Reservoir offset models for Radiocarbon calibration

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Abstract

The purpose of a reservoir offset is to enable the application of calibration data ($\mu(\theta)$, *e.g.* Stuiver *et al.* 1998) developed for one reservoir (primary reservoir) to CRA's from another (secondary reservoir). The usual approach has been to define the activity of the secondary reservoir as some form of constant offset (with error) from the primary reservoir (*e.g.* Stuiver and Braziunas 1993). In this case CRA's from a secondary reservoir are not independent. However, the standard procedure for incorporating offset error into calibrated distributions assumes that the CRA's from secondary reservoirs are independent (*e.g.* Stuiver and Reimer 1993), accordingly the calibrated distributions are incorrect. In many cases this calculation error will be insignificant, however the calculation error will be significant in some situations and approaches such as sample based Bayesian inference need to be adopted if a non independent reservoir offset is applied.

1 Introduction

Reservoir offsets are commonly applied in the calibration of conventional radiocarbon ages (CRA's) deriving from reservoirs for which primary calibration data are unavailable. An offset is itself a measurement and is published with an associated standard error. The contribution of this uncertainty to the final uncertainty in the calibrated date must be taken into account when that CRA is calibrated. Suppose K CRA's

$(y_i, \ i = 1, 2...K)$ are measured, with corresponding standard errors $(\sigma_i, \ i = 1, 2...K)$ and the offset for the reservoir in question has been measured to be δ with standard error σ_δ. Standard practice is to combine the error in the CRA with the error in the offset to produce a single number σ_i' taking into account the two sources of error, *i.e.*

$$\sigma_i' = \sqrt{\sigma_i^2 + \sigma_\delta^2} \tag{1}$$

Surprisingly enough this is not the correct calculation to do. Let R denote the unknown true offset for the reservoir in question. The problem is that the formula above treats R as a quantity which varies independently and normaly from one CRA to the next, with mean equal to the observed offset δ and standard deviation σ_δ. This is not how we imagine the offset behaving. It is indeed assumed to have a normal distribution about δ, but nature has applied exactly the same offset R to each CRA in the R-reservoir. There may be some merit in treating the offset as a quantity that varies from one CRA to another, even within a single reservoir, but this in not in any case the correct way to quantify that variation.

In order to correctly incorporate reservoir offset error into calibrated distributions it is necessary to identify an appropriate observation model.

2   Observation models

Let θ_i denote the calibrated age parameter associated with the ith CRA. The observation model (model 1) which we believe is intended for CRA's $(y_i, \ i = 1, 2 \ldots K)$ from the R-reservoir is

$$y_i = \mu(\theta_i) + \epsilon_i + \epsilon(\theta_i) + R \tag{2}$$
$$\epsilon_i \sim N(0, \sigma_i))$$
$$\epsilon(\theta_i) \sim N(0, \sigma_\mu(\theta_i))$$
$$R \sim N(\delta, \sigma_\delta)$$

According to this model the process mapping a calibrated date θ_i to a CRA y_i is as follows: first there is the deterministic calibration $\mu(\theta)$; this is offset by R to take us from the calibration reservoir to the R-reservoir; to this we add a normal random quantity $\epsilon(\theta_i)$ arising from uncertainty $\sigma_\mu(\theta_i)$ in the calibration curve μ; the CRA measurement yielding y_i adds another normal random quantity ϵ_i. We do not know the values of the quantities R, $\epsilon(\theta_i)$ and ϵ_i, but we do know, or can at least model, their distributions. Thus the random variation $\epsilon(\theta_i)$ arising from uncertainty in the calibration curve is treated a normal random variable with distribution $N(0, \sigma_\mu(\theta_i))$, that is, mean zero and standard deviation $\sigma_\mu(\theta_i)$. These formulae determine an explicit formula for the likelihood of θ_i given the data y_i.

In this case it is not possible to algebraically combine the model error terms to generate an error for y_i after Equation 1. However, the standard approach is to

combine the error terms as given in Equation 1. This calculation follows from an assumption that the CRA's are independent according to the following observation model (model 2).

$$y_i = \mu(\theta_i) + \epsilon_i + \epsilon(\theta_i) + R_i \qquad (3)$$
$$\epsilon_i \sim N(0, \sigma_i))$$
$$\epsilon(\theta_i) \sim N(0, \sigma(\theta_i)$$
$$R_i \sim N(\delta, \sigma_\delta)$$

where the reservoir offset is allowed to differ between each CRA.

Clearly error combination according to Equation 1 is at odds with the assumption that there is a constant reservoir offset (*i.e.* Equation 2). It is easy to visualise the problem. When the uncertainty in the offset is large, and the analysis is done in the usual way, with independent offsets as in model 2, each calibrated date moves independently over the range allowed by the uncertainty in the δ-measurement. However, when the analysis is performed under model 1 all the calibrated dates move as a group over their range: their CRA's suffer a uniform offset, and it is the uncertainty in that common offset leads to uncertainty in the age of the dates as a group. In that case model 2 leads to a greater span in the values of the calibrated dates than model 1.

Unfortunately is is not straightforward to take proper account of the offset measurement uncertainty in model 1 without using simulation-based statistical analysis. However, software packages which implement simulation-based analysis are now in widespread use (*e.g.* BCal Buck, Christen, and James 1999;Oxcal Ramsey 1995;Datelab Jones and Nicholls 1999), and may easily be modified to take account of the issue we raise here.

3   Experiments

In practice the difference between the two methods is slight when the error term associated with the reservoir offset is small. However, where reservoir offset errors are large the differences can be significant. This can be demonstrated via a simple example consisting of two dates. Here we consider two dates (NZ-7755: BP, WK-2548: BP) which relate to the same reservoir and derive from the same archaeological stratum (Anderson, Smith, and Higham 1996). If we artificially vary the associated offset error it can be observed that the degree of correlation between the calibrated dates differs under the two models (Figure 1). In practice this exerts most influence on statistics such as the difference in age, or span, between the two dated samples (Figures 2). Under a constant offset model (model 1, Equation 2) the mean difference in age and distribution of span for the two samples is largely independent of the offset error. However, under an independent offset observation model (model 2, Equation 3) the mean difference in age and span is more strongly correlated with the size of the offset error.

Obviously the degree to which critical differences in statistics and calibrated distributions occur between constant and independent offset models varies depending upon the problem under consideration. While the magnitude of error at which a significant effect occurs in the example presented are not common, more extreme examples can be found. The most significant routine reservoir offset calculation errors are likely to occur with marine samples where reservoir offsets with larger errors occur (Stuiver and Braziunas 1993). It should also be noted that this has implications for the calculation of marine calibration data derived from terrestrial data.

4 Conclusion

In the case that an independant reservoir offset is assumed (*i.e.* Equation 3), the incorporation of offset error into calibration calculations using Equation 1 is appropriate. However, other approaches should be adopted if a constant or other non-independant reservoir offset is applied (*e.g.* Equation 2). The use of a constant offset that renders the observed CRA's non-independant can be calculated via approaches such as sampled based bayesian inference where the appropriate observation model is applied.

Most available calibration software packages only implement reservoir offsets according to the observation model given in Equation 3, and accordingly may not be appropriate for calibration where a constant reservoir offset is assumed. It is currently possible to calibrate CRA's according to the observation models given in both Equations 2 and 3 with the Datelab analysis package (Jones and Nicholls 1999,n.d.;www.car.auckland.ac.nz). Further technical details may be found in (Nicholls and Jones 1998).

Other calibrated likelihood models can be developed to incorporate factors such as temporal variance in the reservoir offset. However, all models other than the one assuming independence require analysis via a method such as sample based Bayesian inference.

A similar issue to the one discussed here appears in the treatment of the standard errors in the calibration curve itself. The problem is that these errors are correlated along the curve. This is considered in (Christen and Nicholls 2000) where it is shown that the effect will be unimportant except possibly for high precision measurements in which several dates have very nearly equal calibrated values.

Acknowledgements

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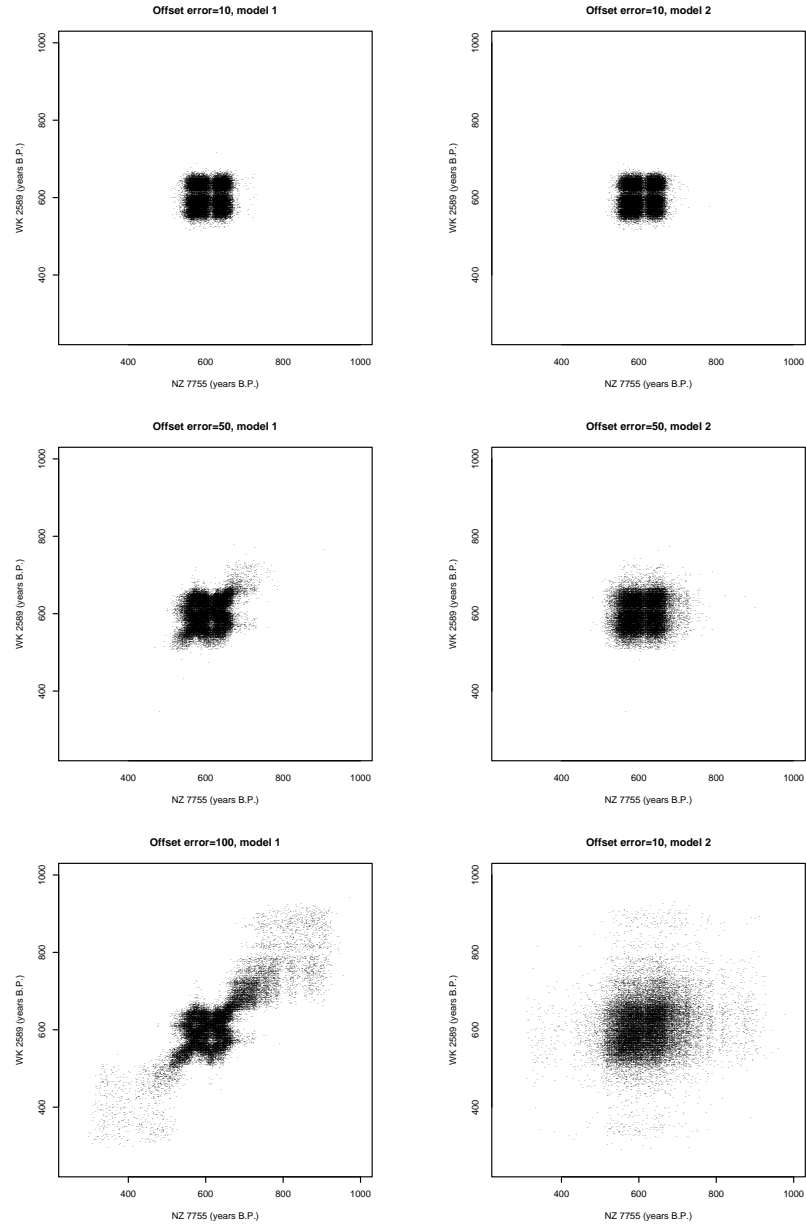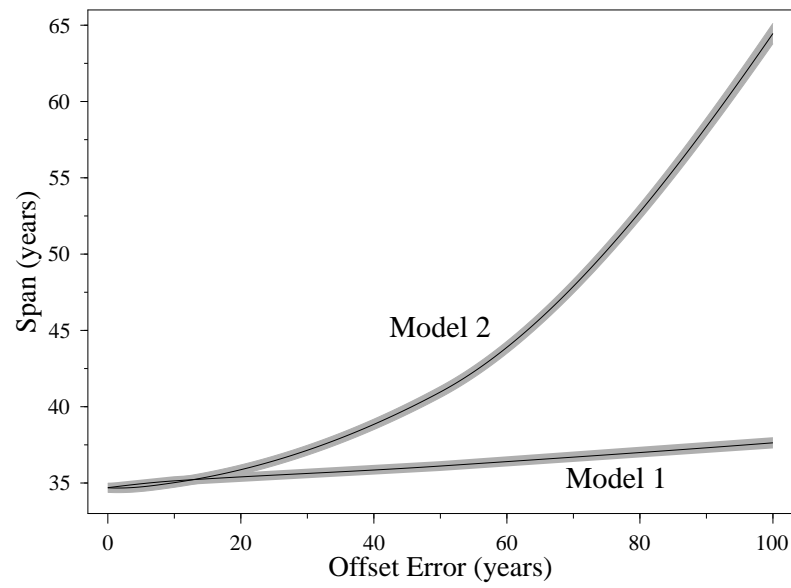
Figure 1: Correlation Plots of NZ 7755 vs WK 2546

Figure 2: Span as a function of offset error under model 1 and model 2