Common Misconceptions Regarding Quantum Mechanics

Daniel F. Styer
Department of Physics
Oberlin College
Oberlin, Ohio 44074

dstyer@physics.oberlin.edu

Reprinted from *American Journal of Physics*
64 (January 1996) 31–34
(Received 20 March 1995; accepted 15 June 1995)

Abstract

This paper lists fifteen commonly held misconceptions concerning quantum mechanics, such as "Energy eigenstates are the only allowed states" and "The wavefunction is dimensionless". A few suggestions are offered to help combat these misconceptions in teaching.

I. Introduction

Effective teaching does not simply teach students what is correct—it also insures that students do not believe what is incorrect. The study of student misconceptions in introductory mechanics has become both an important concern of physics education research and a valuable tool for the practicing physics teacher. (See reference [1] for a review.) This paper concerns misconceptions in the more advanced and abstruse field of quantum mechanics.

In the context of introductory mechanics, misconceptions are often called "preconceptions" because they arise from the everyday observations and generalizations, beginning in childhood, that everyone needs in order to toss a ball, walk down a street, or chew his food. Indeed, professional basketball players probably earn more from their preconceptions concerning classical mechanics than professional physicists earn from their correct conceptions. The origin of misconceptions in quantum mechanics—notoriously far from everyday experience—is less clear. The catalog that follows suggests origins for particular misconceptions, but these

are speculations only and must not be mistaken for well-founded results. The empirical observation is merely that these errors are easy to slip into.

This list of misconceptions is based on my casual observations of students, colleagues, writings, and myself. (At one time or another I have held nearly all of them.) It is not supported by interviews, examinations, or any of the apparatus of the developing field of physics education research. Consequently I am sure that it contains omissions and inaccuracies. To give just one example, it emphasizes conceptual difficulties at the expense of what might be called "operational" difficulties: errors or infelicities frequently made while solving problems, deriving formulas, interpreting experiments, etc.

How can this list be used in teaching? In many cases it is sufficient to simply point out the existence of an error that is easy to slip into... forewarned is forearmed. (In this function the list plays the role of the "dangerous curve" sign found on most highways and in some mathematics books.) In other situations teaching tools such as guided discussions, essay questions, or diagnostic examinations [2] will be more appropriate. A very effective strategy is to assign a traditional quantitative/analytical problem that renders the misconception concrete. Examples are given in items 2 and 14 of the list. (The best such problems are those that lead to one answer if the misconception is followed and to a different one if the correct path is taken. These problems demonstrate that misconceptions are of operational as well as conceptual importance. They are, however, difficult to produce.) A teaching strategy known to be ineffective in combating misconceptions is to ignore them.

II. Misconceptions regarding the idea of quantal state

1. "Energy eigenstates are the only allowed states." This is a particularly common and a particularly galling misconception. If it were true, then quantum mechanics would have no classical limit because, for example, every simple harmonic oscillator, at every instant of time, would have an expected displacement of zero. (Also, every planet would have an expected position in the middle of the sun.) It seems to arise from at least three sources: First, it is similar to the correct statement "Energy eigenvalues are the only allowed energies." Second, the "old quantum theory" of 1900–1925 did indeed have the framework of restricting the "allowed states". Third, quantum mechanics courses usually devote most of their time to the energy eigenproblem, leaving students with the understandable impression that energy eigenstates are in some way blessed by nature rather than by human convenience.

One way to combat this misconception is by devoting more of the course to the time development problem. Although analytically difficult in general, quantal time development can be treated effectively through computer simulation [3, 4] or through analytic tricks in a few special cases [5]. Careful use of terminology can also help prevent this misconception's birth and growth. For example, I always call the "time-dependent Schrödinger equation" simply the "Schrödinger equation" (Schrödinger himself [6] called it the "real wave equation") and I call the "time-independent Schrödinger equation" by the less dignified name of the "energy eigenproblem". I refer to a stationary state as an "energy eigenstate" and to any possible state as an "arbitrary state". For time-independent and time-dependent perturbation theory I

use the terms "perturbation theory for the energy eigenproblem" and "perturbation theory for the time development problem" respectively.

2. "A quantal state $\psi(\mathbf{x})$ is completely specified by its associated probability density $|\psi(\mathbf{x})|^2$." This misconception seems to be particularly prevalent among chemists [7]. It is true that the "probability cloud" is important, but it is not the only thing that is important. It says nothing, for example, about the expected momentum, and therefore nothing about the probability density at future times. A good antidote to this misconception is to assign an exercise calculating the expectation values for the momentum of two Gaussian wavepackets with identical probability densities but different phases and hence with different expected momenta. Exercises like this are commonly assigned early in a quantum mechanics course, and using them to squash a misconception adds to both their utility and their interest.

3. "The wavefunction $\psi(\mathbf{x})$ is a function of regular three-dimensional position space." In fact it is a function of configuration space, which is isomorphic to position space only for one-particle systems. Thus for a two-particle system the wavefunction $\psi(\mathbf{x}_1, \mathbf{x}_2)$ exists in six-dimensional configuration space.

4. "The wavefunction is dimensionless." In fact it carries the dimensions $[\text{length}]^{-dN/2}$ where N is the number of particles in the system and d is the dimensionality of space.

5. "A wavefunction (or state vector) describes an ensemble of classical systems." In the standard Copenhagen interpretation, a state vector describes a single system, e.g. a single particle or, in systems such as the hydrogen atom, a single pair of particles. In tenable statistical interpretations [8], the state vector describes an ensemble of individual systems each of which does not behave classically. The appealing view that a state vector represents an ensemble of classical systems was rendered untenable by tests of Bell's theorem [9], which show that no deterministic[1] model, no matter how complicated, can give rise to all the results of quantum mechanics.

Discussions of quantal scattering are often phrased so as to reinforce this misconception, as when the transmission probability through a barrier is defined as the fraction of particles passing through the barrier when a large number impinge upon it. Definitions of this sort are pedagogically sound and I do not advocate abandoning them. In fact, reminding students that this definition of transmission probability does *not* imply the ensemble view is a good way to spiral back to a previously established point.

6. "A wavefunction (or state vector) describes a single system averaged over some amount of time." In fact it describes a single system at a single instant. (Otherwise what does the time dependence of a wavefunction represent?) Anyone [10, 11] asking the common question "How does a particle get through a node in its wavefunction?" is thinking that the particle is a dot with a definite position and zero width, and probably with definite momentum as well, and that the wavefunction represents some sort of time-average or ensemble-average. In other words, the asker is suffering from either this or the previous misconception or both. An effective pedagogical tool is to invite the passage-through-node question and then guide students in discussion to discover for themselves that the question is unanswerable because it assumes an incorrect classical picture underlying quantum mechanics. (Similar misconceptions probably underlie the thoughts

[1] See erratum, page 8.

of anyone who says that "when a particle tunnels through a potential barrier, it never appears under the barrier... it just disappears from one side and reappears on the other.")

III. Misconceptions regarding measurement

7. "The 'collapse of the wavepacket' involves (or permits) faster-than-light communication." This misconception frequently comes up in connection with Bell's theorem [9], and it almost always involves an implicit acceptance of misconception 3. If permitted to grow unchecked, this misconception can do enormous harm. For example, Larry Dossey [12] invokes this misconception to "explain" faith healing at a distance.

An analogy to electrodynamics helps students here. In the Coulomb gauge, the electric potential at a point in space changes the instant that any source particle moves, regardless of how far away it is [13]. This does not imply that information moves instantly, because electric potential by itself is not measurable. The same applies for wavefunction.

8. "Measurement disturbs the system." In more detail, this misconception holds that each particle really does have definite values for both position and momentum, but these definite values cannot be determined because measurement of, say, a particle's position alters the value of its momentum. (It is related to the idea of a classical picture underlying quantum mechanics mentioned in item 6.) This is a particularly common misconception because some arguments due to Heisenberg ("the gamma ray microscope") and Bohr can be interpreted to support it. It is another attractive idea rendered untenable through tests of Bell's theorem [9]. (This idea is also shown to be incorrect through "quantum eraser" arguments, as in reference [14].)

9. Angular momentum measurements. The misconception (more accurately the class of misconceptions) is that given a system with a definite value of L_z, there exists a definite value of L_x (or of L_y, or both), but that it is changing rapidly (or randomly, or unpredictably) so that when its value is measured the outcome of the measurement cannot be predicted. This misconception is reinforced by the "rotating vector model" often invoked in modern physics courses. This is not to say that the rotating vector model must never be used (although as a matter of taste I do not), only that it must be presented along with cautions concerning its limitations.

Misconceptions 8 and 9 exemplify the overarching misconception that quantum mechanics involves uncertainty, whereas in fact it involves indeterminacy. To combat this misconception, it is desirable to use the term "expected value" rather than "mean value"—the latter suggests that there is only one, correct, value, which is subject to experimental error. Similarly, one can use "indeterminacy in x" rather than "uncertainty in x", and "Heisenberg indeterminacy principle" rather than "Heisenberg uncertainty principle". Whether the payoff in accuracy is worth the investment in syllables is a matter of individual choice.

IV. Misconceptions regarding identical particles

10. "In the two-body expression $\psi(\mathbf{x}_1, \mathbf{x}_2)$, the labels 1 and 2 refer to particles." In fact, they refer to positions. Thus, for two non-identical particles,

$$|\psi(\mathbf{x}_2, \mathbf{x}_1)|^2$$

is the probability density for finding the particle of type 1 at $\mathbf{x}_2$ and the particle of type 2 at $\mathbf{x}_1$. The location within the argument list tells which particle is being considered, while the subscript labels the positions being examined. For two identical particles,

$$|\psi(\mathbf{x}_2, \mathbf{x}_1)|^2$$

is the probability density for finding a particle at $\mathbf{x}_2$ and a particle at $\mathbf{x}_1$. Once again the subscripts refer to positions rather than to particle types, now because both particles are of the same type.

This may seem entirely nitpicky, but if the subscript 1 refers to particle #1, then, in expressions like

$$\psi(\mathbf{x}_1, \mathbf{x}_2) \pm \psi(\mathbf{x}_2, \mathbf{x}_1),$$

how are we to interpret the second member? And if the subscripts label particles, then how can they ever be used with identical particles, which by definition cannot be labelled? To be completely unambiguous, in my teaching I label positions by numbers $(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots)$, wavefunctions by letters $(\psi_A, \psi_B, \psi_C, \ldots)$, and particles by colors (red, orange, yellow,...). This is probably overkill, but once I started I found it impossible to stop.

11. "An antisymmetric wavefunction for two particles is necessarily the difference of products of one-body wavefunctions." One can indeed construct a basis of such states, but an arbitrary state will be a linear combination of such basis states. If the particles interact, then even the energy eigenstates will not (in general) be in the "difference of products" form.

Where does this misconception come from? It is, of course, quite true that any antisymmetric wavefunction may be written as a difference

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = f(\mathbf{x}_1, \mathbf{x}_2) - f(\mathbf{x}_2, \mathbf{x}_1).$$

The trouble is that the function $f(\mathbf{x}_1, \mathbf{x}_2)$ might not factorize. Students, however, almost always see the situation where $f(\mathbf{x}_1, \mathbf{x}_2)$ is a product of one-body energy eigenstates, because they are almost always worried about producing two-body energy eigenstates. A possible antidote is to introduce, perhaps in an exercise, the one-dimensional bivariate Gaussian wavefunction, for which the unsymmetrized "building block" is

$$f(x_1, x_2) = A \exp(-[x_1^2/d_1^2 - 2Gx_1x_2 + x_2^2/d_2^2]) e^{i(k_1x_1 + k_2x_2)},$$

where A is a normalization constant.

12. "Two nearby identical fermions (in the same spin state) repel each other with a force." The effective repulsion (in the classical limit) of such particles is sometimes called an "exchange force". It holds for non-interacting as well as interacting particles. No Hamiltonian—no force—is involved. Ultimately it is not due to strong, electroweak, or gravitational interactions: it is instead a direct consequence of the Pauli principle.

V. Miscellaneous misconceptions

13. "A barrier 'far away' from a particle, where $\psi(x) = 0$, can affect the particle." Quantum mechanics is weird but not that weird. This misconception is presented in Roger Penrose's *The Emperor's New Mind* [15].

14. "The probability current density $\mathbf{j}(\mathbf{x})$ is related to the speed of that part of the particle which is located at position $\mathbf{x}$." In the Born interpretation, a quantal particle behaves somewhat like a classical fluid with density $|\psi(\mathbf{x})|^2$ and velocity field $\mathbf{v}(\mathbf{x}) = \mathbf{j}(\mathbf{x})/|\psi(\mathbf{x})|^2$. Like the rotating vector model, this picture is valuable if its limitations are recognized and dangerous if they are not. It leaves the impression that, although the particle as a whole has neither a definite position nor a definite velocity, it is made up of parts each of which does have definite position $\mathbf{x}$ and definite velocity $\mathbf{v}(\mathbf{x})$. (This misinterpretation is related to misconceptions 5 and 6 in that it imagines some sort of classical undergirding to quantum mechanics.) In fact it is not even correct to speak of "that part of the particle which is located at $\mathbf{x}$"... particles are not made up of parts.

Here is another instance where assigning a traditional analytic problem can help prevent a misconception from taking hold. The mean velocity in the classical fluid picture, and the expected velocity in quantum mechanics, is

$$\int d^3x\, \mathbf{j}(\mathbf{x})$$

in both cases. But the *total* kinetic energy in the classical fluid picture, and the *expected* kinetic energy in quantum mechanics, are

$$\frac{m}{2}\int d^3x\, \frac{\mathbf{j}^2(\mathbf{x})}{|\psi(\mathbf{x})|^2} \quad \text{and} \quad -\frac{\hbar^2}{2m}\int d^3x\, \psi^*(\mathbf{x})\nabla^2\psi(\mathbf{x})$$

respectively. These quantities are not the same either in interpretation or in numerical value. For example, the total kinetic energy in the classical fluid picture vanishes for any energy eigenstate. (It is possible that a more sophisticated classical fluid picture, for example one in which $\mathbf{j}(\mathbf{x})$ represents the mean current of constituent parts that are flowing in many different directions, could make these two expressions match. But it is hard to imagine how such a picture could resolve the differences in interpretation or in setting [namely that one picture takes place in position space and the other in configuration space].)

15. "For any energy eigenstate, the probability density must have the symmetry of the Hamiltonian." If a one-dimensional potential has reflection symmetry about any point, then the probability density functions associated with each energy eigenstate will also posses that symmetry. This fact is so reasonable (and so strongly stressed by many textbooks) that many students slip into believing that it holds for other symmetries, such as the rotational symmetry of the Coulomb problem, where it does not. I have often seen this misconception submerge and propagate unseen until the student takes a course in solid state physics, where it emerges to make the Bloch theorem appear trivial.

Acknowledgments

In preparing this catalog I have benefited from correspondence or conversation with Robert Mills, Ian Johnston, Edwin Taylor, Aaron Titus, and Jeffrey Witmer. I also thank a referee and editor Romer for encouraging me to expand and clarify some of my remarks.

References

[1] Arnold B. Arons, *A Guide to Introductory Physics Teaching* (John Wiley and Sons, New York, 1990).

[2] D. Hestenes, M. Wells, and G. Swackhamer, "Force concept inventory," The Physics Teacher **30**, 141–158 (1992); D. Hestenes and M. Wells, "Mechanics baseline test," The Physics Teacher **30**, 159–166 (1992).

[3] U.E. Kruse, *Quantum Mechanics: Graphical displays of the time-dependent solutions of the Schrödinger equation* (program QMTIME) (Falcon Software, P.O. Box 200, Wentworth, NH 03282, 1988).

[4] J.R. Hiller, I.D. Johnston, and D.F. Styer, *Quantum Mechanics Simulations* (John Wiley and Sons, New York, 1995).

[5] D.F. Styer, "The motion of wavepackets through their expectation values and uncertainties," American Journal of Physics **58**, 742–744 (1990); C.C. Yan, "Soliton like solutions of the Schrödinger equation for simple harmonic oscillator," American Journal of Physics **62**, 147–151 (1994).

[6] E. Schrödinger, *Collected Papers on Wave Mechanics* (Chelsea Publishing Company, New York, 1978) pp. 102–104.

[7] J.E. Douglas, "Visualization of electron clouds in atoms and molecules," Journal of Chemical Education **67**, 42–44 (1990).

[8] L.E. Ballentine, "Resource letter IQM-2: Foundations of quantum mechanics since the Bell inequalities," American Journal of Physics **55**, 785–792 (1987); Leslie E. Ballentine, *Quantum Mechanics* (Prentice Hall, Englewood Cliffs, NJ, 1990).

[9] N.D. Mermin, "Is the moon there when nobody looks? Reality and the quantum theory," Physics Today **38**(4), 38–47 (April 1985); J. Baggott, "Quantum mechanics and the nature of physical reality," Journal of Chemical Education **67**, 638–642 (1990).

[10] R.D. Allendoerfer, "Teaching the shapes of the hydrogenlike and hybrid atomic orbitals," Journal of Chemical Education **67**, 37–39 (1990).

[11] P.G. Nelson, "How do electrons get across nodes?" Journal of Chemical Education **67**, 643–647 (1990).

[12] Larry Dossey, *Healing Words: The power of prayer and the practice of medicine* (Harper, New York, 1994).

[13] O.L. Brill and B. Goodman, "Causality in the Coulomb gauge," American Journal of Physics **35**, 832–837 (1967).

[14] T.F. Jordan, "Disappearance and reappearance of macroscopic quantum interference," Physical Review A **48**, 2449–2450 (1993).

[15] Roger Penrose, *The Emperor's New Mind* (Oxford University Press, Oxford, 1986) figure 6.6, page 234.

Erratum

American Journal of Physics **64** (September 1996) 1202

The commentary for misconception number 5 of this paper should have said that "no local deterministic model, no matter how complicated, can give rise to all the results of quantum mechanics." Due to a blunder on my part, the published paper did not include the necessary qualifier "local". I thank Professors Art Hobson of the University of Arkansas and Gary Bowman of Northern Arizona University for bringing this error to my attention.