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Attenuation in Left-handed Waveguide Structure by Equivalent Current Theory Method

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Abstract In this work, the propagation and attenuation characteristics of both TE and TM waves in a waveguide structure consisting of left handed material (LHM) film by using the equivalent current theory of optical waveguide coupling method have been derived and obtained. The dispersion relations and the attenuation coefficient were numerically solved for a given set of parameters: allowed phase angles; core's thicknesses; and propagation constants. We found that lower attenuation is realized for higher propagation constants. Moreover, attenuation coefficient has same small positive values for all thickness in phase angles range of values ($0^0 - 57^0$). Besides that, the attenuation decreases to negative values with thickness increase in phase angles range of values $57^0 - 90^0$) which means a gain of the wave is achieved for wider buffer layer and at larger phase angles. We also found that, TE waves have lower attenuation than that of TM waves.

Keywords: *attenuation, dispersion, equivalent current theory, left handed material*

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1. Introduction

In light –wave technology many analytical methods and theories have been developed for various optical waveguide components. For example, there are the plane-wave analytical method [1,2] and the reciprocity relation analytical theorem [3]. Many of these analytical methods are famous and of fundamental significance and many calculations formulas obtained by these analytical methods are complex and are not easy to use in calculations and designs. The propagation attenuation of a metal-clad optical waveguide for TE waves is different from that TM waves and recently this large TM to TE loss ratio is used to produce a polarizer or mode analyzer for integrated-optics circuits [4], therefore the analysis of the propagation characteristics of the metal-clad optical waveguide has aroused great interest from many scholars of light-wave technology. Yizun [5] introduced a unified theory called equivalent-current theory that can be used successfully to analyze a variety of optical waveguide components. It was shown that this theory can be used systematically to analyze a series of optical waveguide problems that were previously solved by analytical methods. They applied this theory to the derivation of the general optical waveguide coupling equations and the analysis of a grating coupler and a metal-clad optical waveguide. A new fundamental formula is obtained for calculating the attenuation coefficient of the metal-clad optical waveguide. The obtained results are difficult to obtain by other analytical methods. This formula make the calculation and design of polarizers easy[6]. Rectangular waveguides are employed extensively in microwave receiver [7] since they are much easier to manipulate than circular waveguides and also offer significantly lower cross polarization component. Much studies have been carried out on waveguide structure by using the analytical method directly from the Maxwell equations. Yeap et.al.[8] had proposed a fundamental technique to compute the attenuation in rectangular waveguides with finite conductivity walls. Mousa and Shabat [9,10,11] have examined the propagation characteristics of both nonlinear TE and TM surface waves in a left-handed material (LHM) or metamaterial waveguide structures. These metamaterials grasped great attention of many researchers' worldwide, because of the peculiar characteristics and novel devices which can be built upon. Interest is focused on the propagation of electromagnetic waves in LHM with negative index of refraction: materials which are designed to exhibit both negative permeability and permittivity over predetermined range of frequencies [9]. Metamaterials or left handed materials are used in many potential applications as fabricating Transmission lines, Mi-crostrip Resonators, wave division multiplexors (WDM), Couplers, Resonators, and Antennas [12,13]. S. Zhang et.al. [14] numerically demonstrated a metamaterial with both negative permittivity and negative permeability over an over lapping near-infrared wavelength range resulting in a low loss negative-index material and thus a much higher transmission, which will lead to more extensive applications. Furthermore, the proposed structure has a minimum feature size of ~ 100 nm. In this paper, we use this LHM in clad film waveguide to calculate the attenuation coefficient of both TE and TM mode by using equivalent current theory of optical waveguide coupling.

2. TE Analysis of a LHM-clad Optical Waveguide

As shown in Fig.(1), the proposed structure consists of four dielectric layers of refractive indices n_1, n_2, n_3, n_4. The clad layer of the waveguide is LHM. It has complex refractive index $n_4 = -\sqrt{\varepsilon_h \mu_h}$ +2i where ε_h is its electric permittivity and μ_h is its magnetic permeability.

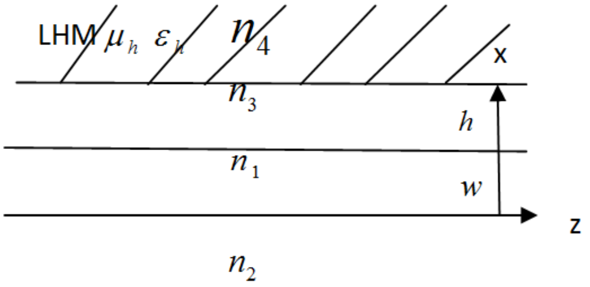

Figure 1. LHM clad waveguide structure

Assuming that Transverse electric (TE) waves propagate in the z direction with a propagation wave constant in the form $\exp[-i(\beta_1 z)]$. The guided wave field of the film waveguide without the LHM cover can be expressed as[5]:

$$E_1 = E_{1y} = \begin{bmatrix} E_0 \cos(k_1 x - \varphi_0) e^{-i\beta_1 z} & 0 \prec x \prec w \\ E_0 \cos(\varphi_0) e^{k_2 x} e^{-i\beta_1 z} & x \prec 0 \\ E_0 \cos(k_1 w - \varphi_0) e^{-k_3(x-w)} e^{-i\beta_1 z} & x \succ w \end{bmatrix} \quad (1)$$

where

$$k_1^2 = n_1^2 k_0^2 - \beta_1^2, k_2^2 = \beta_1^2 - n_2^2 k_0^2, k_3^2 = \beta_1^2 - n_3^2 k_0^2, \quad (2)$$

and β_1 is the propagation constant, φ_0 is the initial phase angle and E_0 is the field normalization parameter. It is found as:

$$E_0 = \left[\frac{4\omega\mu_0}{\beta_1 \left(w + \frac{1}{k_2} + \frac{1}{k_3} \right)} \right]^{1/2}$$

According to Maxwell's equations, the magnetic field is $H_z = (i/\omega\mu_0\mu)\frac{\partial E_{1y}}{\partial x}$. The layer of refractive index n_1 and that of n_2 and n_3 are nonmagnetic where the magnetic permeability $\mu = 1$. The continuity of H_z at the boundary $x = 0$ leads to the following :

$$k_1 \sin(\varphi_0) = k_2 \cos(\varphi_0) \quad (3a)$$

Where the dispersion equation is:

$$\tan(\varphi_0) = \frac{k_2}{k_1} \quad (3b)$$

By substituting Eq.(2) into Eq.(3b), Eq.(3b) is written as:

$$\beta_1^2 = \frac{\tan^2(\varphi_0) n_1^2 k_0^2 + n_2^2 k_0^2}{1 + \tan^2(\varphi_0)} \quad (4)$$

According to equivalent current theory of optical waveguide coupling, The electromagnetic field of the LHM-clad waveguide shown in Figure 1 is equal to the total result of the field excited by the equivalent wave ($\vec{E}^-$) and the guided wave (E_1), the attenuation coefficient is [5]:

$$\alpha = \operatorname{Re}\left[-\frac{i\omega\varepsilon_0}{4}\left(n_3^2 - n_4^2\right) \iint \vec{E}_1 . \vec{E}^- ds \right].$$

The surface integral is evaluated only in LHM region. The field outside the LHM ($x \le w + h$) remains the same as expressed by Eq.(1) and that the field inside the LHM ($x \succ w + h$) is taken as the transmission wave (E_t) when the field of the guide wave of Eq.(1) is incident upon the LHM boundary. Therefore the fields outside and inside the LHM boundary are obtained as:[5]

$$\begin{aligned} E_{iy} &= E_{0i} \exp[-k_3(x-w)] e^{-i\beta_1 z} && x \le w+h \\ E_{ry} &= E_{0r} \exp[k_3(x-h-w)] e^{-i\beta_1 z} && x \le w+h \quad (5) \\ E_{ty} &= E_{0t} \exp[-ik_4 x] e^{-i\beta_1 z} && x \ge w+h \end{aligned}$$

Where E_{ry} is the reflection wave and E_{iy} is the incident wave with $E_{0i} = E_0 \cos(k_1 x - \varphi_0)$. The continuity of the tangential components E_y and H_z at the boundary $(x = w + h)$ leads to the following equations:

$$E_{iy} + E_{ry} = E_{ty}, H_{iz} + H_{rz} = H_{tz}. \quad (6)$$

And

$$E_{0i} \exp[-k_3 h] + E_{0r} = E_{0t} \exp[-ik_4(w+h)] \quad (7)$$

$$-k_3 E_{0i} \exp[-k_3 h] + k_3 E_{0r} = \frac{-ik_4}{\mu_h} E_{0t} \exp[-ik_4(w+h)] \quad (8)$$

By Eq.(7) and Eq.(8), one obtains:

$$\frac{E_{0t}}{E_{0i}} = T_e \frac{\exp[-k_3 h]}{\exp[-ik_4(w+h)]} \quad (9)$$

With $T_e = \dfrac{2\mu_h k_3}{\mu_h k_3 + ik_4}$.

By substituting Eq.(9) into Eq.(5) one gets:

$$E = E_{ty} = T_e E_{0i} \exp[-k_3 h] \exp[-ik_4(x - w - h)] e^{-i\beta_1 z} \quad (10)$$

With $E_{0i} = E_0 \cos(k_1 x - \varphi_0)$.

For the film waveguide the attenuation becomes:

$$\alpha = \operatorname{Re}\left[-\frac{i\omega\varepsilon_0}{4}\left(n_3^2 - n_4^2\right) \int_{h+w}^{\infty} E_{1y} E_{ty} ds \right] \quad (11)$$

By substituting E_{ty} from Eq.(10) and E_{1y} from Eq.(1) into Eq.(11), the attenuation is:

$$\alpha_{TE} = \frac{k_0^2}{\beta_1\left(w+\frac{1}{k_2}+\frac{1}{k_3}\right)} \frac{k_1^2}{k_1^2+k_3^2} \frac{\exp(-2k_3 h)}{2k_3} \operatorname{Im}(n_3^2-n_4^2)T_e^2 \quad (12)$$

3. TM Analysis of a LHM-clad Optical Waveguide

Assuming that the guided wave field of the film waveguide without the LHM cover can be expressed as: [6]

$$H_{1y} = \begin{bmatrix} H_0\cos(k_1x-\varphi_0)e^{-i\beta_1 z} & 0\le x\le w \\ H_0\cos(\varphi_0)e^{k_2x}e^{-i\beta_1 z} & x\le 0 \\ H_0\cos(k_1w-\varphi_0)e^{-k_3(x-w)}e^{-i\beta_1 z} & x\ge w \end{bmatrix} \quad (13a)$$

The components of electrical field vector $\vec{E}_1$ are:

$$E_{1z} = \begin{bmatrix} \frac{-ik_1}{\omega\varepsilon_0 n_1^2}H_0\sin(k_1x-\varphi_0)e^{-i\beta_1 z} & 0\le x\le w \\ \frac{-ik_2}{\omega\varepsilon_0 n_2^2}H_0\cos(\varphi_0)e^{k_2x}e^{-i\beta_1 z} & x\le 0 \\ \frac{ik_3}{\omega\varepsilon_0 n_3^2}H_0\cos(k_1w-\varphi_0)e^{-k_3(x-w)}e^{-i\beta_1 z} & x\ge w \end{bmatrix},$$

$$E_{1x} = \begin{bmatrix} \frac{\beta_1}{\omega\varepsilon_0 n_1^2}H_0\cos(k_1x-\varphi_0)e^{-i\beta_1 z} & 0\le x\le w \\ \frac{\beta_1}{\omega\varepsilon_0 n_2^2}H_0\cos(\varphi_0)e^{k_2x}e^{-i\beta_1 z} & x\le 0 \\ \frac{\beta_1}{\omega\varepsilon_0 n_3^2}H_0\cos(k_1w-\varphi_0)e^{-k_3(x-w)}e^{-i\beta_1 z} & x\ge w \end{bmatrix} \quad (13b)$$

The field outside the LHM ($x\le w+h$) remains the same as expressed by Eq.(13) and that the field inside the LHM ($x \succ w+h$) is taken as the transmission wave (H_t) when the field of the guide wave of Eq.(13) is incident upon the LHM boundary. Therefore the fields outside and inside the LHM boundary are obtained as [6]:

$$\begin{aligned} H_{iy} &= H_{0i}\exp[-k_3(x-w)]e^{-i\beta_1 z} && x\le w+h \\ H_{ry} &= H_{0r}\exp[k_3(x-h-w)]e^{-i\beta_1 z} && x\le w+h \quad (14) \\ H_{ty} &= H_{0t}\exp[-ik_4x]e^{-i\beta_1 z} && x\ge w+h \end{aligned}$$

Where H_{ry} is the reflection wave and H_{iy} is the incident wave with $H_{0i} = H_0\cos(k_1x-\varphi_0)$. The continuity of tangential components H_y and E_z at the boundary $(x=w+h)$ leads to the following equations

$$H_{iy}+H_{ry} = H_{ty}, E_{iz}+E_{rz} = E_{tz}. \quad (15)$$

$$H_{0i}\exp[-k_3h]+H_{0r} = H_{0t}\exp[-ik_4(w+h)] \quad (16)$$

$$\begin{aligned} &-k_3H_{0i}\exp[-k_3h]+k_3H_{0r} \\ &= \frac{-ik_4\varepsilon_3}{\varepsilon_h}E_{0t}\exp[-ik_4(w+h)] \end{aligned} \quad (17)$$

By Eq.(16) and Eq.(17), one gets:

$$\frac{H_{0t}}{H_{0i}} = T_m\frac{\exp[-k_3h]}{\exp[-ik_4(w+h)]} \quad (18)$$

With

$$T_m = \frac{2\varepsilon_h k_3}{\varepsilon_h k_3+\varepsilon_3 ik_4} = \frac{2k_3}{k_3+\frac{n_3^2}{n_4^2}ik_4}$$

where $\frac{\varepsilon_3}{\varepsilon_h} = \frac{n_3^2}{n_4^2}$, and ε_3 is the permittivity of the third layer. By substituting Eq.(18) into Eq.(14) one gets:

$$H_{ty} = T_mH_{0i}\exp[-k_3h]\exp[-ik_4(x-w-h)]e^{-i\beta_1 z} \quad (19)$$

With $H_{0i} = H_0\cos(k_1x-\varphi_0)$.

The components of the transmitted electric field $\vec{E}$ are:

$$\begin{aligned} E_z &= \frac{-i}{\omega\varepsilon_0 n_4^2}\frac{\partial H_{ty}}{\partial x} = \frac{-k_4}{\omega\varepsilon_0 n_4^2}H_{ty}, \\ E_x &= \frac{i}{\omega\varepsilon_0 n_4^2}\frac{\partial H_{ty}}{\partial z} = \frac{\beta_1}{\omega\varepsilon_0 n_4^2}H_{ty} \end{aligned} \quad (20)$$

According to equivalent current theory of optical waveguide coupling, the attenuation becomes[6]:

$$\alpha = \operatorname{Im}\left[\frac{\omega\varepsilon_0}{2}\left(n_3^2-n_4^2\right)\int_{h+w}^{\infty}\vec{E}.\vec{E}_1^- ds\right] \quad (21)$$

By substituting $\vec{E}$ from Eq.(20) and $\vec{E}_1$ from Eq.(13b) into Eq.(21), the attenuation is:

$$\alpha_{TM} = \frac{2k_1^2}{D\beta_1}\frac{\exp(-2k_3h)}{k_1^2+\left(\frac{n_1}{n_3}\right)^4 k_3^2}\operatorname{Im}\left[\frac{(n_3^2-n_4^2)}{n_3^2n_4^2}\frac{\beta_1^2+ik_3k_4}{k_3+ik_4}T_m\right] \quad (22)$$

where

$$D = \frac{w}{n_1^2}+\frac{1}{n_2^2k_2}\left(\frac{k_1^2+k_2^2}{k_1^2+\left(\frac{n_1^4}{n_2^4}\right)k_2^2}\right)+\frac{1}{n_3^2k_3}\left(\frac{k_1^2+k_3^2}{k_1^2+\left(\frac{n_1^4}{n_3^4}\right)k_3^2}\right).$$

4. Numerical Results and Discussion

In the present work, The parameters were used in carrying out the numerical calculations are: $n_1=1.95, n_2=1, n_3=1.45$. Near infrared frequencies, such as 162 THz ($\lambda=1.85\mu m$), LHM has $\varepsilon_h=-14$, $\mu_h=-1$, $n_4=-3.47+i2$ [14]. The dispersion relation, Equation (4), numerically solved to compute the propagation constant for different values of phase angle φ_0 in a complete cycle. Figure(2) shows that the propagation constant β_1 increases to the value of $6.4x10^6 m^{-1}$ with φ_0 increase up to the value of 90^0 and then decreases to the value of $3.3x10^6 m^{-1}$ at $\varphi_0 = 180^0$. The same behavior of β_1 is repeated for φ_0 values of 180^0 to 360^0. In Fig.(3), we plot the attenuation coefficient of TE waves α_{TE} versus the propagation constant β_1 at different buffer layer's thickness h. It is noticed that log (α_{TE}) decreases to the value of $0.7cm^{-1}$ with β_1 increase to the value of $4.8x10^6 m^{-1}$ in the phase angle range of values ($0^0 - 57^0$) and then increases sharply to the value of $2.4cm^{-1}$ where there is depression in α_{TE} around β_1 of value $4.8x10^6 m^{-1}$ which means the Lower attenuation is realized for higher propagation constants. Besides that, Fig. 3 shows that α_{TE} values decrease and turns to negative with thickness h increase to the values of ($0.4\mu m, 0.7\mu m, 1\mu m, 1.5\mu m$)in the phase angle range of values ($57^0 - 90^0$). Negative α_{TE} means that there is a gain in the wave which achieved for wider buffer layer and at larger phase angles and higher propagation constants. Figure(4) describes the variation of α_{TE} versus the buffer layer's thickness h for a series values of φ_0. It displays the value of α_{TE} coefficient decreases with buffer layer's thickness increase and turns to have negative values. At $h = 2\mu m$ as φ_0 increases to the values of ($40^0, 50^0, 80^0$), log(α_{TE}) decreases to the values of ($1cm^{-1}$,-0.8 cm^{-1}, -4.4 cm^{-1}). This confirms the previous results. The attenuation coefficient of TM waves α_{TM} versus the propagation constant β_1 for increasing buffer layer's thickness h is shown in Fig.(5). As a comparison between Fig.(3) and Fig.(5), TE waves have lower attenuation than TM waves where at $h = 1.5\mu m$, $\beta_1 = 4.9x10^6 m^{-1}$, log(α_{TE}) = 1.1 cm^{-1} and log(α_{TM}) = 2.6 cm^{-1}. This is because of the more deeply inverse relation between TE attenuation and β_1 than that of TM and β_1 as observed by Eq.(12) and Eq.(22). The behavior of α_{TM} seems to be similar to α_{TE} except there is another depression in α_{TM} at β_1 of value $4.5x10^6 m^{-1}$.

5. Conclusions

By using the equivalent current theory, we derived the modal dispersion relation and attenuation coefficient for both TE and TM modes for lossy left-handed material (LHM) waveguide in THz range of electromagnetic wave. The numerical solutions showed that the attenuation decreased with propagation constants increase. The waveguide structure offers good guiding for all thickness in the phase angles range ($0^0 - 90^0$) . This waveguide structure is a good candidate for coupling or guiding TE electromagnetic waves more than TM waves.

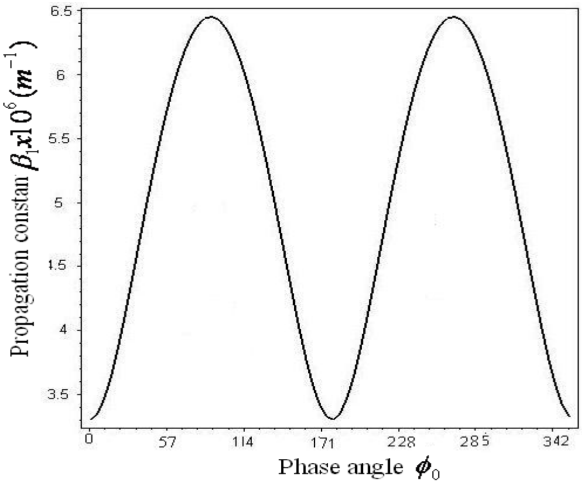

Figure 2. Dispersion curve of TE waves for $n_1 = 1.95, n_2 = 1,$ $n_3 = 1.45$ $n_4 = -3.47 + i2$, $w = 0.5\mu m,$ $\lambda = 1.9\mu m$

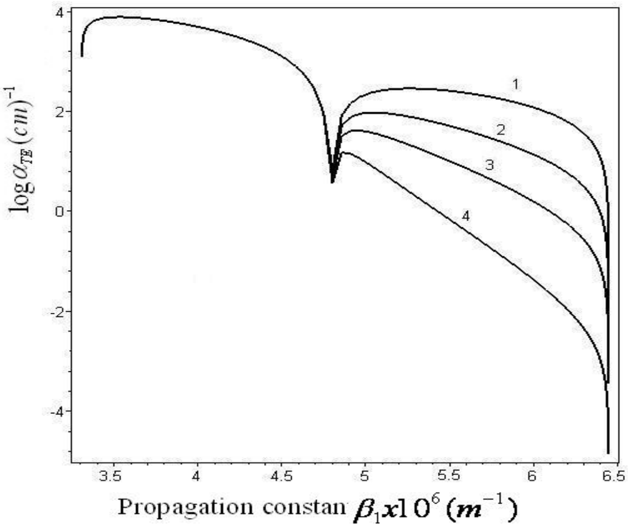

Figure 3. Attenuation of TE waves versus propagation constant β_1 (1) $h = 0.4\mu m,$ (2) $h = 0.7\mu m,$ (3) $h = 1\mu m$ (4) $h = 1.5\mu m,$ $\lambda = 1.9\mu m,$ $w = 0.5\mu m,$ $n_1 = 1.95, n_2 = 1, n_3 = 1.45$, $n_4 = -3.47 + i2$

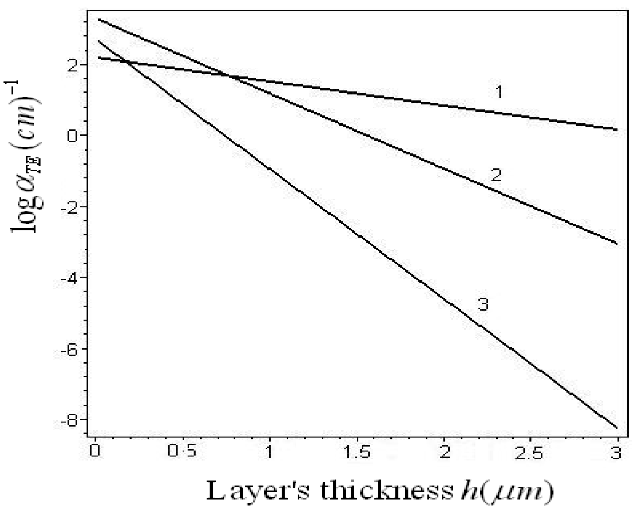

Figure 4. Attenuation of TE waves versus layer's thickness for (1) $\varphi_0 = 40^0$, (2) $\varphi_0 = 50^0$, (3) $\varphi_0 = 80^0$ $n_1 = 1.95,$ $n_2 = 1,$ $n_3 = 1.45$, $n_4 = -3.47 + i2$, $w = 0.5\mu m$, $\lambda = 1.9\mu m$

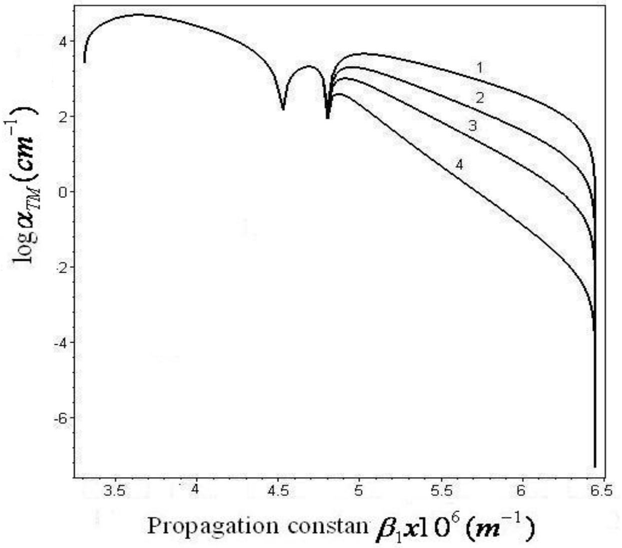

Figure 5. Attenuation of TM waves versus propagation constant β_1 for (1) $h = 0.4\mu m$, $(2) h = 0.7\mu m$, $(3) h = 1\mu m$ $(4) h = 1.5\mu m$, $\lambda = 1.9\mu m$, , $w = 0.5\mu m$, $n_1 = 1.95$, $n_2 = 1$, $n_3 = 1.45$, $n_4 = -3.47 + i2$

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