5. MECHANICAL PROPERTIES AND PERFORMANCE OF MATERIALS

Samples of engineering materials are subjected to a wide variety of mechanical tests to measure their strength, elastic constants, and other material properties as well as their performance under a variety of actual use conditions and environments. The results of such tests are used for two primary purposes: 1) engineering design (for example, failure theories based on strength, or deflections based on elastic constants and component geometry) and 2) quality control either by the materials producer to verify the process or by the end user to confirm the material specifications.

Because of the need to compare measured properties and performance on a common basis, users and producers of materials use standardized test methods such as those developed by the American Society for Testing and Materials (ASTM) and the International Organization for Standardization (ISO). ASTM and ISO are but two of many standards-writing professional organization in the world. These standards prescribe the method by which the test specimen will be prepared and tested, as well as how the test results will be analyzed and reported. Standards also exist which define terminology and nomenclature as well as classification and specification schemes.

The following sections contain information about mechanical tests in general as well as tension, hardness, torsion, and impact tests in particular.

Mechanical Testing

Mechanical tests (as opposed to physical, electrical, or other types of tests) often involves the deformation or breakage of samples of material (called test specimens or test pieces). Some common forms of test specimens and loading situations are shown in Fig 5.1. Note that test specimens are nothing more than specialized engineering components in which a known stress or strain state is applied and the material properties are inferred from the resulting mechanical response. For example, a strength is nothing more than a stress "at which something happens" be it the onset of nonlinearity in the stress-strain response for yield strength, the maximum applied stress for ultimate tensile strength, or the stress at which specimen actually breaks for the fracture strength.

Design of a test specimen is not a trivial matter. However, the simplest test specimens are smooth and unnotched. More complex geometries can be used to produce conditions resembling those in actual engineering components. Notches (such as holes, grooves or slots) that have a definite radius may be machined in specimens. Sharp notches that produce behaviour similar to cracks can also be used, in addition to actual cracks that are introduced in the specimen prior to testing.

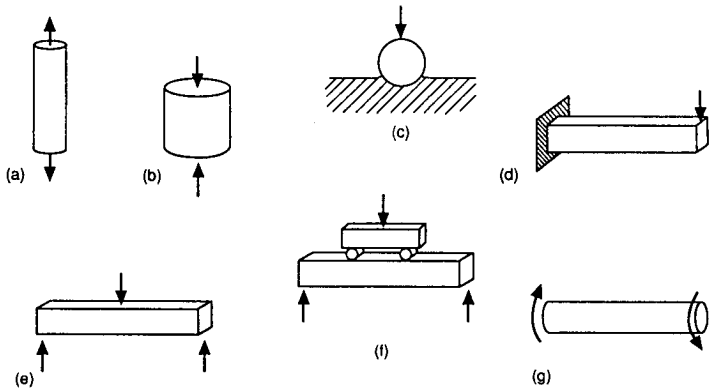


Figure 5.1 Geometry and loading scenarios commonly employed in mechanical testing of materials. a) tension, b) compression, c) indentation hardness, d) cantilever flexure, e) three-point flexure, f) four-point flexure and g) torsion

Equipment used for mechanical testing range from simple, hand-actuated devices to complex, servo-hydraulic systems controlled through computer interfaces. Common configurations (for example, as shown in Fig. 5.2) involve the use of a general purpose device called a universal testing machine. Modern test machines fall into two broad categories: electro (or servo) mechanical (often employing power screws) and servo-hydraulic (high-pressure hydraulic fluid in hydraulic cylinders). Digital, closed loop

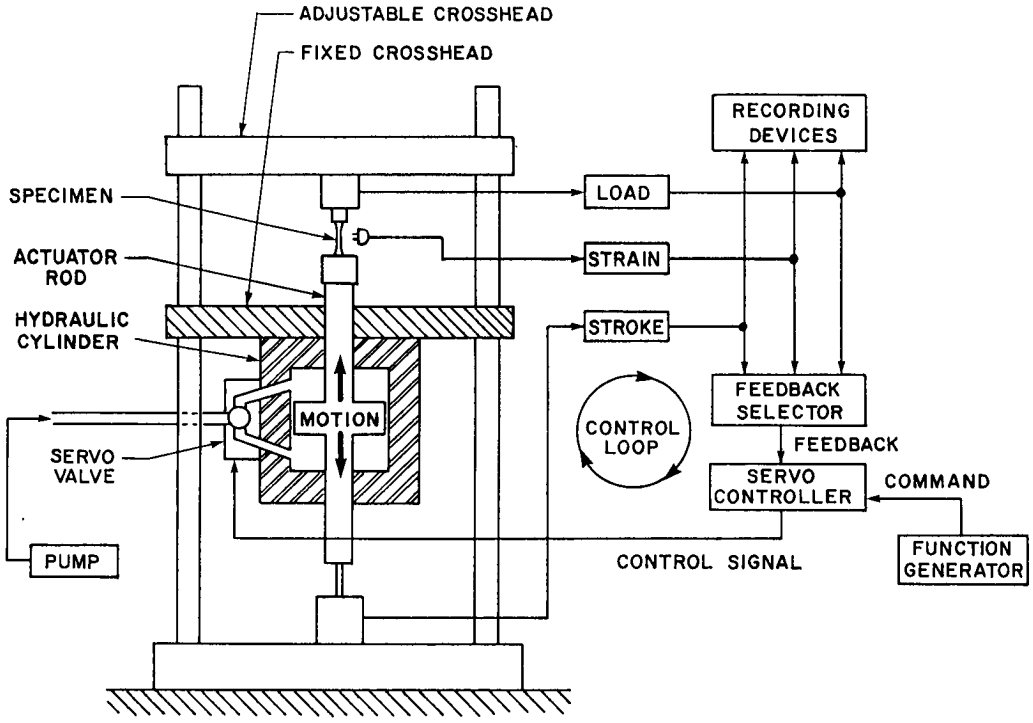


Figure 4.2 Example of a modern, closed-loop servo-hydraulic universal test machine.

control (e.g., force, displacement, strain, etc.) along with computer interfaces and user-friendly software are common. Various types of sensors are used to monitor or control force (e.g., strain gage-based "load" cells), displacement (e.g., linear variable differential transformers (LVDT's) for stroke of the test machine), strain (e.g., clip-on strain-gaged based extensometers). In addition, controlled environments can also be applied through self-contained furnaces, vacuum chambers, or cryogenic apparati. Depending on the information required, the universal test machine can be configured to provide the control, feedback, and test conditions unique to that application.

Tension Test

The tension test is the commonly used test for determining "static" (actually quasi-static) properties of materials. Results of tension tests are tabulated in handbooks and, through the use of failure theories, these data can be used to predict failure of parts subjected to more generalized stress states. Theoretically, this is a good test because of the apparent simplicity with which it can be performed and because the uniaxial loading condition results in a uniform stress distribution across the cross section of the test specimen. In actuality, a direct tensile load is difficult to achieve (because of misalignment of the specimen grips) and some bending usually results. This is not serious when testing ductile materials like copper in which local yielding can redistribute the stress so uniformity exists; however, in brittle materials local yielding is not possible and the resulting non uniform stress distribution will cause failure of the specimen at a load considerably different from that expected if a uniformly distributed load were applied. The typical stress-strain curve normally observed in textbooks with some of the common nomenclature is shown in Fig. 5.3. This is for a typical low-carbon steel specimen. Note that there are a number of definitions of the transition from elastic to plastic behavior. A few of these definitions are shown in Fig. 5.3. Oftentimes the yield point is not so well defined as for this typical steel specimen. Another technique for defining the beginning of plastic behavior is to use an offset yield strength defined as the stress resulting from the intersection of a line drawn parallel to the original straight portion of the stress strain curve, but offset from the origin of this curve by some defined amount of strain, usually 0.1 percent ($\varepsilon = 0.001$) or 0.2 percent ($\varepsilon = 0.002$) and the stress-strain curve itself. The total strain at any point along the curve in Fig. 5.3 is partly plastic after yielding begins. The amount of elastic strain can be determined by unloading the specimen at some deformation, as at point A. When the load is removed, the specimen shortens by an amount equal to the stress divided by elastic modulus (a.k.a., Young's modulus). This is, in fact, the definition of Young's modulus $E = \frac{\Delta\sigma}{\Delta\varepsilon}$ in the elastic region.

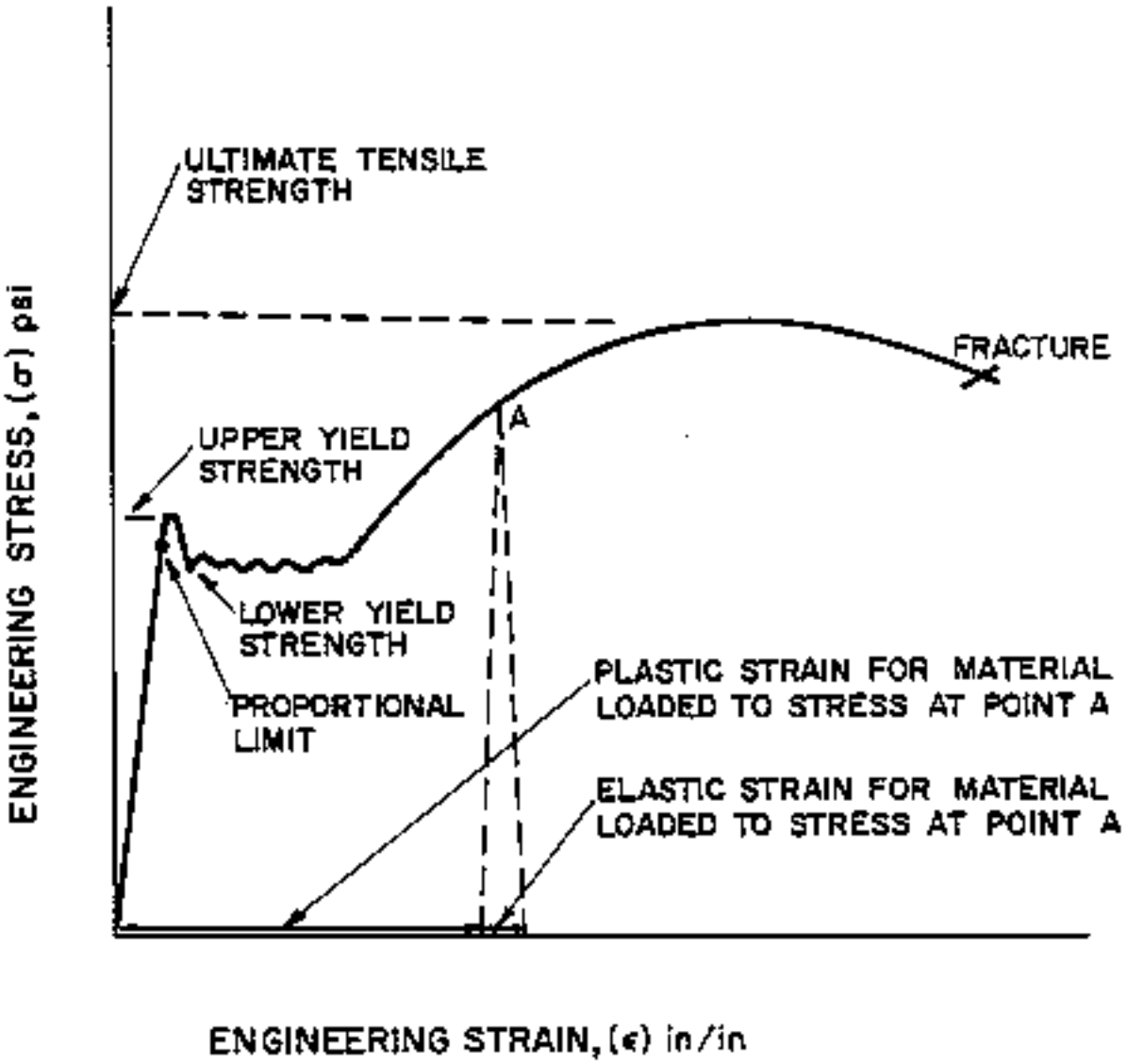


Figure 5.3 Engineering stress-strain diagram for hot-rolled carbon steel showing important properties (Note, Units of stress are psi for US Customary and MPa for S.I. Units of strain are in/in for US Customary and m/m for S.I.

Other materials exhibit stress-strain curves considerably different from carbon-steel although still highly nonlinear. In addition, the stress-strain curve for more brittle materials, such as cast iron, fully hardened high-carbon steel, or fully work-hardened copper show more linearity and much less nonlinearity of the ductile materials. Little ductility is exhibited with these materials, and they fracture soon after reaching the elastic limit. Because of this property, greater care must be used in designing with brittle materials. The effects of stress concentration are more important, and there is no large amount of plastic deformation to assist in distributing the loads.

As shown in Fig. 5.3, often basic stress-strain relations are plotted using engineering stress, σ, and engineering strain, ε. These are quantities based on the original dimensions of the specimen, defined as

$$\sigma = \frac{\text{Load}}{\text{Original Area}} = \frac{P}{A_o} \tag{5.1}$$

$$\varepsilon = \frac{\text{Deformed length - Original length}}{\text{Original length}} = \frac{L - L_o}{L_o} \tag{5.2}$$

The Modulus of Resilience is the amount of energy stored in stressing the material to the elastic limit as given by the area under the elastic portion of the σ - ε diagram and can be defined as

$$U_r = \int_0^{\varepsilon_o} \sigma \, d\varepsilon \approx \frac{\sigma_o \varepsilon_o}{2} \tag{5.3}$$

where σ_o is the proportional limit stress and ε_o is the strain at the proportional limit stress. U_r is important in selecting materials for energy storage such as springs. Typical values for this quantity are given in Table 5.1.

The Modulus of Toughness is the total energy absorption capabilities of the material to failure and is given by the total area under the σ - ε curve such that

$$U_t = \int_0^{\varepsilon_f} \sigma \, d\varepsilon \approx \frac{(\sigma_o + S_u)}{2} \varepsilon_f \tag{5.4}$$

where S_U is the ultimate tensile strength, σ_o is the proportional limit stress and ε_f is the strain at fracture. U_t is important in selecting materials for applications where high overloads are likely to occur and large amounts of energy must be absorbed. This modulus has also been shown to be an important parameter in ranking materials for resistance to abrasion or cavitation. Both these wear operations involve tearing pieces of metal from a parent structure and hence are related to the "toughness" of the material.

Table 5.1 Energy properties of materials in tension

Material	Yield Strength (MPa)	Ultimate Strength (MPa)	Modulus of Resilience, (kJ/m^3)	Modulus of Toughness, (kJ/m^3)
SAE 1020 annealed	276	414	186	128755
SAE 1020 heat treated	427	621	428	91047
Type 304 stainless	207	586	103	195199
Cast iron		172		586
Ductile cast iron	400	503	462	50352
Alcoa 2017	276	428	552	62712
Red brass	414	517	828	13795

The ductility of a material is its ability to deform under load and can be measured by either a length change or an area change. The percent elongation, which is the percent strain to fracture is given by:

$$\%EL = 100\varepsilon_f = 100\left(\frac{L_f - L_o}{L_o}\right) = 100\left(\frac{L_f}{L_o} - 1\right) \tag{5.5}$$

where L_f is the length between gage marks at fracture. We should note that this quantity depends on the gage length used in measuring L, as non uniform deformation occurs in a certain region of the specimen during necking just prior to fracture, hence, the gage length should always be specified. The percent reduction in area is a cross-sectional area measurement of ductility defined as

$$\%RA = 100\left(\frac{A_o - A_f}{A_o}\right) = 100\left(1 - \frac{A_f}{A_o}\right) \tag{5.6}$$

where A_f is the cross-sectional area at fracture. Note %RA is not sensitive to gage length and is somewhat easier to obtain, only a micrometer is required. It should be realized that the stress-strain curves just discussed, using engineering quantities, are fictitious in the sense that the σ and ε are based on areas and lengths that no longer exist at the time of measurement. To correct this situation true stress (σ^T) and true strain (ε^T) quantities are used. The quantities are defined as:

$$\sigma^T = \frac{P}{A_i} \tag{5.7}$$

where A_i is the instantaneous area at the time P is measured. Also

$$\varepsilon^T = \int_{L_o}^{L} \frac{dL}{L} = \ln\frac{L}{L_o} \tag{5.8}$$

or

$$\varepsilon^T = -\int_{A_o}^{A} \frac{dA}{A} = \ln\frac{A_o}{A} \tag{5.9}$$

where L is the instantaneous length between gage mark at the time P is measured.

These two definitions of true strain are equivalent in the plastic range where the material volume can be considered constant during deformation as shown below.

Since

$$AL = A_oL_o \tag{5.10}$$

then

$$L/L_o = A_o/A \tag{5.11}$$

The constant volume condition simply says the stressed volume AL is equal to the original volume A_oL_o. (Note this is only true in the plastic range of deformation, in the

elastic range the change in volume ΔV per unit volume is given by the bulk modulus B (where $B = \frac{E}{3(1-\upsilon)}$ and υ is Poisson's ratio).

Prior to necking, engineering values can be related to true values by noting that

$$\varepsilon^T = \ln\frac{L_i}{L_o} = \ln\frac{L_o + \Delta L}{L_o} \tag{5.12}$$

then

$$\varepsilon^T = \ln(1+\varepsilon) \tag{5.13}$$

and since

$$\frac{A_o}{A} = \frac{L}{L_o} = \frac{L_o + \Delta L}{L_o} \tag{5.14}$$

so

$$A = \frac{A_o}{1+\varepsilon} \tag{5.15}$$

and since

$$\sigma^T = \frac{P}{A} \tag{5.16}$$

then

$$\sigma^T = \frac{P}{A_o}(1+\varepsilon) = \sigma(1+\varepsilon) \tag{5.17}$$

where σ and ε are the engineering stress and strain values at a particular load.

True stress and true strain values are particularly necessary when one is working with large plastic deformations such as large deformation of structures or in metal forming processes. In the elastic region the relation between stress and strain is simply the linear equation

$$\sigma = E\varepsilon \tag{5.18}$$

and also

$$\sigma^T = E\varepsilon^T \tag{5.19}$$

In the plastic region, a commonly used relation to define the relation between stress and strain is given by

$$\sigma^T = K(\varepsilon^T)^n = H(\varepsilon^T)^m \tag{5.20}$$

where strength coefficient, K or H, is the stress when ε^T =1 and m or n is an exponent often called the strain hardening coefficient. Typically values for K or H and m or n are as given in Table 5.2.

Table 4.2 Material constants m or n and K or H for different sheet materials

Material	Treatment	n or m	K or H (MPa)	Specimen Thickness (mm)
0.05% carbon rimmed steel	Annealed	0.261	532	24
0.05/0.07% phosphorus low-carbon steel	Annealed	0.156	644	24
SAE 4130 steel	Annealed	0.118	1168	24
Type 430 stainless steel (17% Cr)	Annealed	0.229	986	32
Alcoa 24-S aluminum	Annealed	0.211	386	258

The last two equations can be written in the form

$$\log \sigma^T = \log E + \log \varepsilon^T \qquad \text{(elastic)} \qquad (5.21)$$

and

$$\log \sigma^T = \log K + m \log \varepsilon^T \qquad \text{(plastic)} \qquad (5.22)$$

by taking logarithms of both sides of the equations.

In this form we see that when plotted on log-log graph paper the following are true.

1. The elastic part of the deformation plots as a 45° line on true stress and true strain coordinates. The extrapolated elastic (45° line) at a value of strain of one corresponds to a stress value equal to the elastic modulus (a.k.a., Young's modulus).
2. the plastic part of the deformation is a straight line of slope m. The strength coefficient, K or H, is that stress existing when the strain is one. This type of graph is shown for a particular aluminum alloy in Fig. 5.4. This type of plot clearly shows the difference in elastic and plastic behavior of ductile materials and the distinct transition between ductile and brittle behavior.

Test specimens used for tensile experiments may be either cut from flat sheet stock or turned from round stock. The round specimens have the advantage of being usable for many types of materials. The specimens typically have a 0.505 in (51 mm) diameter reduced section (giving a cross sectional area of 0.2 in^2.) and may have either a button head or threaded ends for mounting in the machine. The button heads are used more commonly for brittle materials because of less chance of failure in the heads as can occur with threaded specimens.

The load in the specimen is read directly from the testing machine, while the elongation is measured with some type of extensometer. In the elastic region the strains are so small that some type of magnification of the deformations are necessary. There are many ways to achieve this magnification.

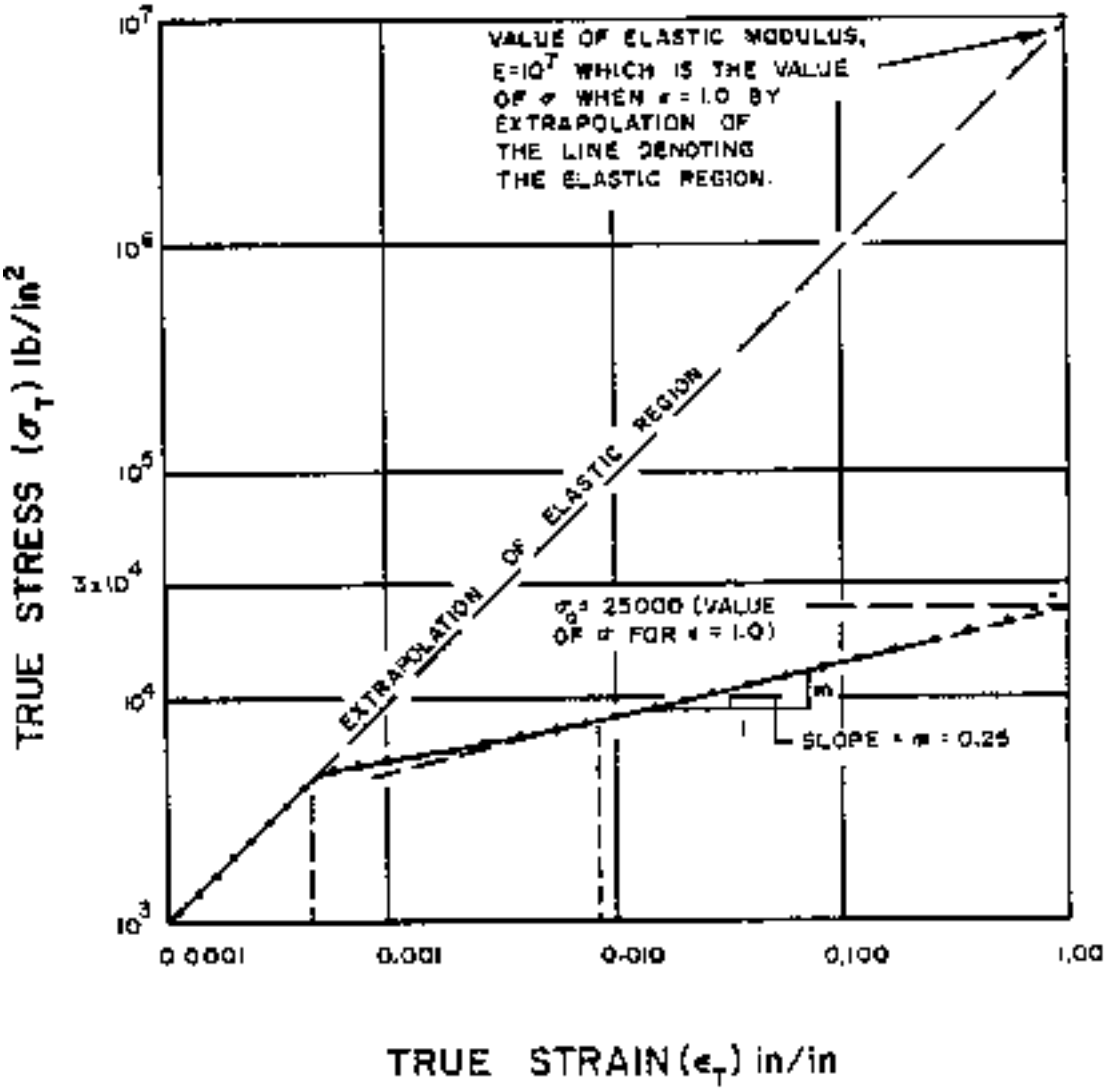


Figure 5.4 Logarithmic true stress-logarithmic (true) strain data plotted on logarithmic coordinates

In the plastic region, the strains become sufficiently large that a finely graduated scale used in conjunction with a pair of dividers to measure linear strain, or a micrometer to measure lateral strain can be used.

In the U.S., generally a 2.0 in (50.8 mm) gage length is used to measure deformations. The 2.0 in (50.8 mm) interval is often marked off with a special tool that marks the interval with punch marks. These punch marks should be very light or fracture will occur at this point. Alternatively an indelible marker can be used to avoid damaging the surface of the test specimen.

Hardness

In the field of engineering, hardness is often defined as the resistance of a material to penetration. Methods to characterize hardness can be divided into three primary categories:

1) Scratch Tests
2) Rebound Tests
3) Indentation Tests

Scratch tests commonly involve comparatively scratching progressively harder materials. In mineralogy, a Mohs hardness scale is used as shown in Fig. 5.5. Diamond, the hardest material, is assigned a value of 10. Decreasing values are assigned to other minerals, down to 1 for the soft mineral, talc. Decimal fractions, such as 9.7 for tungsten carbide, are used for materials intermediate between the standard ones. Where a material lies on the Mohs scale is determined by a simple manual scratch test. If two materials are compared, the harder one is capable of scratching the softer one, but not vice versa. This allows materials to be ranked as to hardness, and decimal values between the standard ones are assigned as a matter of judgment.

Rebound tests may employ techniques to assess the resilience of material by measuring changes in potential energy. For example, the Sceleroscope hardness test employs a hammer with a rounded diamond tip. This hammer is dropped from a fixed height onto the surface of the material being tested. The hardness number is proportional to the height of rebound of the hammer with the scale for metals being set so that fully hardened tool steel has a value of 100. A modified version is also used for polymers.

Indentation tests actually produce a permanent impression in the surface of the material. The force and size of the impression can be related to a quantity (hardness) which can be objectively related to the resistance of the material to permanent penetration. Because the hardness is a function of the force and size of the impression, the pressure (and hence stress) used to create the impression can be related to both the yield and ultimate strengths of materials. Several different types of hardness tests have evolved over the years. These include macro hardness test such as Brinell, Vickers, and Rockwell and micro hardness tests such as Knoop and Tukon.

Brinell Hardness Test In this test, a relatively large steel ball, specifically 10 mm in diameter is used with a relatively large force. The force is usually obtained with either 3000 kg for relatively hard materials such as steels and cast irons or 500 kg for softens materials such as copper and aluminum alloys. For very hard materials, the standard steel ball will deform excessively and a tungsten carbide ball is used. The Brinell

hardness dates from the late 1800's and is probably the most common hardness test in the world.

The Brinell hardness number is obtained by dividing the applied force, P, in kg, by the actual surface area of the indentation which is a segment of a sphere as illustrated in Fig. 5.6 such that:

$$BHN = HB = \frac{P}{\pi Dt} = \frac{2P}{\pi D\left[D - \sqrt{\left(D^2 - d^2\right)}\right]} \qquad (5.23)$$

where D is the diameter of the ball in mm, t is the indentation depth from the surface in mm, and d is the diameter of the indentation at the surface in mm.

Brinell hardness is good for averaging heterogeneities over a relatively large area, thus lessening the influence of scratches or surface roughness. However the large ball size precludes the use of Brinell hardness for small objects or critical components where large indentations may promote failure. Another limitation of the Brinell hardness test is that because of the spherical shape of the indenter ball, the BHN for the same material will not be the same for different loads if the same size ball is used. Thus, geometric similitude must be imposed by maintaining the ratio of the indentation load and indenter diameter such that:

$$\frac{P_1}{D_1} = \frac{P_2}{D_2} = \frac{P_3}{D_3} = etc. \qquad (5.24)$$

The Meyers hardness test is a variation on the Brinell hardness test and addresses this lack of geometric similarity by using the projected area of the indentation such that:

$$MHN = HM = \frac{P}{\pi d^2/4} \qquad (5.25)$$

Although the Meyers hardness is less sensitive to applied load and is a more fundamental measure of hardness, it is rarely used.

Vickers Hardness Test (a.k.a.. diamond pyramid hardness) In this test, the same general principles of the Brinell test are applied. However, a four-sided diamond pyramid is implied as an indenter rather than a ball to promote geometric similarity of indentation regardless of indentation load (see Fig. 5.7). The included angle between the faces of the pyramid is 136° which corresponds to a desired d/D ratio for the Brinell test of 0.25. The resulting Vickers indentation has a depth, h equal to 1/7 of the indentation size, L, measured on the diagonal. The Vickers hardness is obtained by dividing the applied force by the surface area of the paramedical impression such that:

$$VHN = HV = \frac{2P}{L^2}\sin\frac{\alpha}{2} \qquad (5.26)$$

where P is the indentation load which typically ranges from 0.1 to 1 kg but may be as high as 120 kg, L is the diagonal of the indentation in mm and α is the included angle of 136°.

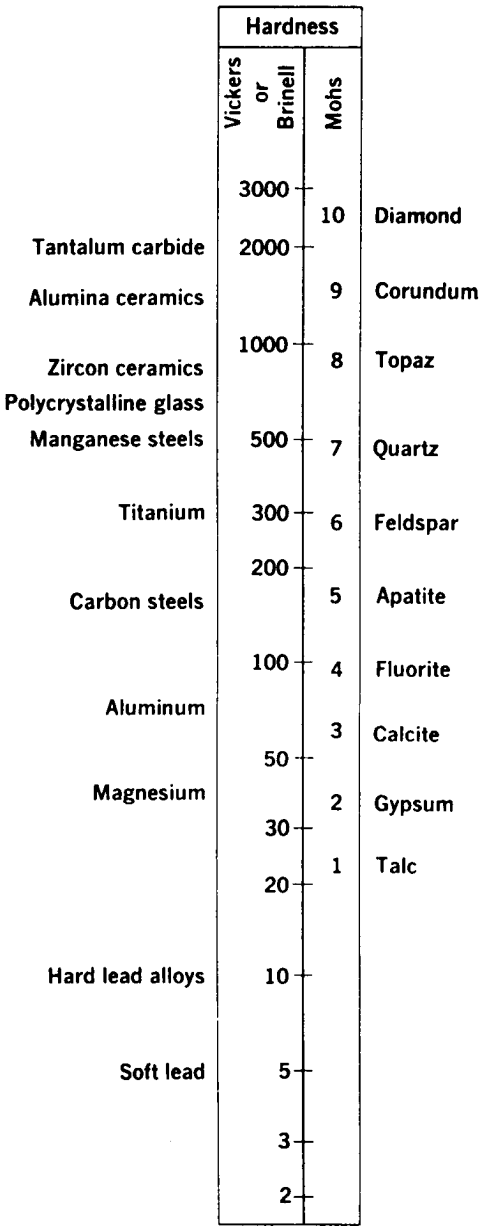


Figure 5.5 Approximate relative hardnesses of metals and ceramics for Mohs scale and indention scales.

The primary advantage of the Vickers hardness is that the result is independent of load. However, disadvantages are that it is somewhat slow since careful surface preparation is required. In addition, the result may be prone to personal error in measuring the diagonal length along with interpretation of anomalies such as "pin cushioning" for soft materials and "barreling/ridging" for hard materials.

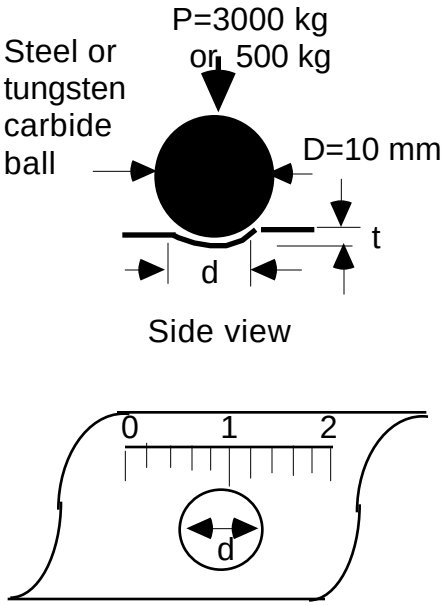


Figure 5.6 Brinell hardness test.

Rockwell Hardness Test The Rockwell test is the most widely accepted hardness test in the United States. In this test, penetration depth is measured, with the hardness reported as the inverse of the penetration depth. A two step procedure is used as illustrated in Fig. 5.8. The first step "sets" the indenter in the material and the second step is the actual indentation test. The conical diamond or spherical indenter tips produce indentation depths, the inverse of which are used to display hardness on the test machine directly. The reported hardness is in arbitrary units, but the Rockwell scale which identifies the indentation load and indenter tip must be reported with the hardness number (otherwise the number is useless). Rockwell scales include those in Table 5.3.

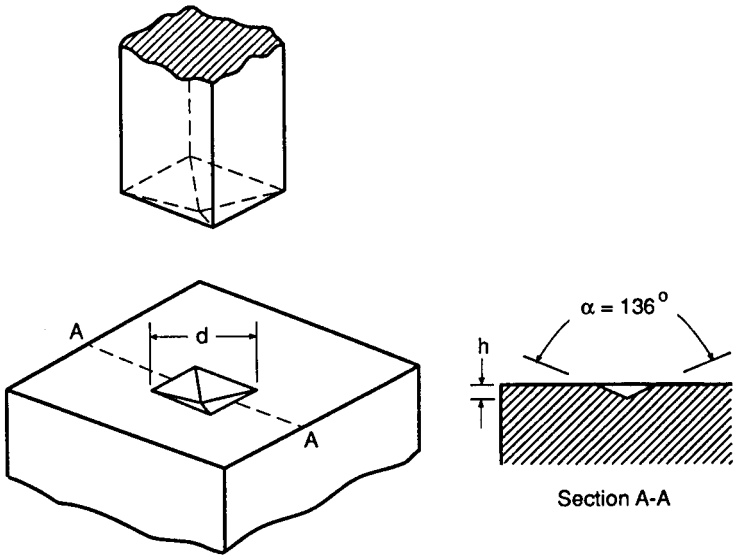


Figure 5.7 Vickers hardness indenter

Table 5.3 Representative Rockwell indenter specifics

Rockwell Scale	Indenter	Major Load (kg)
A	Brale	60
B	1/16" Ball	100
C	Brale	150
D	Brale	100
E	1/8" Ball	100
F	1/16" Ball	60
M	1/4" Ball	100

* Brale is a conical diamond indenter

Some important points concerning Rockwell hardness testing include the following

1) Indenter and anvil should be clean and well seated.
2) Surface should be clean, dry, smooth, and free from oxide
3) Surface should be flat and normal

A primary advantage of the Rockwell hardness test is that it is automatic and self-contained thereby given and instantaneous readout of hardness which lends itself to automation and rapid through put.

Elastic/Plastic Correlations and Conversions

The deformations caused by a hardness indenter can be correlated to those produced at the yield and ultimate tensile strengths in a tensile test. However, an important difference is that the material cannot freely flow outward, sot that a complex triaxial state of stress exists under the indenter (see Fig. 5.9). Nonetheless, various correlations have been established between hardness and tensile properties.

For example, the elastic constraint under the hardness indenture reaches a limiting value of 3 such that the yield strength can be related to the pressure exerted by the indenter tip:

$$S_y = \text{BHN} \times 9.816 \ / \ 3 \qquad (5.27)$$

where S_Y is the yield strength of the material in MPa and BHN is the Brinell hardness in kg/mm^2.

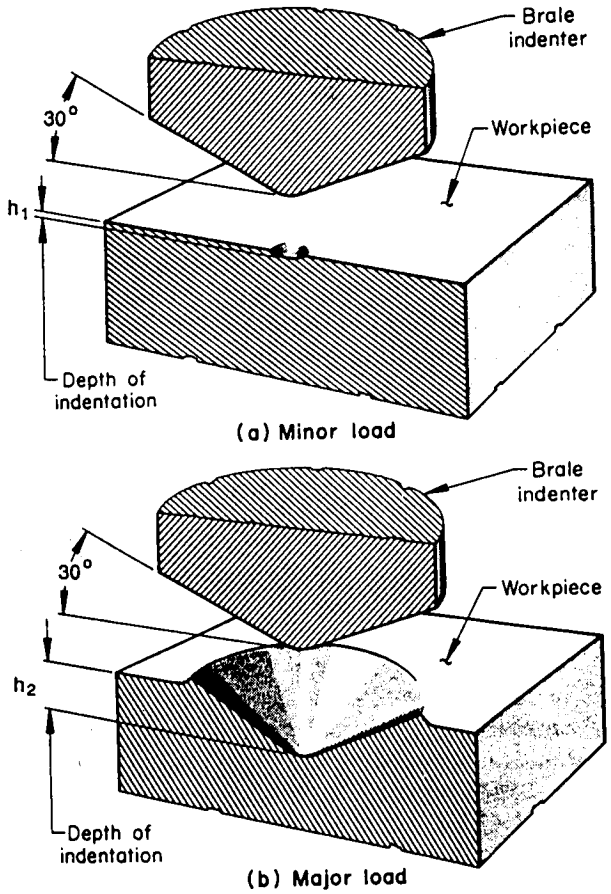


Figure 5.8 Rockwell hardness indentation for a minor load and for a major load.

Empirical relations have also been developed to correlate different hardness number as well as hardness and ultimate tensile strength. For example, for low- and medium carbon and alloy steels,

$$S_u = 3.45 \times \text{BHN} \qquad (5.28)$$

where S_U is the ultimate tensile strength of the material in MPa and BHN is the Brinell hardness in kg/mm^2.

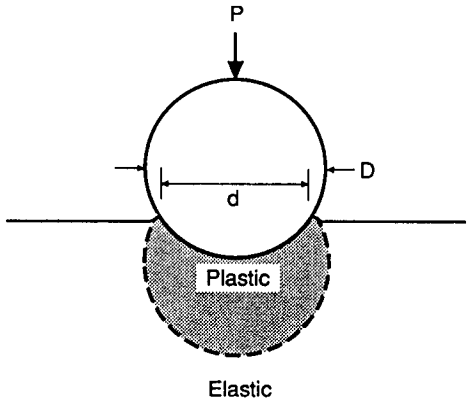


Figure 5.9 Plastic deformation under a Brinell hardness indenter.

Note that for both these relations, there is considerable scatter in actual data, so that these relationships should be considered to provide rough estimates only. For other classes of material, the empirical constant will differ, and the relationships may even become nonlinear. Similarly, the relationships will change for different types of hardness tests. Rockwell hardness correlates well with ultimate tensile strength and with other types of hardness tests, although the relationships can be nonlinear. This situation results from the unique indentation-depth basis of this test. For carbon and alloy steels, conversion charts for estimating various types of hardness from one another as well as ultimate tensile strengths are contained in an ASTM standard, ASM handbooks and information supplied by manufacturers of hardness testing equipment

Torsion

The torsion test is another fundamental technique for obtaining the stress-strain relationship for a metal. Because the shear stress and shear strain are obtained directly in the torsion test, rather than tensile stress and tensile strain as in the tension test, many investigators actually prefer this test to the tension test. Since all deformation of ductile materials is by shear, the torsion test would seem to be the more fundamental of the two.

The torsion test is accomplished by simply clamping each end of a suitable specimen in a twisting machine that is able to measure the torque, T, applied to the specimen. Care must be used in gripping the specimen to avoid any bending. A device called a troptometer is used to measure angular deformation. This device consists of two collars which are clamped to the specimen at the desired gage length. One collar is equipped with a pointer the other with a graduated scale, so the relative twist between the gage marks can be determined. The troptometer is useful for measuring strains up to and slightly past the elastic limit. For larger plastic strains, complete revolutions of the collars are counted.

The test, then, consists of measuring the angle of twist, θ (radians) at selected increments of torque T (N-m). Expressing the twist as $\theta' = \theta/L$, the angular deflection per unit gage length, one is able to plot a T - θ' diagram that is analogous to the load-deflection diagram obtained in the tension test. To be useful for engineering purposes, it is necessary to convert this T - θ' diagram to a shear stress (τ) - shear strain (γ) diagram similar to the previous normal stress (σ) - normal strain (ε) diagram. Of course, one can also convert the τ - γ diagram to a σ - ε diagram as will be shown later.

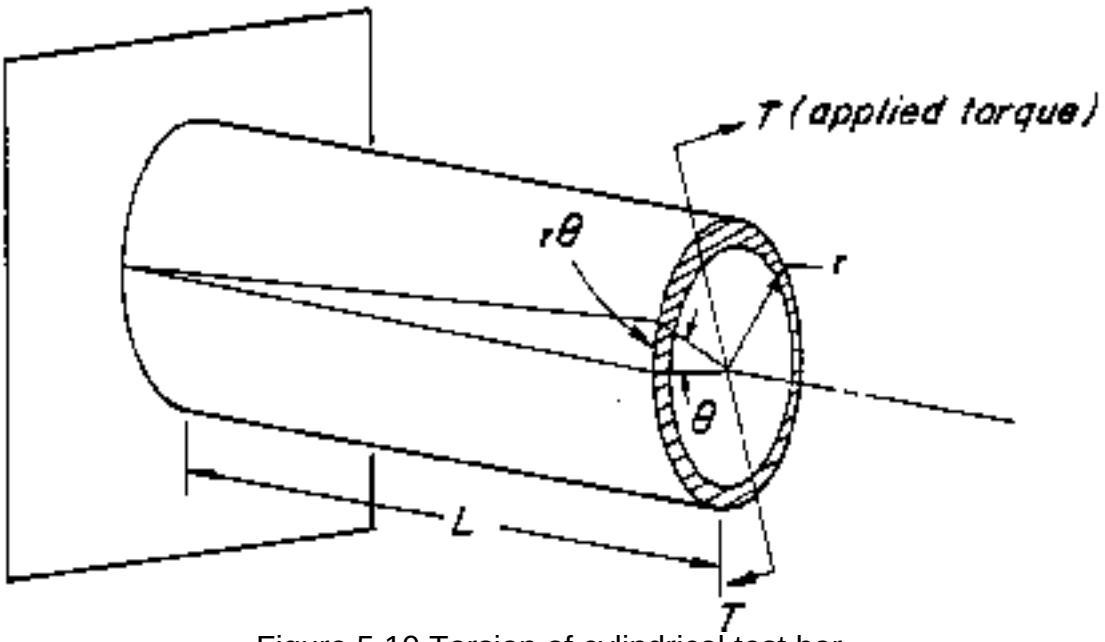


Figure 5.10 Torsion of cylindrical test bar

Two possible approaches are used: 1) a mechanistic approach which requires no a priori knowledge of the properties of the particular material, only the form of the resulting stress-strain relations, and 2) a materials approach which requires a priori knowledge of the properties of the particular material along with the form of the resulting stress-strain relations.

For the mechanistic approach, consider first a circular, thin-walled specimen as shown in Figure 5.10

The shear strain γ is the relative rotation of one circular cross-section with respect to a section one unit length away or:

$$\gamma = \frac{r\theta}{L} \tag{5.29}$$

where θ is in radians. This relation is true in either the elastic or plastic range.

The shear stress τ is simply the average applied force at the tube cross section (T/r) divided by the cross-sectional area. This is so because the stress can be assumed uniformly distributed across the thickness of the tube, t. This gives:

$$\tau = \frac{T}{2\pi r^2 t} \tag{5.30}$$

Using the Eqs. 5.29 and 5.30, the complete τ - γ diagram in the elastic and plastic range can be obtained.

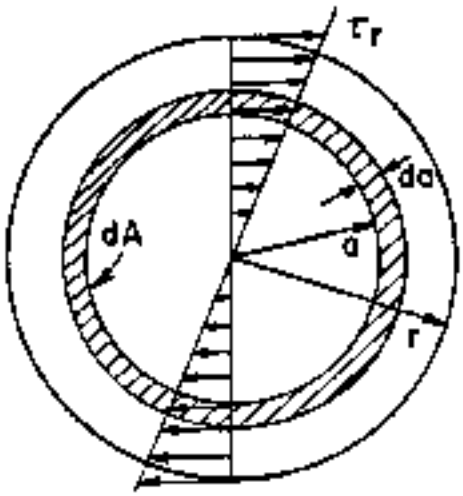


Figure 5.11 Elastic Shear Stress Distribution

The τ - γ diagram can also be obtained from T - θ information obtained using a solid circular test specimen. This specimen has the advantage of being somewhat easier to grip in the testing machine and has no tendency to collapse during twisting. This is the specimen type to be used in this laboratory. For the solid specimen, the shear strain relation remains the same as for the tubular specimen, i.e. $\gamma = \frac{r\theta}{L}$.

The shear stress distribution is somewhat more difficult to obtain because we can no longer assume the stress distribution to be uniform across the section. The derivation for the equation giving τ from T-θ data is as follows:

In the elastic deformation range the stress is distributed uniformly across the section as shown in Fig. 5.11

Considering the very thin circumferential ring shown above, the torque resisted by this ring is given by

$$dT = (\text{shear stress}) \times (\text{area}) \times (\text{lever arm})$$

$$dT = \tau \times \tau_r \times \frac{a}{r} = 2\pi a^2 \tau \, da \qquad (5.31)$$

since

$$dA = 2\pi a da. \qquad (5.32)$$

Since the stress distribution is linear, at any radius, a, the shear stress, γ, is related to the maximum shear stress, τ_r, existing at r by

$$\tau = \tau_r \times \frac{a}{r} \qquad (5.33)$$

so substituting in the equation for dT, the torque on a small area becomes:

$$dT = 2\pi \frac{\tau_r}{r} a^3 da \qquad (5.34)$$

and integrating over the entire cross-sectional area, the total external torque is equal to

$$T=\frac{2\pi\tau_r}{r_o}\int r\,a^3\,da=\frac{2\pi\tau_r}{r}\frac{r^4}{4}=\frac{\pi\tau_r r^3}{2} \tag{5.35}$$

and the shear stress at the outermost fibers is

$$\tau_r=\frac{2T}{\pi r^3} \tag{5.36}$$

Note that Eqs. 5.33 to 5.34 applies only in the elastic region. When the metal starts to deform plastically, the shear stress distribution is no longer linear, but is as shown in Fig. 5.12.

The relation between T and τ is no longer the same. To evaluate this relation we begin as before, noting that the torque at a very thin ring of radius a is again given by

$$dT=2\pi\tau a^2\,da \tag{5.37}$$

So the total external torque resisted across the section is then

$$T=2\pi\int_o^a \tau a^2\,da \tag{5.38}$$

The shear strain relation $\gamma=\frac{a\theta}{L}$ at any radius a is still valid, however, and substituting this in Eq. 5.38, we obtain

$$T=2\pi\int_o^r \frac{\tau\gamma^2L^2}{\theta^2}\frac{L}{\theta}\,d\gamma \tag{5.39}$$

The shear stress T at any radius a is also a function of γ only, i.e.

$$\tau=f(\gamma) \tag{5.40}$$

so the expression for torque T can be written in terms of γ only as

$$T\theta^3=2\pi L^3\int_o^{\gamma_r} f(\gamma)\,\gamma^2 d\gamma \tag{5.41}$$

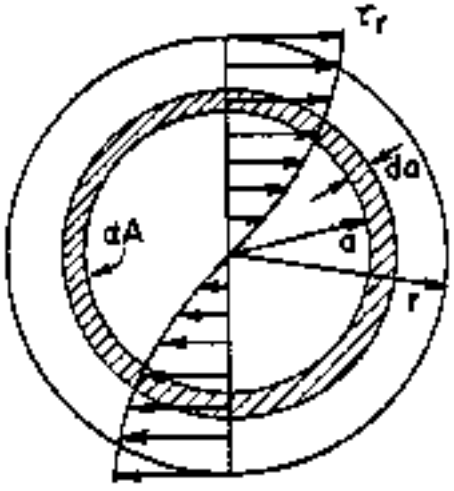


Fig. 5.12 Elastic-Plastic Shear Stress Distribution

Differentiating both sides of this equation with respect to 9, one obtains

$$\frac{d}{d\theta}\left(T\theta^3\right) = 2\pi L^3 f(\gamma_r)\gamma_r^2 \frac{d\gamma_r}{d\theta} \quad (5.42)$$

since $\gamma_r = \frac{r\theta}{L}$ then

$$\frac{d\gamma_r}{d\theta} = \frac{r}{L} \quad (5.44)$$

and substituting these quantities in the equation for $d/d\theta\left(T\theta^3\right)$ and working out the derivative, one obtains

$$3T\theta^2 + \frac{dt}{d\theta}\theta^3 = 2\pi r^3\theta^2 \quad (5.45)$$

and

$$3T + \theta\frac{dt}{d\theta} = 2\pi r^3 \quad (5.44)$$

Solving for the shear stress, the result is

$$\tau = \left(\frac{1}{2\pi r^3}\right)\left(\theta\frac{dt}{d\theta} + 3T\right) \quad (5.45)$$

This was rather a lengthy derivation, but the application is easy. Refer to the typical T - θ diagram as obtained from a torsion test shown in Fig. 5.13.

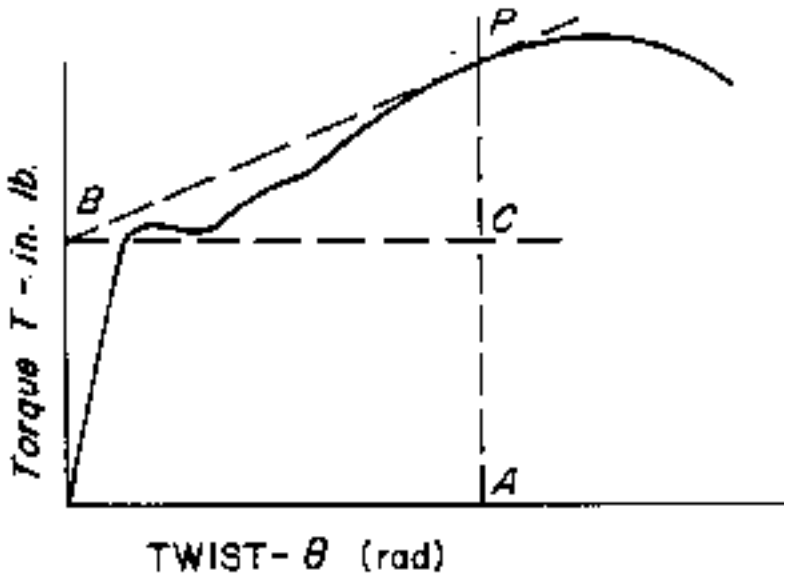


Fig. 5.13 Example of torque-twist curve used for data

At the typical point P at which it is desired to obtain the shear stress, observe that $\theta = BC$ and that $\frac{dT}{d\theta} = \frac{PC}{BC}$ such that T = AP. Substituting these quantities, the result is $\tau = \frac{1}{2\pi r^3}\left[BC\frac{PC}{BC} + 3AP\right]$ or $\tau = \frac{PC + 3AP}{2\pi r^3}$.

With this last relation it is then a simple matter to obtain values of τ at various θ positions of the plastic part of the T - θ curve. Remember that $\gamma = \frac{r\theta}{L}$, the complete τ - γ curve can be obtained.

For the materials approach, it is possible to again make the valid assumption that $\gamma = \frac{r\theta}{L}$. However, τ is determined as one of two functions of γ depending on whether the internal stress state is in the elastic or plastic range. However, calculating this internal stress state requires a priori knowledge of material properties usually determined from a tensile test. In particular, E is required to calculate $G = \frac{E}{2(1+\upsilon)}$, σ_o is required to calculate $\tau_o = \frac{\sigma_o}{\sqrt{3}}$, K and n are required to calculate $K_\tau = \frac{K}{3^{(n+1)/2}}$ and n for shear equals n for tension. Once these relations are established, then it is possible to calculate the shear stress from the shear strain for the elastic or plastic condition as follows.

$$\tau = G\gamma \text{ for } \tau \le \tau_o \text{ and/or } \gamma \le \gamma_o \qquad (5.46)$$

or

$$\tau = K_\tau \gamma^n \text{ for } \tau > \tau_o \text{ and/or } \gamma > \gamma_o \qquad (5.47)$$

where G is the shear modulus, $K_\tau = \frac{K}{3^{(n+1)/2}}$ in which K and n are the strength coefficient and strain hardening exponent from the tension test, respectively. The shear strain at yield can be determined from an effective stress-strain relation from plasticity such that

$$\gamma_o = \frac{\tau_o}{G} \qquad (5.48)$$

where $\tau_o = \frac{\sigma_o}{\sqrt{3}}$ in which σ_o is the "yield" strength from a tension test.

For any given T-θ combination, it is possible to calculate the shear strain at the surface of the specimen (that is, r=R) as $\gamma = \frac{r\theta}{L}$. Comparing this shear strain to that calculated in Eq. 5.48, allows the choice of either Eq. 5.46 or 5.47.

Note that when the shear stress at r=R is plastic, the total torque, T, required to produce the deformation, θ, will have two components: an elastic torque, T_e and a plastic torque, T_p since the shear stress across the cross section of the specimen will have both a

plastic part and an elastic part as shown in Fig. 5.14. The relation for T can then be written as:

$$T_{total} = T_{elastic} + T_{plastic} \tag{5.50}$$

where

$$T_{elastic} = \int_0^{\tau_o} \tau \, dA \, r = \int_0^{\gamma_o} G\gamma \, dA \, r \tag{5.51}$$

$$T_{plastic} = \int_{\tau_o}^{\tau_R} \tau \, dA \, r = \int_{\gamma_R}^{\gamma_R} K_\tau \gamma^n \, dA \, r \tag{5.52}$$

For convenience it is possible to rewrite the integration variables in Eqs. 5.51 and 5.52 in terms for the specimen radius, r, only such that

$$T_{elastic} = \int_0^{r_y} \frac{\tau_y}{r_y} 2\pi r^3 dr \tag{5.53}$$

$$T_{plastic} = \int_{r_y}^{R} \frac{K}{\sqrt{3}} \left(\frac{\sqrt{3}\theta'}{}\right)^n 2\pi r^2 dr \tag{5.54}$$

where $\theta' = \frac{\theta}{L}$ and $r_y = \frac{\theta}{L}\gamma_o = \frac{\theta}{L}\frac{\tau_o}{G} = \theta'\frac{\tau_o}{G}$.

Equations 5.53 and 5.54 can be solved either closed form or numerically for any combination of T and θ.

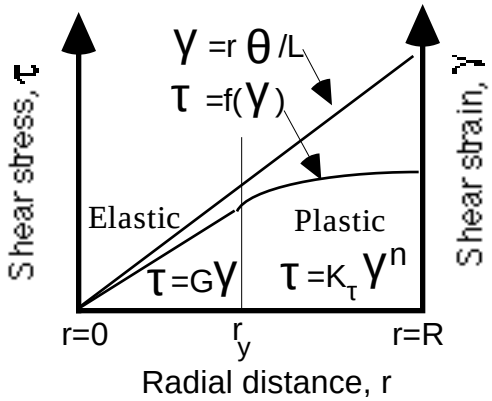


Figure 5.14 Shear stress and shear strain as functions of radial distance

Once the shear stress-strain curve is obtained, engineering properties are easily calculated. A few of the more important quantities will be discussed. As in the tension test, yield strengths for shearing stress can be defined, such as a proportional limit or an offset yield strength. The Modulus of Rupture is the total area under the τ - γ curve determined at r=R and represents the total energy absorption abilities of the material in shear.

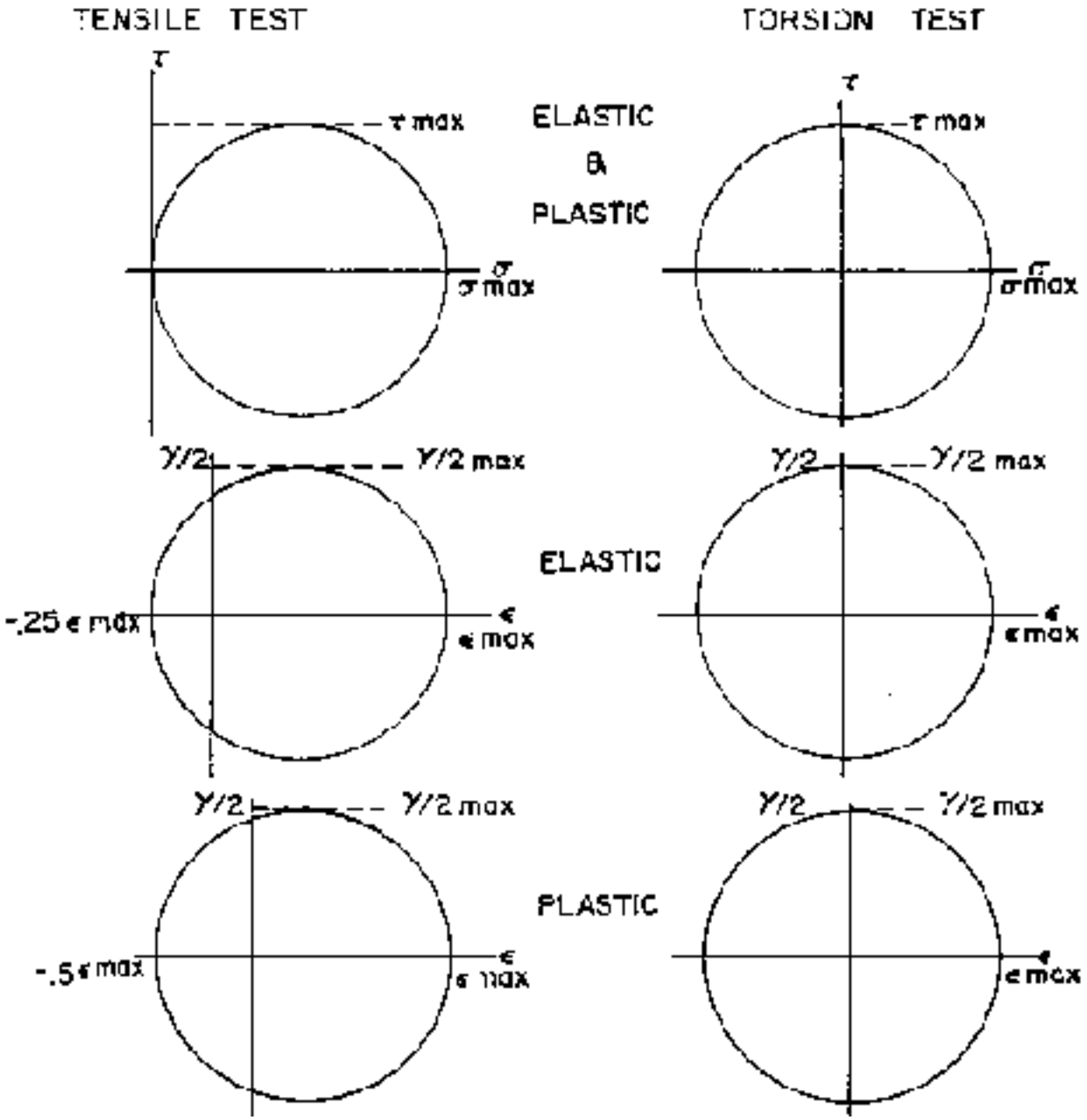


Figure 5.15 Mohr's circles for the tensile test and torsion test

As in the tension test, the Modulus of Resilience is the area under the elastic portion of the τ - γ curve such that

$$U_r = \int_o^{\gamma_o} \tau \, d\gamma \qquad (5.55)$$

Similarly, the Modulus of Toughness is the area under the total τ - γ curve such that

$$U_t = \int_o^{\gamma_f} \tau \, d\gamma \qquad (5.56)$$

The Modulus of Rigidity (or Shear Modulus), G, is the slope of the τ - γ curve in the elastic region and is comparable to Young's Modulus, E, found in tension. Recall that the relation between E and G is $G = \frac{\Delta\tau}{\Delta\gamma} = \frac{E}{2(1+\upsilon)}$.

The true shear stress-strain curve can be compared to the tensile true stress-strain curve by converting the normal values to shear values. The conversion is as follows:

Elastic range: τ equivalent $= \frac{\sigma}{2}$; γ equivalent $= 1.25\varepsilon$ (5.57)

Plastic range: τ equivalent $= \frac{\sigma}{2}$; γ equivalent $= 1.5\varepsilon$ (5.58)

That these values are correct can be seen from Mohr's circle of stress and of strain for the elastic and plastic ranges (Fig. 5.15). Knowledge of Poisson's ratio, υ, is needed for Mohr's circles of strain for the tensile test. For mild steel in the elastic range, υ = .0.30; in the plastic range, υ =0.5 as a result of the constant volume assumption.

Impact

The static properties of materials and their attendant mechanical behavior are very much functions of factors such as the heat treatment the material may have received as well as design factors such as stress concentrations.

The behavior of a material is also dependent on the rate at which the load is applied. Polymeric materials and metals which show delayed yielding are most sensitive to load application rate. Low-carbon steel, for example, shows a considerable increase in yield strength with increasing rate of strain. In addition, increased work hardening occurs at high-strain rates. This results in reduced local necking, hence, a greater overall material ductility occurs. A practical application of these effects is apparent in the fabrication of parts by high-strain rate methods such as explosive forming. This method

results in larger amounts of plastic deformation than conventional forming methods and, at the same time, imparts increased strength and dimensional stability to the part.

In design applications, impact situations are frequently encountered, such as cylinder head bolts, in which it is necessary for the part to absorb a certain amount of energy without failure. In the static test, this energy absorption ability is called "toughness" and is indicated by the modulus of rupture. A similar "toughness" measurement is required for dynamic loadings; this measurement is made with a standard ASTM impact test known as the Izod or Charpy test. When using one of these impact tests, a small notched specimen is broken in flexure by a single blow from a swinging pendulum. With the Charpy test, the specimen is supported as a simple beam, while in the Izod it is held as a cantilever. Figure 5.16 shows standard configurations for Izod (cantilever) and Charpy (three-point) impact tests.

A standard Charpy impact machine is used. This machine consists essentially of a rigid specimen holder and a swinging pendulum hammer for striking the impact blow (see Fig. 5.17). Impact energy is simply the difference in potential energies of the pendulum before and after striking the specimen. The machine is calibrated to read the fracture energy in N-m or J directly from a pointer which indicates the angular rotation of the pendulum after the specimen has been fractured.

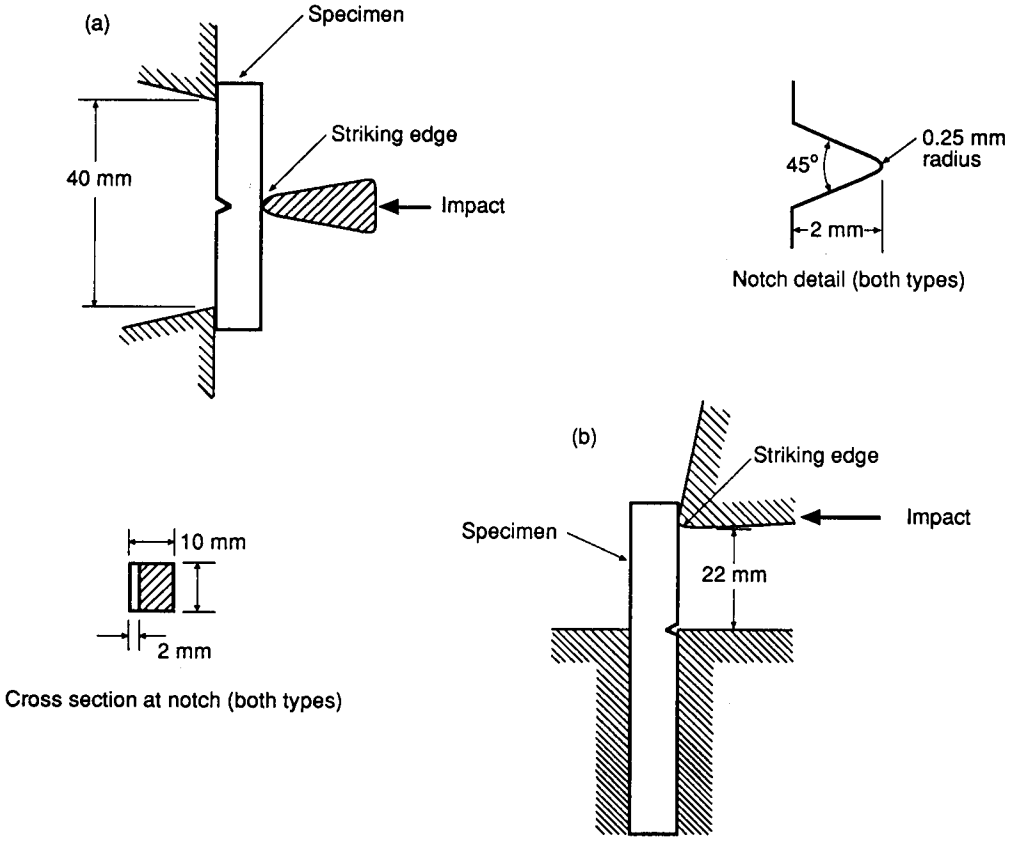
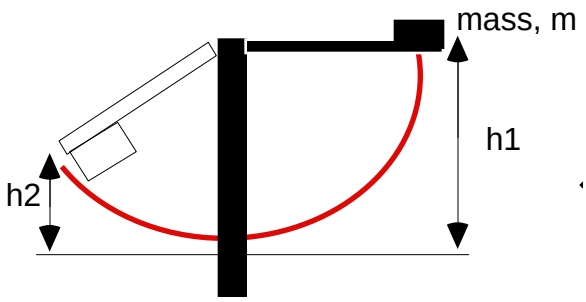


Figure 5.16 Charpy and Izod impact specimens and test configurations



IMPACT ENERGY=mg(h1-h2)

Figure 5.17 Charpy and Izod impact specimens and test configurations

The Charpy test does not simulate any particular design situation and data obtained from this test are not directly applicable to design work as are data such as yield strength. The test is useful, however, in comparing variations in the metallurgical structure of the metal and in determining environmental effects such as temperature. It is often used in acceptance specifications for materials used in impact situations, such as gears, shafts, or bolts. It can have useful applications to design when a correlation can be found between Charpy values and impact failures of actual parts.

Curves as shown in Fig. 5.18 showing the energy to fracture as measured by a Charpy test indicate a transition temperature, at which the ability of the material to absorb energy changes drastically. The transition temperature is that temperature at which, under impact conditions, the material's behavior changes from ductile to brittle. This change in the behavior is effected by many variables. Metals that have a face-centered cubic crystalline structure such as aluminum and copper have many slip systems and are the most resistant to low-energy fracture at low-temperature. Most metals with body-centered cubic structures (like steel) and some hexagonal crystal structures show a sharp transition temperature and are brittle at low temperatures.

Considering steel; coarse grain size, strain hardening, and certain minor impurities can raise the transition temperature whereas fine grain size and certain alloying elements will increase the low temperature toughness. Figure 5.18 shows the effect of heat treatment on alloy steel 3140 and 2340. Note that a transition temperature as high as about 25°C is shown. This material, then should not be in service below temperature of 25°C when impact conditions are likely to exist.

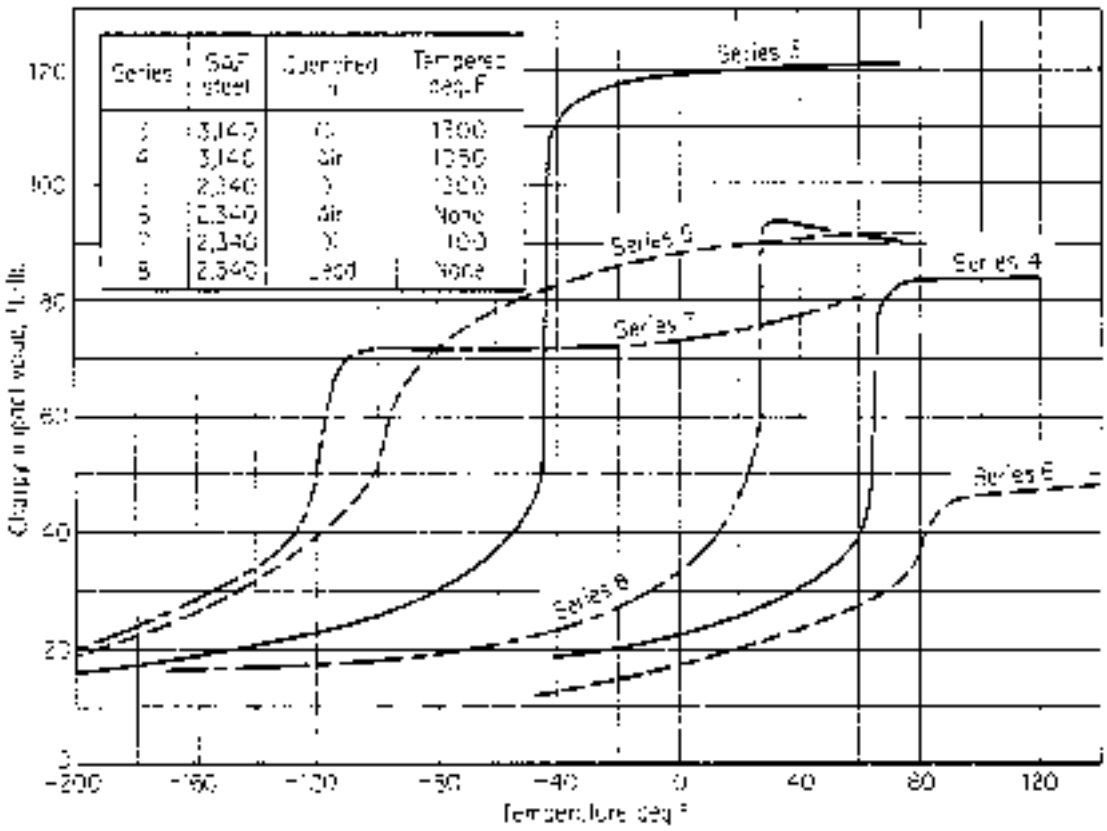


Figure 5.18 Variation in transition-temperature range for steel in the Charpy test

In defining notch "toughness", a number of criteria have been proposed to define the transition temperature. These include:

a. some critical energy level

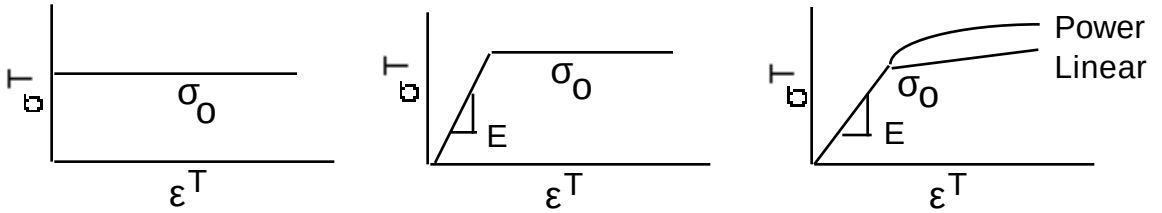
b. a measure of ductility such as lateral contraction of the specimen after fracture

c. fracture surface appearance - the brittle fracture surface has a crystalline appearance, while the portion of the specimen which fracture in a ductile manner will have a so-called fibrous appearance.

Any of these criteria are usable. Perhaps the most direct criteria for a particular metal is to define the transition temperature as that temperature at which some minimum amount of energy is required to fracture. During World War II, allied Victory ships literally broke in two in conditions as mild as standing at the dock because of the use of steel with a high-transition temperature, coupled with high-stress concentrations. It was found that specimens cut from plates of these ships averaged only 9 J. Charpy energy absorption at the service temperature. Ship plates were resistant to failure if the energy absorption value was raised to 20 J at the service temperature by proper control of impurities.

Plasticity Relations

Plasticity can be defined as non recoverable deformation beyond the point of yielding where Hooke's law (proportionality of stress and strain) no longer applies. Flow curves are the true stress vs. true strain curves which describe the plastic deformation. As shown in Fig. 5.19, are several simple approximations made to represent mathematically represent actual plastic deformation.



Rigid-Perfectly Plastic Elastic-Perfectly Plastic Elastic-Linear Hardening
Elastic-Power Hardening

Figure 5.19 Mathematical approximations of plot curves

The hardening- flow curve is the most generally applicable type of flow curve. This type of plastic deformation behaviour has been modeled two different ways: Simple Power Law and Ramberg-Osgood.

In the Simple Power Law model, the stress strain curve is divide into two discrete region, separate at $\sigma = \sigma_o$ such that:

$$\text{Elastic}: \quad \sigma = E\varepsilon \qquad (\sigma \le \sigma_o) \tag{5.59}$$

$$\text{Plastic}: \quad \sigma = H\varepsilon^n \qquad (\sigma \ge \sigma_o) \tag{5.60}$$

In the Ramberg-Osgood relationship the stress-strain curve is modeled as a continuous function such that the total strain is sum of elastic and plastic parts:

$$\varepsilon = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \varepsilon_p \tag{5.61}$$

and

$$\sigma = H\,\varepsilon_p^n \qquad \varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{H}\right)^{\frac{1}{n}} \tag{5.62}$$

For the Ramberg-Osgood relation, σ_o is not distinct "break" in the stress-strain curve, but is instead calculated from the elasticity and plasticity relations such that

$$\sigma_o = E\left(\frac{H}{E}\right)^{\frac{1}{1-n}} \tag{5.63}$$

General stress-strain relations can be developed for deformation plasticity theory such that the effective stress is

$$\bar{\sigma}_{\text{eff}} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \tag{5.64}$$

and the effective total strain is

$$\bar{\varepsilon} = \frac{\bar{\sigma}}{E} + \bar{\varepsilon}_p \tag{5.65}$$

where the effective plastic strain is

$$\bar{\varepsilon}_p = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_{p1}-\varepsilon_{p2})^2+(\varepsilon_{p2}-\varepsilon_{p3})^2+(\varepsilon_{p3}-\varepsilon_{p1})^2} \tag{5.66}$$

The resulting effective stress-effective strain curve is independent of the state of stress and is used to estimate the stress-strain curves for other states of stress. Of particular importance for are equations which allow correlation of plasticity relations for tension and torsion such that:

$$\tau = K_\tau \gamma^n \tag{5.67}$$

where $K_\tau = \frac{K}{3^{(n+1)/2}}$ in which K and n are the strength coefficient and strain hardening exponent from the tension test, respectively. In addition,

$$\tau_o = \frac{\sigma_o}{\sqrt{3}} \tag{5.68}$$

where σ_o is the yield strength from a tension test.

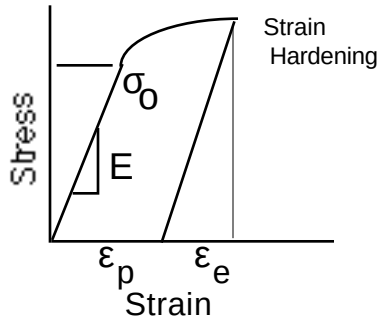


Figure 5.20 Elastic-plastic stress-strain curve