Project Information

Algorithms for Elementary Algebraic Geometry

Math 191, Fall Quarter 2007

The purpose of the project is to provide you with an opportunity to learn new mathematics and to improve your mathematical exposition and verbal explanations. This will involve reading at least one research paper or advanced chapter from a textbook, and writing a summary which includes any necessary background material. Your goal should be to make the paper acessible to other members of the class. Achieving this will take approximately 10 pages.

The paper will go through various stages of revision throughout the quarter. First, you will have to choose a topic, and inform be by email about your choice, on or before 10/29/07. An outline of your project, containing a *title,* an *abstract,* and a *preliminary list of references,* including *primary sources,* is due on 11/02/07. Somewhere between the time you settle on a topic and this date, you should come and see me during office hours to discuss your project with me.

A rough draft of your paper, in *two copies,* will be due on 11/26/07. One of the copies will be read and graded by me, and the other one by a randomly selected member of the class. This person will furnish you with feedback about your paper on or before 12/03/07, and will also provide me with a copy of this sheet, which I will grade. The final version of your paper, which should incorporate the suggestions by me and the reviewer, is due on 12/10/07.

Here is a summary of the various deadlines and grades:

Part of the project	Due date	Weight
Choice of topic	10/29/07	
Outline	11/02/07	20%
Rough draft	11/26/07	20%
Peer review	12/03/07	20%
Final version	12/10/07	40%

You are strongly encouraged to type your assignment. Information on mathematical typesetting software can be found in the class handout given to you at the beginning of the quarter, and, with more details, on the class web page.

A list of possible topics and relevant references.

- *Gröbner bases over principal ideal domains.*
  Chapter 4 of [1] or Section 10.1, parts of Chapters 1 and 2 in [6].
- *Gröbner bases for modules.*
  Chapters in [1], [6], [8], [9], and [12].
- *Universal Gröbner bases.*
  [6], pp. 514–515, and [15], [2].
- *Gröbner bases in power series rings.*
  Standard textbooks on algebra, such as [14], and [3]–[5].
- *Gröbner bases for ideals in localizations of polynomial rings.*
  [8], [11].
- *Gröbner bases of ideals with finitely many zeroes.*
  [8] and [17].
- *Modules, free resolutions, and the Hilbert Syzygy Theorem.*
  [8], [9], and [18].
- *Computations in finitely generated commutative algebras.*
  [7], Chapter 5, Sections 1–3.
- *Applications of Gröbner bases to robotics.*
  [7], Chapter 6, Sections 1–3.
- *Automatic theorem proving.*
  [7], Sections 4 and 5 of Chapter 6, and [13].
- *Primary decomposition of ideals.*
  [7], Chapter 4, Section 6, and [10].
- *The Hilbert function of a variety.*
  [7], Chapter 9, Sections 1–3.
- *Gröbner bases in Weyl algebras.*
  [16]
- *Resultants and proof of the Extension Theorem.*
  [7], Sections 5 and 6 of Chapter 3.

References

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3. Becker, T., Stability and Buchberger criterion for standard bases in power series rings, *J. Pure Applied Algebra*, 66:219–227, 1990a.
4. _____, Standard bases and some computations in rings of power series. *J. Symbolic Comput.*, 10:165–179, 1990b.
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6. Becker, T., and Weispfenning, V., *Gröbner Bases*, Graduate Texts in Mathematics, vol. 141, Springer-Verlag, New York, 1993.
7. Cox, D., Little, J., O'Shea, D., *Ideals, Varieties, and Algorithms,* 3rd ed., Undergraduate Texts in Mathematics, Springer, New York, 2007.

8. Cox, D., Little, J., O'Shea, D., *Using Algebraic Geometry,* 2nd ed., Graduate Texts in Mathematics, vol. 185, Springer-Verlag, New York, 2005.
9. Eisenbud, D., *Commutative Algebra. With a view toward Algebraic Geometry,* Graduate Texts in Mathematics, vol. 150, Springer-Verlag, New York, 1995.
10. Gianni, P., Trager, B., and Zacharias, G., Gröbner bases and primary decomposition of polynomial ideals, *J. Symbolic Comput.*, 6(2–3):149–167, 1988.
11. Greuel, G.-M., and Pfister, G., *A SINGULAR Introduction to Commutative Algebra,* Springer, New York 2002.
12. Kreuzer, M., Robbiano, L., *Computational Commutative Algebra,* vol. 1, Springer-Verlag, Berlin, 2000.
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14. Lang, S., *Algebra,* Revised 3rd ed., Graduate Texts in Mathematics, vol. 211, Springer-Verlag, New York, 2002.
15. Maclagan, D., *Antichains of monomial ideals are finite*, Proc. Amer. Math. Soc. **129** (2001), no. 6, 1609–1615 (electronic).
16. Saito, M., Sturmfels, B., and Takayama, N., *Gröbner Deformations of Hypergeometric Differential Equations,* Algorithms and Computation in Mathematics **6**, Springer-Verlag, Berlin, 2000.
17. Sturmfels, B., *Solving Systems of Polynomial Equations,* CBMS Regional Conference Series in Mathematics, vol. 97, American Mathematical Society, Providence, RI, 2002.
18. Zariski, O., and Samuel, P., *Commutative Algebra,* vol. 2, Graduate Texts in Mathematics, vol. 29, Springer-Verlag, Berlin, 1960.