INTRODUCTION IN SHIP HYDROMECHANICS

Lecture MT519

Draft Edition

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Contents

PREFACE

This preliminary text book is a first attempt to provide a short introduction in ship hydromechanics for Maritime Engineering students (second year) at the Delft University of Technology. Topics have been selected for inclusion based upon their applicability in modern maritime engineering practice. The treatment of these selected topics includes both the background theory and its applications to realistic problems.
The notation used in this book is kept as standard as possible, but is explained as it is introduced in each chapter. In some cases the authors have chosen to use notation commonly found in the relevant literature - even if it is in conflict with a more universal standard or even with other parts of this text. This is done to prepare students better to read the literature if they wish to pursue such maters more deeply. In some other cases, the more or less standard notation and conventions have been adhered to in spite of more common practice.

These lecture notes on an introduction in ship hydromechanics consists of three parts: waves, shipmotions and maneuvering.
For both the waves part and the shipmotions part, use has been made of the extensive new lecture notes on "Offshore Hydromechanics", First Edition, August 2000, by J.M.J. Journée and W.W. Massie, Interfaculty Offshore Technology of the Delft University of Technology; see Internet: http://www.shipmotions.nl.
For the maneuvering part, use has been made of the "Principles of Naval Architecture", 1989, Volume III, Chapter IX on Controllability by C.L. Crane, H. Eda and A. Landsberg, as well as various proceedings of the International Towing Tank Conference (ITTC) on maneuvering.
Existing lecture notes in Dutch on "Waves, Shipmotions and Maneuvering, Parts I and II", Reports 563-K and 487-K of the Ship Hydromechanics Laboratory of the Delft University of Technology, 1989, by J. Gerritsma (retired Professor in Ship Hydromechanics) were a very useful guide for the authors.

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Feel free to contact them with positive as well as negative comments on this text. The authors are aware that this preliminary edition of "Ship Hydromechanics, Part I" is still in progress and is not complete in all details. Some references to materials, adapted from the work of others may still be missing and occasional additional figures and text can improve the presentation as well.

Delft, January 2001.

[0] J.M.J. Journée and J.A. Pinkster, *"SHIP HYDROMECHANICS, Part I: Introduction"*, Draft Edition, January 2001, Ship Hydromechanics Laboratory, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands. For updates see web site: http://www.shipmotions.nl.

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Chapter 1

Introduction

Since about 1950, the industrial and commercial interest for the behavior of ships and other floating structures in seaway has been increased significantly. Nowadays, it becomes more and more usual to judge a ship or other offshore designs on its seakeeping performance. Among others, the behavior of a ship in seaway has a large influence on:
- the safety of passengers, crew, cargo and ship,
- the comfort of passengers and crew,
- the dynamic loads on the floating structure, its cargo or its equipment and
- the sustained sea speed of the ship and the fuel consumption of its propulsion plant.

The motions of a ship or offshore structure in waves are important from a safety point of view. Roll motions combined with lateral wind loads can cause dangerous heeling angles or even capsizing. Because of this, modern (static) stability criteria are based on the (dynamic) roll behavior too.
When compared with the ship's stability characteristics in still water, following waves can cause a considerable reduction of the transverse stability and unacceptable large roll angles can be the result. This can happen with fast ships in following waves with wave lengths equal to about the ship length.
Transverse accelerations on the ship by combined sway and roll motions can cause a shift of cargos like ore or grain. Sea-fastenings of containers at deck can collapse by to large accelerations and vulnerable cargos like fruits can be damaged. Bilge keels, fin stabilizers and anti-rolling flume tanks can be used to reduce the roll motions.
Because of its drilling capability, the design of a floating offshore drilling structure is generally such that its vertical motion responses on waves are small; one tries to design a floating structure with an optimal workability.
An important aspect of ship motions is its convenient or inconvenient behavior for human beings in seaway. Ships with a large initial metacentric height can be very inconvenient; high dynamic acceleration levels are often the result of small natural roll periods. A combination of the acceleration level and its average frequency of oscillation rules the amount of comfort aboard. The human acceptance of these combinations is given in figure 1.1.
A lot of researchers have found a high correlation between the magnitude of the accelerations and the frequency on one hand and the appearance of seasickness on the other

[0] J.M.J. Journée and J.A. Pinkster, *"SHIP HYDROMECHANICS, Part I: Introduction"*, Draft Edition, January 2001, Ship Hydromechanics Laboratory, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands. For updates see web site: http://www.shipmotions.nl.

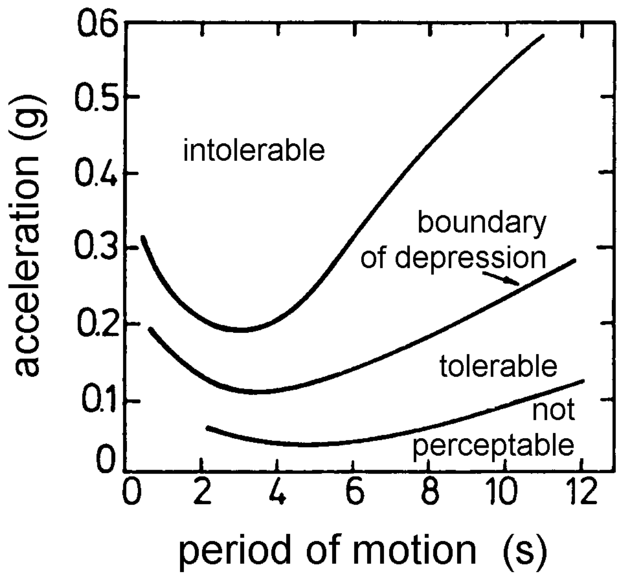

Figure 1.1: Human Appreciations of Accelerations

hand. At low frequencies, as they appear for the motions of a ship in seaway, it is found that the product of the acceleration amplitude and the frequency (so the amplitude of the derivative of the acceleration) is a measure for the experienced hindrance. Seasickness is caused by an excessive stimulation of the human balance organ, particularly that part of this organ that is sensitive for linear accelerations.

Motions of ships and other floating structures have adverse effects on human performance. Degradation can be an important factor when carrying out complicated tasks aboard (navy, offshore).

In the past, static considerations were use to determine the required ship's strength. The ship was placed in a "frozen" wave with a wave length equal to the ship length and a double wave amplitude equal to 1/20 of the length. Dynamic effects caused by ship motions were accounted for in a very simple way; even they were ignored in a lot of cases. The increased ship size and its forward speed have lead to an extension of these standard calculations, accounting for hydrodynamic loads on the hull, vertical accelerations and even fatigue considerations.

Large local loads can appear when the vertical relative accelerations of the forefoot of the ship exceeds a certain threshold value, when hitting the wave surface at re-entrance after bow emergence (slamming). Very high peak pressures (7 times the atmospheric pressure or more) have been measured, which can cause bottom damage. Also, slamming can cause a 2-node vertical vibration of the ship's hull with an increased amidships bending moment; see figure 1.2.

It is very complicated or even impossible to give a general definition of an optimal behavior of a ship or another floating structure in seaway. This optimum can be related to various phenomena, like motions, accelerations, speed loss, fuel consumption, workability at certain locations, degradation, noise, etc. Sometimes, contradictory effects will be found. Attempts to define general criteria for these phenomena can be found in the literature.

[Kent, 1950] has given the following qualitative description of a "sea-kindly ship":

A "sea-kindly ship" is one which rides the seas in rough weather, with decks free of seawater: that is green seas are not shipped and little spray comes inboard. No matter in which direction the wind and waves meet the ship, she will stay on her course with only an occasional use of helm, she will respond quickly to small rudder angles and maintain a fair speed without slamming, abnormal fluctuations in shaft torque, or periodic racing of her engines. Open decks will be easy to transverse in all weathers, without danger or discomfort to her passengers and crew, and her behavior in a seaway - i.e. her rolling, pitching, yawing, heaving, surging and leeway drift - will be smooth and free from baulks of shocks. Expert handling of the ship by her master and crew will always be required to produce a sea-kindly performance by any vessel in rough weather.

In the second part of the 19^{th} century **William Froude**, 1810 - 1879, proposed the British Admiralty to build what would become the world's first towing tank at Torquay in England. He had recently developed scaling laws for predicting the resistance of ships from tests on ship models and he intended to use this tank for the required scale model experiments. The British Admiralty accepted Froude's proposal on the condition that he also used the tank to investigate the ways of reducing the rolling motion of ships. That was when sails were being replaced by steam driven propulsion systems and roll was becoming more of a problem. On March 3 1872, the first indoor professional model test in the world was carried out in a basin of 85 x 11 x 3 meter, as shown in figure 1.3. About half a century later, Froude's towing tank was pulled down.

In 1883, the world's first privately owned towing tank, the Denny tank as shown in figure 1.3 too, was built at Dumbarton in Scotland, using the original design of Froude's tank in Torquay. The Denny tank - with dimensions 93.00 x 6.80 x 2.75 meter - was closed in 1983, but it was re-opened in 1986 under the umbrella of the Scottish Maritime Museum. The tank testing and museum parts are separate. Since 1989, the tank testing facility operates under the management of Strathclyde University's Department of Ship and Marine Technology.

In due course towing tanks were built in many different countries. These were often fitted with wave makers which allowed the behavior of model ships in waves to be studied at leisure and provided, for the first time, a technique for refining a full-scale design to ensure adequate performance in rough weather. These model experiments were usually confined to tests in regular head or following waves with occasional tests at zero speed in beam waves. Tests at other headings or in more realistic irregular waves were impossible because

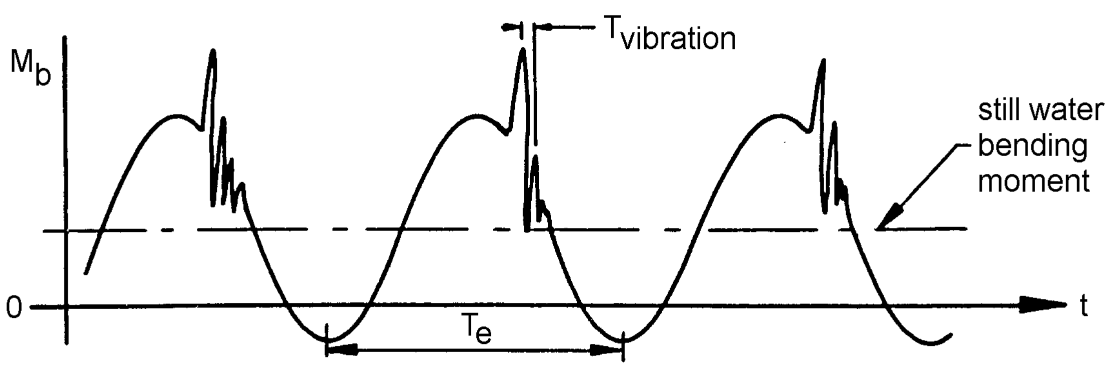

Figure 1.2: Amidships Vertical Bending Moments During Slamming

Froude's Tank (1872) Denny Tank (1883)

Figure 1.3: The World's First Towing Tanks

of the long narrow shape of the towing tanks and the simplicity of the wave makers.

These early model experiments allowed some limited developments in the study of seakeeping but they could not he used to predict the actual performance of ships at sea because no technique for relating the behavior of the model in the regular waves of the laboratory to the behavior of the ship in the chaotic environment of the real ocean was available. This situation prevailed for sixty years or more and the study of seakeeping remained in effective limbo until the publication of a landmark paper by St. Denis and Pierson in 1953. This showed, for the first time, how this problem could be solved using the techniques of spectral analysis borrowed from the field of electromagnetic communications.

St. Denis and Pierson supposed the sea as being a superposition of many simple regular waves, each regular wave with its own frequency (or length), amplitude and direction of propagation. The ship is supposed to respond on each of these individual waves as being a linear system. This holds that doubling the wave amplitude will result in a doubled response amplitude, while the phase lag between the response and the regular wave has not changed. A summation of the responses on each of these individual regular waves provides the irregular response in a seaway.

At about the same time, theoretical methods of predicting the behavior of ships in regular waves were being developed. The breakthrough came in 1949 with Ursell's theory for predicting the characteristics of the flow around a circular cylinder oscillating in a free surface. Classical transformation techniques allowed these results to be applied to a wide range of shapes of ship-like cross-section and the fundamentals of modern ship motion theory were born.

These developments some 50 years ago provided the basic tools required to develop routine techniques for the prediction of ship motions in something approaching the real irregular wave environment of the ocean. It was now possible for the first time to predict the rough weather performance of a ship at the design stage and to allow seakeeping to take its rightful place in the design process. Since that time seakeeping has remained an active field of research, but developments have been in the nature of progressive refinements rather than spectacular advances. Techniques for designing roll stabilizers, criteria determination, prediction of long-term motion statistics and operational effectiveness have all been added to the naval architect's armory of weapons: seakeeping performance prediction should now be a routine in any ship design office.

The history of scientific research on maneuvering of ships started already in 1749 with the classic work of Euler on equations of motion of a ship, but shipbuilding still remained for almost two centuries fully based on a shipbuilder's experience-based knowledge. In about 1920, some first attention on a more analytic approach appeared when single-plated rudders were replaced by flow-line-curved rudders. This development was based on research on airfoils, carried out in the new aviation industry. More systematic work on maneuvering has been started during World War II. Shortly after this War, in 1946, Davidson and Schiff published a paper dealing with maneuvering problems on a (for that time) modern approach.
The first International Symposium on Ship Maneuverability was held in 1960 in Washington in the U.S.A. There, among others, Norrbin gave his view on the "state of the art" of scientific work carried out on maneuverability of ships. Since then, an increasing attention has been paid on research in this particular hydrodynamic field. Especially, (inter)national organizations - such as the IMO (International Maritime Organization), the ITTC (International Towing Tank Conference) and the SNAME (Society of Naval Architects and Marine Engineers) - became more active in the stimulation of research on those aspects of ship maneuverability which are vital for safer shipping and cleaner oceans.
During the last four decades, research on ship maneuverability was also stimulated strongly by the increasing ship size problems (crude oil carriers and container vessels), the related shallow water problems when entering harbors, the increasing ship speed problems (nuclear submarines and fast ferries), newly-developed experimental oscillatory techniques (planar motion mechanisms) and - last but not least - the enormous developments in the computer industry with related new possibilities for computer simulations.

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Chapter 2

Ocean Surface Waves

Ocean surface waves cause periodic loads on all sorts of man-made structures in the sea. It does not matter whether these structures are fixed, floating or sailing and on the surface or deeper in the sea. To understand these loads, a good understanding of the physics of water waves is necessary.
Looking over the sea, one gets the impression that there is an endlessly moving succession of irregular humps and hollows reaching from horizon to horizon. If the winds are light, the irregularities are small. If the winds are heavy, you may be awed by the resulting gigantic stormy seas. Since water moves "easily" and flat calms seldom occur, an undisturbed water surface is rarely found at sea. Even when drifting in glassy calm, one will usually find the ocean heaving itself in a long smooth swell whose source is a storm which may have occurred days before and hundreds of miles away.
Ocean surface waves are generally distinguished in two states: **sea** or wind waves, when the waves are being worked on by the wind that raised them and **swell**, when they have escaped the influence of the generating wind. Sea is usually of shorter period (higher frequency) than swell. As a rule of thumb, a period of about 10 seconds may be taken as separating sea from swell; although one must allow for considerable overlap. Sea is shorter in length, steeper, more rugged and more confused than swell. Since wind-generated waves have their origin in the wind - which is proverbially changeable - they too are changeable, varying both seasonally and regionally.
Wind waves, especially, are short crested and very irregular. Even so, they can be seen as a superposition of many simple, regular harmonic wave components, each with its own amplitude, length (or period or frequency) and direction of propagation. Such a concept can be very handy in many applications; it allows one to predict very complex irregular behavior in terms of the much simpler theory of regular waves.

B. Kinsman wrote *Wind Waves - Their Generation and Propagation on the Ocean Surface* while he was Professor of Oceanography at The Johns Hopkins University in Baltimore, Maryland. The book, published in 1963 was complete for its time; the wit scattered throughout its contents makes is more readable than one might think at first glance.

First, an introduction to the most relevant phenomena is given here for the case of regular deep water waves; the (general) shallow water case will be treated in a next lecture. Then,

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the superposition principle will be used to define the irregular ocean surface waves.

2.1 Regular Waves

Figure 2.1 shows a harmonic wave, ζ, as seen from two different perspectives:

- Figure 2.1-a shows what one would observe in a snapshot photo made looking at the side of a (transparent) wave flume; the wave profile (with wave amplitude ζ_a and wave length λ) is shown as a function of distance x along the flume at a fixed instant in time:

$$\zeta = \zeta_a \cos\left(2\pi \cdot \frac{x}{\lambda}\right) \tag{2.1}$$

- Figure 2.1-b is a time record of the wave profile (with wave amplitude ζ_a and wave frequency ω) observed at one location along the flume; it looks similar in many ways to the first figure, but the angle $2\pi x/\lambda$ has been replaced by ωt:

$$\zeta = \zeta_a \cos\left(\omega \cdot t\right) \tag{2.2}$$

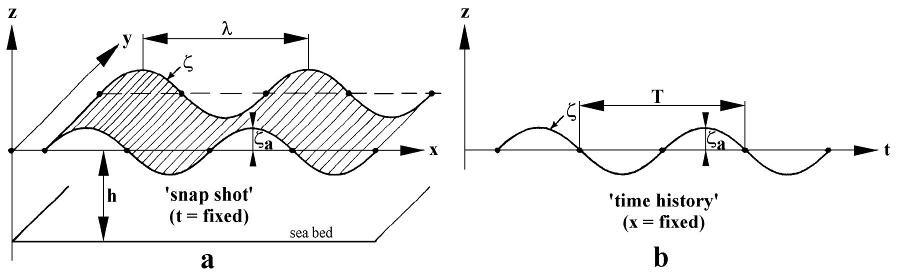


Figure 2.1: Harmonic Wave Definitions

Notice that the origin of the coordinate system is at the still water level with the positive z-axis directed upward; most relevant values of z will be negative. The still water level is the average water level or the level of the water if no waves were present. The x-axis is positive in the direction of wave propagation. The water depth, h, (a positive value and infinite here) is measured between the sea bed ($z = -h$) and the still water level ($z = 0$). The highest point of the wave is called its crest and the lowest point on its surface is the trough. If the wave is described by a sine wave, then its amplitude ζ_a is the distance from the still water level to the crest, or to the trough for that matter. The subscript a denotes amplitude, here. The wave height $H = 2\zeta_a$ is measured vertically from wave trough level to the wave crest level; it is the double amplitude.
The horizontal distance (measured in the direction of wave propagation) between any two successive wave crests is the wave length, λ. The same distance along the time axis is the wave period, T. Since the distance between any two corresponding points on successive sine waves is the same, wave lengths and periods are usually actually measured between two consecutive upward (or downward) crossings of the still water level. Such points are also called zero-crossings, and are easier to detect in a wave record.
Wave heights are always much smaller than wave lengths. The ratio of wave height to wave length is often referred to as the wave steepness, H/λ. When waves become too high, they

become unstable with a tendency to fall apart at the slightest nudge. One theory - that of Stokes - sets the upper limit at $H/\lambda = 1/7$.
Since sine or cosine waves are expressed in terms of angular arguments, the wave length and period are converted to angles using:

$$k\lambda = 2\pi \qquad \text{or:} \qquad \boxed{k = \frac{2\pi}{\lambda}}$$

$$\omega T = 2\pi \qquad \text{or:} \qquad \boxed{\omega = \frac{2\pi}{T}} \tag{2.3}$$

in which k is the wave number (rad/m) and ω is the circular wave frequency (rad/s).
Obviously, the wave form moves one wave length during one period so that its propagation speed or phase velocity, c, is given by:

$$\boxed{c = \frac{\lambda}{T} = \frac{\omega}{k}} \tag{2.4}$$

Fortunately, the water particles themselves do not move with this speed; only the wave form (wave crests or troughs) moves with this phase velocity, c.
If the wave moves in the positive x-direction, the wave profile (shape of the water surface) can now be expressed as a function of both x and t as follows:

$$\boxed{\zeta = \zeta_a \cos(kx - \omega t)} \tag{2.5}$$

2.1.1 Potential Theory

A velocity potential, $\Phi(x, y, z, t)$, is a function - a mathematical expression with space and time variables - which is valid in the whole fluid domain. This potential function has been defined in such a way that it has one very important property: in any point in the fluid, the derivative of this function in a certain direction provides the velocity component of a fluid particle in that point in that direction. This will be explained here by a very simple example.

Suppose a uniform flow with a velocity U in the positive x-direction. Then, the velocity potential of the fluid is $\Phi = Ux$, because the velocity of each fluid particle in the x-direction is $u = d\Phi/dx = U$. For other potential flow elements, such as sources, sinks and vortices, reference is given to lecture notes on fluid dynamics.

The velocity potential of a harmonic oscillating fluid in the x-direction is given by $\Phi = Ux \cdot \cos \omega t$. If the magnitude of the potential Φ - the potential value - will be doubled, then the velocity component will be doubled too; $u = d\Phi/dx = U \cdot \cos \omega t$. This follows directly from the definition of the phenomenon "potential", so the potential has been described by a linear function. This means also that all potential flow elements (pulsating uniform flows, sources, sinks, etc.) may be superposed.

In order to use the linear potential theory for water waves, it will be necessary to assume that the water surface slope is very small. This means that the wave steepness is so small that terms in the equations of motion of the waves with a magnitude in the order of the steepness-squared can be ignored.

Only the final result of the derivation of the velocity potential of a simple harmonic wave is given here. A detailed derivation of this definition will be treated in a next lecture.

Velocity Potential

In order to use this linear theory with waves, it will be necessary to assume that the water surface slope is very small. This means that the wave steepness is so small that terms in the equations of the waves with a magnitude in the order of the steepness-squared can be ignored. Using the linear theory holds here that harmonic displacements, velocities and accelerations of the water particles and also the harmonic pressures will have a linear relation with the wave surface elevation.
The profile of a simple wave with a small steepness looks like a sine or a cosine and the motion of a water particle in a wave depends on the distance below the still water level. This is reason why the wave potential is written as:

$$\Phi_w(x, z, t) = P(z) \cdot \sin(kx - \omega t) \tag{2.6}$$

in which $P(z)$ is an (as yet) unknown function of z.
The velocity potential $\Phi_w(x, z, t)$ of the harmonic waves has to fulfill some requirements, which will be treated in detail in a next lecture. The inviscid irrotational fluid is supposed to be incompressible, from which follows the so-called Continuity Condition and Laplace Equation. The vertical velocity of the fluid particles at the sea bottom is zero; the bottom is impervious. The pressure at the surface of the fluid is equal to the atmospheric pressure; there is no pressure jump between the fluid domain and the air domain.
These requirements lead to a more complete expression for the velocity potential as will be explained in a next lecture:

$$\Phi_w = \frac{\zeta_a g}{\omega} \cdot \frac{\cosh k\,(h + z)}{\cosh kh} \cdot \sin(kx - \omega t) \tag{2.7}$$

Waves are dispersive; they run at speeds which depend on their length (and water depth). The relationship between c and λ in deep water, or equivalently between ω and k, can be established from the condition that fluid particles in the surface of the fluid remain there (the fluid surface is impervious too) as:

$$\omega^2 = k \cdot g \cdot \tanh kh \tag{2.8}$$

These relations are valid for all water depths, but the fact that they contain hyperbolic functions makes them cumbersome to use. Therefore we restrict ourselves in the following to deep water waves.

For deep water, $h \rightarrow \infty$, this expression for the velocity potential and the dispersion relation reduce to:

$$\boxed{\Phi_w = \frac{\zeta_a g}{\omega} \cdot e^{kz} \cdot \sin(kx - \omega t)} \quad \text{and} \quad \boxed{\omega^2 = k \cdot g} \tag{2.9}$$

This last dispersion relation provides other definitions for the phase velocity in equation 2.4 of waves in deep water too:

$$c = \frac{\omega}{k} = \frac{g}{\omega} = \sqrt{\frac{g}{k}} \tag{2.10}$$

Simple and very practical relations between the wave length (m) and frequency (rad/s) or period (s) follow from this:

$$\begin{aligned} \lambda &= \frac{2\pi g}{\omega^2} \approx \frac{61.6}{\omega^2} \\ &= \frac{g\,T^2}{2\pi} \approx 1.56 \cdot T^2 \end{aligned} \tag{2.11}$$

or:

$$\omega \approx \frac{7.85}{\sqrt{\lambda}} \qquad \text{and} \qquad T \approx 0.8 \cdot \sqrt{\lambda} \tag{2.12}$$

Mind you that these relations between the wave length and the wave frequency or the wave period are valid for **regular** deep water waves only. They may not be used as relations between the average values of these phenomena in irregular waves, which are treated in a following section.

2.1.2 Water Particle Kinematics

The kinematics of a water particle is found from the velocity components in the x- and z-directions, obtained from the velocity potential and the dispersion relation.

Velocities

The resulting velocity components - in their most general form - can be expressed as:

$$\begin{aligned} \boxed{u = \frac{\partial \Phi_w}{\partial x} = \frac{dx}{dt}} &= \zeta_a \omega \cdot e^{kz} \cdot \cos(kx - \omega t) \\ \boxed{w = \frac{\partial \Phi_w}{\partial z} = \frac{dz}{dt}} &= \zeta_a \omega \cdot e^{kz} \cdot \sin(kx - \omega t) \end{aligned} \tag{2.13}$$

An example of a velocity field is given in figure 2.2.
Beneath the crest of a wave, the water movement is with the wave. Beneath the trough it is against the wave. This can be seen easily by watching a small object, such as a bottle, floating low in the water. It will move more or less with the water particles.
The combined motions in the x- and z-directions become circles, as will be treated hereafter. The circular outline velocity or orbital velocity follows from the two equations 2.13:

$$\begin{aligned} V_o &= \sqrt{u^2 + w^2} \\ &= \zeta_a \omega \cdot e^{kz} \end{aligned} \tag{2.14}$$

This velocity, which consists of two harmonic contributions, is in this case not harmonic; in this case it is constant.

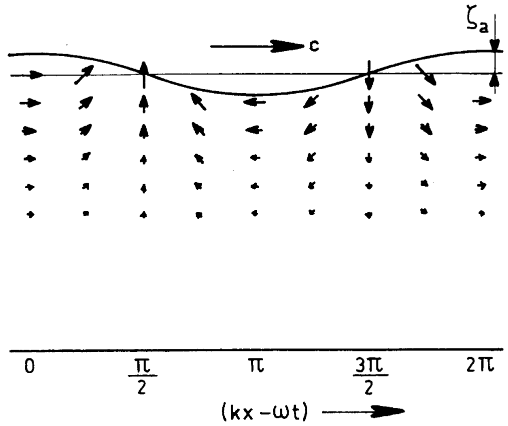


Figure 2.2: Velocity Field in a Deep Water Wave

Displacements

Because of the small steepness of the wave, x and z in the right hand side of the equations 2.13 can be replaced by the coordinates of the mean position of the considered water particle: x_1 and z_1. Hence the distances $x - x_1$ and $z - z_1$ are so small that differences in velocities resulting from the water motion position shifts can be neglected; they are of second order. Then, an integration of velocity equations over t yields the water displacements:

$$\boxed{x = \int_0^t u \cdot dt} = -\zeta_a \cdot e^{kz} \cdot \sin(kx_1 - \omega t) + C_1$$

$$\boxed{z = \int_0^t w \cdot dt} = +\zeta_a \cdot e^{kz} \cdot \cos(kx_1 - \omega t) + C_2 \tag{2.15}$$

Trajectories

It is obvious that the water particle carries out an oscillation in the x- and z-directions about a point (C_1, C_2). This point will hardly deviate from the situation in rest, so: $C_1 \approx x_1$ and $C_2 \approx z_1$.

The trajectory of the water particle is found by an elimination of the time, t, by using:

$$\sin^2(kx_1 - \omega t) + \cos^2(kx_1 - \omega t) = 1 \tag{2.16}$$

which provides:

$$(x - x_1)^2 + (z - z_1)^2 = \left(\zeta_a \cdot e^{kz_1}\right)^2 \tag{2.17}$$

This equation shows that the trajectories of water particles are circles, with a radius, $\zeta_a e^{kz_1}$, which decreases exponentially with distance below the surface.

Figure 2.3 shows an example of these trajectories.

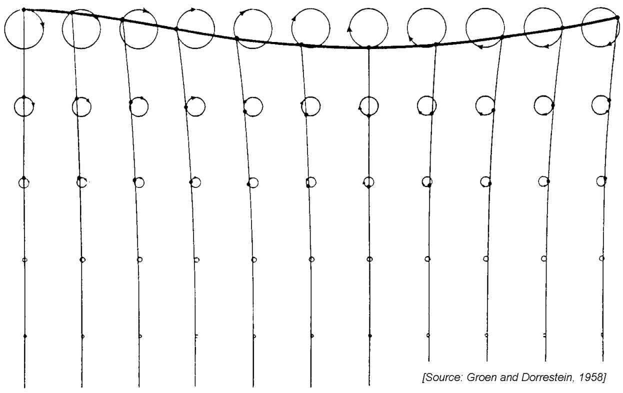


Figure 2.3: Trajectories of Water Particles in a Deep Water Wave

The trajectory radius, r, decrease very fast with increasing distance below the free surface as a result of the negative value for z_1 in the exponential term e^{kz_1}. This is illustrated here:

$$\begin{array}{lllll} z & = & 0 & \longrightarrow r = \zeta_a \cdot e^{kz} = \zeta_a \cdot e^{0} & = 1.000 \cdot \zeta_a \\ z & = & -0.50\lambda & \longrightarrow r = \zeta_a \cdot e^{kz} = \zeta_a \cdot e^{-\pi} & = 0.043 \cdot \zeta_a \\ z & = & -\lambda & \longrightarrow r = \zeta_a \cdot e^{kz} = \zeta_a \cdot e^{-2\pi} & = 0.002 \cdot \zeta_a \end{array}$$

One should remember from kinematics that the velocity of the water particles will have a constant magnitude in this case and that it is always directed tangentially to the trajectory circle.

Accelerations

The water particle accelerations follow directly from a differentiation of the velocity components, which yields:

$$\boxed{\dot{u} = \frac{du}{dt}} = +\zeta_a \omega^2 \cdot e^{kz} \cdot \sin(kx - \omega t)$$

$$\boxed{\dot{w} = \frac{dw}{dt}} = -\zeta_a \omega^2 \cdot e^{kz} \cdot \cos(kx - \omega t) \qquad (2.18)$$

Relative to the velocity components, the accelerations have amplitudes which have been multiplied by ω; also their corresponding phases have been shifted by 90 degrees as well. It is wise to note that in a circular motion, the magnitude of the acceleration is constant and that it is always directed toward the center of the circle. This acceleration vector is therefore also always perpendicular to the velocity vector.

2.1.3 Pressure

The pressure, p, in the first order wave theory follows from the Bernoulli equation:

$$\boxed{\frac{\partial \Phi_w}{\partial t} + \frac{1}{2}\left(u^2 + w^2\right) + \frac{p}{\rho} + gz = 0} \tag{2.19}$$

or:

$$p = -\rho g z - \rho \frac{\partial \Phi_w}{\partial t} - \frac{1}{2}\rho\left(u^2 + w^2\right) \tag{2.20}$$

The definition of the wave potential, the expressions for the water particle velocities and some algebra lead to:

$$p = \quad -\rho g z \quad + \rho g \zeta_a e^{kz} \cdot \cos(kx - \omega t) \quad - \frac{1}{2}\rho \zeta_a^2 \omega^2 e^{2kz} \tag{2.21}$$

Three parts can be distinguished in this expression for the pressure:

1. The first part, $p^{(0)} = -\rho g z$, is the (zero$^{\text{th}}$ order) hydrostatic pressure as used in Archimedes' law and treated in the hydrostatic stability theory for floating bodies.

2. The second part, $p^{(1)} = +\rho g \zeta_a e^{kz} \cdot \cos(kx - \omega t)$, is the (first order) dynamic pressure due to the wave form and will be used in a next chapter when treating "first order wave loads" on floating bodies.

3. The third part, $p^{(2)} = -\frac{1}{2}\rho \zeta_a^2 \omega^2 e^{2kz}$, is the (second order) pressure due to the local kinetic energy in the waves - also called radiation pressure - and will be used in a next chapter when treating "second order wave drift forces" on floating bodies.

2.1.4 Wave Energy

The theory about wave energy will be treated in detail in a next lecture; only the final theoretical results are given here.
If one drops a stone in still water, shortly later radial waves come into existence and they travel away from the point of impact. A certain amount of energy has been put into the water by the impact. Part of this energy is stored in the deformation of the water surface as potential energy and, part of it is found in the motion of the deformation as kinetic energy.
The total energy in the waves can be written as:

$$\boxed{E = \frac{1}{2}\rho g \zeta_a^2} \qquad \text{per unit horizontal sea surface area} \tag{2.22}$$

This total energy will be used in a following section on wave energy spectra.

2.1.5 Numerical Exercises

Consider a towing tank filled with fresh water with a length of 150 meters, a width of 5 meters and a wave maker in one end that generates long-crested regular waves. Assume in this tank a generated regular deep water wave with an amplitude $\zeta_a = 0.20$ meter and a wave period $T = 2.0$ seconds.

Determine:

1. The circular wave frequency, ω, the wave number, k, and the wave length, λ.
2. The maximum fluid particle velocity in the tank.
3. The path of a fluid particle in the surface of the wave.
4. The path of a fluid particle at 0.50 meter below the still water level.
5. The maximum pressure at 0.50 meter below the still water level.
6. The phase velocity, c, of these waves.
7. The time between passing an observer along the tank side of two successive wave crests.
8. The phase of the wave elevation 2.50 meter closer to the wave maker relative to the position of an observer.

Solutions:

1. $\omega = 3.14$ rad/s, $k = 1.0$ m^{-1} and $\lambda = 6.25$ m.
2. $u_{\max} = v_{\max} = 0.63$ m/s.
3. Circular motion with a radius of 0.20 m.
4. Circular motion with a radius of 0.12 m.
5. $p_{\max} = 0.62 \cdot 10^4$ N/m^2.
6. $c = 3.12$ m/s.
7. $t = T = 2.0$ s.
8. $\varepsilon = -2.5$ rad $= -144°$.

2.2 Irregular Waves

Ocean surface waves can be classified into two basic categories:

- Sea
  A sea is a train of waves driven by the prevailing local wind field. The waves are short-crested with the lengths of the crests only a few (2-3) times the apparent wave length. Also, sea waves are very irregular; high waves are followed unpredictably by low waves and vice versa. Individual wave crests seem to propagate in different directions with tens of degrees deviation from the mean direction. The crests are fairly sharp and sometimes even small waves can be observed on these crests or there are dents in the larger wave crests or troughs. The apparent or virtual wave period, $\widetilde{T}$, varies continuously, as well as the virtual or apparent wave length, $\widetilde{\lambda}$.

- Swell
  A swell is waves which have propagated out of the area and local wind in which they were generated. They are no longer dependent upon the wind and can even propagate for hundreds of kilometers through areas where the winds are calm. Individual waves are more regular and the crests are more rounded than those of a sea. The lengths of the crests are longer, now several (6-7) times the virtual wave length. The wave height is more predictable, too. If the swell is high, 5 to 6 waves of approximately equal heights can pass a given point consecutively. If the waves are low, they can stay low for more than a minute even though the surface elevation remains irregular.

In the general case, the total irregular waves are a superposition of the sea and the swell at that location; they can be added as is shown in the following sections.

2.2.1 Simple Statistical Analysis

Figure 2.4 shows a part of a simple time history of an irregular wave. When such a time history is available, then a simple analysis can be carried out to obtain statistical data from this record. The length of this time history should be at least 100 times the longest wave period, to obtain reliable statistical information. The analysis presented in this section can be carried out by hand

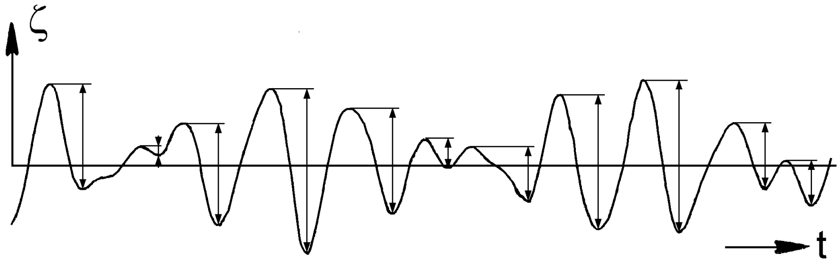


Figure 2.4: Time History of a Seaway

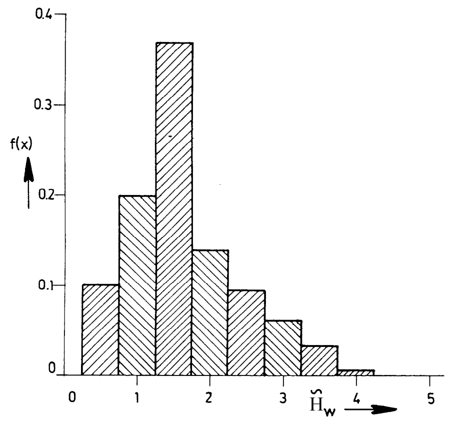


Figure 2.5: Histogram of Wave Heights

Average Wave Period

The average wave period, $\bar{T}$, can be found easily (by hand) from the average zero up-crossing period or from the average period of the wave crests or troughs. The simplest way to do this is to divide the record duration one less than the number of upward (or downward) zero crossings found.

Wave Height Statistics

Obtaining an average wave height requires more work. Successive wave heights are measured and classified in groups with intervals of, for instance, 0.5 meter. The number of wave heights in each group is then counted. These counts for each group are divided by the total number of wave heights to obtain the frequency quotients or the probability density function, $f(x)$. Finally these frequency quotients are added cumulatively to obtain the cumulative frequency quotients. The probability density function of wave heights, $f(x)$, is given in figure 2.5 as a histogram. As one uses even more waves to determine this function, the function converges to a stable form.

Statistical information can be obtained from the probability density function, $f(x)$. For example, the probability that the wave height, $\widetilde{H}_w$, exceeds a certain threshold value, a, in this record is given by:

$$P\left\{\widetilde{H}_w > a\right\} = \int_a^{\infty} f(x) \cdot dx \qquad (2.23)$$

A numerical example of this approach is given in the table below.

wave height intervals (m)	wave height average (m)	number of waves n	frequency quotient $f(x)$
0.25-0.75	0.5	15	0.100
0.75-1.25	1.0	30	0.200
1.25-1.75	1.5	55	0.367
1.75-2.25	2.0	21	0.140
2.25-2.75	2.5	14	0.093
2.75-3.25	3.0	9	0.060
3.25-3.75	3.5	5	0.033
3.75-4.25	4.0	1	0.007
total		150	1.000

The table shows that, if $a = 3.25$ meter, the probability of finding a wave higher than that threshold value is given by this integral (or in this case):

$$P\left\{\widetilde{H}_w > 3.25\right\} = \frac{5+1}{150} = 0.04 \qquad \text{or:} \qquad P\left\{\widetilde{H}_w > 3.25\right\} = 0.033 + 0.007 = 0.04$$

Significant Wave Height

The so-called significant wave height, $H_{1/3}$, defined as the average (centroid) of the highest 1/3 of the waves in the record. Thus, in this case:

$$\begin{aligned} H_{1/3} &= \frac{2.0 \cdot 21 + 2.5 \cdot 14 + 3.0 \cdot 9 + 3.5 \cdot 5 + 4.0 \cdot 1}{50} \\ &= 2.51 \text{ m} \end{aligned}$$

or from $f(x)$:

$$\begin{aligned} H_{1/3} &= \frac{2.0 \cdot 0.140 + 2.5 \cdot 0.093 + 3.0 \cdot 0.060 + 3.5 \cdot 0.033 + 4.0 \cdot 0.07}{1/3} \\ &= 2.51 \text{ m} \end{aligned}$$

The significant wave height, $H_{1/3}$, plays an important role in many practical applications of wave statistics. Often there is a fair correlation between the significant wave height and a visually estimated wave height. This comes, perhaps, because higher waves make more impression on an observer than do the smallest ones.

2.2.2 Superposition

Wind waves, especially, are very irregular. Even so, they can be seen as a superposition of many simple, regular harmonic wave components, each with its own amplitude, length, period or frequency and direction of propagation. Such a concept can be very handy in many applications; it allows one to predict very complex irregular behavior in terms of much simpler theory of regular waves. This so-called superposition principle, first introduced in hydrodynamics by [St. Denis and Pierson, 1953], is illustrated in figure 2.6.

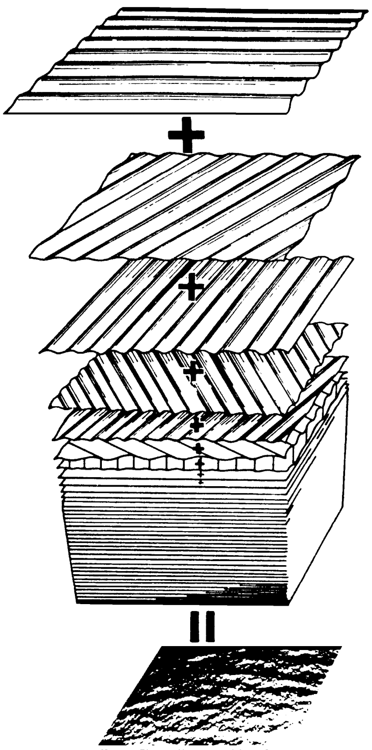


Figure 2.6: A Sum of Many Simple Sine Waves Makes an Irregular Sea

This means that the irregular waves can be written as:

$$\boxed{\zeta(x,t) = \sum_{n=1}^{N} \zeta_{a_n} \cos(\omega_n t - k_n x + \varepsilon_n)}$$

see figure 2.7.

2.2.3 Energy Density Spectrum

Suppose a time history, as given in figure 2.8, of the wave elevation during a sufficient long but arbitrary period:

$$\tau = N \cdot \Delta t$$

The instantaneous wave elevation has a Gaussian distribution and zero mean. The amplitudes ζ_{a_n} can be obtained by a Fourier analysis of the signal. However, for each little time shift of the time history one will find a new series of amplitudes ζ_{a_n}. Luckily, a mean square value of ζ_{a_n} can be found: $\overline{\zeta_{a_n}^2}$.
When $\zeta(t)$ is an irregular signal without prevailing frequencies, the average values $\overline{\zeta_{a_n}^2}$ close to ω_n will not change much as a function of the frequency; $\overline{\zeta_a^2}$ is a continuous function.
The variance σ_ζ^2 of this signal is equal to the average value of the squares of the wave elevation:

$$\sigma_\zeta^2 = \overline{\zeta^2}$$

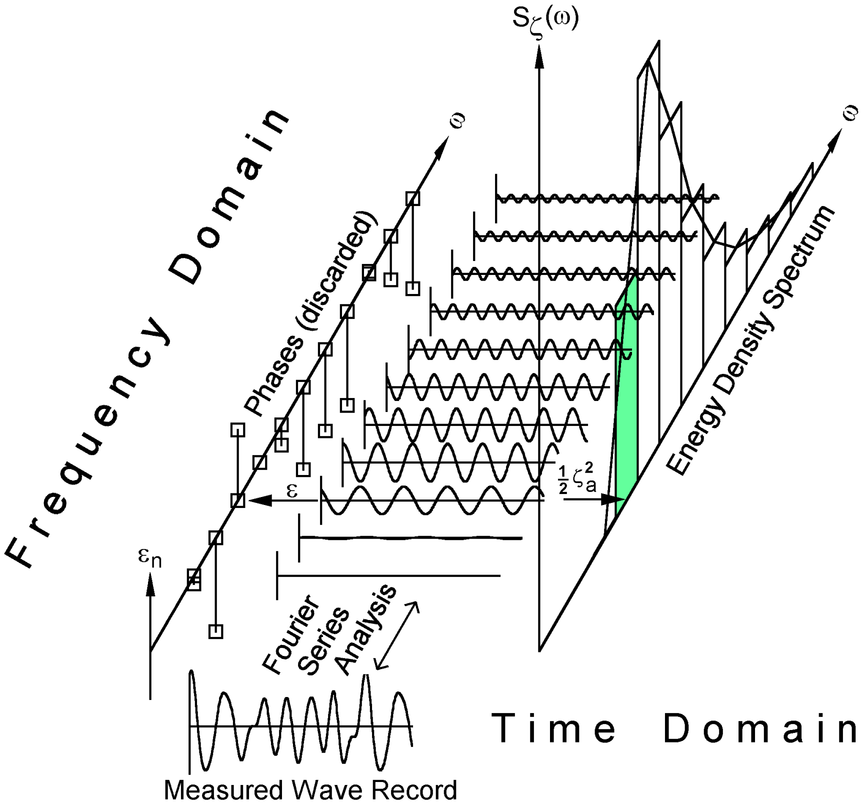


Figure 2.7: Wave Record Analysis

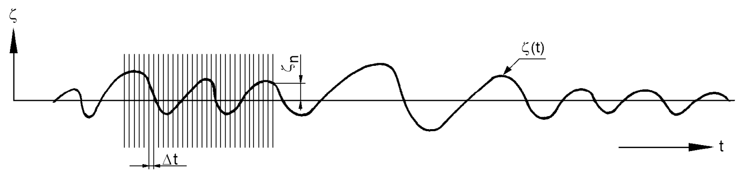


Figure 2.8: Registration and Sampling of a Wave

$$
\begin{aligned}
&= \frac{1}{N}\sum_{n=1}^{N}\zeta_n^2 = \frac{1}{N\cdot\Delta t}\sum_{n=1}^{N}\zeta_n^2\cdot\Delta t \\
&= \frac{1}{\tau}\int_0^{\tau}\zeta^2(t)\cdot dt = \frac{1}{\tau}\int_0^{\tau}\left\{\sum_{n=1}^{N}\zeta_{a_n}\cos(\omega_n t - k_n x + \varepsilon_n)\right\}^2\cdot dt \\
&= \sum_{n=1}^{N}\frac{1}{2}\zeta_{a_n}^2
\end{aligned}
\qquad (2.24)
$$

The wave amplitude, ζ_{a_n}, can be expressed by a wave spectrum, $S_\zeta(\omega_n)$:

$$
\text{Definition:}\qquad S_\zeta(\omega_n)\cdot\Delta\omega = \sum_{\omega_n}^{\omega_n+\Delta\omega}\frac{1}{2}\zeta_{a_n}^2(\omega) \qquad (2.25)
$$

where $\Delta\omega$ is a constant difference between two successive frequencies; see figure 2.9. Multiplied with ρg, this expression is the energy per unit area of the waves in the frequency interval $\Delta\omega$, which will be treated in detail in a next lecture.

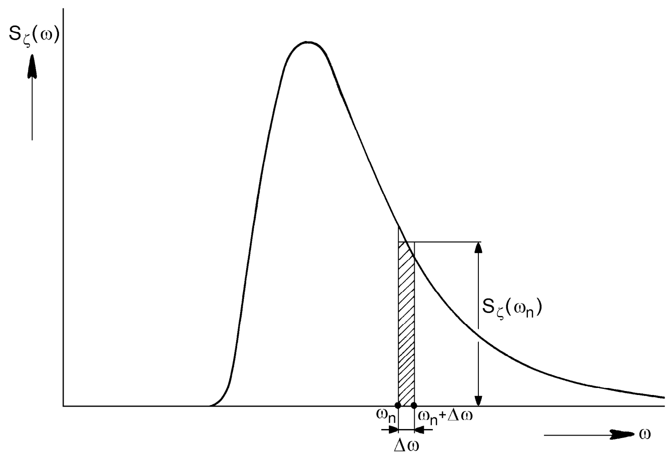


Figure 2.9: Definition of Spectral Density

Letting $\Delta\omega \rightarrow 0$, the definition of the wave energy spectrum, $S_\zeta(\omega)$, becomes:

$$\boxed{S_\zeta(\omega_n) \cdot d\omega = \frac{1}{2}\zeta_{a_n}^2} \tag{2.26}$$

and the variance, σ_ζ^2, of the water surface elevation is simply equal to the area under the spectrum:

$$\sigma_\zeta^2 = \int_0^\infty S_\zeta(\omega) \cdot d\omega \tag{2.27}$$

Figure 2.7 gives a graphical interpretation of the meaning of a wave spectrum and how it relates to the waves. The irregular wave history, $\zeta(t)$, in the time domain at the lower left hand part of the figure can be expressed via Fourier series analysis as the sum of a large number of regular wave components, each with its own frequency, amplitude and phase in the frequency domain. These phases will appear to be rather random, by the way. The value $\frac{1}{2}\zeta_a^2(\omega)/\Delta\omega$ - associated with each wave component on the ω-axis - is plotted vertically in the middle; this is the wave energy spectrum, $S_\zeta(\omega)$. This spectrum, $S_\zeta(\omega)$, can be described nicely in a formula; the phases are usually thrown away.

Wave Height and Period

Relationships with statistics can be found from computing the moments of the area under the spectrum with respect to the vertical axis at $\omega = 0$.
If m denotes a moment of properties of the wave spectrum with respect to the vertical axis at $\omega = 0$, then $m_{n\zeta}$ denotes the n^{th} order moment given in this case by:

$$\boxed{m_{n\zeta} = \int_0^\infty \omega^n \cdot S_\zeta(\omega) \cdot d\omega} \tag{2.28}$$

This means that $m_{0\zeta}$ is the area under the spectral curve, $m_{1\zeta}$ is the first order moment (static moment) of this area and $m_{2\zeta}$ is the second order moment (moment of inertia) of this area.
As has already been indicated, $m_{0\zeta}$ is an indication of the variance squared, $m_{0\zeta} = \sigma_\zeta^2$, of the water surface elevation. Of course this $m_{0\zeta}$ can also be related to the various wave amplitudes and heights:

$$\boxed{\sigma_\zeta = RMS = \sqrt{m_{0\zeta}}} \qquad \text{(Root Mean Square of the water surface elevation)}$$

$$\boxed{\zeta_{a_{1/3}} = 2 \cdot \sqrt{m_{0\zeta}}} \qquad \text{(significant wave amplitude)}$$

$$\boxed{H_{1/3} = 4 \cdot \sqrt{m_{0\zeta}}} \qquad \text{(significant wave height)} \tag{2.29}$$

Characteristic wave periods can be defined from the spectral moments:

$$\begin{array}{rcll} m_{1\zeta} & = & \omega_1 \cdot m_{0\zeta} & \text{with } \omega_1 \text{ is spectral centroid} \\ m_{2\zeta} & = & \omega_2^2 \cdot m_{0\zeta} & \text{with } \omega_2 \text{ is spectral radius of inertia} \end{array} \tag{2.30}$$

as follows:

$$\boxed{T_1 = 2\pi \cdot \frac{m_{0\zeta}}{m_{1\zeta}}} \qquad \text{(mean centroid wave period)}$$

$$\boxed{T_2 = 2\pi \cdot \sqrt{\frac{m_{0\zeta}}{m_{2\zeta}}}} \qquad \text{(mean zero-crossing wave period)} \tag{2.31}$$

The mean zero-crossing period, T_2, is sometimes indicated by T_z. One will often find the period associated with the peak of the spectrum, T_p, in the literature as well.

Rayleigh Distribution

Expressed in terms of $m_{0\zeta}$, the Rayleigh distribution is given by:

$$f(x) = \frac{x}{m_{0\zeta}} \cdot \exp\left\{-\frac{x^2}{2 \cdot m_{0\zeta}}\right\} \qquad \text{(Rayleigh distribution)} \tag{2.32}$$

in which x is the variable being studied and $m_{0\zeta}$ is the area under the spectral curve.
This distribution can be used for more or less narrow spectra. This is true for normal wave spectra; they are not too wide. Generally, its frequencies vary between $\omega = 0.2$ and $\omega = 1.5$-2.0.
With this distribution, the probability that the wave amplitude, ζ_a, exceeds a chosen threshold value, a, can be calculated using:

$$\begin{aligned} P\left\{\zeta_a > a\right\} &= \int_a^\infty f(x) \cdot dx \\ &= \frac{1}{m_{0\zeta}} \int_a^\infty x \cdot \exp\left\{-\frac{x^2}{2 \cdot m_{0\zeta}}\right\} \cdot dx \end{aligned} \tag{2.33}$$

or:

$$\boxed{P\left\{\zeta_a > a\right\} = \exp\left\{-\frac{a^2}{2 \cdot m_{0\zeta}}\right\}} \tag{2.34}$$

The number of times per hour that the threshold value, a, will be exceeded by the waves is this probability, $P\left\{\zeta_a > a\right\}$, times the number of oscillations per hour, $3600 / T_2$:

$$\boxed{N_{hour} = P\left\{\zeta_a > a\right\} \cdot \frac{3600}{T_2}} \tag{2.35}$$

2.2.4 Standard Wave Spectra

Investigators have attempted to describe a wave frequency spectrum in a standard form. Two important ones often found in the literature are described here. The mathematical formulations of these normalized uni-directional wave energy spectra are based on two parameters: the significant wave height, $H_{1/3}$, and average wave periods $\overline{T} = T_1$, T_2 or T_p:

$$\boxed{S_\zeta(\omega) = H_{1/3}^2 \cdot f\left(\omega, \overline{T}\right)} \tag{2.36}$$

Note that this definition means that the spectral values are proportional to the significant wave height squared; in other words $S_\zeta(\omega)/H_{1/3}^2$ is a function of ω and $\overline{T}$ only.

Bretschneider Wave Spectra

One of the oldest and most popular wave spectra was given by Bretschneider. It is especially suited for open sea areas and mathematically given by:

$$S_\zeta(\omega) = \frac{173 \cdot H_{1/3}^2}{T_1^4} \cdot \omega^{-5} \cdot \exp\left\{\frac{-692}{T_1^4} \cdot \omega^{-4}\right\} \tag{2.37}$$

For not-truncated wave spectra, other wave period definitions can be used by substituting:

$$T_1 = 1.086 \cdot T_2 \qquad \text{or} \qquad T_1 = 0.772 \cdot T_p$$

JONSWAP Wave Spectra

In 1968 and 1969 an extensive wave measurement program, known as the Joint North Sea Wave Project (JONSWAP) was carried out along a line extending over 100 miles into the North Sea from Sylt Island. Analysis of the data yielded a spectral formulation for fetch-limited wind generated seas.
The following definition of a Mean JONSWAP wave spectrum is advised by the 17th ITTC in 1984 for fetch limited situations:

$$S_\zeta(\omega) = \frac{320 \cdot H_{1/3}^2}{T_p^4} \cdot \omega^{-5} \cdot \exp\left\{\frac{-1950}{T_p^4} \cdot \omega^{-4}\right\} \cdot \gamma^A \tag{2.38}$$

with:

$$\begin{aligned}
\gamma &= 3.3 \quad \text{(peakedness factor)} \\
A &= \exp\left\{-\left(\frac{\frac{\omega}{\omega_p}-1}{\sigma\sqrt{2}}\right)^2\right\} \\
\omega_p &= \frac{2\pi}{T_p} \quad \text{(circular frequency at spectral peak)} \\
\sigma &= \text{a step function of } \omega\text{:} \;\; \text{if } \omega < \omega_p \text{ then: } \sigma = 0.07 \\
& \qquad\qquad\qquad\qquad\quad\; \text{if } \omega > \omega_p \text{ then: } \sigma = 0.09
\end{aligned}$$

Taking $\gamma^A = 1.522$ results in the formulation of the Bretschneider wave spectrum with the peak period, T_p. Sometimes, a third free parameter is introduced in the JONSWAP wave spectrum by varying the peakedness factor, γ.
For not-truncated wave spectra, other wave period definitions can be used by substituting:

$$T_p = 1.200 \cdot T_1 \qquad \text{or} \qquad T_p = 1.287 \cdot T_2$$

Wave Spectra Comparison

Figure 2.10 compares the Bretschneider and mean JONSWAP wave spectra for three sea states with a significant wave height, $H_{1/3}$, of 4 meters and peak periods, T_p, of 6, 8 and 10 seconds, respectively. The figure shows the more pronounced peak of the JONSWAP spectrum, while the areas under all spectral density curves are the same.

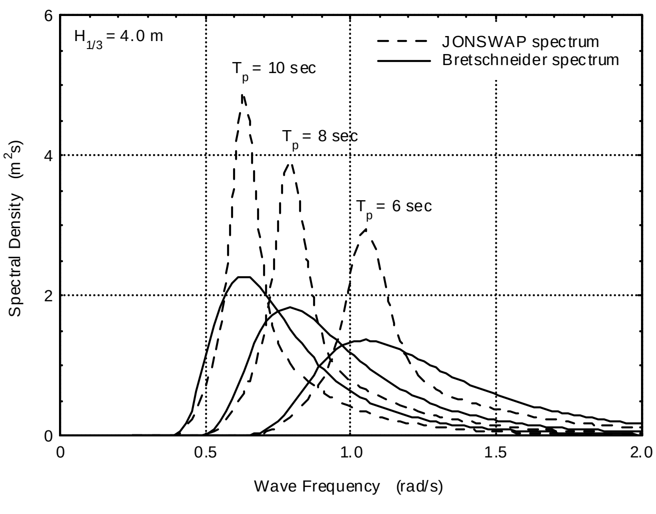


Figure 2.10: Comparison of Two Spectral Formulations

2.2.5 Wave Prediction and Climatology

In 1805, the British Admiral Sir Francis Beaufort devised an observation scale for measuring winds at sea. His scale measures winds by observing their effects on sailing ships and waves

and is still used today by many weather stations. A definition of this Beaufort wind force scale is given in figure 2.11.
The pictures in figure 2.12 give a visual impression of the sea states in relation to Beaufort's scale.

Short Term Wave Data

An entire storm can be characterized by just two numbers: one related to the wave height and one to the wave period. It now becomes important to predict these values from other data - such as geographical and meteorological information. Figure 2.13 for "Open Ocean Areas" and "North Sea Areas" gives an indication of an average relationship between the Beaufort wind scale and the significant wave height $H_{1/3}$ and the average wave periods T_1 and T_2, defined before.
Notice that these short term or storm wave data are linked here to wind data. This is quite common in practice since wind data is often much more available or can be predicted rather easily from other available meteorological data. But, these relations are an indication only. Fixed relations between wave height and period does not exist; the history and duration of the wind plays an important role.

Long Term Wave Data

Longer term wave climatology is used to predict the statistical chance that for instance a given wave-sensitive offshore operation - such as lifting a major topside element into place - will be delayed by sea conditions which are too rough. Sets of characteristic wave data values can be grouped and arranged in a table such as that given below for all wave directions in the winter season in areas 8, 9, 15 and 16 of the North Atlantic Ocean. A "storm" here is an arbitrary time period - often of 3 or 6 hours - for which a single pair of values has been collected. The number in each cell of this table indicates the chance that a significant wave height is between the values in the left column and in the range of wave periods listed at the top of the table.

Winter Data of Areas 8, 9, 15 and 16 of the North Atlantic (Global Wave Statistics)												
	T_2 (s)											
H_s (m)	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5	12.5	13.5	Total
14.5	0	0	0	0	2	30	154	362	466	370	202	1586
13.5	0	0	0	0	3	33	145	293	322	219	101	1116
12.5	0	0	0	0	7	72	289	539	548	345	149	1949
11.5	0	0	0	0	17	160	585	996	931	543	217	3449
10.5	0	0	0	1	41	363	1200	1852	1579	843	310	6189
9.5	0	0	0	4	109	845	2485	3443	2648	1283	432	11249
8.5	0	0	0	12	295	1996	5157	6323	4333	1882	572	20570
7.5	0	0	0	41	818	4723	10537	11242	6755	2594	703	37413
6.5	0	0	1	138	2273	10967	20620	18718	9665	3222	767	66371
5.5	0	0	7	471	6187	24075	36940	27702	11969	3387	694	111432
4.5	0	0	31	1586	15757	47072	56347	33539	11710	2731	471	169244
3.5	0	0	148	5017	34720	74007	64809	28964	7804	1444	202	217115
2.5	0	4	681	13441	56847	77259	45013	13962	2725	381	41	210354
1.5	0	40	2699	23284	47839	34532	11554	2208	282	27	2	122467
0.5	5	350	3314	8131	5858	1598	216	18	1	0	0	19491
Total	5	394	6881	52126	170773	277732	256051	150161	61738	19271	4863	999995

These wave scatter diagrams can be used to determine the long term probability for storms exceeding certain sea states. Each cell in this table presents the probability of occurrence of its significant wave height and zero-crossing wave period range. This probability is equal

WIND SPEEDS — BEAUFORT WIND FORCE SCALE

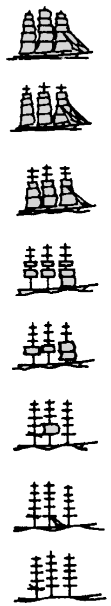
Beaufort Number	Knots	miles per hr. (U.S. Statute)	meters per sec.	km per hr.	Wind Press. N/m^2	Beaufort description for square rigged ships 1806	Racing Sailor's description (C.A. Marchay, 1964)	U.S. Weather Service description	Dutch KNMI description	Beaufort Number
0	0 1		0 0.5	0 2				Calm	Windstil	0
1	1 3	1 3	0.5 1.5	2 6	0.14 1.4	Just Steerage Way	Boredom	Light air		1
2	4 6	4 7	2.1 3.1	7 11	2.4 5.7	1-3 knots close hauled	Mild pleasure	Light breeze	zwakke	2
3	7 10	8 12	3.6 5.1	13 19	7.7 16	4-5 knots close hauled	Pleasure	Gentle breeze		3
4	11 16	13 18	5.7 8	20 30	19 41	6-7 knots close hauled	Great Pleasure	Moderate breeze	matige	4
5	17 21	19 24	9 11	32 39	46 67	Hull Speed Full Sail 	Delight	Fresh breeze	vrij krachtige	5
6	22 27	25 31	11 14	41 50	77 115		Delight tinged with anxiety	Strong breeze	krachtige	6
7	28 33	32 38	14 17	52 61	125 172		Anxiety tinged with fear	Moderate Gale	harde	7
8	34 40	39 46	18 21	63 74	182 250		Fear tinged with terror	Gale	stormachtige	8
9	41 47	47 54	21 24	76 87	270 350		Great terror	Strong Gale	storm	9
10	48 55	55 63	25 28	89 102	360 480		Panic	Whole Gale	zware storm	10
11	56 63	64 75	29 33	104 120	500 630		I want my mummy!!	Storm	zeer zware storm	11
12	above 63	above 75	above 33	above 120	above 630	bare poles	Yes, Mr. Jones	Hurricane	orkaan	12

Figure 2.11: Beaufort's Wind Force Scale

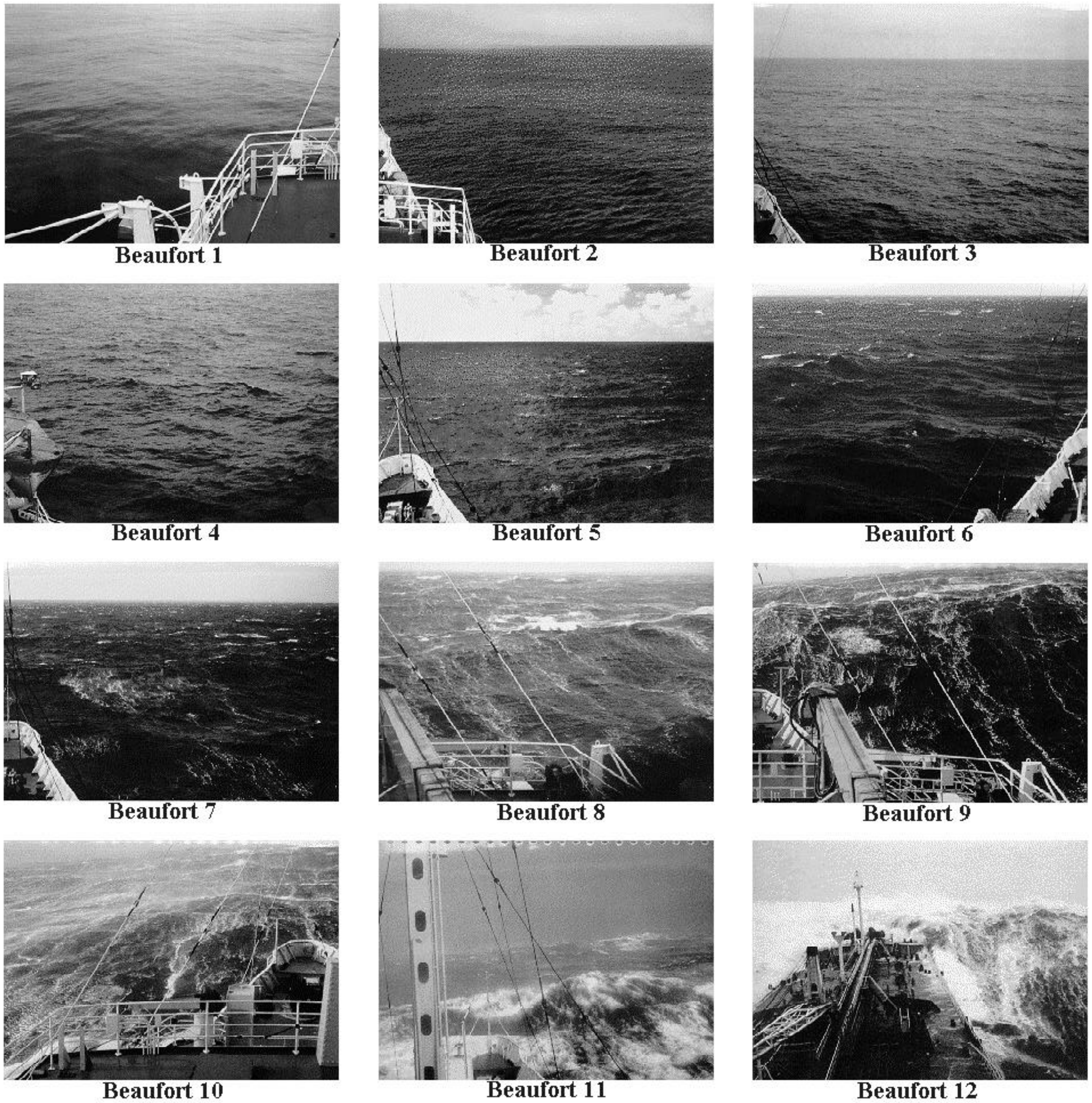


Figure 2.12: Sea State in Relation to Beaufort Wind Force Scale

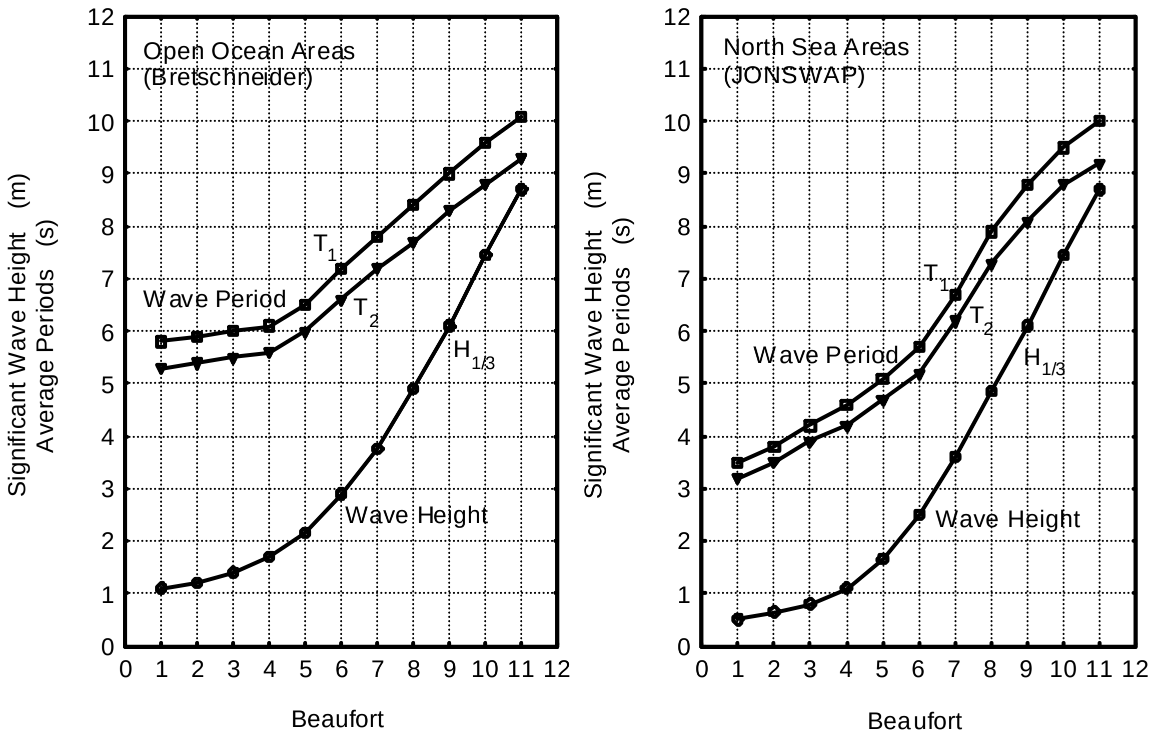


Figure 2.13: Wave Spectrum Parameter Estimates

to the number in this cell divided by the sum of the numbers of all cells in the table, for instance:

$$\Pr\left\{4 < H_{1/3} < 5 \text{ and } 8 < T_2 < 9\right\} = \frac{47072}{999996} = 0.047 = 4.7\%$$

For instance, the probabilty on a storm with a significant wave height between 4 and 6 meters with a zero-crossing period between 8 and 10 seconds is:

$$\Pr\left\{3 < H_{1/3} < 5 \text{ and } 8 < T_2 < 10\right\} = \frac{47072 + 56347 + 74007 + 64809}{999996} = 0.242 = 24.2\%$$

The probabilty for storms exceeding a certain significant wave height is found by adding the numbers of all cells with a significant wave height larger than this certain significant wave height and dividing this number by the sum of the numbers in all cells, for instance:

$$\Pr\left\{H_{1/3} > 10\right\} = \frac{6189 + 3449 + 1949 + 1116 + 1586}{999996} = 0.014 = 1.4\%$$

2.2.6 Numerical Exercises

Exercise 1

The following histogram of wave heights has been derived from a time history of the surface elevation of an irregular wave system.

wave height $\widetilde{H}_w$ (m)	number n (-)
< 0.25	0
0.25 - 0.75	30
0.75 - 1.25	60
1.25 - 1.75	110
1.75 - 2.25	42
2.25 - 2.75	28
2.75 - 3.25	18
3.25 - 3.75	10
3.75 - 4.25	2
> 4.25	0

1. Give a definition of the conception "significant wave height, $H_{1/3}$" and calculate this value.

2. What is the probability in this storm that the double wave amplitude, $\widetilde{H}_w$, will exceed a value of 2.75 meter?

3. What is the probability that the significant wave height, $H_{1/3}$, will be exceeded?

Solutions:

1. $H_{1/3} = 2.51$ m.

2. $P\left\{\widetilde{H}_w > 2.75 \text{ m}\right\} = 0.10.$

3. $P\left\{\widetilde{H}_w > H_{1/3}\right\} \approx 0.146.$

Exercise 2

A simplified wave energy spectrum is given by:

ω (1/s)	0.5	0.7	0.9	1.1	1.3	1.5
$S_\zeta(\omega)$ m^{2}s	0.00	0.75	0.95	0.43	0.12	0.00

1. Calculate (by using the trapezoid rule) the significant wave height, $H_{1/3}$, and the mean wave periods T_1 and T_2. Give a physical explanation of each of these phenomena.

2. Determine the probability, P, of exceeding a wave height of 4.0 meter in this wave spectrum by using the Rayleigh probability density function.

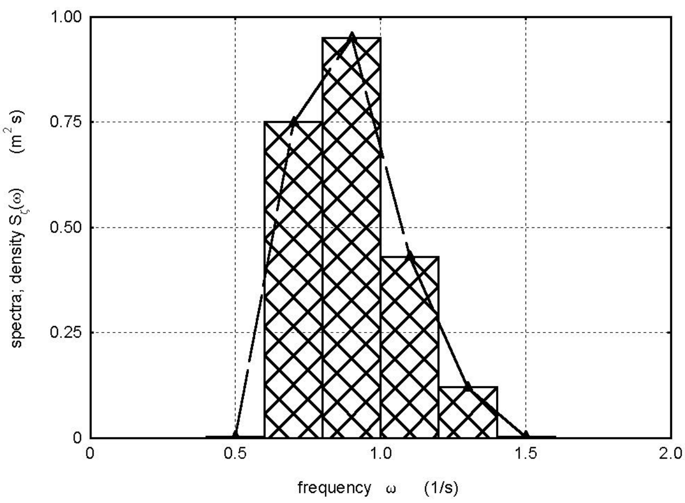


Figure 2.14: Simplified Wave Energy Spectrum

3. Determine also the number of times per hour that this wave height will be exceeded.
4. Explain the term m_0 in the Rayleigh probability density function.
   Which restriction should be kept in mind when using this probability density function?
5. What is the probability that the significant wave height, $H_{1/3}$, will be exceeded?
6. What is the disadvantage of the use of T_2 when analyzing a measured wave spectrum?

Solutions:

1. $H_{1/3} = 2.7$ m, $T_1 = 7.0$ s and $T_2 = 6.9$ s.
2. $P = 1.2$ %
3. 6.3 times per hour. Note: Use T_2 for this and not T_1!
4. The magnitude of m_0 in the Rayleigh probability density function is the spectral area of the considered time-dependent variable. This Rayleigh distribution is valid for more or less narrow spectra.
5. $P\left\{2\zeta_a > H_{1/3}\right\} = e^{-2} \approx 0.135$.
6. The more or less uncertain or truncated "tail" of the spectrum has a larger influence on T_2 than it has on T_1.

Chapter 3

Behavior of Structures in Waves

The dynamics of rigid bodies and fluid motions are governed by the combined actions of different external forces and moments as well as by the inertia of the bodies themselves. In fluid dynamics these forces and moments can no longer be considered as acting at a single point or at discrete points of the system. Instead, they must be distributed in a relatively smooth or a continuous manner throughout the mass of the fluid particles. The force and moment distributions and the kinematic description of the fluid motions are in fact continuous, assuming that the collection of discrete fluid molecules can be analyzed as a continuum.

Two very good and readable books are advised here as a supplement to these lectures.
A.R.J.M. Lloyd published in 1989 his book on ship hydromechanics, *Seakeeping, Ship Behaviour in Rough Weather*, ISBN 0-7458-0230-3, Ellis Horwood Limited, Market Cross House, Cooper Street, Chichester, West Sussex, P019 1EB England.
O.M. Faltinsen, Professor of Marine Technology at the Norwegian University of Science and Technology is the author of *Sea Loads on Ships and Offshore Structures,* published in 1990 in the Cambridge University Press Series on Ocean Technology.

3.1 Behavior in Regular Waves

When on board a ship looking toward the **bow** (front end) one is looking **forward**. The **stern** is **aft** at the other end of the ship. As one looks forward, the **starboard** side is one's right and the **port** side is to one's left.
The motions of a ship, just as for any other rigid body, can be split into three mutually perpendicular translations of the center of gravity, G, and three rotations around G:

- three translations of the ship's center of gravity (**CoG** or G) in the direction of the x-, y- and z-axes:
  - surge in the longitudinal x-direction, positive forwards,
  - sway in the lateral y-direction, positive to port side, and
  - heave in the vertical z-direction, positive upwards.

[0] J.M.J. Journée and J.A. Pinkster, *"SHIP HYDROMECHANICS, Part I: Introduction"*, Draft Edition, January 2001, Ship Hydromechanics Laboratory, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands. For updates see web site: http://www.shipmotions.nl.

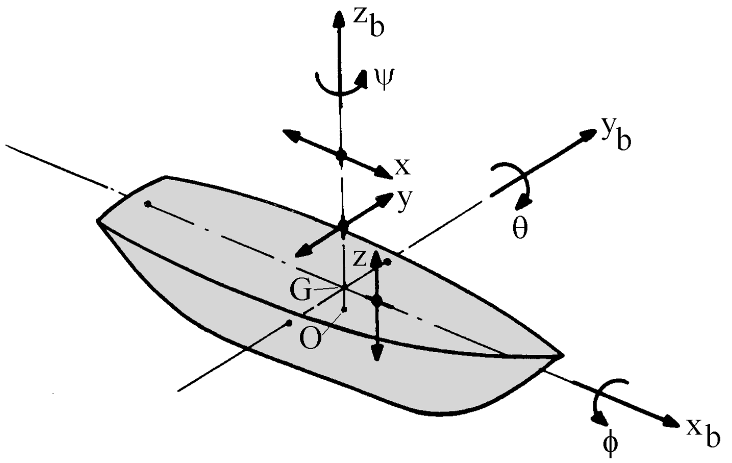


Figure 3.1: Definition of Ship Motions in Six Degrees of Freedom

- three rotations about these axes:
  - roll about the x-axis, positive right turning,
  - pitch about the y-axis, positive right turning, and
  - yaw about the z-axis, positive right turning.

  These definitions have been visualized in figure 3.1.

In many cases these motion components will have small amplitudes. Any ship motion is build up from these basic motions. For instance, the vertical motion of a bridge wing is mainly build up by heave, pitch and roll motions.
Another important motion is the vertical relative motion, defined as the vertical wave elevation minus the local vertical motion of the ship. Thus, this is the motion that one observes when looking over the rail downwards to the waves.

3.1.1 Axis Conventions

Three right-handed orthogonal coordinate systems are used to define the ship motions:

- An **earth-bound** coordinate system $S(x_0, y_0, z_0)$.
  The (x_0, y_0)-plane lies in the still water surface, the positive x_0-axis is in the direction of the wave propagation; it can be rotated at a horizontal angle μ relative to the translating axis system $O(x, y, z)$ as shown in figure 3.2. The positive z_0-axis is directed upwards.

- A **body–bound** coordinate system $G(x_b, y_b, z_b)$.
  This system is connected to the ship with its origin at the ship's center of gravity, G. The directions of the positive axes are: x_b in the longitudinal forward direction, y_b in the lateral port side direction and z_b upwards. If the ship is floating upright in still water, the (x_b, y_b)-plane is parallel to the still water surface.

- A **steadily translating** coordinate system $O(x, y, z)$.
  This system is moving forward with a constant ship speed V. If the ship is stationary, the directions of the $O(x, y, z)$ axes are the same as those of the $G(x_b, y_b, z_b)$ axes. The (x, y)-plane lies in the still water surface with the origin O at, above or under the time-averaged position of the center of gravity G. The ship is supposed to carry out oscillations around this steadily translating $O(x, y, z)$ coordinate system.

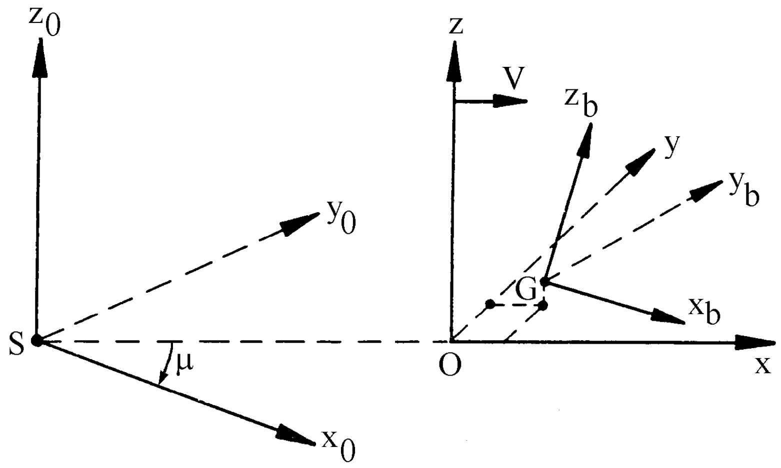


Figure 3.2: Coordinate Systems

The harmonic elevation of the wave surface ζ is defined in the earth-bound coordinate system by:

$$\zeta = \zeta_a \cos(\omega t - k x_0) \tag{3.1}$$

in which:

ζ_a	=	wave amplitude (m)
$k = 2\pi/\lambda$	=	wave number (rad/m)
λ	=	wave length (m)
ω	=	circular wave frequency (rad/s)
t	=	time (s)

3.1.2 Frequency of Encounter

The wave speed c, defined in a direction with an angle μ (wave direction) relative to the ship's speed vector V, follows from:

$$\boxed{c = \frac{\omega}{k} = \frac{\lambda}{T}} \qquad \text{(see chapter 2)} \tag{3.2}$$

The steadily translating coordinate system $O(x, y, z)$ is moving forward at the ship's speed V, which yields:

$$\boxed{x_0 = Vt \cos\mu + x \cos\mu + y \sin\mu} \tag{3.3}$$

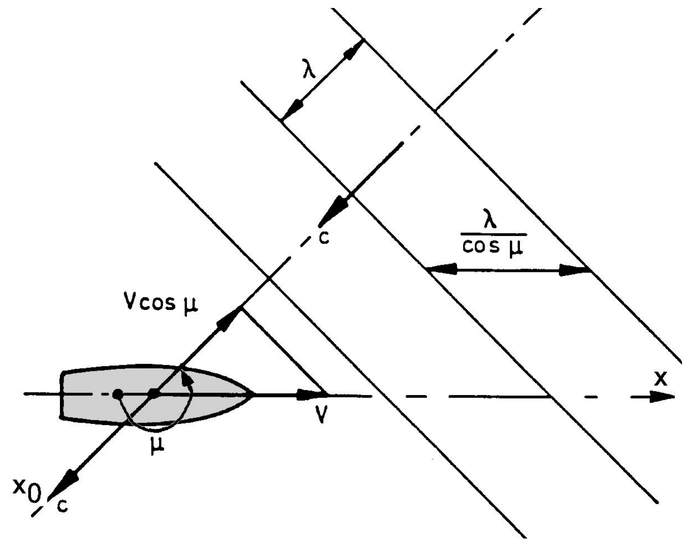


Figure 3.3: Frequency of Encounter

When a ship moves with a forward speed, the frequency at which it encounters the waves, ω_e, becomes important. Then the period of encounter, T_e, see figure 3.3, is:

$$T_e = \frac{\lambda}{c + V\cos(\mu - \pi)} = \frac{\lambda}{c - V\cos\mu} \tag{3.4}$$

and the circular frequency of encounter, ω_e, becomes:

$$\omega_e = \frac{2\pi}{T_e} = \frac{2\pi\,(c - V\cos\mu)}{\lambda} = k\,(c - V\cos\mu) \tag{3.5}$$

Note that $\mu = 0$ for following waves.
Using $k \cdot c = \omega$ from equation 3.2, the relation between the frequency of encounter and the wave frequency becomes:

$$\boxed{\omega_e = \omega - kV\cos\mu} \tag{3.6}$$

Note that at zero forward speed ($V = 0$) or in beam waves ($\mu = 90°$ or $\mu = 270°$) the frequencies ω_e and ω are identical.
In deep water, with the dispersion relation $k = \omega^2/g$, this frequency relation becomes:

$$\omega_e = \omega - \frac{\omega^2}{g}V\cos\mu \qquad \text{(deep water)} \tag{3.7}$$

Using the frequency relation in equation 3.6 and equations 3.1 and 3.3, it follows that the wave elevation can be given by:

$$\boxed{\zeta = \zeta_a\cos(\omega_e t - kx\cos\mu - ky\sin\mu)} \tag{3.8}$$

3.1.3 Motions of and about CoG

The ship motions in the steadily translating $O(x, y, z)$ system are defined by three translations of the ship's center of gravity (CoG) in the direction of the x-, y- and z-axes and three rotations about them as given in figure 3.1:

$$
\begin{array}{rcl}
\text{Surge} & : & x = x_a \cos(\omega_e t + \varepsilon_{x\zeta}) \\
\text{Sway} & : & y = y_a \cos(\omega_e t + \varepsilon_{y\zeta}) \\
\text{Heave} & : & z = z_a \cos(\omega_e t + \varepsilon_{z\zeta}) \\
\text{Roll} & : & \phi = \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta}) \\
\text{Pitch} & : & \theta = \theta_a \cos(\omega_e t + \varepsilon_{\theta\zeta}) \\
\text{Yaw} & : & \psi = \psi_a \cos(\omega_e t + \varepsilon_{\psi\zeta})
\end{array} \tag{3.9}
$$

in which each of the ε values is a different phase angle.
Knowing the motions of and about the center of gravity, G, one can calculate the motions in any point on the structure using superposition.
The phase shifts of these motions are related to the harmonic wave elevation at the origin of the steadily translating $O(x, y, z)$ system, the average position of the ship's center of gravity - even though no wave can be measured there:

$$
\text{Wave elevation at } O \text{ or } G: \qquad \zeta = \zeta_a \cos(\omega_e t) \tag{3.10}
$$

3.1.4 Displacement, Velocity and Acceleration

The harmonic velocities and accelerations in the steadily translating $O(x, y, z)$ coordinate system are found by taking the derivatives of the displacements.
For roll:

$$
\begin{array}{rcl}
\text{Displacement} & : & \phi = \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta}) \\
\text{Velocity} & : & \dot{\phi} = -\omega_e \phi_a \sin(\omega_e t + \varepsilon_{\phi\zeta}) = \omega_e \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta} + \pi/2) \\
\text{Acceleration} & : & \ddot{\phi} = -\omega_e^2 \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta}) = \omega_e^2 \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta} + \pi)
\end{array} \tag{3.11}
$$

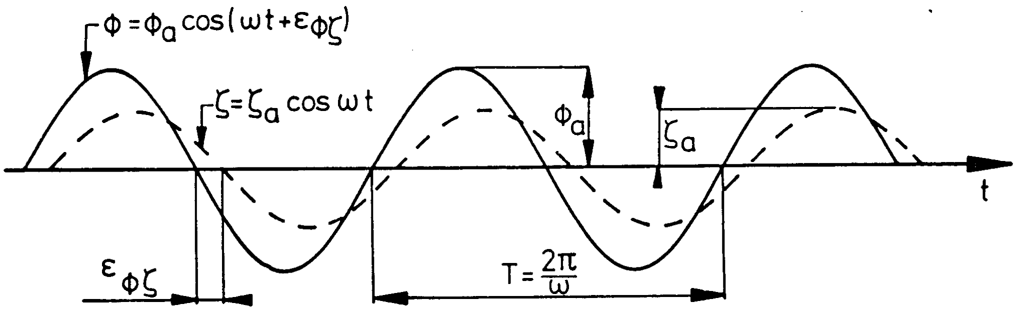


Figure 3.4: Harmonic Wave and Roll Signal

The phase shift of the roll motion with respect to the wave elevation, $\varepsilon_{\phi\zeta}$ in figure 3.4, is positive, here because when the wave elevation passes zero at a certain instant, the roll motion already has passed zero. Thus, if the roll motion, ϕ, comes before the wave elevation, ζ, then the phase shift, $\varepsilon_{\phi\zeta}$, is defined as positive. This convention will hold for all other responses as well of course.
Figure 3.5 shows a sketch of the time histories of the harmonic angular displacements, velocities and accelerations of roll. Note the mutual phase shifts of $\pi/2$ and π.

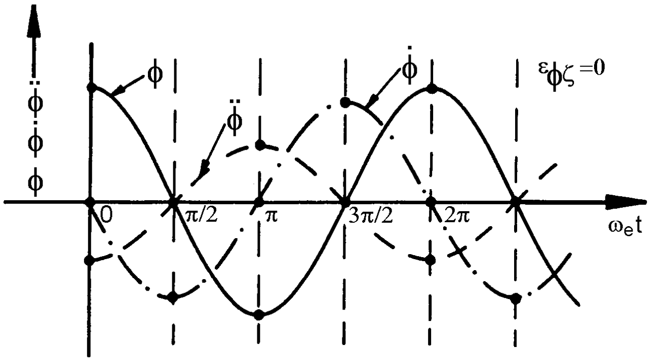


Figure 3.5: Displacement, Acceleration and Velocity

3.1.5 Motions Superposition

Knowing the motions of and about the center of gravity, G, one can calculate the motions in any point on the structure using superposition.

Absolute Motions

Absolute motions are the motions of the ship in the steadily translating coordinate system $O(x, y, z)$. The angles of rotation ϕ, θ and ψ are assumed to be small (for instance < 0.1 rad.), which is a necessity for linearizations. They must be expressed in radians, because in the linearization it is assumed that:

$$\boxed{\sin\phi \approx \phi} \qquad \text{and} \qquad \boxed{\cos\phi \approx 1.0} \tag{3.12}$$

For small angles, the transformation matrix from the body-bound coordinate system to the steadily translating coordinate system is very simple:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -\psi & \theta \\ \psi & 1 & -\phi \\ -\theta & \phi & 1 \end{pmatrix} \cdot \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \tag{3.13}$$

Using this matrix, the components of the **absolute harmonic motions** of a certain point $P(x_b, y_b, z_b)$ on the structure are given by:

$$\begin{array}{c} \boxed{x_P = x - y_b\psi + z_b\theta} \\ \boxed{y_P = y + x_b\psi - z_b\phi} \\ \boxed{z_P = z - x_b\theta + y_b\phi} \end{array} \tag{3.14}$$

in which x, y, z, ϕ, θ and ψ are the motions of and about the center of gravity, G, of the structure.
As can be seen in equation 3.14, the vertical motion, z_P, in a point $P(x_b, y_b, z_b)$ on the floating structure is made up of heave, roll and pitch contributions. When looking more detailed to this motion - it is called here now $h(\omega_e, t)$ for convenient writing - it can be found:

$$h(\omega_e, t) \;=\; z - x_b\theta + y_b\phi$$

$$
\begin{aligned}
&= z_a \cos(\omega_e t + \varepsilon_{z\zeta}) - x_b \theta_a \cos(\omega_e t + \varepsilon_{\theta\zeta}) + y_b \phi_a \cos(\omega_e t + \varepsilon_{\phi\zeta}) \\
&= \quad \{+z_a \cos \varepsilon_{z\zeta} - x_b \theta_a \cos \varepsilon_{\theta\zeta} + y_b \phi_a \cos \varepsilon_{\phi\zeta}\} \cdot cos(\omega_e t) \\
&\quad - \{+z_a \sin \varepsilon_{z\zeta} - x_b \theta_a \sin \varepsilon_{\theta\zeta} + y_b \phi_a \sin \varepsilon_{\phi\zeta}\} \cdot sin(\omega_e t)
\end{aligned} \tag{3.15}
$$

As this motion h has been obtained by a linear superposition of three harmonic motions, this (resultant) motion must be harmonic as well:

$$
\begin{aligned}
h\,(\omega_e, t) &= h_a \cos(\omega_e t + \varepsilon_{h\zeta}) \\
&= \{h_a \cos \varepsilon_{h\zeta}\} \cdot \cos(\omega_e t) - \{h_a \sin \varepsilon_{h\zeta}\} \cdot \sin(\omega_e t)
\end{aligned} \tag{3.16}
$$

in which h_a is the motion amplitude and $\varepsilon_{h\zeta}$ is the phase lag of the motion with respect to the wave elevation at G.
By equating the terms with $\cos(\omega_e t)$ in equations 3.15 and 3.16 ($\omega_e t = 0$, so the $\sin(\omega_e t)$-terms are zero) one finds the in-phase term $h_a \cos \varepsilon_{h\zeta}$; equating the terms with $\sin(\omega_e t)$ in equations 3.15 and 3.16 ($\omega_e t = \pi/2$, so the $\cos(\omega_e t)$-terms are zero) provides the out-of-phase term $h_a \sin \varepsilon_{h\zeta}$ of the vertical displacement in P:

$$
\begin{aligned}
h_a \cos \varepsilon_{h\zeta} &= +z_a \cos \varepsilon_{z\zeta} - x_b \theta_a \cos \varepsilon_{\theta\zeta} + y_b \phi_a \cos \varepsilon_{\phi\zeta} \\
h_a \sin \varepsilon_{h\zeta} &= +z_a \sin \varepsilon_{z\zeta} - x_b \theta_a \sin \varepsilon_{\theta\zeta} + y_b \phi_a \sin \varepsilon_{\phi\zeta}
\end{aligned} \tag{3.17}
$$

Since the right hand sides of equations 3.17 are known, the amplitude h_a and phase shift $\varepsilon_{h\zeta}$ become:

$$
\boxed{h_a = \sqrt{(h_a \sin \varepsilon_{h\zeta})^2 + (h_a \cos \varepsilon_{h\zeta})^2}}
$$

$$
\boxed{\varepsilon_{h\zeta} = \arctan \left\{ \frac{h_a \sin \varepsilon_{h\zeta}}{h_a \cos \varepsilon_{h\zeta}} \right\} \quad \text{with: } 0 \leq \varepsilon_{h\zeta} \leq 2\pi} \tag{3.18}
$$

The phase angle $\varepsilon_{h\zeta}$ has to be determined in the correct quadrant between 0 and 2π. This depends on the signs of both the numerator and the denominator in the expression for the arctangent. If the phase shift $\varepsilon_{h\zeta}$ has been determined between $-\pi/2$ and $+\pi/2$ and $h_a \cos \varepsilon_{h\zeta}$ is negative, then π should be added or subtracted from this $\varepsilon_{h\zeta}$ to obtain the correct phase shift.

For ship motions, the relations between displacement or rotation, velocity and acceleration are very important. The vertical velocity and acceleration of point P on the structure follow simply from the first and second derivative with respect to the time of the displacement in equation 3.15:

$$
\begin{aligned}
\dot{h} &= -\omega_e h_a \;\sin(\omega_e t + \varepsilon_{h\zeta}) = \{\omega_e h_a\} \cdot \cos(\omega_e t + \{\varepsilon_{h\zeta} + \pi/2\}) \\
\ddot{h} &= -\omega_e^2 h_a \cos(\omega_e t + \varepsilon_{h\zeta}) = \{\omega_e^2 h_a\} \cdot \cos(\omega_e t + \{\varepsilon_{h\zeta} + \pi\})
\end{aligned} \tag{3.19}
$$

The amplitudes of the motions and the phase shifts with respect to the wave elevation at G are given between braces {...} here.

Vertical Relative Motions

The vertical relative motion of the structure with respect to the undisturbed wave surface is the motion that one sees when looking overboard from a moving ship, downwards toward the waves. This relative motion is for instance of importance for shipping water on deck. The **vertical relative motion** s at $P(x_b, y_b)$ is defined by:

$$\boxed{s = \zeta_P - h} = \zeta_P - z + x_b\theta - y_b\phi \tag{3.20}$$

with for the local wave elevation:

$$\zeta_P = \zeta_a \cos(\omega_e t - kx_b \cos\mu - ky_b \sin\mu) \tag{3.21}$$

where $-kx_b \cos\mu - ky_b \sin\mu$ is the phase shift of the local wave elevation relative to the wave elevation in the center of gravity.
The amplitude and phase shift of this relative motion of the structure can be determined in a way analogous to that used for the absolute motion.

3.1.6 Equations of Motion

Consider a seaway with irregular waves of which the energy distribution over the wave frequencies (the wave spectrum) is known. These waves are input to a system that possesses linear characteristics. These frequency characteristics are known, for instance via model experiments or computations. The output of the system is the motion of the floating structure. This motion has an irregular behavior, just as the seaway that causes the motion. The block diagram of this principle is given in figure 3.6.

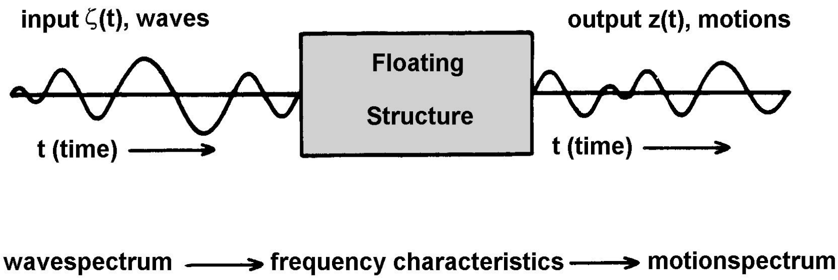


Figure 3.6: Relation between Motions and Waves

The first harmonics of the motion components of a floating structure are often of interest, because in many cases a very realistic mathematical model of the motions in a seaway can be obtained by making use of a superposition of these components at each of a range of frequencies; motions in the so-called frequency domain will be considered here.
In many cases the ship motions have mainly a linear behavior. This means that, at each frequency, the ratios between the motion amplitudes and the wave amplitudes and also the phase shifts between the motions and the waves are constant. Doubling the input (wave) amplitude results in a doubled output amplitude, while the phase shifts between output and input does not change.
As a consequence of the linear theory, the resulting motions in irregular waves can be obtained by adding together results from regular waves of different amplitudes, frequencies and possibly propagation directions. With known wave energy spectra and the calculated frequency characteristics of the responses of the ship, the response spectra and the statistics of these responses can be found.

Kinetics

A rigid body's equation of motions with respect to an earth-bound coordinate system follow from Newton's second law. The vector equations for the translations of and the rotations about the center of gravity are respectively given by:

$$\vec{F} = \frac{d}{dt}\left(m\vec{U}\right) \qquad \text{and} \qquad \vec{M} = \frac{d}{dt}\left(\vec{H}\right) \tag{3.22}$$

in which $\vec{F}$ is the resulting external force acting in the center of gravity (N), m is the mass of the rigid body (kg), $\vec{U}$ is the instantaneous velocity of the center of gravity (m/s), $\vec{M}$ is the resulting external moment acting about the center of gravity (Nm), $\vec{H}$ is the instantaneous angular momentum about the center of gravity (Nms) and t is the time (s). The total mass as well as its distribution over the body is considered to be constant during a time which is long relative to the oscillation period of the motions.

Loads Superposition

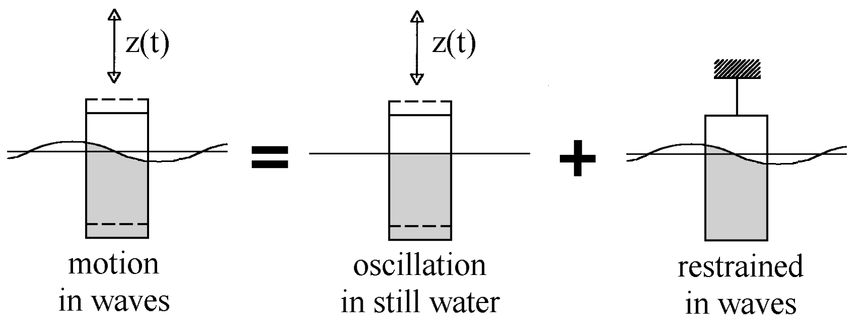


Figure 3.7: Superposition of Hydromechanical and Wave Loads

Since the system is linear, the resulting motion in waves can be seen as a superposition of the motion of the body in still water and the forces on the restrained body in waves. Thus, two important assumptions are made here for the loads on the right hand side of the picture equation in figure 3.7:

a. The so-called **hydromechanical forces and moments** are induced by the harmonic oscillations of the rigid body, moving in the undisturbed surface of the fluid.
b. The so-called **wave exciting forces and moments** are produced by waves coming in on the restrained body.

The vertical motion of the body follows from:

$$\frac{d}{dt}\left(\rho\nabla \cdot \dot{z}\right) = \rho\nabla \cdot \ddot{z} = F_h + F_w \tag{3.23}$$

in which ρ is the density of water (kg/m^3), ∇ is the volume of displacement of the body (m^3), F_h is the hydromechanical force in the z-direction (N) and F_w is the exciting wave force in the z-direction (N).
This superposition will be explained in more detail for a circular cylinder, floating in still water with its center line in the vertical direction, as shown in figure 3.8.

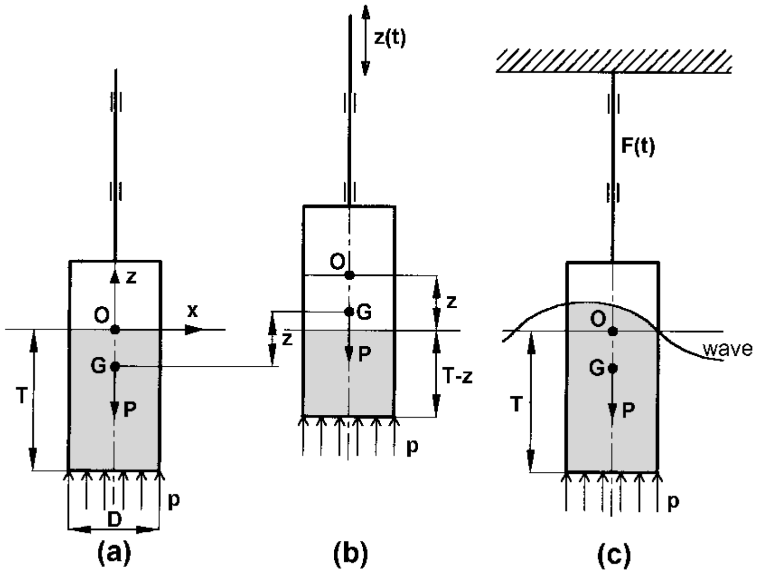


Figure 3.8: Heaving Circular Cylinder

Hydromechanical Loads

First, a free decay test in still water will be considered. After a vertical displacement upwards (see 3.8-b), the cylinder will be released and the motions can die out freely. The vertical motions of the cylinder are determined by the solid mass m of the cylinder and the hydromechanical loads on the cylinder.
Applying Newton's second law for the heaving cylinder:

$$\begin{aligned} m\ddot{z} &= \text{sum of all forces on the cylinder} \\ &= -P + pA_w - b\dot{z} - a\ddot{z} \\ &= -P + \rho g\left(T - z\right)A_w - b\dot{z} - a\ddot{z} \end{aligned} \tag{3.24}$$

With Archimedes' law $P = \rho g T A_w$, the linear equation of the heave motion becomes:

$$(m + a)\,\ddot{z} + b\dot{z} + cz = 0 \tag{3.25}$$

in which z is the vertical displacement (m), $P = mg$ is the mass force downwards (N), $m = \rho A_w T$ is the solid mass of cylinder (kg), a is the hydrodynamic mass coefficient ($\text{Ns}^2/\text{m} = \text{kg}$), b is the hydrodynamic damping coefficient ($\text{Ns/m} = \text{kg/s}$), $c = \rho g A_w$ is the restoring spring coefficient ($\text{N/m} = \text{kg/s}^2$), $A_w = \frac{\pi}{4}D^2$ is the water plane area (m^2), D is the diameter of the cylinder (m) and T is the draft of the cylinder at rest (s).
The terms $a\ddot{z}$ and $b\dot{z}$ are caused by the hydrodynamic reaction as a result of the movement of the cylinder with respect to the water. The water is assumed to be ideal and thus to behave as in a potential flow.

The vertical oscillations of the cylinder will generate waves which propagate radially from it. Since these waves transport energy, they withdraw energy from the (free) buoy's oscillations; its motion will die out. This so-called wave damping is proportional to the velocity of the cylinder $\dot{z}$ in a linear system. The coefficient b has the dimension of a mass per unit of

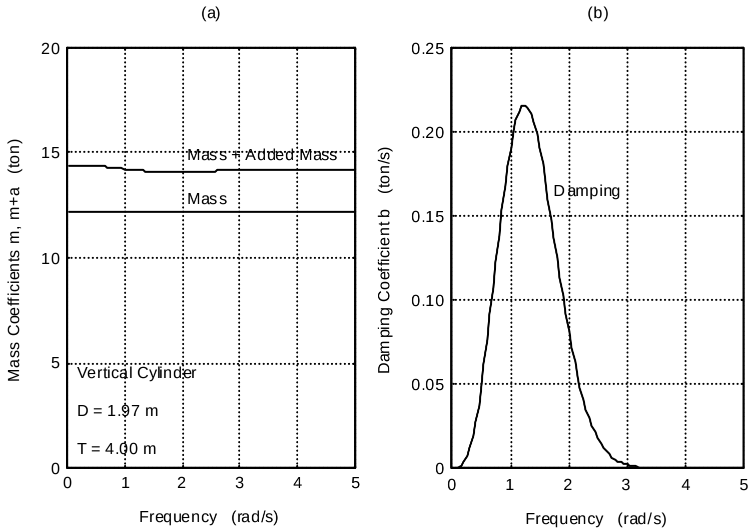


Figure 3.9: Mass and Damping of a Heaving Vertical Cylinder

time and is called the **(wave or potential) damping** coefficient. Figure 3.9-b shows the hydrodynamic damping coefficient b of a vertical cylinder as a function of the frequency of oscillation. In an actual viscous fluid, friction also causes damping, vortices and separation phenomena. Generally, these viscous contributions to the damping are non-linear, but they are usually small for most large floating structures; they are neglected here for now.
The other part of the hydromechanical reaction force $a\ddot{z}$ is proportional to the vertical acceleration of the cylinder in a linear system. This force is caused by accelerations that are given to the water particles near to the cylinder. This part of the force does not dissipate energy and manifests itself as a standing wave system near the cylinder. The coefficient a has the dimension of a mass and is called the **hydrodynamic mass** or **added mass**. Figure 3.9-a shows the hydrodynamic mass a of a vertical cylinder as a function of the frequency of oscillation.
It appears from experiments that in many cases both the acceleration and the velocity terms have a sufficiently linear behavior at small amplitudes; they are linear for practical purposes. The hydromechanical forces are the total reaction forces of the fluid on the oscillating cylinder, caused by this motion in initially still water:

$$m\ddot{z} = F_h \qquad \text{with:} \quad F_h = -a\ddot{z} - b\dot{z} - cz \tag{3.26}$$

and the equation of motion of the decaying cylinder in still water becomes:

$$(m + a) \cdot \ddot{z} + b \cdot \dot{z} + c \cdot z = 0 \tag{3.27}$$

Wave Loads

Waves are now generated in the test basin for a new series of tests. The object is restrained so that one now measures (in this vertical cylinder example) the vertical wave load on the fixed cylinder. This is shown schematically in figure 3.8-c.

The classic theory of deep water waves yields:

$$\begin{array}{lll} \text{wave potential} & : & \Phi = \frac{-\zeta_a g}{\omega} e^{kz} \sin(\omega t - kx) \\ \text{wave elevation} & : & \zeta = \zeta_a \cos(\omega t - kx) \end{array} \quad (3.28)$$

so that the pressure, p, on the bottom of the cylinder ($z = -T$) follows from the linearized Bernoulli equation:

$$\begin{aligned} p &= -\rho \frac{\partial \Phi}{\partial t} - \rho g z \\ &= \rho g \zeta_a e^{kz} \cos(\omega t - kx) - \rho g z \\ &= \rho g \zeta_a e^{-kT} \cos(\omega t - kx) + \rho g T \end{aligned} \quad (3.29)$$

Assuming that the diameter of the cylinder is small relative to the wave length ($kD \approx 0$), so that the pressure distribution on the bottom of the cylinder is essentially uniform, then the pressure becomes:

$$p = \rho g \zeta_a e^{-kT} \cos(\omega t) + \rho g T \quad (3.30)$$

Then the vertical force on the bottom of the cylinder is:

$$F = \left\{ \rho g \zeta_a e^{-kT} \cos(\omega t) + \rho g T \right\} \cdot \frac{\pi}{4} D^2 \quad (3.31)$$

where D is the cylinder diameter and T is the draft.
The harmonic part of this force is the regular harmonic wave force, which will be considered here. More or less in the same way as with the hydromechanical loads (on the oscillating body in still water), this wave force can also be expressed as a spring coefficient c times a reduced or effective wave elevation ζ^*:

$$F_{FK} = c \cdot \zeta^* \quad \text{with:} \quad \begin{array}{lll} c &=& \rho g \frac{\pi}{4} D^2 \qquad \text{(spring coeff.)} \\ \zeta^* &=& e^{-kT} \cdot \zeta_a \cos(\omega t) \qquad \text{(deep water)} \end{array} \quad (3.32)$$

This wave force is called the **Froude-Krilov force**, which follows from an integration of the pressures on the body in the undisturbed wave.
Actually however, a part of the waves will be diffracted, requiring a correction of this Froude-Krilov force. Using the relative motion principle described earlier in this chapter, one finds additional force components: one proportional to the vertical acceleration of the water particles and one proportional to the vertical velocity of the water particles.
The total wave force can be written as:

$$F_w = a\ddot{\zeta}^* + b\dot{\zeta}^* + c\zeta^* = F_a \cos(\omega t + \varepsilon_{F\zeta}) \quad (3.33)$$

in which the terms $a\ddot{\zeta}^*$ and $b\dot{\zeta}^*$ are considered to be corrections on the Froude-Krilov force due to diffraction of the waves by the presence of the cylinder in the fluid.
The "reduced" wave elevation is given by:

$$\begin{aligned} \zeta^* &= \zeta_a e^{-kT} \cos(\omega t) \\ \dot{\zeta}^* &= -\zeta_a e^{-kT} \omega \sin(\omega t) \\ \ddot{\zeta}^* &= -\zeta_a e^{-kT} \omega^2 \cos(\omega t) \end{aligned} \quad (3.34)$$

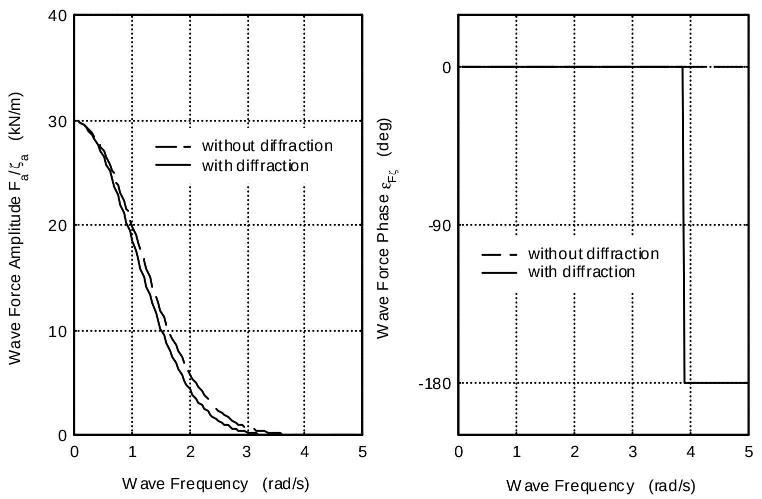


Figure 3.10: Vertical Wave Force on a Vertical Cylinder

The wave force amplitude, F_a, is proportional to the wave amplitude, ζ_a, and the phase shift $\varepsilon_{F\zeta}$ is independent of the wave amplitude, ζ_a; the system is linear.
Figure 3.10 shows the wave force amplitude and phase shift as a function of the wave frequency. For low frequencies (long waves), the diffraction part is very small and the wave force tends to the Froude-Krilov force, $c\zeta^*$. At higher frequencies there is a small influence of diffraction on the wave force on this vertical cylinder. There, the wave force amplitude remains almost similar to the Froude-Krilov force. Diffraction becomes relatively important after the total force has become small; an abrupt phase shift of $-\pi$ occurs quite suddenly, too.

Equation of Motion

The addition of the exciting wave loads from equation 3.33 to the right hand side of equation 3.25, gives the equation of motion for this heaving cylinder in waves:

$$(m+a)\,\ddot{z} + b\dot{z} + cz = a\ddot{\zeta}^* + b\dot{\zeta}^* + c\zeta^* \tag{3.35}$$

The heave response to the regular wave excitation is given by:

$$\begin{aligned} z &= z_a \cos(\omega t + \varepsilon_{z\zeta}) \\ \dot{z} &= -z_a\omega \sin(\omega t + \varepsilon_{z\zeta}) \\ \ddot{z} &= -z_a\omega^2 \cos(\omega t + \varepsilon_{z\zeta}) \end{aligned} \tag{3.36}$$

Some algebra results in the heave amplitude:

$$\frac{z_a}{\zeta_a} = e^{-kT}\sqrt{\frac{\left\{c - a\omega^2\right\}^2 + \left\{b\omega\right\}^2}{\left\{c - (m+a)\,\omega^2\right\}^2 + \left\{b\omega\right\}^2}} \tag{3.37}$$

and the phase shift:

$$\varepsilon_{z\zeta} = \arctan\left\{\frac{-mb\omega^3}{(c - a\omega^2)\left\{c - (m+a)\,\omega^2\right\} + \left\{b\omega\right\}^2}\right\} \quad with: 0 \leq \varepsilon_{z\zeta} \leq 2\pi \tag{3.38}$$

The requirements of linearity is fulfilled: the heave amplitude z_a is proportional to the wave amplitude ζ_a and the phase shift $\varepsilon_{z\zeta}$ is not dependent on the wave amplitude ζ_a.

3.1.7 Frequency Characteristics

Generally, the amplitudes and phase shifts in the previous section are called:

$$\left.\begin{array}{rcr} \frac{F_a}{\zeta_a}(\omega) \text{ and } \frac{z_a}{\zeta_a}(\omega) & = & \text{amplitude characteristics} \\ \varepsilon_{F\zeta}(\omega) \text{ and } \varepsilon_{z\zeta}(\omega) & = & \text{phase characteristics} \end{array}\right\} \textbf{frequency characteristics}$$

The response amplitude characteristics $\frac{z_a}{\zeta_a}(\omega)$ are also referred to as **Response Amplitude Operator** (RAO).

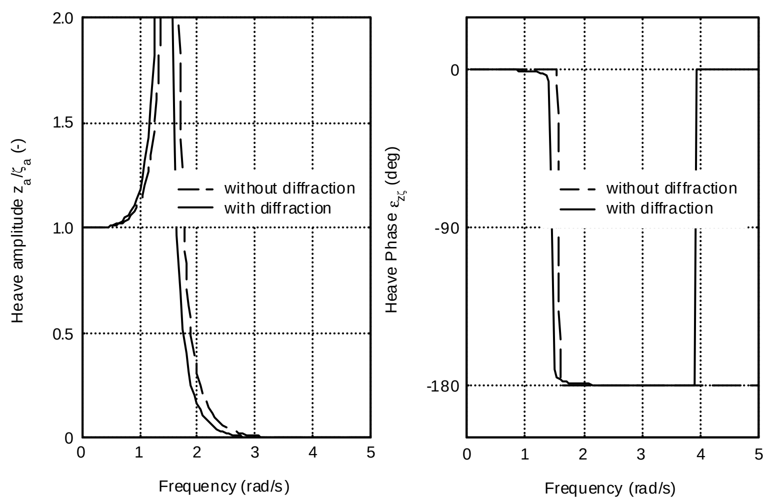


Figure 3.11: Heave Motions of a Vertical Cylinder

Figure 3.11 shows the frequency characteristics for heave together with the influence of diffraction of the waves. The annotation "without diffraction" in these figures means that the wave load consists of the Froude-Krilov force, $c\zeta^*$, only. A phase shift of $-\pi$ occurs at the natural frequency. This phase shift is very abrupt here, because of the small damping of this cylinder. A second phase shift appears at a higher frequency. This is caused by a phase shift in the wave load.
Equation 3.37 and figure 3.12 show that with respect to the motional behavior of this cylinder three frequency areas can be distinguished:

1. the low frequency area ($\omega^2 \ll c/(m+a)$), with motions dominated by the restoring spring term,

2. the natural frequency area ($c/(m+a) \lesssim \omega^2 \lesssim c/a$), with motions dominated by the damping term and

3. the high frequency area($\omega^2 \gg c/a$), with motions dominated by the mass term.

Also, equation 3.37 shows that the vertical motion tends to the wave motion as the frequency decreases to zero.
Figure 3.13 shows the speed dependent transfer functions of the roll motions in beam waves and the pitch motions in head waves of a container ship. Notice the opposite effect of forward speed on these two angular motions, caused by a with forward speed strongly increasing lift-damping of the roll motions.

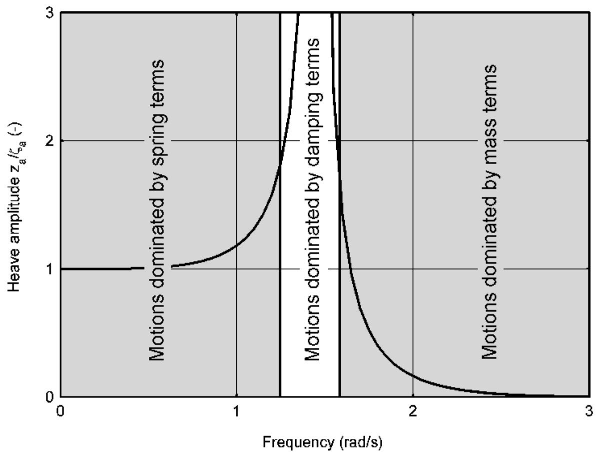


Figure 3.12: Frequency Areas with Respect to Motional Behavior

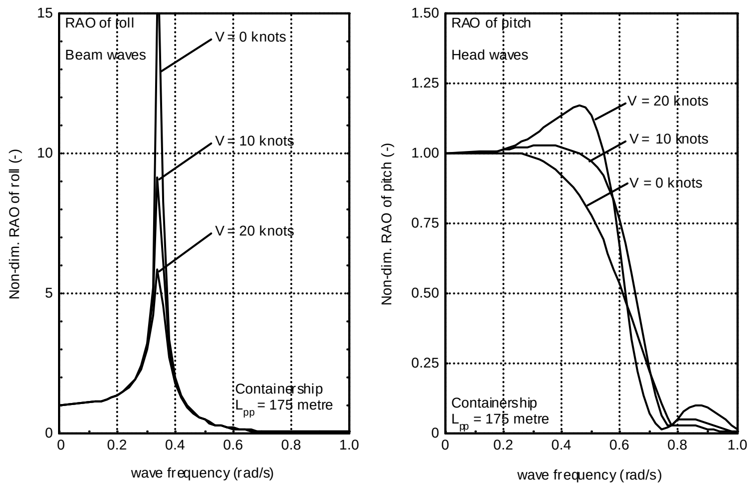


Figure 3.13: RAO's of Roll and Pitch of a Containership

Figure 3.14 shows the speed dependent transfer functions of the absolute and the relative vertical bow motions of a container ship in head waves. Note the opposite characteristics of these two motions in very short and in very long waves.

The resonance frequency of a motion does not necessarily coincides with the natural frequency. A clear example of this is given by [Hooft, 1970], as shown in figure 3.15, for a semi-submersible platform with different dimensions of the under water geometry. This geometry has been configured in such a way that the responses are minimal at the natural frequency.

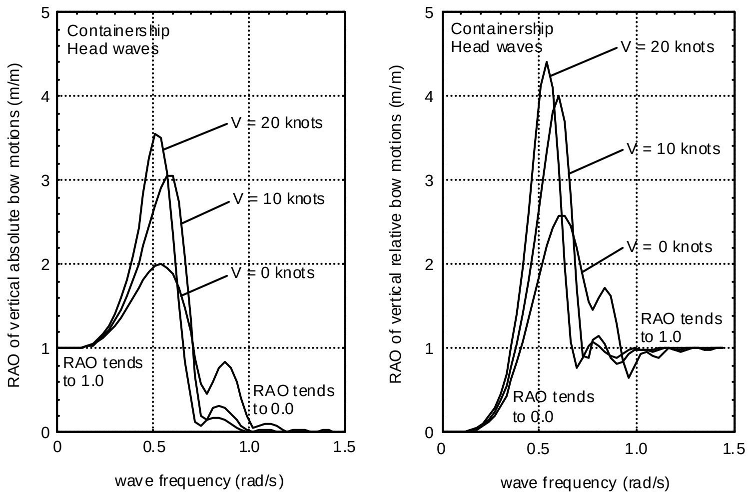
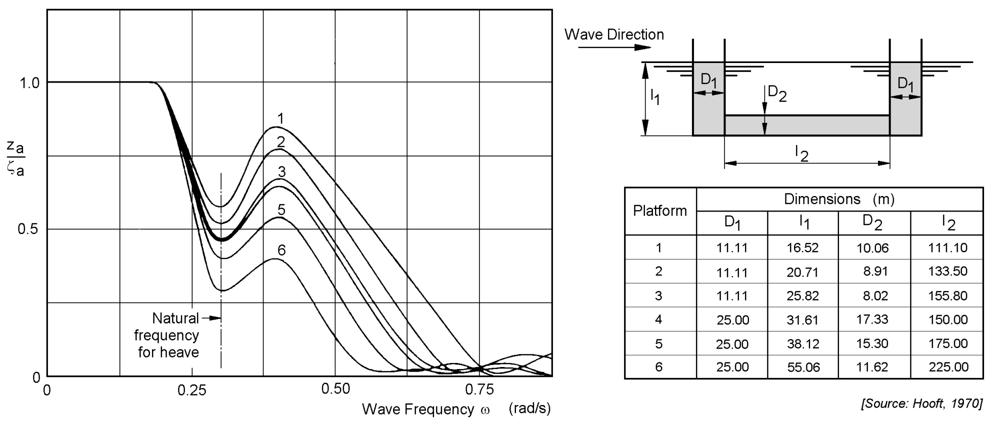


Figure 3.14: Absolute and Relative Vertical Motions at the Bow



Platform	Dimensions (m)			
	D_1	l_1	D_2	l_2
1	11.11	16.52	10.06	111.10
2	11.11	20.71	8.91	133.50
3	11.11	25.82	8.02	155.80
4	25.00	31.61	17.33	150.00
5	25.00	38.12	15.30	175.00
6	25.00	55.06	11.62	225.00

Figure 3.15: Heave Responses of Semi-Submersible Platforms in Waves

3.2 Behavior in Irregular Waves

When information on the irregular waves is available, now the first order motions can be determined.
The wave energy spectrum was defined by:

$$S_\zeta(\omega) \cdot d\omega \;=\; \frac{1}{2}\,\zeta_a^2(\omega) \tag{3.39}$$

Analogous to this, the energy spectrum of the heave response $z(\omega, t)$ can be defined by:

$$\begin{aligned} S_z(\omega) \cdot d\omega &= \frac{1}{2} z_a^2(\omega) \\ &= \left| \frac{z_a}{\zeta_a}(\omega) \right|^2 \cdot \frac{1}{2}\zeta_a^2(\omega) \\ &= \left| \frac{z_a}{\zeta_a}(\omega) \right|^2 \cdot S_\zeta(\omega) \cdot d\omega \end{aligned} \tag{3.40}$$

Thus, the heave response spectrum of a motion can be found by using the transfer function of the motion and the wave spectrum by:

$$S_z(\omega) = \left| \frac{z_a}{\zeta_a}(\omega) \right|^2 \cdot S_\zeta(\omega) \tag{3.41}$$

The principle of this transformation of wave energy to response energy is shown in figure 3.16 for the heave motions being considered here.
The irregular wave history, $\zeta(t)$ - below in the left hand side of the figure - is the sum of a large number of regular wave components, each with its own frequency, amplitude and a random phase shift. The value $\frac{1}{2}\zeta_a^2(\omega)/\Delta\omega$ - associated with each wave component on the ω-axis - is plotted vertically on the left; this is the wave energy spectrum, $S_\zeta(\omega)$. This part of the figure can be found in chapter 2 as well, by the way.
Each regular wave component can be transferred to a regular heave component by a multiplication with the transfer function $z_a/\zeta_a(\omega)$. The result is given in the right hand side of this figure. The irregular heave history, $z(t)$, is obtained by adding up the regular heave components, just as was done for the waves on the left. Plotting the value $\frac{1}{2}z_a^2(\omega)/\Delta\omega$ of each heave component on the ω-axis on the right yields the heave response spectrum, $S_z(\omega)$.
The moments of the heave response spectrum are given by:

$$m_{nz} = \int_0^\infty S_z(\omega) \cdot \omega^n \cdot d\omega \qquad \text{with: } n = 0, 1, 2, ... \tag{3.42}$$

where $n = 0$ provides the area, $n = 1$ the first moment and $n = 2$ the moment of inertia of the spectral curve.
The significant heave amplitude can be calculated from the spectral density function of the heave motions, just as was done for waves. This significant heave amplitude, defined as the mean value of the highest one-third part of the amplitudes, is:

$$\bar{z}_{a_{1/3}} \;=\; 2 \cdot RMS \;=\; 2 \cdot \sqrt{m_{0z}} \tag{3.43}$$

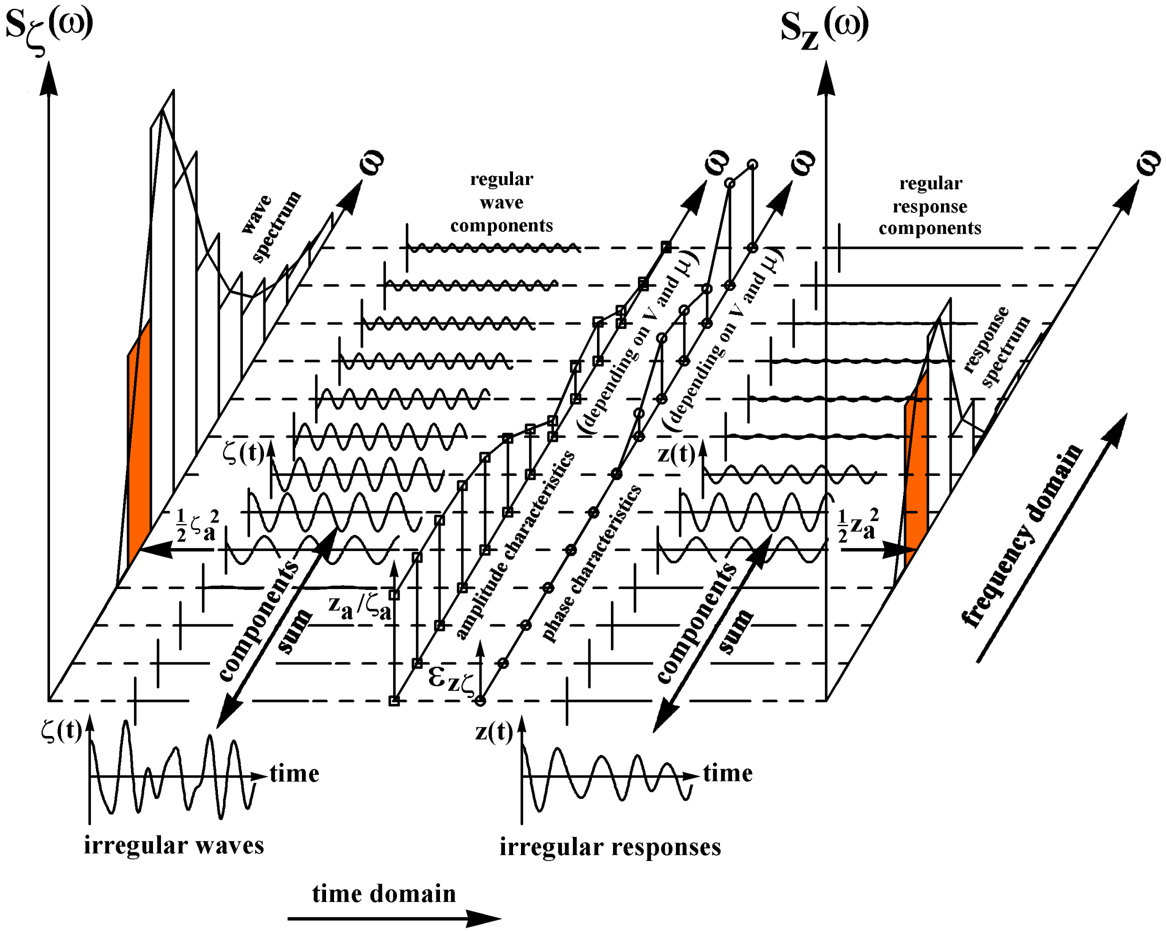


Figure 3.16: Principle of Transfer of Waves into Responses

in which RMS ($=\sqrt{m_{0z}}$) is the Root Mean Square value.
A mean period, T_{1z}, can be found from the centroid of the spectrum or a period, T_{2z}, equivalent to the average zero-crossing period, found from the spectral radius of gyration:

$$T_{1z} = 2\pi \cdot \frac{m_{0z}}{m_{1z}} \qquad \text{and} \qquad T_{2z} = 2\pi \cdot \sqrt{\frac{m_{0z}}{m_{2z}}} \tag{3.44}$$

Figure 3.17 shows an example of the striking influence of the average wave period on a response spectrum. This response is the heave motion of a 175 meter container ship, sailing with a speed of 20 knots in head waves with a significant wave height of 5.0 meters.
For the wave spectrum with an average period of 6.0 seconds, the transfer function has very low values in the wave frequency range. The response spectrum becomes small; only small motions result. As the average wave period gets larger (to the right in figure 3.17), the response increases dramatically.
A similar effect will be obtained for a larger range of average wave periods if the transfer function of the motion shifts to the low frequency region. A low natural frequency is required to obtain this. This principle has been used when designing semi-submersibles, which have a large volume under water and a very small spring term for heave (small water plane area). However, such a shape does not make much of a wave when it oscillates; it has little potential damping. This results in large (sometimes very large) RAO's at the natural

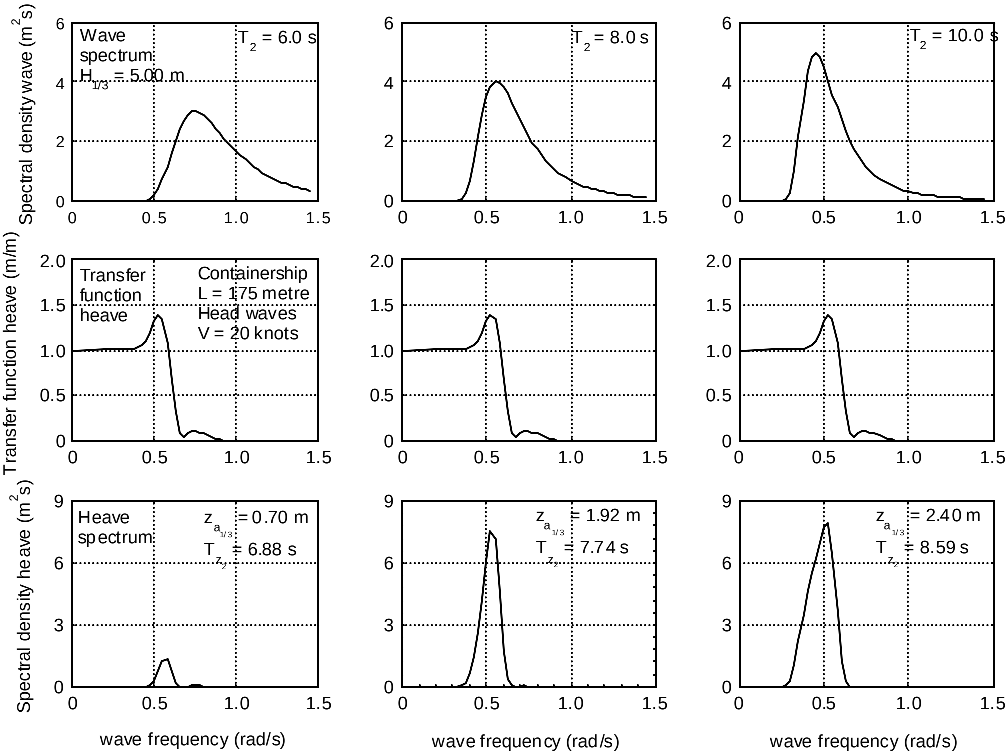


Figure 3.17: Effect of Wave Period on Heave

frequency. As long as there is (almost) no wave energy at this frequency, the response spectrum will remain small.

Figure 3.18 shows a wave spectrum with sketches of *RAO*'s for heave of three different types of floating structures at zero forward speed:

- The pontoon has a relatively large natural frequency and as a result of this significant *RAO* values over a large part of the normal wave frequency range. Almost all wave energy will be transferred into heave motions, which results in a large motion spectrum. An extreme example is the wave buoy, which has (ideally) an *RAO* of 1.0 over the whole frequency range. Then the response spectrum becomes identical to the wave spectrum, which is of course the aim of this measuring tool. It should follow the water surface like a sea gull!

- The ship, with a lower natural frequency, transfers a smaller but still considerable part of the wave energy into heave motions.

- The semi-submersible however, with a very low natural frequency (large mass and small intersection with the water line), transfers only a very small part of the wave energy; very low first order heave motions will appear; it remains essentially stable in the waves.

One can conclude that the natural frequency is a very important phenomenon which dictates (to a significant extent) the behavior of the structure in waves. Whenever possible, the natural frequency should be shifted out of the wave frequency region.

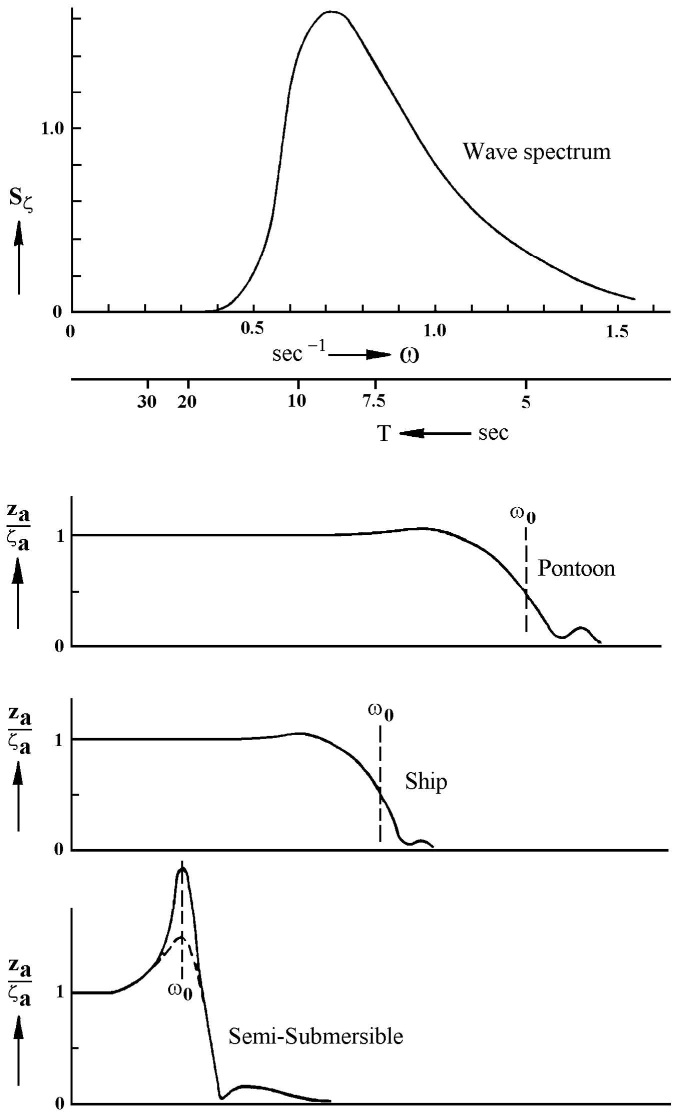


Figure 3.18: Effect of Natural Period on Heave Motions

Chapter 4

Maneuvering of Ships

The history of scientific research on maneuvering of ships started already in 1749 with the classic work of Euler on equations of motion of a ship, but shipbuilding still remained for almost two centuries fully based on a shipbuilder's experience-based knowledge. In about 1920, some first attention on a more analytic approach appeared when single-plated rudders were replaced by flow-line-curved rudders. This development was based on research on airfoils, carried out in the new aviation industry. More systematic work on maneuvering has been started during World War II. Shortly after this War, [Davidson and Schiff, 1946] published a paper dealing with maneuvering problems on a (for that time) modern approach.
The first International Symposium on Ship Maneuverability was held in 1960 in Washington in the U.S.A. There, among others, [Norrbin, 1960] gave his view on the "state of the art" of scientific work carried out on maneuverability of ships. Since then, an increasing attention has been paid on research in this particular hydrodynamic field. Especially, (inter)national organizations - such as the IMO (International Maritime Organization), the ITTC (International Towing Tank Conference) and the SNAME (Society of Naval Architects and Marine Engineers) - became more active in the stimulation of research on those aspects of ship maneuverability which are vital for safer shipping and cleaner oceans. Results of this (and other) research are treated in this Chapter.
During the last four decades, research on ship maneuverability was also stimulated strongly by the increasing ship size problems (crude oil carriers and container vessels), the related shallow water problems when entering harbors, the increasing ship speed problems (nuclear submarines and fast ferries), newly-developed experimental oscillatory techniques (planar motion mechanisms) and - last but not least - the enormous developments in the computer industry with related new possibilities for computer simulations.

In some parts of this Chapter on Maneuvering of Ships, very fruitful use has been made of the following references:

- The "Principles of Naval Architecture", 1989, Chapter IX on Controllability by C.L. Crane, H. Eda and A. Landsberg.

- Various Proceedings of the International Towing Tank Conference; particularly those

[0] J.M.J. Journée and J.A. Pinkster, *"SHIP HYDROMECHANICS, Part I: Introduction"*, Draft Edition, January 2001, Ship Hydromechanics Laboratory, Delft University of Technology, Mekelweg 2, 2628 CD Delft, The Netherlands. For updates see web site: http://www.shipmotions.nl.

of the 22nd ITTC (1999), which presents detailed procedures on conducting full scale maneuvering trial tests.

- A paper on "IMO's Activities on Ship Maneuverability" in the Proceedings of the International Conference on Marine Simulation and Ship Maneuverability (MARSIM) in October 1993 at St. John's in Canada by S.D. Srivastava.

For more detailed information on maneuvering of ships, reference is given to these publications.

4.1 Introduction

Controllability encompasses all aspects of regulating a ship's trajectory, speed and orientation at sea, as well as in restricted waters where positioning and station keeping are of particular concern. Controllability includes starting, steering a steady course, turning, slowing, stopping and backing. In the case of submarines, diving has to be added to these controllability tasks too.

The study of the complex subject of controllability is usually divided into three distinct areas of functions:

- Course keeping (or steering)
  This aspect yields the maintenance of a steady mean course or heading. Interest centers on the ease with which the ship can be held to the course.

- Maneuvering
  This aspect yields the controlled change in direction of motion; turning or course changing. Interest centers on the ease with which change can be accomplished and the radius and distance required to accomplish the change.

- Speed changing
  This aspect yields the controlled change in speed including stopping and backing. Interest centers on the ease, rapidity and distance covered in accomplishing changes.

Performance varies with water depth, channel restrictions and hydrodynamic interference from nearby vessels and obstacles. Course keeping and maneuvering characteristics are particularly sensitive to the ship's trim. For conventional ships, the two qualities of course keeping and maneuvering may tend to work against each other; an easy turning ship may be difficult to keep on course whereas a ship which maintains course well may be hard to turn. Fortunately, a practical compromise is nearly always possible.
Since controllability is so important, it is an essential consideration in the design of any floating structure. Controllability is, however, but one of many considerations facing of naval architects and involves compromises with other important characteristics. Some solutions are obtained through comparison with the characteristics of earlier successful designs. In other cases, experimental techniques, theoretical analyses, and rational design practices must all come into play to assure adequacy.

Three tasks are generally involved in producing a ship with good controllability:

- Establishing realistic specifications and criteria for course keeping, maneuvering and speed changing.
- Designing the hull, control surfaces, appendages, steering gear and control systems to meet these requirements and predicting the resultant performance.
- Conducting full-scale trials to measure performance for comparison with required criteria and predictions.

This chapter will deal more or less with each of these three tasks. Its goal is to give an introduction to the basics of controllability analysis and some of its many facets. In a next lecture, manners will be given that should lead to the use of rational design procedures to assure adequate ship controllability.

4.2 Requirements and Tools

In this section, some basic information is given on maneuvering requirements, rudder types, required rudder areas and rudder control systems.

4.2.1 General Requirements

Each sailing ship must have a certain amount of course stability; i.e. it must be able to maintain a certain direction or course. During sailing, the drift angle (the angle between the path of the center of gravity and the middle line plane of the ship) may not show large fluctuations. The phenomena "course" and "drift angle" have been defined in figure 4.1. The rudder angle, required to compensate for external disturbances by wind and waves, may not be too large.

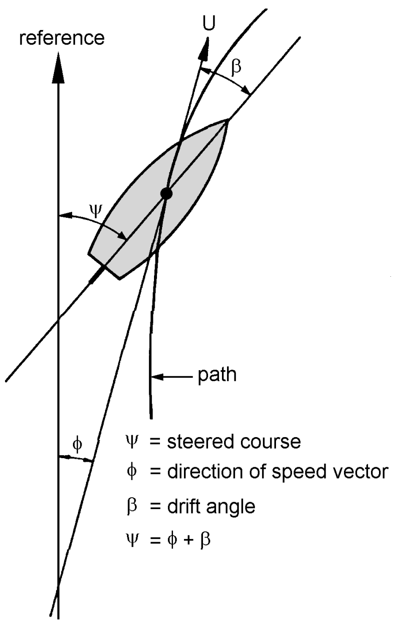


Figure 4.1: Definition of Course and Drift Angle

The sailing ship must be able to change its course relatively fast, with small overshoots. Also, the path overshoot (or path width) has to stay within certain limits. The ship must be able to carry out turning maneuvers within limits, defined later. It must remain well-maneuverable during accelerating and decelerating the ship and it must have an acceptable stopping distance. Also, the ship must be able to maneuver at low speeds without assistance of tugs.
For this, certain demands on design, performance and control of the rudder are required.

4.2.2 Rudder Types

The type of rudder and its location and placement relative to the propeller have significant influence on rudder effectiveness and ship controllability. Rudders should be located near the stern and should be located in the propeller stream for good controllability.
Figure 4.2 shows the major rudder types available to the designer. Its performance characteristics are of major importance to the controllability of the ship.

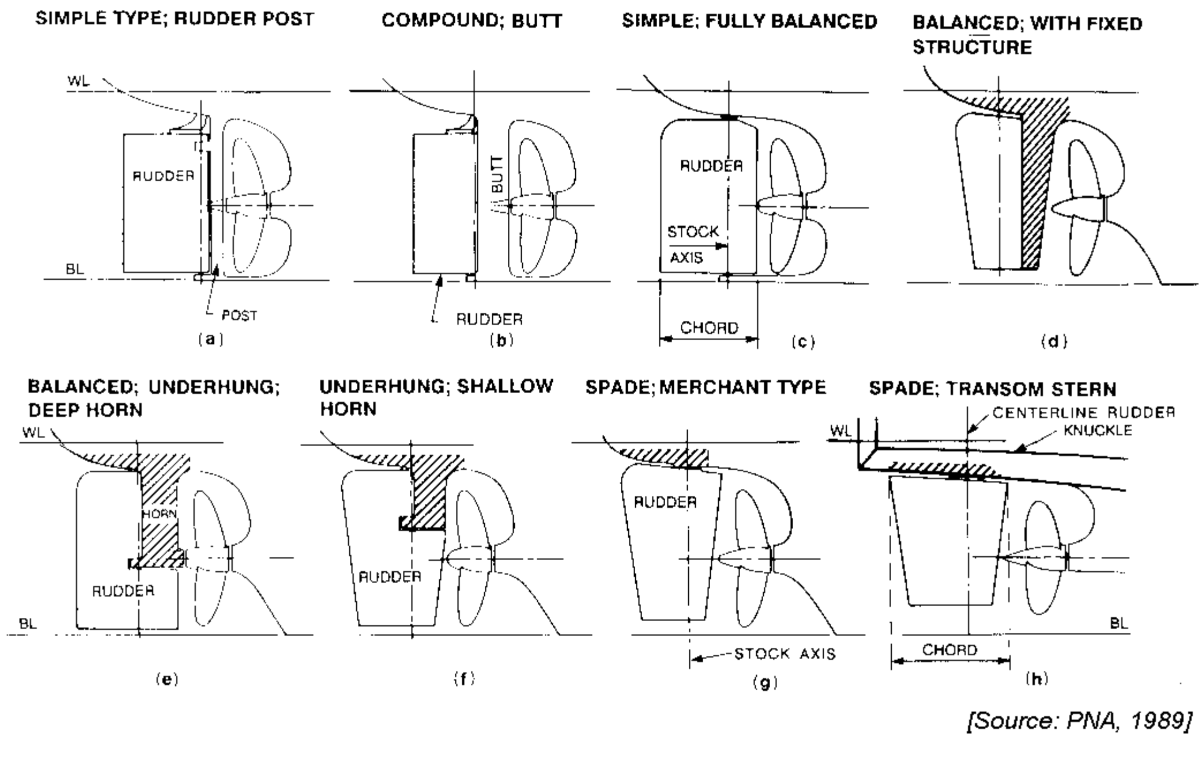


Figure 4.2: Various Rudder-Fin Arrangements

All-movable rudders are desirable for their ability to produce large turning forces for their size. The required rudder moment is strongly influenced by a careful choice of the balance ratio; this is the rudder area forward of the rudder stock divided by the total rudder area. Usually, this balance ratio varies between 0.25 for ships with a small block coefficient and 0.27 for ships with a large block coefficient.
Structural considerations, costs, the need for additional stabilizing side forces provided by a horn and the considerations may require use of other types of rudders such as the semi-suspended (or horn) rudder. The horn type is also favored for operations in ice.

4.2.3 Rudder Size

The rudder area should be determined and verified during the initial ship arrangement study. A good first step is to use the 1975-Rules value of Det Norske Veritas for a minimum projected rudder area:

$$A_r \approx \frac{d \cdot L_{pp}}{100} \left\{ 1.0 + 25.0 \cdot \left(\frac{B}{L_{pp}} \right)^2 \right\} \tag{4.1}$$

in which:

$$\begin{aligned} A_r &= \text{projected rudder area} \\ L_{pp} &= \text{length between perpendiculars} \\ B &= \text{beam} \\ d &= \text{draft} \end{aligned}$$

This formula 4.1 applies only to rudder arrangements in which the rudder is located directly behind the propeller. For any other rudder arrangement Det Norske Veritas requires an increase in the rudder area by - at least - 30 percent. A twin screw (or more) arrangement should be combined with rudders located directly behind the propellers for maximum low-speed maneuverability. A single rudder placed between two propellers may be inadequate, because the rudder blade does not swing sufficiently into the flow of a propeller to generate the needed turning moment.

4.2.4 Rudder Forces and Moments

A rudder behind a ship acts as an airfoil or a wing; it produces lift and drag in a proper flow. A cross section of a rudder (or a lifting surface) is such that at a rudder angle (or angle of attack of the flow) a relative large force perpendicular to the flow direction comes into existence.
Figure 4.3 shows the components of the force produced by the rudder. The rudder profile has been placed with an angle of attack α in a homogeneous flow with a constant velocity V. Practically, V is defined as the velocity of the fluid far before the rudder. For small angles of attack, α, the total force P on the profile acts at about $e \approx 0.25 \cdot c$, provided that the span width s is large with respect to the chord length c. This force P can be decomposed in a lift force L perpendicular to the flow and a drag force D in the direction of the flow. The total force P can be decomposed in a normal force N and a tangential force T, too.
Rudder forces are made dimensionless by the stagnation pressure $\frac{1}{2}\rho V^2$ and the projected rudder area A_r:

$$\begin{aligned} C_L &= \frac{L}{\frac{1}{2}\rho V^2 A_r} \quad \text{and} \quad C_D = \frac{D}{\frac{1}{2}\rho V^2 A_r} \\ C_N &= \frac{N}{\frac{1}{2}\rho V^2 A_r} \quad \text{and} \quad C_T = \frac{T}{\frac{1}{2}\rho V^2 A_r} \end{aligned} \tag{4.2}$$

From the force components, as presented in figure 4.3, follows:

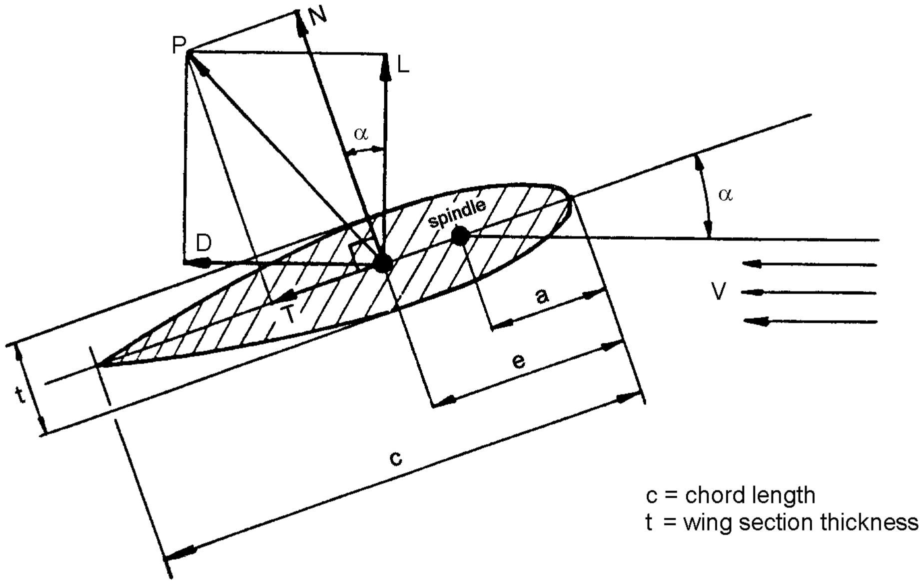


Figure 4.3: Forces on a Rudder Section

$$\begin{aligned} P &= \sqrt{L^2 + D^2} = \sqrt{N^2 + T^2} \\ N &= L\cos\alpha + D\sin\alpha \qquad \text{and} \qquad C_N = C_L\cos\alpha + C_D\sin\alpha \\ T &= D\cos\alpha - L\sin\alpha \qquad \text{and} \qquad C_T = C_D\cos\alpha - C_L\sin\alpha \end{aligned} \tag{4.3}$$

For the rudder moment, it is important to know about which point it has been defined. The moment M_e about the front (or nose) of the rudder is:

$$M_e = N \cdot e \qquad \text{and} \qquad C_{M_e} = \frac{M_e}{\frac{1}{2}\rho V^2 A_r \cdot c} \tag{4.4}$$

Thus the chord length c has been used for making the rudder moment dimensionless, so:

$$C_{M_e} = \frac{N \cdot e}{\frac{1}{2}\rho V^2 A_r \cdot c} = C_N \cdot \frac{e}{c} \qquad \text{or:} \qquad \frac{e}{c} = \frac{C_{M_e}}{C_N} \tag{4.5}$$

An example of these characteristics has been given in figure 4.4. The lift coefficient increases almost linearly with the angle of attack until a maximum value is reached, whereupon the wing is said to "stall". At small angles of attack, the center of the lift forces acts at about $e/c \approx 0.25$. At higher angles of attack - before stalling of the wing - the flow starts to separate at the suction side of the wing and the center of the lift force shifts backwards, for instance to about $e/c \approx 0.40$.

The moment M_{rs} about the rudder stock is:

$$M_{rs} = N \cdot (e - a) \tag{4.6}$$

A rudder moment is called positive here, when it is right turning. Sometimes, a definition of the rudder moment about a point fixed at $e = 0.25 \cdot c$ will be found in the literature.

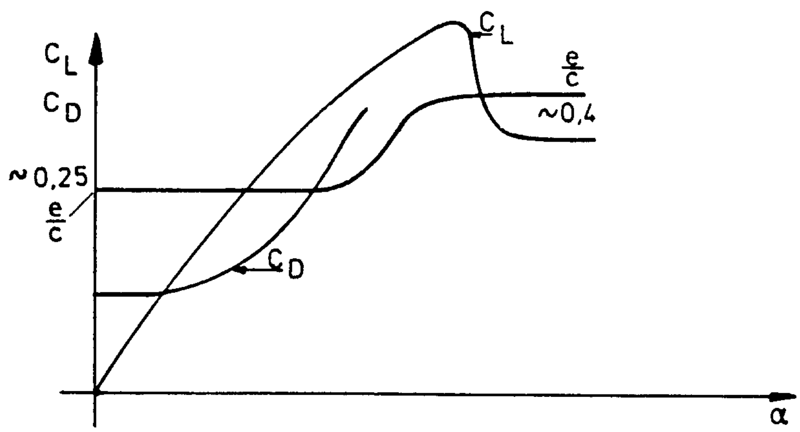


Figure 4.4: Lift, Drag and Moment Characteristics of a Wing Section

The aspect ratio s/c of the rudder is an important parameter. The geometrical aspect ratio is defined by the ratio of the mean span width $\bar{s}$ and the mean chord length $\bar{c}$:

$$AR = \frac{\bar{s}}{\bar{c}} \tag{4.7}$$

For an arbitrary rudder plan form can be written:

$$\bar{c} = \frac{A_r}{\bar{s}} \qquad \text{thus:} \qquad AR = \frac{\bar{s}^2}{A_r} \tag{4.8}$$

Because of the finite aspect ratio, three-dimensional effects will appear at the upper and lower side of the rudder; see figure 4.5. A fluid flow around the corners appears, of which the effect on the lift coefficient will increase with a decreasing aspect ratio.

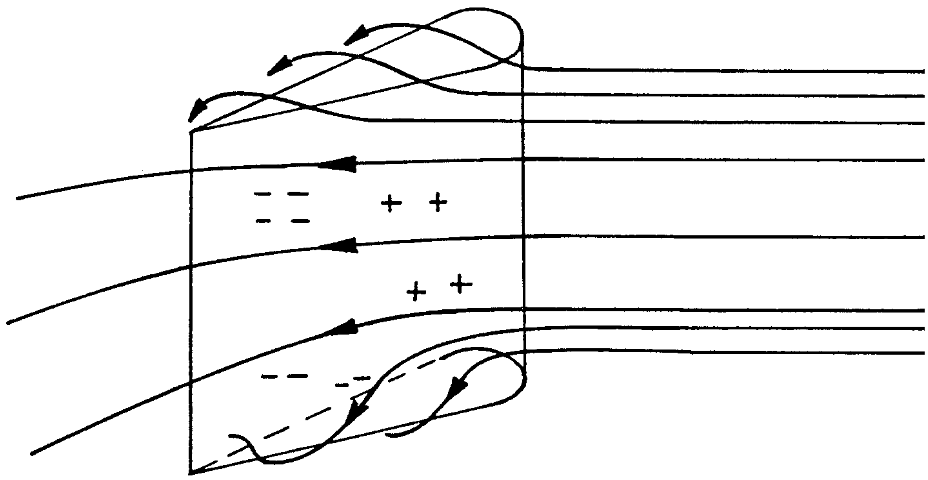


Figure 4.5: Flow Around Corners of a Lifting Surface

At larger aspect ratios, the flow becomes more two-dimensional over a larger part of the rudder surface. The flow around the corners can be avoided by walls or end-plates. A rudder which almost joints the stern of the ship at the upper side of the rudder, has a considerably increased effective aspect ratio. Then, theoretically, the effective aspect ratio is twice the geometrical aspect ratio.

A flat plate has a relative large increase of the lift coefficient with the angle of attack, so a high $\partial C_L/\partial\alpha$ value. From the lifting surfaces theory, see [Abbott and von Doenhoff, 1958], follows:

$$\frac{\partial C_L}{\partial \alpha} = 2\pi \qquad \text{for a flat plate (with } \alpha \text{ in radians)}$$

The great disadvantage of a flat plate as a rudder is the fast flow separation; the flow will separate already at small angles of attack. A flat plate stalls already at a small angle of attack. A wing section thickness, t, of 9 percent of the chord length, c, is often considered as a practical minimum value, so $t/c \geqslant 0.09$.

The lift, drag and moment coefficients (C_L, C_D and C_M) of symmetrical NACA (National Advisory Committee for Aeronautics) wing sections for $0.06 \leqslant t \leqslant 0.18$ and an aspect ratio $AR = 6$ - as given by [Abbott and von Doenhoff, 1958] - are given in the following table.

$AR = 6$	NACA 0006	NACA 0009	NACA 0012	NACA 0015	NACA 0018
α			C_L		
-4^0			-0.30	-0.30	
-2^0	-0.13	-0.15			-0.13
0^0	0.00	0.00	0.00	0.00	0.00
4^0	0.32	0.32	0.30	0.30	0.30
8^0	0.61	0.61	0.61	0.61	0.61
12^0	0.80	0.91	0.91	0.91	0.89
16^0	0.89	1.20	1.20	1.19	1.14
20^0	0.85	1.04	1.43	1.40	1.42
24^0	0.83	0.90	1.12	1.20	1.30
30^0	0.82	0.81	0.90	0.90	0.95
C_L(max)		1.28	1.52	1.53	1.50
at $\alpha =$		18.0^0	22.2^0	22.5^0	22.5^0
α			C_M		
-4^0			-0.075	-0.075	
-2^0	-0.030	-0.040			
0^0	0.000	0.000	0.000	0.000	0.000
4^0	0.080	0.080	0.075	0.075	0.075
8^0	0.150	0.150	0.150	0.150	0.150
12^0	0.200	0.225	0.225	0.225	0.220
16^0	0.300	0.300	0.300	0.300	0.185
20^0	0.270	0.330	0.360	0.360	0.360
24^0	0.260	0.345	0.360	0.360	0.375
30^0	0.245	0.341	0.355	0.335	0.340
C_M(max)		0.420	0.380	0.380	0.375
at $\alpha =$		18.0^0	22.2^0	22.5^0	22.5^0
α			C_D		
-4^0			0.018	0.018	
-2^0	0.005	0.010	0.012	0.015	0.015
0^0	0.008	0.008	0.010	0.010	0.015
4^0	0.017	0.017	0.018	0.019	0.018
8^0	0.039	0.032	0.037	0.037	0.036
12^0	0.150	0.060	0.059	0.059	0.060
16^0	0.260	0.098	0.098	0.098	0.098
20^0	0.320	0.270	0.140	0.140	0.141
24^0	0.395	0.390	0.320	0.279	0.240
28^0			0.400	0.380	0.400

For calculating the lift and drag coefficients from known data of a profile with another aspect ratio, use can be made of the formulas of Lanchester and Prandtl:

$$\begin{aligned} C_{D_{(2)}} &= C_{D_{(1)}} + \frac{C^2_{L_{(1)}}}{\pi} \left(\frac{1}{AR_{(2)}} - \frac{1}{AR_{(1)}} \right) \\ \alpha_{(2)} &= \alpha_{(1)} + \frac{C_{L_{(1)}}}{\pi} \left(\frac{1}{AR_{(2)}} - \frac{1}{AR_{(1)}} \right) \qquad \text{(with } \alpha \text{ in radians)} \\ C_{L_{(2)}}\left(\alpha_{(2)}\right) &= C_{L_{(1)}}\left(\alpha_{(1)}\right) \end{aligned} \tag{4.9}$$

This last relation has been visualized in figure 4.6.

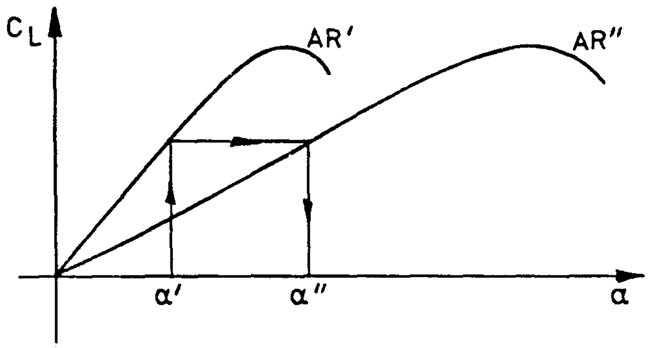


Figure 4.6: Aspect Ratio Transformation of Lift Coefficients

Starting from two-dimensional properties, so $AR_{(1)} = \infty$, the formulas of Lanchester and Prandtl reduce to:

$$\begin{aligned} C_{D_{(2)}} &= C_{D_{(ar=\infty)}} + \frac{C^2_{L_{(AR=\infty)}}}{\pi \cdot AR_{(2)}} \\ \alpha_{(2)} &= \alpha_{(AR=\infty)} + \frac{C_{L_{(AR=\infty)}}}{\pi \cdot AR_{(2)}} \\ C_{L_{(2)}}\left(\alpha_{(2)}\right) &= C_{L_{(AR=\infty)}}\left(\alpha_{(AR=\infty)}\right) \end{aligned} \tag{4.10}$$

The practical value of these formulas of Lanchester and Prandtl will be demonstrated in figure 4.7. The lift and drag coefficients for $AR = 1 - 7$ have been transformed in this figure to those for $AR = 5$. All transformed coefficients fit within a small band.

4.2.5 Rudder Control

For surface ships, course-keeping, speed changing and maneuvering involve primarily forces, moments and motions acting in all directions in the horizontal plane. For submarines, the third dimension also comes into play. Hydrodynamic motion forces and interactions acting on the vessel's hull, rudder and other appendages are of first consideration and difficulty. However, it is important to recognize that the responses of a number of mechanical, electronic, environmental and, most importantly, human factors all influence controllability.

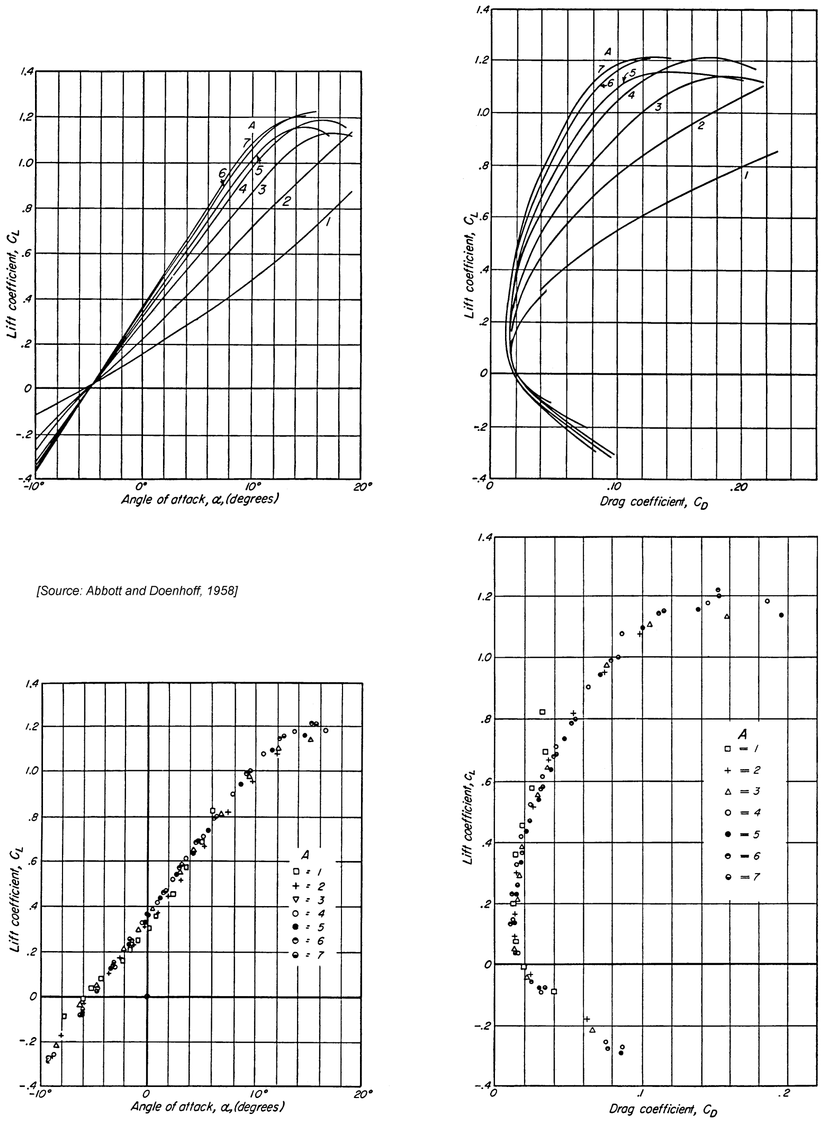


Figure 4.7: Lift and Drag Coefficient Transformations

The following discussion of controllability - based on the concept of control loops - illustrates the role of these many factors.

Consider the closed-loop directional control system, shown in figure 4.8. Starting at the left of this figure, there is a desired path or trajectory that the ship's conning officers wants to follow either under conditions of steady steaming at sea or in maneuvering. In an idealistic case, the desired path would be displayed for use by the helmsman (or alternatively by an autopilot). Simultaneously, again for the idealistic case, the path actually being traversed would also be shown on the display. If these two paths do not coincide, corrective action is taken by the helmsman or autopilot by changing the helm in a direction that will tend to correct the path error. This action activates the steering gear which changes the rudder position which in turn exerts a control force on the ship. This control force acts to induce on the hull an angle of attack, an angular velocity and other motions. These motions of the hull introduce the major hydrodynamic forces and moments that effect the change in

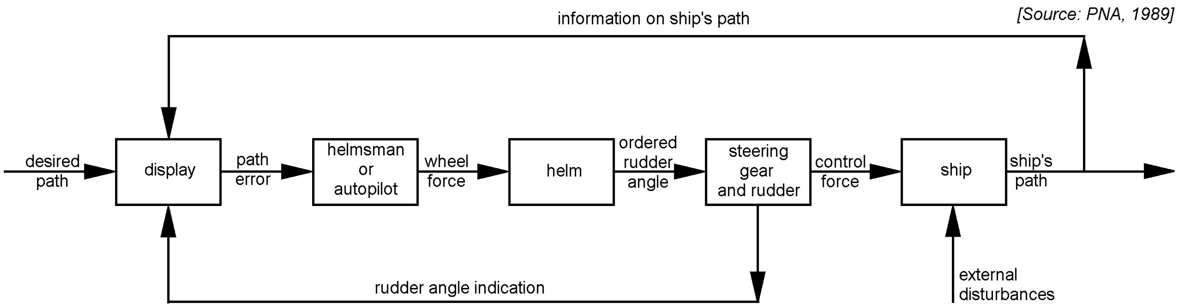


Figure 4.8: Closed-Loop Control System of Ship Controllability

heading and path.

In addition to control and hydrodynamic forces and moments, external disturbances such as wind, current or waves may also be simultaneously acting on the ship. Ideally, the resultant heading and actual path coordinates can be fed continuously back to the helmsman's display. This last step closes the control loop.

In the real case, all of the information concerning the actual instantaneous path is rarely known. On most ships only the heading and sometimes the rate of turn are continuously determinable, while the coordinates are available only occasionally. In spite of the shortcomings of the real case, the control loop still functions as shown in figure 4.8, but with less than complete information available to the conning officer and helmsman or autopilot. Relatively large-scale changes in position are determined or deduced in restricted water conditions from visual observations, a radar display or from visual cross bearings. Modern electronic navigation systems, such as Loran C and Global Positioning System (GPS) also offer a means for determining large-scale changes in position.

Consideration of the control loop of figure 4.8 shows that each of its elements plays a vital role in the overall controllability of the ship. The last two elements of the loop - the "ship" and the "steering gear and rudder" - are of the greatest concern to the naval architect although the human factors present must be continually reviewed to develop a successful design.

Whereas the directional control loop of figure 4.8 functions to determine the path, a second loop of interest - the speed control loop - functions to determine the speed along the path. The only common link between the two loops is the conning officer, who issues the orders in both cases. In the case of the speed control loop, an operator or engineer receives the orders of the conning officer and manipulates the power output and direction of rotation of the main propulsion machinery to maintain, accelerate, slow down, stop or reverse the speed of the ship.. In the open sea where decisions can be made in a more leisure manner, the conning officer has ample time to issue the necessary orders to both control loops. In restricted and congested waters, however, orders to both control loops may have to be issued simultaneously. With development of automation, the integration of these loops and the elimination of the intermediate roles of the helmsman on the bridge and the operator in the engine room is becoming commonplace.

With today's sophisticated drill rigs and track-keeping vessels, the automatic controller has indeed become quite advanced. Heading and speed, plus transverse position error and fore and aft position error are used to compute vector thrusts required of the various force efforts (propellers, rudders, thrusters, etc.) and proper distribution of correction forces and moments is automatically ordered.

4.3 Motion Stability Definitions

The concept of path keeping is strongly related to the concept of course stability or stability of direction. A body is said to be stable in any particular state of equilibrium in rest or motion if, when momentarily disturbed by an external force or moment, it tends to turn, after release from the disturbing force, to the state of equilibrium existing before the body was disturbed. In the case of path keeping, the most obvious external disturbing force would be a wave or a gust of wind. For optimum path keeping, it would be desirable for the ship to resume its original path after passage of the disturbance, with no intervention by the helmsman. Whether this will happen depends on the kind of motion stability possessed.
The various kinds of motion stability associated with ships are classified by the attributes of their initial state of equilibrium that are retained in the final path of their centers of gravity. For example, in each of the cases in the next figures, a ship is initially assumed to be travelling at a constant speed along a straight path.
Figure 4.9 shows the **straight-line or dynamic stability**; the final path after release from a disturbance retains the straight-line attribute of the initial state of equilibrium, but not its direction.
Figure 4.10 shows the **directional or course stability**; the final path after release from a disturbance retains not only the straight-line attribute of the initial path, but also its direction. In some cases the ship does not oscillate after the disturbance, but passes smoothly to the final path.
Figure 4.11 shows the **path or positional motion stability**; the ship returns to the original path, i.e. the final path not only has the same direction as the original path, but also its same transverse position relative to the surface of the earth.

The foregoing kinds of stability form an ascending hierarchy. Achieving straight-line stability is the designer's usual goal for most ships when steered by hand. The other cases require various degrees of automatic control.
All these kinds of stability have meaning with control surfaces (rudders) fixed at zero, with control surfaces free to swing, or with controls either manually or automatically operated. The first two cases involve only the last two elements of the control loop of figure 4.8, whereas the last case involves all of the elements of the control loop. In normal marine usage, the term stability implies controls-fixed stability; however, the term can also have meaning the controls working.

The following examples indicate distinctions:

- A surface ship sailing a calm sea possesses positional motion stability in the vertical plane (and therefore directional and straight line stability in this plane) with controls fixed. This is an example of the kind of stability shown in figure 4.11. In this case, hydrostatic forces and moments introduce a unique kind of stability which in the absence of these forces could only be introduced either by very sophisticated automatic controls or by manual control. The fact that the ship's operator and designer can take for granted is that this remarkable kind of stability does not detract from its intrinsic importance.

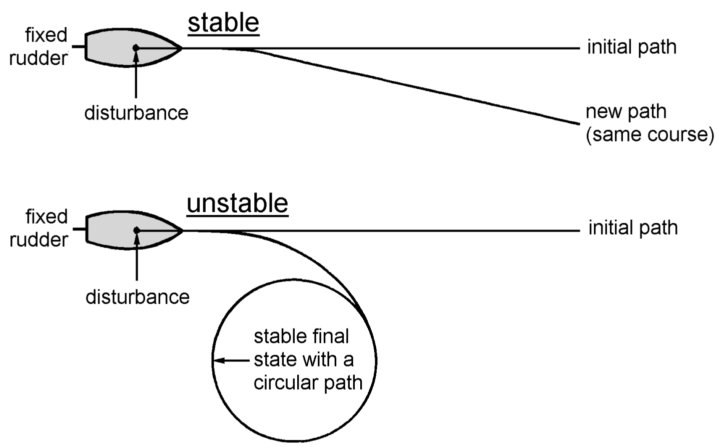


Figure 4.9: Straight-Line or Dynamic Stability

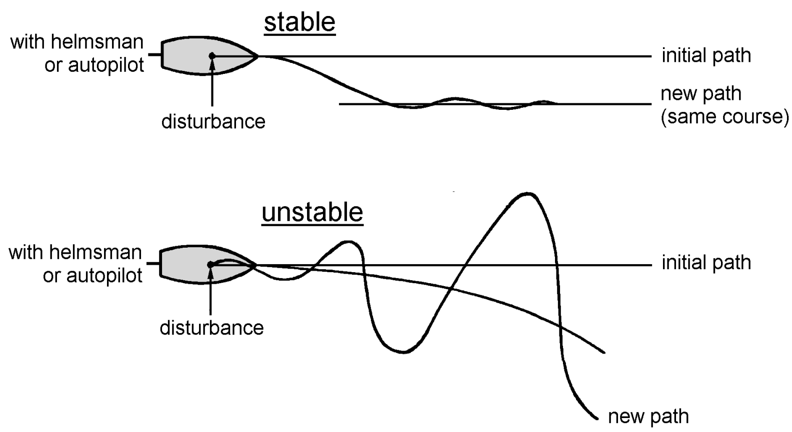


Figure 4.10: Directional or Course Stability

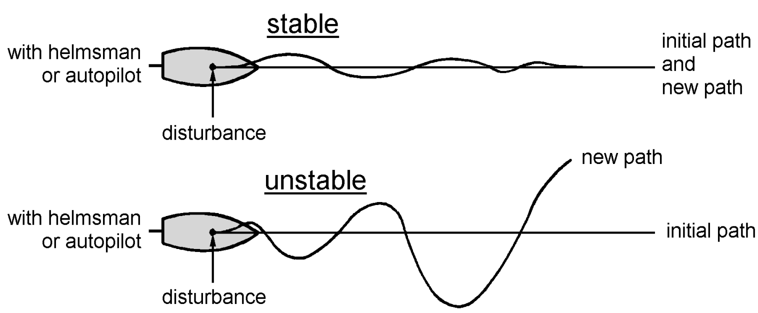


Figure 4.11: Path or Positional Motion Stability

- In the horizontal plane in the open sea with stern propulsion, a self-propelled ship cannot possess either positional or directional stability with controls fixed, because the changes in buoyancy that stabilize the vertical plane are non-existent in the horizontal plane. However, a ship must possess both of these kinds of stability with controls working either under manual or automatic guidance. Possible exceptions include sailing vessels, some multi-hull ships, and foil or planing craft, but not other surface effect ships.

- The only kind of motion stability possible with self-propelled ships in the horizontal plane with controls fixed, is straight-line stability. In fact, many ships do not possess it. Often - whenever controls-fixed stability is mentioned - the intended meaning is controls-fixed straight-line stability. This kind of stability is desirable, but not mandatory.

With each of the kinds of controls-fixed stability, there is associated a numerical index which by its sign designates whether the body is stable or unstable in that particular sense and by its magnitude designates the degree of stability or instability. To show how these indexes are determined, one must resort to the differential equations of motion.

4.4 Maneuverability Activities of IMO

During the last three decades, the IMO (International Maritime Organization) has been active in dealing with the following aspects on ship maneuverability, which are vital to achieve its objectives of safer shipping and cleaner oceans:

1. Maneuvering information aboard ships in order to enhance the safety of navigation.

2. Impaired maneuverability of tankers to reduce the risk of marine pollution.

3. Maneuvering standards for ship designers to ensure that no ships have maneuvering properties that may constitute a safety risk.

Many resolutions with respect to maneuverability of ships were initiated by the IMO Sub-Committee on Ship Design and Equipment and by the IMO Marine Safety Committee, which were adopted by the IMO Assembly; for detailed information see a paper of [Srivastava, 1993] and references given there. However, the IMO gives recommendations and guidelines only; they can not make international laws, the final decision has to be made by the individual Governments.

4.4.1 Maneuverability Information On-Board Ships

The value of readily available maneuvering information on the ship's bridge can not be overemphasized as it is of crucial importance to the master, navigating officers and pilots for discharging their duties efficiently and enhancing the safety of navigation.
Having regard to the variety of circumstances that a ship may encounter and the ship's characteristic maneuvering capabilities, the IMO Assembly adopted in 1968 in Resolution A.160 on "Recommendation on Data Concerning Maneuvering Capabilities and Stopping Distances of Ships". The Governments were urged to ensure that the master and the

officers should have readily available on the bridge all necessary data in accordance with this recommendation under various conditions of draft and speed.
This was the first attempt by the IMO to address and recommend the contents of the manoeuvering information on board ships. As a result, essential maneuvering information in some detail is now available on the bridge for use by the master, navigating officers and pilots. It contributes to safety of navigation under various circumstances in which the ship operates.

4.4.2 Impaired Maneuverability of Tankers

The IMO Marine Safety Committee approved in 1983 a first text on "Recommendation on Emergency Towing Requirements for Tankers" which was adopted by the IMO Assembly by Resolution A.535. This recommendation requires that tankers greater than 50,000 tons deadweight should be fitted with emergency towing arrangements at the bow and stern. About 10 years later, this tonnage number was decreased in the recommendations to 20,000 tons deadweight.
The provision of technologically advanced equipment and its ready availability in a pre-rigged condition would ensure that the risk of marine pollution is minimized from a tanker involved in an accident and resultant impaired maneuverability.

4.4.3 Estimation of Maneuverability in Ship Design

All ships should have maneuvering qualities which permit them to keep course, to turn, to check turns, to operate at acceptable low speeds and to stop. All these should be performed in a satisfactory manner. Since most maneuvering qualities are inherent in the design of the hull and machinery, they should be consciously estimated during the design process.
In 1985, interim guidelines have been developed to define the specific maneuvering characteristics which quantify maneuverability and recommended estimation of these characteristics during design both for the fully loaded and the test conditions in deep water. They also outline full scale tests to confirm the maneuvering performance in test conditions. They are intended to estimate the maneuvering performance of all new ships greater than 100 meters in length. The guidelines are intended to apply to ships with rudders of conventional design. Ships with unconventional steering arrangements should be included by the use of an equivalent control setting.

4.4.4 Standards for Ship Maneuverability

There is a clear link between safety and maneuverability of ships. Maneuverability must be considered during the design stage already; minimum standards of performance must be included in the design process. All maneuvering performance criteria and measures should be simple, relevant, comprehensible, measurable and practicable. The maneuvering characteristics addressed in the (interim) maneuverability standards of IMO are typical features of the performance quality and handling capacity of the ship, which are considered to be of direct nautical interest. Moreover, they may all be reasonably well predicted at the design stage and they can be measured or evaluated from accepted trial-type maneuvers. The IMO Sub-Committee has refined values of criteria on abilities on (initial) turning, yaw checking, course keeping and stopping. These (interim) standards were adopted by the IMO

Assembly in 1993 in Resolution A.751 on "Interim Standards for Ship Maneuverability" and Governments were invited to collect data obtained by application of these standards. The following "Interim Standards for Ship Manoeuvrability" are given by IMO:

1 Principles

1.1 The standards should be used with the aim of improving ship manoeuvring performance and with the objective of avoiding building ships that do not comply with the criteria.

1.2 The standards contained in this document are based on the understanding that the manoeuvrability of ships can be evaluated from the characteristics of conventional trial manoeuvres. The following two methods can be used to demonstrate compliance with these standards:

1.2.1 Scale model tests and/or computer predictions using mathematical models can be performed to predict compliance at the design stage. In this case full-scale trials should be conducted to validate these results. The ship should then be considered to meet these standards regardless of full-scale trial results, except where the Administration determines that the prediction efforts were substandard and/or the ship performance is in substantial disagreement with these standards;

1.2.2 The compliance with the standards can be demonstrated based on the results of the full-scale trials conducted in accordance with the standards. If a ship is found in substantial disagreement with the interim standards, then the Administration may require remedial action.

1.3 The standards presented herein are considered interim for a period of 5 years from the date of their adoption by the Assembly. The standards and method of establishing compliance should be reviewed in the light of new information and the results of experience with the present standards and ongoing research and developments.

2 Application

2.1 The standards should be applied to ships of all rudder and propulsion types, of 100 m in length and over, and chemical tankers and gas carriers regardless of the length, which are constructed on or after 1 July 1994.

2.2 In case ships referred to in paragraph 2.1 undergo repairs, alterations and modifications which in the opinion of the Administration may influence their manoeuvrability characteristics the continued compliance with the standards should be verified.

2.3 Whenever other ships, originally not subject to the standards, undergo repairs, alterations and modifications, which in the opinion of the Administration are of such an extent that the ship may be considered to be a new ship, then that ship should comply with these standards. Otherwise, if the repairs, alterations and modifications in the opinion of the Administration may influence the manoeuvrability characteristics, it should be demonstrated that these characteristics do not lead to any deterioration of the manoeuvrability of the ship.

2.4 The standards should not be applied to the high speed craft as defined in the relevant Code.

3 Definitions

3.1 Geometry of the ship:

3.1.1 Length (L) is the length between the aft and forward perpendiculars;

3.1.2 Midship point is the point of the centre line of a ship midway between the aft and forward perpendiculars;

3.1.3 Draught (T_a) is the draught at the aft perpendicular;

3.1.4 Draught (T_f) is the draught at the forward perpendicular;

3.1.5 Mean draught (T_m) is defined as $T_m = (T_a + T_f)/2$.

3.2 Standard manoeuvres and associated terminology
Standard manoeuvres and associated terminology are as defined below:

3.2.1 The test speed (V) used in the standards is a speed of at least 90 % of the ship's speed corresponding to 85 % of the maximum engine output.

3.2.2 Turning circle manoeuvre is the manoeuvre to be performed to both starboard and port with 35^0 rudder angle or the maximum rudder angle permissible at the test speed, following a steady approach with zero yaw rate.

3.2.3 Advance is the distance travelled in the direction of the original course by the midship point of a ship from the position at which the rudder order is given to the position at which the heading has changed 90^0 from the original course.

3.2.4 Tactical diameter is the distance travelled by the midship point of a ship from the position at which the rudder order is given to the position at which the heading has changed 180^0 from the original course. It is measured in a direction perpendicular to the original heading of the ship.

3.2.5 Zig-zag test is the manoeuvre where a known amount of helm is applied alternately to either side when a known heading deviation from the original heading is reached.

3.2.6 $10^0/10^0$ zig-zag is performed by turning the rudder alternatively by 10^0 to either side following a heading deviation of 10^0 from the original heading in accordance with the following procedure:

3.2.6.1 after a steady approach with zero yaw rate, the rudder is put over to 10^0 to starboard/port (first execute);

3.2.6.2 when the heading has changed to 10^0 off the original heading, the rudder is reversed to 10^0 to port/starboard (second execute);

3.2.6.3 after the rudder has been turned to port/starboard, the ship will continue turning in the original direction with decreasing turning rate. In response to the rudder, the ship should then turn to port/starboard. When the ship has reached a heading of 10^0 to port/starboard of the original course, the rudder is again reversed to 10^0 to starboard/port (third execute).

3.2.7 The first overshoot angle is the additional heading deviation experienced in the zig-zag test following the second execute.

3.2.8 The second overshoot is the additional heading deviation experienced in the zig-zag test following the third execute.

3.2.9 $20^0/20^0$ zig-zag test is performed using the procedure given in 3.2.6 above using 20^0 rudder angles and 20^0 change of heading, instead of 10^0 rudder angles and 10^0 change of heading, respectively.

3.2.10 Full astern stopping test determines the track reach of a ship from the time an order for full astern is given until the ship stops in the water.

3.2.11 Track reach is the distance along the path described by the midship point of a ship measured from the position at which an order for full astern is given to the position at which the ship stops in the water.

4 Standards

4.1 The standard manoeuvres should be performed without the use of any manoeuvring aids, which are not continuously and readily available in normal operation.

4.2 Conditions at which the standards apply
In order to evaluate the performance of a ship, manoeuvring trials should be conducted to both port and starboard and at conditions specified below:

4.2.1 deep, unrestricted water;

4.2.2 calm environment;

4.2.3 full load, even keel condition;

4.2.4 steady approach at the test speed.

4.3 Criteria
The manoeuvrability of a ship is considered satisfactory, if the following criteria are complied with:

4.3.1 Turning ability
The advance should not exceed 4.5 ship lengths (L) and the tactical diameter should not exceed 5 ship lengths in the turning circle manoeuvre;

4.3.2 Initial turning ability
With the application of 10^0 rudder angle to port/starboard, the ship should not have travelled more than 2.5 ship lengths by the time the heading has changed 10^0 from the original heading;

4.3.3 Yaw checking and course keeping abilities

4.3.3.1 The value of the first overshoot angle in the $10^0/10^0$ zig-zag test should not exceed:
- 10^0, if L/V is less than 10 seconds;
- 20^0, if L/V is 30 seconds or more; and
- $(5 + L/V/2)$ degrees, if L/V is 10 seconds or more but less than 30 seconds,
where L and V are expressed in m and m/s, respectively;

4.3.3.2 The value of the second overshoot angle in the $10^0/10^0$ zig-zag test should not exceed the above criterion values for the first overshoot by more than 15^0;

4.3.3.3 The value of the first overshoot angle in the $20^0/20^0$ zig-zag test should not exceed 25^0;

4.3.4 Stopping ability

The track reach in the full astern stopping test should not exceed 15 ship lengths. However, this value may be modified by the Administration where ships of large displacement make this criterion impracticable.

5 Additional considerations

5.1 In case the standard trials are conducted at a condition different from those specified in 4.2.3 necessary corrections should be made in accordance with the guidelines contained in the explanatory notes on the standards for ship manoeuvrability developed by the Organization.

5.2 Where standard manoeuvres indicate dynamic instability, alternative test may be conducted to define the degree of instability. Guidelines for alternative tests such as a spiral test or pull-out manoeuvre are included in the explanatory notes on the standards for ship manoeuvrability developed by the Organization.

4.4.5 Prediction Technology

Recognizing that it is not currently possible at the design stage to predict maneuvering characteristics with sufficient accuracy, the primary concern of the IMO Sub-Committee was that a ship - even though predicted to be in compliance with the standards by employing the best prediction method available - might fail to do so on trials and then be disqualified. The IMO Sub-Committee agreed that it would be permissible to demonstrate compliance with the standards by predicting trial performance through model tests and/or computer simulation. Moreover, when acceptable methods of prediction have demonstrated compliance with the standards, the results of full scale trials would not disqualify a ship.

4.5 Full Scale Maneuvering Trials

There are no definitive international standards for conducting maneuvering trials with ships. Many ship yards have developed their own procedures driven by their experience and with consideration to the efforts made by the International Towing Tank Conference (ITTC, Proceedings 1963 - 1975) and other organizations or institutes.
The Society of Naval Architects and Marine Engineers (SNAME) has produced three guidelines: "Code on Maneuvering and Special Trials and Tests" (1950), "Code for Sea Trials" (1973) and "Guide for Sea Trials" (1989).
The Norwegian Standard Organization has produced "Testing of New Ship, Norsk Standard" (1985).
The Japan Ship Research Association (JSRA) has produced a "Sea Trial Code for Giant Ships, JSRA" (1972) for maneuvering trial procedure and analysis of measurements.
IMO Resolution A.601 (1987) and IMO Resolution A.751 (1993), were adopted by the IMO Assemblies to address ship maneuverability. The last one provides (interim) criteria for turning, initial turning, stopping, course keeping and course changing ability, as well as information for the execution of the maneuvering trials.

Accounting for the IMO (interim) standards, the 22^{nd} International Towing Tank Conference - held in Seoul (Korea) and Shanghai (China) in September1999 - has adopted new guidelines for performing maneuvering trials. For an important part, the text in this section on maneuvering trial tests has been taken from the Proceedings of that particular Conference.

It is recommended (and nowadays almost usual) that all experimental data should be acquired by a computer-based system, because almost all of the maneuvering trials require time history data. The system should be able to collect, record and process real-time trial data. A data sampling of 0.5 - 2.0 samples per second is advised.
Particulars of each intended test have to be denoted in the trial agenda. A trial report should be made, which contains items on ship characteristics, loading condition, instrumentation, trial site, environmental conditions, trial program and trial results.

4.5.1 Loading Condition

Ship maneuverability can be significantly affected by the draft and trim condition. The IMO (interim) standards are applied to the full load and even keel conditions. This full load draft is chosen, based on the general understanding that the poorest maneuvering performance (poor course keeping capability) of a ship occurs at this full load condition. When it is impractical to conduct trials at full load because of the ship type (bulk carrier, etc.), draft and trim must be as close as possible to the specified trial condition and/or the condition in which model tests have been carried out. This allow for predicting the full load maneuvering performance, using a reliable correction method (based on model tests or proven computer simulations) that ensures satisfactory extrapolation of trial results as suggested by IMO Resolution A751. In case of any corrections, the procedure used must be clearly referenced and documented.

4.5.2 Trial Site Criteria

Maneuvering of a ship is strongly affected by interactions with the bottom, banks and passing vessels. The trial site should therefore be located in waters of adequate depth with as low a current and tidal influence as possible. Maneuvering trials are generally conducted at the same site as the speed/power trials. Therefore, the same trial depth criteria should be used. IMO standards require that the water depth should exceed four times the mean draft of the ship.

4.5.3 Environmental Conditions

Environmental conditions such as wind, waves and current should always be reported even though they may be considered to have no influence on the ship behavior. Maneuvering trials should be performed in the calmest possible weather conditions.

IMO Resolution A.751 (1993) prescribed the maximum environmental condition for carrying out the maneuvering trials as follows:

1. Wind
   The trials should not be conducted with a true wind speed greater than Beaufort 5. This means that the true wind speed should be less than about 19 knots.

2. Waves
   The trials should be carried out in sea states less than 4. This means that the significant wave height should be less than about 1.90 meter, with most probable peak periods of about 8.8 seconds.

3. Currents
   No specific information on current limitations is given.

Wind, waves and currents can significantly affect the ship maneuverability. However, the IMO Resolution A.751 suggests a method to account for environmental effects during turning tests only.
The effect of the current velocity on the path of a ship during turning circles is shown in figure 4.12. This type of figures can be used to obtain some information on current velocity and direction.

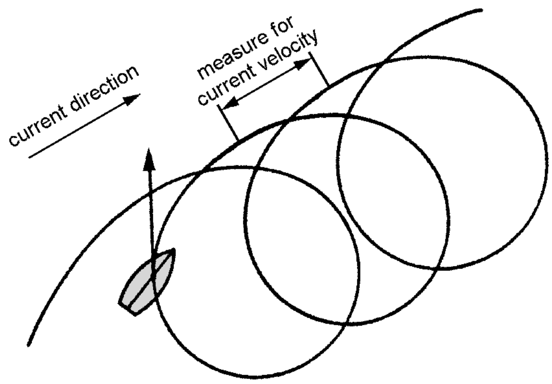


Figure 4.12: Effect of Current on Path of Turning Circles

4.5.4 Selection of Tests

In selecting tests, consideration should be given to their purpose. Some are intended to demonstrate performance of vital machinery and satisfactory regulatory requirements. Some are essential to verify that the vessel has satisfactory basic course keeping and course changing qualities and emergency maneuverability, while others are intended to obtain maneuverability data to be used in establishing operating regulations.

The choice of a test is based on its purpose; it is based on the subject of interest:

1. Course Keeping Qualities
   The suitable test method for this are the direct spiral test, the reverse spiral test, the Z-maneuver test with small rudder angle, the modified Z-maneuver test (also called: zig-zag test) and the pull-out test.

2. Course Changing Qualities
   The Z-maneuver test, the initial turning test and the (new) course keeping test can considered to be suitable test methods for this.

3. Emergency Maneuver Qualities
   The suitable test methods for this are the turning test with maximum rudder angle, stopping test (crash stop astern and stopping inertia test), man-overboard test and parallel course maneuver test.

Table 4.1 summarizes the different types of maneuvering tests recommended or required by various organizations.

	Type of Test	IMO A.601	IMO A.751	ITTC 1975	SNAME 1989	Norsk Standard	Japan RR
1	Turning Test	✓	✓	✓	✓	✓	✓
2	Z-Maneuver Test (Kempf)	✓	✓	✓	✓	✓	✓
3	Modified Z-Maneuver Test						✓
4	Direct Spiral Test (Dieudonné)			✓	✓	✓	✓
5	Reverse Spiral Test (Bech)			✓	✓	✓	✓
6	Pull-Out Test	✓		✓	✓		
7	Stopping Test	✓	✓	✓	✓	✓	✓
8	Stopping Inertia Test	✓				✓	✓
9	New Course Keeping Test	✓					✓
10	Man-Overboard Test	✓					
11	Parallel Course Maneuver Test	✓					
12	Initial Turning Test				✓		
13	Z-Maneuver Test at Low Speed	✓			✓		✓
14	Accelerating Turning Test	✓		✓			
15	Acceleration/Deceleration Test	✓			✓		
16	Thruster Test	✓		✓	✓	✓	
17	Minimum Revolution Test	✓			✓	✓	
18	Crash Ahead Test	✓			✓	✓	✓

Table 4.1. Recommended Maneuvering Tests by Various Organizations

The 1999-ITTC has described the purpose of each individual test and also gave (new) procedures on how to conduct the tests. These (detailed) descriptions have been given in the Sections below, where the general notations are defined by:

$$\begin{aligned} \psi &= \text{course angle} \\ \dot{\psi} = r &= \text{rate of turn} \\ \delta &= \text{rudder angle} \\ t &= \text{time} \end{aligned}$$

1. Turning Test

Purpose The purpose of the turning test is to determine the turning characteristics of the ship. Basic information obtained from the turning test are advance, transfer, tactical diameter, steady turning diameter, final ship speed and turning rate in the steady state; see figure 4.13. It is suggested to perform turning tests at different speeds and for different rudder angles.

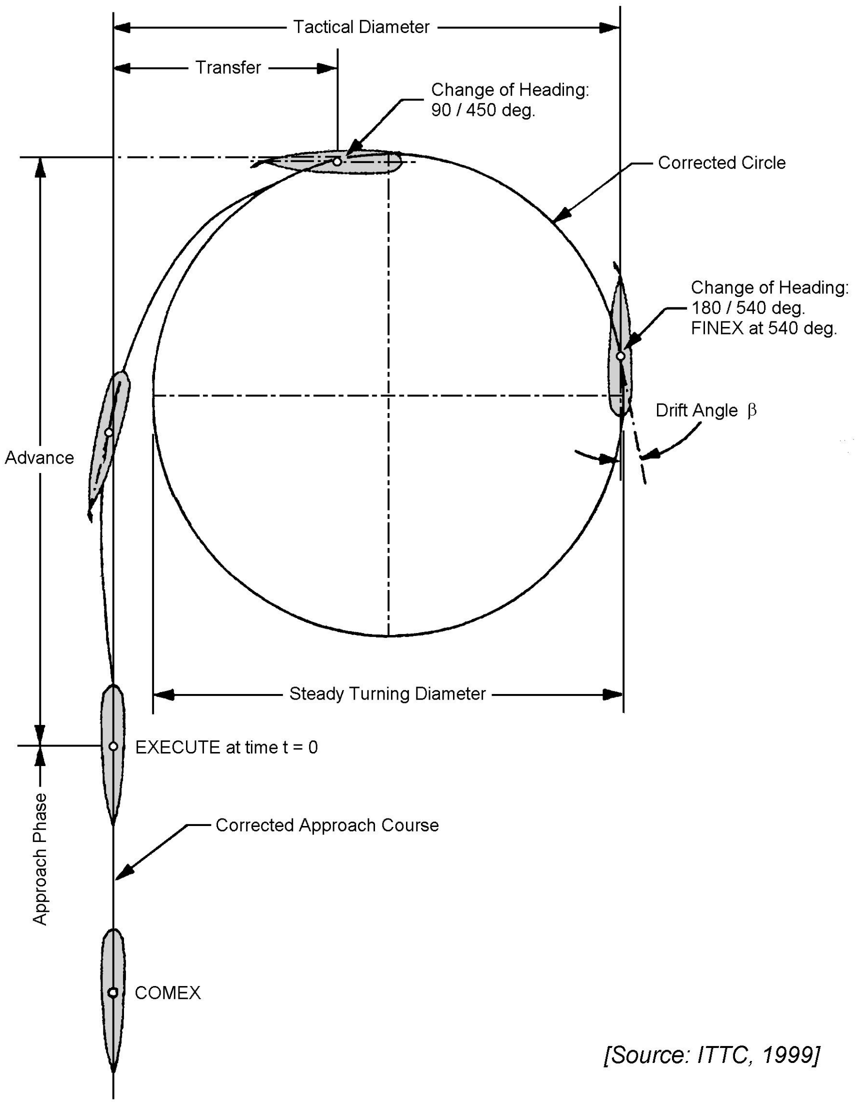


Figure 4.13: Drift-Corrected Turning Circle

Execution The turning test should be carried out as follows:

1. Establish steady ship speed to the conditions indicated in the trial agenda and adjust the ship's heading to a steady course prior to the point of required initial rudder execution. At this time (COMEX, commence execution), no further adjustment to the engine order is to be made until the conclusion (FINEX, finish execution) of the test. Start the data acquisition system.

2. Steer with the prescribed minimum rudder angle. Record the time at which the rudder is moved to the angle denoted in the trial agenda. Once the rudder is set, no further adjustment should be made, even if the rudder angle achieved deviates to some degree from the rudder angle prescribed in the agenda.

3. The test is concluded when the change of heading reaches an angle between 540^0 and 720^0 (the larger the angle the better). At the conclusion of the test, steer the ship on a straight course in readiness for the subsequent test. A suitable approach

distance should be chosen to establish the desired approach speed specified in the trial agenda.

4. The normal rudder angle to be used is 35^0 or maximum rudder right and left, supplemented with 15^0 right and left rudder angles.

5. The approach for right and left turns should preferably be conducted at the same headings.

2. Z-Maneuver Test

Purpose The Z-maneuver or zig-zag test (Kempf, 1944) is used to express course changing (yaw checking) and course keeping qualities. The essential information to be obtained is the initial turning time, time to second execute, the time to check yaw and the angle of overshoot. Values of the steering indices K (gain constant) and T (time constant) associated with the linearized response model are also obtained.

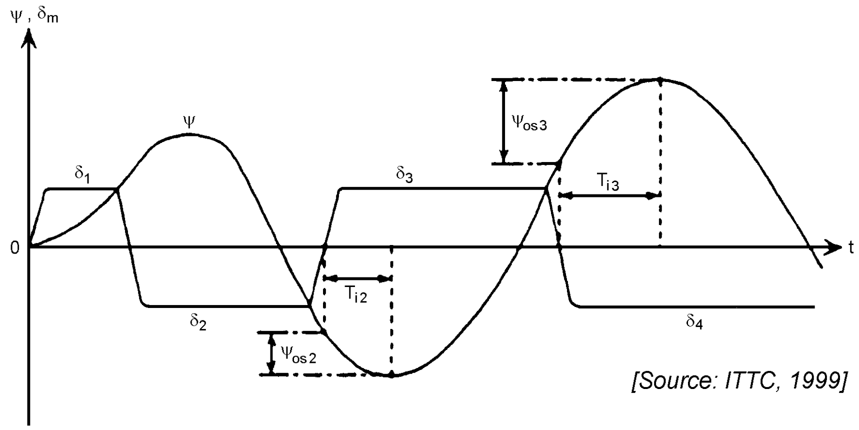


Figure 4.14: Z-Maneuver or Zig-Zag Test

Execution The Z-maneuver test should be carried out as follows:

1. Establish steady ship speed at the conditions indicated in the trial agenda and adjust the ship's heading to a steady course prior to the point of required initial rudder execution. At this time, no further adjustments to the engine order is to be made until the conclusion of the test. Start the acquisition system.

2. Order "10^0 right rudder". Maintain rudder at the ordered angle. In the case of a neutral angle, the rudder commands should be issued relative to the neutral angle.

3. When the change of heading nears to 10^0 to starboard, order "stand by for 10^0 left rudder". Upon attainment of 10^0 to starboard heading, order "execute". Reverse rudder at normal rate from 10^0 right to 10^0 left and thereafter maintain rudder at 10^0 left.

4. The ship will keep turning to starboard for a while until reaching the overshoot angle. When the change of heading nears to 10^0 to port from the original approach heading, order "stand by for 10^0 right rudder". Repeat step 3 above with left and right reversed.

5. Upon attainment of approach heading, the test is terminated and the ship is reset to a steady course.

6. Bring the ship to the same initial heading as in the previous run. Establish the next steady ship speed indicated in the trial agenda. Repeat steps 2 to 6 above with term "left" and "right" reversed. (The initial procedure steps 2 to 6 starting with right rudder is hereafter referred to as "$+10^0$" and the reversed procedure starting with left rudder "-10^0").

7. IMO requires Z-maneuver tests be performed at $\pm 10^0$ and $\pm 20^0$. For large full ships it is recommended to also perform tests at $\pm 5^0$ and $\pm 15^0$ rudder angles.

The rudder commands during these Z-maneuver tests should be issued for a rudder angle relative to the neutral angle. The neutral angle is the mean rudder angle at which the ship keeps steady course at approach heading during an extended seaway.
The SC recommends that the procedure be extended to include four additional rudder throws to determine the repeatability of the overshoot headings and angles. GPS should be used to determine the ship's track.

3. Modified Z-Maneuver Test

Purpose The modified Z-maneuver test is used to express course keeping qualities under an actual operation.

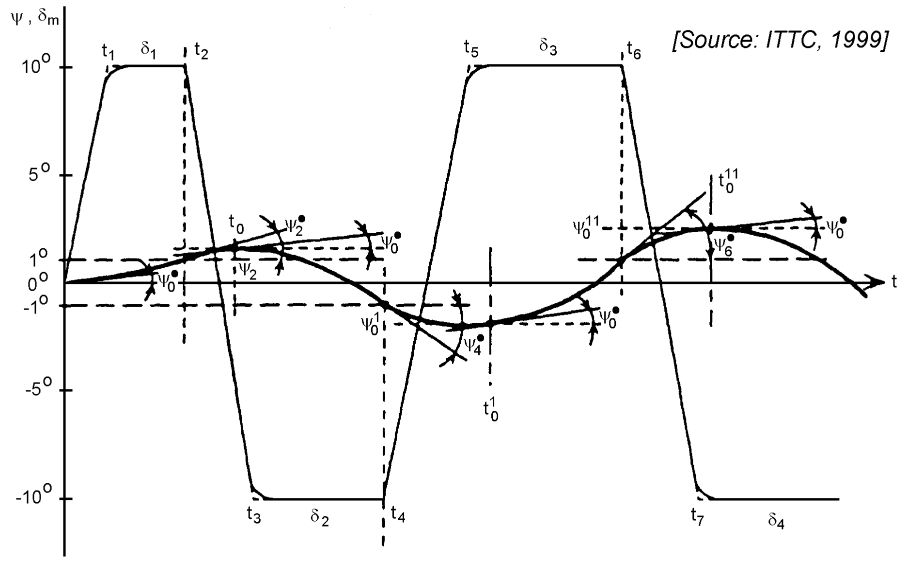


Figure 4.15: Modified Z-Maneuver Test

Execution The test procedure is the same as the Z-maneuver test, except that the rudder angle is only 5^0 or 10^0 and the rudder changes when the change of heading becomes 1^0.

4. Direct Spiral Test

Purpose The purpose of the direct spiral test (Dieudonné, 1953) is to determine whether the ship is directional stable or not. Data defining the relationship between rudder angle and steady turning rate are presented. Important parameters are width and height of the loop for an unstable ship.

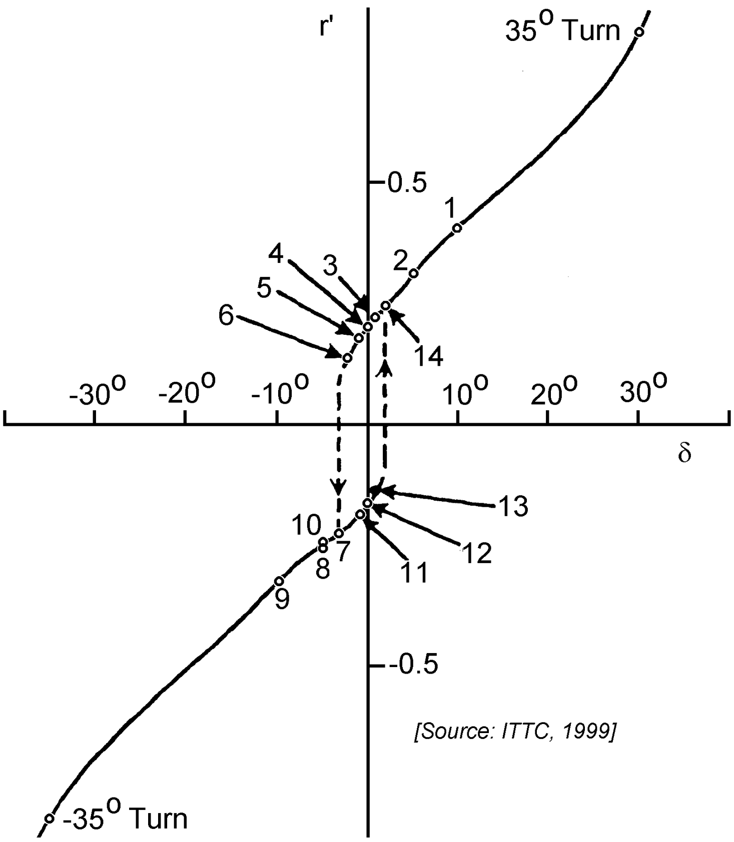


Figure 4.16: Direct Spiral Test

Execution The direct spiral test should be carried out as follows:

1. Set ship to steady speed and to steady course, then order "10^0 right rudder". Actuate rudder at normal steering rate to 10^0 right. Maintain rudder at ordered angle.

2. When a steady rate of change of heading is achieved, record the rate of turn, rudder angle and propeller revolution.

3. Order "shift rudder to right 5^0" and repeat measurements as in step 2 above.

4. Order "shift rudder to right 1^0" and repeat the step 2 measurements.

5. Order "shift rudder to amidships". Repeat the step 2 measurements. The ship will normally maintain a steady rate of turn to starboard for a while. Record this residual rate of turn.

6. Order "1^0 left rudder" and repeat the step 2 measurements.

7. If the ship still keeps turning to starboard, order "2^0 left ...", "3^0 left ...". Incrementally increase the rudder angle by 1^0 until the ship starts turning to port. At every rudder angle, repeat the step 2 measurements.

8. When the ship starts to turn to port,wait until the rate of turn becomes steady and record both the rate of turn and the rudder angle.

9. Order "5^0 left rudder". Repeat the step 2 measurements. Proceed to "10^0 left rudder" and repeat the step 2 measurements. If step 7 above has been attained only beyond 5^0 port, start the present step 1 at "10^0 left rudder".

10. Repeat step 4 to 5, or 3 to 5, whichever is relevant. (Left and right are reversed in step 6.)

11. Order "1^0 right rudder" and repeat the step 2 measurements.

12. If the ship still keeps turning to port, order "2^0 right ...", "3^0 right ...", incrementally increase the rudder angle by 1^0 until the ship starts turning to starboard. Repeat the step 2 measurements.

13. When the ship starts to turn to starboard,wait until the rate of turn becomes steady and record both the rate of turn and the rudder angle. Terminate the test.

These direct spiral tests should preferably be conducted in a sea state below 2 or 3. Reliable measurements can not be expected in a sea state beyond 4.
If a test has to be temporarily suspended, it should be resumed using the rudder angle associated with the step preceding that at which the suspension took place.

5. Reversed Spiral Test

Purpose The purpose of the reversed spiral test (Bech, 1968) is to obtain for an unstable ship the complete loop, when it is not determined by the direct spiral test.

Execution The reversed spiral test should be carried out as follows:

1. The ship shall proceed at the prescribed heading and speed. The yaw rate setting is adjusted to a predefined value. The ship is steered around this value according to the following scheme.

2. Rudder is turned to left 5^0 when the indicator passes through the 0 point from the port to starboard side, and to 5^0 right in the converse case. If the time to steer left 5^0 proves to be shorter than the corresponding time for 5^0 right rudder, substitute 5^0 left with 4^0 left ,and 5^0 right with 6^0 right. Proceed in similar gradations until the times of both steering become balanced.

3. When the second step is completed, a rough regular yawing motion is obtained upon which the measurements at the defined setting are determined.

Increase yaw angular rate inevitably increases the difference between left and right rudder angle. In such cases, the procedure may be started using the angle set in the preceding cycle.

6. Pull-Out Test

Purpose The pull-out test represents a simpler way to give the height of the loop for an unstable ship than the direct spiral test and the reversed spiral test.

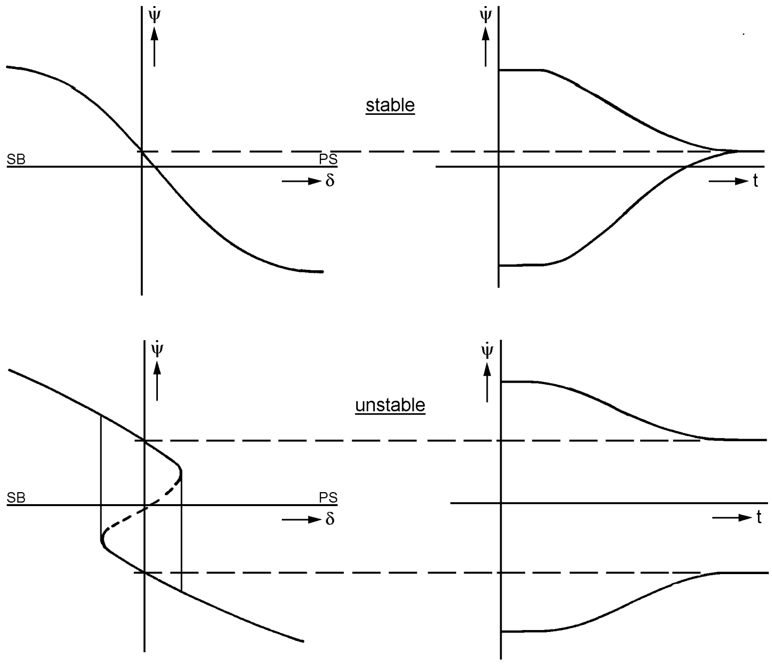


Figure 4.17: Results of a Pull-Out Test for a Stable and an Unstable Ship

Execution The pull-out test should be carried out as follows:

1. Establish steady speed and course conditions, then order "right rudder" prescribed in the trial agenda. Actuate the rudder at a normal rate to the prescribed rudder angle. Maintain the rudder at the ordered angle.

2. When the ship's turning rate has steadied, order "rudder amidships".

3. Maintain the rudder at the amidships position until the ship shows a steady residual rate of turn.

4. Repeat steps 1, 2 and 3 using "left rudder".

Tests should preferably be conducted in a sea state below 2 or 3. Reliable measurements can not be expected in a sea state beyond 4. The pull-out tests should be performed as a continuation of the turning tests. The pull-out test must be performed using both left and right rudder to show possible asymmetry.

7. Stopping Test

Purpose The stopping test is used to express the qualities for emergence maneuvers. The obtainable information are stopping time and stopping distance of the ship from the time at which an astern engine order is given until the ship is stopped dead in the water. These tests include both crash stop astern and crash stop ahead.

Execution The stopping test should be carried out as follows:

1. Establish steady ahead ship speed at the condition noted in the trial agenda and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the engine order is initiated, start the data acquisition system.

2. Order "engine astern" at the prescribed position noted in the trial agenda (full, half, slow).

3. With the rudder at amidships, the test will proceed until the ship is stopped dead in the water.

At the end of the ahead stopping test, the test should be repeated with the ship initially moving at an initial steady astern ship speed and using an ahead engine order to stop the ship.

8. Stopping Inertia Test

Purpose The stopping inertia test is used to determine the running distance and time for the ship from the time at which an order for engine stop is given until the ship achieves a defined minimum speed. IMO Resolution A.601 (1987) requires that the stopping performance information using maximum rudder angle be displayed in the wheelhouse.

Execution The stopping inertia test should be carried out as follows:

1. Establish a steady ship speed in accordance with the trial agenda and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the engine order is initiated, start the acquisition system.

2. Order "engine stop" and move the rudder to either amidships or 35^0 port or starboard.

3. The test will proceed until the ship achieves a defined minimum speed.

When the prescribed rudder angle is "0^0" the stopping inertia is named "free stop". When the prescribed rudder angle is "35^0", left or right, the stopping inertia is called "IMO stop".

9. New Course Keeping Test

Purpose The new course keeping test is used to provide information for changing a ship course. The obtained data are presented in an operational diagram of ship heading versus advance and transfer.

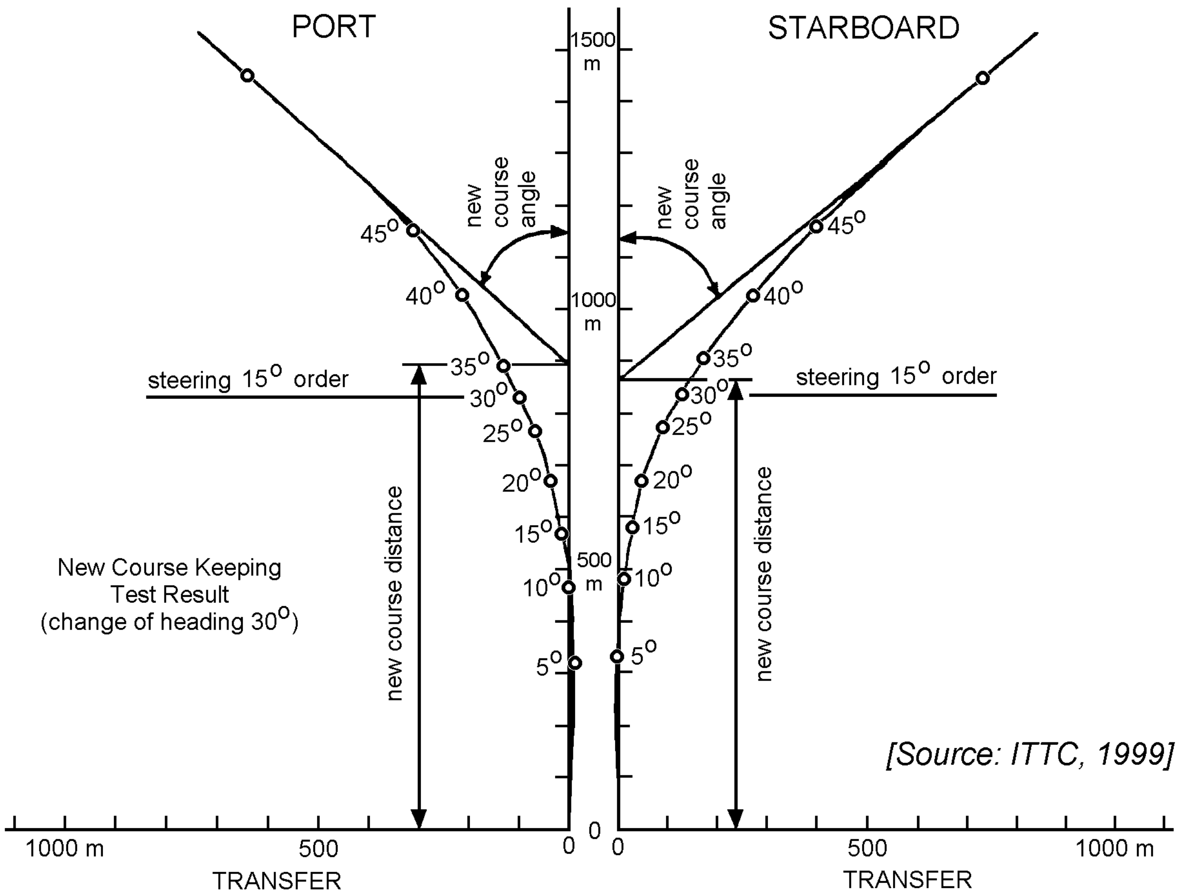


Figure 4.18: New Course Keeping Test

Execution The new course keeping test should be carried out as follows:

1. Establish a steady ship speed in accordance with the trial agenda and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the rudder movement is initiated, start the acquisition system.

2. Move the rudder to 15^0 right and maintain that rudder angle.

3. When the change of heading reaches 10^0 starboard from the initial approach heading, quickly move the rudder to 15^0 left.

4. When the rate of turn reaches 0 degree/second, return the rudder to amidships.. This completes the test.

5. Repeat the test using an initial rudder angle of left 15^0 port. Repeat steps 2 and 3 with the term "left" and "right" reversed.

6. Repeat the test using initial rudder angles of 20^0 and 30^0 (total of 6 runs).

Move the rudder to the prescribed angle as accurate as possible. Ensure that speed and heading are steady during the approach.

10. Man-Overboard Test

Purpose The man-overboard test is used to simulate the rescue of a man who has fallen overboard. Rudder angle and ship trajectory are checked throughout this test.

Execution The man-overboard test should be carried out as follows:

1. Establish a steady ship speed in accordance with the trial agenda and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the rudder movement is initiated, start the acquisition system.

2. Order 35^0 rudder angle, left or right.

3. When the ship has turned between 20^0 and 60^0 from the initial base course, order 35^0 opposite rudder and hold the rudder until the ship has turned 120^0 to 150^0 from the initial course.

4. Gradually reduce the rudder angle until the ship's heading is reversed 180^0 from the initial approach course.

11. Parallel Course Maneuver Test

Purpose Parallel course maneuver test provides the information for the emergency avoidance of a passing ship when maximum rudder angle is used. The obtainable data are an operational diagram of rudder angle and lateral displacement from the original base course.

Execution The parallel course maneuver test should be carried out as follows:

1. Establish a steady ship speed in accordance with the trial agenda and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the rudder movement is initiated, start the acquisition system.

2. Order 35^0 rudder angle.

3. When the ship has turned 30^0, order 35^0 opposite rudder and steer the ship to regain the original course.

The maneuver is repeated with the sequence of left and right commands reversed.
Additional tests are conducted where the ship will be turned after 60^0 and at 90^0 course deviations have been achieved.
Further tests may be performed with the rudder angle modified from 35^0 to 30^0, 25^0 and so on.

12. Initial Turning Test

Purpose The initial turning test is used to provide the information on the transient change of heading after application of a moderate rudder rate.

Execution The initial turning test should be carried out as follows:

1. Establish a steady ship speed in accordance with the trial agenda and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the rudder movement is initiated, start the acquisition system.

2. Order the rudder move in accordance to the values noted in the trial agenda. Once the rudder angle is set, no further adjustment is made, even if the rudder angle achieved deviates to some degree from the desired rudder angle.

3. When the ship has reached a steady turning rate, the test is completed and the ship is made ready for the subsequent test.

The normal rudder angle to be ordered is 10^0 and 20^0 for both port and starboard turns. It is desirable that the approach headings for port and starboard turns be the same.

13. Z-Maneuver Test at Low Speed

Purpose The Z-maneuver test at low speed is used to determine the minimum speed at which the ship does not respond to the helm.

Execution The Z-maneuver test at low speed should be carried out as follows:

1. Establish a steady ship speed in accordance with the trial agenda and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the rudder movement is initiated, start the acquisition system.

2. Stop the engine and let the propeller idle. Order the rudder to be moved to 35^0 right.

3. When the ship turns to 1^0 starboard heading, reverse the rudder angle to 35^0 left.

4. After the ship reverses turning direction and the approach heading reaches 1^0 port, reverse the rudder angle to right 35^0.

5. Repeat steps 3 and 4 until the rudder is no longer active and the ship does not respond to the rudder. At this point the test is complete.

14. Accelerating Turning Test

Purpose This test provides the turning characteristics of the ship accelerating from dead in the water. Maximum engine output are applied simultaneously.

Execution The accelerating turning test should be carried out as follows:

1. Bring the ship dead in the water with the engine set to "stop engine". Start the acquisition system.

2. Order the rudder angle prescribed in the trial agenda. Simultaneously order "ahead" at the prescribed engine order. Once the rudder is set, no further adjustment is made, even if the rudder angle achieved deviates to some degree from the desired rudder angle.

3. When the change of heading reaches an angle between 540^0 and 720^0, the test is complete.

The normal rudder angle to be ordered is 35^0 or maximum rudder angle left and right. It is desirable that the approach headings for left and right turns be the same.

15. Acceleration/Deceleration Test

Purpose These tests determine speed and reach along the projected approach path versus elapsed time for a series of acceleration/deceleration runs using various engine order combinations.

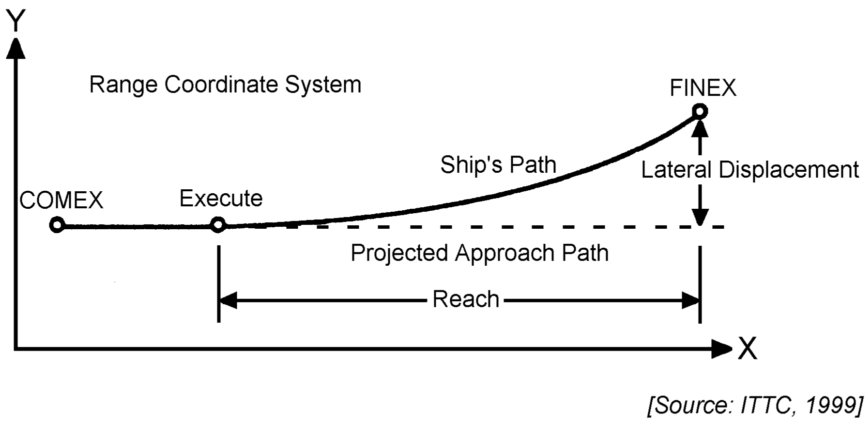


Figure 4.19: Acceleration/Deceleration Test

Execution The acceleration/deceleration test should be carried out as follows:

1. Establish a steady ship speed in accordance with the trial agenda and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the engine order is initiated, start the acquisition system.

2. Execute the prescribed engine order.

3. When the ship attains the steady terminal speed stated in the trial agenda, the test is complete.

Use minimum rudder throughout the run to maintain heading. It is desirable to conduct all of the acceleration/deceleration runs from the same initial base heading and general location. The trial agenda details the combination of approach and execute engine orders that should be used in this series of tests.

16. Thruster Test

Purpose The thruster test is used to determine the turning qualities using thruster(s) with the ship dead in the water or running at a given speed.

Execution The thruster test should be carried out as follows:

1. With the ship dead in the water at the heading prescribed in the trial agenda and the engine set to "stop engine", start the data acquisition.
2. Order the bow thruster(s) to full thrust.
3. When the thruster(s) operate over 10 minutes or the ship's heading reaches 30^0, the test is complete. The ship is brought back to a dead in the water condition at the desired heading, in readiness for the subsequent test (reverse bow thruster).

The same procedure is applied using an initial forward speed designated in the trial agenda (starting point being a stable prescribed speed). Approach headings for left and right turns should be the same.
With the ship in trial ballast condition, it should be noted that reduced thrust may result unless the thruster is not properly submerged. The thruster should be submerged so that its axis is at a depth of at least 0.8 times the thruster diameter.
Bow thruster tests for dry cargo ships in the trial ballast condition are severely influenced by sea and wind and should be conducted only in protected areas or in the open sea when the sea conditions are exceptionally smooth.
Refer to Guide for sea Trials, SNAME (1989) for other thruster tests using combinations of rudder.

17. Minimum Revolution Test

Purpose The ability to proceed at steady slow speed is determined from the ship's speed associated with the lowest possible engine revolutions per minute.

Execution The minimum revolution test should be carried out as follows:

1. Establish a steady ship speed in accordance with the trial agenda (normally maximum speed in harbor) and adjust the ship's heading to a steady course. At a position roughly one ship length before the point where the rpm is changed, start the acquisition system.
2. Main shaft revolution is gradually decreased until the minimum revolution necessary to keep the engine running is found.
3. A confirmation run at minimum revolution is carried out for about one minute.

18. Crash Ahead Test

Purpose The crash ahead test is used to express the qualities for emergency maneuvers similar to the stopping test. An order for full ahead is given when the ship is moving steadily astern. The obtainable information are stopping time and stopping distance of the ship from the time at which an order for full ahead is given until the ship is stopped dead in the water.

Execution The crash ahead test is a part of the stopping test, whose procedure is described in a previous section on the stopping test.

4.5.5 Stability Characteristics

Dieudonné's direct spiral maneuver identifies the directional stability characteristics of a ship. The numerical measures, obtained from this spiral maneuver, are the steady yawing rates as a function of rudder angle. A plot of these values is indicative of the stability characteristics of a ship. For example, if the plot is a single line going from starboard rudder to port rudder and back again - as shown for Ship A in figure 4.20 - the ship possesses controls-fixed straight-line stability. If, however, the plot consists of two branches joined together to form a "hysteresis" loop - as shown for Ships B and C in figure 4.20 - the ship is unstable. The solid line curves show the steady turning rates. Arrows along the curves show the sequence of results of the spiral tests. Dotted lines indicate the jump in steady-turning rate during spiral tests of the dynamically unstable Ships B and C.

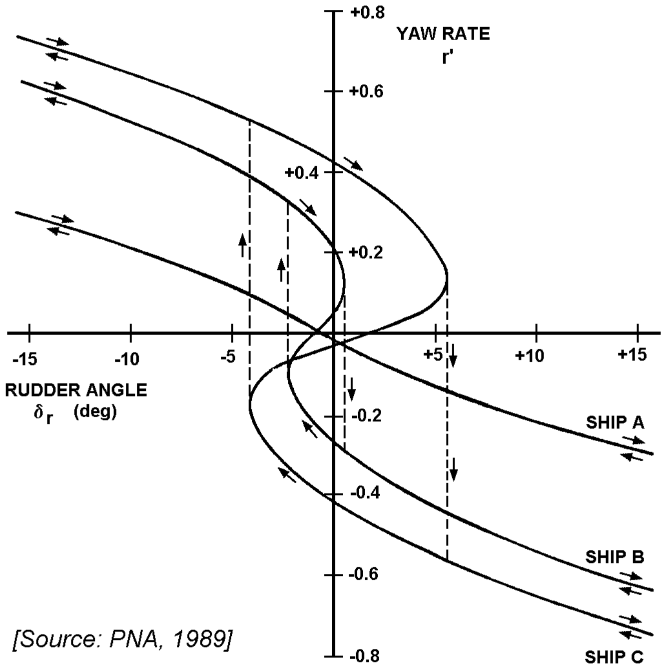


Figure 4.20: Steady Turning Rate Versus Rudder Angle

For unstable vessels, a hysteresis loop is identifiable, although a definite relationship is indicated within the loop. This is because the test condition is no longer controls-fixed. The results still provide the shape of the loop for evaluation of the degree of instability.
A properly calibrated rate-gyro and an accurate rudder angle indicator are required, although in certain cases the test may be performed with automatic steering devices available on board. If manual steering is used, the instantaneous rate of turn must be visually displayed for the helmsman, either on a recorder or on a rate-of-turn indicator. Using the reverse spiral test technique, point on the curve of yaw rate versus rudder angle may be taken in any order.
In addition, the height and width of the loop are numerical measures of the degree of instability; the larger the loop, the more instable the ship. The slope of the yaw-rate curve at zero rudder angle is a measure of the degree of stability or instability.

It may be noted here already that the linear theory, as will treated in the next section, is unable to predict the characteristics of the hysteresis for unstable ships. For this purpose, a non-linear theory is essential; this will be treated in another lecture.

Although not commonly in use, the pull-out tests provide an indication of a ship's stability on a straight course. The ship is first made to turn with some rate of turn in either direction. The rudder is then returned to amidships (neutral position). If the ship is stable, the rate of turn will decay to zero for turns to both port and starboard. If the ship is only moderate unstable, the rate of turn will reduce to some residual rate. The pull-out tests should be performed to both port and starboard to show possible asymmetry. Normally, pull-out tests can easily be performed in connection with other tests being run.

One of the main reasons for speed reduction when sailing a (part of a) turning circle will be caused by an inertia term and a drift angle, see figure 4.21, which causes an additional longitudinal resistance term.

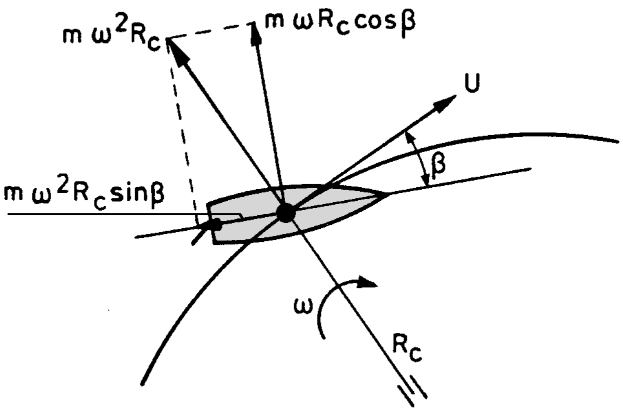


Figure 4.21: Effect of Drift Angle on the Resistance of a Ship

Figure 4.22 gives an indication of the speed reduction of a ship as a function of the turning diameter - ship length ratio and the block coefficient.

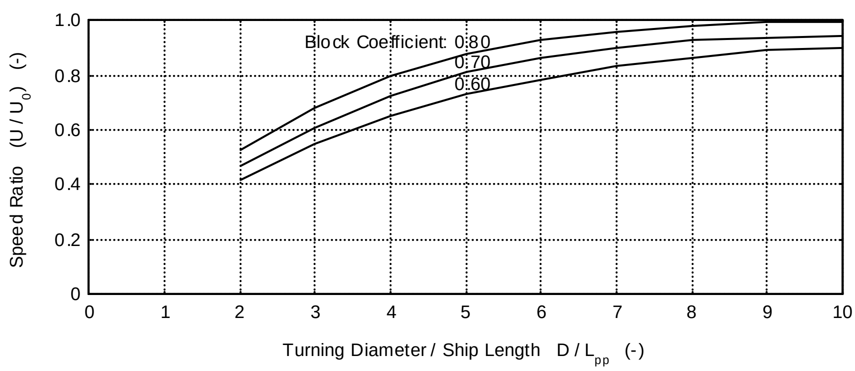


Figure 4.22: Speed Reduction as Function of Diameter, Ship Length and C_B

Figure 4.23 gives an indication of the required rate of turn of a ship at low speeds, when using special maneuvering tools like bow propellers and active rudders.

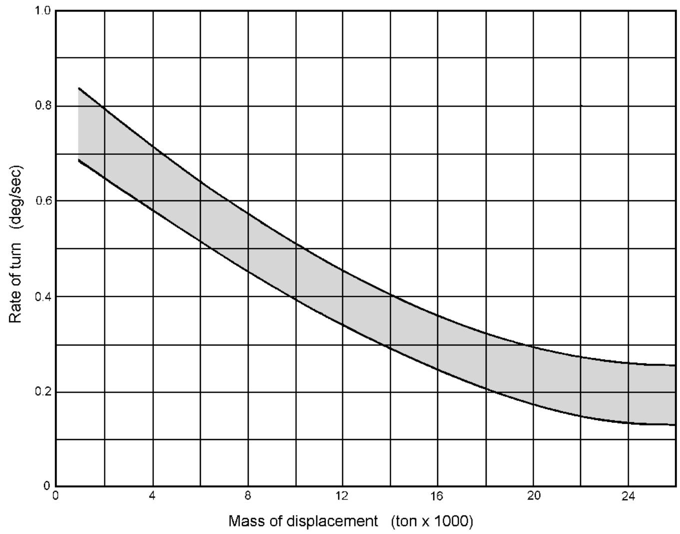


Figure 4.23: Rate of Turn at Low Speeds with Special Maneuvering Tools

4.6 Mathematical Maneuvering Models

Mathematical maneuvering models are used mainly when ship maneuvering is to be assessed with he aid of ship maneuvering simulator, training of navigating officers on a maneuvering simulator or for the development of a ship auto-pilot dedicated to a certain ship design. The model might be rather complicated, consisting of three non-linear, coupled first order differential equations in the horizontal plane, describing the angular velocity and the forward velocity respectively.
When only directional stability and maneuverability is in concern, the only purpose of a mathematical model is to describe yaw and sway as accurately as necessary only for this purpose. Then, a simple but practical model is sufficient.
The simplest example of such model is the linear first order model of Nomoto.

$$\boxed{T\ddot{\psi} + \dot{\psi} = K\delta_r} \quad \text{or} \quad \boxed{T\dot{r} + r = K\delta_r} \tag{4.11}$$

where ψ is the course angle and r is the rate of turn.
To understand this uncoupled yaw equation better, one should write it in the following form:

$$I_e\ddot{\psi} + N_e\dot{\psi} = M_e\delta_r \tag{4.12}$$

with:

$$T = \frac{I_e}{N_e} \quad \text{and} \quad K = \frac{M_e}{N_e}$$

in which:

I_e = total mass moment of inertia coefficient
N_e = hydrodynamic damping coefficient
M_e = external moment coefficient

The indices e of the coefficients indicate here that this are "effective" coefficients, which include coupling effects between sway and yaw.

T and K are simplified characteristic constants of ship; they are often called the "K-T Indices" or the "Nomoto Steering Indices". A large time constant T will be obtained in case of a large moment of inertia I_e and a small damping N_e, like is the case for large crude oil carriers. A large K is caused by a large rudder moment M_e and a small damping N_e. Figure 4.24 shows some of these effects on the path of a ship during a turning test.

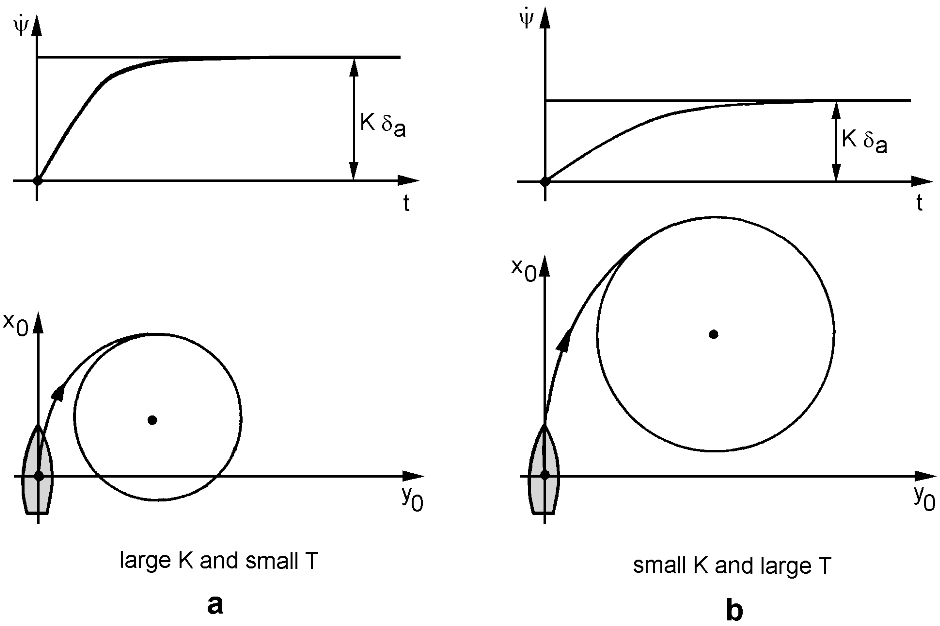


Figure 4.24: Effect of K and T on the Path during a Turning Test

Figure 4.24-a shows that a large K thus provides greater steady state turning ability and a smaller T provides a quicker response to helms; quick response implied good course changing ability. Poor turning and slow response or bad course stability is associated with small K and large T, see figure 4.24-b. Large full ships have a relative large K and a large T.

4.7 Estimation of Nomoto's K and T Indices

Nomoto's first order indices K and T will be determined now from experimental results of two different kinds of test trials:

- turning test data and
- zig-zag maneuver data.

Generally, no linear relation will be found between $\ddot{\psi}$ or $\dot{\psi}$ and δ_r when analyzing maneuvering test data, because of a non-linear behavior of the ship. This means that both K and T will depend on the rudder angle δ.

4.7.1 Use of Turning Test Data

Nomoto's K and T indices in equation:

$$T\ddot{\psi} + \dot{\psi} = K\delta_r \tag{4.13}$$

will be determined here for a turning test with a rudder action as given in figure 4.25.

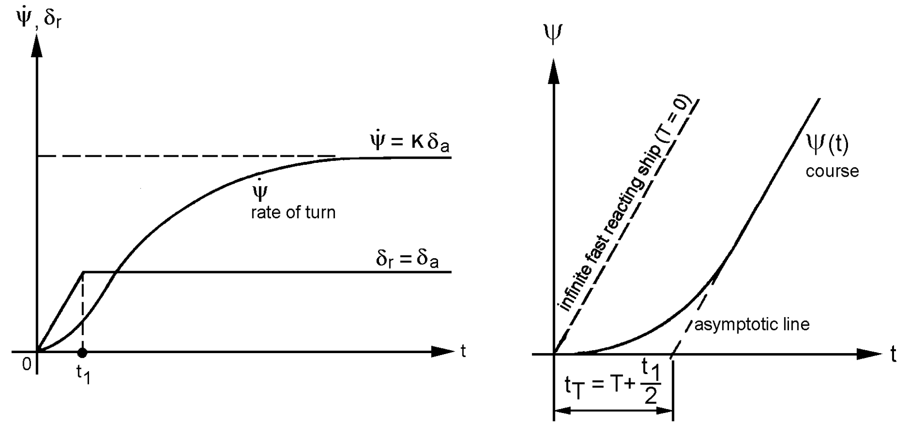


Figure 4.25: Course and Rate of Turn during a Turning Test

Two time interval regions have to be considered here:

1. Time interval $0 \leqq t \leqq t_1$, with a time-depending rudder angle $\delta_r = \frac{\delta_a}{t_1} \cdot t$
   Here, Nomoto's first order equation 4.13 for yaw becomes:

   $$T\ddot{\psi} + \dot{\psi} = \frac{K\delta_a}{t_1} \cdot t \qquad \text{for: } 0 \leqq t \leqq t_1 \tag{4.14}$$

   with solution:

   $$\dot{\psi} = \frac{KT\delta_a}{t_1} \cdot \left(e^{-\frac{t}{T}} - 1 + \frac{t}{T} \right) \qquad \text{for: } 0 \leqq t \leqq t_1 \tag{4.15}$$

2. Time interval $t \geqq t_1$, with a constant rudder angle $\delta = \delta_a$
   Here, Nomoto's first order equation 4.13 for yaw becomes:

   $$T\ddot{\psi} + \dot{\psi} = K\delta_a \qquad \text{for: } t \geqq t_1 \tag{4.16}$$

   with solution:

   $$\dot{\psi} = C_e \cdot e^{-\frac{t}{T}} + K\delta_a \qquad \text{for: } t \geqq t_1 \tag{4.17}$$

For $t = t_1$ both solutions should have the same value:

$$\dot{\psi} = \frac{KT\delta_a}{t_1} \cdot \left(e^{-\frac{t_1}{T}} - 1 + \frac{t_1}{T} \right) = C_e \cdot e^{-\frac{t_1}{T}} + K\delta_a \qquad \text{for: } t = t_1 \tag{4.18}$$

from which follows the so far unknown constant C_e:

$$C_e = \frac{KT\delta_a}{t_1} \left(e^{-\frac{t_1}{T}} - 1 \right) \tag{4.19}$$

This yields as solution for $\dot{\psi}(t)$ at interval $t \geqq t_1$:

$$\dot{\psi}(t) = \frac{KT\delta_a}{t_1} \cdot \left(e^{-\frac{t}{T}} - \frac{e^{\frac{t_1}{T}}}{e^{\frac{t}{T}}} + \frac{t_1}{T} \right) \qquad \text{for: } t \geqq t_1 \tag{4.20}$$

For a large t, this solution of the rate of turn reduces to:

$$\dot{\psi} = K\delta_a \qquad \text{for: } t \to \infty \tag{4.21}$$

and the turning capacity index K in Nomoto's first order equation 4.13 for yaw becomes:

$$\boxed{K = \frac{\dot{\psi}(t \to \infty)}{\delta_a}} \tag{4.22}$$

This relation can also be found directly by applying equation 4.13 to the final part of the time history of a turning test, where the ship is sailing in a steady turning circle already. There: $\ddot{\psi} = 0$, where equation 4.13 reduces to: $\dot{\psi} = K\delta$, from which K can be found.

The time history of the course $\psi(t)$ can be found from an integration of the rate of turn $\dot{\psi}(t)$:

$$\begin{aligned} \psi(t) &= \int_0^t \dot{\psi}(t^*)\, dt^* \\ &= \int_0^{t_1} \dot{\psi}(t^*)\, dt^* + \int_{t_1}^{t} \dot{\psi}(t^*)\, dt^* \\ &= \frac{KT\delta_a}{t_1} \left\{ \int_0^{t_1} \left(e^{-\frac{t^*}{T}} - 1 + \frac{t^*}{T} \right) dt^* + \int_{t_1}^{t} \left(e^{-\frac{t^*}{T}} - \frac{e^{\frac{t_1}{T}}}{e^{\frac{t^*}{T}}} + \frac{t_1}{T} \right) dt^* \right\} \\ &= \ldots \quad \ldots \quad \ldots \end{aligned} \tag{4.23}$$

After some algebra, this integral provides for $t \geqq t_1$:

$$\psi(t) = K\delta_a \cdot \left\{ t - \left(T + \frac{t_1}{2} \right) + \frac{T^2}{t_1} \left(e^{-\frac{t_1}{T}} - 1 \right) e^{-\frac{t}{T}} \right\} \qquad \text{for: } t \geqq t_1 \tag{4.24}$$

An example of the time histories of the rudder angle $\delta(t)$, the course $\psi(t)$ and the rate of turn $\dot{\psi}(t)$ during a turning test is given in figure 4.25.

Equation 4.24 can also be written as:

$$\psi(t) = a + bt + ce^{-\frac{t}{T}} \tag{4.25}$$

with:

$$\begin{aligned} a &= -KT\delta_a \cdot \left(1 + \frac{t_1}{2T}\right) \\ b &= K\delta_a \\ c &= \frac{KT^2\delta_a}{t_1} \cdot \left(e^{\frac{t_1}{T}} - 1\right) \end{aligned}$$

For $t \to \infty$, this equation reduces to its asymptotic function:

$$\psi(t) = a + bt \qquad \text{for: } t \to \infty \tag{4.26}$$

This asymptotic function $\psi(t) = a + bt$ intersects $\psi(t) = 0$ at time t_T:

$$\begin{aligned} t_T &= \frac{-a}{b} \\ &= \frac{KT\delta_a \cdot \left(1 + \frac{t_1}{2T}\right)}{K\delta_a} \\ &= T + \frac{t_1}{2} \end{aligned} \tag{4.27}$$

Then, the time constant T in Nomoto's first order equation 4.13 for yaw follows from:

$$\boxed{T = t_T - \frac{t_1}{2}} \tag{4.28}$$

Thus, Nomoto's first order indices K and T can be found very easy when applying the equations 4.22 and 4.28 to the time history of a turning circle as given in figure 4.25.

4.7.2 Use of Zig-Zag Maneuver Data

Now, Nomoto's K and T indices in equation:

$$T\ddot{\psi} + \dot{\psi} = K(\delta_c + \delta_r) \tag{4.29}$$

will be determined for a zig-zag maneuver. The correction angle δ_c in here is a required rudder angle to compensate for an asymmetry of the ship, when sailing a straight line. At time $t = 0$, the ship has a rate of turn equal to $\dot{\psi}_0$. An integration of equation 4.29 between the boundaries $t = 0$ and $t = t_e$, with $\dot{\psi} = \dot{\psi}_0$ at both boundaries, results in:

$$T\dot{\psi}\Big|_{\dot{\psi}_0}^{\dot{\psi}_0} + \psi\big|_0^{\psi_e} = K\delta_c t_e + K\int_0^{t_e} \delta_r dt \tag{4.30}$$

or:

$$\psi_e = K\delta_c t_e + K\int_0^{t_e} \delta_r dt \tag{4.31}$$

In a similar way, it can be found that:

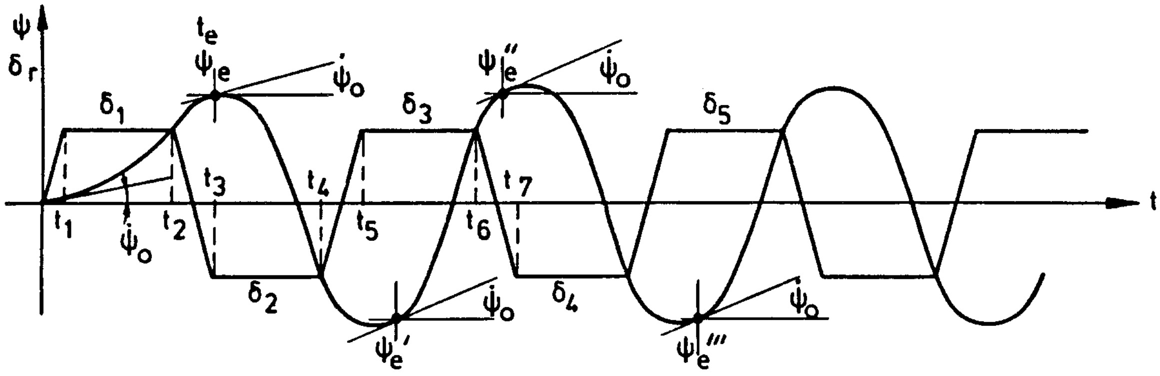


Figure 4.26: Integration Boundaries for Analysing Zig-Zag Maneuvers

$$\psi_e^{'} = K\delta_c t_e^{'} + K \int_0^{t_e^{'}} \delta_r dt \qquad (4.32)$$

$$\psi_e^{''} = K\delta_c t_e^{''} + K \int_0^{t_e^{''}} \delta_r dt \qquad (4.33)$$

The integral $\int_0^{t_e} \delta_r dt$ can be determined by a numerical integration of the time history of the rudder deflection angle. The unknowns K and δ_c can be solved from equations 4.32 and 4.33. Generally, these values does not fullfil equation 4.31, because the initial conditions will play a role at the beginning of a zig-zag maneuver.

The time constant T can be determined from an integration of equation 4.29 over the trajectory between t_2 and t_e (alternatively between t_4 and $t_e^{'}$ or between t_6 and $t_e^{''}$), because of a relatively large response, so:

$$T\dot{\psi}\Big|_{\dot{\psi}_2}^{\dot{\psi}_0} + \psi|_{\psi_2}^{\psi_e} = K\delta_c t|_{t_2}^{t_e} + K \int_{t_2}^{t_e} \delta_r dt \qquad (4.34)$$

$$T\dot{\psi}\Big|_{\dot{\psi}_4}^{\dot{\psi}_0} + \psi|_{\psi_4}^{\psi_e^{'}} = K\delta_c t|_{t_4}^{t_e^{'}} + K \int_{t_4}^{t_e^{'}} \delta_r dt \qquad (4.35)$$

$$T\dot{\psi}\Big|_{\dot{\psi}_6}^{\dot{\psi}_0} + \psi|_{\psi_6}^{\psi_e^{''}} = K\delta_c t|_{t_6}^{t_e^{''}} + K \int_{t_6}^{t_e^{''}} \delta_r dt \qquad (4.36)$$

or:

$$T\left(\dot{\psi}_2 - \dot{\psi}_0\right) = (\psi_e - \psi_2) - K\delta_c (t_e - t_2) - K \int_{t_2}^{t_e} \delta_r dt \quad (4.37)$$

$$T\left(\dot{\psi}_4 - \dot{\psi}_0\right) = \left(\psi_e^{'} - \psi_4\right) - K\delta_c \left(t_e^{'} - t_4\right) - K \int_{t_4}^{t_e^{'}} \delta_r dt \quad (4.38)$$

$$T\left(\dot{\psi}_6 - \dot{\psi}_0\right) = \left(\psi_e^{''} - \psi_6\right) - K\delta_c \left(t_e^{''} - t_6\right) - K \int_{t_6}^{t_e^{''}} \delta_r dt \quad (4.39)$$

Again, equation 4.37 will be affected by initial conditions, so equations 4.38 and 4.39 can be used to determine the turning capacity coefficient K.

A simple method for determining Nomoto's manoeuvering indices K and T from full-scale zigzag trial data is given by [Journée, 1970].

4.7.3 Experimental Data on K and T

The maneuvering indices K and T are made dimensionless with ship length L and initial forward speed U_0 by:

$$\boxed{K' = \frac{L}{U_0} \cdot K} \quad \text{and} \quad \boxed{T' = \frac{U_0}{L} \cdot T} \quad (4.40)$$

Figure 4.27 shows a graphic presentation of K' and T' indices in relation to the size of the ship for various types, as they have been published by Kensaku Nomoto on the First Symposium on Ship Maneuverability in Washington in 1960. Nevertheless the spreading of the data points, the figure shows that K' does not depend very much on the ship size, but that T' increases with the ship size.

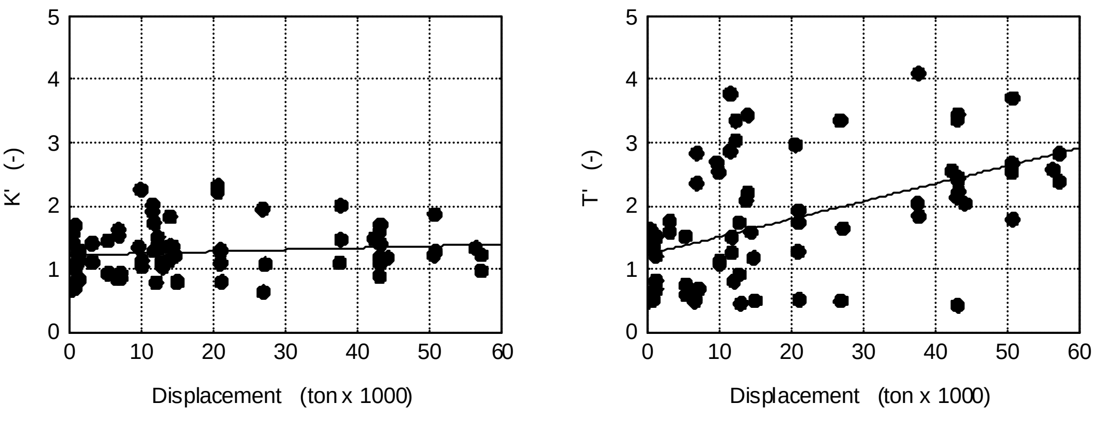


Figure 4.27: Non-Dimensional Presentation of K and T Indices of Nomoto

Figure 4.28 shows the data points for K' and T', obtained from various zig-zag trials, with the upper and lower curves set at 1.25 and 0.75 times the mean. In the literature, sometimes the overshoot angle is basis for analyzing results of zig-zag maneuvers. However, Nomoto has pointed out that the overshoot angle is nearly proportional to K'/T', with the rudder angle δ_r fixed. Figure 4.28 confirms Nomoto's suggestion that the parameters K' and T' provide a more useful basis for analyzing results of zig-zag maneuvers than does the overshoot angle.

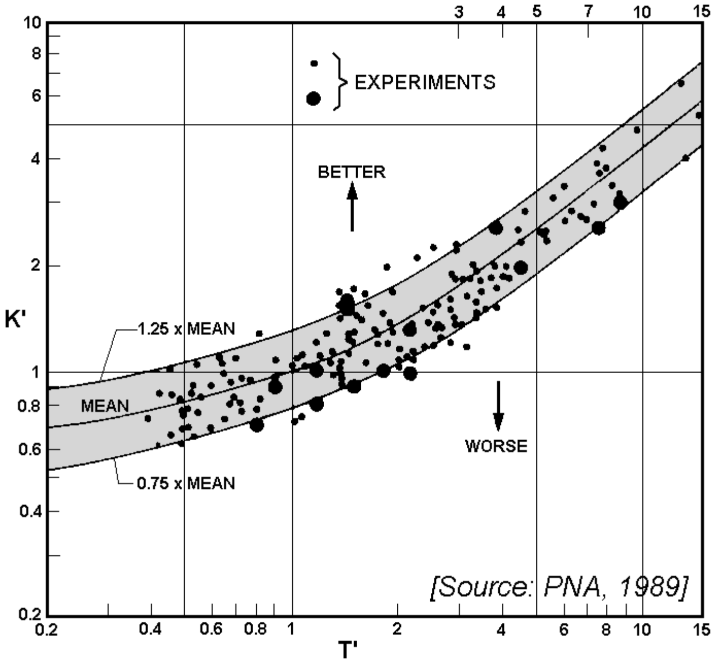


Figure 4.28: Relationship of Steering Quality Indices K' and T'

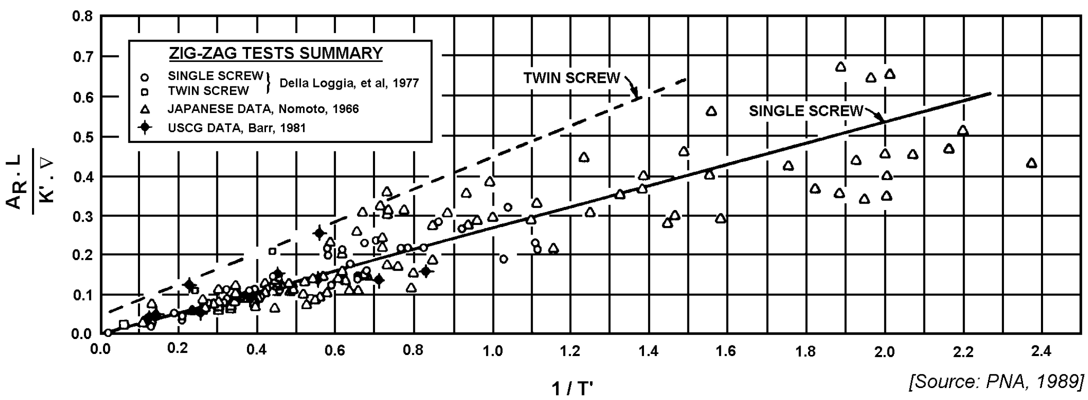


Figure 4.29: Correlation of Zig-Zag Maneuver Performance Data

Figure 4.29 shows the relationship between the parameters K' and T' and the rudder area, A_R. Together with formula 4.1 of Det Norske Veritas, this relationship may be useful as design tools in selecting a rudder area.

4.8 Speed Changing

Stopping, coasting, backing and accelerating are important ship maneuvers; the first three particularly when near land, other vessels and fixed structures. However, the interactions between hull and propeller(s) during these maneuvers are quite complex. Because of this and the transient character of maneuvering, empirical calculations of the characteristics of these maneuvers are sometimes used when adequate motion equation coefficients are not available for simulation.

Stopping is decelerating the ship speed from any given ahead speed until the ship comes to rest. When discussing stopping capabilities, at least two ahead speeds should be considered; a crash stop from "full-ahead sea-speed" and a stop from "harbor speed". Harbor speed may be 12 knots for a slow ship, such as a tanker, or about 15 knots for a fast ship, such as a container vessel. Although in practice "emergency full astern" is almost never ordered from "full-ahead sea-speed"; it is a customary machinery acceptance trial and the results are useful as a relative measure of stopping ability.

Coasting refers to decelerating without using backing power. Time and distance required for a ship to decelerate to a slower speed is often of interest in ship handling. Decelerating more generally means that engine power ahead is insufficient to maintain steady forward speed. In that case, the unbalanced longitudinal force (i.e., thrust is less than resistance) then causes the ship to decelerate until resistance again equals thrust, at some lower speed. Rarely will a ship-handler coast a ship to near dead-in-the-water, because of the very long time it takes. However, decelerating at the least sustainable ahead power at which the ship will steer is very important to the ship handler. The distance required to thus decelerate is critical to getting a ship's speed down from the harbor approach velocity to a speed regime at which tugs can be effective in controlling the ship. In harbors where berthing may be located near the harbor entrance, this figures in harbor design, siting of terminals and in the selection and use of tugs. In some places it has led to the use of braking ships.

Backing a ship is a maneuver of accelerating from rest to a given astern speed or distance. A backing propeller, on the other hand is one in which the blades are turning with negative angle of attack, producing astern thrust.

Accelerating means increasing ship speed from rest or from a particular ahead speed to a higher ahead speed.

The principal performance indexes of these maneuvers reflect the time and distance from the initiation to completion. To simplify analyses, we often assume that the ship travels on a straight line during stopping. This is generally not true except in the case of some multi-screw ships with opposite rotating propellers, in the absence of appreciable wind, current and rudder angle and with controls-fixed straight-line stability. For the backing or stopping of ships with single-screws or uni-rotating multi-screws, the rotation of the propeller tends to swing the stern to port if the propellers are right-handed (positive rotation according to sign convention) and to starboard if they are left-handed (negative rotation). Other factors may cause the ship to veer in the opposite direction.
When a ship deviates from its straight path during a stopping or backing maneuver, the distance traveled is measured along its curved track. But the projections of this distance - termed head reach and side reach - are generally of greater importance as performance indexes.

4.8.1 Stopping

Suppose a ship sailing in calm water at a straight path, with a speed U_0. Its stopping distance and stopping time can be obtained by stationary model tests. At a range of forward speeds U, the remaining longitudinal force $X(U, n)$ is measured at a range of propeller rates n; see figure 4.30. This force consists of resistance and thrust contributions.

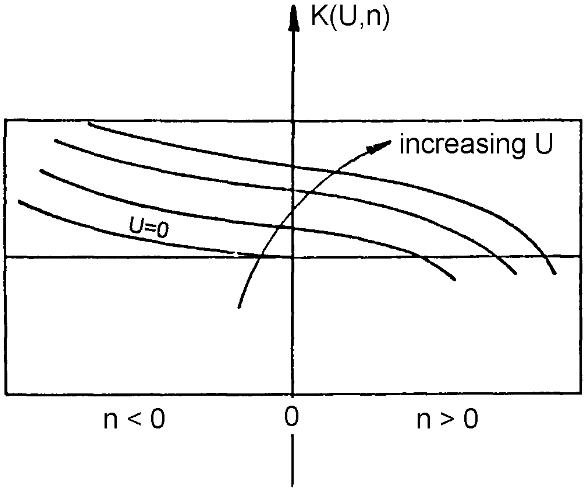


Figure 4.30: Breaking Force as Function of Forward Speed and Propeller Rate

Newton's second law provides the following equation of motion in the longitudinal direction:

$$\frac{d}{dt}\left\{(m - X_{\dot{u}}) \cdot U\right\} = -X(U, n) \tag{4.41}$$

In here, $-X_{\dot{u}}$ is the added mass in the longitudinal direction, which is assumed to constant here.
Then:

$$t - t_0 = -(m - X_{\dot{u}}) \int_{U_0}^{U} \frac{du}{X(U, n)} \tag{4.42}$$

$$s - s_0 = -(m - X_{\dot{u}}) \int_{U_0}^{U} \frac{U du}{X(U, n)} \tag{4.43}$$

In here, $t - t_0$ is the time span required to reduce speed from U_0 to U. The covered distance during that time span is $s - s_0$.
From equation 4.41 can be found:

$$\left[m - X_{\dot{u}}\right]_t \cdot U - \left[m - X_{\dot{u}}\right]_{t_0} \cdot U_0 = -\int_{t_0}^{t} X \cdot dt'$$

From this follows for $U = 0$:

$$[m - X_{\dot{u}}]_t = \frac{1}{U_0} \int_{t_0}^{t} X \cdot dt'$$

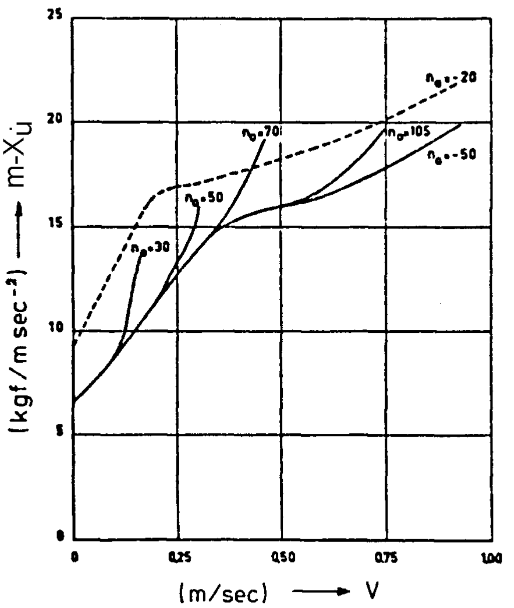


Figure 4.31: Virtual Mass as Function of Forward Speed and Propeller Rate

This added mass, $-X_{\dot{u}}$, is depending on forward ship speed and propeller rate, as has been shown in figure 4.31.
Figure 4.32 shows an example of the measured and computed stopping distances at several initial speeds of a tanker in full load and in ballast condition.

4.8.2 Coasting

Coasting with the propeller "windmilling" consists of reducing the ahead power to that level necessary to cause the propeller to rotate without producing any thrust. In that case, the ship would be slowed solely by its hull resistance. When coasting with the propeller stopped, the ship would be slowed by its hull resistance plus the resistance of the locked propeller. In practice, the propeller's rpm is likely to be slightly less than its zero-slip value, so that it exerts some sternward thrust. With feedback engine control it may circle between very slow ahead and astern.

4.8.3 Backing

Many operators feel that backing time should be established primarily on the basis of maneuverability around docks. In the case of clearing a ship slip, the astern speed achieved after the ship has traveled one ship length may be an adequate criterion for judging backing speed. However, a poll of the operators did not suggest what the speed should be, but

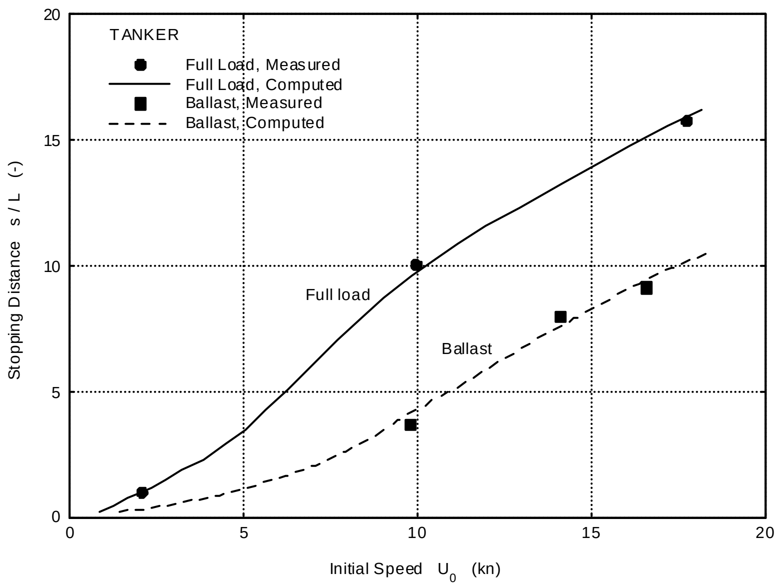


Figure 4.32: Measured and Computed Stopping Distances of a Tanker

rather indicated that experience and the particular hydrographic conditions would dictate the desirable astern speed.

The astern speed, U, reached in differential time at starting from rest may be obtained by equating the product of the instantaneous accelerating force, X, times the distance traveled, s, to the kinetic energy of the ship at speed U. Thus:

$$s \cdot X = \frac{1}{2} (m - X_{\dot{u}}) \, U^2 \tag{4.44}$$

4.8.4 Accelerating

Acceleration ahead is important for naval ships that may have a change position rapidly in a task force or accelerate suddenly for other tactical reasons. The acceleration may be found from:

$$(m - X_{\dot{u}}) \cdot \dot{u} = T \, (U, n) \cdot \{1 - t\} - R \, (U) \tag{4.45}$$

with the variables: thrust T, thrust deduction fraction t and resistance R.

Figure 4.33 shows an example of the acceleration force and the acceleration as functions of the ship speed.

Figure 4.33-a shows typical relationships amongst R, T, X and speed for a steam turbine. The thrust curves 1 and 2 in the figure apply to the case where the initial speed is greater than zero. In the case shown, because the thrust is greater than resistance at the initial speed, there is no equilibrium and the ship accelerates. At "execute", the thrust is increased rapidly to the amount desired. Then thrust curve 1 applies. Of course, the time to reach total equilibrium at maximum speed will be quite long. This is because resistance will very gradually approach the thrust curve as speed approaches maximum and there is gradually diminishing unbalanced thrust remaining to cause acceleration.

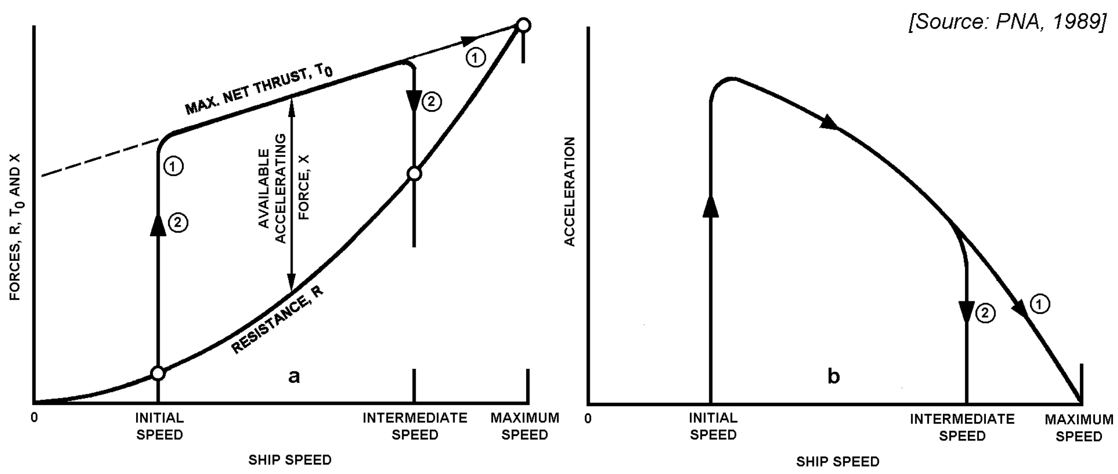


Figure 4.33: Acceleration Force and Acceleration as Functions of Ship Speed

If the final desired speed after acceleration is less than the maximum speed, then much less time and distance are needed. Then the maximum available thrust of curve 1 is utilized until the desired speed is reached and the thrust is appropriately reduced to equal the resistance at the desired speed, curve 2. This technique is useful in conducting maneuvering trials to shorten the time needed to "steady up" on the approach course.
Figure 4.33-b shows the relationship defined by equation 4.45 between $\dot{u}$ and the ship speed, U, corresponding to the thrust curves 1 and 2. The relationship between time, velocity and distance is as follows:

$$\dot{u} = \frac{dU}{dt} \qquad dt = \frac{1}{\dot{u}} \cdot dU \qquad t = \int \frac{1}{\dot{u}} \cdot dU \qquad s = \int U \cdot dt \tag{4.46}$$

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