BASIC MATH

Student Manual

AIDT
ISO 9001: 2008
A Member of the Alabama Community College System

AIDT
BASIC MATH MANUAL
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I. BASIC MATH

A. BASIC ARITHMETIC

Learner Objectives: Upon completion of this unit, the student will have a general concept of numbers and digits, kinds of numbers, mathematical signs, and symbols.

Mathematics is the foundation upon which modern day life depends. Diesel transportation, high speed computers, jet planes, submarines, telephones, and televisions are just a few of the limitless number of products and mechanisms that depend upon mathematics for development and production.

Arithmetic is the simplest form of mathematics and is used everyday to solve most of the common problems encountered in work, play, and living. Basic arithmetic includes addition, subtraction, multiplication, and division. The four different operations of arithmetic are easy to recognize because of the use of signs to indicate the type of operation being performed. (See below).

1. + plus sign
2. - minus sign
3. x multiplication sign
4. ÷ division sign

The equal sign (=) is used to show equal or even values. For example, two plus two equals four, or stated another way 2 + 2 = 4. The values on each side of the equal sign are equal.

1. Beginning Terminology

Numbers

A number is a symbol or word commonly used to express value or quantity. However, any number expressed belongs to a system of numbers. The Arabic number system is the one used most often in the United States. This system includes ten numerals; 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These numerals may be combined to express any desired number.

Digits

In the Arabic number system, the location or position of a numeral (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) in the written whole number expresses its value. The word digit is a name given to the place or position of each numeral in a whole number. The first numeral in the extreme right place (digit) of a whole number is said to be in the ones column. The numeral in the next position (or digit) to the left is in the tens column; the third digit, hundreds; the fourth digit, thousands; etc. as shown in Figure 1-1.

FIGURE 1-1
Number Sequence

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones
8	7	6	5	4	3	2

A whole number like 432 is a simple way of saying 400 + 30 + 2. The number 432 has three digits. Words are formed by combining letters; whole numbers are formed by combining digits. Refer to Figure 1-1 and review the sequence in which digits are arranged.

2. Kinds of Numbers

Whole Numbers

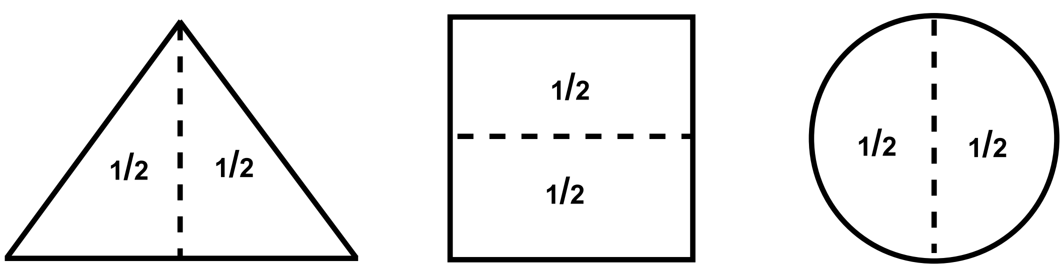
Whole numbers refer to complete units where there is no fractional part. Numbers such as 30 and 50 and quantities such as 140 machine screws, 10 spools, and 43 outlets are examples of types of whole numbers. Whole numbers are also used to describe measurements such as 75 feet or 12 inches, and other material values such as $810. All of these examples represent whole numbers because the values do not contain a fraction.

Whole numbers may be written in the form of words: three hundred fifty-seven, or four thousand six hundred ninety-eight.

Fractions

A fraction is a part of a whole unit or quantity. For example, if a square, triangle, or circle is divided into two equal parts, one of these parts is a fraction of the whole square, triangle, or circle. (see Figure 1-2).

FIGURE 1-2
Description of Common Fractional Forms



Decimal Numbers

A decimal number is a type of fraction which can be written on one line as a whole number. The difference in decimal numbers and whole numbers is the position of a period directly in front of the number. (see Figure 1-3)

FIGURE 1-3
Position of Period and Decimal Digits

	Tenths	Hundredths	Thousandths
.	0	0	5

B. WHOLE NUMBERS

Learner Objectives: Upon completion of this unit, the student will solve addition problems of simple and complex whole numbers.

Simple addition of whole numbers results in single digit answers in each column. Examples of simple addition include the following:

3	322	11132
+ 6	+ 132	12136
9	454	+ 74211
		97479

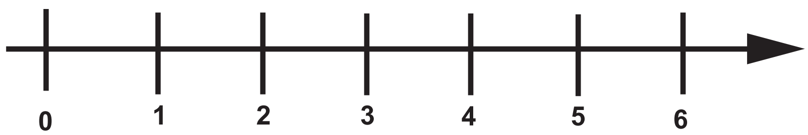
Notice that in each of these examples the added columns result in a single digit, not larger than 9.

Complex addition consists of adding digits which result in column answers higher than 9 or double digits. This double digit column answer must be regrouped mentally to the next column.

1. Addition

A number line can be used to show how numbers are added together. A number line is a picture that shows numerals in order of value.

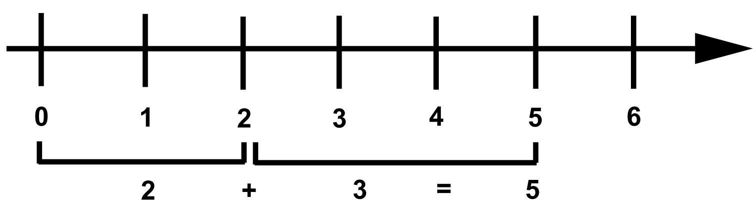
Example:



The number 2 can be added with the number 3 on the number line.

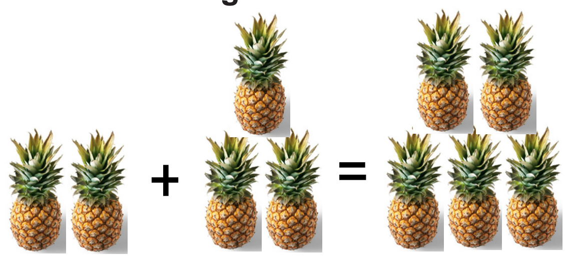
Example:

2 + 3 = 5



Adding whole numbers can also be explained by adding pictures which show the added amount. An example is shown in Figure 1-4.

FIGURE 1-4
Adding with Pictures



Example: $2 + 3 = 5$

The same numbers can be added still another way by the column method.

Example:

$$\begin{array}{r} 2 \\ +\ 3 \\ \hline 5 \end{array}$$

This method uses no equal sign. Whole numbers are usually added this way. This method can be used for simple addition as well as complex addition.

Example:

$$\begin{array}{r} 5 \\ +\ 5 \\ \hline 10 \end{array} \qquad \begin{array}{r} 897 \\ +\ 368 \\ \hline 1265 \end{array} \leftarrow \text{"SUM"}$$

Simple Addition **Complex Addition**

Addition is nothing more than a procedure of adding all the numbers in each column in a problem. The answer to the addition problem is called the **sum**. Whole numbers can be easily added using a few basic rules. Review the following example and **remember** each step of the problem.

Example: Add 2765 + 972 + 857 + 1724.

Step 1. Arrange each number in column form with all ones in the ones column, all tens in the tens column, all hundreds in the hundreds column, etc.

Step 2. Mentally add the numerals in the **ones** column, from top to bottom (5 + 2 + 7 + 4 = 18) Place 8 in the **ones** column's answer section and mentally regroup the 1 to the **tens** column.

Step 3. Mentally add the numerals in the **tens** column and remember to add the 1 carried over from the **ones** column (6 + 7 + 5 + 2 + 1 = 21). Place 1 in the answer section of the **tens** column and mentally regroup the 2 to the **hundreds** column.

Step 4. Perform the same addition steps for the **hundreds** and **thousands** columns.

FIGURE 1-5
Table of Digits

Millions	Hundred-Thousands	Ten-Thousands	Thousands	Hundreds	Tens	Ones
			2	7	6	5
				9	7	2
				8	5	7
			1	7	2	4
			6	3	1	8

```
  2765
   972
   857
+ 1724
  6318
```

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Addition Practice Exercises

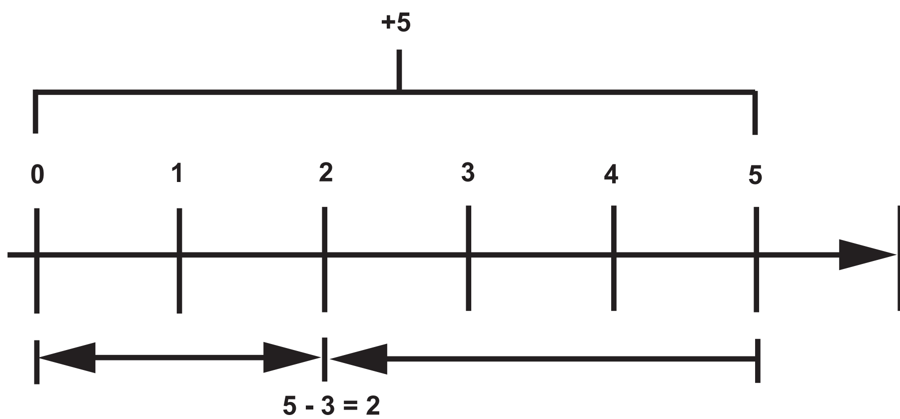
Add the following whole numbers.

1.	a.	222 + 222	b.	318 + 421	c.	611 + 116	d.	1021 + 1210
2.	a.	813 + 267	b.	924 + 429	c.	618 + 861	d.	411 + 946
3.	a.	813 222 + 318	b.	1021 611 + 421	c.	611 96 + 861	d.	1021 1621 + 6211

2. Subtraction

The subtraction of whole numbers can be shown on a number line in a way similar to addition. An example showing the subtraction of five minus three (5 - 3) is shown below.

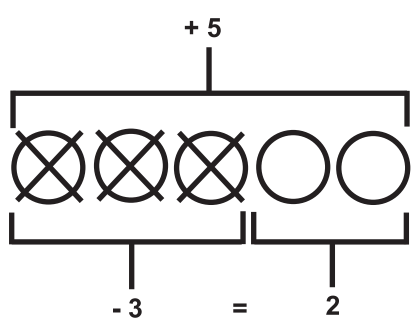
Example:



After taking **three** intervals from the **five** intervals, **two** intervals remain between 0 and 5.

Subtracting whole numbers can also be explained through the use of a picture.

Example:



Whole numbers are usually subtracted by the column method. This method is used for simple and complex subtraction. Simple subtraction, involving just two numerals in one column, is shown in the following examples:

$$\begin{array}{r} 7 \\ -\ \underline{2} \\ 5 \end{array} \qquad \begin{array}{r} 9 \\ -\ \underline{5} \\ 4 \end{array} \qquad \begin{array}{r} 8 \\ -\ \underline{4} \\ 4 \end{array}$$

Simple subtraction consists of deducting smaller digits from larger digits. Review the examples below.

Larger digits	7	389	968431
Smaller digits	- 5	- 276	- 952120
	2	113	16311

Notice that in each example the subtracted digits in each column are smaller than those from which they are subtracted.

Complex subtraction consists of subtracting larger digits from smaller digits. To solve problems of this type, it is necessary to **borrow** from the next column.

Example: Subtract 397 from 538.

Step 1. Position the larger number above the smaller number making sure the digits are in line forming straight columns.

$$\begin{array}{r} 538 \\ -\ \underline{397} \end{array}$$

Step 2. Subtract the right hand column. (8 - 7 = 1)

$$\begin{array}{r} 538 \\ -\ \underline{397} \\ 1 \end{array}$$

Step 3. Now subtract the column next to the right hand column (3- 9). Since 9 cannot be subtracted from 3, 1 must be **borrowed** from the 5, the next digit to the left of the 3. Now place the 1 next to the 3, making it 13 (13 - 9 = 4).

$$\begin{array}{r} {}^{4}\not{5}\,{}^{1}38 \\ -\ \ \underline{397} \\ 41 \end{array}$$

Step 4. Subtract the next column (4 - 3), remembering that 1 was **borrowed** from 5 making it a 4.
Therefore, 4 - 3 = 1.

$$\begin{array}{r} {}^{4}\not{5}\,{}^{1}38 \\ -\ \ \underline{397} \\ 141 \end{array}$$

Solution: The difference between 538 and 397 is 141.

Test your skills in subtraction by solving the problems which appear in the subtraction practice exercise on the following page.

Subtraction Practice Exercises

1.	a.	6 - 3	b.	8 - 4	c.	5 - 2	d.	9 - 5	e.	7 - 3
2.	a.	11 - 6	b.	12 - 4	c.	28 - 9	d.	33 - 7	e.	41 - 8
3.	a.	27 - 19	b.	23 - 14	c.	86 - 57	d.	99 - 33	e.	72 - 65

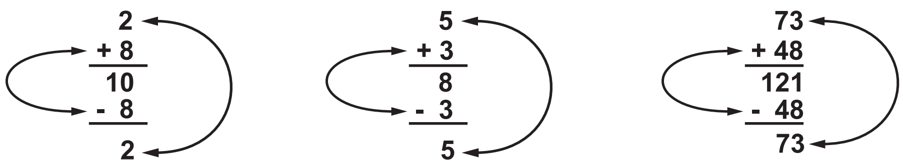
4.	a.	387 - 241	b.	399 - 299	c.	847 - 659	d.	732 - 687
5.	a.	3472 - 495	b.	312 - 186	c.	419 - 210	d.	3268 - 3168
6.	a.	47 - 38	b.	63 - 8	c.	47 - 32	d.	59 - 48
7.	a.	372 - 192	b.	385 - 246	c.	219 - 191	d.	368 - 29

3. Checking Addition and Subtraction

To make sure that you have not made a careless mistake, always **check** your addition or subtraction answers.

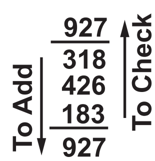
Check addition answers by subtracting one of the added numbers from the sum or total of the numbers added. This subtraction should produce the other added number.

Example:



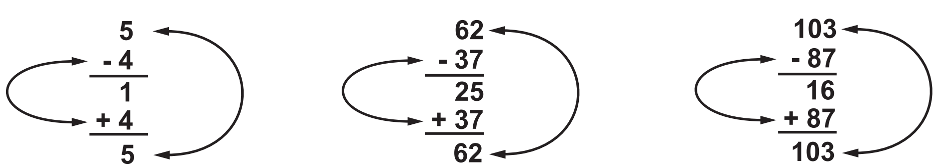
To check three or more numbers which are added, add the numbers from the bottom to top. The following example shows figures which were added from top to bottom and then checked by adding from bottom to top.

Example:



Subtraction answers can be checked by adding. The answer is added to the number subtracted (the smaller of the two numbers) to produce the larger number.

Example:



Checking Addition and Subtraction Practice Exercises

(Check Answers by the Addition and Subtraction Method Described in this Manual)

1. a. $\begin{array}{r} 6 \\ +\ 8 \\ \hline 13 \end{array}$ b. $\begin{array}{r} 9 \\ +\ 5 \\ \hline 14 \end{array}$ c. $\begin{array}{r} 18 \\ +\ 18 \\ \hline 26 \end{array}$ d. $\begin{array}{r} 109 \\ +\ 236 \\ \hline 335 \end{array}$

2. a. $\begin{array}{r} 87 \\ -\ 87 \\ \hline 1 \end{array}$ b. $\begin{array}{r} 291 \\ -\ 192 \\ \hline 99 \end{array}$ c. $\begin{array}{r} 367 \\ -\ 212 \\ \hline 55 \end{array}$ d. $\begin{array}{r} 28 \\ -\ 5 \\ \hline 24 \end{array}$

3. a. $\begin{array}{r} 34 \\ +\ 12 \\ \hline 46 \end{array}$ b. $\begin{array}{r} 87 \\ 13 \\ 81 \\ +\ 14 \\ \hline 195 \end{array}$ c. $\begin{array}{r} 103 \\ 212 \\ 439 \\ +\ 195 \\ \hline 746 \end{array}$ d. $\begin{array}{r} 21 \\ +\ 83 \\ \hline 104 \end{array}$

4. a. $\begin{array}{r} 28 \\ -\ 16 \\ \hline 22 \end{array}$ b. $\begin{array}{r} 361 \\ -\ 361 \\ \hline 0 \end{array}$ c. $\begin{array}{r} 2793142 \\ -\ 1361101 \\ \hline 1432141 \end{array}$

4. Multiplication

In arithmetic, multiplication is indicated by a **"times"** sign (x). To work multiplication problems such as 3 x 4, 5 x 12, 4 x 4, or 126 x 26, you must know the **multiplication** (or "times") **table**. This table is shown in Figure 1-6.

FIGURE 1-6
Multiplication Table

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

To read this table, find the number on the left and the number across the top which are to be multiplied. The point on the table at which the numbers meet in the columns is the multiplied answer.

Example: 6 x 8 = 48

Find the 6 on the left and the 8 across the top. Note that two #'s intersect at 48.

When working a multiplication problem, you cannot count on having a "times table" available. For this reason, learn the multiplication table.

Rules of Multiplication

- Any number multiplied by 0 is 0. For example, 7 x 0 = 0; 64 x 0 = 0; 31 x 0 = 0; and 103 x 0 = 0. Any number multiplied by 1 is equal to the number multiplied. For example, 1 x 1 = 1, 7 x 1 = 7, 4 x 1 = 4, and 106 x 1 = 106.

- Multiplication is basically adding a number to itself a certain number of times. For example, 3 x 4 means three added to three four times or 3 + 3 + 3 + 3. If you cannot remember that 3 x 4 = 12, but you do know that 3 x 3 = 9, simply add 3 to the 9. The answer is 12. If you know that 8 x 5 = 40, but cannot remember 8 x 6, add 8 to 40; 8 x 6 = 48.

- It is best not to rely on tricks or "gimmicks." Learn the 1 through 12 "times" tables.

<u>Simple Multiplication</u>

Like addition and subtraction, simple multiplication is done by the column method.

Example:

6	8	9	10	6
x 3	x 8	x 7	x 6	x 6
18	64	63	60	36

Simple multiplication is done through knowledge of the "times" tables.

Complex Multiplication

Solve more difficult multiplication problems using these steps;

Example: 6 x 18

Step 1. In multiplication, first multiply the digit on the right (6 x 8 = 48). Place the 8 under the 6 as shown.

$$\begin{array}{r} {}^{+4}18 \\ \underline{x\ \ \ 6} \\ 8 \end{array}$$

Step 2. Next multiply the digit on the left (6 x 1 = 6). To this 6 add the 4 from the answer of 48 which resulted from the multiplication of 6 x 8 in the first step.

(6 x 1) + 4 = 6 + 4 = 10

This 10 becomes the second and third digits of the answer, 108.

$$\begin{array}{r} {}^{+4}18 \\ \underline{x\ \ \ 6} \\ 108 \end{array}$$

Example: 48 x 23

Step 1. In this problem, first multiply 3 x 8. The digit 4, from your answer of 24, should be placed below the 3 as shown. Add the 2 from your answer to the next column.

$$\begin{array}{r} {}^{+2}48 \\ \uparrow \\ \underline{x\ \ 23} \\ 4 \end{array}$$

Step 2. Next multiply 3 times the next digit to the left, the 4. 3 x 4 equals 12. To this 12 add the 2 from the answer of 24 which resulted from the multiplication of 8 x 3 in the first step. (3 x 4) + 2 = 12 + 2 = 14.

```
 +2 48
    ↑
 x  23
 ------
   144
```

Step 3. Now multiply the 48 by the second digit to the left, the 2. Multiply the top digit on the right first. 2 x 8 = 16. **Place the 6 under the second digit from the right** as shown. Place the 1 from your answer in the next column.

```
 +1 48
    ↑
 x  23
 ------
   144
    60 ←—— (Zero is space holder)
```

Step 4. Now multiply 2 x 4. To the answer of 8, add the 1 from the answer of 16 which resulted from the multiplication of 2 x 8 in the earlier step. Therefore (2 x 4) + 1 = 8 + 1 = 9. Place the 9 to the left of the 6 as shown.

```
 +1 48
    ↑
 x  23
 ------
   144
   960
```

Step 5. To get the final answer, add the two multiplication answers. Do not be confused by the out-of-line numbers; add just as you would in regular addition. The four is brought down. 6 + 4 is 10. Place the 0 to the left of the 4 and "regroup" the 1.

```
      48
x     23
  +1 144
    960
      04
```

Step 6. Continue adding. 9 + 1 + the 1 which was carried = 11. Place the 11 as the last digits to the left. 48 x 23 = 1104.

```
      48
x     23
  +1 144
    960
    1104
```

The same process is followed in multiplying three or four-digit problems. Examples are given below. Follow these examples to check your understanding of multiplication.

Example 1:

```
   275          275          275
x   24       x   24       x   24
  1100         1100         1100
               550          550
                            6600
```

Example 2:

364	364	364
x 217	x 217	x 217
2548	2548	2548
	3640	3640
	72800	72800
		78988

NOTE: Each answer is recessed, or moved to the left one digit as you work the problem. Adding the zeros (shaded above) as space holders helps maintain alignment.

To check your understanding of multiplication, work the problems on the next pages.

Multiplication Practice Exercises

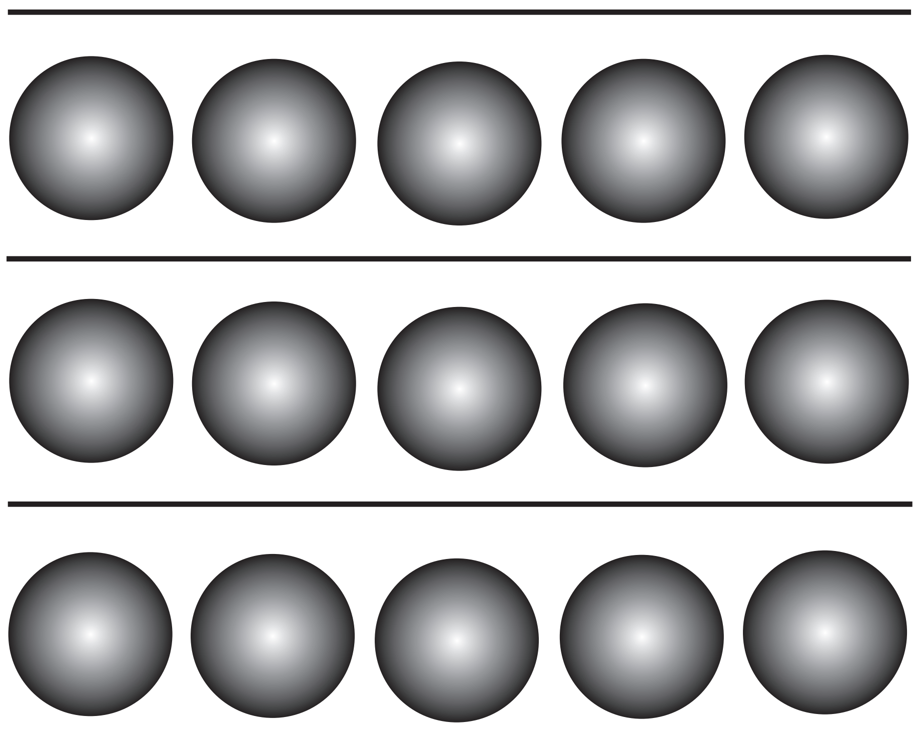
1.	a.	21 x 4	b.	81 x 9	c.	64 x 5	d.	36 x 3
2.	a.	87 x 7	b.	43 x 2	c.	56 x 0	d.	99 x 6
3.	a.	24 x 13	b.	53 x 15	c.	49 x 26	d.	55 x 37
4.	a.	94 x 73	b.	99 x 27	c.	34 x 32	d.	83 x 69

5.	a.	347 x 21		b.	843 x 34		c.	966 x 46
6.	a.	360 x 37		b.	884 x 63		c.	111 x 19
7.	a.	493 x 216		b.	568 x 432		c.	987 x 654

5. Division

You have learned that multiplication is adding a number to itself a certain number of times. For example, 5 x 3 means 5 + 5 + 5. Division is finding out how many dividers are in a whole number. An expression used in division is “goes into”; for example, 5 “goes into” 15 (3) times. Using another example, 15 balls can be divided to give 3 groups of 5 balls in each group. (see Figure 1-7).

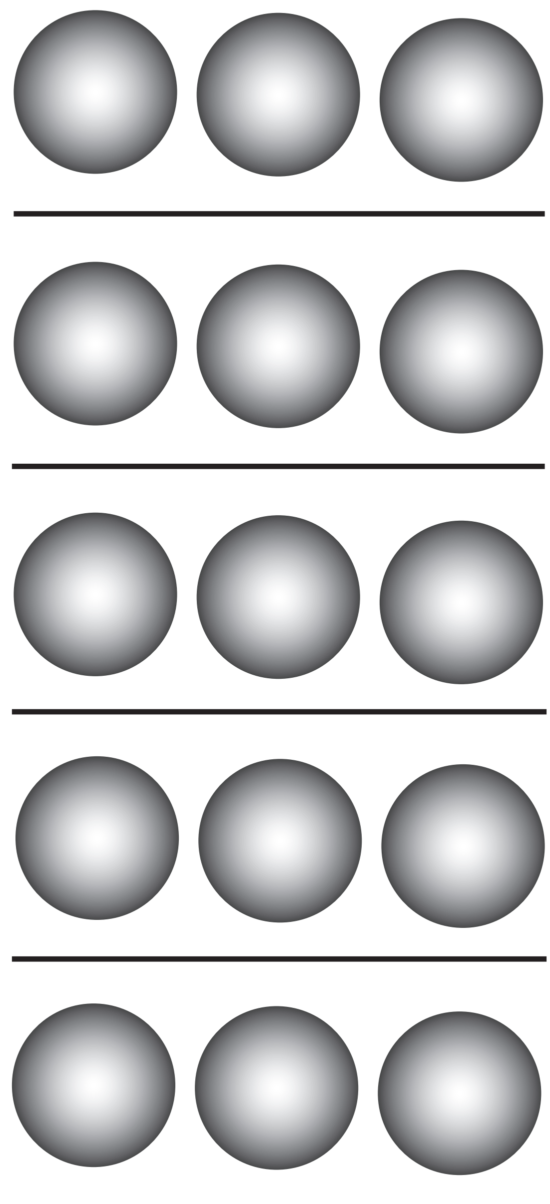
FIGURE 1-7
$15 \div 5 = 3$



They can also be divided to give 5 groups of 3 balls. (see Figure 1-8.) Likewise, 3 balls can be multiplied 5 times (3 x 5) to produce 15 balls. Also, 5 balls can be multiplied 3 times (5 x 3) to produce 15 balls. These examples show that to divide you must know the multiplication table.

FIGURE 1-8

15 ÷ 3 = 5



Division is shown by the following signs. The division sign (÷) means "divided by" and is used for problems such as 12 ÷ 3 = 4. This reads, "Twelve divided by three is, or equals, four." A straight bar (-) or a slanted bar (/) can be used to show division.

Using the straight bar,

$$\frac{12}{3}$$

would read "twelve divided by three".

Using the diagonal bar,

12/4

would read "twelve divided by four."

Division problems usually use the sign $\overline{)\quad\quad}$.

Using this, "twelve divided by two" is represented by $2\overline{)12}$.

There are two forms of division, **long division** and **short division**. Long division is a process for finding out how many times one number may be divided by another, or how many times one number "goes into" another. The long division is shown in the example below. How may times does 48 "go into" 5040 or what is 5040 divided by 48? As you follow this example, remember that in division, the number to be divided is referred to as the **dividend**, and the number by which it is divided is the **divisor**.

Example: $48\overline{)5040}$

Step 1. Set up the problem by placing the division sign over the dividend (5040), with the divisor (48) to the left.

$$48\overline{)5040}$$

Step 2. Start on the left of the dividend and determine the smallest number that the divisor will divide into. In this case, the number is 50 (50 ÷ 48 = 1+). The quotient (1) is placed above the division sign directly over the 0 of 50.

$$\begin{array}{r} 1 \\ 48\overline{)5040} \end{array}$$

Step 3. Multiply the quotient (1) and place the answer under the 50 in the dividend found in Step 2.

$$\begin{array}{r} 1 \\ 48\overline{)5040} \\ \underline{48} \end{array}$$

Step 4. Subtract the product (48) from 50.

$$\begin{array}{r} 1 \\ 48\overline{)5040} \\ \underline{48} \\ 2 \end{array}$$

Step 5. Bring down the next number in the dividend to form a partial remainder (24).

$$\begin{array}{r} 1 \\ 48\overline{)5040} \\ \underline{48} \\ 24 \end{array}$$

Step 6. Divide the divisor (48) into the partial remainder (24), and place the quotient above the division line. In this case, 48 cannot be divided into 24 and a zero is placed above the sign after the 1.

$$\begin{array}{r} 10 \\ 48\overline{)5040} \\ \underline{48} \\ 24 \end{array}$$

Step 7. The next number in the dividend is brought down to form a new partial remainder (240).

$$
\begin{array}{r}
10 \\
48\overline{)5040} \\
\underline{48} \\
240
\end{array}
$$

Step 8. The divisor is divided into the partial remainder (240) and the quotient placed above the division line.

$$
\begin{array}{r}
105 \\
48\overline{)5040} \\
\underline{48} \\
240
\end{array}
$$

Step 9. Multiply the divisor (48) by the quotient (5) and place the product below the partial remainder.

$$
\begin{array}{r}
105 \\
48\overline{)5040} \\
\underline{48} \\
240 \\
240
\end{array}
$$

Step 10. Subtract the product (240) from the previous partial remainder (240). This step completes the operations needed to solve this problem.

$$
\begin{array}{r}
105 \\
48\overline{)5040} \\
\underline{4800} \\
240 \\
\underline{240} \\
0
\end{array}
$$

This problem ended with a partial remainder and a product which, when subtracted in the final step, resulted in 0. If in the final subtraction of the product and the partial remainder the answer is not 0, the answer is said to have a remainder. The remainder cannot be larger than the divisor. If it is, a mistake has been made in the division. The following example shows a division problem with a remainder.

Example:

$$\begin{array}{r} 105 \\ 48\overline{)5042} \\ \underline{4800} \\ 242 \\ \underline{240} \\ 2 \end{array}$$

When the product (240) is subtracted from the previous partial remainder (242) the answer is 2. This 2 is the **remainder**. The answer to the problem 5042 divided by 48 is 105 with a **remainder** of 2. This answer is written 105 *2/48*. Remember, if the remainder is larger than the divisor, a mistake has been made.

In **short division** the process of multiplying the quotient and subtracting the remainder is not used. With short division the problem is solved "in your head." Examples of short division follow.

$$\begin{array}{r} 303 \\ 3\overline{)909} \end{array} \qquad \begin{array}{r} 90 \\ 6\overline{)540} \end{array}$$

Use the **short division** method when the answer is easily determined without using the long division process. If you are uncertain of an answer, use the long division method to ensure accuracy.

To test your understanding of division, work the following problems.

Division Practice Exercises

1. a. $4\overline{)844}$ b. $7\overline{)434}$ c. $9\overline{)828}$

2. a. $9\overline{)117}$ b. $12\overline{)3720}$ c. $10\overline{)1010}$

3. a. $23\overline{)5888}$ b. $56\overline{)38472}$

4. a. $98\overline{)9604}$ b. $13\overline{)871}$

5. a. $50\overline{)2500}$ b. $789\overline{)97047}$

6. a. $21\overline{)147}$ b. $3\overline{)27000}$

7. a. $32\overline{)1952}$ b. $88\overline{)8888}$

8. a. $87\overline{)5848}$ b. $15\overline{)12883}$

9. a. $994\overline{)12883}$ b. $352\overline{)8073}$

C. FRACTIONS

Learner Objectives: Upon completion of this unit, the student will learn about fractions and solve some fractions problems.

Whenever anything is divided into equal parts, each part is smaller than the whole from which it was taken. Each smaller part is called a fraction of the whole. A fraction is one or more of the equal parts of a unit or, in math, equal parts of a whole number. Fractions are written with one number over the other, divided by a line which indicates division, as shown below:

$$\frac{3}{8} \qquad \frac{11}{16} \qquad \text{or } 3/8 \text{ and } 11/16$$

The denominator, which indicates the total number of parts the unit is divided into, is written below the line. The numerator, written above the line, indicates the number of parts being considered. The top number (numerator) should always be read first, then the denominator. The fraction 3/8 would be read "three eighths" and means that three out of eight equal parts are being considered.

Remember: Any fraction indicates equal parts of the whole number 1, because 1 is the smallest whole number. Therefore, any number smaller than one must be a fraction.

1. Changing Whole Numbers to Fractions

Any whole number can be changed to a fraction, that is, be divided into equal parts. This is done by multiplying the number by the number of parts being considered, and placing the product over the number of parts. For example, changing the number 4 to sixths would be done as shown below:

$$4 = \frac{4 \text{ x } 6}{6} = \frac{24}{6} \qquad \text{or } 24/6$$

Each whole unit contains 6 sixths. Four units will contain 4 x 6 sixths, or 24 sixths.

Changing Whole Numbers to Fractions Exercises

Change these whole numbers to fractions.

1) 49 to sevenths

2) 40 to eighths

3) 54 to ninths

4) 27 to thirds

5) 12 to fourths

6) 130 to fifths

2. Proper and Improper Fractions

A proper fraction is one in which the numerator is a smaller number than the denominator.

When the numerator is greater than or equal to the denominator, the fraction is called an improper fraction. Examples of improper fractions would be $6/4$, $9/8$, $3/3$ or $17/1$. Improper fractions can also be written as mixed numbers.

3. Mixed Numbers

Mixed numbers are combinations of a whole number and a proper fraction. For example, $3\ 1/2$ consists of the whole number 3 and the fraction $1/2$. It is read "three and one half," because the whole number and the fraction are added together. Any mixed number can be changed into an improper fraction.

4. Changing Mixed Numbers to Fractions

A mixed number can be changed into an improper fraction by the following steps. In this example, the fraction $3\ 7/8$ will be changed into an improper fraction:

Step 1: Each whole unit contains 8 eighths. The whole number, 3, is multiplied by the denominator of the fraction, 8, resulting in a product of 24.

$$3 \times 8 = 24$$

Three whole units, then, contain 24 eighths.

Step 2: The 24 eighths contained in the three whole units must be combined with, or added to, the 7 existing eighths in order to make one fraction. The numerator of the fraction, 7, is added to the product of the whole number multiplied by the denominator, 24, with a sum of 31.

$$24 + 7 = 31$$

Step 3: Mixed number converted to improper fraction:

After Step 2, all that remains to be done is to write the new fraction in its proper form. The sum, 31, is written over the denominator of the original fraction, resulting in the fraction 31 eighths.

$$3\ ^7/_8 = {}^{31}/_8$$

Summary of Steps:

Multiply the whole number by the denominator of the fraction, add the numerator, then write the result over the denominator.

Changing Mixed Numbers to Fractions Exercises

Change these mixed numbers to fractions, showing all work.

1) $4\ ^1/_2$

2) $8\ ^3/_4$

3) $19\ ^7/_{16}$

4) $7\ ^{11}/_{12}$

5) $6\ ^9/_{14}$

6) $5\ ^1/_{64}$

5. Changing Improper Fractions to Whole/Mixed Numbers

Changing improper fractions into whole or mixed numbers is simply a matter of dividing the numerator by the denominator, as shown in the following examples:

$^{18}/_{3} = 18 \div 3 = 6$

The answer is 6, a whole number.

$^{19}/_{3} = 19 \div 3 = 6$, remainder 1.

The answer is 6 $^{1}/_{3}$, a mixed number.

Changing Improper Fractions to Whole/Mixed Numbers Exercises

Change the following improper fraction into whole or mixed numbers, showing all work.

1) $^{37}/_{7}$

2) $^{44}/_{4}$

3) $^{23}/_{5}$

4) $^{43}/_{9}$

5) $^{240}/_{8}$

6) $^{191}/_{6}$

6. Reducing Fractions

Fractions can be reduced, or changed, to different terms. Terms is a name for the numerator and denominator of a fraction when considered together.

Fractions can be changed to either higher or lower terms. The most common application, however, is reducing a fraction to its lowest terms.

Remember: Reducing a fraction does not change the value of the original fraction.

7. Reducing to Lower Terms

Reducing a fraction to lower terms is simply a matter of dividing both the numerator and denominator by the same number. Reducing to lower terms usually makes a large fraction easier to work with. For example, the fraction $\frac{3}{9}$ can be reduced to lower terms as follows:

$$\frac{3 \div 3 = 1}{9 \div 3 = 3}$$

The fractions $\frac{3}{9}$ and $\frac{1}{3}$ both have the same value.

It is also possible to reduce a fraction to lower terms with a given denominator. This is done by dividing the denominator of the fraction by the required denominator, then dividing the numerator and the denominator of the fraction by this quotient. For example, the fraction $\frac{12}{16}$ could be reduced to fourths as follows:

$$16 \div 4 = 4$$

$$\frac{12 \div 4 = 3}{16 \div 4 = 4}$$

The fractions $\frac{12}{16}$ and $\frac{3}{4}$ each have the same value.

8. Reducing to Lowest Terms

A fraction is reduced to its lowest terms if 1 is the only number which evenly divides both the numerator and the denominator. The process of reducing a fraction to its "lowest" terms is the same as reducing to lower terms, but takes the process to its lowest possible extreme.

For example, the fraction $^{16}/_{32}$ can be reduced to its lowest terms as follows:

a) $\frac{16 \div 2 = 8}{32 \div 2 = 16}$ b) $\frac{8 \div 2 = 4}{16 \div 2 = 8}$ c) $\frac{4 \div 2 = 2}{8 \div 2 = 4}$ d) $\frac{2 \div 2 = 1}{4 \div 2 = 2}$

As shown above, $^{16}/_{32}$ has not been reduced to its lowest terms until d), because the 1 and 2 in $^{1}/_{2}$ have no common divisor other than 1.

Reducing to Lowest Terms Exercises

1) Reduce the following fractions to lower terms as indicated.

a) $^{15}/_{20}$ to 4ths

b) $^{36}/_{40}$ to 10ths

c) $^{24}/_{36}$ to 6ths

d) $^{12}/_{36}$ to 9ths

e) $^{30}/_{45}$ to 15ths

f) $^{16}/_{76}$ to 19ths

2) Reduce the following fractions to their lowest terms.

a) $^{6}/_{10}$

b) $^{3}/_{9}$

c) $^{6}/_{64}$

d) $^{13}/_{32}$

e) $^{32}/_{48}$

f) $^{76}/_{152}$

9. Common Denominator

If two or more fractions have the same denominator, the fractions are said to have a common denominator. For example, $\frac{1}{8}$, $\frac{2}{8}$, $\frac{6}{8}$ and $\frac{7}{8}$ all have the same denominator, or a common denominator of 8.

For any group of fractions, a common denominator can be found by simply multiplying each of the denominators together. For example, a common denominator for the fractions $\frac{1}{6}$, $\frac{3}{8}$, $\frac{2}{9}$, $\frac{5}{12}$, $\frac{5}{18}$, $\frac{7}{24}$ and $\frac{1}{36}$ can be found as follows:

$$6 \times 8 \times 9 \times 12 \times 18 \times 24 \times 36 = 80{,}621{,}568$$

One common denominator for these fractions, then, is 80,621,568. It would not, however, be the only common denominator, and certainly not the easiest to work with when adding and subtracting fractions.

10. Least Common Denominator

The least common denominator (LCD), is the smallest number into which the denominators of a group of two or more fractions will divide evenly.

The easiest method for finding the LCD is shown in the following steps, and is basically a matter of finding the lowest prime factors of each denominator. This example will use the same fractions as in the preceding example: $\frac{1}{6}$, $\frac{3}{8}$, $\frac{2}{9}$, $\frac{5}{12}$, $\frac{5}{18}$, $\frac{7}{24}$ and $\frac{1}{36}$.

Step 1: The prime factors for each denominator are found.

$\frac{1}{6}$	$\frac{3}{8}$	$\frac{2}{9}$	$\frac{5}{12}$	$\frac{5}{18}$	$\frac{7}{24}$	$\frac{1}{36}$
2 x 3	2 x 2 x 2	3 x 3	2 x 3 x 2	2 x 3 x 3	3 x 2 x 2 x 2	2 x 2 x 3 x 3

Step 2: Each set of factors is examined, and the most number of times any single factor appears in a set is multiplied by the most number of times any other factors appear. In this example, in no set does the factor 2 appear more than 3 times (2 x 2 x 2) and the greatest number of times the factor 3 appears is twice (3 x 3). These sets of factors are then multiplied together to obtain the LCD.

$$(2 \times 2 \times 2) \times (3 \times 3) = 72$$

Remember: If a denominator is a prime number, it can't be factored except by itself and 1.

Least Common Denominator Exercises

Find the LCD of the following groups of fractions.

1) $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$

2) $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{24}$

3) $\frac{3}{10}$, $\frac{4}{15}$, $\frac{7}{20}$

---

11. Reducing to LCD

Reducing a group of fractions to their LCD can only be done after the LCD is itself known. Once the LCD has been determined, the entire group of fractions can be converted to that common denominator by dividing the LCD by each denominator in turn and multiplying each term (both numerator and denominator) of the fraction by the result.

In the preceding LCD example, the LCD of the fractions $\frac{1}{6}$, $\frac{3}{8}$, $\frac{2}{9}$, $\frac{5}{12}$, $\frac{5}{18}$, $\frac{7}{24}$ and $\frac{1}{36}$ was found to be 72. The conversion of the first fraction, $\frac{1}{6}$, is shown in the following example:

$$72 \div 6 = 12$$

$$\frac{1 \times 12}{6 \times 12} = \frac{12}{72}$$

The remaining fractions are handled in the same way.

Reducing to Lowest Common Denominator Exercises

Reduce each set of fractions to their LCD.

1) $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$

2) $\frac{1}{12}$, $\frac{1}{16}$, $\frac{1}{24}$

3) $\frac{3}{10}$, $\frac{4}{15}$, $\frac{7}{20}$

12. Addition of Fractions

To add any group of fractions, they must have the same denominators. When the denominators are the same, the adding process simply involves adding the numerators.

For example, the fractions $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ can be added as follows:

$$\frac{1}{4} + \frac{2}{4} + \frac{3}{4} = \frac{6}{4}$$

The sum should always be reduced to its lowest terms. Therefore:

$$\frac{6}{4} = 6 \div 4 = 1\,\frac{2}{4} = 1\,\frac{1}{2}$$

When fractions have unlike denominators, a common denominator must be found before addition can take place. Unlike the methods already discussed for finding the LCD and converting fractions to their LCD, the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{6}{8}$ and $\frac{5}{13}$ can be added as shown in the following steps:

Step 1: The LCD is determined first.

$\frac{1}{2}$	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{5}{13}$
2	2 x 2	2 x 2 x 2	13

2 x 2 x 2 x 13 = 104 (LCD)

Step 2: The fractions are reduced to their LCD.

a) $104 \div 2 = 52$

$\frac{1}{2} \times \frac{52}{52} = \frac{52}{104}$

c) $104 \div 8 = 13$

$\frac{6}{8} \times \frac{13}{13} = \frac{78}{104}$

b) 104 ÷ 4 = 26

$\frac{3}{4} \times \frac{26}{26} = \frac{78}{104}$

d) 104 ÷ 13 = 8

$\frac{5}{13} \times \frac{8}{8} = \frac{40}{104}$

Step 3: The numerators are added together.

$$\frac{52 + 78 + 78 + 40}{104} = \frac{248}{104}$$

Step 4: Answer: The answer is reduced to its lowest terms.

$$104 \overline{)248} = 2 \quad \text{remainder } 40 \qquad (248 - 208 = 40)$$

$$= 2\ \frac{40}{104} = 2\ \frac{5}{13}$$

13. Addition of Mixed Numbers

Remember: A **mixed number** consists of a whole number and a fraction.

When mixed numbers are to be added, the whole numbers are added first, then the fractions are added as already described. If the sum of the fractions is an improper fraction, it must be reduced to a mixed number and the whole number added to the sum of the whole numbers.

For example, the problem $3\ \frac{1}{3} + 2\ \frac{1}{2} + 6\ \frac{1}{6} = ?$ can be solved in the following steps:

Step 1: The whole numbers are added together first.

3 + 2 + 6 = 11

Step 2: The LCD is determined for the fractions.

$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$
3	*2*	*2 x 3*

$$3 \times 2 = 6 \text{ (LCD)}$$

Step 3: The fractions are reduced to their LCD.

a) $6 \div 3 = 2$	b) $6 \div 2 = 3$	c) $6 \div 6 = 1$
$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$	$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}$	$\frac{1 \times 1}{6 \times 1} = \frac{1}{6}$

Step 4: The numerators of the fractions are added together, and the answer reduced to its lowest terms.

$$\frac{2 + 3 + 1}{6} = \frac{6}{6} = 1$$

Step 5: Answer: The sum of the fractions is added to the sum of the whole numbers.

$$11 + 1 = 12$$

Adding Fractions and Mixed Numbers Exercises

Add the following fractions and mixed numbers, reducing the answers to their lowest terms.

1) $\frac{3}{4} + \frac{3}{4}$

2) $\frac{2}{5} + \frac{7}{10}$

3) $\frac{9}{32} + \frac{15}{16}$

4) $5\frac{2}{5} + 1\frac{3}{4}$

14. Subtraction of Fractions

Subtracting fractions is similar to adding fractions in that a common denominator must be found first. To subtract fractions with the same denominator, the subtraction process consists of subtracting one numerator from another numerator.

Fractions with different denominators should be reduced to their LCD, then one numerator subtracted from the other.

For example, subtracting the fraction $^{14}/_{24}$ from $^{20}/_{24}$ can be done as follows:

$$^{20}/_{24} - {}^{14}/_{24} = {}^{6}/_{24}$$

The following steps show how fractions with different denominators can be subtracted. In this example, $^{1}/_{4}$ will be subtracted from $^{5}/_{16}$.

Step 1: The LCD is found first.

$$^{5}/_{16} \quad - \quad ^{1}/_{4}$$

$$2 \text{ x } 2 \text{ x } 2 \text{ x } 2 \qquad 2 \text{ x } 2$$

$$2 \text{ x } 2 \text{ x } 2 \text{ x } 2 = 16 \text{ (LCD)}$$

Step 2: The fractions are changed to their LCD.

$$16 \div 16 = 1 \qquad 16 \div 4 = 4$$

$$^{5}/_{16} \text{ x } ^{1}/_{1} = {}^{5}/_{16} \qquad ^{1}/_{4} \text{ x } ^{4}/_{4} = {}^{4}/_{16}$$

Step 3: The numerators are subtracted,

$$^{5}/_{16} - {}^{4}/_{16} = {}^{1}/_{16}$$

resulting in the answer, $^{1}/_{16}$.

15. Subtraction of Mixed Numbers

In the subtraction of mixed numbers, the fractions are subtracted first, then the whole numbers and finally the remainders from the fractions and whole numbers.

For example, 4 $\frac{1}{2}$ can be subtracted from 10 $\frac{2}{3}$ as shown in the following steps:

10 $\frac{2}{3}$ - 4 $\frac{1}{2}$

Step 1: The LCD is determined first, and the fractions changed to their LCD.

First, multiply the denominators of the two fractions.

3 x 2 = 6 (LCD)

Next, divide the LCD by the denominator of each fraction.

6 ÷ 3 = 2 and 6 ÷ 2 = 3

Then, multiply both the numerator and denominator of the fractions by their respective numbers.

$\frac{2}{3}$ x $\frac{2}{2}$ = $\frac{4}{6}$ $\frac{1}{2}$ x $\frac{3}{3}$ = $\frac{3}{6}$

The result is two fractions, both with a common denominator.

$\frac{4}{6}$ and $\frac{3}{6}$

Step 2: The numerators of the fractions are subtracted, and the resulting fraction reduced to its lowest terms if necessary.

$\frac{4}{6}$ – $\frac{3}{6}$ = $\frac{1}{6}$

Step 3: The whole numbers are subtracted.

10 – 4 = 6

Step 4: The whole number and fraction are added together to form the complete answer.

$$6 + 1/6 = 6\ 1/6$$

It is also possible to borrow units as in basic subtraction of whole numbers, as shown when 3⅜ is subtracted from 5 1/16 in the example below:

5 1/16 - 3⅜

Step 1: The LCD is determined, and the fractions changed to their LCD.

1/16 3/8

2 x 2 x 2 x 2 2 x 2 x 2

2 x 2 x 2 x 2 = 16 (LCD)

16 ÷ 16 = 1 16 ÷ 8 = 2

1/16 x 1/1 = 1/16 3/8 x 2/2 = 6/16

Step 2: Because 6/16 cannot be subtracted from 1/16, 1 unit is borrowed from the 5 units, leaving 4 units. Because 1 unit is equal to 16/16, the borrowed 16/16 must be added to the existing 1/16, for a sum of 17/16.

4 ~~5~~ 1/16 + 16/16 = 4 17/16

Step 3: The numerators are subtracted, and the result reduced to its lowest terms if possible. In this case, 11/16 will not reduce.

$$17/16 - 6/16 = 11/16$$

Step 4: The whole numbers are subtracted.

$$4 - 3 = 1$$

Step 5: The whole number and the fraction are added together to form the complete answer.

$1 + \frac{11}{16} = 1\frac{11}{16}$

Subtracting Fractions and Mixed Numbers Exercises

Subtract the following fractions and mixed numbers, reducing the answers to their lowest terms.

1) $\frac{2}{5} - \frac{1}{3}$

4) $33\frac{1}{3} - 15\frac{2}{5}$

2) $\frac{5}{8} - \frac{3}{12}$

5) $101\frac{1}{4} - 57\frac{15}{16}$

3) $47\frac{2}{5} - 28\frac{1}{3}$

6) $14\frac{3}{4} - 10\frac{5}{12}$

16. Multiplying Fractions

A common denominator is not required in the multiplication of fractions. Instead, the numerators are multiplied together and the denominators are multiplied together. For example, multiplying ¾ x 4⁄16 can be done as shown in the following steps:

Step 1: First the numerators are multiplied.

$$\frac{3}{4} \times \frac{4}{16} = \frac{12}{\;}$$

Step 2: Then, the denominators are multiplied.

$$\frac{3}{4} \times \frac{4}{16} = \frac{12}{64}$$

Step 3: The answer is reduced to its lowest terms if possible.

$$\frac{12}{64} \div \frac{4}{4} = \frac{3}{16}$$

17. Multiplication of Fractions and Whole/Mixed Numbers

Whole and mixed numbers must be changed to improper fractions before being multiplied by fractions. This is true when a fraction is multiplied by either a whole number or mixed number, when a mixed number is multiplied by a whole number or when two mixed numbers are multiplied together.

Remember: An improper fraction is a fraction whose numerator is larger than its denominator.

Whole numbers changed to improper fractions all have a denominator equal to 1. For example, the problem ¾ x 4 would be solved as follows:

Step 1: The whole number (4) is changed to an improper fraction

$$\left(\frac{4}{1}\right).$$

Step 2: The fractions are multiplied, numerator by numerator and denominator by denominator.

$$\frac{3}{4} \times \frac{4}{1} = \frac{12}{4}$$

Step 3: The fraction is reduced to its lowest terms.

$$\frac{12}{4} \div \frac{4}{4} = \frac{3}{1} = 3$$

Mixed numbers can be changed to improper fractions by multiplying the denominator by the whole number, then adding the product to the numerator. For example, $2\frac{2}{3}$ can be changed to an improper fraction as shown here below:

$$2\frac{2}{3} = \frac{6}{3} + \frac{2}{3} = \frac{8}{3}$$

Once a mixed number has been changed to an improper fraction, it can be multiplied by other fractions as shown in the earlier examples.

18. Cancellation

Multiplying fractions can be made easier by using the cancellation method. If the numerator of one of the fractions being multiplied and the denominator of the other fraction can be evenly divided by the same number, they can be reduced, or cancelled. In the following example, $\frac{8}{3}$ is multiplied by $\frac{5}{16}$. Notice that the 8 and the 16 can both be divided by 8.

$$\frac{8}{3} \times \frac{5}{16} = \frac{\overset{1}{\cancel{8}}}{3} \times \frac{5}{\underset{2}{\cancel{16}}} = \frac{1}{3} \times \frac{5}{2} = \frac{5}{6}$$

In the above example, 8 ÷ 8 = 1, and 16 ÷ 8 = 2.

Remember: Reducing fractions to their lowest terms often makes multiplying those fractions easier.

Multiplying Fractions, Whole and Mixed Numbers Exercises

Multiply the following fractions, whole numbers and mixed numbers, reducing to lowest terms.

1) $\frac{3}{4} \times \frac{4}{16}$

2) $26 \times \frac{1}{26}$

3) $\frac{4}{5} \times 3$

4) $\frac{9}{5} \times \frac{2}{3}$

5) $\frac{35}{4} \times \frac{4}{35}$

6) $\frac{9}{10} \times \frac{3}{5}$

7) $\frac{1}{6} \times \frac{7}{12}$

8) $\frac{2}{3} \times \frac{5}{11}$

9) $5 \times \frac{77}{15}$

19. Division of Fractions

Division of fractions is actually done by multiplication. Division problems are simply changed into multiplication problems and multiplied, by inverting the divisor.

It is important to recognize which value is the divisor in any division problem, and particularly in the division of values involving fractions.

Remember: the sign ÷ means "divided by," and the fraction to the right of this sign is always the divisor. The problem $\frac{3}{4} \div \frac{2}{3}$ would be read, $\frac{3}{4}$ divided by $\frac{2}{3}$. Division may also be indicated with a "bar" as shown below:

$$\frac{3/4}{2/3}$$

When dividing fractions, the basic rule is to invert the divisor and multiply. When inverted, the positions of the numerator and denominator are reversed: the numerator is moved to the bottom of the fraction (below the line) and the denominator is moved to the top (above the line). For example, the problem $\frac{3}{4} \div \frac{1}{5}$ is solved in the following steps:

Step 1: The divisor, $\frac{1}{5}$, is inverted, changing it to $\frac{5}{1}$. The division sign is changed to multiplication.

$\frac{3}{4} \div \frac{1}{5}$ becomes $\frac{3}{4} \times \frac{5}{1}$

Step 2: The problem is solved by multiplication, and the answer reduced to its lowest term, in this case, a mixed number.

$\frac{3}{4} \times \frac{5}{1} = \frac{15}{4} = 3\frac{3}{4}$

20. Division of Fractions and Whole/Mixed Numbers

Whole and mixed numbers must be changed to improper fractions before they can be divided by, or into, fractions and other mixed numbers before inverting and multiplying. For example, the problem $3\,{}^{3}/_{16} \div 2\,{}^{1}/_{8}$ can be solved in the following steps:

Step 1: Mixed numbers are changed to improper fractions,

$$3\,{}^{3}/_{16} \div 2\,{}^{1}/_{8} =$$

$$16 \times 3 + {}^{3}/_{16} = {}^{51}/_{16} \quad \text{and} \quad 8 \times 2 + {}^{1}/_{8} = {}^{17}/_{8}$$

and the problem becomes:

$${}^{51}/_{16} \div {}^{17}/_{8} =$$

Step 2: The divisor (${}^{17}/_{8}$) is inverted, and the division sign changed to multiplication.

$${}^{51}/_{16} \div {}^{17}/_{8} \text{ becomes } {}^{51}/_{16} \times {}^{8}/_{17}$$

Step 3: Cancellation is used to reduce this multiplication problem to its lowest terms. Note: 51 ÷ 17 = 3, 17 ÷ 17 = 1; 16 ÷ 8 = 2, 8 ÷ 8 = 1.

$${}^{51}/_{16} \times {}^{8}/_{17} = {}^{3\,\cancel{51}}/_{\cancel{16}\,2} \times {}^{1\,\cancel{8}}/_{\cancel{17}\,1} = {}^{3}/_{2} \times {}^{1}/_{1}$$

Step 4: The problem is solved by multiplication, and the answer reduced to its lowest terms.

$${}^{3}/_{2} \times {}^{1}/_{1} = {}^{3}/_{2} = 1\,{}^{1}/_{2}$$

Dividing Fractions, Whole and Mixed Numbers Exercises

Solve the following division problems, reducing the answer to their lowest terms.

1) $\frac{5}{8} \div \frac{3}{6}$

2) $5\frac{1}{16} \div \frac{3}{8}$

3) $18 \div \frac{1}{8}$

4) $15 \div \frac{7}{12}$

5) $1\frac{4}{3} \div \frac{7}{4}$

D. DECIMAL NUMBERS

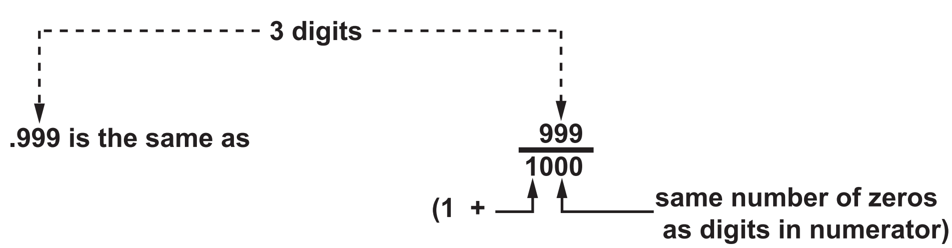
Learner Objectives: Upon completion of this unit, the student will solve problems of addition, subtraction, multiplication, and division using decimal numbers.

Precision measurement is a constant companion of machine, hand, and assembly operations in industry. Specifications on drawings, blueprints, and sketches are commonly expressed in terms of hundredths, thousandths, and ten-thousandths for extremely accurate work. Steel rules, micrometers, indicators, and many other precision measurement instruments are available for use based on the decimal system.

1. Decimal System

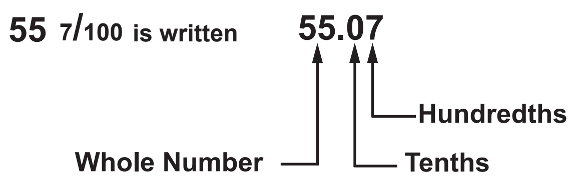
The decimal system is a system of numbers based on ten. A decimal fraction is a fraction whose denominator is 10, 100, 1000, or another value which can be obtained by multiplying 10 by itself a specified number of times. The decimal fraction differs from a common fraction in that it is written on one line as a whole number with a period in front of it. This is possible because the denominator is always one (1) followed by zeros. By placing a period before the number which appears in the numerator, the denominator may be omitted. This period is called a decimal point. For example, the common fraction 5/10 is written as the decimal .5; 5/100 as 0.05; and 5/1000 as 0.005.

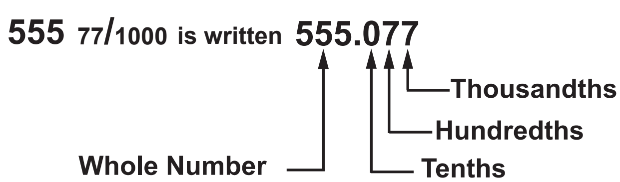
Any whole number with a decimal point in front of it is a decimal fraction. The numerator is the number which is to the right of the decimal point. The denominator is one (1) with as many zeros after it as there are digits in the numerator. For example, the decimal fraction .999 is the same as 999/1000. To illustrate further,



2. Reading and Writing Decimals

Just as mixed numbers such as 5 7/10 are used to express a whole number and a fraction, whole numbers and decimal fractions can be similarly written. Three examples are given to show how different quantities may be expressed.





A decimal is read like a whole number except that the name of the last column to the right of the decimal point is added. Decimals are read to the right of the decimal point as follows:

Examples:

.63 is read "sixty-three **hundredths**".

.136 is read "one hundred thirty-six **thousandths**".

.5625 is read "five thousand six hundred twenty-five **ten-thousandths**".

3.5 is read "three and five **tenths**".

2.15625 is read "two and fifteen thousand six hundred twenty-five **hundred-thousandths**".

Dimensions involving whole numbers and decimals are frequently expressed in abbreviated form. Examples of this simplified method of reading decimals are shown below.

Examples:

1. A dimension such as 6.625 is spoken of as “six, point six two five".

2. A dimension such as 21.312 is spoken of as “twenty-one, point three one two".

When there is need to make accurate computations in addition, subtraction, multiplication, and division of fractions with denominators of 10, 100, 1000, etc., decimal fractions provide an easy method for solving problems.

One place	.0	tenths
Two places	.00	hundredths
Three places	.000	thousandths
Four places	.0000	ten-thousandths
Five places	.00000	hundred-thousandths

3. Addition of Decimals

In the machine shop, drafting lab, and many other industrial settings, computation of dimensions from drawings and sketches often requires the addition of two or more decimals. The typical example is determining the distance between two points when each point is presented in decimal form. The addition of these decimals is the same as addition of regular whole numbers except the location of the decimal point demands additional considerations.

Rules for Adding Decimals

Example: Add .865 + 1.3 + 375.006 + 71.1357 + 735

- Align each given number so that all decimal points are in a vertical column.
- Place a decimal point to the right of all whole numbers.
- Add each column the same as for regular addition of whole numbers.
- Locate the decimal point in the answer by placing it in the same column in which it appears with each number.

Step 1. Write the numbers under each other to insure that all decimal points are in a vertical line.

$$\begin{array}{r} .865 \\ 1.3 \\ 375.006 \\ 71.1357 \\ \underline{+\ 735.} \end{array}$$

Step 2. Add each column.

NOTE: Zeros are sometimes added to the numbers so that they all have an equal number of places after the decimal point. This practice may help eliminate errors.

$$\begin{array}{r} .8650 \\ 1.3000 \\ 375.0060 \\ 71.1357 \\ \underline{+\ 735.0000} \\ 1183.3067 \end{array}$$

Step 3. Locate the decimal point in the answer in the same column in which it appears with the numbers being added.

```
      .8650
     1.3000
   375.0060
    71.1357
 + 735.0000
 ----------
  1183.3067
```

4. Subtraction of Decimals

Subtraction of one decimal dimension from another is a common and necessary practice for completion in most industrial jobs. The process is the same as subtraction of whole numbers with the exception of providing for an accurate placement of the decimal point.

Rules for Subtracting Decimals

- Write numbers so that the decimal points are under each other.
- Subtract each column the same as for regular whole numbers.
- Locate the decimal point in the answer by placing it in the same column in which it appears in the problem.

Review the following example to further explain this procedure.

Example: Determine the difference of the following dimensions: 62.1251 square inches and 24.102 square inches.

Step 1. Write the two dimensions so the smaller is under the larger with decimal points aligned in the same vertical column.

NOTE: Includes zeros for ease of subtraction if desired.

```
   62.1251
 - 24.1020
 ---------
```

Step 2. Subtract the numbers as you would in finding the difference between whole numbers starting with the last digit on the right.

Step 3. Locate the decimal in the answer in the same vertical position that it appeared in the problem.

$$\begin{array}{r} 62.1251 \\ \underline{-\ 24.1020} \\ 38.0231 \end{array}$$

5. Multiplication of Decimals

Multiplying decimals is a convenient and somewhat simplified way of adding them. Instead of taking one number and listing it, then adding it to itself a certain number of times, it is easier and there is less chance for error if the two numbers are multiplied.

With the exception of locating or pointing off the decimal places in the answer, the entire multiplication process is identical with that used for whole numbers.

Rules for Multiplying Decimals

- Multiply the same as with whole numbers.
- Count the number of decimal places to the right of the decimal point in both numbers being multiplied.
- Locate the decimal point in the answer by starting at the extreme right digit and counting as many places to the left as there are in the total number of decimal places found in both numbers being multiplied.

Example: Multiply 38.639 x 2.08

Step 1. Multiply 38.639 by 2.08 as if they were whole numbers.

Step 2. Total the number of digits to the right of the decimal place in both numbers being multiplied (5).

Step 3. Start at the extreme right in the answer and count off five decimal places to the left and place the decimal point here.

```
   3 8.6 3 9     (3 decimal places)
 x   2.0 8       (2 decimal places)
 3 0 9 1 1 2
7 7 2 7 8
8 0.3 6 9 1 2
    (5 decimal places)
```

6. Division of Decimals

Division is a process for determining how many times one number is contained in another. The division of decimals, like all other mathematical processes for decimals, is essentially the same as for whole numbers except provisions must be made for the location of the decimal in the answer.

Rules for Dividing Decimals

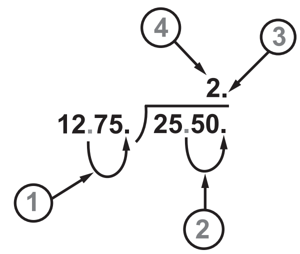
- Place the number to be divided (called dividend) inside the division box.
- Place the divisor outside.
- Move the decimal point in the divisor to the extreme right. The divisor then becomes a whole number.
- Move the decimal point the same number of places to the right in the dividend.

  NOTE: Zeros are added in the dividend if needed.
- Mark the position of the decimal point in the quotient directly above the decimal point in the dividend.
- Divide as whole numbers and place each figure in the quotient directly above the digit involved in the dividend.

- Add zeros after the decimal point in the dividend if it cannot be divided evenly by the divisor.

- Continue division until the quotient has as many places as are required for the answer.

Example 1: 25.5 ÷ 12.75



Step 1. Move the decimal point in the divisor to the right two places.

Step 2. Move the decimal point in the dividend to the right the same number of places (2).

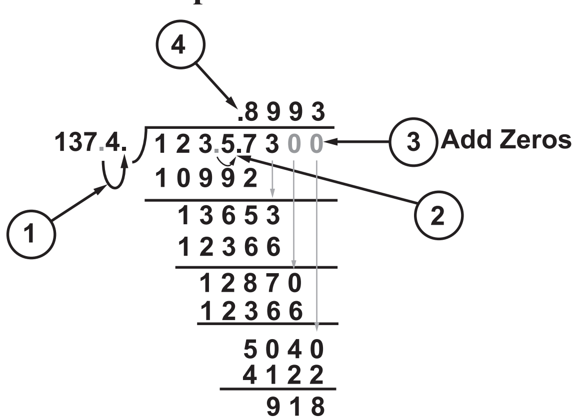
NOTE: Since there is only one digit after the decimal, add a zero to it.

Step 3. Place the decimal point in the quotient.

Step 4. Divide as whole numbers.

Example 2: 123.573 ÷ 137.4

Pay close attention to the slide.



NOTE: The steps necessary to compute the answer are the same as those used in Example 1.

Decimal Numbers Practice Exercises

1. Add the following decimals.

a. .6 + 1.3 + 2.8 =

b. 72.8 + 164.02 + 174.01 =

c. 185.7 + 83.02 + 9.013 =

d. 0.93006 + 0.00850 + 3315.06 + 2.0875 =

2. Subtract the following decimals.

a. 2.0666 - 1.3981 =

b. 18.16 - 9.104 =

c. 1.0224 - .9428 =

d. 1.22 - 1.01 =

e. 0.6 - .124 =

f. 18.4 - 18.1 =

g. 1347.008 - 108.134 =

h. 111.010 - 12.163 =

i. 64.7 - 24.0 =

3. Multiply the decimals listed below.

a.
$$\begin{array}{r} 3.01 \\ \times\ 6.20 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 21.3 \\ \times\ 1.2 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 1.6 \\ \times\ 1.6 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 83.061 \\ \times\ 2.4 \\ \hline \end{array}$$

e.
$$\begin{array}{r} 1.64 \\ \times\ 1.2 \\ \hline \end{array}$$

f.
$$\begin{array}{r} 44.02 \\ \times\ 6.01 \\ \hline \end{array}$$

g.
$$\begin{array}{r} 63.12 \\ \times\ 1.12 \\ \hline \end{array}$$

h.
$$\begin{array}{r} 183.1 \\ \times\ .23 \\ \hline \end{array}$$

i.
$$\begin{array}{r} 68.14 \\ \times\ 23.6 \\ \hline \end{array}$$

4. Divide the following decimals.

a. $1.4\overline{)42.7}$

b. $.8\overline{)4.63}$

c. $1.2\overline{)620.4}$

d. $6\overline{)6.6786}$

e. $1.1\overline{)110}$

E. CHANGING FRACTIONS TO DECIMALS

Decimals are another way to represent a fraction. You can show this by reviewing the tenths, hundredths and thousandths on the base 10 number system. Decimals are simply so many tenths or hundredths or thousandths.

A fraction can be changed to a decimal number by dividing the numerator by the denominator. This will usually require the addition of a decimal point and trailing zeros.

¾ (three-quarters) as a decimal

Divide the numerator by the denominator $4\overline{)3}$

Add the decimal point plus the trailing zeros $4\overline{)3.0}$

Answer = .75

Figure 1-9 is a fraction-to-decimal conversion chart.

FIGURE 1-9
Fraction-to-Decimal Conversion Chart

$^{1}/64$	= .015625	$^{11}/32$	= .34375	$^{43}/64$	= .671875
$^{1}/32$	= .03125	$^{23}/64$	= .359375	$^{11}/16$	= .6875
$^{3}/64$	= .046875	**$^{3}/8$**	**= .375**	$^{45}/64$	= .703125
$^{1}/16$	= .0625	$^{25}/64$	= .390625	$^{23}/32$	= .71875
$^{5}/64$	= .078125	$^{13}/32$	= .40625	$^{47}/64$	= .734375
$^{3}/32$	= .09375	$^{27}/64$	= .421875	**$^{3}/4$**	**= .750**
$^{7}/64$	= .109375	$^{7}/16$	= .4375	$^{49}/64$	= .765625
$^{1}/8$	**= .125**	$^{29}/64$	= .453125	$^{25}/32$	= .78125
$^{9}/64$	= .140625	$^{15}/32$	= .46875	$^{51}/64$	= .796875
$^{5}/32$	= .15625	$^{31}/64$	= .484375	$^{13}/16$	= .8125
$^{11}/64$	= .171875	**$^{1}/2$**	**= .500**	$^{53}/64$	= .838125
$^{3}/16$	= .1875	$^{33}/64$	= .515625	$^{27}/32$	= .84375
$^{13}/64$	= .203125	$^{17}/32$	= .53125	$^{55}/64$	= .859375
$^{7}/32$	= .21875	$^{35}/64$	= .546875	**$^{7}/8$**	**= .875**
$^{15}/64$	= .234375	$^{9}/16$	= .5625	$^{57}/64$	= .890625
$^{1}/4$	**= .250**	$^{37}/64$	= .578125	$^{29}/32$	= .90625
$^{17}/64$	= .265625	$^{19}/32$	= .59375	$^{59}/64$	= .921875
$^{9}/32$	= .28125	$^{39}/64$	= .609375	$^{15}/16$	= .8125
$^{19}/64$	= .296875	**$^{5}/8$**	**= .625**	$^{61}/64$	= .953125
$^{5}/16$	= .3125	$^{41}/64$	= .640625	$^{31}/32$	= .96875
$^{21}/64$	= .328125	$^{21}/32$	= .65625	$^{63}/64$	= .984375

Decimal Conversion Practice Exercises

1. Write the fractions/mixed numbers below as decimals.

   a. 6/10

   b. 3/5

   c. 4/5

   d. 1/5

   e. 1/2

   f. 8/20

   g. 7/20

   h. 15/20

   i. 7/25

   j. 12/25

   k. 17/20

   l. 49/50

   m. 1-9/10

   n. 1-1/25

   o. 6-15/25

F. PERCENTAGES

Learner Objectives: Upon completion of this unit, the student will solve problems using percentages with a high rate of accuracy.

1. Percents

Percents are used on numerous occasions to show how many parts of a total are taken out. Percents are used to make comparisons and to compute interest, taxes, discounts, and gains and losses in industry, business, the home, and school.

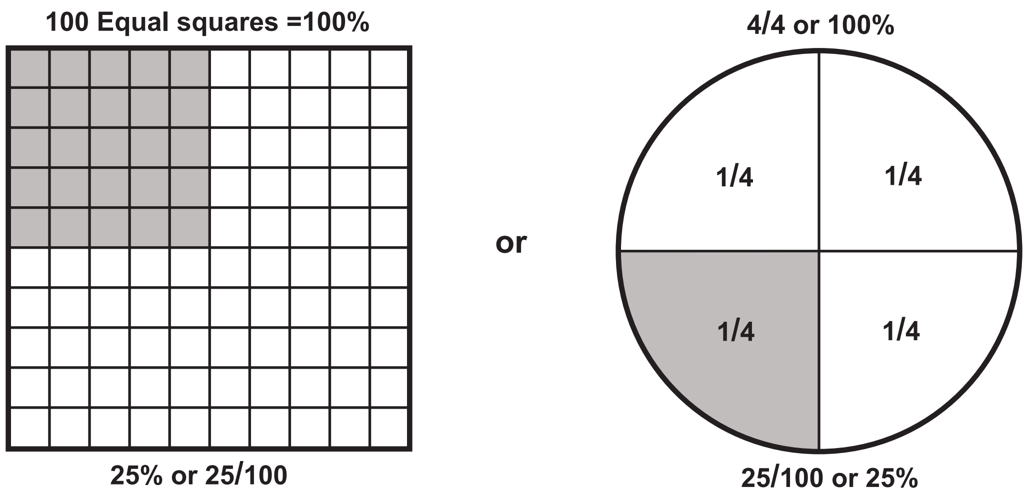
The word percent is a short way of saying "by the hundred or hundredths part of the whole." A percent refers to a given number of parts of the whole which is equal to 100 percent. The symbol % is used to indicate percent. Thus 10 % is the same as writing "ten percent."

Percent can be examined through the use of diagrams which indicates the number of equal fractional parts neededd to determine the percentage desired (see Figure 1-10).

The square is divided into 100 equal parts. Each square is 1/100 of 100 %=1 %. By the same reasoning, the 25 shaded squares are 25/100 of the total or 25 %. In the circle, the shaded area is 1/4 of the whole circle or 1/4 of 100% or 25%.

Percent is another way to represent fractional parts of a whole number. One hundred percent of a number is the same as one hundred hundredths (100/100) or 1. It represents the whole number or total. Hence, 100 % of a number is the number itself.

FIGURE 1-10
Percent



To change a decimal to a %, move your decimal to the right two places and write the percent sign.

.15 = 15%

.55 = 55%

.853 = 85.3%

1.02 = 102%

In this example, there is only one digit after the decimal. Add the zero to hold the place.

.8 = 80%

Remember: Zeros may be needed to hold your place.

.06 = 6%

When we move the decimal to the right 2 places, we get 06.
06 and 6 are the same number.

To change a percent to a decimal number, move your decimal to the left 2 places.

34% = .34

5% = .05

75.4% = .754

Percents Practice Exercises

Write as a decimal.

1. 35 % = ________
2. 14 % = ________
3. 58.5 % = ________
4. 17.45 % = ________
5. 5 % = ________

Write as a percent.

6. .75 = ______ %
7. 0.40 = _____ %
8. 0.4 = ______%
9. .4 = ______%

Rule for Any Equivalent

Convert the number to its decimal equivalent by multiplying it by 0.01.

NOTE: For fractions and improper fractions, change the given number to a decimal number by dividing the numerator by the denominator and multiplying the results by 0.01.

Example: Change 6 1/4% to its decimal equivalent.

Step 1. Change the mixed number into an improper fraction and then divide the numerator by the denominator.

6 1/4 = 25/4 = 6.25

Step 2. Now multiply the answer (6.25) by 0.01.

6.25 x 0.01 = 0.0625

Answer (0.0625)

Rules For Finding Any Percent of Any Number

- Convert the percent into its decimal equivalent.
- Multiply the given number by this equivalent.
- Point off the same number of spaces in the answer as there are in the two numbers multiplied.
- Label the answer with the appropriate unit measure where applicable.

Examples: Find 16% of 1028 square inches

Step 1. Change 16% to a decimal.

16 x .01 = .16

Step 2. Multiply the given number (1028) by the decimal (.16).

1028 x 0.16 = 164.48

Step 3. Point off two decimal places in the answer.

164.48

Step 4. Label the answer.

164.48 square inches

2. Percentage

Percentage refers to the value of any percent of a given number. In all problems of percentage, three values are involved. The first number is called the **base** because a definite percent is to be taken of it. The second number, the **rate**, refers to the percent that is to be taken of the base. The third number is the **percentage**. The relationship between each of these values may be stated in the rule; **the product of the base times the rate equals the percentage.**

In simplified form, **Percentage = Base x Rate**.

In an effort to allow for ease of operation, the letter B can be used for base, R for Rate, and P for percentage. Using letters instead of words, the rule may be written as **P = B x R**

NOTE: The rate must always be in decimal form.

Using the example "Find 16% of 1028 square inches," we can now identify each value. Once 16% is converted to a decimal (.16), it becomes the rate (R). Because 1028 is the value that the definite percent or rate is to be taken out of, it is obviously the base (B). The product of these two values (.16 x 1028) equals the percentage (P) or the value (164.48 square inches).

To solve percent problems, we can make use of a simple formula:

Rate = Percentage x Base

Example: R = 50% of 16

Step 1. The **rate** is the quantity in question;
The **percentage** is the ratio of amount to base;
The **base** is the quantity being compared to.

Step 2. We can further simplify this by using a memory device. To see how to find the missing quantity, cover it and see what is left.

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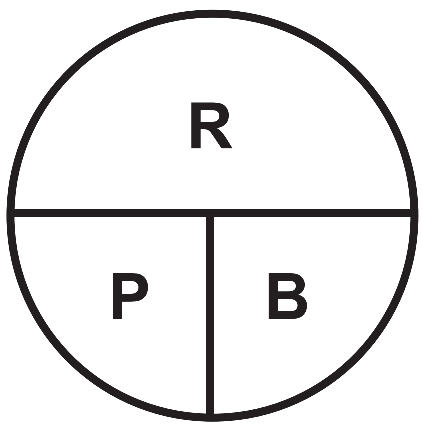


Step 3. Only three types of percent problems exist.

1. Find the amount or rate $R = P \times B$

2. Find the percentage $P = \frac{R}{B}$

3. Find the base $B = \frac{R}{P}$

Finding the Amount or Rate

R = P x B or Rate = Percentage x Base

Example: What number is 60% of 135?

Step 1. R = P x B

Step 2. R = 0.60 x 135

Step 3. R = 81

Finding the Percentage

$P = \frac{R}{B}$ or $\text{percentage} = \frac{\text{rate}}{\text{base}}$

Example: 18 is what percent of 75?

Step 1. $\frac{R}{B} = P = \frac{18}{75} = 0.24 = \frac{24}{100} = 24\%$

Answer = 24%

Here, 75 is the base since it is the quantity being compared to.

Finding the Base

$B = \frac{R}{P}$ or $\text{base} = \frac{\text{rate}}{\text{percentage}}$

Example: 54 is 36% of what number?

Step 1. $B = \frac{R}{P} = \frac{54}{0.36} = 150$

Answer = 150

Percentage Practice Exercises

1. Determine the rate or amount for each problem A through E for each value given. Only use calculators if instructed to do so. Round answers to the nearest 100th.

	A.	B.	C.	D.	E.
BASE	2400 lbs	1875 gallons	148 feet	3268.5 square inches	$875.00
PERCENTAGE	80 %	45 %	15 %	4 1/2 %	19.5 %
	A.__________	B.__________	C. __________	D. __________	E.__________

2. The labor and material for renovating a building totaled $25,475. Of this amount, 70% went for labor and the balance for materials. Determine: (a) the labor cost, and (b) the material cost.

3. Find 35 % of 82.

4. What is 14 % of 28?

5. Sales tax is 9 %. Your purchase is $4.50. How much do you owe?

6. You have 165 seconds to finish your task. At what point are you 70 % finished?

7. You make $14.00 per hour. In March you receive a cost of living raise of 5 %. How much raise per hour did you get. How much are you making per hour now?

G. APPLYING MATH TO THE REAL WORLD

MATH. Do we really need it? Everyone needs to be able to work simple math problems in the real world. Some say they don't really need to be able to multiply, divide, or know percentages. But do they?. Solve the following math problems. These are the kind of math problems that everyone will face from time to time in the real world.

Real World Math Practice Exercises

1. You are presented a sheet of steel with a grid pattern or holes punched in it. There are 18 holes punched lengthwise down the sheet, and 12 holes punched across the width (18 rows of 12 holes). How many holes are punched in the sheet?

2. A box contains 240 rivets. How many floor panels can you make with that one box, if each floor panel contains 8 rivets?

3. An empty waste oil drum can hold 55 gallons. Over the course of a week, it is filled with3.5 gallons, 8.5 gallons, 12 gallons, 2.5 gallons, and 15 gallons. How much more waste oil can the drum hold?

4. In order for a rivet to be properly installed, it must extend from the hole 1.5 times its diameter. If a rivet's diameter is 0.8 mm, what length must extend from the hole?

5. In order for a nut to be correctly screwed on to a bolt, the bolt must extend through the hole by 20 % of its length. How much must extend through the hole if the bolt is 5 inches long?

6. There are six people on your work-team. The customer needs 2,400 dashboard mounts made by the end of the week (5 days). How many mounts must be made each day by each team member to meet this order?

7. A can of dye states that it will cover 400 square feet of metal. How many cans would it take to cover a strip of rolled metal 6 feet wide and 200 feet long?

8. You are supposed to drill several holes with a diameter of 2mm. When you drill them, you are allowed to vary from this diameter a maximum of $\pm$ 3%. What are the minimum and maximum diameters allowed? Hint: The minimum would be 97% of the ideal diameter and the maximum would be 103%.

H. METRIC SYSTEM

1. Metrication

The word **"Metrication"** denotes the process of changing from the English weights and measurements system to the metric system. Metrication is in current use across the nation.

The United States is the only major country today which does not use metric as the standard for weights and measurements. Because of modern means of travel, and as countries depend more on imports and exports, a single standard for weights and measurements is needed throughout the world. This standard will be the metric system.

Some industries in the United States, such as the drug and film industries, have been using the metric system for many years. Other industries are in the process of changing to metric. Several states have recently passed laws requiring schools to teach metric education.

Effect of Change to Industry

Industrial workers will need to learn the new metric system of weights and measurements to replace the present system of English weights and measurements. Millimeters, centimeters, meters, and Kilometers will be used for linear measurements. The milligram, gram, and Kilogram will be used to measure weights; the milliliter and liter will be used to measure liquids.

Basic Principles of the Metric System

The metric system will make measuring and using measurements easier than the methods now used in the United States. Here are a few basic principles of the metric system which should be remembered.

a. All the divisions of the basic units, or prefixes, are the same regardless of what you might be measuring. Although distance is measured in meters, weight in grams, and liquid in liters, the prefixes of these measurements are the same.

These prefixes are:

Kilo	=	1000 units
Hecto	=	100 units
Deka	=	10 units
deci	=	0.1 unit (one-tenth of the unit)
centi	=	0.01 (one-hundredth of the unit)
milli	=	0.001 (one-thousandth of the unit)

The most commonly used prefixes are **Kilo**, **centi**, and **milli**.

Place Value Table:

Place Value	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
Prefix	Kilo	Hecto	Deka	base unit	deci	centi	milli

Examples to show students that the same prefixes are used for different types of measurement would be:

A weight measurement:
9 milligrams = nine-thousandths of a gram

A distance measurement:
9 millimeters = nine-thousandths of a meter

A liquid measurement:
9 milliliters = nine-thousandths of a liter

b. Metric weights and measurements are based on the decimal system. This should make the metric system easier to teach and use because there are no fractions or mixed numbers. Two simple math problems may help point out this advantage to students.

Example 1:

Using three pieces of masking tape of the following English measurement lengths: 4 1/8 inches, 7 6/16 inches, and 2 3/4 inches, determine the total length of the tape.

Solution:

Step 1: Find the least common denominator (16). This must be done because unequal fractions cannot be added.

Step 2: Convert all fractions to the least common denominator.

$$4\tfrac{1}{8} = 4\tfrac{2}{16}$$
$$7\tfrac{9}{16} = 7\tfrac{9}{16}$$
$$2\tfrac{3}{4} = 2\tfrac{12}{16}$$

Step 3: Add to find the sum. → $13\tfrac{23}{16} =$

Step 4: Change the sum to the nearest whole number. → $14\tfrac{7}{16}$

Example 2:

Using three pieces of masking tape of the following lengths: 85 mm, 19.4 cm, and 57 mm, determine the total length of tape.

Solution:

Millimeters and centimeters cannot be added. The conversion to all millimeters or centimeters, however, is very easy because of the decimal system. The problem can be solved in only two steps.

Step 1: Convert to all millimeters or centimeters.

Step 2: Add to find the sum.

85 mm	=	85 mm
19.4 cm	=	194 mm
57 mm	=	57 mm
		336 mm

or

8.5 cm
19.4 cm
5.7 cm
33.6 cm

336 thousandths of a meter is the same as 33.6 hundredths of a meter.

2. Metric Abbreviations

Drawings are of little value unless they contain dimensions. Since it would be time-consuming for the draftsman and would take too much space to print the words “inches” or “feet” after each dimension, symbols are used instead. In the English system of measurement the symbols for inches(") and for feet (') are used.

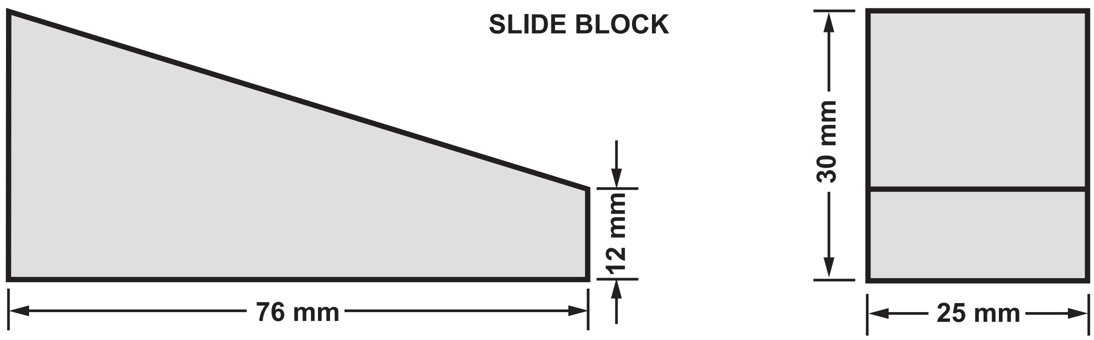
In the metric system of measurements, the words are even longer; therefore, abbreviations are used throughout. The basic unit for length, the meter, is abbreviated by using the small m.

Abbreviations for metric linear measurements are:

mm	=	millimeter	=	one-thousandth of a meter
cm	=	centimeter	=	one-hundredth of a meter
Km	=	Kilometer	=	one thousand meters

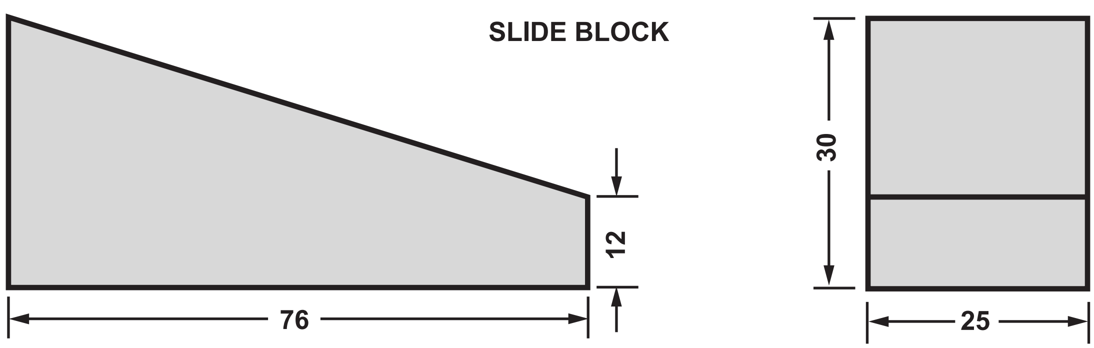
In most drawings, millimeters or centimeters are used. All rules of dimensioning are the same. A dimensioned drawing would appear as shown in Figure 1-11.

FIGURE 1-11
Dimensioned Drawing



If all of the dimensions on a drawing are in the same unit, e.g., all in millimeters, then a notation to that effect can be made on the drawings. When this is done, all abbreviations following the number are omitted. Such a dimensioned drawing would appear as in Figure 1-12.

FIGURE 1-12
Dimensioned Drawing with Note for Standard Unit Application



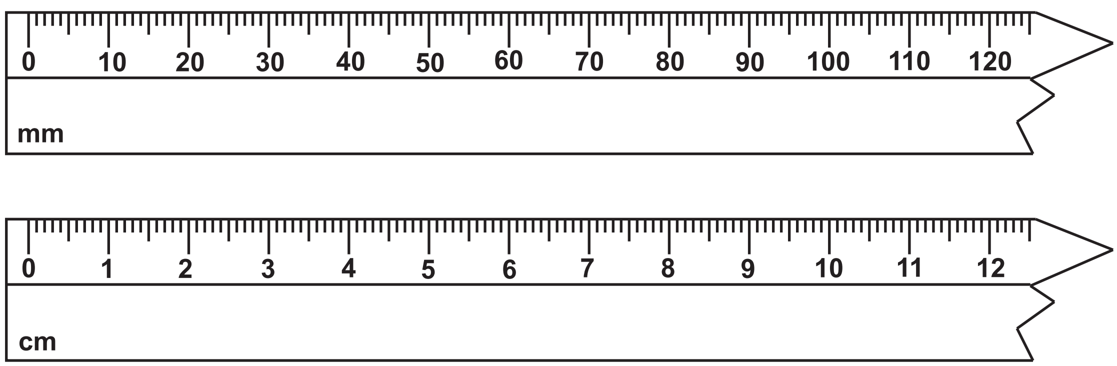
NOTE: All dimensions are in millimeters.

3. The Metric Scale

Metric measurements are based upon the decimal system. This makes measuring with a metric scale relatively easy because there are no fractions.

The metric scale is graduated (marked off) into millimeters. The short lines shown on the scales in Figure 1-13 represent millimeters. The longer lines are the centimeter marks (10 millimeters = 1 centimeter).

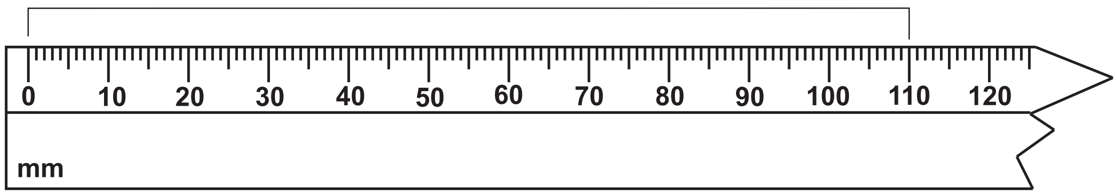
FIGURE 1-13
Metric Scales



Notice that both scales are graduated the same, but the numbering on them is different. On the top scale, the millimeters are numbered, whereas the centimeters are numbered on the bottom scale. The abbreviations mm for millimeter or cm for centimeter are always marked on the left side of the scale close to the 0 number. When using a metric scale, always look for this abbreviation to know how it is numbered.

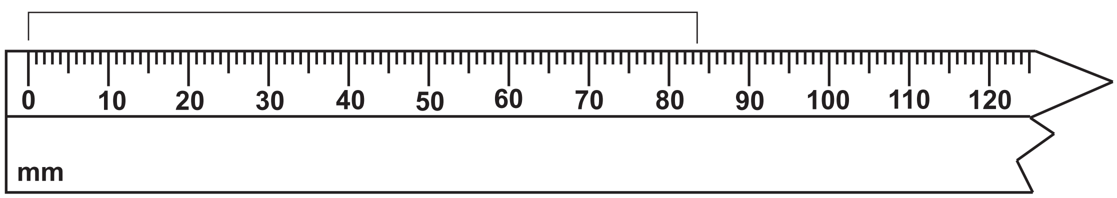
When measuring a distance with a metric scale, always place the 0 at the starting point and read the measurement at the end point. Study Figure 1-14. The line is 110 mm long (or 11 cm).

FIGURE 1-14
Application of Metric Scale



What happens when the end of the line being measured does not fall on one of the millimeter marks on the scale? Study Figure 1-15. Notice that the line ends halfway between 83 mm and 84 mm. The length of the line is 83.5mm (8.35cm).

FIGURE 1-15
Application of Metric Scale



Metric Measurement Practice Exercises

Measure the lines below and indicate their length in the space provided. Use a metric scale provided by the instructor.

Mark Answers Here

a.____________ mm

b.____________ mm

c.____________ cm

d.____________ mm

e.____________ cm

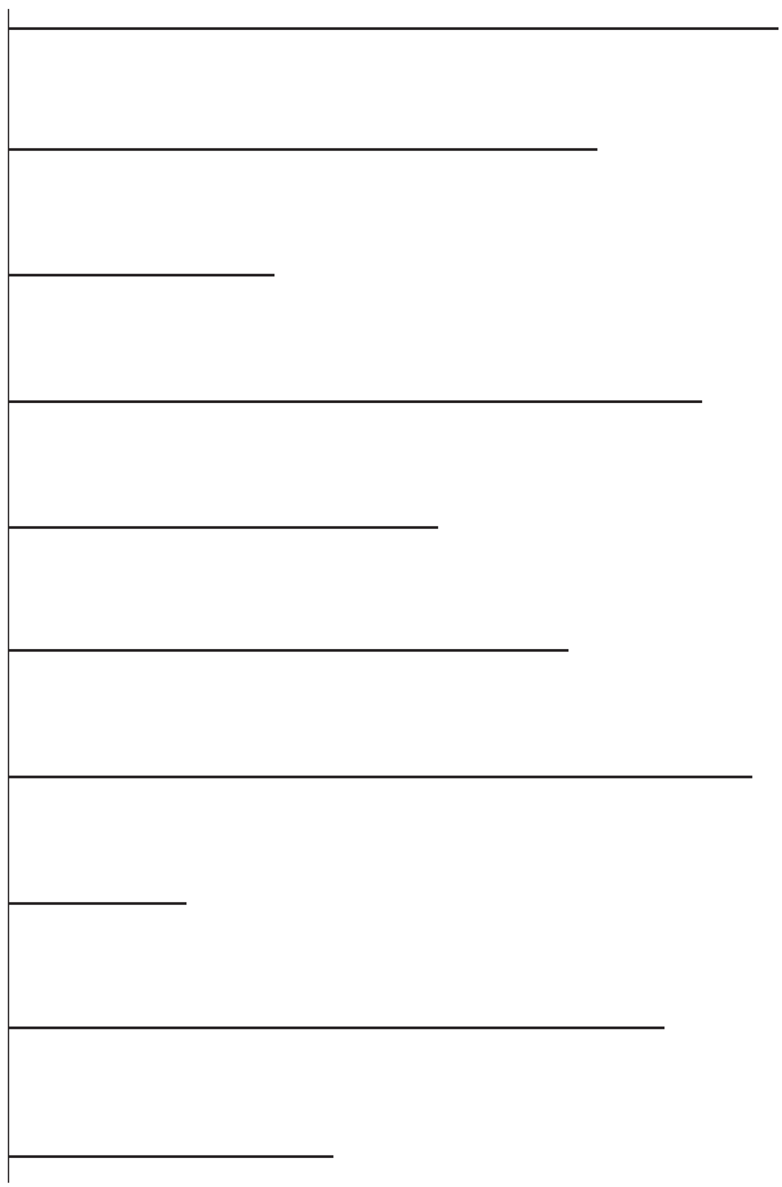
f.____________ mm

g.____________ cm

h.____________mm

i.____________ mm

j.____________ cm



4. Comparisons and Conversions

Manufacturing is a global business now and you will likely encounter the metric system in nearly every plant. It will be very useful to know how to convert from one system to the other.

Compare the following:

One Yard: About the length between your nose and the end of your right hand with your arm extended.

One meter: About the length between your left ear and the end of your right hand with your arm extended.

One centimeter: About the width of the fingernail on your pinky finger.

One Inch: About the length between the knuckle and the end of your index finger.

U.S. Customary and Metric Comparisons

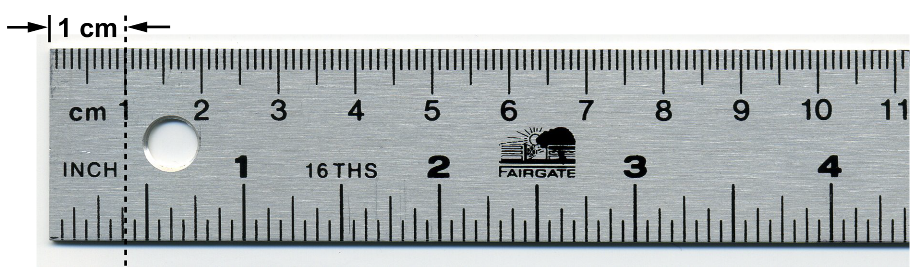
Length

A Kilometer — A little over 1/2 mile - .62 miles to be more precise.

1 mile

1 Km

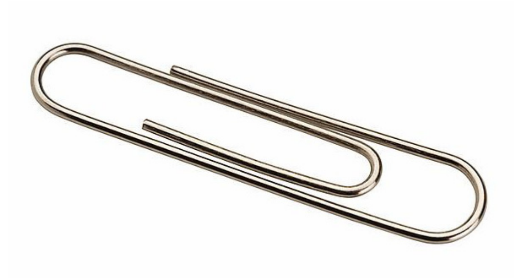
A centimeter is about 3/8 inch.



Weight

A gram

A paper clip weighs about a gram.



A nickel weighs about 5 grams.

A Kilogram

2.2 pounds-2 packages of butter plus about 1 stick.

Capacity

A liter

Everybody is familiar with a 2-liter drink which is equivalent to approximately 2 quarts. 1 liter and 1 quart are approximately the same.

A milliliter

There are 5 milliliters in a teaspoon.

Pressure is measured in newton meter (nm) rather than in foot pounds.

Equivalent Units

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths	← Place Value
Kilo	Hecto	Deka	base unit	deci	centi	milli	← Prefix

Shortcut -

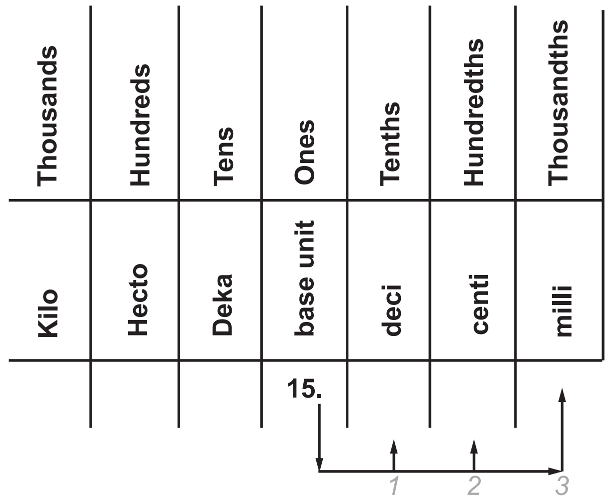
To change to a smaller unit, move the decimal to the right (multiply). →

To change to a larger unit, move the decimal to the left (divide). ←

Changing to a Smaller Unit

Example: 15 1iters = _________ milliliters (ml)

Look at the prefix chart.



Count the number of spaces from 1 to ml. There are 3 to the right. Move the decimal 3 places to the right.

15 1iters = 15.000 1iters

Move your decimal to the right 3 places.

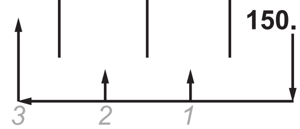
15 1iters = 15000. milliliters (ml)

Changing to a Larger Unit

Example: 150 grams (g) = ________ Kilograms (Kg)

Look at the prefix chart.

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
Kilo	Hecto	Deka	base unit	deci	centi	milli
			150.			

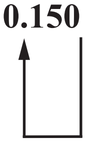


Count the number of spaces from grams to Kilo. There are 3 to the left. Move the decimal 3 places to the left.

150 grams (g) = 150. grams (g)

Move your decimal to the left 3 places.

150 g = 0.150 Kg



Comparisons and Conversions Practice Exercises

Convert the following:

1. 1 liter = __________ ml
2. 6000 ml = __________ liters
3. 10 cm = __________mm
4. 500 cm = __________ m
5. 4 Kg = __________ g
6. 55 ml = __________ liters
7. 8.5 Km = __________ m
8. 6.2 cm = __________ mm
9. 0.562 mm = __________ cm
10. 75 cm = __________ mm

5. Conversion Factors

Fractions to Decimal Equivalents

To change a common fraction to a decimal equivalent, divide the numerator by the denominator and write the quotient in decimal form. For example, 3/5 can be converted to a decimal equivalent by dividing 3.0 by 5, which equals .6.

Conversion Table for Length

Conversion charts are used to show equivalents from one standard of measurement to another. Some of the metric equivalents of U.S. standard measures for length are shown in the table below. Powers of ten are used to represent some of the values.

Conversion Table for Length

	mm	cm	meter	Km	inch	feet
1 millimeter =	1	.01	.001	.000001	25.4	.0394
1 centimeter =	10	1	10^{-2}	10^{-5}	.394	3.28×10^{-2}
1 meter =	1000	100	1	10^{-3}	39.4	3.28
1 Kilometer =	10^{6}	10^{5}	1000	1	3.94×10^{3}	3280
1 inch =	25.4	2.54	2.54×10^{-2}	2.54×10^{-5}	1	8.33×10^{-2}
1 foot =	305	30.5	.305	3.05×10^{-4}	12	1

Conversion Table for Area

Square measure is used to determine the area of a surface. Square measure involves the dimensions of length and width. A table for converting area measurements is shown on the next page.

English

1 square foot = 144 square inches
1 square yard = 9 square feet
1 square rod = 30.25 square yards

Metric

1 sq. meter = 10,000 sq. centimeters
1 sq. meter = 1,000,000 sq. millimeters
1 sq. centimeter = 100 sq. millimeters
1 sq. centimeter = .0001 sq. meter
1 sq. Kilometer = 1,000,000 sq. meters

Conversion Table for Area

	$meter^2$	cm^2	$inch^2$	$feet^2$
sq. meter =	1	10^4	10.8	1550
sq. centimeter =	10^{-4}	1	1.08×10^{-3}	0.155
sq. foot =	9.29×10^2	929	144	1
sq. inch =	6.45×10^{-4}	6.45	1	6.94×10^{-3}

Conversion of Volume

Volume measure is used to determine the total space occupied by three-dimensional objects or substances. Volume of six-sided spaces is easily calculated by multiplying **length** x **width** x **height** *(Volume of spheres and cylinders is more complicated)*. The term "cubic" is used because it is a mathematical function involving 3 factors. For example, a box that is 2 feet wide, 4 feet long, and 3 feet high has a total volume of:

2 x 4 x 3 = 24 Cubic Feet

English

1 cubic inch = 1 cubic inch
1 cubic foot = 1728 cubic inches (12 x 12 x 12)
1 cubic yard = 27 cubic feet (3 x 3 x 3)

Metric

1 cubic meter = 1,000,000 cubic centimeters (100 x 100 x 100)
so much for centimeters

Remember that 1 foot = .305 meters
and
1 meter = 3.28 feet

so, the 24 cubic foot box described above can be converted to metric by first converting the English "feet" measurements to metric:

2ft x .305m = .610m
4ft x .305m = 1.22m
3ft x .305m = .915m

Then, multiply .610 x 1.22 x .915 = .680943 cubic meters

Conversion Table for Pressure

A table for converting pressure is shown below.

Conversion Table for Pressure

	Nt./meter ^{2}	lb./in. ^{2}	lb./ft. ^{2}
1 Newton per meter	1	1.45×10^{-4}	2.09×10^{-2}
1 pound per inch	6.90×10^{3}	1	144
1 pound per foot	47.9	6.94×10^{-3}	1

Conversion Table for Weight

A table for converting weight is shown below.

Conversion Table for Weight

TO CONVERT	MULTIPLY BY	TO CONVERT	MULTIPLY BY
Grams to ounces	0.353	Ounces to grams	28.35
Grams to pounds	0.0022	Pounds to grams	453.592
Kilograms to pounds	2.2046	Pounds to kilograms	0.4536
Kilograms to tons	0.00098	Tons to kilograms	1016.05
Tonnes to tons	0.9842	Tons to tonnes	1.016

Conversion Table for Temperature

To convert between Celsius and Fahrenheit, use the following equations:

Farenheit to Celsius . . . . . **(^{O}F-32) x 5/9 = ^{O}C**

Celsius to Farenheit . . . . . **(^{O}C x 9/5) + 32 = ^{O}F**

Conversion Table for Temperature

CELSIUS ^{0}C	FARENHEIT ^{0}F
-30	-22
-20	-4.0
-10	14
0	32.0
1	33.8
2	35.6
3	37.4
4	39.2
5	41.0
6	42.8
7	44.6
8	46.4
9	48.2

Metric System Practice Exercises

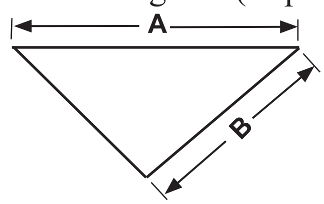
1. Which one of the following is **not** a metric measurement unit?

   a. millimeter
   b. centimeter
   c. square feet
   d. cm

2. Milli - is the prefix for which one of the following?

   a. 100 ones
   b. 0.001 unit
   c. 0.0001 unit
   d. 0.00001 unit

3. How long are lines A and B in this figure? (Express in metric units.)



4. How long is the line below? (Express in metric units.)

5. Convert the following:

   a. 1 meter = _______ millimeters

   b. 5 cm = _______ millimeters

   c. 12 mm = _______ centimeters

   d. 7 m = _______ centimeters

H. THE CALCULATOR

The electronic calculator is a tool used to solve mathematical problems. It is, like any other tool, capable of working correctly only when it is used correctly. Therefore, the user must understand its features and functions.

Calculator functions vary from manufacturer-to-manufacturer, brand-to-brand. Some calculators have only the most basic functions, such as addition, subtraction, multiplication, division, square roots and percentages. Other more advanced scientific calculators are designed for more complicated work, such as statistical analysis or other complex mathematical and engineering applications. These more advanced calculators will also perform trigonometric functions.

Solar calculators are also available. These never need batteries, and are powered either by sunlight or normal indoor lighting fixtures.

This section will deal only with the basic calculator and its functions.

Figure 1-16 shows a basic electronic calculator.

FIGURE 1-16
Calculator

1. The Basic Keys

The keys which control operations on the calculator determine the type of mathematical operation used to solve a problem. These keys and their functions are shown as follows:

ON Key: This is the ON and it turns the calculator on.

C/AC Key: This is the Clear Entry key. Pressing this key one time erases the last entry made in the calculator. Pressing it twice erases all entries.

OFF Key: This key shuts the calculator off. It should be used whenever work on the calculator is finished in order to save batteries. If your calculator is solar powered, it will not have an OFF key but will automatically shut of after several seconds on inactivity.

÷ **Key**: This key controls the **division** function.

X Key: This key controls the **multiplication** function of the calculator.

- Key: This key controls the **subtraction** function of the calculator.

\+ **Key**: This key controls the **addition** function of the calculator.

√ **Key**: This key controls the **square root** function of the calculator.

Many calculators have a memory function, giving the user the ability to store, calculate and recall numbers for further use. While the following keys may differ in name from calculator-to-calculator, every calculator with a memory will have keys that perform these functions:

M+ Key: When pressed after a numeric key or calculation the Memory Plus key adds the numbers or the result to the memory register.

M- Key: When pressed after a numeric key or calculation, the Memory

Minus key subtracts the numbers or the result from the memory register.

MR Key: The Recall Memory recalls the calculator's memory to the display screen.

MC Key: The Memory Clear key clears or erases all the contents from the memory.

% Key: The Percent Key controls the percentage functions of the calculator.

2. Calculator Functions

Solving problems by using a calculator is simply a matter of pressing the proper keys. The calculator does the rest, but it cannot come up with the correct answer if it has been given the wrong information.

In working with decimals, for example, it is important that the decimal point be placed properly within each number. It is also important that the correct numbers and function key be entered, or the answer may not be the correct one.

If at any time a wrong number is entered, the C/AC key should be pressed. This will erase the number and cause 0 to appear on the screen. Pressing the last function key entered will provide the latest total, and then the correct number can be entered. This allows corrections to be made in entries without the need for re-entering an entire calculation.

Calculators will provide a running total of the operations entered into them. As more numbers are added, subtracted, multiplied or divided, for example, a new total will appear on the display screen. It is not necessary, then, to press the = key after every operation. Pressing the last desired function key will provide the most recent total. This will be shown in the following step-by-step examples:

Addition

Add 3, 8, 9 and 14.

Step 1: Press the 3 Key.
The number 3 appears on the screen.

Step 2: Press the + Key.
The number 3 remains on the screen.

Step 3: Press the 8 Key.
The number 8 appears on the screen.

Step 4: Press the + Key.
The number 11 appears on the screen.

Step 5: Press the 9 Key.
The number 9 appears on the screen.

Step 6: Press the + Key.
The number 20 appears on the screen.

Step 7: Press the 1 and 4 Keys.
The number 14 appears on the screen.

Step 8: Press the = Key.
The number 34 appears on the screen. This is the answer.

Note that a new total was displayed each time the + key was pressed. In step 8, pressing the + key would have provided the latest total, or answer.

Calculator Addition Exercise

Use the calculator to add the following.

1)	.06783 .49160 .76841 .02134 + .87013	2)	154758 3906 4123 5434 + 76	3)	12.54 + 932.67 + 13.4

Subtraction

Subtract 25 from 187.

Step 1: Press the 1, 8 and 7 keys.
The number 187 appears on the screen.

Step 2: Press the - key.
The number 187 remains on the screen.

Step 3: Press the 2 and 5 keys.
The number 25 appears on the screen.

Step 4: Press the = key.
The number 162 appears on the screen. This is the answer.

Note that pressing the - key instead of the = key in Step 4 would have provided the latest total or answer.

Calculator Subtraction Exercise

Use the calculator to subtract the following.

1) .0543 - .0532

2) .0578 - .0463

3) 179853 - 4327

Multiplication

Multiply 342 by 174.

Step 1: Press the 3, 4 and 2 keys.
The number 342 appears on the screen.

Step 2: Press the x key.
The number 342 remains on the screen.

Step 3: Press the 1, 7 and 4 keys.
The number 174 appears on the screen.

Step 4: Press the = key.
The number 59508 appears on the screen. This is the answer.

Calculator Multiplication Exercise

Use the calculator to multiply the following.

1) 2.45 x 16

2) 60.8 x 19

3) 12.8976 x 43.7 x 12.01

Division

Divide 66 by 12.3

Step 1: Press the 6 key twice.
The number 66 appears on the screen.

Step 2: Press the ÷ key.
The number 66 remains on the screen.

Step 3: Press the 1, 2, . (decimal) and 3 keys.
The number 12.3 appears on the screen.

Step 4: Press the = key.
A number that rounds to 5.3659 appears on the screen. This is the answer.

Calculator Division Exercise

Use the calculator to divide the following.

1) .2961 ÷ 5 2) 13.5678 ÷ 11.1 3) .1765 ÷ .5

Percentages

Find 1.3% of 50.

Step 1: Press the 5 and 0 keys.
The number 50 appears on the screen.

Step 2: Press the x key.
The number 50 remains on the screen.

Step 3: Press the 1,the . and 3 keys.
The number 1.3 appears on the screen.

Step 4: Press the % key.
The number .065 appears on the screen. This is the answer.

Calculator Percentages Exercise

Use the calculator to find the following percentages.

1) Find 5% of the following numbers:
a) 150 b) 675 c) 100

2) Find 10% of the following numbers:
a) 1250 b) 871 c) 202

3) Find 26% of the following numbers:
a) 260 b) 212 c) 1817

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