Mathematics as Language:
Fact or Metaphor?
CEMELA Short Course Package

<u>Course Outline</u>:
Recommended Timeline:
- **Section I**: About 2 hours for developing the overview of formal language
- **Section II**: About 1 hour discussing the ideas from the Moses excerpt
- **Section III**: About 1 hour discussing the relevance of formal language in K-8 Mathematics Education (this is where majority of optional resources are based, so it can last much longer than one hour if you decide to use more of the recommended resources).

Required Resources:
- (Section I) Article: excerpt from David Pimm's book (1)
- (Section I) Activity: The Angle Problem
- (Section II) Article: excerpt from Moses's book (3)
- (Section II) Activity: the Pompeii Problem
- (Section III) Article: excerpt from Tarski's book (5)
- (Setion III) Article: paper by Khisty and Chval (6)

Recommended Resources:
- (Section I) Article: excerpt from D. Marker (2)
- (Section I) Article: paper by Baldwin (11)
- (Section I) Article: excerpt from J. Barwise and J. Etchemendy (4)
- (Section III) Article: paper by K. Koedinger and M. Nathan (7)
- (Section III) Article: paper by L. Healy and C. Hoyles (8)
- (Section III) Article: paper by K. Falkner, L. Levi, and T. Carpenter (9)
- (Section III) Article: slideshow by J. Baldwin (10)

Short Course Abstract:
Courses in mathematical logic routinely study formal languages and develop first order logic as a language adequate for formalizing essentially all of mathematics. In contrast, Pimm, in Speaking Mathematically, insists that `mathematics as a language' is only a metaphor. We discuss the language of arithmetic and algebra as a formal language. Then we will observe how this formulation can inform teaching in K-12. Examples will come from such sources as K-12 texts, Pimm, and the CGI literature on equality. The goal of the presentation is a) to describe the formal language of mathematics and b) to explore how this description can be useful in analyzing classroom discourse and planning curriculum and lessons.

Contents

This package contains two slide show presentations, suggested readings for discussion, and two working activities to motivate the course. The articles included in the package are:

Articles

1. D. Pimm: 1987, *Speaking Mathematically*. London: Routledge. (Abridged readings relevant to this class: page xiv, page 20, pages 95-96, pages 161-169).
2. D. Marker: 2002. *Model Theory*. Springer Press (Abridged readings for this class: pages 7-14)
3. R. Moses: 2001. *Radical Equations: Math Literacy and Civil Rights*. Beacon Press. (Abridged readings for this class: Appendix).
4. J. Barwise and J. Etchemendy: 1999. *Logic, Proof, and Logic*:. CSLI Publications. (Abridged readings relevant to this class: pages 1-5).
5. A. Tarski: 1941. *Introduction to Logic and to the methodology of the deductive sciences*. Oxford University Press. (Abridged readings for this class: chapter 1: On the use of variables pages 3-14).
6. L. L. Khisty and K. Chval: 2002. *Pedagogic Discourse and Equity in Mathematics: When Teachers Talk Matters*. Math Ed. Research Journal (14) 2002 pp 154-168.
7. K. Koedinger and M. Nathan: 2004. *The Real Story Behind Story Problems: Effects of Representations on Quantitative Reasoning*. Journal of the Learning Sciences, v13 n2 p129-164.
8. L. Healy and C. Hoyles: 2000. *A Study of Proof Conceptions in Algebra*. Journal for Research in Mathematics Education (31) pp 396-428.
9. K. Falkner, L. Levi, and T. Carpenter: 1999. *Children's understanding of Equality: A foundation for Algebra*. Teaching Children Mathematics 6(4), 232-236.
10. J. Baldwin. Geometry and Proof (Slideshow based on a series of geometry talks given to in-service teachers)
11. J. Baldwin. Variables: Syntax, Semantics, and Situations (preprint) , http://www2.math.uic.edu/~jbaldwin/mathed.html

The two activities included in this package are:

1. The Angle Problem- provided to motivate discussion on the possibility of misinterpreting mathematical notation
2. The Pompeii Problem- provided to motivate discussion around the ideas brought up in the excerpt by R. Moses and to focus on the necessity for algebra teachers to label variables correctly.

Notes for the slides

This is a section-by-section summary of the slides. The slides are fairly modular and can be moved around or deleted to better suit the needs of your course. You can use this summary as a script when presenting, or a quick reference to help decide which slides to include or exclude.

Framing the issues (slides 1-6)

- *A Language of / for mathematics*: The quote from Pimm was given to contrast his view with that expounded in the lectures. In these lectures the notion of a mathematical language is taken literally.
- *Alternatively*: The quote from Moses provides a different perspective of mathematical language as being regimented discourse and describes one step on the path from natural language to fully formalized mathematics.
- *Goals*: The goals of this short class are listed on slide 4 of the first slide set;, any of the three goals can be given greater focus and attention.
- *Outline*: (self explanatory)
- *History of Formal Mathematics*: Formal languages are purely syntactical notions, and need not have any meaning. They are formed by a set (the alphabet) and a formal grammar (a means of combining the alphabet). The classification of formal languages are (now a days) more commonly studied in logic, computer science, and linguistics. The specific formal languages of first order logic are designed to encompass mathematics. They involve both a syntactic and a semantic component (assigning meaning to the syntactical expressions). To formalize mathematics requires two further notions: axioms and rules of proof. Paper 11 sketches some of the history of formal mathematics, particularly the notion of variable.

Section I

The majority of the slides in the Formal Language .pdf file address formal language. The slides and readings are meant to provide a definition for formal language which mathematicians, specifically logicians, use, and introduce how formal language is used in establishing truth, proof, and validity.

Slide 7: The Gallup poll example is to show that people do not interpret natural language using a compositional account of truth.

Slide 11 emphasizes the distinction between the names we use for numbers and the numbers. This distinction exists regardless of the ontological commitments various philosophical views may make.

Slide 14 introduces the crucial distinction between expressions that denote numbers and those that have truth values.

The crucial distinction of Slide 15 is between elements of the language (syntax) and the underlying numbers (semantics). The warning on slide 16 is to not get bogged down in the words numeral and number.

Slide 19 begins the distinction between sentences and open formulas.

The notion of a formal language is then explained precisely. A more introductory version to this topic (specific to high school algebra) was written later and is in reference 11.

Slide 31 begins the discussion of the distinction between two uses of the equal sign: an evaluation operator and a relation. The references to the cognitively guided instruction analysis (reference 9) emphasize the significance of this distinction at the earliest level.

Section II

This section begins with Slide 37 of the first set of slides.

This section draws heavily on the appendix from Moses's book. The main goal of this section is to compare everyday, feature, and mathematical language.

Slide 62 goes back to the formal development with a short explanation of Godel's completeness theorem. This material is repeated and expanded in slides 18-42 of the second set of slides.

Slides 69-74 are supplementary.

Section III

This brings the notion of formal language back into the K-8 classroom as it moves the discussion to how understanding formal language is used in mathematics can lead to richer learning environments in the classroom.

Slides 1-17 reprise the formal background.

18-42 of the second set of slides reprise the formal development.

Slide 42-51 provide examples of the difficulty of translating from natural to formal language.

Anyone interested in further details or developing such a course can contact John Baldwin at jbaldwin@uic.edu.