Optimal Minimum Wage Policy in Competitive Labor Markets*

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Abstract

This paper provides a theoretical analysis of optimal minimum wage policy in a perfectly competitive labor market and obtains two key results. First, we show that a binding minimum wage – while leading to unemployment – is nevertheless desirable if the government values redistribution toward low wage workers and if unemployment induced by the minimum wage hits the lowest surplus workers first. Importantly, this result remains true in the presence of optimal nonlinear taxes and transfers. In that context, a binding minimum wage enhances the effectiveness of transfers to low-skilled workers as it prevents low-skilled wages from falling through incidence effects. Second, when labor supply responses are along the extensive margin only, the co-existence of a minimum wage with a positive tax rate on low-skilled work is always (second-best) Pareto inefficient. A Pareto improving policy consists of reducing the pre-tax minimum wage while keeping constant the post-tax minimum wage by increasing transfers to low-skilled workers, and financing this reform by increasing taxes on higher paid workers. Overall, our results imply that the minimum wage and subsidies for low-skilled workers are complementary policies.

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1 Introduction

The minimum wage is a widely used but controversial policy tool. A minimum wage can increase low-skilled workers' wages at the expense of other factors of production–such as higher skilled workers or capital–and hence can be potentially useful for redistribution. However, it may also lead to involuntary unemployment, thereby worsening the welfare of workers who lose their jobs. A large empirical literature has studied the extent to which the minimum wage affects the wages and employment of low-skilled workers (see e.g., Card and Krueger (1995), Brown (1999), or Neumark and Wascher (2006) for extensive surveys). The normative literature on the minimum wage, however, is much less extensive.

This paper provides a normative analysis of optimal minimum wage policy in a conventional competitive labor market model, using the standard social welfare framework adopted in the optimal tax theory literature. Our goal is to use this framework to illuminate the trade-offs involved when a government sets a minimum wage, and to shed light on the appropriateness of a minimum wage in the presence of optimal taxes and transfers.

The first part of the paper considers a competitive labor market with no taxes/transfers. Although simple, this analysis does not seem to have been formally derived in the previous literature. We show that a binding minimum wage is desirable as long as the government values redistribution from high- to low-wage workers, the demand elasticity of low-skilled labor is finite, the supply elasticity of low-skilled labor is positive, and most importantly, that the unemployment induced by the minimum wage is efficient, i.e. unemployment hits workers with the lowest surplus first. The intuition is extremely simple: starting from the competitive equilibrium, a small binding minimum wage has a first order effect on distribution but only a second order effect on efficiency as only marginal workers initially lose their job.

The second part of the paper considers the more realistic case where the government also uses taxes and transfers for redistribution. In our model, we abstract from the hours of work decision and focus only on the job choice and work participation decisions. Such a model can capture both participation decisions (the extensive margin) as well as decisions whereby individuals can choose higher paying occupations by exerting more effort (the intensive margin). In that context, the government observes only earnings, but not the utility work costs incurred by individuals.[1] In such a model, we show that a minimum wage is desirable if unemployment

[1]We also show in Section 5 that our results extend to a model with variable hours of work. The theoretical

induced by the minimum wage is efficient and the government values redistribution toward low-skilled workers. The intuition for this result is the following. A binding minimum wage enhances the effectiveness of transfers to low-skilled workers as it prevents low-skilled wages from falling through incidence effects. Theoretically, the minimum wage under efficient rationing sorts individuals into employment and unemployment based on their unobservable cost of work. Thus, the minimum wage partially reveals costs of work in a way that the tax system cannot.[2] Unsurprisingly, if rationing is uniform (i.e., unemployment hits randomly and independently of surplus), then the minimum wage does not reveal anything on costs of work and it cannot improve upon the optimal tax/transfer allocation.

Finally, when labor supply responses are solely along the participation margin, a realistic assumption supported by the empirical labor supply literature, we show that imposing a positive tax rate on the earnings of minimum wage workers is second-best Pareto inefficient. Reducing the minimum wage and compensating low-skilled workers with higher transfers financed by extra taxes on high-skilled workers leads to a Pareto improvement. This result remains true even if rationing is inefficient. Consistent with our result, many OECD countries, which initially had significant minimum wages and high tax rates on low-skilled work, have moved in this direction by reducing payroll taxes on low-skilled work and expanding in-work benefits along the model of the US Earned Income Tax Credit. This result can also be applied to other situations where low skilled wages are downward rigid, for example because of Unions.

There are two strands in the recent normative literature on the minimum wage. The first, most closely associated with labor economics, focuses on efficiency effects of the minimum wage in the presence of labor market imperfections such as monopsonistic competition (Robinson 1933, Manning, 2003, Cahuc and Laroque, 2009), efficiency wages (Drazen, 1986, Jones, 1987, Rebitzer and Taylor, 1995), bargaining models (Cahuc, Zylberberg, and Saint-Martin, 2001), signalling models (Lang, 1987, Blumkin and Sadka, 2005), search models (Swinnerton, 1996, Acemoglu 2001, Flinn, 2006, Hungerbuhler and Lehmann, 2007), Keynesian macro models (Foellmi and Zweimuller, 2007), or endogenous growth models (Cahuc and Michel, 1996). In

drawback of the model with variable hours of work is that although the government observes wage rates to impose the minimum wage, it does not use this information to design the optimal tax, which creates an informational inconsistency in the government decision making.

[2]This result can also be seen as an application of the Guesnerie (1981) and Guesnerie and Roberts (1984) theory of quantity controls in second best economies. When the government values redistribution toward low-skilled workers, the optimal tax system over-encourages the supply of low-skilled labor. A minimum wage effectively rations over-supplied low-skilled labor, which is socially desirable.

many of those of situations, a minimum wage can improve efficiency absent any redistributive consideration. These studies are complementary to our analysis that focuses on the equity-efficiency trade-off under perfect competition.

A second smaller literature in public economics investigates, as we do, whether the minimum wage is desirable for redistributive reasons on top of optimal taxes and transfers. In the context of the two-skilled Stiglitz (1982) model with endogenous and competitive wages, Allen (1987) and Guesnerie and Roberts (1987) show that a minimum wage can sometimes usefully supplement an optimal *linear* tax but is never useful in the presence of an optimal nonlinear tax even in the most favorable case where unemployment is efficiently shared. This result is obtained because, in the Stiglitz (1982) model, a minimum wage does not in any way prevent high-skilled workers from imitating low-skilled workers. A shortcoming of the Stiglitz model, however, is that in the absence of the minimum wage, all individuals work; the participation elasticity of low-skilled workers is essentially zero, whereas a non-trivial participation decision of low-skilled workers strikes us as central to the minimum wage problem in the real world, particularly in how it interacts with a tax and transfer system.[3] Our occupational choice model can naturally focus on the employment participation margin and the economic intuitions behind our results relate to the employment and incidence effects of the minimum wage studied in the empirical literature.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 analyzes optimal minimum wage policy with no taxes. Section 4 extends the analysis to the case with optimal taxes and transfers. Since our main analysis abstracts from the hours-of-work decision, in Section 5, we discuss how our results extend to a model with variable hours of work. Interestingly, efficient rationing of low-skilled work is no longer an assumption, but instead naturally occurs in this context. Section 6 concludes. Formal proofs of our propositions are presented in the appendix.

[3]Marceau and Boadway (1994) build upon those earlier contributions and show that a minimum wage can be desirable when a participation constraint for low-skilled workers is introduced. Although Marceau and Boadway do not explicitly model this participation constraint using fixed costs of work as we do, their paper can be seen as a first step in incorporating the labor force participation decision in the problem.

2 The Model

For our main analysis, we utilize a discrete labor type model of the economy. The supply side follows the optimal tax model presented in Saez (2002), which can be shown to be a discrete version of the seminal optimal tax model of Mirrlees (1971). The key difference is that we explicitly model the demand side to allow for imperfect substitution of labor types. We consider this a necessary extension since, as we discuss below, perfect substitutability – apart from being unrealistic, and unsupported by empirical evidence – trivially results in the minimum wage being undesirable. Importantly however, introducing imperfect substitution in production does not affect the optimal tax analysis (without minimum wage), as originally shown by Diamond and Mirrlees (1971).

2.1 Demand Side

We consider a simple model with two labor inputs where production of a unique consumption good $F(h_1, h_2)$ depends on the number of low-skilled workers h_1 and the number of high-skilled workers h_2. We assume constant returns to scale in production. As we shall see, the production function can be generalized to many labor inputs without affecting the substance of our results. We assume perfectly competitive markets so that firms take wages (w_1, w_2) as given. The production sector chooses labor demand (h_1, h_2) to maximize profits: $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, which leads to the standard first order conditions where wages are equal to marginal product:

$$w_i = \frac{\partial F}{\partial h_i}, \tag{1}$$

for $i = 1, 2$. We will assume that we always have $w_1 < w_2$ in the equilibria we consider.[4]

2.2 Supply Side

Each individual faces costs $\theta = (\theta_1, \theta_2)$ of working in occupations 1 and 2 respectively and has three labor supply options: (1) not work and earn zero, which we henceforth denote occupation 0, (2) work in occupation 1 in the low-skilled sector and earn w_1 at cost θ_1, (3) work in occupation 2 in the high-skilled sector and earn w_2 at cost θ_2. The cost of work vector θ is smoothly distributed across individuals in the population with cumulative distribution $F(\theta)$ and support

[4]This is not a strong assumption. If we had $w_2 < w_1$, then the definition of high-skilled vs. low-skilled would naturally be reversed.

Θ. Heterogeneity in θ reflects heterogeneity in ability and taste for work. Costs θ can also represent the costs of acquiring skills through education so that our framework can also capture long-term human capital investments.

We realistically assume that the government can observe earnings outcomes $(0, w_1, w_2)$, but not the individual costs of work θ. Therefore, the government can condition tax and transfers only on observable earnings outcomes. As there are only three outcomes, we can denote by T_i with $i = 0, 1, 2$ the tax on each of the three possible earnings outcomes $w = 0, w_1, w_2$ and by $c_i = w_i - T_i$ the disposable income in each situation. This represents a fully general nonlinear income tax on earnings as in Saez (2002).

We rule out income effects by assuming that utility is linear.[5] Each individual chooses an occupation $i = 0, 1, 2$ which maximizes utility $u_i = c_i - \theta_i$ (assuming $\theta_0 = 0$ with a slight abuse of notation). Let $\Theta_i = \{\theta \in \Theta | u_i = \max_j u_j\}$ denote the subset of individuals choosing occupation i. Let $h_i(c) = |\Theta_i|$, be the aggregate supply function, i.e., the fraction of the population working in occupation i as a function of the disposable income vector $c = (c_0, c_1, c_2)$. We assume that the distribution $F(\theta)$ is regular enough so that the aggregate supply functions $h_i(c)$ are also smooth. Importantly, our simple discrete labor supply model captures both the extensive and intensive labor supply margins. The extensive margin is for individuals choosing between working and not working, and the intensive margin is for individuals choosing between occupation 1 and occupation 2.

2.3 Competitive Equilibrium

If there is no minimum wage, for given tax parameters T_0, T_1, T_2 defining a tax and transfer system, combining the demand side and the supply side defines the competitive equilibrium $(h_1^*, h_2^*, w_1^*, w_2^*)$.

It is useful to depict the competitive equilibrium for low-skilled labor in our model using the standard supply and demand curve representation as in Figure 1. Both the demand $D_1(w_1)$ and supply $S_1(w_1)$ curves in the low-skilled labor market are defined assuming that the market clears in the high-skilled labor market as we describe in Appendix A.1. Therefore, Figure 1 implicitly captures general equilibrium effects as well. This representation is useful for the analysis of the minimum wage because the demand equation always holds as employers cannot be forced to

[5] Our results do not change if we consider income effects with concave utilities of the form $u(c_i) - \theta$; see appendix B.3 of Lee and Saez, 2008. We rule out effects to simplify the exposition.

hire or layoff workers. The low-skilled labor demand elasticity η_1 is defined as:

$$\eta_1 = -\frac{w_1}{h_1} \cdot D_1'(w_1), \tag{2}$$

where the minus sign normalization is used so that $\eta_1 > 0$.

2.4 Government Social Welfare Objective

Assuming no exogenous spending requirement, the government budget constraint is:[6]

$$h_0 c_0 + h_1 c_1 + h_2 c_2 \leq h_1 w_1 + h_2 w_2. \tag{3}$$

We will denote by λ the Lagrange multiplier associated with this constraint in the government's optimization problem.

As is standard in the optimal taxation literature, we assume that the government evaluates outcomes using a social welfare function of the form: $SW = \int G(u)dF(\theta)$ where $u \to G(u)$ is an increasing and concave transformation of the individual money metric of individual utilities $u = c_i - \theta_i$. The concavity of $G(.)$ represents either individuals' decreasing marginal utility of money and/or the social preferences for redistribution. Given the structure of our model, we can write social welfare as:

$$SW = (1 - h_1 - h_2)G(c_0) + \int_{\Theta_1} G(c_1 - \theta_1)dF(\theta) + \int_{\Theta_2} G(c_2 - \theta_2)dF(\theta), \tag{4}$$

It is useful for our analysis to introduce the concept of social marginal welfare weights for each occupation. Formally, we define $g_0 = G'(c_0)/\lambda$ and $g_i = \int_{\Theta_i} G'(c_i - \theta_i)dF(\theta)/(\lambda \cdot h_i)$ as the average social marginal welfare weight of individuals in occupation $i = 1, 2$. Intuitively, g_i measures the social marginal value of redistributing one dollar uniformly across all individuals in occupation i. In our model, because all individuals have a choice to not work and receive payoff c_0, workers will always be better off than non-workers in equilibrium. Hence concavity of $G(.)$ implies $g_0 > g_1$ and $g_0 > g_2$.

Because of no income effects in labor supply, when the government sets c_0, c_1, c_2 optimally, we have:

$$h_0 g_0 + h_1 g_1 + h_2 g_2 = 1. \tag{5}$$

[6]None of our results would be changed if we assumed a positive exogenous spending requirement for the government.

This can proven as follows. Suppose that the government simultaneously increases c_0, c_1, c_2 all by \$1. Because of no income effects, there are no behavioral responses so that the total fiscal cost is equal to the mechanical fiscal cost of \$1. The social welfare gain (expressed in terms of government funds) is by definition equal to $h_0 g_0 + h_1 g_1 + h_2 g_2$, which proves (**??**).

3 Desirability of the Minimum Wage with no Taxes

Starting from the market equilibrium $(w_1^*, w_2^*, h_1^*, h_2^*)$ illustrated in Figure 1, we introduce a *small* minimum wage just above the low-skilled wage w_1^*, which we denote by $\bar{w} = w_1^* + d\bar{w}$. As shown on Figure 1, the minimum wage creates loss of employment in the low-skilled labor market. Those losing their job because of the minimum wage either become unemployed and earn zero or shift to the high-skilled sector and earn w_2 depending on which occupation was their second-best option. Conceptually, the minimum wage creates an allocation problem: which workers lose their low-skilled job due to the minimum wage? Let us introduce the important assumption of *efficient rationing*.

Assumption 1 ***Efficient Rationing****: Workers who involuntarily lose their low-skilled jobs due to the minimum wage are those with the least surplus from working in the low-skilled sector.*

Obviously, the case with efficient rationing is the most favorable to minimum wage policy, and we will discuss how our results change when this assumption is relaxed. Can the efficient rationing assumption be justified based on empirical evidence? Evidence of unemployment effects of the US minimum wage is stronger among teenagers and secondary earners (Neumark and Wascher 2006) who are likely to be more elastic - and hence have a lower surplus - suggesting that rationing might be efficient. More directly, Luttmer (2007) used variation in state minimum wages and showed that (proxies for) reservation wages do not increase following an increase in the minimum wage, suggesting that minimum wage induced rationing is efficient.[7] We note, however, that even if rationing is found to be efficient empirically, it is still possible that significant resources (such as queuing or search costs) have been dissipated to reach this efficient outcome.[8]

[7] This is in contrast to a situation with low turnover, such as in the housing market with rent control, as in Glaeser and Luttmer (2003).

[8] Therefore, in the presence of significant search frictions, we cannot directly apply our theoretical results and a micro-founded search modelling approach along the lines of Hungerbuhler and Lehmann (2007) is required.

An important theoretical point is that if employers reduce employment by reducing hours of work across the board – instead of laying off workers – then efficient rationing will automatically hold and we would obtain the same conclusions as in our discrete choice labor supply model. We outline such a model in Section 5. Furthermore, we will discuss the relevance of efficient rationing, and how even an infinitesimal degree of labor turnover costs can lead to the efficient rationing outcome in the case with taxes and transfers in Section 4.

Proposition 1 *With no taxes/transfers, if (1) Assumption 1 holds (efficient rationing); (2) the government values redistribution from high-skilled workers toward low-skilled workers ($g_1 > g_2$); (3) the demand elasticity η_1 for low-skilled workers is finite; and (4) the supply elasticity of low-skilled workers is positive, then introducing a minimum wage increases social welfare.*

The formal proof is in Appendix A.2 but a graphical and intuitive proof is provided in Figure 1. The small minimum wage creates changes dw_1, dw_2, dh_1, dh_2 in our key variables of interest. By definition, $dw_1 = d\bar{w}$. From $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2$, we have $d\Pi = \sum_i [(\partial F/\partial h_i) dh_i - w_i dh_i - h_i dw_i] = -h_1 dw_1 - h_2 dw_2$ using (1). The no profit condition $\Pi = 0$ then implies $d\Pi = 0$ and hence:

$$h_1 dw_1 + h_2 dw_2 = 0. \tag{6}$$

Equation (6) is fundamental and shows that the earnings gain of low-skilled workers $h_1 dw_1 > 0$ (the shaded rectangle on Figure 1) due to a small minimum wage is entirely compensated by an earnings loss of high-skilled workers $h_2 dw_2 < 0$. If $g_2 < g_1$, i.e., the government values redistribution from high-skilled workers to low-skilled workers, such a transfer has a first order gain in social welfare equal to $[g_1 - g_2] h_1 dw_1$.[9]

Under efficient rationing, as can be seen in Figure 1, as long as the supply elasticity is positive (non-vertical supply curve) and the demand elasticity is finite (non-horizontal demand curve), those who lose their low-skilled job because of $d\bar{w}$ have infinitesimal surplus. Therefore, the welfare loss due to involuntary unemployment caused by the minimum wage is second order and represented by the shaded triangle, exactly as in the standard Harberger deadweight burden analysis. □

It is useful to briefly analyze the desirability of the minimum wage when any of the four conditions required in Proposition 1 does not hold.

[9]Formally, with no taxes or transfers, the government multiplier λ is not defined. However, we can always define λ such that equation (**??**) holds as λ does not affect the relative ranking of g_1 and g_2.

First and most importantly, if the efficient rationing assumption condition (1) is replaced by uniform rationing (i.e., unemployment strikes independently of surplus), then a small minimum wage creates a first order welfare loss. In that case, a minimum wage may or may not be desirable depending on the parameters of the model (see Lee and Saez, 2008 for a formal analysis of that case).

Condition (2) is necessary. It obviously fails if the government does not care about redistribution at all ($g_1 = g_2$). It also fails in the extreme case where the government has Rawlsian preferences and only cares about those out of work, meaning it values the marginal income of low- and high-skilled workers equally ($g_1 = g_2 = 0$). Therefore, a minimum wage is desirable only for intermediate redistributive tastes.[10]

Condition (3) is also necessary. If the demand elasticity is infinite, which in our model is equivalent to assuming low- and high-skilled workers are perfect substitutes, (so that $F = a_1 h_1 + a_2 h_2$ with fixed parameters a_1, a_2), then any minimum wage set above the competitive wage $w_1^* = a_1$ will completely shut down the low-skilled labor market and therefore cannot be desirable. A large body of empirical work suggests that the demand elasticity for low-skilled labor is not infinite (see Hamermesh, 1996 for a survey). In addition, evidence of a spike in the wage density distribution at the minimum wage also implies a finite demand elasticity (Card and Krueger, 1995).

When condition (4) breaks down and the supply elasticity is zero, then there are no marginal workers with zero surplus from working. Therefore, the unemployment welfare loss is no longer second order. In that context, whether or not a minimum wage is desirable depends on the parameters of the model (specifically, the reservation wages of low-skilled workers and the size of demand elasticity).[11] Empirically, however, a large body of work has shown that there are substantial participation supply elasticities for low-skilled workers (see e.g., Blundell and MaCurdy, 1999 for a survey).

The logic of Proposition 1 easily extends to a more general model with many labor inputs (including a continuum with a smooth wage density), a capital input or pure profits, and many consumption goods. In those contexts, g_2 is the average social welfare weight across each factor

[10] Of course, one way in which condition (1) might fail in practice is if minimum wage workers belong to well-off families (for example teenagers or secondary earners). Kniesner (1981), Johnson and Browning (1983) and Burkhauser, Couch, and Glenn (1996) empirically analyze this issue in the United States.

[11] The well known result that a minimum wage cannot be desirable if $\eta_1 > 1$ is based on such a model with fixed labor supply (see e.g. Freeman, 1996; Dolado, Felgueroso, and Jimeno, 2000).

bearing the incidence of the minimum wage increase.

Finally, it is important to note that a minimum wage cannot be replicated with taxes and transfers. Returning to Figure 1, it is tempting to think that a small tax on low-skilled workers creates the same wedge between supply and demand as the minimum wage. However, to replicate the welfare consequences of the minimum wage, this small tax would have to be rebated lump-sum to low-skilled workers only. But if the tax were rebated to low-skilled workers, those who dropped out of low-skilled work because of the tax would want to come back to work. Thus, without a rationing mechanism preventing this labor supply response, taxes and transfers cannot achieve the minimum wage allocation.

4 Minimum Wage with Taxes and Transfers

In this section, we assume that the government can also use taxes and transfers, i.e., set T_0, T_1, T_2, or equivalently set c_0, c_1, c_2, to maximize social welfare.[12] As mentioned above, absent the minimum wage, this is an optimal income tax model with discrete occupational choices as in Saez (2002). In addition, the model has endogenous wages w_1, w_2 instead of fixed wages as in Saez (2002) but, as is well known since Diamond and Mirrlees (1971), this does not affect optimal tax formulas. Therefore, the optimal tax formulas derived in Saez (2002) continue to apply in our model.

4.1 Minimum Wage Desirability under Efficient Rationing

Proposition 2 *Under Assumption 1 (efficient rationing), assuming $\eta_1 < \infty$, if $g_1 > 1$ at the optimal tax allocation (with no minimum wage), then introducing a minimum wage is desirable. Furthermore, at the joint minimum wage and tax optimum, we have:*

- $g_1 = 1$ *(Full redistribution to low-skilled workers)*
- $h_0 g_0 + h_1 g_1 + h_2 g_2 = 1$ *(Social welfare weights average to one)*

[12] Cahuc and Laroque (2009) point out that a minimum wage can be replicated by a knife-edge nonlinear income tax function such that $T(w) = w$ for $0 < w < \bar{w}$. Under such a scheme, nobody would want to work in a job paying less than $\bar{w}$, employers would be forced to pay at least $\bar{w}$ to attract workers, and there would over supply of work at wage $\bar{w}$. As Cahuc and Laroque do, we consider that such a knife-edge nonlinear tax would effectively be a minimum wage. Our occupation specific tax formulation rules out such knife-edge income taxes. Therefore, we think the definition of the tax and minimum wage tools we use is the most illuminating to understand the problem of joint minimum wage and tax optimization.

The formal proof is in Appendix A.3 (along with a standard (w, c) diagram) but a intuitive illustration of the result is provided in Figure 2. In Panel a, without the minimum wage, an attempt to increase c_1 by dc_1while keeping c_0 and c_2 constant through an increased work subsidy provides incentives for some of the non-workers to start working in occupation 1 (extensive labor supply response) and for some of workers in occupation 2 to switch to occupation 1 (intensive labor supply response). This leads to the familiar result of a reduction in w_1 through demand side effects (as long as $\eta_1 < \infty$).

But consider the same attempt to increase c_1 when the minimum wage was initially set at set at $\bar{w} = w_1^T$, where (w_i^T, c_i^T) is the the optimal tax and transfer system which maximizes social welfare absent the minimum wage. As illustrated in Panel b, in the presence of a minimum wage $\bar{w}$ set at w_1^T, w_1 cannot fall, implying that the labor supply responses are effectively blocked. The assumption of efficient rationing is key here as individuals willing to shift to occupation 1 are precisely those with the lowest surplus from working in occupation 1 relative to their next best option.

Given that the labor supply channel is effectively shut down by the minimum wage, the dc_1 change is like a lump-sum tax reform and its net welfare effect is simply $[g_1 - 1]h_1 dc_1$. This implies that if $g_1 > 1$, introducing a minimum wage improves upon the tax/transfer optimum allocation which proves the first part of the proposition.

Because increasing c_1 is a pure lumpsum transfer, it should be carried out until $g_1 = 1$ which proves the second part. The third part is obtained as in the standard case by considering distributing an extra dollar to every individual. This costs \$1 to the government but generates social welfare equal to $h_0 g_0 + h_1 g_1 + h_2 g_2$ so that the equality holds at the new optimum with minimum wage as well.

Note that Figures 2 (and Figure 3 below) are meant to illustrate the main intuition behind the result, but does not adequately illustrate general equilibrium effects. A rigorous proof, as well as a standard (w, c) diagram are provided in Appendix A.3. □

The proof presented here shows that Proposition 2 remains true even if the starting tax and transfer system is not initially optimized. As long as $g_1 > 1$, the reform described is desirable. The results also naturally carry over to a model with many occupations (instead of two), capital input factors, or many consumption goods, as long as the government can specifically adjust the net price of low-skilled work c_1.

In terms of practical policy recommendation, Proposition 2 implies that additional transfers to low-skilled workers are more effective when low-skilled wages are downward rigid because of a binding minimum wage. In the absence of a rigid wage, exactly as stated in our proof, low-skilled wages would fall and high-skilled wages would rise through wage incidence effects, partially offsetting the initial transfer. Empirically, Rothstein (2009) shows that those incidence effects are not small in the case of the EITC expansions of the 1990s in the United States. With his preferred estimates, he finds that the EITC increases after-tax incomes of low-skilled workers by only \$0.73 per dollar spent. With a binding minimum wage, and under the strong assumption of efficient rationing, Proposition 2 implies that an EITC expansion would increase after-tax incomes of low-skilled workers dollar for dollar.

Compared with the case with no taxes in Section 3, we note that when $g_2 < 1$, the condition $g_1 > 1$ is stronger than the earlier condition $g_1 > g_2$. $g_2 < 1$ is a natural assumption as higher skilled workers are better off than the average. However, if the government desires more redistribution at the no-minimum wage equilibrium, then $g_1 > 1$ is a weak condition as the low-skilled sector can be chosen to represent the very lowest income workers.[13] This also implies that, in the presence of many factors of production or many output goods, the incidence of the minimum wage on other factors (captured by the term g_2 in the case with no taxes) becomes irrelevant. In particular, whether the minimum wage creates neo-classical spill-over effects on slightly higher wages and whether the minimum wage increases prices of goods disproportionately consumed by low income families is irrelevant when assessing the desirability of the minimum wage in the presence of optimal taxes. The only relevant factor is whether the government values redistribution to minimum wage workers relative to an across the board lump-sum redistribution (i.e., the condition $g_1 > 1$).

Finally, the desirability of the minimum wage hinges again crucially on the "efficient rationing" assumption. Under "uniform rationing", where unemployment strikes independently of surplus, the minimum wage cannot improve upon the optimal tax allocation, a point formally proven in Lee and Saez, 2008. Indeed, with efficient rationing, a minimum wage effectively reveals the marginal workers to the government. Since costs of work are unobservable, this is valuable because it allows the government to sort workers into a more socially (albeit not privately) efficient set of occupations, making the minimum wage desirable. In contrast, with

[13]Our model includes only two skills for simplicity but all the results carry over with no change if we assume a large number of skills so that the bottom skill represents very low paid workers.

uniform rationing, as unemployment strikes randomly, a minimum wage does not reveal anything about costs of work. As a result, it only creates (privately) inefficient sorting across occupations without revealing anything of value to the government. It is not surprising that a minimum wages would not be desirable in this context.

It is important to note, however, that "uniform rationing" may not have real-world relevance if there exists some labor turnover costs. That is, even if there is an infinitesimal amount of hiring and firing costs, firms are no longer indifferent about who is rationed: it is always better to adjust to new employment levels with the minimal amount of turnover. Consider the situation of Proposition 2 above, where we began with the just-binding minimum wage and then increased the subsidy. In the initial allocation, only those workers with positive surplus were employed. As we demonstrated above, a marginal increase in c_1 would not lead to a change in h_1. Therefore, in moving to the new equilibirum, any firm that wanted to minimize turnover costs, would not choose to layoff existing workers and replace them with the same number of identical workers. Instead they would keep the same workers – these are precisely the workers that remain employed under efficient rationing.

Finally, our previous result that the optimal minimum wage follows an inverted U-shape pattern with the strength of redistributive tastes also carries over to the case with optimal taxes. Extreme redistributive (Rawlsian) tastes imply that $g_1 = 0 < 1$ and thus no minimum wage is desirable to supplement optimal tax policy. Conversely, no redistributive tastes imply that $g_0 = g_1 = g_2 = 1$, a situation where no minimum wage (nor any tax or transfer) is desirable.

4.2 Pareto Improving Reform

Proposition 2 shows that in the presence of a minimum wage, redistribution to low-skilled workers can be made "lumpsum" in nature, and hence is more desirable. This suggests that the minimum wage and low-skilled work subsidies (such as the EITC in the United States) might be complementary. As we shall see, this is indeed the case when labor supply responses are solely along the extensive margin.

Let us therefore consider a model with only extensive labor supply responses where workers cannot switch from occupation 1 to occupation 2 (and vice-versa). For example, workers are of two types: educated or uneducated. Educated workers can only work in the high-skilled sector and uneducated workers can only work in the low-skilled sector but there remains heterogeneity

in costs of work within education classes. Empirical labor supply studies suggest that the extensive labor supply margin is most important, particularly at the bottom of the distribution (see e.g., Blundell and MaCurdy 1999 for a recent survey), which makes this particular case highly relevant in practice. In this model, we can define the participation tax rate on low-skilled work τ_1 as $1-\tau_1 = (c_1 - c_0)/w_1$ or equivalently $c_1 = c_0 + (1-\tau_1)w_1$, i.e., low-skilled individuals keep only a fraction $1-\tau_1$ of their earnings when they work and earn w_1. We can then prove the following result:

Proposition 3 *In a model with extensive labor supply responses only, a binding minimum wage associated with a positive tax rate on minimum wage earnings ($\tau_1 > 0$) is second-best Pareto inefficient. This result remains a-fortiori true when rationing is not efficient.*

The formal proof is presented in Appendix A.4, along with a standard (w, c) diagram. The intuition behind Proposition 3 is illustrated in Figure 3 which depicts a situation with a binding minimum wage and a positive tax rate on low-skilled work $\tau_1 > 0$. Suppose that the government reduces the minimum wage ($d\bar{w} < 0$) while keeping c_0, c_1, c_2 constant. Reducing the minimum wage leads to a positive employment effect $dh_1 > 0$ as involuntary unemployment is reduced, improving the welfare of the newly employed workers and increasing tax revenue as $\tau_1 > 0$. The increase $dh_1 > 0$ also leads to a change $dw_2 > 0$. However, because $h_1 d\bar{w} + h_2 dw_2 = 0$ through the no-profit condition (6), the mechanical fiscal effect of $d\bar{w}$ and dw_2, keeping c_1 and c_2 constant, is zero. Because c_0, c_1, c_2 remain constant, nobody's welfare is reduced.[14] The increase in welfare due to the reduction in unemployment remains a-fortiori true if rationing is not efficient.[15] Therefore, this reform is a second-best Pareto improvement. □

Note that if workers respond along the intensive margin, the minimum wage generates not only involuntary unemployment, but also involuntary over-work as high-skilled workers are also rationed out. In that case, a minimum wage decrease would induce some high-skilled workers to become minimum wage workers, reducing government revenue so that Proposition 3 would not necessarily hold anymore. However, the fact that the minimum wage can create over-work is hardly ever discussed in empirical studies, suggesting that the intensive response channel is

[14] Because, $c_2 - c_0$ remains constant, h_2 does not change either.

[15] Formally, this requires assuming that the pecking order for the rationing mechanism does not change when the tax system or minimum wage change. If this is not the case and there is a entirely new draw in the rationing allocation, some of the formerly employed low-skilled workers could loose their job. Even in that case, any social welfare objective that is neutral with respect to the pecking order for rationing would increase.

unimportant empirically. Furthermore, even if intensive responses are allowed (for example, workers decide how much to invest in education early in life) but we make the additional assumption that the rationing generated by the minimum wage hits only low-skilled individuals (and never prevents higher skilled workers from taking minimum wage work), then Proposition 3 remains valid.[16]

Proposition 3 implies that, when labor supply responses are concentrated along the extensive margin, a minimum wage should always be associated with low-skilled work subsidies such as the US EITC or the British Family Credit. Proposition 3 may have wide applicability because many OECD countries, especially in continental Europe, combine significant minimum wages (OECD 1998, Immervoll 2007) with very high tax rates on low-skilled work (Immervoll et al. 2007). The high tax rates are generated by substantial payroll tax rates (financing social security benefits) and by the high phasing-out rates of traditional means-tested transfer programs.

In practice, the reform described in Proposition 3 could be achieved by cutting the employer payroll taxes for low income workers which lowers the (gross) minimum wage without affecting the net minimum wage after taxes and transfers.[17] Such a policy should stimulate low-skilled employment and increase high-skilled wages. Thus, the direct loss in tax revenue due to the payroll tax cut on low-skilled workers could be recouped by adjusting upward taxes on high earning workers (without hurting high earning workers on net).[18] A number of OECD countries have already implemented such policy reforms over the last 20 years. For example, France started reducing the employer payroll tax on low income workers in the early 1990s (see Crpon and Desplatz, 2002 for an empirical analysis).

The US policy in recent decades of letting inflation erode the minimum wage while expanding the Earned Income Tax Credit is closely related. The EITC expansions compensate minimum wage workers (at least those minimum wage workers eligible for the EITC, i.e. primarily single mothers) for the erosion in the minimum wage and attracts previously unemployed workers into the labor force increasing their welfare and increasing tax revenue (assuming $\tau_1 > 0$ because of

[16]Note that this assumption violates efficient rationing. However, it may be realistic that employers could preferentially hire the most qualified workers even for minimum wage jobs.

[17]Politically, it is extremely difficult to directly cut the legal minimum wage.

[18]In our formal model, we have only two factors of production, low- and high-skilled work. In reality, there are many factors of production. The additional tax should be spread to other factors that benefit from the minimum wage reduction. In the short-run, employers and hence profits are the most likely to benefit from the minimum wage cut. In the long-run, as employment adjusts, higher skilled wages will benefit from the minimum wage cut as in our basic model.

the phasing-out of welfare programs and payroll taxes). In principle, the direct fiscal cost of the EITC expansion (which maintains c_1 constant) can be recouped by increasing τ_2 as w_2 increases (so that c_2 also stays constant).

4.3 Other Sources of Wage Rigidity

Low-skilled wages can be rigid and above their market clearing equilibrium for other reasons than a minimum wage. For example, unions and wage bargaining agreements may lead to low-skilled wages set above their equilibrium. Similarly, downward wage rigidity might prevent low-skilled wages from falling after a downturn or a technological shock favoring high-skilled workers. If we assume that employers hire low-skilled labor to maximize profits so that the competitive demand equations always hold, the model is identical to the minimum wage model we have presented and therefore our results carry over as follows.

Proposition 2 implies that the government would welcome downward wage rigidity for low-skilled labor as a way to enhance the redistributive power of low-skilled work subsidies, under the strong assumption that rationing due to the rigidity is efficient. As we discussed above, EITC expansions would generate a greater transfer to low-skilled workers (per dollar spent) if, for example, unions were protecting low-skilled wages from falling through standard incidence effects.

Perhaps more importantly, Proposition 3 implies that, when low-skilled work is taxed and has a rigid wage above its market clearing level, the government could implement a Pareto improving policy if it can bring down low-skilled wages while at the same time adjusting the tax and transfer system. In the case of a union keeping w_1 above equilibrium for example, that would require the government to directly bargain with the union for an increase of in-work benefits (paid for by higher taxes on high-skilled workers) in exchange for lower wage increases demands from the union. A cut in employer payroll taxes for low wage earners achieves this dual goal without requiring formal Union agreement (as long as Unions do not correspondingly ask for higher wages to absorb the payroll tax cut).

5 Labor Supply with Variable Hours of Work

A potential concern is that our results are artifacts of our choice to abstract from the hours-of-work decision. We show that this is not the case by considering a conventional hours-of-work

model of labor supply.

An important fact about an hours-of-work model is that it naturally produces efficient rationing, in the case that employers cut low-skilled work by *uniformly* reducing hours across low-skilled employees in the presence of a minimum wage. That is, if employees are choosing their optimal level of hours of work, then they receive no surplus from their marginal hour of work: a small reduction in hours of work has no first order effect on welfare.[19]

We modify our model from the previous sections as follows. Suppose that an individual of skill i can supply l hours of work in occupation i (and solely in occupation i) with utility function $u_i(c, l) = c - v_i(l)$ where disutility of work $v_i(l)$ is increasing and convex in l. We assume away fixed costs of work so that everybody works and we assume that skills are exogenously set for simplicity.

- **Case with no Taxes**

Proposition 1 remains valid in this model. The analysis of Figure 1 carries over by re-interpreting h_1 as hours of work per low-skilled worker (instead of number of low-skilled workers). A small minimum wage produces a desirable first order transfer from high-skilled workers to low-skilled workers (if $g_1 > g_2$) and has only a second order welfare effect, since low-skilled workers get no surplus from their marginal hour of work.

- **Case with Optimal Taxes**

In this model with elastic hours of work and no occupational choice, the government can achieve complete redistribution at no efficiency costs by conditioning taxes on wage rates (as opposed to income). In that case, no minimum wage would be required. The traditional assumption since Mirrlees (1971) is that the government cannot observe wage rates w_i and hence has to condition taxes on income. However, this traditional assumption is not consistent with the ability of implementing a minimum wage. As recognized by the previous literature, there is no fully satisfactory way to address this informational inconsistency between tax policy and minimum wage policy in a model with elastic hours of work.[20] In practice, governments simultaneously

[19] In fact, it is possible that the failure to detect strong employment effects of the minimum wage in the United States is due in part to the fact that employers adjust hours of work rather than number of employees. It is easy to show that, in a model with both hours of work and participation labor supply responses, if employers ration hours per job rather than number of jobs, a small minimum wage increase can actually increase employment (as some individuals may decide to start working) while reducing hours per job and total hours.

[20] The occupational model developed earlier avoided this informational inconsistency as there was no choice of

impose minimum wages based on wage rates and income and payroll taxes based on earnings. Therefore, in the spirit of our previous analysis, the natural assumption is that the government can impose specific taxes on each occupation but that those taxes have to be proportional to earnings within each occupation.[21] Let us denote by τ_i the tax rate on earnings in occupation i and by c_0 the lump-sum grant redistributed to everybody so that $c_i = w_i(1-\tau_i)l + c_0$.

Proposition 2 remains valid in this model. The logic of the result can again be seen on Figure 3 and Figure A.1. Suppose that the government sets the minimum wage at the initial w_1 and then decreases τ_1 slightly. Labor supply in low skilled sector cannot increase (as this would lead to a fall in w_1). Hence, the tax decrease is again a pure lumpsum transfer to low skilled workers which is desirable as long as $g_1 > 1$.

It is useful to contrast this positive result with the results of Allen (1987) and Guesnerie (1987) obtained in the Stiglitz (1982) model where the minimum wage is not desirable in the presence of optimal nonlinear income taxation. In the Stiglitz model, starting from the optimal income tax allocation, if a minimum wage is set at w_1, and taxes on low skilled workers are reduced, then high skill workers will find it more attractive to imitate the low skilled by reducing their hours of work while staying in the high skilled sector. Therefore, the reform proposed worsens the incentive compatibility constraint and is not desirable in the Stiglitz model. In the hours of work model we sketched above, in contrast, high-skilled labor supply is determined solely by $w_2 \cdot (1-\tau_2)$ and hence is not affected directly by the minimum wage. As a result, a minimum wage helps improving redistribution toward low-skilled workers.

We believe that the channel of response to the minimum wage in the Stiglitz (1982) model is not first order in practice as empirical studies on the minimum wage do not discuss the possibility of higher workers reducing their hours of work in the presence of the minimum wage. In contrast, the responses within the low skilled sector, either through hours of work as in the model of this Section, or perhaps more importantly through participation as in Section 4, are certainly central to the empirical debate on the effects of the minimum wage.[22]

hours.

[21] Although individual income tax systems do not differentiate across sectors, governments sometimes differentiate tax rates across sectors using differential payroll tax rates.

[22] As noted in introduction, when introducing fixed costs of work in the Stiglitz model, Marceau and Boadway (1994) show that a minimum wage could be desirable even with optimal taxes. Our analysis shows that this point is central and that the economic logic can be transparently understood in the models we have proposed.

6 Conclusion

Our paper proposes a theoretical analysis of optimal minimum wage policy for redistribution purposes in a perfectly competitive labor market, considering both the case with no taxes/ transfers and the case with optimal taxes/ transfers. In light of the previous literature on this topic, we find that the standard competitive labor market model offers a surprisingly strong case for using the minimum wage when we adopt the efficient rationing assumption. The minimum wage is a useful tool if the government values redistribution toward low wage workers, and this remains true in the presence of optimal nonlinear taxes/transfers. In that context, our model of occupational choice abstracting from hours of work allows us to focus on the labor force participation decision as well as overcome the informational inconsistency that plagued previous work analyzing minimum wage policy with optimal income taxation.

When low-skilled labor supply is driven by the extensive margin, as empirical studies suggest, a minimum wage should always be associated with in-work subsidies: the co-existence of minimum wages and positive participation tax rates for low-skilled workers is (second-best) Pareto inefficient. In that situation (common in most OECD countries) a cut in employer payroll taxes decreasing the gross minimum wage while keeping the net minimum wage constant, combined with an offsetting tax increase on higher skilled workers is Pareto improving.

There are a number of issues that we have abstracted from in our very stylized model that are worth pointing out as caveats and potential avenues for future research.

First, a minimum wage rationing mechanism operates very differently from a tax and transfer that alters prices, but lets markets freely clear. The rationing induced by the minimum wage creates an allocation problem with no natural market. It is conceivable that rationing and the ensuing involuntary unemployment would create additional psychological costs (such as feelings of low self-worth) that are not captured in standard models (including those with search frictions), which would make minimum wage policies less attractive in practice.

Second and related, by the same logic, rationing out-of-work benefits would be desirable if such rationing could be made efficient (i.e., benefits would go to those with the highest costs of working so that those with low costs of working would remain the work force). In that case, however, the government would have to set up a direct rationing scheme (as opposed to indirectly letting private agents work out a rationing scheme as under a minimum wage).

Re-trading of out-of-work benefits can make the allocation efficient but such re-trading could worsen inequality and hence social welfare. Tackling this issue could connect the theoretical literature on quotas following Neary and Roberts (1980), Guesnerie (1981), and Guesnerie and Roberts (1984) to the more applied literature on optimal ordeals or screening devices for welfare programs following Nichols and Zeckhauser (1982) and Besley and Coate (1992).

Finally, economies generate involuntary unemployment through other channels than minimum wage. Recent experience shows that macro-economic downturns can generate substantial unemployment. In such situations as in the case of minimum wage induced unemployment studied here, the labor supply margin becomes irrelevant as too many workers are chasing too few jobs, which can significantly change the calculus of optimal tax and transfers. We leave it for future work to develop a theory of optimal transfers in more general contexts with involuntary unemployment.

A Appendix: Formal Proofs

A.1 Theoretical Underpinnings of Figure 1

Constant returns to scale and demand equations (1) imply that $w_2/w_1 = F_2(1, h_2/h_1)/F_1(1, h_2/h_1)$. Constant returns to scale along with decreasing marginal productivity along each skill implies that the right-hand-side is a decreasing function of h_2/h_1. Therefore, the function is invertible and the ratio h_2/h_1 can be written as a function of the wage ratio w_2/w_1: $h_2/h_1 = r(w_2/w_1)$ with $r(.)$ a decreasing function. For example, in the case of a CES production function, $F(h_1, h_2) = [a_1 h_1^{(\sigma-1)/\sigma} + a_2 h_2^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$, we have $h_2/h_1 = (a_2/a_1)^\sigma \cdot (w_2/w_1)^{-\sigma}$.

Constant returns to scale also implies that there are no profits in equilibrium. Hence $\Pi = F(h_1, h_2) - w_1 h_1 - w_2 h_2 = 0$ so that $w_1 + w_2 \cdot r(w_2/w_1) = F(1, r(w_2/w_1))$, which defines a decreasing mapping between w_1 and w_2 so that we can express w_2 as a decreasing function of w_1: $w_2(w_1)$. For example, in the case of a CES production function, the equation defining $w_2(w_1)$ is $a_1^\sigma w_1^{1-\sigma} + a_2^\sigma w_2^{1-\sigma} = 1$.

Differentiating $w_1 + w_2 \cdot r(w_2/w_1) = F(1, r(w_2/w_1))$ implies that $dw_1 + r dw_2 + w_2 dr = F_2 dr$ so that, using $w_2 = F_2$, we have $dw_1 + r dw_2 = 0$. Therefore, $dw_2/dw_1 = -1/r(w_2/w_1) = -h_1/h_2$ which proves (6).

In Figure 1, the supply function $S_1(w_1)$ is defined as follows. For a given w_1, the demand side defines a unique $w_2 = w_2(w_1)$ as discussed above. For given tax parameters T_0, T_1, T_2, the supply side function is defined as $S_1(w_1) = h_1(-T_0, w_1 - T_1, w_2(w_1) - T_2)$.

The definition of the demand function $D_1(w_1)$ depends on the rationing mechanism. Let us work out the case with efficient rationing.[23] For a given w_1, the demand side defines a unique $w_2 = w_2(w_1)$ and pins down the ratio $h_2/h_1 = r(w_2(w_1)/w_1)$. Under efficient rationing, if the wage w_1 is above (below) its competitive level, then there exist $\delta > 0$ (< 0) such that only those with surplus above δ work in occupation 1. In that case, the population will be distributed across the 3 occupations according to the functions $h_i(-T_0, w_1 - T_1 - \delta, w_2 - T_2)$ (instead of $h_i(-T_0, w_1 - T_1, w_2 - T_2)$ with no rationing). Obviously, h_1 decreases with δ and h_2 increases with δ (or is constant if there are no intensive labor supply responses). Therefore, for a given w_1, as $w_2(w_1)$ and $h_2/h_1 = r(w_2(w_1)/w_1)$ are pinned down, there exists a single δ such that $h_2(-T_0, w_1 - T_1 - \delta, w_2(w_1) - T_2)/h_1(-T_0, w_1 - T_1 - \delta, w_2(w_1) - T_2) = r(w_2(w_1)/w_1)$. This δ is a

[23] A similar derivation can be made for any other form of rationing but the formulas of course depend on the rationing form chosen. The reader can easily work out the case with uniform rationing.

function of w_1, and hence we can formally define $D_1(w_1) = h_1(-T_0, w_1 - T_1 - \delta(w_1), w_2(w_1) - T_2)$.

A.2 Proof of Proposition 1

At the competitive equilibrium (w_1, w_2) with no taxes and transfers, social welfare is given by:

$$SW = [1 - h_1 - h_2]G(0) + \int_{\Theta_1} G(w_1 - \theta_1)dF(\theta) + \int_{\Theta_2} G(w_2 - \theta_2)dF(\theta),$$

where w_2 is a function of w_1 through the demand side as discussed in Appendix A.1 above. Consider introducing a small minimum wage $d\bar{w}$ above w_1. We have,

$$\frac{dSW}{d\bar{w}} = \left[-\frac{dh_1}{d\bar{w}} - \frac{dh_1}{d\bar{w}}\right] \cdot G(0) + \frac{dh_1}{d\bar{w}}G(0) + \frac{dh_2}{d\bar{w}}G(0)$$

$$+ \int_{\Theta_1} G'(w_1 - \theta_1)dF(\theta) + \frac{dw_2}{d\bar{w}} \int_{\Theta_2} G'(w_2 - \theta_2)dF(\theta).$$

The second and third terms are obtained because of the efficient rationing assumption whereby those loosing their low-skilled job and shifting to no-work are those with the least surplus, namely zero, for having a low-skilled job.[24] Therefore, the first three terms cancel out, and, using $dw_2/d\bar{w} = -h_1/h_2$ we obtain finally,

$$\frac{dSW}{d\bar{w}} = h_1\lambda[g_1 - g_2] > 0,$$

which proves the proposition. □

A.3 Proof of Proposition 2

Social welfare at the tax and transfer optimum with no minimum wage is given by:

$$SW = [1 - h_1(c) - h_2(c)]G(c_0) + \int_{\Theta_1} G(c_0 + \Delta c_1 - \theta_1)dF(\theta) + \int_{\Theta_2} G(c_0 + \Delta c_2 - \theta_2)dF(\theta),$$

where $\Delta c_i = c_i - c_0$ and with budget constraint $h_1 \cdot (w_1 - \Delta c_1) + h_2 \cdot (w_2 - \Delta c_2) \geq c_0$ (multiplier λ). Forming the Lagrangian $L = SW + \lambda[h_1 \cdot (w_1 - \Delta c_1) + h_2 \cdot (w_2 - \Delta c_2) - c_0]$, let us consider a variation dc_1 with a binding minimum wage $\bar{w}$ set at the initial equilibrium as depicted on Figure A.1 in a standard (w, c) diagram. We have no change in w_1 by definition, hence no change in w_2 either as w_2 and w_1 are related by $w_2(w_1)$ based on demand constraints (Appendix A.1). Hence, there is no change in $h_2/h_1 = r(w_2/w_1)$. Because h_1 and h_2 cannot increase (resp.

[24] The small minimum wage also induces some low-skilled workers to shift to the high-skilled sector but this has no first order welfare effect as those workers are indifferent between the two occupations to start with.

decrease) simultaneously, this implies no change in both h_1 and h_2. As depicted on Figure A.1., with the crossed-off curly arrows, the labor supply response that would occur because of dc_1 cannot happen because of the minimum wage. Therefore, we obtain

$$\frac{dL}{dc_1} = \int_{\Theta_1} G'(c_0 + \Delta c_1 - \theta_1) dF(\theta) - \lambda h_1 = \lambda[g_1 - 1]h_1.$$

This proves the first part of the proposition. At the full optimum with taxes and transfers and the minimum wage, the condition above must be zero which implies that $g_1 = 1$. The first order condition with respect to c_0, keeping Δc_1, Δc_2, and $\bar{w}$ constant implies:

$$0 = \frac{dL}{dc_0} = [1 - h_1(c) - h_2(c)]G'(c_0) + \int_{\Theta_1} G'(c_0 + \Delta c_1 - \theta_1) dF(\theta) + \int_{\Theta_2} G'(c_0 + \Delta c_2 - \theta_2) dF(\theta) - \lambda$$
$$= \lambda[h_0 g_0 + h_1 g_1 + h_2 g_2 - 1],$$

which completes the proof. □

A.4 Proof of Proposition 3

As depicted on Figure A.2. using a standard (w, c) diagram, the proposed reform imposes $dw_1 = d\bar{w} < 0$ while keeping c_0, c_1, c_2 constant. The utility of those who do not switch jobs is therefore not affected. From the demand side (Appendix A.1), we have $w_2(w_1)$ with $dw_2/dw_1 = -h_1/h_2 < 0$ and hence $dw_2 > 0$. This implies that relative demand for high-skilled work $h_2/h_1 = r(w_2/w_1)$ decreases as $r(.)$ is decreasing (Appendix A.1). Because $c_2 - c_0$ remains constant, and labor supply is only along the extensive margin, the supply of high-skilled workers is unchanged so that $dh_2 = 0$, which then implies that $dh_1 > 0$ (depicted with the curly arrow on Figure A.2). Those dh_1 individuals shifting from no work to low-skilled work are better-off because they were by definition rationed by the minimum wage. The government budget is $h_1 \cdot (w_1 - \Delta c_1) + h_2 \cdot (w_2 - \Delta c_2) - c_0 \geq 0$. Therefore the net effect of the reform on the budget is: $dh_1 \cdot (w_1 - \Delta c_1) + h_1 dw_1 + h_2 dw_2 = dh_1 \tau_1 w_1 > 0$. Thus, with $\tau_1 > 0$, the reform creates a budget surplus which can be used to increase c_0 and improve everybody's welfare (with no behavioral response effects), a Pareto improvement. □

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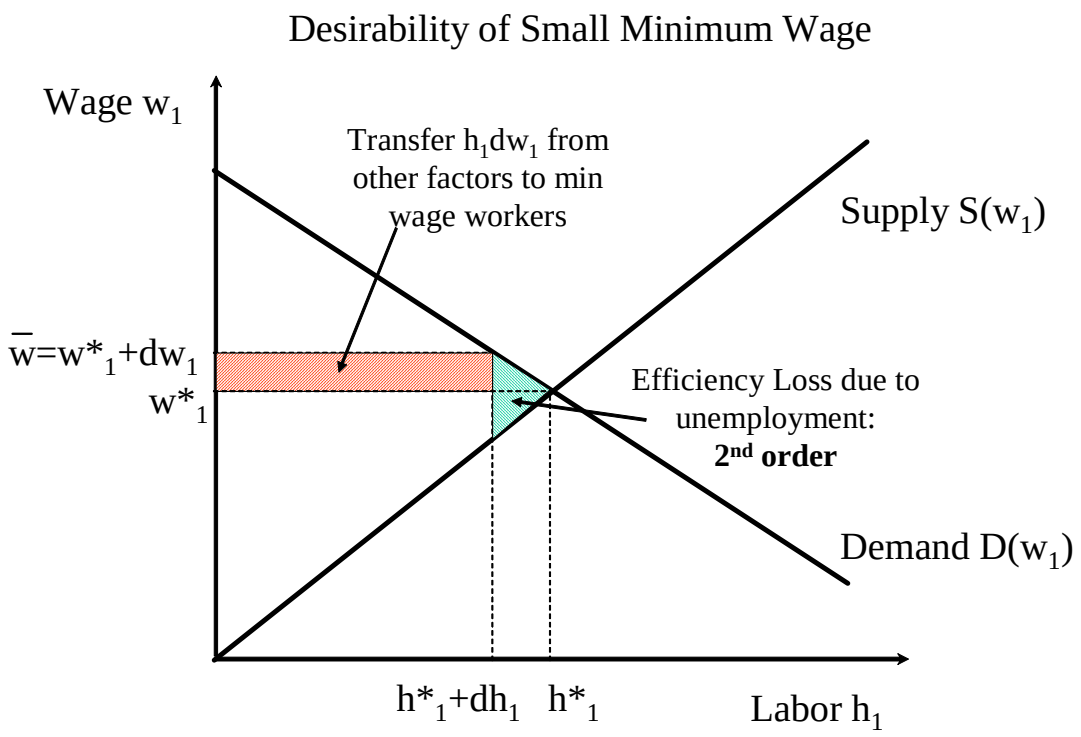
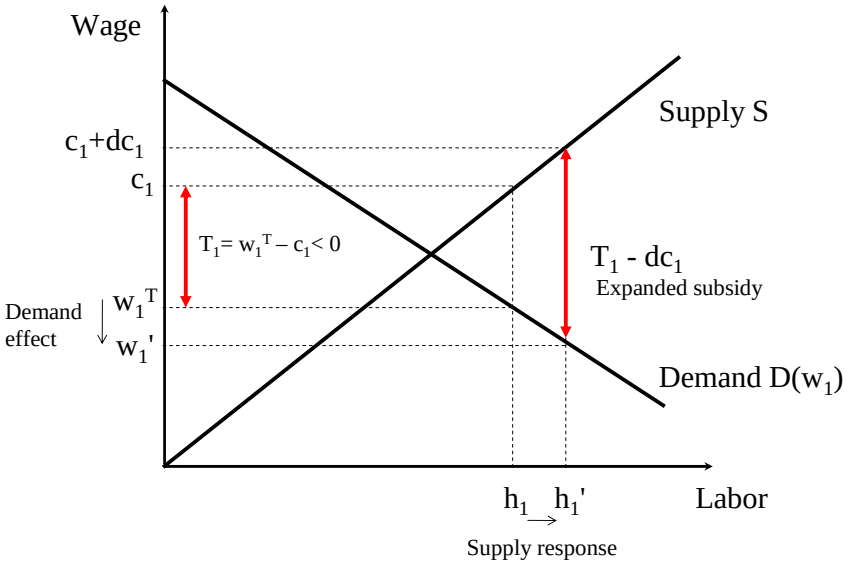


Figure 1. Minimum Wage with no Taxes and Transfers
The figure depicts the desirability of introducing a small minimum wage starting from the competitive equilibrium. A small minimum wage creates a first order transfer to low skilled workers from other factors and a second order welfare low due to involuntary unemployment (under the key assumption of efficient rationing).

a. Response to a low-skilled subsidy with no minimum wage



b. Low-skilled subsidy with binding minimum wage

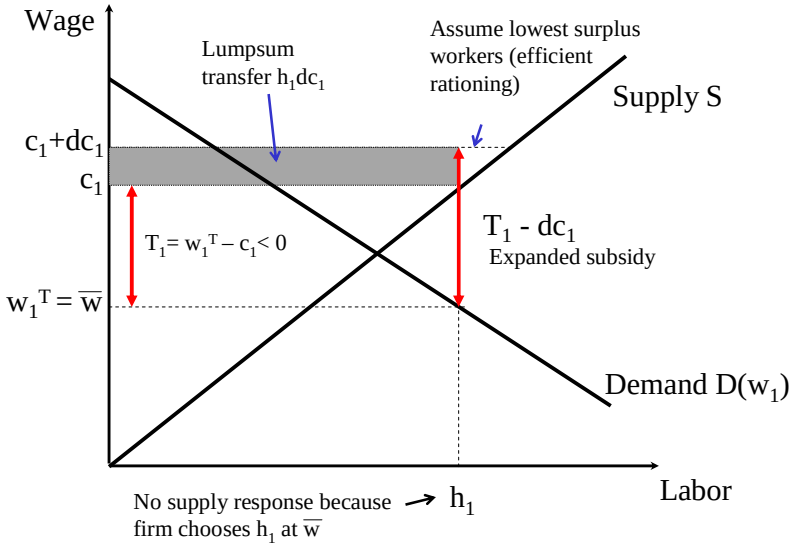


Figure 2. Desirability of the Minimum Wage with Optimal Taxes and Transfers
Panel a shows that, in the absence of the minimum wage, an increase in the work subsidy by dc_1 triggers a supply response, and therefore leads to the usual demand side effect on wages. As usual, the incidence is shared by other factors of production.
Panel b shows that when the minimum wage is binding at the initial optimum, an increase in the work subsidy by dc_1 is not accompanied by a supply response. The wage cannot fall below the minimum wage, and firms will optimize by staying at h_1. As long as "marginal" workers are shut out, the increase in dc_1 is an effective lump-sum transfer to low-skilled workers, which is desirable whenever $g_1>1$.

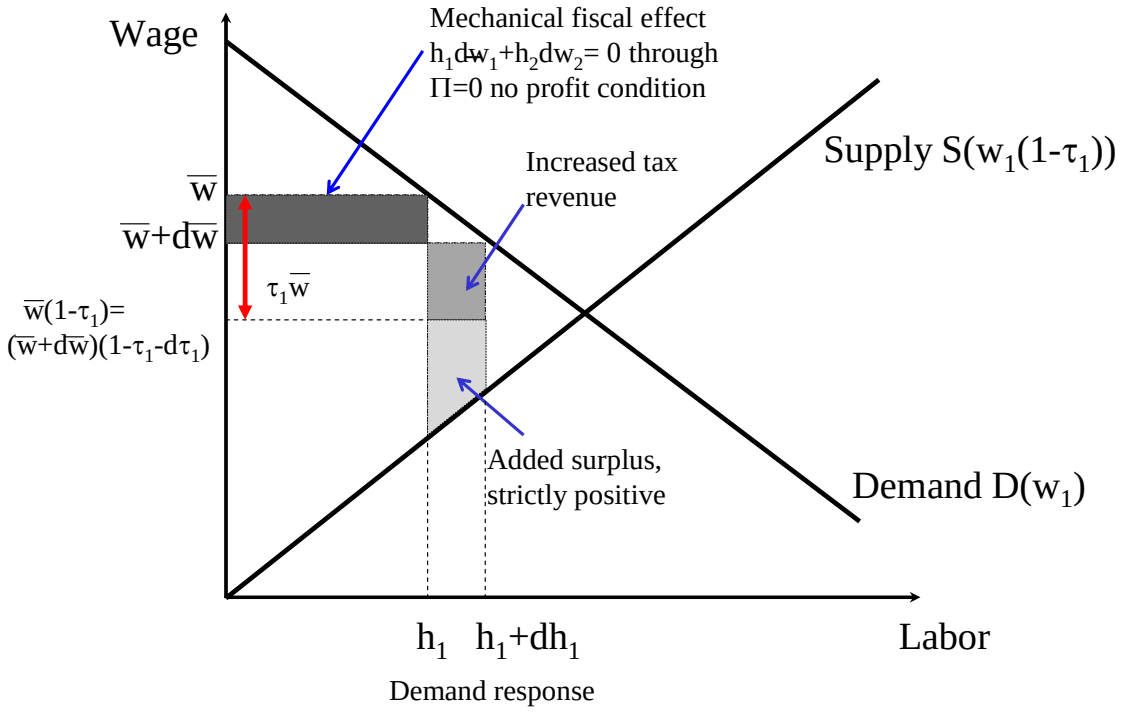


Figure 3. Improving Policy when $\tau_1>0$ and the Minimum Wage Binds
The minimum wage is reduced while the tax τ_1 on low skilled workers is correspondingly reduced to keep c_1 constant. Because c_2 is kept constant, the reduction in tax revenue from the inframarginal low-skilled workers (h_1dw_1) is exactly offset by an increase in tax revenue via an increase in w_2 because of the non-profit condition $h_1dw_1+h_2dw_2=0$. Due to an increase in employment previously rationed, there will be marginal gains in tax revenue as well as strictly positive surplus from marginal workers, leading to a Pareto improvement.

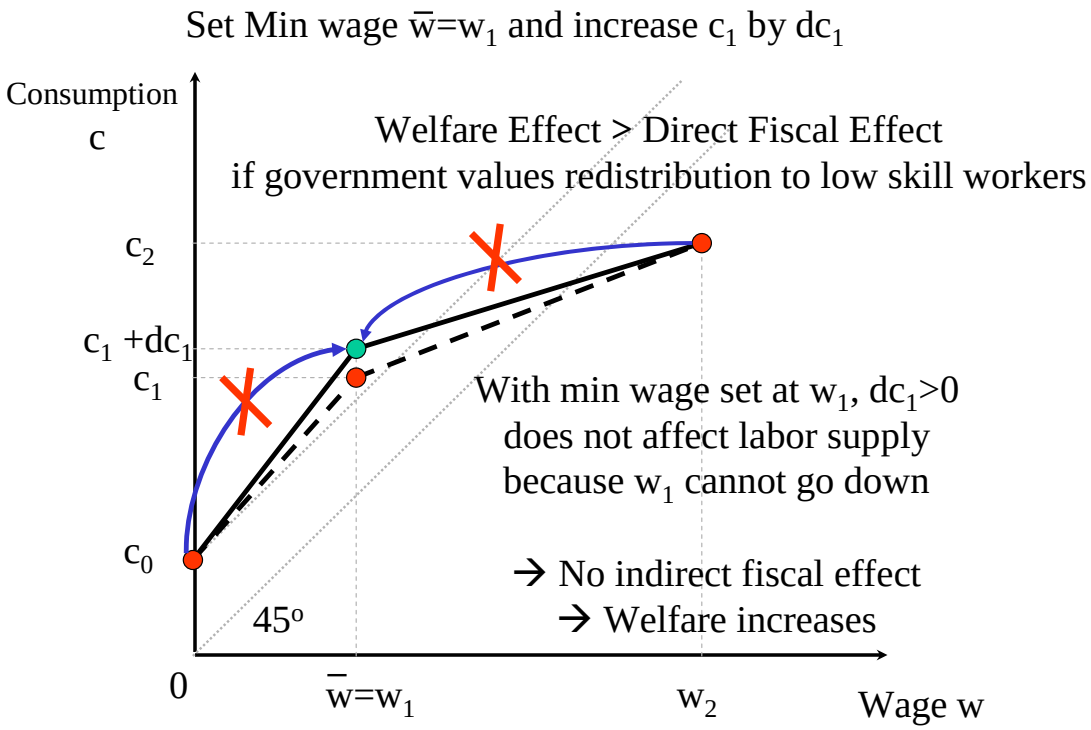


Figure A1. Desirability of a Minimum Wage with Optimal Taxes and Transfers
The figure shows that, starting from the tax optimum with no minimum wage, introducing a minimum wage (equal to w_1) and increasing c_1 by dc_1 improves welfare when $g_1>1$. The initial tax and transfer system is depicted in dashed line while the system after the reform is depicted is solid line. Absent the minimum wage, labor supply responses would increase the number of low skilled workers (curly arrows) and hence drive w_1 down through demand effects. Because the minimum wage is set at w_1, such behavioral responses cannot happen, making the proposed reform welfare increasing because $g_1>1$.

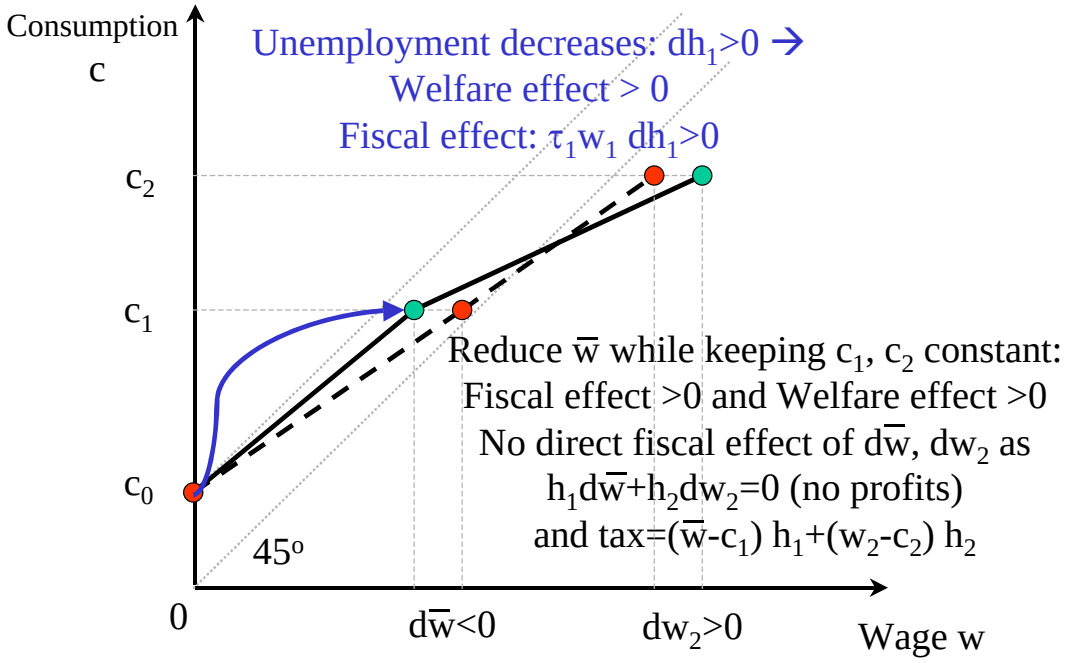


Figure A2. Pareto Improving Policy when $\tau_1>0$ and the Minimum Wage Binds
The Figure starts from a situation with a positive tax rate on low skilled work ($\tau_1>0$) in dashed line along with a binding minimum wage creating involuntary unemployment. From that situation, consider lowering the minimum wage while keeping c_0, c_1, and c_2 constant as depicted by the solid line. Through demand effects, dw_2 adjusts so that $h_1dw_1+h_2dw_2=0$, which implies that the loss in tax revenue raised from low skilled workers is exactly compensated by an increase in tax revenue from high skilled workers. Furthermore, this reform reduces involuntary unemployment (curly arrow), hence increases welfare of the newly employed and increasing tax revenue as the newly employed pay higher taxes. Therefore, this reform is a Pareto improvement.