to columns. Most of the precast concrete has exposed aggregate finish; only the inside of channel beams is smooth and painted white for a striking contrast and to enhance natural lighting by reflection.

Max-Eyth school Schöntal, Germany
Architect: P. M. Kaufmann
Engineer: W. Böck

Located on the edge of the Jagst valley with its rich history, the school is named after the poet-engineer Max-Eyth. The terracing follows the natural grade and adjacent vineyards. Clear story light floods the central atrium and stair. The atrium provides visual continuity. Terracing creates dynamic space composition. The structural grid provides vertical continuity for load paths and installations. A concrete moment frame of 8.4/8.4/3.6 m resists gravity and lateral loads. Exposed two-way joists, spaced 2.4 m, span the square modules and support a two-way concrete slab. The exposed concrete frame and precast exterior wall panels are contrasted by wood partitions

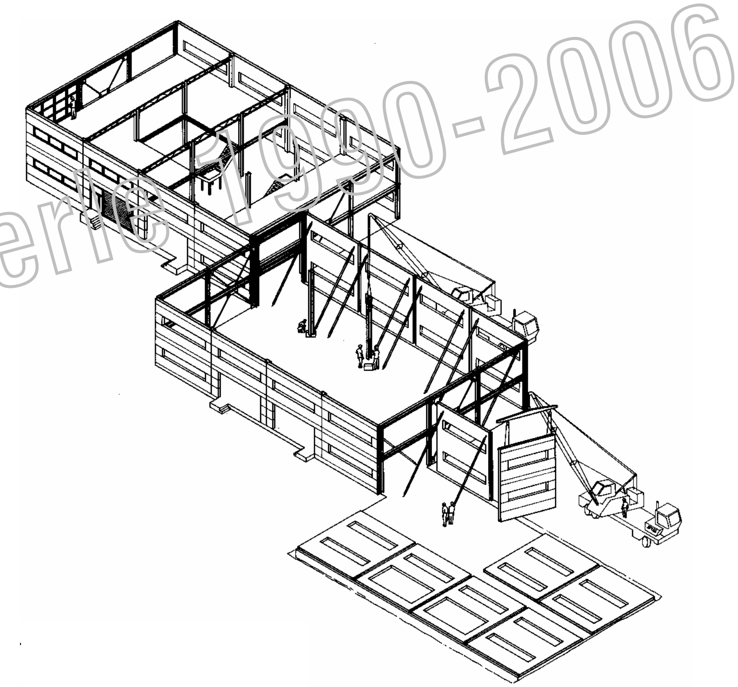


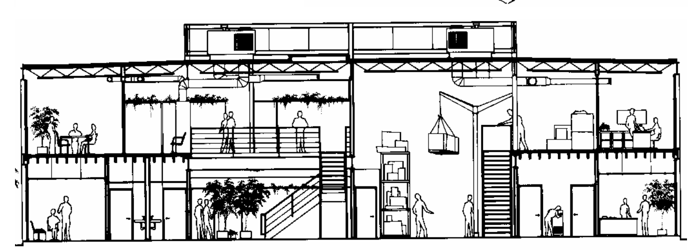




Hampshire national building, Culver City, California
Architect: James Tyler
Engineer: Dimitry Vergun

This two-story facility of three 60x100 feet units is designed on a 25/25 feet module. Tilt-up concrete panels on the long sides are joined to steel moment frames on the short sides to resist gravity load and lateral load in length and width direction, respectively. Interior steel columns carry gravity load only. The 6 inch tilt-up wall panels, 25/25 feet are spliced with poured-in-place concrete, and welded to steel columns at four corners. Steel truss joists support floor and roof metal decks. The tilt-up panels were poured on the concrete floor, adjacent to their erected positions. The panel's outside was sandblasted for a textured gravel finish.





24

Cable and Fabric

Material

Tent membranes have been around since ancient history, notably in nomadic societies. However, contemporary membrane structures have only evolved in the last forty years. Structural membranes may be of fabric or cable nets. Initial contemporary membrane structures consisted of

- Natural canvass for small spans
- Cable nets for large spans

Industrial fabric of sufficient strength and durability was not available prior to 1970. Contemporary membrane structures usually consist of synthetic fabric with edge cables or other boundaries. Cables and fabric are briefly described.

Fabric for contemporary structures consists of synthetic fibers that are woven into bands and then coated or laminated with a protective film

Common fabrics include:

- Polyester fabric with PVC coating
- Glass fiber fabric with PTFE coating
- Glass fiber fabric with silicon coating
- Fine mesh fabric, laminated with PTFE film

Fabric properties are tabulated on the next page. Foils included are only for very short spans due to low tensile strength. Unfortunately the elastic modulus of fabric is no longer provided by fabric manufacturers, though it is required for design and manufacture of fabric structures. The elastic modulus of fabric is in the range of:

E = 2000 lb/in, 11492 kPa/m to
E = 6000 lb/in, 34475 kPa/m

Cables may be single strands or multiple strand wire ropes as shown on following pages. Cables consist of steel wires, protected by one of the following corrosion resistance:

- Zinc coating (most common)
- Hot-dip galvanizing
- Stainless steel (expensive)
- Plastic coating (used at our cable nets at Expo64 Lausanne)

Depending on corrosion protection needs, zinc coating comes in four grades: type A, type B (double type A), type C (triple type A), type D (four times type A). Cables are usually prestressed during manufacture to increase their stiffness.

Elastic modulus of cables:

E = 20,000 ksi, 137900 MPA	(wire rope)
E = 23,000 ksi, 158,585 MPa	(strand > 2.5 inch diameter)
E = 24,000 ksi, 165,480 MPa	(strand < 2.5 inch diameter)

Fabric

Type	Makeup	Common use	Tensile strength	Fire rating ++ incombustible + low flammability 0 none	UV light resistance ++ very good + good	Translucency	Durability
Coated fabric*	Polyester fabric PVC coating	Permanent + mobile Internal + external	40 to 200 kN/m 228 to 1142 lb/in	+	+	0 to 25 %	15 to 20 years
Coated fabric*	Glass fiber fabric PTFE coating	Permanent Internal + external	20 to 160 kn/m 114 to 914 lb/in	++	++	4 to 22 %	> 25 years
Coated fabric	Glass fiber fabric Silicone coating	Permanent Internal + external	20 to 100 kN/m 114 to 571 lb/in	++	++	10 to 20 %	> 20 years
Laminated fabric*	Fine mesh fabric Laminated with PTFE film	Permanent Internal + external	50 to 100 kN/m 286 to 571 lb/in	++	++	35 to 55 %	> 25 years
Foil	PVC foil	Permanent internal Temporary external	6 to 40 kN/m 34 to 228 lb/in	0	+	Up to 90 %	15 to 20 years internally
Foil*	Flouropolymer foil ETFE	Permanent Internal + external	6 to 12 kN/m 34 to 69 lb/in	++	++	Up to 96 %	> 25 years
Coated or uncoated fabric*	PTFE fabric (good qualities for sustainability)	Permanent + mobile Internal + external	40 to 100 kN/m 228 to 571 lb/in	++	++	15 to 40 %	> 25 years
Coated or uncoated fabric*	Flouropolymer fabric	Permanent + mobile Internal + external	8 to 20 kN/m 46 to 114 lb/in	++	++	Up to 90 %	> 25 years

* Self-cleaning properties

SI-to-US unit conversion:
1 kN/m = 5.71 lb/in

Maximum fabric span*

Tensile strength	Maximum span
500 lb/in	60 ft
1000 lb/in	120 ft

* Assuming:
Live load = 20 psf, 956 Pa (wind or snow)
Safety factor = 4
Fabric span/sag ratio = 10

Cables

Cables may be of two basic types and many variations thereof. The two basic types are strands and wire ropes.

Strands have a minimum of six wires twisted helically around a central wire. Strands have greater stiffness, but wire ropes are more flexible. To limit deformation, strands are usually used for cable stayed and suspension structures.

Wire ropes consist of six strands twisted helically around a central strand. They are used where flexibility is desired, such as for elevator cables.

Metallic area, the net area without air space between wires, defines the cable strength and stiffness. Relative to the gross cross section area, the metallic area is about: 70% for strands and 60% for wire ropes. To provide extra flexibility, some wire ropes have central cores of plastic or other fibers which further reduce the metallic area.

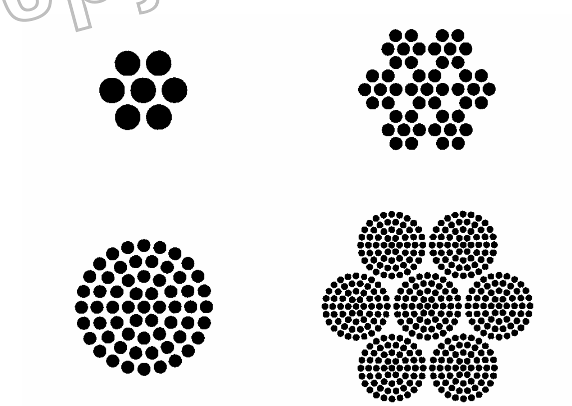
1 Strand (good stiffness, low flexibility)
E = 22,000 to 24,000 ksi; 70% metallic

2 Wire rope (good flexibility, low stiffness)
E = 12,000 to 20,000 ksi; 60% metallic

Cable fittings

Cable fitting for strands and wire ropes may be of two basic types: adjustable and fixed. Adjustable fittings allow to adjust the length or to introduce prestress by shortening. The amount of adjustment varies from a few inches to about four feet

3 Bridge Socket (adjustable)
4 Open Socket (non-adjustable)
5 Wedged Socket (adjustable)
6 Anchor Stud (adjustable)
A Support elements
B Socket / stud
C Strand or wire rope



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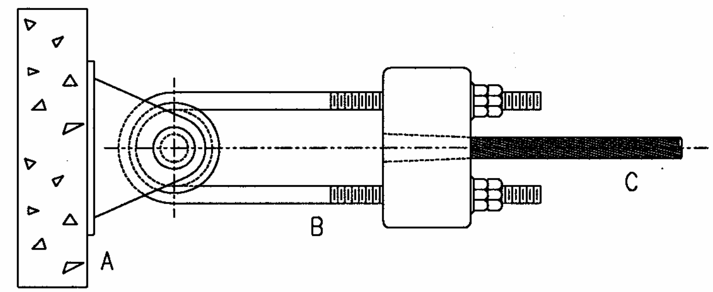

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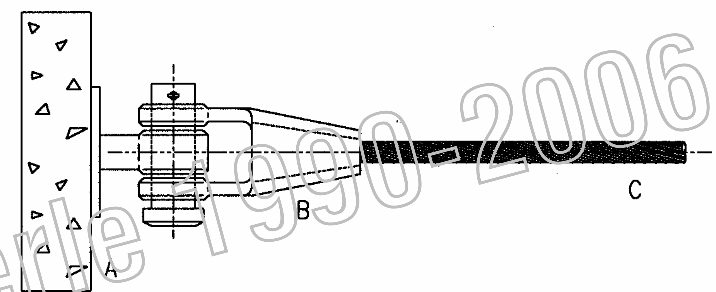

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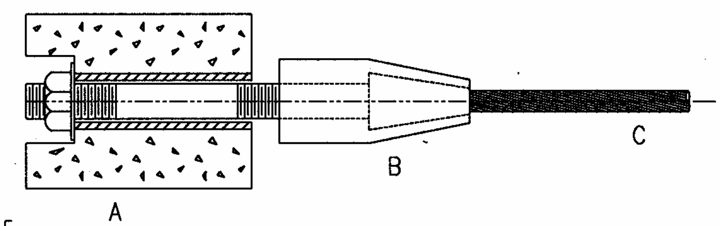

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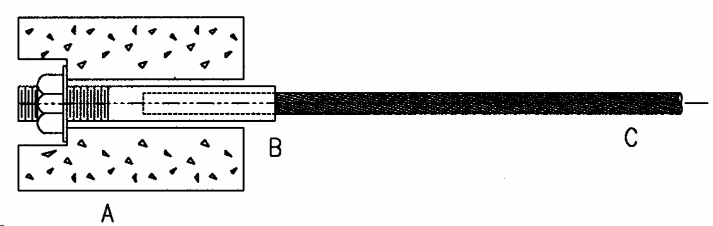

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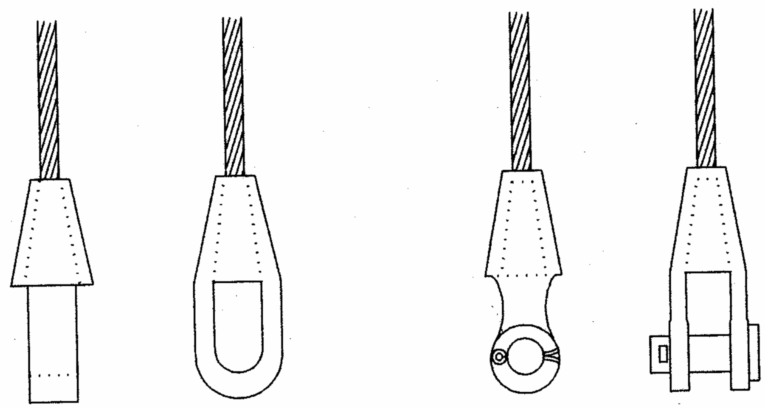
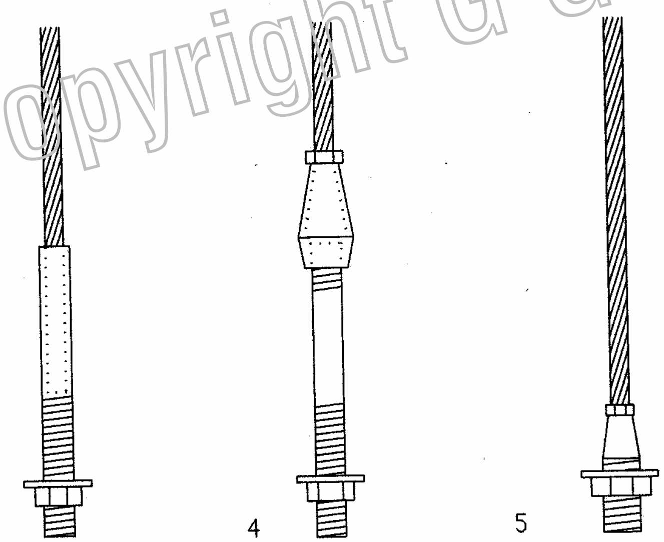


1 2

1 Closed socket for strand or wire rope
Length about 10 times cable size
Width about 3 to 4 times cable size

2 Open socket for strand or wire rope
Length about 10 times cable size
Width about 3 to 4 times cable size

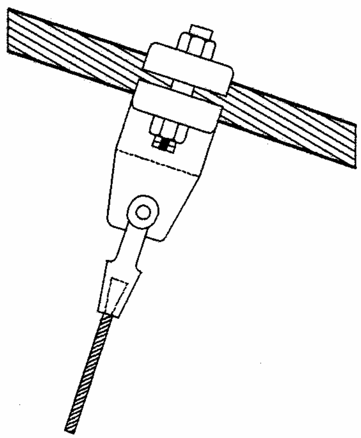


3 4 5

3 Threaded stud fir strand
swaged fitting
Adjustment about 2 to 3 time cable size

4 Composite socket with threaded stud
Customized adjustment

5 Threaded socket for strand or wire rope
Limited adjustment for ½” to 4” cable size



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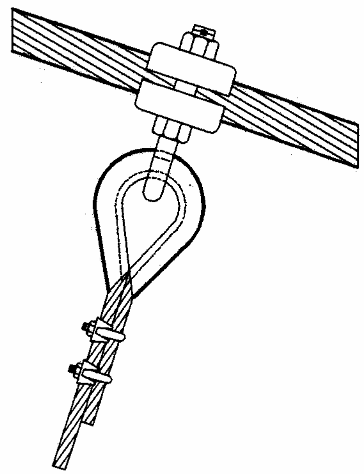

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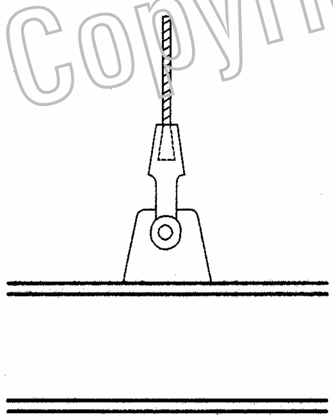

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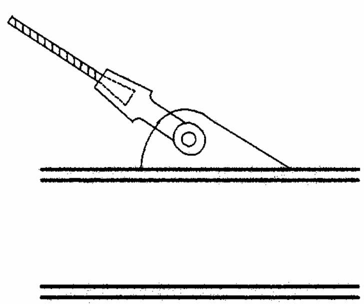


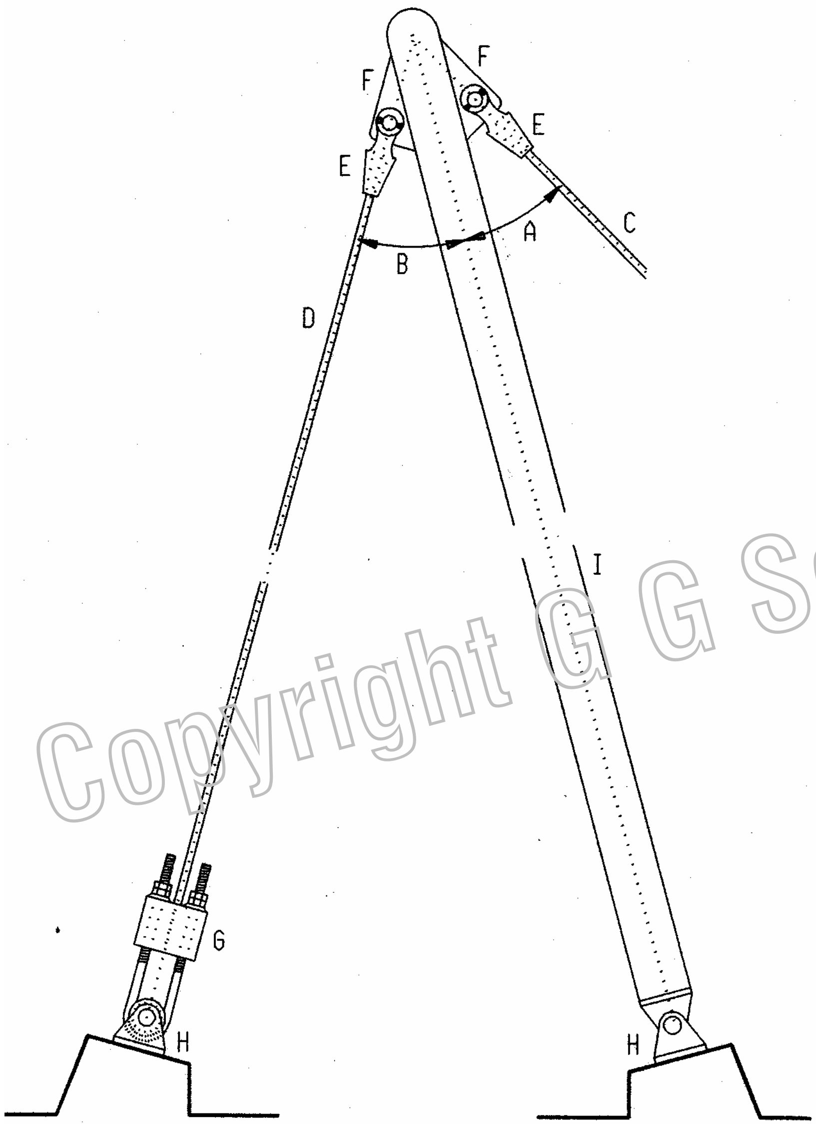
4

1 Cable-to-cable connection with integral strand fitting

2 Cable-to-cable connection with wire rope thimble

3 Open socked connection, perpendicular
Trapezoidal gusset plate for synergy of form and reduced weld stress

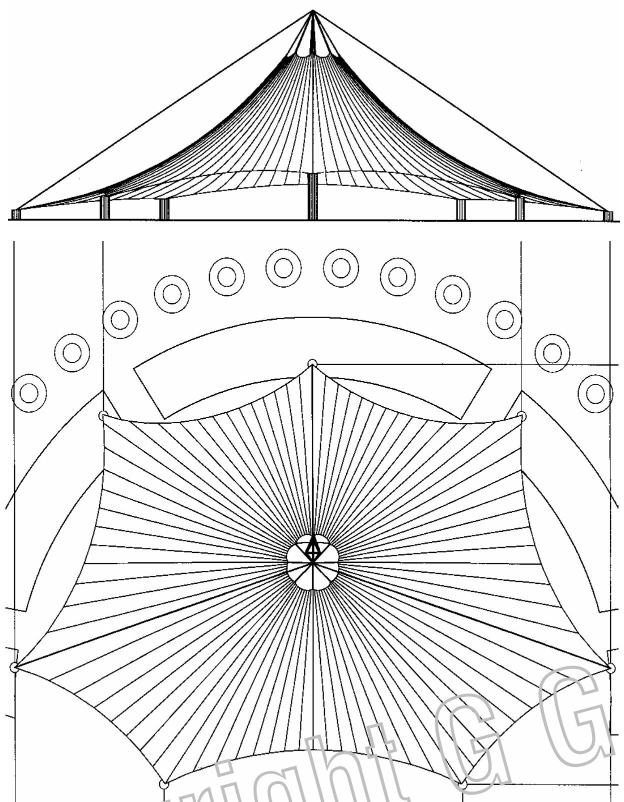
4 Open socked connection, angled
Sloping gusset plate for synergy of form and uniform weld stress distribution



Mast / cable details

The mast detail demonstrates typical use of cable or strand sockets. A steel gusset plate usually provides the anchor for sockets. Equal angles A and B cause equal forces in strand and guy, respectively.

- A Mast / strand angle
- B Mast / guy angle
- C Strand
- D Guy
- E Sockets
- F Gusset plates
- G Bridge socket (to adjust prestress)
- H Foundation gusset (at strand and mast)
- I Mast



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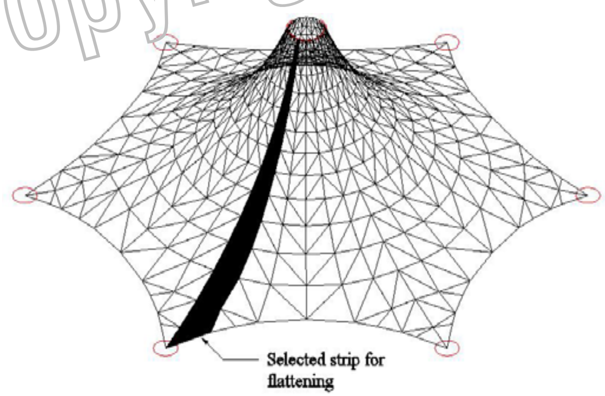

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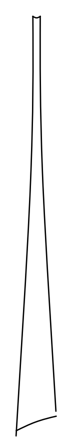

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Production process

Fabric pattern

To assume surface curvature, fabric must be cut into patterns which usually involve the following steps:

- Develop a computer model of strips representing the fabric width plus seems
- Transform the computer model strips into a triangular grids
- Develop 3-D triangular grids into flat two-dimensional patterns

The steps are visualized ad follows:

1 Computer model with fabric strips
2 Computer model with triangular grid
2 Fabric pattern developed from triangular grid

Pattern cutting

Cutting of patterns can be done manually of automatic.
The manual method requires drawing the computer plot on the fabric
The automatic method directs a cutting laser or knife from the computer plot

Note:
For radial patterns as shown at left, cutting two patterns from one strip, juxtaposing the wide and narrow ends, minimizes fabric waste.

Pattern joining

Fabric patterns are assembled by one of three methods:

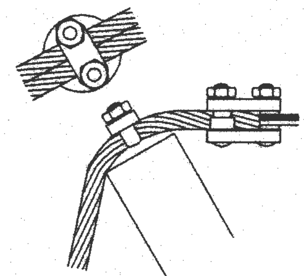
- Welding (most common)
- Sewing
- Gluing

Edge cables

Unless other boundaries are used, edge cables are added, either embedded in fabric sleeves or attached by means of lacing.

Fabric panels

For very large structures the fabric may consist of panels that are assembled in the field, usually by lacing. Laced joints are covered with fabric strips for waterproofing.



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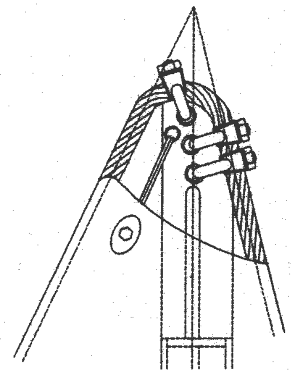

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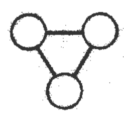

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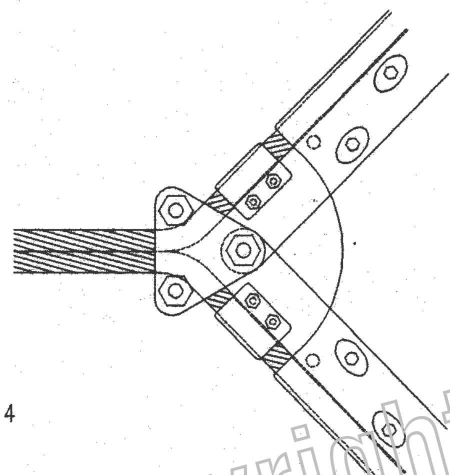

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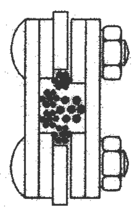

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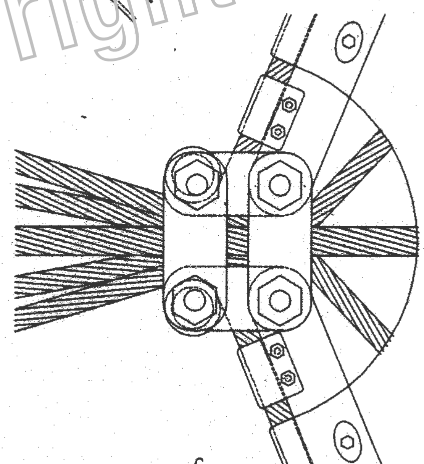


6

Cable/ fabric details (1 to 6 Frei Otto details)

1 Continued cable over mast
U-bolt connect cable to mast

2 mast top with continuing membrane edge cable
U-bolts connect cables to mast

3 Mast cross section
Three pipes joined by plate bars

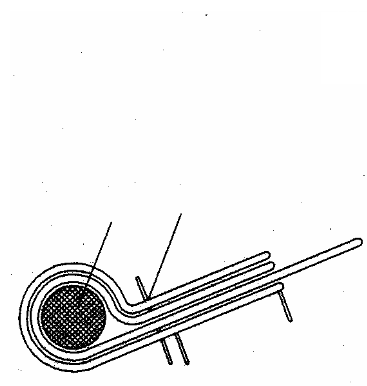
4 Edge cable/ fabric corner
Twin triangular plates join edge cables at fabric corner

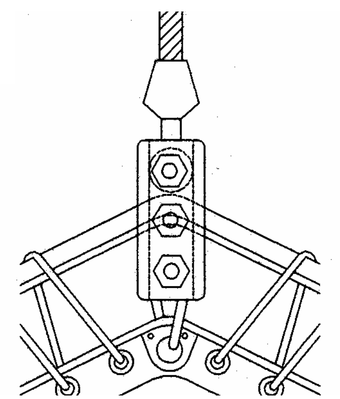
5 Cable clamp cross section

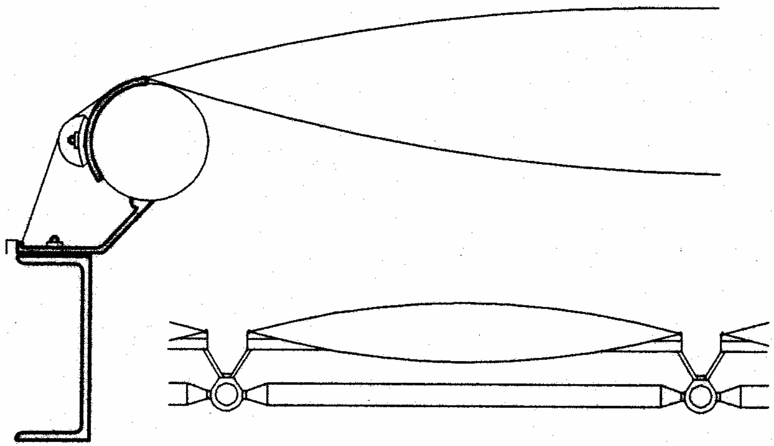
6 Fabric corner
Cable transfer at fabric corner

7 Edge cable/ membrane sleeve

8 Membrane laced to edge cable

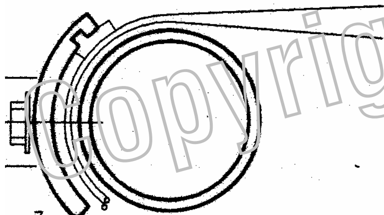






1 2

1, 2 Pneumatic cushions joint to space truss of Osaka Festival Plaza



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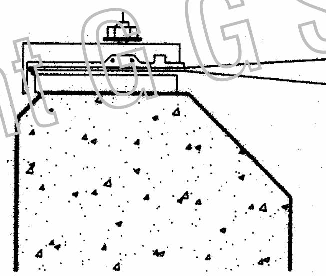

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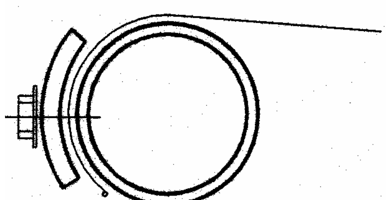
3 Pneumatic cushion connected to pipe with synthetic gasket

4 Pneumatic cushion attached to concrete with twin plates and synthetic gaskets



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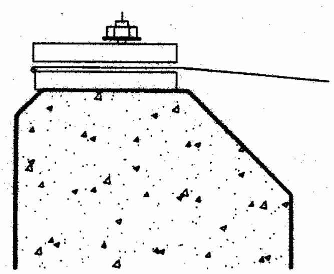

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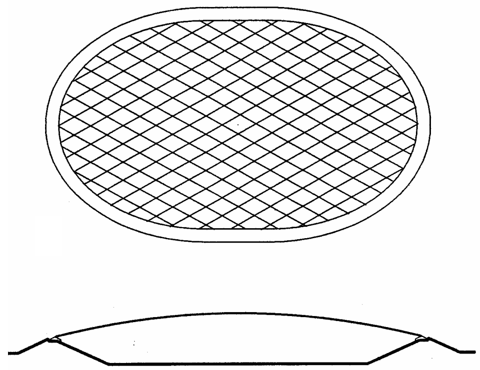
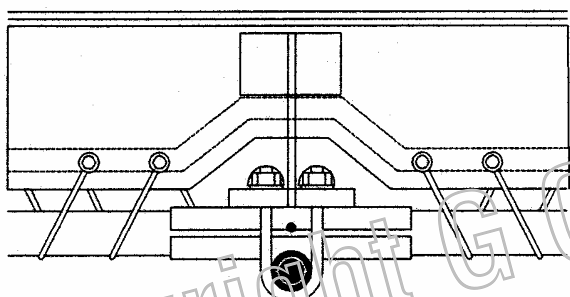
5 Membrane attached to pipe

6 Membrane attached to concrete with twin plates





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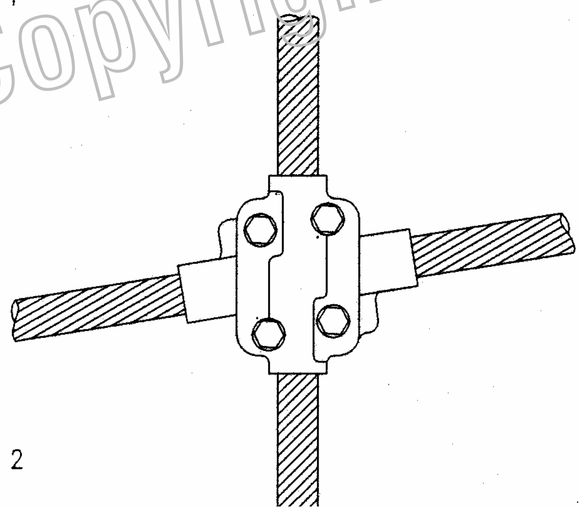


2

Projects

US Pavilion, Expo 70, Osaka
Architect: Davis, Brody, Chermayeff, Geismar, De Harak
Engineer: David Geiger

The pneumatic structure was supported by diagonal cables

1 Fabric / cable detail

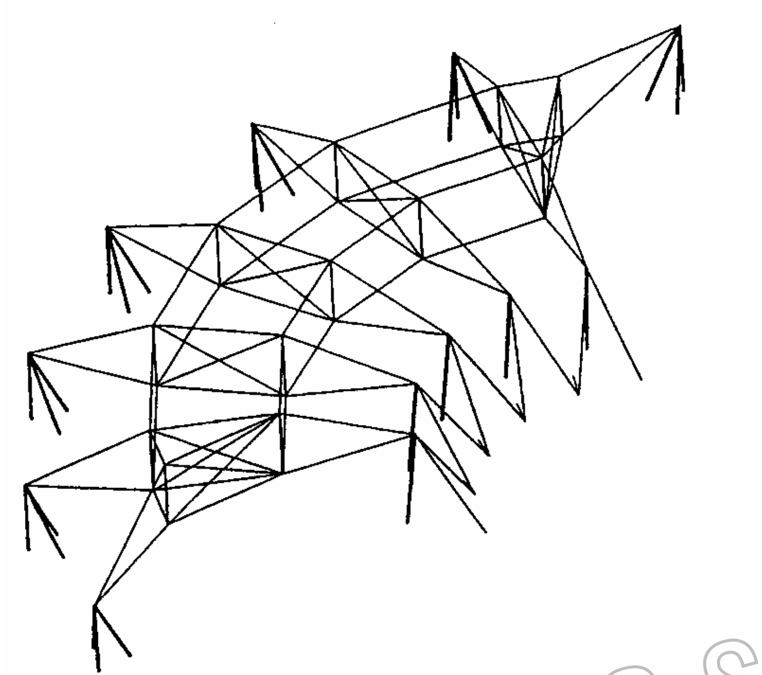
2 Cable crossing clamp

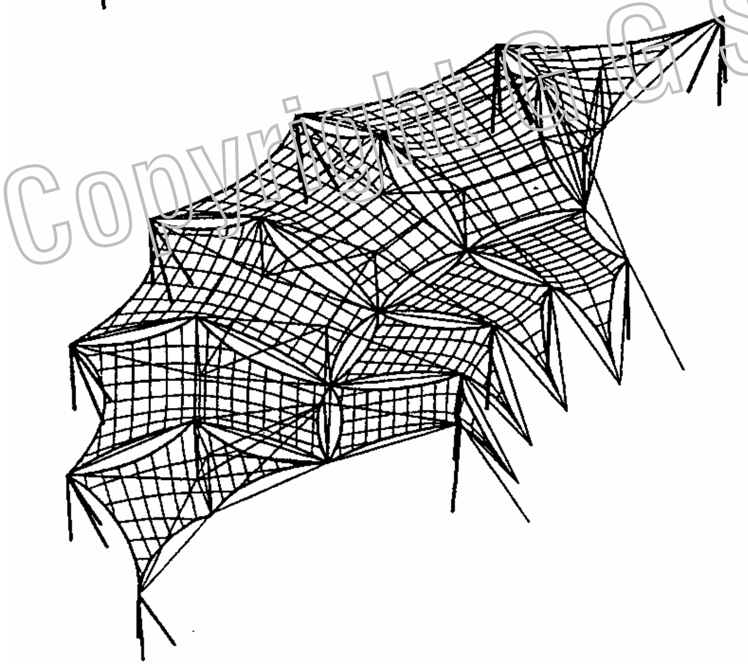
Watts Towers Canopy

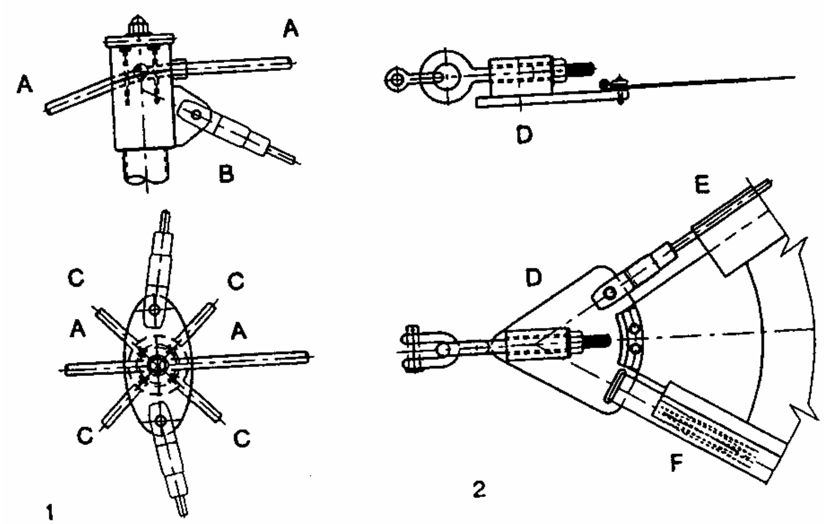
Architect: G G Schierle with Joe Addo
Engineer: ASI

A transparent membrane suspended from radial cable trusses is designed to provide sun shading for occasional performances at the Watts towers. The crescent-shaped roof follows the crescent-shaped seating below. The cable trusses minimize bulk for optimal view of the towers and facilitate fast erection and removal at annual events. The truss depth provides desired curvature for the anticlastic membrane panels. Two membranes provide shading for spectators and performers over the respective areas. The architectural design is shown below. The final computer drawings are shown at right.

1 Strut top
2 Fabric corner
A Top chord strand
B Diagonal strand
C Fabric attachment
D Metal plate at fabric corner, adjustable to induce prestress
E Edge cable
F Edge webbing

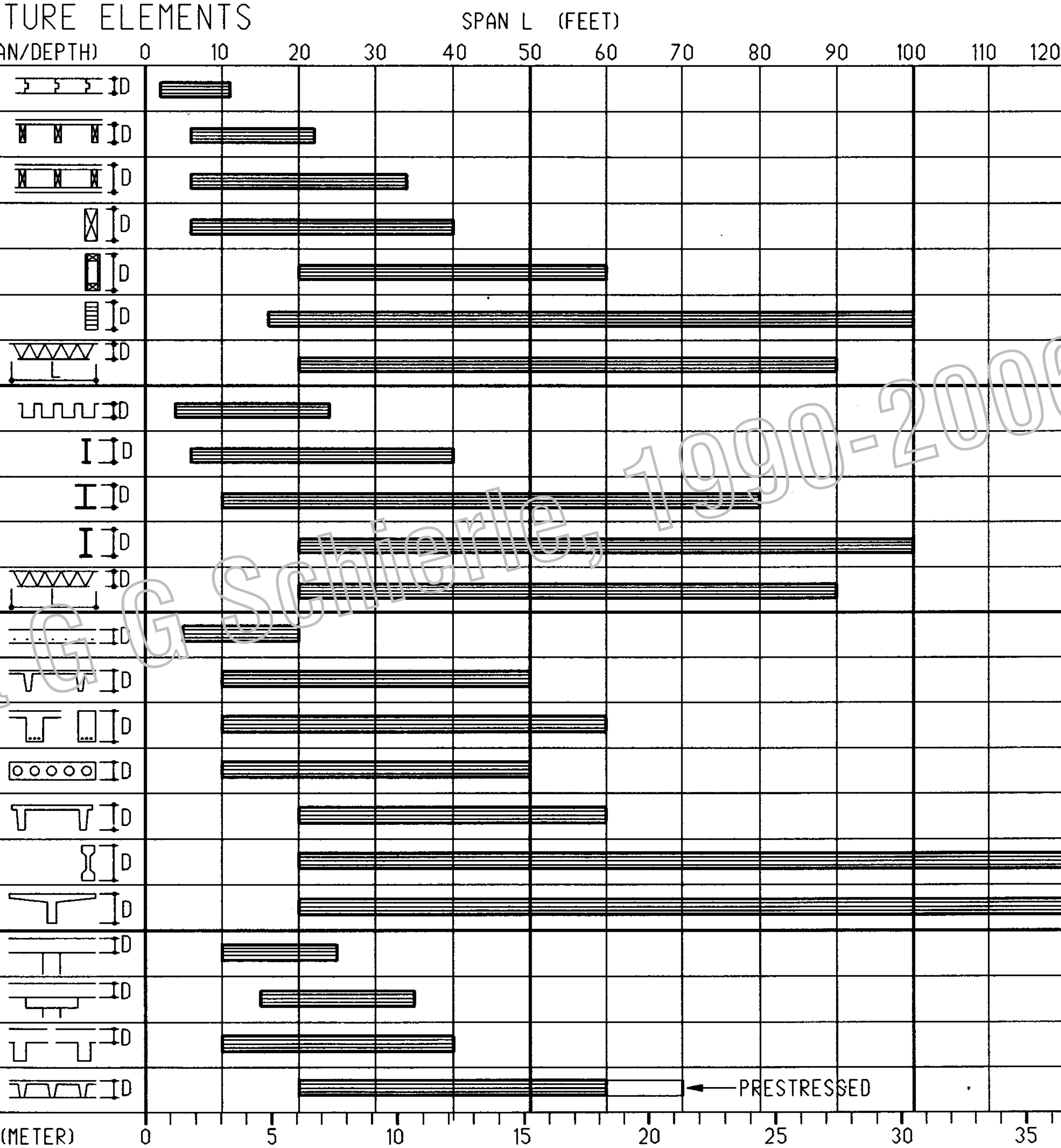






SPAN RANGE FOR STRUCTURE ELEMENTS

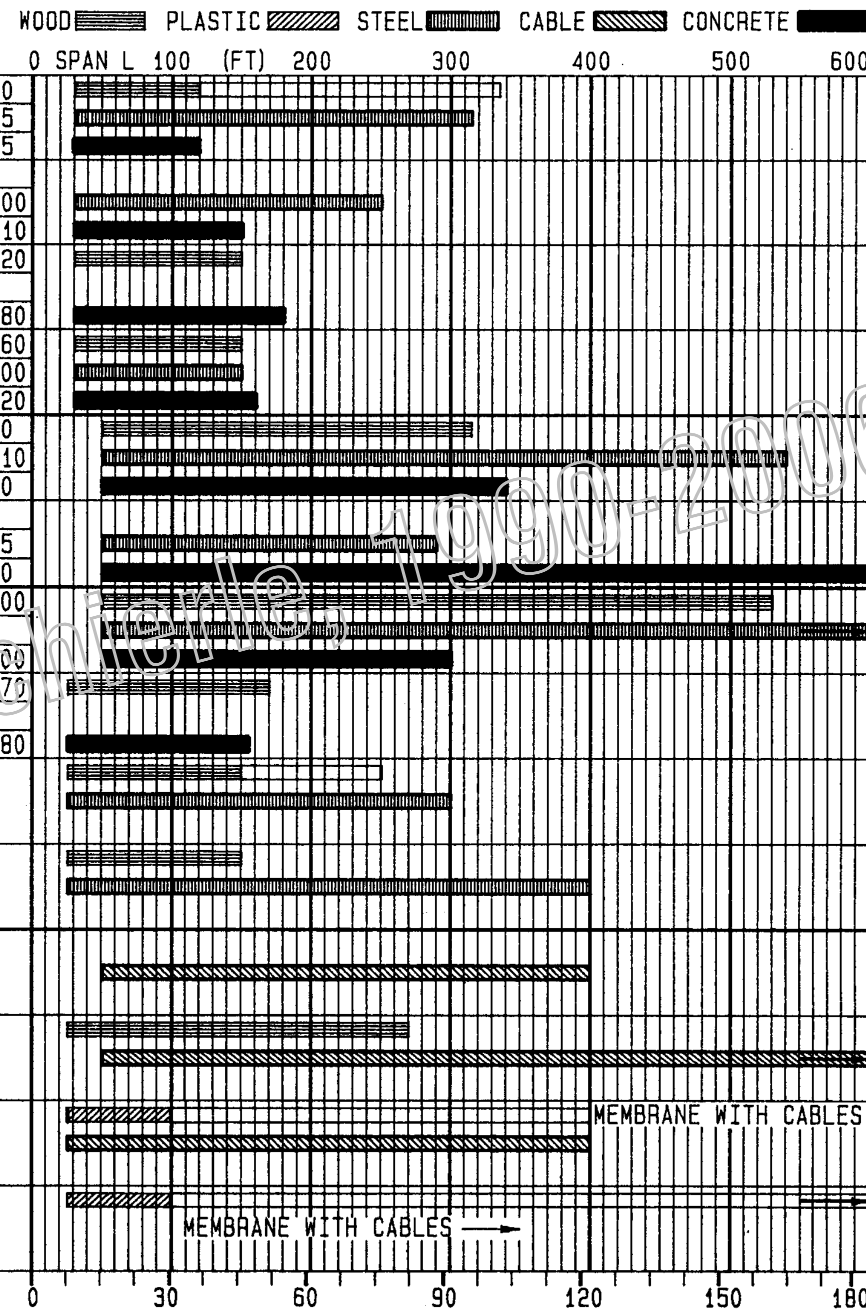
			L/D (SPAN/DEPTH)
WOOD	ONE-WAY	DECK	24-48
		JOIST	16-20
		STRESSED SKIN	20-40
		BEAM	16-24
		BOX BEAM	16-20
		LAMINATED BEAM	16-24
		TRUSS JOIST	10-20
STEEL	ONE-WAY	METAL DECK	30-50
		JOIST	16-24
		BEAM	16-24
		PLATE GIRDER	16-24
		TRUSS JOIST	12-24
CONCRETE	ONE-WAY	SLAB	24-36
		RIB SLAB	20-24
		BEAM	12-24
		PRECAST PLANK	25-40
		PRECAST CHANNEL	16-26
		PRECAST I	15-30
		PRECAST T	20-40
	TWO-WAY	FLAT PLATE	30-36
		FLAT SLAB	30-36
		SLAB ON 4 BEAMS	30-36
		WAFFLE SLAB	16-32



SPAN RANGE FOR STRUCTURE SYSTEMS

SPAN: 1=ONE WAY 2=TWO WAY

PRIMARY STRESS		SPAN	SYSTEM	L/D	L/T
BENDING		1	FRAME	3-6	20-40 15-35 20-25
BENDING		1 OR 2	VIERENDEEL FRAME	10-20	50-200 30-110
BENDING		1	CYLINDRICAL SHELL	10-16	100-120 180-480
BENDING		1	FOLDED PLATE	8-16	120-160 150-200 160-320
COMPRESSION		1	ARCH	3-6	40-50 100-110 40-50
COMPRESSION		1	VAULT	3-8	45-55 40-50
COMPRESSION/TENSION		2	DOME	4-8 2-8 4-8	180-300 30-50 120-600
COMPRESSION/TENSION		2	HYPAR SHELL	4-12	80-470 240-480
COMPRESSION/TENSION		1	TRUSS	8-16 10-20	
COMPRESSION/TENSION		2	SPACE TRUSS	10-18 10-24	
TENSION		1 OR 2	CABLE TRUSS	10-20	
TENSION		1 OR 2	SUSPENSION SYSTEM	8-20	
TENSION		2	SOFT SHELL	8-16	
TENSION		1 OR 2	PNEUMATIC	2-20	



About the book

Structures not only support gravity and other loads, but are essential to define form and space. To design structures in synergy with form and space requires creativity and an informed intuition of structural principles. The objective of this book is to introduce the principles as foundation of creative design and demonstrate successful application on many case studies from around the world. Richly illustrated, the book clarifies complex concepts without calculus yet also provides a more profound understanding for readers with an advanced background in mathematics. The book also includes structural details in wood, steel, masonry, concrete, and fabric to facilitate design of structures that are effective and elegant. Many graphs streamline complex tasks like column buckling or design for wind and seismic forces. The graphs also visualize critical issues and correlate US with metric SI units of measurement. These features make the book useful as reference book for professional architects and civil engineers as well as a text book for architectural and civil engineering education. The book has 613 pages in 24 chapters. http://www.usc.edu/structures

About the author

Professor Schierle, FAIA, has PhD and Master of Architecture degrees from UC Berkeley and a Dipl-Ing degree from Stuttgart, Germany. He was founding Director of USC's Graduate Program of Building Science and teaches structures at the USC School of Architecture. Prior to USC he taught at UC Berkeley and the Stanford University. He has been Visiting Professor at UCLA and EPFL Lausanne, lectured at AIA National Conventions and these universities: Arizona, Carnegie-Mellon, Harvard, MIT, Utah, Braunschweig, Delft, EPFL, Stuttgart; Mexico; and Sydney. He has received several grants from the National Science Foundation, the Department of Housing and Urban Development, and FEMA for research on seismic safety. His research on lightweight structures and seismic safety is widely published. Dr. Schierle chaired the architectural license examination on structures and serves on the *Fabric Architecture* advisory board. He also served on the *Journal for Architectural Education* Editorial Board and the Fabric Structures Awards Jury. His architecture practice includes major projects in America, Asia, and Europe. http://www-rcf.usc.edu/~schierle

Structures in Architecture **G G Schierle**