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Model Order Reduction Using Routh Approximation and Cuckoo Search Algorithm

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Abstract In this paper, a large order system is reduced by using the Cuckoo Search Algorithm (CSA) to a reduced order approximate model. The denominator coefficients of a desired reduced order system are determined by Routh approximation method while the numerator coefficients are determined using CSA based on integral square error minimization as an objective function pertaining to a unit step as input. The efficacy of the proposed method is tested with three SISO test systems to get a corresponding reduced order system and extended to a MIMO system. The results are satisfactory in terms of minimum error with the proposed method as compared to Routh Pade approximation and weighted sum multi-objective harmony search based reduced models.

Keywords: *ISE minimization approach, large scale systems, Model order reduction, Optimization using Cuckoo search algorithm, Routh approximation*

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1. Introduction

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A system is a large scale when its dimensions are so high, such that conventional techniques of modeling, analysis control, design and computation fail to give accurate solutions with reasonable computational efforts. Thus, Model reduction of a high order system is an important problem in analysis, as well as in controller synthesis of a practical system [1]. To reduce large system to its low-order many techniques are available in literature using time domain and frequency domain methods [2,3,4,5].

In a frequency domain, one important order reduction method is Pade approximation reported in [4,6,7]. It is reported that the Pade approximation method does not guarantee the stability of the reduced-order model; therefore, many other alternative methods, such as the Mihailov stability criterion [8], Routh approximation [6,9], stability equation method [10,11,12], Routh stability array method [11,13] and Hurwitz polynomial approximation [14] have been reported time to time. Shamash [15] have considered the effect of including Markov parameters along with time moments to ensure the initial time response. A modified version Routh Pade approximation method is reported in [3]. The stability equation method application is presented in [16] and Routh array method as in [17]. Stability preservation using differentiation method is presented in [18] and extended to MIMO systems in [19].

Soloklo and Farasangi in 2013 [1], have introduced the multiobjective weighted sum approach for model reduction by Routh Pade approximation using Harmony search algorithm. The advantage of stability equation method along with genetic algorithm (GA) is used to reduce the order of a system in [20]. A 10^{th} order two input and two output linear time invariant power system model is reduced using PSO in [21]. Other evolutionary methods based reduced order model is reported in [22] , H∞ Model Reduction in [23,24,25] and Hankel singular based in [26,27]. Moreover, the application of model order reduction methods has been applied in the field of power system [9,28,29,30]. The application of bat algorithm for model order reduction is presented in [31].

The organizations of the contents are as problem formulation in section 2, review on Cuckoo Search algorithm and the modifications made in it are included in section 3, the test systems considered for the reduction and comparison to existing reduced models are incorporated in section 4 and the concluding remarks are made in section 5, followed by references.

2. Problem Formulation

Consider a higher order system of order n and is represented by equation (1).

$$G(s)=\frac{Y(s)}{R(s)}=\frac{a_1 s^{n-1}+a_2 s^{n-2}+...+a_n}{s^n+b_1 s^{n-1}+b_2 s^{n-2}+...+b_n} \quad (1)$$

Where, a_i and b_i are constants for $i=1,2,\dots,n$.

If r represents a reduced order as of lesser order than n, then, the reduced order model of the system in equation (1) is represented as in equation (2). The important and principal requirement of the reduced order model is to posses all important specifications of the original system.

$$G_r(s) = \frac{Y_r(s)}{R_r(s)} = \frac{c_1 s^{r-1} + c_2 s^{r-2} + \ldots + c_r}{s^r + d_1 s^{r-1} + d_2 s^{r-2} + \ldots + d_r} \quad (2)$$

Where, c_r and d_r are unknown constants and are to be determined by using cuckoo search algorithm subjected to minimization of integral square error defined in equation (3). Error denotes difference of the unit step responses by original system and the reduced model to a unit-step function as shown in Figure 1.

$$J = \int_{t=0}^{t=\infty} |e(t)|^2 dt \quad (3)$$

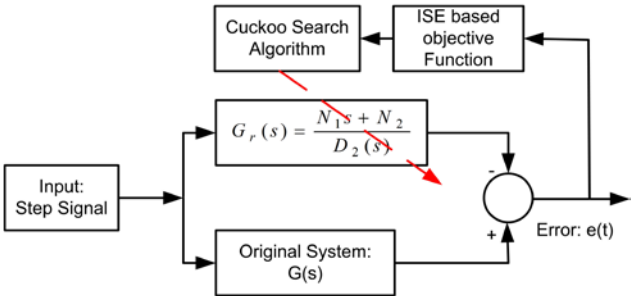

Figure 1. Scheme for optimization of free coefficients of reduced order model

3. Cuckoo Search Algorithm

The literature on cuckoo search is expanding rapidly. There has been a lot of attention and recent studies using cuckoo search with a diverse range of applications. Walton et *al*. improved the algorithm by formulating a modified cuckoo search algorithm [32], while Yang and Deb extended it to multi-objective optimization problems [33].

As the Cuckoos lay their eggs in the nest of other birds and respective host birds take care of the cuckoo's chicks [34]. It is mainly inspired by the obligate brood parasitism of cuckoos by laying their eggs in the nests of other host birds. The infringing cuckoos are in direct contest with the host birds. The host bird discovers the eggs of other birds and may throw these out of nest or may construct another nest elsewhere. The Parasitic cuckoos generally selects a nest in which the host bird just laid its own eggs [34]. The Cuckoo eggs generally hatch somewhat earlier than their host eggs [35]. As soon as, cuckoo chick is hatched starts to evict y blindly propelling the eggs out of the nest to reduce the share of food. Cuckoo chick starts to mimic the voice call of host chicks to gain more opportunity of feeding [34].

An algorithm provides a set of output variables on application of input variables. An optimization algorithm generates/produces a new set of solution x^{t+1} to a given problem from a given solution x^t at time t or iteration.

$$x^{t+1} = A\{x^t, p(t)\} \quad (4)$$

Where, the new solution vector x^{t+1} is nonlinearly mapped through A to given d-dimensional vector x^t. Let the variables of the problem are k and are represented as $p(t) = p_1, p_2, \ldots, p_k$ which may be time dependent and can be tuned by A. Let an optimization problem is S with states as ψ then according to predefine criterion D, the optimal solution x_{os} selects the desired states as φ as in equation (5).

$$S(\psi) \xrightarrow{A(t)} S\{\varphi(x_{os})\} \quad (5)$$

Thus, the final found/converged state φ represents to an optimal solution x_{os} of the problem of interest. Here, the system states are selected in the design space by running the optimization algorithm A. Thus, the performance of the algorithm is depended /controlled by the initial solution $x^{t=0}$, the parameters p, and stopping criterion *D*.

3.1. Procedural Steps

The cuckoo search algorithm is based on the brood parasitism of some cuckoos such as the ani and Guira and is enhanced by use of *Levy flights* [36], not just by simple isotropic random walks. The Cuckoos are special birds not only because of the beautiful sounds but also because of their aggressive reproduction strategy. Cuckoos engage the obligate brood parasitism by laying their eggs in the nests of other host birds. The ani and Guira as the species of cuckoos used to lay their eggs in other bird's nests and they may remove others' eggs to increase the *hatching probability* of their own eggs. It is necessary to make assumptions as followings:

Assumptions

- At a time each cuckoo lays one egg and dumps it in a randomly selected nest
- The nests with high-quality eggs are selected and being carried over to the next generations
- The available number of nests (of hosts) is kept fixed (as n), and the probability of cuckoo egg detection by the host bird is fixed as $P_a \in [0,1]$. As above, the host bird may get rid of the egg or may even abandon the nest to build a new nest i.e. a fraction *Pa* of the *n* host nests that are replaced by new nests [34].

Further, as an implementation, it should be assumed that the solution refers to an egg in a nest, and each cuckoo can lay only one egg. Thus, there is no distinction between cuckoo, egg or nest because as each nest consists one egg which corresponds to one cuckoo. CS algorithm uses a combination of a local random walk (for local search) and the global random walk (for global search) and is controlled by a switching parameter P_a.

3.1.1. Local Random Walk

Let two different solutions selected by random permutation are as x_j^t and x_k^t, Heaviside function as $H(P_a - \in)$, random number drawn from a uniform distribution as $\in$, and the step size as '**s**'. Then, the local random walk can be represented as.

$$x_i^{t+1} = x_i^t + \alpha\, s \otimes H(P_a - \in) \otimes (x_j^t - x_k^t) \quad (6)$$

Here, $\alpha > 0$ is the step size related to the scales of the problem of interests. It is generally selected as $\alpha = 1$.The product " $\oplus$ " means entry-wise walk during multiplications.

3.1.2. Global Random Walk

The global random walk is carried out by using Levy flights in which the step-lengths are distributed according to a heavy-tailed probability distribution [34]. On completion of large number of steps the random walk tends to a stable distribution as compared to its origin. The final solution can be represented by equation (7) as following.

$$x_i^{t+1} = x_i^t + \alpha\, L(s, \lambda) \quad (7)$$

Where,

$$L(s,\lambda) = \frac{\lambda \Gamma(\lambda) \sin(\pi\lambda/2)}{\pi} \frac{1}{s^{1+\lambda}} \quad (8)$$
$$(s >> s_0 > 0)$$

The equation (7) is the stochastic representation for a random walk. The random walk is a Markov chain; whose next location directly depends on the current location and the transition probability. An appropriate value of new solutions generated by randomization and their locations should be far enough from the best solution (current) to make sure not be trapped in a local optimum [35,37]. The local search exists about to 1/4 of the search time (with Pa = 0.25), while global search exists for 3/4 of the total search time.

3.1.3. Levy Distribution

Levy flights are characterized by infinite mean and variance therefore, CSA can explore the search space more efficiently as compared to standard Gaussian process. Thus, CSA guaranteed global convergence and highly efficient [37,38,39].

In Lévy flight the step-lengths are distributed according to the probability distribution as in equation (9), which provides a random walk while the random step length is drawn from a Levy distribution [35].

$$Levy(u) = t^{-\lambda}, (1 < \lambda \le 3) \quad (9)$$

3.1.4. Improved Cuckoo Search

As above the α introduced in the CSA is to find locally improved solutions, while P_a and λ to find global solution. P_a and α parameters play a vital role in tuning of solution vectors. In original CSA, P_a and α are kept fixed and cannot be altered during new generations, therefore, the number of iterations kept large to get optimal solutions. With large value of P_a and small value of α, the convergence speed is high but unable to find required solutions. To mitigate the problem of adjusting the value of P_a and α, these are considered as variables in improved CSA. The values of P_a and α must be large enough to make capable the algorithm to increase the diversity of solution vectors during early generations and decreased in final generations to result in a better fine-tuning of solution vectors [34]. Thus, P_a and α are dynamically changed with the number of generation and expressed in equations (10) – (12), where *NI* and *gn* are the number of total iterations and the current iteration, respectively [34].

$$P_a^{gn} = P_{a\max} - \frac{(P_{a\max} - P_{a\min})}{NI}.\, gn \quad (10)$$

$$\alpha\,(gn) = \alpha_{\max}.\exp\,(c.gn) \quad (11)$$

$$c = \frac{Ln\left(\alpha_{\min} / \alpha_{\max}\right)}{NI} \quad (12)$$

The performance of the algorithm may deteriorate by an increase in the maximum value of α as in [34], therefore, the suitable values are $0.005 \le P_a \le 1.0$ and $0.05 \le \alpha \le 0.5$. The considered values of P_a and α are 0.25 and 0.25, respectively.

4. Results and Discussions

4.1. Example-1: SISO System

Considering a fourth order single input and single output (SISO) system [20,40,41] and is described by the transfer function as in equation (13). The second order ROM as proposed is represented by equation (14) with free coefficients as N_1 and N_2. The range of these free coefficients are considered as $0.1 \le N_1 \le 1.0$ and $0.1 \le N_2 \le 2.0$, thus the lower and upper bounds are defined in CSA accordingly in vector format. The other necessary parameters of CSA as number of nests i.e. different solutions (*n*) as 25, discovery rate of alien eggs (P_a) as 0.25 and maximum iterations as 200 are selected to compute the free coefficients of equation (14). The computed coefficients N_1 and N_2 are obtained as 0.8130 and 0.7945, respectively and resulting equation (15) and the variation of these free coefficients for 200 iterations by CSA is shown in Figures 2(b) - 2(c). The performance of CSA during coefficient computation in terms of fitness function value for 200 iterations is shown in Figure 2(a).

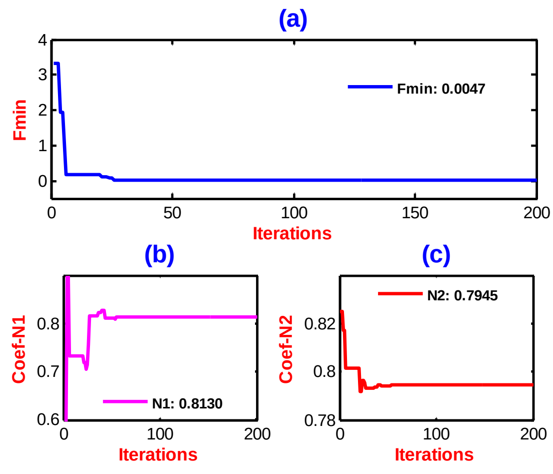

Figure 2. Performance of Cuckoo search algorithm for test system - 1 for (a) Plot of fitness function, (b) Variation of free coefficient N_1, and (c) Variation of free coefficient N_2

The reduced second order model by Desai [41] using Routh approximation and Big Bang Big Crunch is represented by equation (16). In this method the numerator coefficients are determined by Big Bang Big Crunch algorithm and the denominator elements are determined by Routh approximation. The original system is reduced to second order, where in [20], the denominator is reduced using stability equation method and the numerator coefficients are found using Genetic algorithm and represented in equation (17). Another approach proposed by Boby and Philip in [42], where in denominator found by dominant pole method and the numerator by Big Bang Big Crunch algorithm and represented by equation (18).

$$G(s)\Big|_{Original} = \frac{s^3+7s^2+24s+24}{s^4+10s^3+35s^2+50s+24} \quad (13)$$

$$G_r(s) = \frac{N_1 s + N_2}{s^2+1.6560s+0.7947} \quad (14)$$

$$G(s)\Big|_{\Pr oposed} = \frac{0.8130s+0.7945}{s^2+1.6560s+0.7947} \quad (15)$$

$$G(s)\Big|_{Desai} = \frac{0.8058s+0.7944}{s^2+1.65s+0.7944} \quad (16)$$

$$G(s)\Big|_{Parmar} = \frac{0.7442575s+0.6991576}{s^2+1.45771s+0.6997} \quad (17)$$

$$G(s)\Big|_{Boby} = \frac{0.9315s+1.6092}{s^2+2.75612s+1.6092} \quad (18)$$

Step response of original system, proposed reduced, and other published reduced models are compared in Figure 3 and other performance parameters as overshoot, rise time, settling time and performance indices such as integral time multiplied absolute error (ITAE), integral absolute error (IAE) and integral square error (ISE) are enlisted in Table 1 and defined in [43,44,45]. All three performance indices (PIs) for proposed reduced order model (ROM) are lesser in magnitude as compared to other published ROMs [20,40,41], proving superior performance. In Desai and Prasad, 2013[11], the 2nd order reduced model of the considered system in equation (13) is $(0.8s+0.686)/(s^2+1.47s+0.686)$ with ISE value 3.5×10^{-4} which is greater than the proposed 2.15×10^{-4}; proving effectiveness of the proposed methodology.

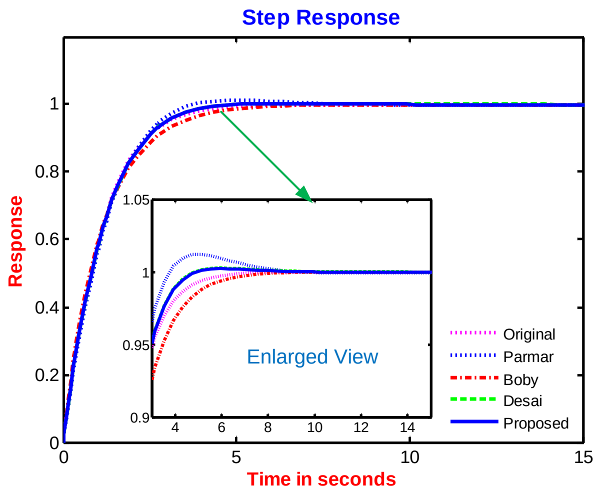

Figure 3. Step response of Original system, 2nd order reduced (Proposed) and other comparing reduced models

Table 1. Performance parameters of original test system -1 and reduced models (proposed ROM and as in [20,40,41])

Para-meters	Original [20,40,41]	Pro-posed	Desai [41]	Parmar [20]	Boby [42]
Over-shoot	0	0	0.2738	1.3007	0.2738
Rise Time	2.2602	2.2767	2.2787	2.189	2.2787
Settling Time	3.9307	3.6120	3.6199	3.222	3.6199
ISE	-	2.15E-4	2.17E-4	1.64E-3	1.7E-3
ITAE	-	0.1758	0.1770	0.5226	0.2575
IAE	-	0.0430	0.0447	0.1147	0.0933

4.2. Example-2: SISO System

Considering a test system of order six as in [1] and presented in equation (19). Considering second order reduced order model as in equation (20), where, N_1,and N_2 are the free parameters to be optimized by CSA while the denominator is obtained using Routh approximation as in [41,46]. The parametric bound as lower and upper value for CSA are considered as $0.1 \le N_1 \le 1$and $0.1 \le N_2 \le 1.0$ respectively. The performance of CSA in optimization for 200 iterations is shown in Figure 4. The fitness function variation over 200 iterations is shown in Figure 4(a), while the free coefficient variation is shown in Figure 4(b) for N_1 and Figure 4(b) for N_2. The resulting value of fitness function at completion of 200 iterations is 0.0791 and the values of N_1 and N_2 parameters are 0.0910 and 0.0109, respectively. Thus resulting 2nd order reduced model (proposed) is represented in equation (21).

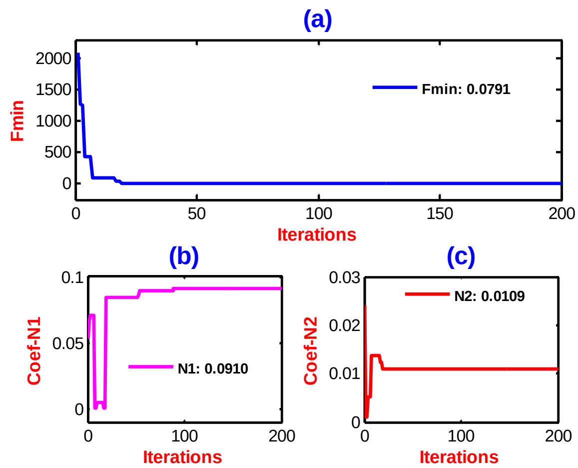

Figure 4. Performance of Cuckoo search algorithm for test system - 2 for (a) Plot of fitness function, (b) Variation of free coefficient N1, and (c) Variation of free coefficient N2

$$G(s)\Big|_{original} = \left[\frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1} \right] \quad (19)$$

$$G_r(s) = \frac{N_1 s + N_2}{s^2 + 0.2012s + 0.01099} \quad (20)$$

$$G(s)\Big|_{Proposed} = \left[\frac{0.0910s + 0.0109}{s^2 + 0.2012s + 0.01099} \right] \quad (21)$$

The other second order reduced models as typical Pade approximant [1] and Multi objective Harmony search algorithm [1] based are as in equation (22) and equation (23), respectively.

$$G(s)\Big|_{Pade} = \left[\frac{6.87815s + 1.09228}{89.54625s^2 + 12.96860s + 0.99732} \right] \quad (22)$$

$$G(s)\Big|_{Soloklo} = \left[\frac{7.80016s + 0.81849}{87.58712s^2 + 12.43314s + 0.81657} \right] \quad (23)$$

Table 2. Performance parameters of original test system-2 and reduced models

Para-meters	Original [1]	Proposed	Pade [1]	Soloklo[1]
Over-shoot	0	0.0193	7.46	7.61
Rise Time	22.7	21.4609	15.1	14.3
Settling Time	40	36.0165	50.4	53
ITAE	-	30.4949	585.0442	138.72
IAE	-	0.5987	12.7028	4.906
ISE	-	0.0052	2.8887	0.5377

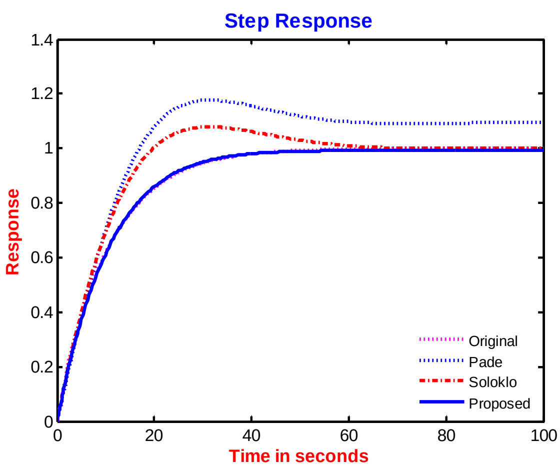

Figure 5. Step response of Original system, 2nd order reduced (Proposed) and other comparing reduced models as in [1]

The Step response of Original system, 2nd order reduced (Proposed) and other comparing reduced models of [1] as in equation (19) and equations (21) - (23) is shown in Figure 5. The step response information and PIs based performance is summarized in Table 2, where in, the proposed ROM possesses least value of the PI values resulting to superior performance.

4.3. Example-3: SISO System

Considering a test system as boiler system of order nine as in [47] and presented in equation (24). Here, the desired third order reduced model is represented by equation (25). The parameter bounds for free coefficients in CSA are as $120 \le N_1 \le 160$, $4000 \le N_2 \le 4500$ and $4400 \le N_3 \le 5000$ for 200 iterations and the denominator is determined by typical Routh approximation as in [41,46]. The performance of Cuckoo Search algorithm for determining free coefficients is shown in Figure 6(a) in terms of fitness function plot and coefficients in Figures 6(b) - 6(d). Using optimized parameters, the third order reduced model by CSA is shown in equation (26). The other third order model as in [1] as typical Pade and HSA based multi-objective are represented in equations (27) - (28), respectively.

The step signal based response of these systems is shown in Figure 7. The step response by proposed method is able to mimic the original system more appropriately as compared to others. The other comparing parameters as overshoot, rise time, settling time and performance indices as ISE (Integral square error), ITAE (Integral time multiplied absolute error) and IAE (Integral absolute error) of these step responses are determined and enlisted in Table 3. The performance parameters with proposed reduced model in terms of ISE, ITAE and IAE are having least values as compared to other methods, proving superior performance.

$$G(s) = \left[\frac{146.4s^8 + 9.81\times10^4 s^7 + 5.999\times10^7 s^6 + 3.206\times10^{10} s^5 + 3.582\times10^{12} s^4 + 1.113\times10^{14} s^3 + 1.154\times10^{15} s^2 + 3.971\times10^{15} s + 3.063\times10^{15}}{s^9 + 659.8 s^8 + 4.136\times10^5 s^7 + 2.13\times10^8 s^6 + 2.422\times10^{10} s^5 + 8.737\times10^{11} s^4 + 1.523\times10^{13} s^3 + 1.221\times10^{14} s^2 + 3.636\times10^{14} s + 2.406\times10^{14}} \right] \quad (24)$$

$$G_r(s) = \left[\frac{N_1 s^2 + N_2 s + N_3}{s^3 + 29.8634s^2 + 399.8925s + 355.9565} \right] \quad (25)$$

$$G_r(s) = \left[\frac{144.50s^2 + 4119.30s + 4531.50}{s^3 + 29.8634s^2 + 399.8925s + 355.9565} \right] \quad (26)$$

$$G(s)\big|_{Pade} = \left[\frac{145.36242\,s^2 + 431231031\,s + 4701.85734}{s^3 + 23.23900\;s^2 + 420.38264\,s + 371.29177}\right] \quad (27)$$

$$G(s)\big|_{Soloklo} = \left[\frac{148.12856\,s^2 + 4398.96963\,s + 4725.72521}{s^3 + 29.90996\,s^2 + 429.17178\,s + 371.99085}\right] \quad (28)$$

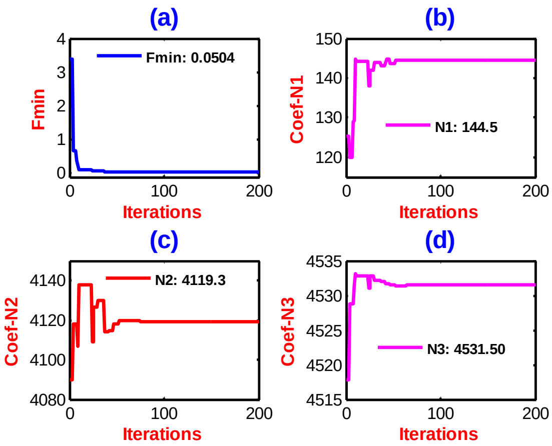

Figure 6. Performance of Cuckoo search algorithm for test system - 3 for (a) Plot of fitness function, (b) Variation of free coefficient N_1, (c) Variation of free coefficient N_2, and (c) Variation of free coefficient N_3

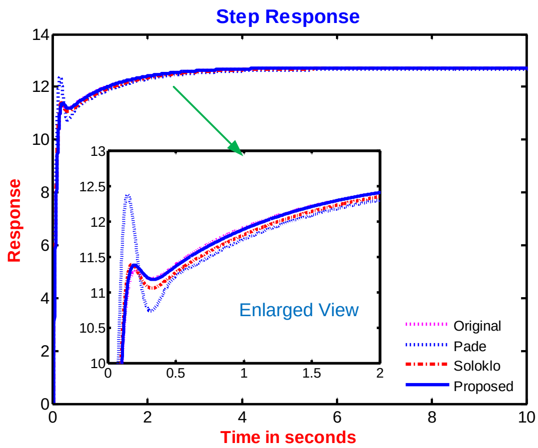

Figure 7. Step response of Original system, 2nd order reduced (Proposed) and other comparing reduced models as in [1]

Table 3. Performance parameters of original and reduced models

Para-meters	Original [47]	Proposed	Pade [1]	Soloklo [1]
Over-shoot	0	0	0	0
Rise Time	0.543	0.5613	0.0918	0.612
Settling Time	2.28	2.2520	2.39	2.36
ITAE	-	0.2374	53.9939	21.622
IAE	-	0.0719	3.0904	1.2432
ISE	-	0.0027	0.3561	0.0269

4.4. Example-4: MIMO system

Let us consider a Multi-Input Multi-Output (MIMO) system as in [41,48] of sixth order having the following transfer matrix as in equations (29) - (30). The system in sub-system format is represented in equations (31) - (34). The proposed second order reduced model is represented by equation (35).

$$G(s) = \begin{bmatrix} \dfrac{2(s+5)}{(s+1)(s+10)} & \dfrac{(s+4)}{(s+2)(s+5)} \\ \dfrac{(s+10)}{(s+1)(s+20)} & \dfrac{(s+6)}{(s+2)(s+3)} \end{bmatrix} \quad (29)$$

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \quad (30)$$

$$G_{11}(s) = \left[\frac{2s^5 + 70s^4 + 762s^3 + 3610s^2 + 7700s + 6000}{s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000}\right] \quad (31)$$

$$G_{12}(s) = \left[\frac{s^5 + 38s^4 + 459s^3 + 2182s^2 + 4160s + 2400}{s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000}\right] \quad (32)$$

$$G_{21}(s) = \left[\frac{s^5 + 30s^4 + 331s^3 + 1650s^2 + 3700s + 3000}{s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000}\right] \quad (33)$$

$$G_{22}(s) = \left[\frac{s^5 + 42s^4 + 601s^3 + 3660s^2 + 9100s + 6000}{s^6 + 41s^5 + 571s^4 + 3491s^3 + 10060s^2 + 13100s + 6000}\right] \quad (34)$$

$$G(s) = \frac{\begin{bmatrix} N_{11}(s) & N_{12}(s) \\ N_{21}(s) & N_{22}(s) \end{bmatrix}}{D_2(s)} \quad (35)$$

The 6th order denominator of the system is reduced to 2nd order model by applying Routh approximation method as in [41] and is being represented by equation (36).

$$D_2(s) = s^2 + 1.5480\; s + 0.7091 \quad (36)$$

Since, the denominator of equations (31) - (34) is same; therefore, the denominator would remain same. The numerators are different, would be determined by using Cuckoo search algorithm. The general form of numerator with free coefficient can be represented as N_1s+N_2. The performance of CSA in terms of fitness function is shown in Figure 8 and the plot of the free coefficients for $N_{11}(s)$, $N_{12}(s)$, and $N_{21}(s)$ and $N_{22}(s)$ is shown in Figure 9. The optimized numerators are shown in equation (37) - (40) and the resulting reduced MIMO system is represented by equation (41).

$$N_{11}(s) = 0.0672\; s + 0.7086 \quad (37)$$

$$N_{12}(s) = 0.5223\; s + 0.2831 \quad (38)$$

$$N_{21}(s) = 0.4552\ s + 0.3543 \quad (39)$$

$$N_{22}(s) = 1.1398\ s + 0.7083 \quad (40)$$

$$G_2(s) = \frac{\begin{bmatrix} 0.0672\ s + 0.7086 & 0.5223\ s + 0.2831 \\ 0.4552\ s + 0.3543 & 1.1398\ s + 0.7083 \end{bmatrix}}{s^2 + 1.5480s + 0.7091} \quad (41)$$

The reduced second order transfer matrix by Prasad [48] is given as in equation (42).

$$R_2(s) = \frac{\begin{bmatrix} 1.18156s + 3.65079 & 1.04664s + 1.46031 \\ 0.49819s + 1.82539 & 1.6911s + 3.65079 \end{bmatrix}}{s^2 + 4.3374s + 3.65079} \quad (42)$$

The reduced second order transfer matrix by Desai [41] is given as in equation (43).

$$R_2(s) = \frac{\begin{bmatrix} 0.9475s + 0.7091 & 0.4892s + 0.2837 \\ 0.455s + 0.3546 & 1.126s + 0.7091 \end{bmatrix}}{s^2 + 1.548267s + 0.7091} \quad (43)$$

The reduced second order transfer matrix by Parmar [20] is given as in equation (44).

$$R_2(s) = \frac{\begin{bmatrix} \begin{pmatrix} 0.8503087s \\ +0.6171331 \end{pmatrix} & \begin{pmatrix} 0.4617562s \\ +0.2466069 \end{pmatrix} \\ \begin{pmatrix} 0.4093304s \\ +0.3086095 \end{pmatrix} & \begin{pmatrix} 0.9976611s \\ +0.6171125 \end{pmatrix} \end{bmatrix}}{s^2 + 1.34952s + 0.6181} \quad (44)$$

The step response for original system, proposed reduced model and other published reduced models [20,41,48] are shown in Figures 10 - 13 and the corresponding performance indices (ITAE, IAE and ISE) are shown in Table 4 – Table 7. The step response for G_{11} as in Figure 10, shown the best mimic operation as compared to others while in Figure 11 (for G_{12}, G_{21} and G_{22}) is Vishwakarma and Prasad. As the performance indices are quantitative representation of the responses and minimum value represents the best performance. By observation of Table 4 – Table 7, the all PIs (ITAE, IAE and ISE) found to be minimum with the proposed reduced model. It is found that the proposed method is able reduce manual calculative complexity as in the pole-clustering and other method.

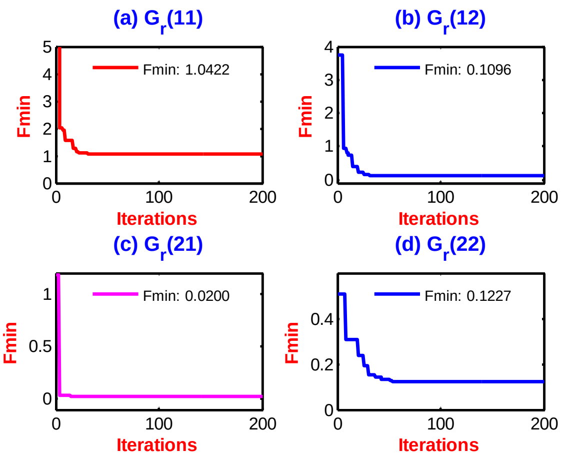

Figure 8. Fitness function for optimization of free coefficients using CSA for denominator by Routh approximation method

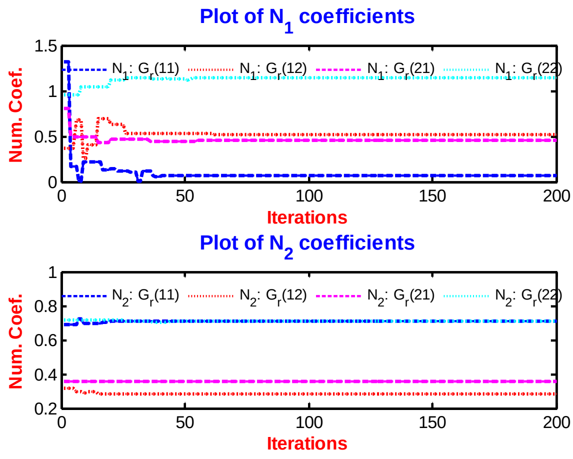

Figure 9. Plot of free coefficients N_1 and N_2 for $G_r(11)$, $G_r(12)$, $G_r(21)$ and $G_r(22)$ sub-systems of proposed reduced models using CSA.

Table 4. Performance indices (IAE, ITAE and ISE) for step response of different reduced models of $G_{11}(s)$

Methods	$R_{11}(s)$		
	IAE	ITAE	ISE
Proposed	0.35	0.5411	0.000825
Desai [41]	1.25	2.9822	0.00672
Prasad [48]	1.21	2.6621	0.001515
Parmar [20]	1.31	3.4544	0.014498

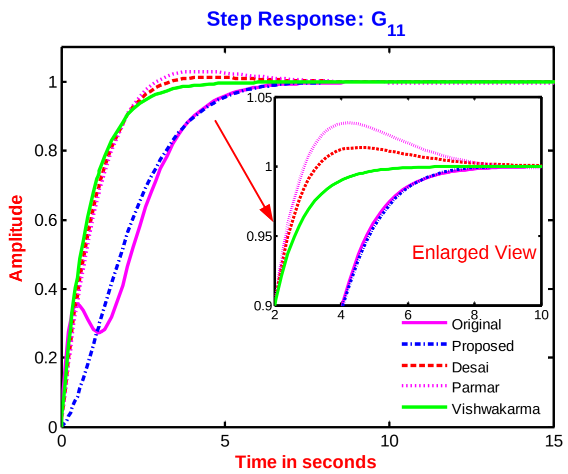

Figure 10. Step response of Original system, $G_{11}(s)$ and different reduced models.

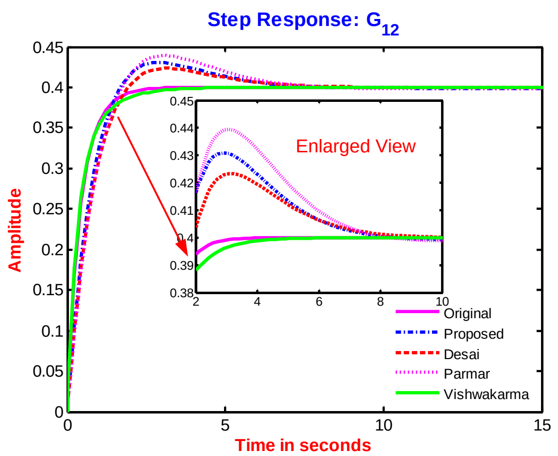

Figure 11. Step response of Original system, $G_{12}(s)$ and different reduced models

Table 5. Performance indices (IAE, ITAE and ISE) for step response of different reduced models of $G_{12}(s)$

Methods	$R_{12}(s)$		
	IAE	ITAE	ISE
Proposed	0.1665	0.5	6.006E-3
Desai [41]	0.1559	0.37	0.005918
Prasad [48]	0.0168	3E-2	7.845E-5
Parmar [20]	0.2169	0.70	0.008744

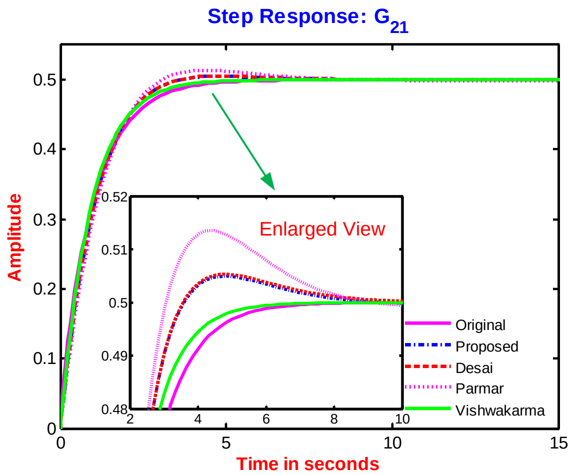

Figure 12. Step response of Original system, $G_{21}(s)$ and different reduced models.

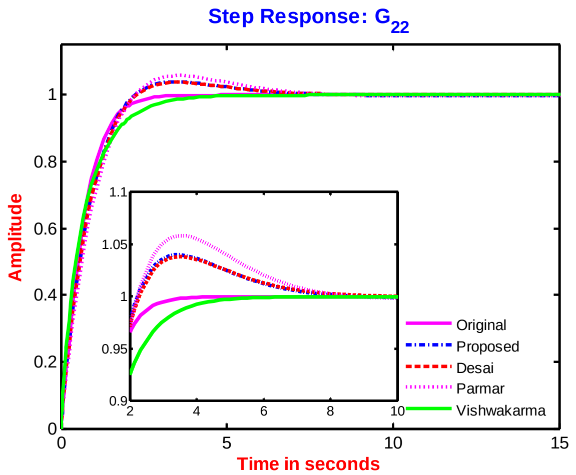

Figure 13. Step response of Original system, $G_{22}(s)$ and different reduced models

Table 6. Performance indices (IAE, ITAE and ISE) for step response of different reduced models of $G_{21}(s)$

Methods	$R_{21}(s)$		
	IAE	ITAE	ISE
Proposed	0.0763	0.2594	1.005E-3
Desai [41]	0.0766	0.2387	0.000958
Prasad [48]	0.0342	0.0717	0.000299
Parmar [20]	0.1288	0.4807	0.002538

Table 7. Performance indices (IAE, ITAE and ISE) for step response of different reduced models of $G_{22}(s)$

Methods	$R_{22}(s)$		
	IAE	ITAE	ISE
Proposed	0.196	0.7638	0.005123
Desai [41]	0.191	0.6182	0.005896
Prasad [48]	0.125	0.2207	0.004681
Parmar [20]	0.315	1.1704	0.015741

5. Conclusion

The Cuckoo Search algorithm based model order reduction is designed with minimization of ISE pertaining to a unit step input for optimization of numerator free coefficients. The denominator of the original systems for SISO and MIMO systems are reduced using Routh approximation method. The optimization process is for bounded constraints varying as per examples and the order of reduction. The performance of the Cuckoo Search algorithm based reduced order model for different systems is compared to Routh approximation and Big Bang Big Crunch, Dominat Pole clustering and Big Bang Big Crunch, Stability equation and Genetic algorithm, typical Pade and HSA based multi-objective ROM and outperforms in terms of performance indices as ITAE, IAE and ISE.

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