REVIEWS

Crop Circles – The Hidden Form

By Nick Kollerstrom

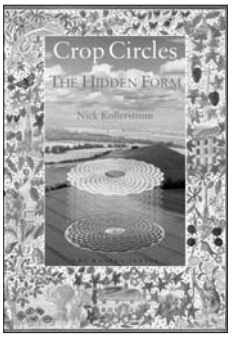
'For the Schoolchildren of tomorrow. Let us imagine a maths teacher who could not endure the thought that Art and Science should be separate; and who demanded that Delight and Wonder should be the guiding principles.'

So starts this wonderful book that gives the reader an insight into the mathematics of sixty crop circles, three-quarters of which were laid down between 1997 and 2000. The crop circles are described with beautiful colour photographs and diagrams. Every formation has a name and its date of construction and the location are given along with a description and an interesting commentary. There are circles, circles within circles, intersecting circles, touching circles, interlinked circles not to mention triangles, squares, pentagons, hexagons, heptagons and spirals. Moving into 3-dimensions we find cubes and spheres, fractals and interference patterns, a wealth of mathematics from basic shape, symmetry, rotation and ratio to trigonometry and hyperbolic functions. All this sparks the readers intrigue of the where and when of crop circle formation along with the much more baffling and philosophical how and why?

This is certainly a book to inspire your students and thoughts for lessons that immediately came into my head included – Can you describe the patterns made? Can you compare them? What features are the same? What features are different? Can you construct on paper the patterns made? How about in a field? There is an opportunity for statistical analysis of the dates, days of the week, and location when the crop circles were formed and an extended investigation could include the weather at the time. Another could be to investigate the crop circles formed since this book was written and update the statistical analysis. There are so many questions and activities just waiting to be investigated.

A really lovely book full of inspiration and fascinating detail about crop circles from the very simplest to complex moiré interference patterns. The author, quite deliberately, does not discuss any of the possibilities as to how crop circles appear, leaving the reader to further investigate if they so choose. It certainly offers a lovely way to teach mathematics in addition to promoting students' spiritual, moral, social and cultural development and a must for any school library.

Julie Gibbon is a part time mathematics teacher at Hassockfield Secure Training Centre, Durham.



Crop Circles – The Hidden Form, Nick Kollerstrom, Wessex Books ISBN 1903035112

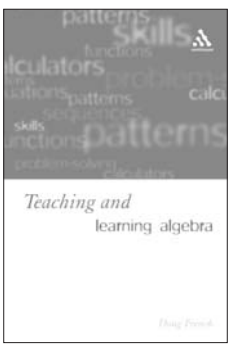
A set of six A2 full-colour posters taken from the book is available at £16 from: Wessex Books, 2 Station Cottages, Newton Toney, Salisbury Wiltshire SP4 0HD; Telephone 01980 629349; **info@wessexbooks.co.uk**
Wessex Books are giving away a **FREE SET** of posters to the first person pulled out of the hat on the 1st of May. To enter, send your name and address on a postcard or e-mail the ATM office, Unit 7 Prime Industrial Park, Shaftesbury Street, Derby DE23 8YB. **admin@atm.org.uk**

Teaching and Learning Algebra

Doug French

We have been waiting for a comprehensive book on this topic for a long time. There have been many research papers and theses written on the topic, but as yet no one has come up with the 'right way' to teach algebra. There is, however, a good deal of evidence of how algebra should *not* be taught. Doug French has done an excellent job in bringing together research evidence and in providing useful suggestions for the classroom.

We can go back to the work of



Teaching and learning algebra, Doug French, Continuum, 2002, ISBN 0 8264 5222 1, PB £18.99, telephone 020 7922 0914, www.continuumbooks.com

Alan Bell and Ken Collis to find early criticisms of 'fruit salad algebra' and a plea for the teaching of algebra to be underpinned by an understanding of how letters and symbols are used in mathematics. It is of no use to offer pupils tasks of 'collecting like terms' until they have a grasp of what the 'like terms' signify. *They never signify 'apples and pears'*. This book has tackled the problem head on and allows no excuses for the typical exercises in current text books such as in ***Key Maths Book 7*** pages 178-9, where the old apples and bananas routine is dragged out again. This teaching strategy is described by Tanner et al. (2002) [1] as 'quick and dirty' because it fails to make mathematical sense, but merely 'keeps the class quiet' for a while. How does this strategy permit the later explanation of the meaning of 2ab?

French recognises that most pupils react negatively towards symbols which they do not understand and which appear to have no useful purpose. Only a few accept the challenge to strive for understanding and realise subsequently the power of algebra to solve all sorts of problems. In his book, French describes all the reasons why algebra is a difficult subject to teach and to learn but provides suggestions for overcoming these difficulties. He gives many examples of appropriate introductory activities as well as harder topics relating to later content from the algebra curriculum.

Chapter 1 describes the crux of the problem and how algebra confirms many pupils' view of mathematics, that it is a set of rules to be remembered rather than as something to be engaged with and thought about.

Chapter 2 provides a review of research referring particularly to the work of Mason, Skemp, Booth and the CSMS team. The main misconceptions in learning algebra are discussed and this background forms the basis for identifying useful strategies for teaching it, one main emphasis being to link numerical and algebraic ideas. A typical example starts with the expression $x^2 - 1$ and equates this to $(x + 1)(x - 1)$. This in turn relates to a graph, then to a picture proof of the identity, and lastly a list of number facts is compared ($8 \times 8 = 64$, $7 \times 9 = 63$, and so on). The chapter concludes with a list summarising the hazards and pitfalls as well as useful advice.

Chapter 3 suggests some ways to start. French has classified these starting points, which include one of my favourites, John Mason's (et al. 1985) [2] THOANs (Think of a number problems), into four introductory approaches:

- expressions and rules for operations
- equation solving
- formulae
- patterns leading to functions

Chapter 4 discusses how we might develop the required algebraic skills in pupils, but emphasises that these skills must not be presented as an end in themselves, they must be seen clearly as a tool for solving problems (which otherwise would be too difficult to solve).

Chapter 5 onwards deal with a variety of topics from the curriculum describing the difficulties which pupils may face and provide suggestions of approaches to take in presenting these topics. The subtitle for Chapter 5 is 'Explaining and proving' and French incorporates some nice pictorial 'proofs' of algebraic expressions. Chapter 7 deals in more depth with puzzles and creating and solving equations. Chapter 11 provides some useful discussion on sequences and series and includes some adapted 'pictures' from Nelsen (1993) [3] to demonstrate and reinforce the structure of formulae

In Chapter 12, French argues that it may be desirable to introduce differentiation and integration quite separately, as ways of calculating gradients of curves and areas under curves respectively, and allowing the relationship between the two concepts to emerge and to come at pupils as a surprise. This sounds like an interesting idea. I await news from a teacher who has tried this strategy (or a pupil

References

1 H. Tanner, S. Jones & A. Davies: ***Developing numeracy in the secondary school***, Fulton, 2002

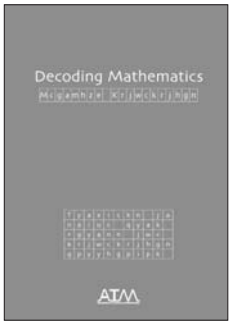
2 J. Mason, A. Graham, D. Pimm & N. Gowar: ***Routes to, roots of algebra***, Open University, 1985

3 R.B. Nelsen: ***Proofs without words: exercises in visual thinking***, Mathematical Association of America, 1993

who has experienced it).

I would urge everyone to read this excellent book, some of the ideas may be familiar ideas, but there are also many new ones, especially on more advanced topics. But crucially, it will serve to remind us all to present algebra, not as the popular text-books do (who do not have the pupils' interests at heart), but how our beliefs and expertise tell us it should be done.

Heather McLeay teaches at the School of Education, University of Wales, Bangor.



Decoding mathematics, Derek Ball, published by ATM, 2003; £10 (£8 for personal members of ATM) ISBN 1 898611 21 1

Decoding Mathematics
Derek Ball

So who's the author here? No mention of a name on the front. On the inside page? By Mcycb Xrii – never heard of him. Some impenetrable Hungarian mathematician? But no…the decoding here starts on the cover. Mcycb Xrii unscrambles to ATM's very own Derek Ball. (Decoding and Derek begin the same way…)

Derek adopts a wide vision of codes, noting that his task here is the reverse of their usual use, which is to introduce a lack of clarity – he is attempting to employ codes to make the mathematics they code clearer to students. And of course, both cracking a code and composing a code can be great fun.

So, to real students with this material. I tried this example very happily with a new AS class.

$$\_\_^2 - \_\_^2 = 280$$

However often I go over the difference of two squares, there are always some students who never quite take it on board. This exercise really does develop an understanding of the factorisation involved, and it brings simultaneous equations in too. Excellent.

Then to a carousel of these activities for my retake Intermediate GCSE class. Not the easiest of groups to win over, yet for the most part they were hooked. A multiplication table with coded digits drew out some fine logical thinking, as did the arithmetic sequences with missing digits. The cross-number puzzles really testing their understanding of vocabulary like *prime*, *multiple* and *cube*, providing a powerful motivation to learn these.

Some comments were revealing: "R times D is MH, so radius times diameter is…" The substitution code problems here are perhaps better approached with Simon Singh's code-breaking CD – with this the students can watch letters being substituted on the computer immediately, without too much drudgery on their part. I should say that the hidden text puzzles in ***Decoding mathematics*** were the least popular: if you got stuck, there was no real logic that you could employ to get any further. And throughout, if a trio of code came close to spelling something rude, then that inevitably rather overtook the maths involved!

There is far more in Derek's book than I have covered here: geometry, fractions, even music. It contains an exciting mix of problems that I shall use again and again. A winner.

Jonny Griffiths, Paston College, North Walsham, Norfolk.

Decoding mathematics contains many different activities using codes to give students mathematically challenging activities. The book begins with some multiplication tables that have been coded using a simple substitution code – ie, each digit has been coded with a unique letter. The tables are not written in order. The students need to work out which letters stand for which digit. This activity would represent an interesting challenge across many different year groups. I expect that a bright Y6 student would cope well, whilst a Y10 student is probably not so familiar with their times tables and could easily find it too hard!

The book continues with some deceptively simple looking cross numbers (with clues such as 'prime number' and 'multiple of 4'). There are some more recognisable coding

activities – pieces of text that have been coded and the reader is invited to decode.

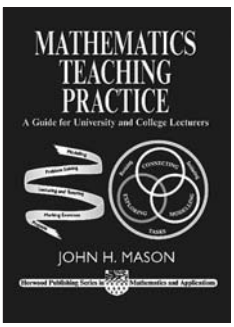
There were a few pages with the title of hidden text where the reader needs to find a missing letter for each box on the page (to decode sentences such as *there are 60 seconds in 1 minute*). I am not quite so convinced of the merit of this section. The activities owe more to literacy than mathematics, but perhaps this is an area we ought to be making more of?

The final few problems relate to the ATM program *Counter* which I have not seen, but the instructions are clear enough and the problems are sufficiently interesting without the program.

Finally the book concludes with some hints for each page of problems. These hints will be sufficient for any student or teacher to be able to solve the problem.

All in all this is an excellent publication and I cannot recommend it strongly enough. I am waiting for the right moment to use it this year – whether with my lower set in Y10 to help us through a long double lesson, or with a Y6 master class session next term.

Peter Hall, Head of Mathematics, Tonbridge Grammar School, Kent.
Go to www.atm.org.uk for more reviews of this publication.



Mathematics teaching practice: a guide for university and college lecturers, John Mason, Horwood Publishing, 2002, ISBN 1898563799, 218 pages, £30

Mathematics Teaching Practice: A Guide for University and College Lecturers

John Mason

John Mason's work with the OU and his considerable influence on teachers' awareness of mathematical thinking and problem solving will be known to many teachers. In this book he draws on his experience over many years to provide a fascinating range of suggestions for making small and medium sized changes in teaching style. It is difficult to imagine any lecturer who would not find a great deal here to stimulate, revitalise and extend his or her own practice.

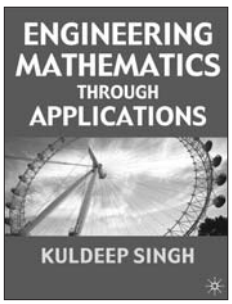
This book is not about teaching practice as the term is used in initial teacher training but about the practice of teaching mathematics, particularly at university level. It assumes that good mathematics teaching should actively engage students in mathematical thinking and should stimulate them to make sense of content. It is not designed to be read from cover to cover but provides a cross-referenced compendium of suggestions for action.

There are sections on lecturing, tutoring, constructing tasks and marking as well as on wider issues that may arise. In all of these the format is to provide a partial diagnosis of student behaviours, to illustrate common teaching problems and to provide tactics. Tactics is a key term in the book and is used to indicate specific actions within teaching, eg, giving students two minutes in pairs to clarify the meaning of a term just introduced.

In the lecturing chapter I found the consideration of 'punctuation' particularly useful. I had always presumed that I largely spoke in sentences and that I left significant pauses to emphasise key points. The use of some of the tactics provided has forced me to question these assumptions and alter my own practice. One very simple tactic suggested to revitalise teaching and force yourself to think on your feet (and to articulate your thinking to students) is simply to change the algebraic notation used in theorems.

The text is peppered with pertinent quotes. My favourite is from Paul Halmos: '*My job is to lecture and yours is to listen. Let me know if you finish first!*' Perhaps greater use of the tactics in this book might help lecturers and students to finish at the same time.

Alan Mclean, Faculty of Education, University of Plymouth.



Engineering mathematics through applications, Kuldeep Singh, ISBN 0-333-92224-7, £24.99

Engineering Mathematics through Applications

Kuldeep Singh

Kuldeep Singh has written this book for anyone doing mathematics as part of their engineering or science undergraduate studies. The unique quality of this book is the wealth of examples applying the mathematical techniques taught here. These examples span mechanics, aerodynamics, electronics, engineering, fluid dynamics and other areas of applied mathematics. These are not just the usual examples involving differential equations and equations of motion, but real and thoughtful applications that will be relevant to the student.

For example, in the chapter, *Visualizing engineering formulas*, the author uses the set of equations $y=x^2+c$ to sketch streamlines in fluid mechanics and the set $y=kx^2$ to illustrate the power dissipated in a resistor. The chapter, *Matrices*, shows how to set up a system of 6 simultaneous equations for a heat transfer problem, and then how to rewrite them as a matrix problem. In very practical engineering fashion (and splendid mathematical practice as far as I am concerned), the student is then exhorted to store the resulting matrix in a graphical calculator and hence obtain the solution.

An engineer studying mathematics with this book would be in no doubt of the worth of mastering particular techniques. These techniques are set out clearly with worked examples, and in addition, complete solutions to all exercises are provided on the book's website: www.palgrave.com/engineering/singh. For this reason, the book is particularly suitable for students working alone or with limited access to tutorial advice.

I am very enthusiastic about this book of over 800 well set out, and readable pages. Not only is the mathematical presentation sound, but I am sure any mathematician would be fascinated by the scope of applications shown here. It would help 'A' level teachers to demonstrate the use and applicability of the mathematics they are teaching and they will find plenty of useful examples here. I am not at all surprised to read that Kuldeep Singh spent six years writing this book. You can find out what it can offer you by looking at sample sections and the worked examples on the web site.

Jill Russell, Part-time Assistant Lecturer with the Open University.

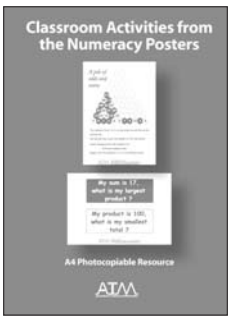
Classroom Activities from the Numeracy Posters – An A4 Photocopiable Masters Pack

ATM

Classroom activities from the numeracy posters contains twelve good quality A4 single sided sheets. This publication stems from the two numeracy poster packs: Set B *Using numbers* VIS007, £9.50 (personal members £7.50) and Set C *How To…* VIS008, £12.50 (personal members £10) – both available from ATM). The use of yellow and pink colouring makes them both bold and attractive. The support sheet gives some good ideas for how to use the posters to stimulate discussions, for thinking about number patterns and developing strategies. There are also suggestions of how the activities might be extended. These would be particularly useful for non-specialist teachers. In the pack there is an emphasis on the development of language through discussion work, in groups and as a class.

The first six posters encourage the pupils to think about how to carry out the task and what they are aiming to find out. For instance in the arithmogons activity they need not only to have a strategy for finding the answer but to recognise that fractions and decimals may be required to get a solution.

The use of the posters is extended as there is a chance for pupils to develop their questioning skills by



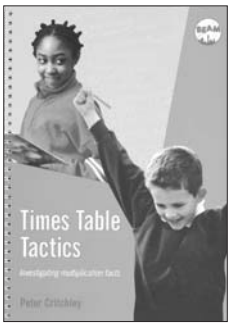
Classroom activities from the numeracy posters – an A4 photocopiable masters pack, ATM, code VIS019, containing 12 posters + support sheet, £9.50; personal members £7.60

posing their own questions as they do not in themselves answer the question, but allow for further development, whether by adding more solutions (eg poster 9 The answer is 2), posing different 'What if…' questions (eg poster 5 Birthday stickers) or being used as a framework (eg poster 7 If… $78 \times 14 = 1092$).

I have used the full size posters as a stimulus for further work with Y6 and Y7 pupils, but found it difficult to get them all to work from the same poster. Having them as photocopiable A4 sheets will overcome this problem, this size giving the option of making them into overheads, being laminated or put into plastic wallets. There is also the benefit in that this allows pupils to write on them without damaging the originals, hence making them reusable and more cost effective.

They would be a useful addition to the resources in both primary and lower school secondary mathematics departments. The posters can be used with different ages/abilities, the formats could be used as a framework for differentiated work or different topics by changing the numbers, including as a style for assessment.

Alison Parish, Suffolk mathematics teacher



Times tables tactics: investigating multiplication facts, Peter Critchley, Published by BEAM Education, Code TTT, ISBN 1 903142 03 2, 96 pages, Price: £14.50 Publisher BEAM e-mail: grahambarker@beam.co.uk Telephone: 020 7684 3323

Times Tables Tactics Investigating multiplication facts

Peter Critchley

Times tables tactics develops the theme that to know your tables you do not need to learn them by rote. The book is full of ideas to help pupils become familiar with multiplication facts to 10×10 and to develop strategies through the relationship of numbers.

The book is divided into two sections (supported with twenty-seven resource sheets), which are cross-referenced to learning objectives from Y3 to Y6. The first section consists of twenty short activities that might take up to 30 minutes and are designed to be used within a lesson. In second part there are nineteen longer activities that are written as plans for the main part of the lesson.

Each activity is accompanied by a heading with a purpose, learning objective and the equipment needed. The tasks themselves are open-ended, and involve the pupils in doing activities as a class, in pairs or individually within the lesson. There are suggested teacher prompt questions, encouraging pupils to think and to develop their own questions and to talk about the mathematics involved in doing the tasks. The longer activities include some suggestions for pupils to work on at home. Beside the activities there is a commentary with extra instructions and helpful hints.

The hints column contains further explanations as to what one might expect.

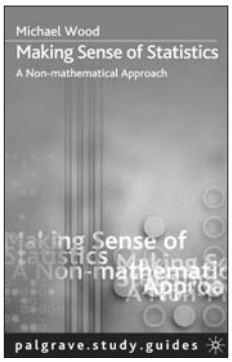
I particularly like the way that pupils are encouraged to manipulate numbers and that they are encouraged to develop strategies (eg Building Blocks (p.32) & Heading into the Great Unknown (p.34) for multiplying two digit numbers by one digit)) that they can extend to dealing with larger numbers. I also liked the ISBN numbers (p.62) to show where multiplication is in practical use and for the fact that calculator-based activities can enhance the confidence in number. Many of the ideas in the book can be adapted or extended for work with different groups.

A sample of activities has been tried with a range of pupils from Y3 to Y8. They enjoyed the alternative approach to what could be one of the drier areas of mathematical learning. Testing is not forgotten, however, but it is made more interesting by using a variety of methods that allow for both individual and whole-class testing (p.42) and assessments to be made.

A very useful resource for primary schools and would certainly not be out of place in high schools for work with the younger and less able pupils and could be used by specialist and non-

specialist teachers alike. A friend in a primary school also tried some of the tasks and said she will buy a copy for her school. A good recommendation. There will certainly be a copy on our bookshelf.

Alison Parish, Suffolk mathematics teacher.



Making sense of statistics: a non-mathematical approach, Michael Wood, Palgrave Study Guide, ISBN 1403901074, £11.99

Making Sense of Statistics: A non-mathematical approach

Michael Wood

This is a very interesting book. Wood is attempting to teach statistics in a non-mathematical way. He claims that 'mathematical statistics does not work satisfactorily, except for the experts'. In this book Wood aims to help the reader to 'get the answer with formulae by using concepts and methods that are simpler and more direct than the conventional ones'.

From the beginning Wood discusses probability and uses the simple concept of balls in a bucket to make his point. He gives a clear explanation using his balls-in-bucket model to give some understanding behind the classic problem 'How many people do you need so that the probability that two share the same birthday is 95%?' He relies a great deal on the reader's understanding of Microsoft Excel (or SPSS) and he provides example data on the associated website.

The book explains the concept of modelling very well indeed, and would make it worth buying just for this part alone. Wood also explains, very clearly, some common statistical misconceptions – such as the Aldermaston cancer cluster and the Sally Clark murder trial. The reader should understand clearly the difficulties involved in conditional probability.

However, I found some of the later chapters rather harder to grasp. When discussing quartiles Wood writes, 'Excel and SPSS have exact definitions of quartiles and percentages built in … you will probably find that you get different answers because these packages use more subtle methods than the method above'. At this point the mathematician in me became rather annoyed. My experience of non-mathematicians is that they tend to prefer mathematics to produce a single right answer. Wood completely skips over the difficulties involved in calculating quartiles, and why different packages should use a different method. (Incidentally the Excel method does take some explaining – and doesn't appear to agree with the standard A-level method.)

In the correlation chapter he uses the Kendal correlation co-efficient. He makes a good attempt to explain it, but again the mathematician in me is crying out for a formula (and sadly since Excel doesn't know how to calculate it, I couldn't experiment easily). Wood is very clear with the interpretation of the co-efficient so perhaps it doesn't really matter which one he uses.

In all, this book is fascinating and I would recommend it wholeheartedly. It would make an excellent book to lend to weaker students to cope with A-level statistics modules, or perhaps to those studying geography, economics or psychology who would like to understand statistics a little more, but whose mathematics is not really up to the conventional explanations. There are clearly far too many misunderstandings concerning statistics and this book presents a very clear way of overcoming them.

Peter Hall, Head of Mathematics, Tonbridge Grammar School, Kent.

The Association of Teachers of Mathematics
for mathematics educators
primary, secondary and higher

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