Chapter 8 Beta Decay

8.1 Introduction

We have seen that many thousands of nuclei can be produced and studied in the lab. However, only less than 300 of these nuclei are stable, the rest are radioactive. We have also seen that the degree of instability grows with the "distance" a given nuclide is from the stable nuclide with the same mass number. In the previous chapter we considered the process of a decay in which heavy nuclei emit alpha particles to reduce their mass and move towards stability. The Coulomb barrier limits this process to those regions where the Q-value provides sufficient energy to tunnel through the barrier. The vast majority of unstable nuclei lie in regions in which α decay is not important and the nuclei undergo one or another form of beta decay in order to become more stable. In a certain sense, the stable nuclei have a balance between the numbers of neutrons and protons. Nuclei are said to be unstable with respect to β decay when these numbers are "out of balance." In a very qualitative way β decay "converts" a neutron into a proton (or vice versa) inside a nucleus, which becomes more stable while maintaining a constant mass number. The β decay process is more complicated than α emission and we will provide an overview and a discussion of its basic features in this chapter.

Beta decay is named for the second most ionizing rays that were found to emanate from uranium samples. The naturally occurring beta rays were identified as fast moving (negative) electrons relatively easily but it took many years to obtain a full understanding of the emission process. The difficulty lies in the fact that two particles are "created" during the decay as compared to the "disruption" of a heavy nucleus in alpha decay. In contrast to alpha decay, angular momentum plays a crucial role in understanding the process. Let us consider the simplest form of β decay to illustrate the difficulties. The proton and the neutron are the two possible isobars for A=1. We know that the neutron has a larger mass than the proton and is thus unstable with respect to the combination of a proton and an electron. A free neutron will undergo β decay with a half-life of approximately 10 minutes. We might expect to write the decay equation as:

$$^{1}_{0}n \rightarrow ^{1}_{1}p + ^{0}_{-1}e + Q(incomplete)$$

However, all three particles in this equation are fermions with intrinsic spins S=1/2 ħ. Therefore, we *cannot* balance the angular momentum in the reaction as written. The spins of the proton and the electron can be coupled to 0 or 1 ħ and can also have relative angular momenta with any integral value from the emission process. This simple spin algebra will never yield the half-integral value on the left-hand side of the equation. Another fermion must be present among the products.

Another feature of β decay that was puzzling at first but really pointed to the incompleteness of the previous equation is that the β-rays have a continuous energy distribution. That is, electrons are emitted from a source with a distribution of energies that extends from a maximum at the Q value down to zero. Recall that if there are only two products from a reaction then they will precisely share the decay energy according to conservation of momentum. We have clearly seen such sharp energy spectra in α decay. (The continuous energy distribution is not an instrumental artifact nor does it come from electron scattering.) Quite dramatic pictures of the tracks of charged particles from beta decay show events in which the particles move in one direction in clear violation of conservation of linear momentum. The way out of this mounting paradox with violations of very strongly held conservation laws is to introduce another conservation law and recognize that another unseen particle must be created and emitted. The conservation law is conservation of the number of "particles" in a reaction and the unseen particle is a form of neutrino, literally the little neutral one in Italian.

8.2 Neutrino Hypothesis

Enrico Fermi on his voyage to the new world postulated that a third particle was needed to balance the emission of the electron in β decay. However, the existing conservation laws also had to be satisfied so there were a number of constraints on the properties of this new particle. Focusing on the decay of a neutron as a specific example, the reaction is already balanced with respect to electric charge, so any additional particle must be neutral. The electrons were observed with energies up to the maximum allowed by the decay Q value so the mass of the particle must be smaller that the instrumental uncertainties. Initially this limit was <1 keV but this has been reduced to <10 eV in recent work. Recent experiments have shown that the neutrinos have mass (Chapter 12). The third constraint on the neutrino from the decay is that it must be an "antiparticle" in order to cancel or compensate for the creation of the electron, a "particle." The fourth constraint is that the neutrino must have half-integral spin and be a fermion in order to couple the total final angular momentum to the initial spin of ½ $\hbar$

Combining all of these constraints we can now rewrite the previous equation properly as:

$${}_{0}^{1}n \rightarrow {}_{1}^{1}p + {}_{-1}^{0}e + {}_{0}^{0}\overline{\nu_e} + Q$$

where we have used the notation of placing a bar over the Greek character nu to indicate that the neutrino is an antiparticle and a subscript indicating the neutrino is an electron neutrino (Chapter 1). As indicated in Chapter 1, the existence of antiparticles and antimatter extends quite generally and we produce and observe the decays of antielectrons (positrons), antiprotons, antineutrons, etc., and even combine positrons and antiprotons to make antihydrogen!

The spins of all of the final products can be combined in two ways and still couple to the initial spin of the neutron. Focusing on the spins of the created particles, they can vector couple to S_β=1 in a parallel alignment or to S_β=0 in an anti-parallel alignment. Both of these can combine with S=1/2 of the neutron for a resultant vector of 1/2. The two possible relative alignments of the "created" spins are labeled as Fermi (F) (S_β=0) and Gamow-Teller (GT) (S_β=1) decay modes after the people that initially described the mode. Both modes are very often possible and a source will produce a mixture of relative spins. In some cases, particularly the decay of even-even nuclei with N=Z (the so-called mirror nuclei), the neutron and protons are in the same orbitals so that 0+ to 0+ decay can only take place by a Fermi transition. In heavy nuclei with protons and neutrons in very different orbitals (shells) the GT mode dominates. In complex nuclei, the rate of decay will depend on the overlap of the wave functions of the ground state of the parent and the state of the daughter. The final state in the daughter depends on the decay mode. Notice that in the example of neutron decay, the difference between the two modes is solely the orientation of the spin of the bare proton relative to the spins of the other products. The decay constant can be calculated if these wave functions are known. Alternatively, the observed rate gives some indication of the quantum mechanical overlap of the initial and final state wave functions.

The general form of β^- *decay* of a heavy parent nucleus, ^{A}Z, can be written as:

$$^{A}Z_{N} \rightarrow {}^{A}(Z+1)^{+}_{N-1} + e^{-} + \overline{\nu_{e}} + Q_{\beta^{-}}$$

where we have written out the charges on the products explicitly. Notice that the electron can be combined with the positive ion to create a neutral atom (with the release of very small binding energy). This allows us to use the masses of the neutral atoms to calculate the Q value, again assuming that the mass of the antineutrino is very small. Thus,

$$Q_{\beta^-} = M\,[^{A}Z] - M\,[^{A}\,(Z + 1)]$$

Up to this point we have concentrated on the β decay process in which a neutron is converted into a proton. There are a large number of unstable nuclei that have more protons in the nucleus than the stable isobar and so will decay by converting a proton into a neutron. We can write an equation for *β^+ decay* that is exactly analogous to the previous equation.

$$^{A}Z_{N} \rightarrow {}^{A}(Z-1)^{-}_{N+1} + e^{+} + \nu_{e} + Q_{\beta^{+}}$$

where we have replaced both the electron and the electron antineutrino with their respective antiparticles, the positron and the electron neutrino. Note in this case, in contrast to β^- decay, the charge on the daughter ion is negative. This means that there is an extra electron present in the reaction compared to that with a neutral daughter atom. Thus, the Q value must reflect this difference:

$$Q_{\beta+} = M\,[^{A}Z] - (M\,[^{A}\,(Z-1)] + 2m_ec^2)$$

where m_e is the electron mass. Recall that particles and antiparticles have identical masses. This equation shows that spontaneous β^+ decay requires that the mass difference between the parent and daughter atoms be greater than $2m_ec^2$ = 1.022 MeV. Nature takes this to be an undue restriction and has found an alternative process for the conversion of a proton into a neutron (in an atomic nucleus). The process is the capture of an orbital electron by a proton in the nucleus. This process, called *electron capture*, is particularly important for heavy nuclei. The reaction is written:

$$^{A}Z_{N} \rightarrow {}^{A}(Z-1)^{+}_{N+1} + \nu_{e} + Q_{EC}$$

where all of the electrons are implicitly understood to be present on the atoms. This process also has the property that the final state has only two products so conservation of momentum will cause the neutrino to be emitted with precise energies depending on the binding energy of the captured electron and the final state of the daughter nucleus.

To summarize, there are three types of decay, all known as beta decay. They are

$$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + \beta^{-} + \overline{\nu}_{e}$$

$$^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + \beta^{+} + \nu_{e}$$

$$e^{-} + {}^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + \nu_{e}$$

indicating β^- decay of neutron rich nuclei, β^+ decay of proton rich nuclei and electron capture decay of proton rich nuclei. Neglecting the electron binding energies in computing the decay energetics, we have

$$Q_{\beta^-} = (M_P - M_D)c^2$$

$$Q_{\beta^+} = (M_P - M_D)c^2 - 2m_ec^2$$

$$Q_{EC} = (M_P - M_D)c^2$$

where M is the atomic mass of the nuclide involved and m_e is the electron mass. Typical values of Q_{β^-} are ~0.5 – 2 MeV, Q_{β^+} ~2-4 MeV and Q_{EC} ~ 0.2 – 2 MeV.

As a final point in the introduction, it is interesting to note that the analogous process of positron capture by neutron excessive nuclei should be possible in principle. However, such captures are hindered by two important facts: first, the number of positrons available for capture is vanishingly small in nature, and second, both the nucleus and the positron are positively charged and will repel one another. Compare this to the situation for electron capture in which the nucleus is surrounded by (negative) electrons that are attracted to the nucleus, of course, and the most probable position to find any s-electrons is at the nucleus (r=0).

Example of Equation balancing

Write the balanced equation for positron capture on the beta-unstable nucleus, ^{24}Na. Calculate the Q-value for this process.

On the left-hand side of the equation we assume that we have a ^{24}Na nuclide (with 11 electrons) and a single positron, which is an antilepton. The conservation rules imply that the mass number of the product will be 24, the atomic number will be Z=11 + 1, the 11 electrons will carry over, and an antilepton has to be created to conserve lepton number. Thus:

$$^{24}Na + e^{+} \rightarrow {}^{24}Mg^{+} + \overline{\nu}_e + Q_{pc}$$

We must be careful about the numbers of electrons on both sides of the equation when we calculate the Q value. If we use mass excesses rather than the masses and assume a zero-mass neutrino, then:

$$Q_{pc} = (\Delta(^{24}Na) + m_e c^2) - (\Delta(^{24}Mg) - m_e c^2)$$

or

$$Q_{pc} = (\Delta(^{24}Na) + 2m_e c^2) - \Delta(^{24}Mg)$$

$$Q_{pc} = (-8.417 + 1.022) - -13.933 = +4.494\ MeV$$

8.3 Derivation of the Spectral Shape

Beta decay is clearly a process that follows first order kinetics and the rate of decay should be described by a single decay constant. Experimentally, β decay has been observed with a huge range of half-lives, from a few milliseconds (and no shorter) to ~ 10^{16} years. This large range is reminiscent of the range for alpha decay and we should expect that the nuclear structure of the parent, ground state and the available daughter states will play important roles in determining the half-life. We should also recognize that the calculation of the rate will require a full quantum mechanical approach because the decay process involves the creation of two particles and the kinetic energy spectrum is continuous for the relativistic electron because $Q_\beta \sim m_e c^2$.

Fermi developed a quantum mechanical theory of beta decay building on the foundation of the theory for the spontaneous emission of photons by systems in excited states. At first blush these may seem unrelated but in both cases a system in a very well defined single state that has excess energy releases the energy spontaneously by the creation of a particle (or particles). The decay constant for the emission of a photon was shown in the appendix to be given by the general expression:

$$\lambda = \frac{2\pi}{\hbar}\left|\int \Psi^*_{final} V_p \Psi_{initial} d\tau\right|^2 \rho(E_f)$$

which is also called Fermi's Golden Rule. The wave functions, Ψ, represent the complete initial and final states of the *entire* system and V_p is a (very) small perturbative interaction that stimulates the transition. The form and the strength of the perturbation will have to be determined. Fermi assumed that the interaction responsible for β decay is different from the gravitational, Coulomb and nuclear forces. This interaction between the nucleons, electron and neutrino is called the *weak interaction* and a new constant expressing its strength, like e and G, can be defined. This constant, g, has the numerical value of 0.88 x 10^{-4} MeV/fm^3, which is approximately 10^{-3} of the electromagnetic force constant. .The last factor, $\rho(E_f)$, is the density of states that are available to the system after the transition and is often written as dn/dE where n is the number of states per unit energy interval. In this case the final energy is the decay Q value. The initial wave function contains only the parent nucleus, whereas the final wave function will have parts for all the resultant particles. Specifically for beta decay $\Psi_{initial} = \phi_{gs}$ (^{A}Z), the complete wave function for the parent in its ground state. The final wave function will have three parts, $\Psi^*_{final} = \phi_j^*$ (^{A}Z)ϕ^*(e)$\phi^*(\nu)$, a part for the daughter nucleus in the appropriate state j, a part for the traveling wave of the electron and a part for the corresponding traveling wave of the neutrino, all of which must be coupled so that energy is conserved.

The quantum mechanical problem can be separated into two parts, the determination of $\rho(E_f)$ and the matrix element $\left|\int \varphi^*_{final} V_p \varphi_{initial} d\tau\right|^2$, to make the calculation tractable. The determination of the density of final states, dn/dE, is done using quantum statistical mechanics. It is basically the problem of counting the number of ways the decay energy can be divided amongst the electron and the neutrino, neglecting for the moment, the recoiling daughter nucleus. Classically, the number of states of a free electron with momentum between p_e and $p_e + dp_e$ in a volume V is $\frac{V 4\pi p_e^2 dp_e}{h^3}$. (This is the volume of a spherical shell in phase space where the volume of a unit cell is h^3.) Similarly for the neutrino, the number of states of the free neutrino with momentum between p_v and p_v +dp_v in a volume V is $\frac{V4\pi p_v^2 dp_v}{h^3}$. The total number of states is the product of these two factors

$$dn = \frac{16\pi^2 V^2 p_e^2 p_v^2 dp_e dp_v}{h^6}$$

If we assume the neutrino has zero rest mass

$$p_v = \frac{T_v}{c} = \frac{Q - T_e}{c}$$

$$dp_v = \frac{dQ}{c}$$

Then, substituting, we get

$$dn = \frac{16\pi^2 V^2}{h^6 c^3}(Q - T_e)^2 p_e^2 dp_e dQ$$

$$\frac{dn}{dQ} = \frac{16\pi^2 V^2}{h^6 c^3}(Q - T_e)^2 p_e^2 dp_e$$

(One must understand this equation expresses the variation of the number of final states with changes in the Q value of the decay and does not represent differentiation with respect to a constant Q).

The electron and neutrino wave functions can be written as plane waves as

$$\phi_e(r) = Ae^{ik_e r} = \frac{1}{\sqrt{V}} e^{ik_e r}$$

$$\phi_v(r) = Be^{ik_v r} = \frac{1}{\sqrt{V}} e^{ik_v r}$$

where we have applied a normalization condition to determine the constants A and B. We can expand the exponentials for r ~ 0 (the nuclear volume) as

$$e^{ikr} = 1 + ikr + \ldots \cong 1$$

Thus

$$\phi_e(r \sim 0) \cong \frac{1}{\sqrt{V}}$$

$$\phi_\nu(r \sim 0) \cong \frac{1}{\sqrt{V}}$$

The probability of emitting an electron with a momentum p_e between p_e and dp_e becomes

$$\lambda(p_e)dp_e = \frac{1}{2\pi^3\hbar^7c^3}\left|M_{fi}\right|^2 g^2 (Q - T_e)^2 p_e^2 dp_e$$

where $\left|M_{if}\right|^2$ is a *nuclear* matrix element representing the overlap between the initial and final *nuclear* states. This matrix element must be evaluated with the detailed nuclear wave functions, for example, those available from the shell model.

Collecting all constants for a given decay, the probability of a decay as a function of the electron momentum is:

$$\lambda(p_e)dp_e = (constants)(Q - T_e)^2 p_e^2 dp_e$$

This form (even though it is mixed with a momentum part and an energy part for the electron) clearly goes to zero at p_e=0 and also at T_e=Q and has a maximum in between. The shape of this function is shown in figure 8-1. This function is often called the statistical or phase space factor for the decay.

We should be sure to note that we have made a big approximation in ignoring the charge on the emitted electron. Positively charged β-particles (positrons) will be repelled by the nucleus and shifted to higher energies while negatively charged β-particles (electrons) will be attracted and slowed down. These effects were incorporated by Fermi by using Coulomb-distorted wave functions and are contained in a spectrum distortion expression called the Fermi function, F (Z_D, p_e), where Z_D is the atomic number of the daughter nucleus. The β spectrum thus has the form:

$$\lambda(p_e)dp_e = (constants)F(Z_D, p_e)p_e^2(Q - T_e)^2 dp_e$$

The effects of the Coulomb distortion can be seen in the measured spectra from the decay of ^{64}Cu shown in figure 8-2. This odd-odd nucleus undergoes both β^- and β^+ decay to its even-even neighbors with very similar Q values.

Relaxing the restriction that the neutrino rest mass is zero, we get (Heyde)

$$\lambda(p_e)dp_e = \frac{|M_{fi}|^2}{2\pi^3\hbar^7c^3} g^2 F(Z_D, p_e)p_e^2(Q - T_e)^2\left(1 - \frac{m_\nu^2c^4}{(Q - T_e)^2}\right)^{1/2} dp_e$$

8.4 Kurie Plots

We have seen that the β spectrum has an endpoint at the Q value, but the form of equation for the spectrum does not allow us to easily identify the endpoint. Notice that with a little rearrangement this spectrum can be represented as:

$$\left(\frac{\lambda(p_e)}{p_e^2 F(Z_D, p_e)}\right)^{1/2} \propto (Q - T_e)|M_{fi}|^2$$

If the nuclear matrix element does not depend on the electron kinetic energy, as we have assumed so far, then a plot of the reduced spectral intensity, the left-hand-side, versus the electron kinetic energy will be a straight line that intercepts the abscissa at the Q-value. Such a graph is called a Kurie plot and an example is shown in figure 8-3. This procedure applies to allowed transitions (see below). There are correction terms that need to be taken into account for forbidden transitions.

8.5 Beta Decay Rate Constant

The differential form of the spectrum can be integrated over all electron momenta to obtain the total decay constant. The expression, for a constant nuclear matrix element, to be integrated is:

$$\lambda = \frac{g^2 |M_{fi}|^2}{2\pi^3 \hbar^7 c^3} \int_0^{p_{\max}} F(Z_D, p_e) p_e^2 (Q - T_e)^2 \, dp$$

Note that an appropriate relativistic substitution for T in terms of the momentum is still needed. This integral has been shown to only depend on the atomic number of the daughter and the maximum electron momentum. The integral, called the Fermi integral, f (Z_D, Q), is complicated but numerical expressions or tables of the solutions are available. Note that the differential *Fermi function*, F (Z_D, p_e), contains the momentum and the *Fermi integral*, f (Z_D, Q), contains the Q value. The Fermi integral is a constant for a given beta decay and has been presented in many forms. For example, curves of the Fermi function are shown in figure 8-4.

The decay constant is now reduced to an expression with the nuclear matrix element, M ($\equiv |M_{fi}|$), and the strength parameter, g, written:

$$\lambda = \frac{g^2 |M|^2 m_e^5 c^4}{2\pi^3 \hbar^7} f(Z_D, Q)$$

or in terms of the half-life of the parent, $t_{1/2}$

$$f t_{1/2} = \ln 2 \frac{2\pi^3 \hbar^7}{g^2 |M|^2 m_e^5 c^4} \propto \frac{1}{g^2 |M|^2}$$

The left hand side of this equation is called the comparative half-life, or "ft value" because this value can be readily measured in experiments and should only depend on the nuclear matrix element and the beta decay strength constant. Recall that β decay half-lives span many orders of magnitude so the ft values will span a similarly large range. It is therefore convenient to use the common logarithm of the ft value (with $t_{1/2}$ in seconds) to characterize observed β decays.

Values of log ft may be calculated from the nomograph and curves in figure 8-4, which are due to Moszkowski (Phys. Rev. **82**, 35 (1951). Log ft values can be calculated for β^-, β^+ and EC decay. These ft values fall into groups which can be correlated with the spin and parity change in the decay (see below) and can, then, be used to assign spins and parities in nuclei whose structure is not known.

Example of ft Values

Using the graph of the Fermi integral in fig. 8-4, estimate the log ft value for the decay of ^{32}P ($t_{1/2}$=14.28 d).

1) This is a neutron rich nucleus and undergoes …- decay, thus:

$$Q_{\beta-} = M(^{32}P) - M(^{32}S) = \Delta(^{32}P) - \Delta(^{32}S)$$

$$Q_{\beta-} = (-24.305) - (-26.015)\ MeV = +1.71\ MeV$$

2) From the figure, Z=15, Q=1.71 MeV, $\log(ft)=\log(f_0 t)+\log(C)$ =7.8 +0.2 = 8.0

The creation of relative angular momentum in beta decay is even more difficult than that in alpha decay and causes more severe "hindrance" for each unit of relative angular momentum. The difficulty is easy to see with a simple calculation. We can write the relative angular momentum for two bodies as the *cross product* L= r x p where r is the radius of emission and p is the momentum. Taking a typical nuclear radius of 5 fm and a typical beta decay energy of 1 MeV, we find the maximum of the cross product to be L = 5 fm (1.4 MeV/c) = 7.90 MeV fm/c or 0.035 $\hbar$ units. Log ft values increase by an average of 3.5 units for each unit, of orbital angular momentum or degree of forbiddenness. Such an increase in the lifetime indicates a hindrance of ~3 x 10^{-4} for each unit of angular momentum. There is a large spread in the values, however, due to the strong effect of the nuclear overlap on the decay.

The quantum mechanical selection rules for beta decay with no relative angular momentum in the exit channel (ℓ=0) are ΔI=0,1 and $\Delta\pi$=0. The two values for the spin change come directly from the two possible couplings of the spins of the electron and neutrino. Some representative "allowed" beta decays are described in Table 8-1 along with their ft values and the character of the decay.

Table 1: Representative allowed beta decays

Parent	Daughter	Half life (sec)	Q_β(MeV)	log ft	Character
^{6}He (0+)	^{6}Li(1+, gs)	0.808	3.5097	2.42	Gamow-Teller
^{14}O(0+)	^{14}N(0+, 2.313)	71.1	1.180	2.81	Fermi
n (1/2+)	p (1/2+)	612	0.7824	-0.27	mixed
^{14}O(0+)	^{14}N(1+, gs)	1.16x 10^{4}	4.123	7.36	Gamow-Teller

The decay of ^{14}O to the 0+ excited state of ^{14}N can only take place by a Fermi decay where the created spins couple to zero. This parent nucleus also has a weak branch to the 1+ ground state that takes place by a Gamow-Teller transition. In contrast, the decay of ^{6}He to the ground state of ^{6}Li must take place by a Gamow-Teller transition in order to couple the total resultant angular momentum to zero. As mentioned earlier, the decay of the neutron into a proton can take place with no change in angular momentum between the spin 1/2 particles and the angular momentum coupling rules allow both decay modes.

The decay of the neutron into the proton is an important example of decay between mirror nuclei. In the β decay of mirror nuclei, the transformed nucleons (neutron → proton or proton → neutron) must be in the same shell and have very similar wave functions. This gives rise to a large matrix element $|M_{fi}|^2$ and a very small log ft value. For the beta decay of mirror nuclei to their partners, log ft values are about 3, which is unusually small. Such transitions are called *super-allowed* transitions.

When the initial and final states in beta decay have opposite parities, decay by an "allowed" transition cannot occur. However such decays can occur, albeit with reduced probability compared to the allowed transition. Such transitions are called "forbidden" transitions even though they do occur. The forbidden transitions can be classified by the spin and parity changes (and the corresponding observed values of log ft) as in Table 8-2

Table 8-2. Classifications of β-decay Transitions

Transition Type	Log ft	L_β	Δπ	Fermi ΔJ	Gamow-Teller ΔJ
Superallowed	2.9 – 3.7	0	No	0	0
Allowed	4.4 – 6.0	0	No	0	0,1
First forbidden	6 – 10	1	Yes	0,1	0,1,2
Second forbidden	10 – 13	2	No	1,2	1,2,3
Third forbidden	> 15	3	Yes	2,3	2,3,4

Remember that in β decay,

$$\vec{J}_P = \vec{J}_D + \vec{L}_\beta + \vec{S}_\beta$$

$$\pi_P = \pi_D (-1)^{L_\beta}$$

where the subscripts P, D refer to the parent and daughter, $L_{...}$ is the orbital angular momentum carried away by the emitted electron and S_β is the coupled spin of the electron-neutrino pair (S_β = 0 for a Fermi transition and S_β = 1 for a Gamow-Teller transition).

8.6 Electron Capture Decay

When the decay energy is less than 1.02 MeV ($2m_ec^2$) the β decay of a proton-rich nucleus to its daughter must take place by electron capture (EC). For decay energies greater than 1.02 MeV, EC and β^+ decay compete. In EC decay, only one particle, the neutrino, is emitted with an energy M_Pc^2-M_Dc^2-B_e where B_e is the binding energy of the captured electron. The decay constant for electron capture can be written, assuming a zero neutrino rest mass, as

$$\lambda_{EC} = \frac{g^2 |M_{fi}|^2 T_\nu^2}{2\pi^2 c^3 \hbar^3} |\varphi_K(0)|^2$$

where we have assumed that the capture of a 1s (K) electron will occur because the elecdtron density at the nucleus is the greatest for the K electrons. The K electron wave function can be written as

$$\varphi_K(0) = \frac{1}{\sqrt{\pi}} \left(\frac{Zm_e e^2}{4\pi\varepsilon_0 \hbar^2} \right)^{3/2}$$

Thus

$$\lambda_{K-EC} = \frac{g^2 Z^3 |M_{fi}|^2 T_\nu^2}{cons\tan ts}$$

Comparison of the decay constants for EC and β^+ decay shows

$$\frac{\lambda_K}{\lambda_{\beta^+}} = cons\tan ts \frac{Z^3 T_\nu^2}{f(Z_D, Q)}$$

Thus EC decay is favored for high Z nuclei. Of course, the decay energy must be greater than 1.02 MeV for β^+ decay, a situation found mostly in low Z nuclei where the slope of the walls of the valley of β stability is large (see Figure 2.8) and decay energies of > 1.02 MeV occue.

Electron capture decay produces a vacancy in the atomic electron shells and secondary processes that lead to filling that vacancy by the emission of x-rays and Auger electrons occur. These x-rays permit the detection of EC decays.

8.7 Parity Nonconservation

In Chapter 1, we introduced the concept of parity, the response of the wave function to an operation in which the signs of the spatial coordinates were reversed. As we indicated in our discussion of α-decay, parity conservation forms an important selection rule for α-decay. Emission of an α-particle of orbital angular momentum ℓ carries a parity change $(-1)^\ell$ so that $1^+ \rightarrow 0^+$ or $2^- \rightarrow 0^+$ α-decays are forbidden. In general, we find that parity is conserved in strong and electromagnetic interactions.

In the late 1950s, it was found (Wu et al.) that parity was not conserved in weak interaction processes such as nuclear beta decay. Wu et al. measured the spatial distribution of the β^- particles emitted in the decay of a set of polarized ^{60}Co nuclei (Fig. 8-6). When the nuclei decay, the intensity of electrons emitted in two directions, I_1 and I_2, was measured. As shown in

Figure 8-6, application of the parity operator will not change the direction of the nuclear spins, but will reverse the electron momenta and intensities, I_1 and I_2. If parity is conserved, we should not be able to tell the difference between the "normal" and "parity reversed" situations, *i.e.*, $I_1 = I_2$. Wu et al. found that $I_1 \neq I_2$, *i.e.*, that the β-particles were preferentially emitted along the direction opposite to the ^{60}Co spin. (God is "left-handed")

8.8 Neutrinos

A number of studies have been undertaken of the interaction of neutrinos with nuclei, to determine the neutrino mass, and to show that neutrinos and antineutrinos are produced in β^+ and β^- decay, respectively. Neutrinos also provide important information about stellar nuclear reactions because they have a very low probability for interacting with matter and come directly out from the stellar interior.

Starting with the simple equation for the β^- decay of the neutron and the β^+ decay of the proton, we can write two closely related reactions that are induced by neutrinos:

$$\bar{\nu}_e + p \rightarrow n + e^+$$

$$\nu_e + n \rightarrow p + e^-$$

These reactions, called inverse beta decay, were obtained by adding the antiparticle of the electron in the normal beta decay equation to both sides of the reaction. When we did this we also canceled (or annihilated) the antiparticle/particle pair. Notice that other neutrino induced reactions such as $\bar{\nu}_e + n \rightarrow p + e^-$ do not conserve lepton number because an antilepton, $\bar{\nu}_e$, is converted into a *lepton*, e^-. Proving that this reaction does not take place, for example, would show that there is a difference between neutrinos and antineutrinos. One difficulty with studying these reactions is that the cross sections are extremely small, of order 10^{-19} barns, compared to typical nuclear reaction cross sections, of order 1 barn (10^{-24} cm^2).

The combination of two studies of inverse beta decay clearly showed that the neutrinos emitted in β^- and β^+ decay were different. Both used nuclear reactors to provide strong sources of antineutrinos. Recall that nuclear fission produces very neutron-rich products that undergo a series of rapid beta decays emitting antineutrinos. In the first experiment, performed by Reines and Cowen, a large volume of liquid scintillator was irradiated and protons in the organic solution were reacted into a neutron and a positron. The positron was rapidly annihilated with an electron providing the first signal of an interaction. The neutron was captured within a few microseconds by Cd nuclei that were added to the scintillator and provided a second correlated signal. The flux of neutrinos from the reactor was sufficient to produce a few events per hour in a 1 m^3 volume of scintillator.

In the second study, Ray Davis and coworkers, irradiated a large volume of liquid $CC1_4$ with antineutrinos from a reactor. The putative reaction, $\bar{\nu}_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^-$, could be detected by periodic purging of the liquid, collection of the noble gas, and then detection of the induced activity (^{37}Ar is unstable, of course). The reaction was not observed to occur. Thus, they concluded that the reactor emits antineutrinos and that lepton number is conserved in the reactions.

Antineutrino Flux

Estimate the flux of antineutrinos from an operating nuclear power reactor. For this estimate assume the power plant produces 1 GW of thermal power, that fission produces 200 MeV per event and that there are approximately 6 rapid β decays per fission.

There is one antineutrino per β^- decay, of course, so this is really a problem in dimensional analysis:

$$Rate = 1\ GW\ (10^6\ J/s)/GW)(1\ fission\ /200\ MeV)(1\ MeV/1.602x10^{-13})(6\ \bar{\nu}_e/fission)$$

$$Rate = 2 \times 10^{17} antineutrinos/sec$$

8.8 Beta-delayed Radioactivities

The central feature of β-decay is that, for example in the β^- direction, the decay converts a neutron into a proton at a constant mass number. This conversion will clearly change the number of pairs of like nucleons in the nucleus and we have already seen that unpaired nucleons influence the overall stability. Beta decay in even mass chains will convert odd-odd nuclei into the even-even isobar with potentially large Q-values due to a gain of twice the pairing energy. The large Q-values lead to high-energy beta-particles and rapid decays but the relative stability of the daughter may be less than that of the parent. The large Q-values also allow the population of higher lying states in the daughter. If the nuclei are far from the (most) stable isobar, the decay may have sufficient energy to populate states in the daughter that are above the binding energy.

^{90}Sr provides an example of a change in relative stability following beta decay. This even-even parent is an important fission product that has a 29-year half-life. It decays to the odd-odd ^{90}Y, which decays to the stable isobar ^{90}Zr with a half-life of only 64 hours. Thus, a pure preparation of ^{90}Sr will come into equilibrium with its daughter after about a week and the observed activity will be the sum of the two decays. A chemical separation can be used to strip out the daughter activity. The daughter will decay away in the separated sample and will grow back into the parent sample. There are several examples of these parent/daughter pairs that provide convenient sources of short-lived activities. For example, the 66 hour ^{99}Mo decays predominantly to a 6 hour excited state in ^{99}Tc because the decay to ground state would require a very large spin change. The daughter, ^{99}Tc^{m} is used extensively in nuclear medicine.

The natural decay chains have several examples of short, lived alpha activities that are "delayed" by a longer-lived parent. In fact, the existence of these activities on earth is possible by the fact that the "head" of the chain has a half-life on the order of the age of the earth. Another more practical example near the end of the 4n chain is ^{212}Pb with a half-life of 10.6 hours that decays to ^{212}Bi. The daughter rapidly decays by alpha or beta emission. The lead nucleus is also preceded by a short-lived Rn parent, which can produce very thin sources of alpha particles by emanation.

The beta decay of nuclei far from the bottom of the valley of beta stability can feed unbound states and lead to direct nucleon emission. This process was first recognized during the discovery of fission by the fact that virtually all the neutrons are emitted promptly but on the order of one percent are delayed with respect to the fission event. These delayed neutrons play a very important role in the control of nuclear reactors. The fission products are very neutron-rich and have large beta-decay energies. For example, ^{87}Br is produced in nuclear fission and decays with a half-life of 55 seconds to ^{87}Kr with a Q-value of 6.5 MeV The decay populates some high lying states in the krypton daughter, notice that ^{87}Kr has 51 neutrons, one more than the magic number 50, and the neutron separation energy of 5.1 MeV is less than the Q-value. Thus, any states that lie above the neutron separation energy will be able to rapidly emit a neutron and form ^{86}Kr.

Example of Beta-delayed Neutron Emission

An important delayed neutron emitter in nuclear fission is ^{137}I. This nuclide decays with a half-life of 25 seconds and emits neutrons with an average energy of 0.56 MeV and a total probability of approximately 6%. Estimate the energy of an excited state in ^{137}Xe that would emit a 0.56 MeV neutron. First obtain the Q-value for the neutron emission reaction. This is the minimum amount of energy necessary to `unbind' the 83rd neutron and should be negative, of course.

$$^{137}Xe \rightarrow n + {}^{136}Xe + Q_n$$
$$Q_n = \Delta(^{137}Xe) - [\Delta(^{136}Xe) + \Delta(n)]$$
$$Q_n = -82.218 - [-86.425 + 8.0714] = -3.86 \text{ MeV}$$

The average energy of the excited state will be Q_n plus the kinetic energies of the particles, that is the neutron plus the energy of the recoil. In this case the recoil energy is very small and could have been ignored. The recoil energy is obtained by conservation of momentum in the two-body decay.

$$E^* = -Q_n + T_n + T_n\,(1/137) = 3.86 + 0.57 = 4.43 \text{ MeV}$$

Now as a check, obtain the Q-value for the beta decay and verify that it is more than the excitation energy.

$$^{137}I \rightarrow {}^{137}Xe + \bar{\nu} + Q_\beta$$

$$Q_{beta} = \Delta(^{137}I) - \Delta(^{137}Xe) = -76.72 - -82.21 = 5.49 \text{ MeV}$$

The population of high lying unbound states by beta decay is an important feature of nuclei near the drip lines. Beta-delayed proton emission and beta-delayed neutron emission have been studied extensively and provide important insight into the structure of exotic nuclei.

8.9 Double Beta-decay

The periodic variation of the mass surface caused the pairing energy also causes a large number of even-even nuclei to be unstable with respect to two successive beta decays. This process is called double beta decay and extensive searches have been carried out for it. The difficulty is that the probability of a double transition is extremely low. A gross estimate can be made by squaring the rate constant obtained above, and the number of decays from even large samples is at best one per day and at worst a few per year.

Two reactions have been studied as possible candidates for double beta decay. The first reaction is simply two times the normal beta decay process:

$$^{A}Z \rightarrow {}^{A}(Z-2) + 2e^{-} + 2\bar{\nu}$$

and thus follows the conservation laws. A second, more exotic reaction has been proposed as a test of weak interaction theory and proceeds without creation of neutrinos:

$$^{A}Z \rightarrow {}^{A}(Z-2) + 2e^{-}$$

Instrumental searches for this latter neutrino less process have been made but there is no strong evidence for its existence. The former two-neutrino decay has been observed with a variety of techniques that were carefully tuned to detect the rare products.

As an example of the process, the ^{86}Kr nucleus just mentioned above in delayed neutron emission is stable with respect to single β^- decay to ^{86}Rb having a Q-value of -0.526 MeV However, ^{86}Kr is unstable with respect to the double-beta decay to ^{86}Sr as it has a Q-value of 1.249 MeV. In this case decay to the intermediate state is energetically forbidden and only the simultaneous emission of two beta particles can take place. To obtain the gross estimate we can rewrite the expression for the decay constant:

$$\lambda = \left(\frac{m_e c^2}{\hbar}\right)\left(\frac{|\mathrm{M}|^2 m_e^4 c^2}{2\pi^3 \hbar^6} g^2 f(Z_D, Q)\right)$$

The first term is 8×10^{20}/sec and the second term reflects the details of the decay. Using $|\mathrm{M}| = \sqrt{2}$ for the decay from the 0+ ground state, to the 0+ ground state of the daughter, the second term is 1.5×10^{-25} f. For this case, log (f)~1.5, then taking the first term times the square of the second for double beta decay, we get $\lambda \sim 10^{-26}$ /sec or, $\sim 10^{-19}$ per year! Given a mole of this gas has $\sim 10^{24}$ atoms, we expect about one decay per day in the entire sample.

The techniques used to observe double beta decay fall into three general categories, geochemical, radiochemical, and instrumental. The geochemical studies rely on assumptions that are similar to those used in geochemical dating (see Chapt.3). A sample of an ore containing the parent nuclide is processed; the daughter atoms are chemically extracted and then assayed, for example with a mass spectrometer. The number of daughter atoms is then compared to the number of parent atoms and with an estimate of the life-time of the ore, the double beta decay half-life can be calculated. Difficulties with this technique are discussed in the Chap. 3. The radiochemical searches for double beta decay relied on chemically separating and identifying a radioactive daughter of the process. Such cases are relatively rare but the decay $^{238}U \rightarrow ^{238}Pu$ was observed by chemically separating a uranium ore and observing the characteristic alpha decay of the plutonium isotope. The successful instrumental searches for double beta decay have used time-projection chambers in which sample of the parent were introduced into the active volume of the detector. The tracks of the two coincident beta particles can be observed providing a clear signal for the exotic process.

References

Some useful general summaries include:

K.S. Krane, Introductory Nuclear Physics (Wiley, New York, 1988)

W.E. Meyerhof, Elements of Nuclear Physics (McGraw-Hill, New York, 1967).

R.D. Evans, The Atomic Nucleus (McGraw-Hill, New York, 1956)

J.R. Lamarsh, Introduction to Nuclear Reactor Theory (Addison-Wesley, Reading, 1967)

M. Moe and P. Vogel, Ann. Rev. Nucl. Sci. **44**, 247 (1994).

Some advanced discussions include:

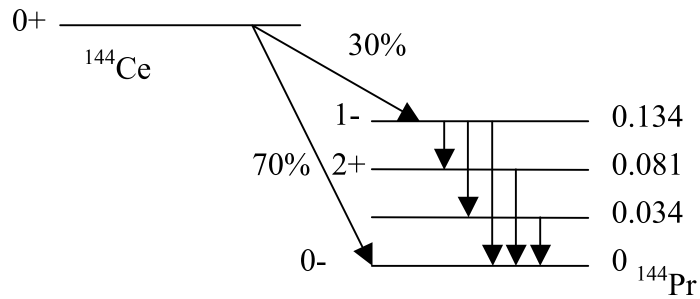
C.S. Wu and S. A. Moszkowski, Beta Decay (Wiley, New York, 1966).

K. Siegbahn, Alpha, Beta and Gamma Ray Spectroscopy (North-Holland, Amsterdam, 1966).

C.S. Wu et al., Phys. Rev. **105**, 1413 (1957).

Problems

1. The β^- -decay of ^{144}Ce is shown below.



(a) What log ft value should we expect for the β-decay to the 1^- state of ^{141}Pr? (b) Why is there no β decay to the 2+ level?

2.Sketch quantitatively the shape of the neutrino energy spectrum for the following types of decay. Label all axes carefully and indicate the types of neutrinos involved.

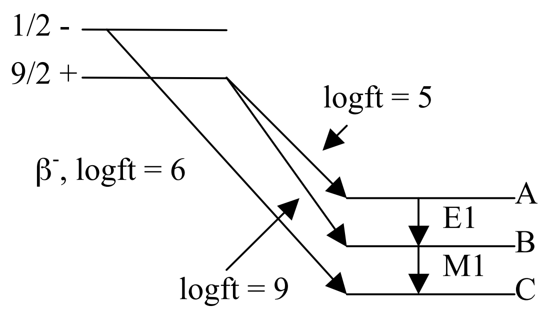
(a) The electron capture decay of ^{207}Bi, Q_{EC} = 2.40 MeV.

(b) The β^+ decay of ^{22}Na, Q_β = 2.842 MeV

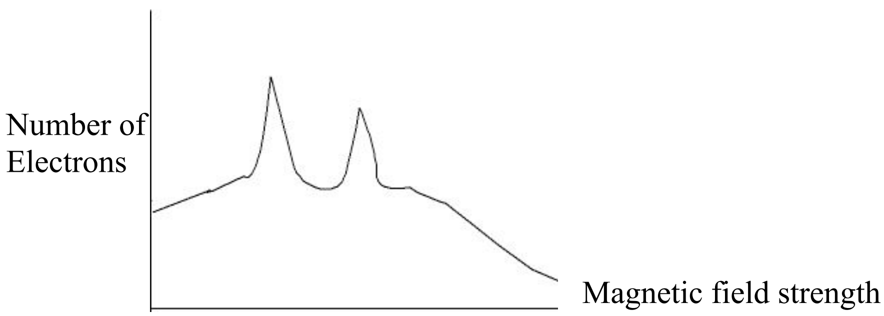
(c) The β^- decay of ^{14}C, Q_β = 0.156 MeV.

3. Suppose a state in a Bi isotope decays by EC to the 2+ state of an e-e Pb nucleus in which the three lowest states are the 0+. 2+, and 4+, with E_{EC} =1.0 MeV. Assume Q_{EC} = 4 MeV, $t_{1/2}$ = 4 sec. Calculate Jπ for the initial state of the Bi nucleus

4. Given the β decay scheme shown below for the decay of a pair of isomers to three excited states A, B. and C of the daughter nucleus. List the spins and parities of the three levels A. B. and C.



5. The results of some measurements with a beta ray spectrometer of the radiation coming from a given radionuclide are shown below.



The two sharp peaks were labeled K and L by the experimenter. Explain what the labels K and L mean. Which peak is the K peak? Why?

6. A 1- excited state of a Lu isotope decays to a 0+ state of a Yb isotope with a maximum β^+ energy of 4.6 MeV. Estimate $t_{½}$ for the transition. Do not neglect electron capture.

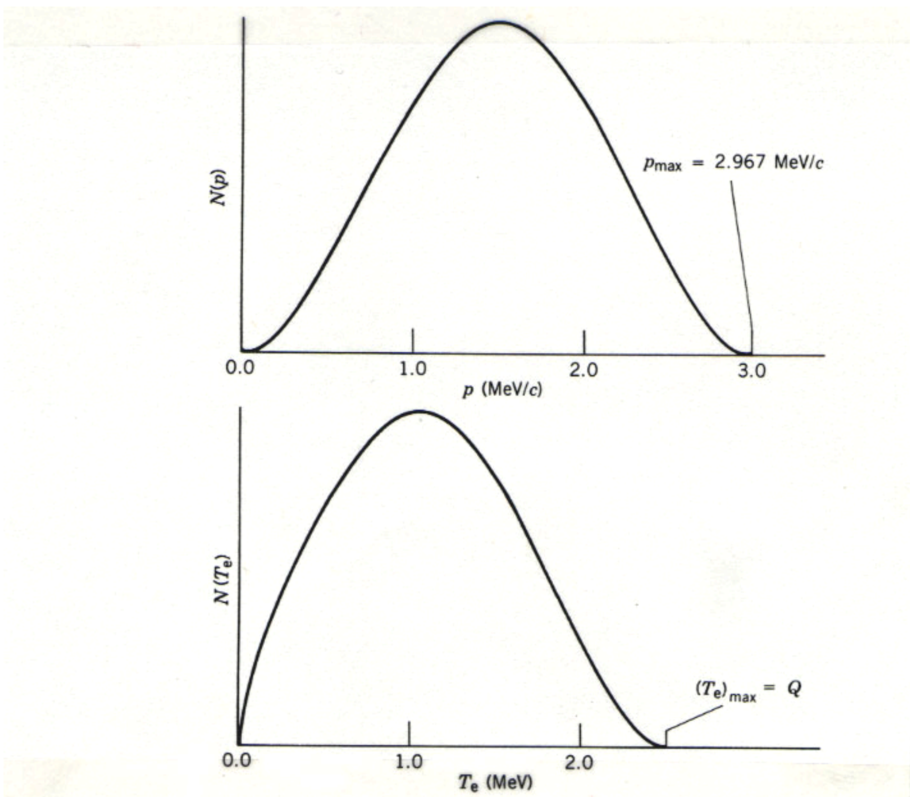


Figure 8-1 The shape of the statistical factor for beta decay, which represents the expected shape of the electron momentum distribution before distortion by the Coulomb potential.

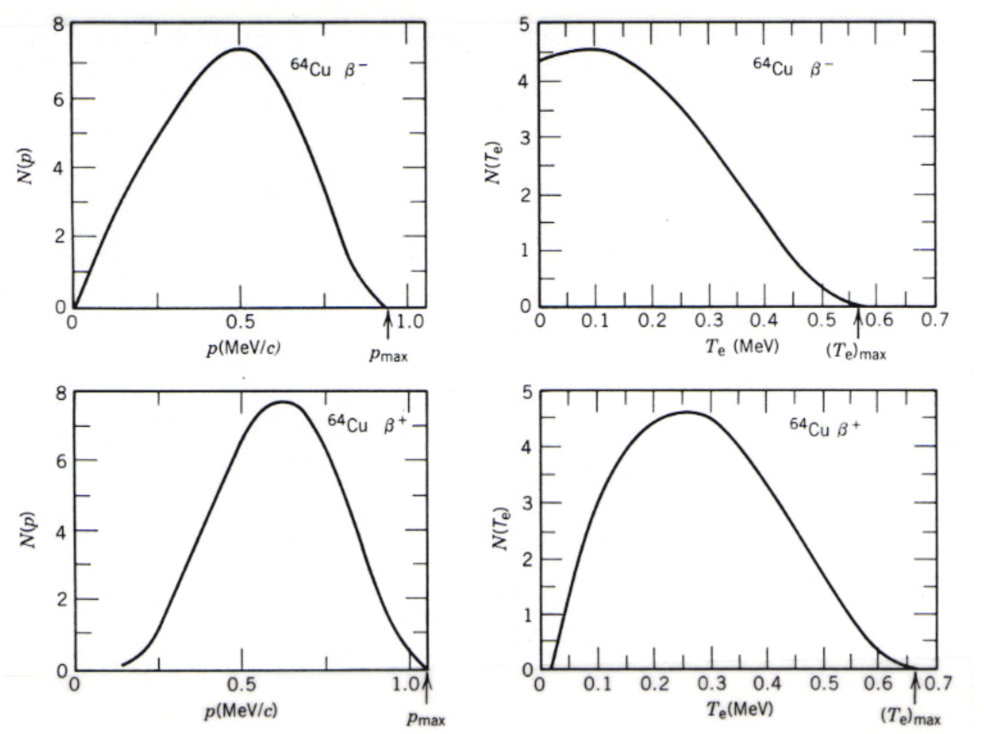


Figure 8-2 The momentum and energy spectra from the decay of ^{64}Cu for β^- and β^+ decay. The Q values for these decays are 0.5782 and 0.6529 MeV, respectively.

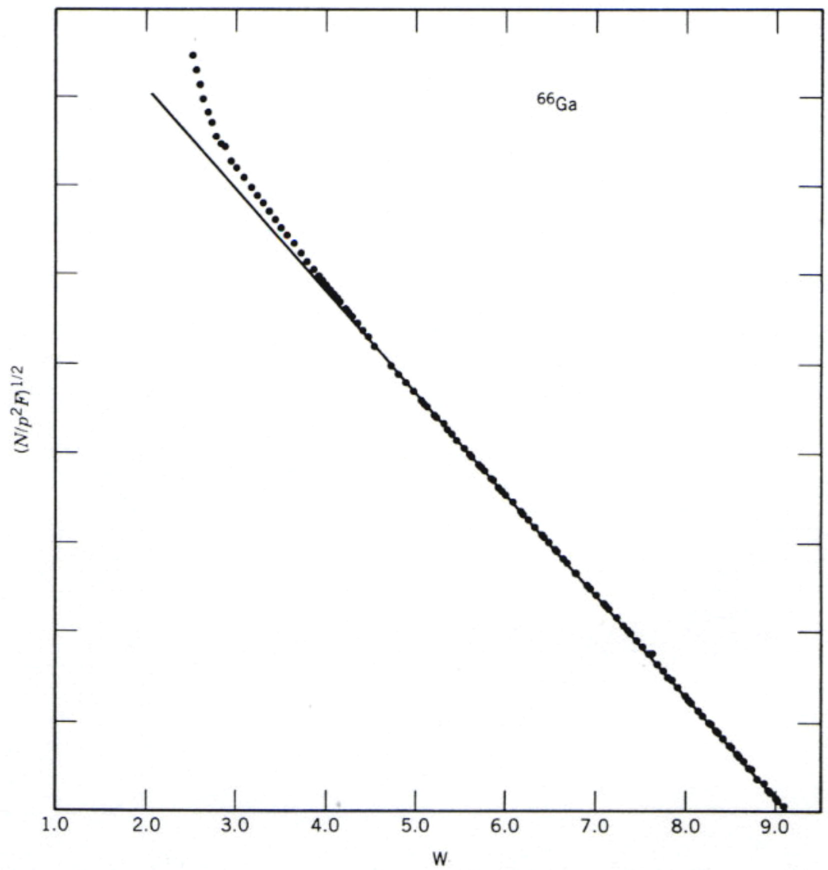
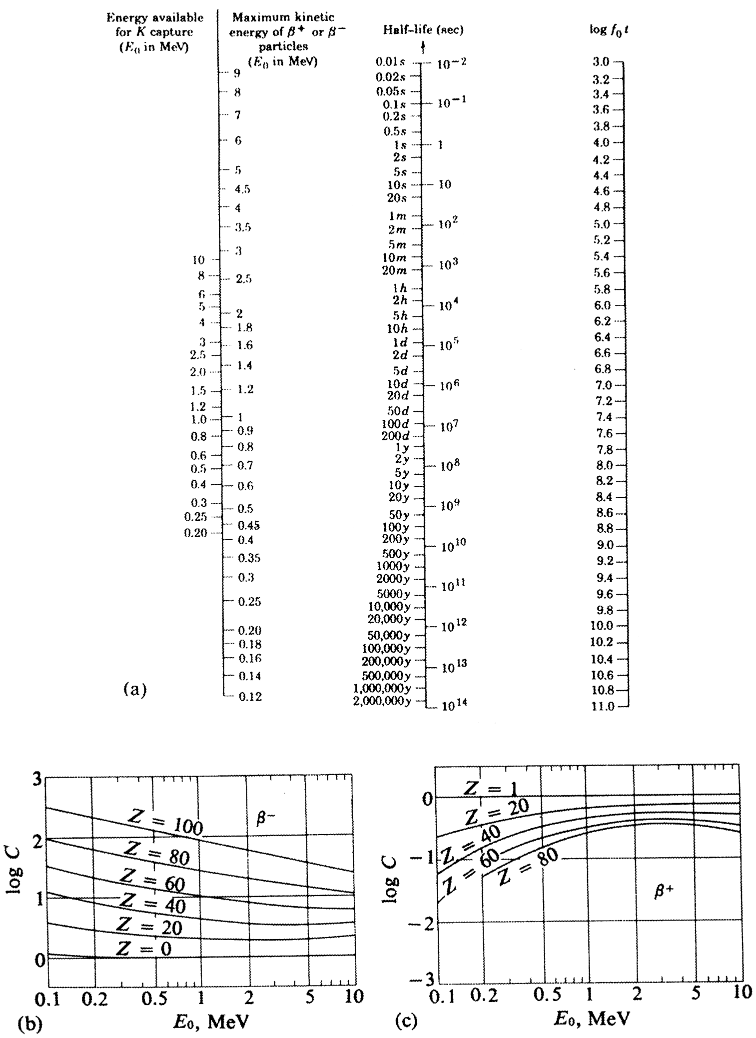


Figure 8-3. An example of a Kurie plot. (From D.C. Camp and L.M. Langer, Phys. Rev. **129**


Energy available for K capture (E_0 in MeV)
Maximum kinetic energy of β^+ or β^- particles (E_0 in MeV)
Half-life (sec)
log $f_0 t$
(a)
log C
Z = 100
Z = 80
Z = 60
Z = 40
Z = 20
Z = 0
β^-
E_0, MeV
(b)
Z = 1
Z = 20
Z = 40
Z = 60
Z = 80
β^+
(c)

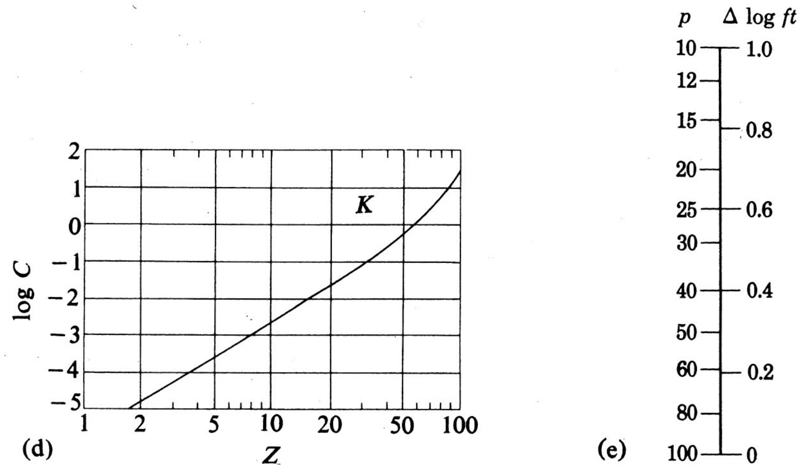


Figure 8-4 Rapid method for determining $\log_{10}(ft)$ values (From S.A. Moszowski, Phys. Rev. **82**, 35 (1951).

The above figures permit the rapid calculation of log (ft) for a given type of decay, given energy, branching ratio, etc. Notation: E_0 for β^- emission is the maximum kinetic energy of the particles in MeV; E_0 for K electron capture is the Q value in MeV. When a β^+ emission and K electron capture go from and to the same level, E_0 for the K capture = E_0 for β^+ emission + 1.02 MeV.. Z is the atomic number of the parent, t is the total half life and p is the percentage of decay occurring in the mode under consideration. When no branching occurs, p = 100. To obtain log (ft), obtain log (f_0t) using part (a). Read off log (c) from parts (b), (c), and (d) for β^-, β^+, and K EC, respectively. Get Δ log(ft0 from part (e) if p < 100. For p = 100, Δ log (ft0 =0. $\log(ft)=\log(f_0t)+\log(C)+\Delta \log(ft)$.

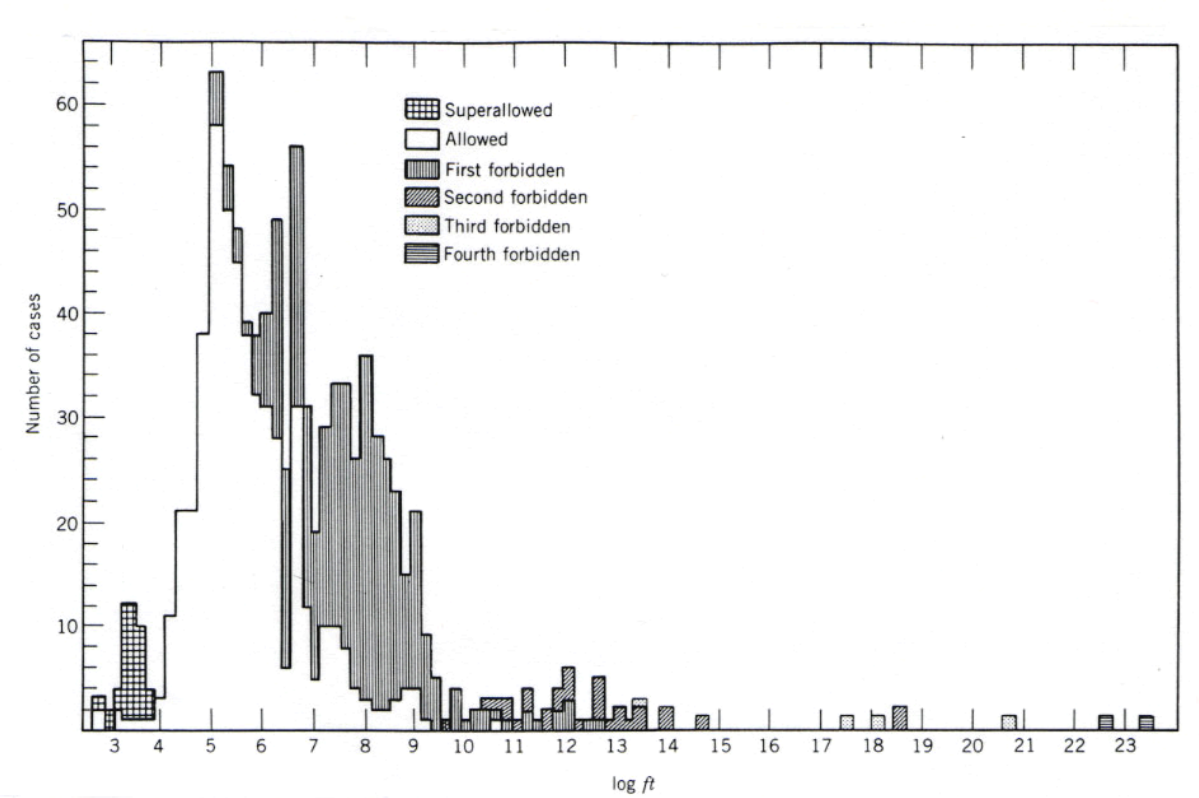


Figure 8-5. Systematics of experimental log ft values. From W. Meyerhof, Elements of Nuclear Physics (New York: McGraw-Hill, 1967).

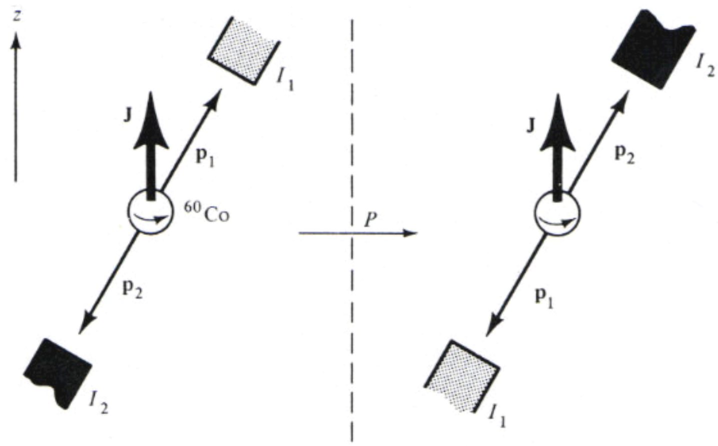


Figure 8-6. Schematic diagram of the Wu et al. apparatus. (From Frauenfelder and Henley). A polarized nucleus emits electrons with momenta p_1 and p_2 that are detected with intensities I_1 and I_2. The left figure shows the "normal" situation while the right figure shows what would be expected after applying the parity operator. Parity conservation implies the two situations cannot be distinguished experimentally (which was not the case).