Bubbles

Abstract

Bubbles refer to asset prices that exceed an asset's fundamental value because current owners believe that they can resell the asset at an even higher price in the future. There are four main strands of models that identify conditions under which bubbles can exist. The first class of models assumes that all investors have rational expectations and identical information. These models generate the testable implication that bubbles have to follow an explosive path. In the second category of models investors are asymmetrically informed and bubbles can emerge under more general conditions because their existence need not be commonly known. A third strand of models focuses on the interaction between rational and behavioral traders. Bubbles can persist in these models since limits to arbitrage prevent rational investors from eradicating the price impact of behavioral traders. In the final class of models, bubbles can emerge if investors hold hetereogenous beliefs, potentially due to psychological biases, and they agree to disagree about the fundamental value. Experiments are useful to isolate, distinguish and test the validity of different mechanisms that can lead to or rule out bubbles.

Bubbles are typically associated with dramatic asset price increases followed by a collapse. Bubbles arise if the price exceeds the asset's fundamental value. This can occur if investors hold the asset because they believe that they can sell it at an even higher price to some other investor even though the asset's price exceeds its fundamental value. Famous historical examples are the Dutch Tulip Mania (1634-7), the Mississippi Bubble (1719-20), the South Sea Bubble (1720) and the "Roaring 20's" that preceded the 1929 crash. More recently, internet share prices (CBOE Internet Index) surged to astronomical heights until March 2000, before plummeting by more than 75% by the end of 2000.

Since asset prices affect the real allocation of an economy, it is important to understand the circumstances under which these prices can deviate from their fundamental value. Bubbles have long intrigued economists and led to several strands of models, empirical tests, and experimental studies.

We can broadly divide the literature into four groups. The first two groups of models analyze bubbles within the rational expectations paradigm, but differ in their assumption whether all investors have the same information or are asymmetrically informed. A third group of models focuses on the interaction between rational and non-rational (behavioral) investors. In the final group of models traders' prior beliefs are heterogeneous, possibly due to psychological biases, and consequently they agree to disagree about the fundamental value of the asset.

Rational bubbles under symmetric information are studied in settings in which

all agents have rational expectations and share the same information. There are several theoretical arguments that allow us to rule out rational bubbles under certain conditions. Tirole (1982) uses a general equilibrium reasoning to argue that bubbles cannot exist if it is commonly known that the initial allocation is interim Pareto efficient. A bubble would make the seller of the "bubble asset" better-off, which – due to interim Pareto efficiency of the initial allocation – has to make the buyer of the asset worse off. Hence, no individual would be willing to buy the asset. Partial equilibrium arguments alone are also useful in ruling out bubbles. Simply rearranging the definition of (net) return, $r_{t+1,s} := (p_{t+1,s} + d_{t+1,s})/p_t - 1$, where $p_{t,s}$ is the price and $d_{t,s}$ is the dividend payment at time t and state s, and taking rational expectations yields

$$p_t = E_t \left[\frac{p_{t+1} + d_{t+1}}{1 + r_{t+1}} \right]. \tag{1}$$

That is, the current price is just the discounted expected future price and dividend payment in the next period. For tractability assume that the expected return that the marginal rational trader requires in order to hold the asset is constant over time, $E_t[r_{t+1}] = r$, for all t. Solving the above difference equation forward, i.e. replacing p_{t+1} with $E_{t+1}[p_{t+2} + d_{t+2}]/(1+r)$ in Equation (1) and then p_{t+2} and so on, and using the law of iterated expectations, one obtains after T - t - 1 iterations

$$p_t = E_t \left[\sum_{\tau=1}^{T-t} \frac{1}{(1+r)^{\tau}} d_{t+\tau} \right] + E_t \left[\frac{1}{(1+r)^{T-t}} p_T \right].$$

The equilibrium price is given by the expected discounted value of the future dividend

stream paid from t+1 to T plus the expected discounted value of the price at T. For securities with finite maturity, the price after maturity, say T, is zero, $p_T = 0$. Hence, the price of the asset, p_t , is unique and simply coincides with the expected future discounted dividend stream until maturity. Put differently, finite horizon bubbles cannot arise as long as rational investors are unconstrained from selling the desired number of shares in all future contingency. For securities with infinite maturity, $T \to \infty$, the price p_t only coincides with the future expected discounted future dividend stream, call it fundamental value, v_t , if the so-called transversality condition, $\lim_{T\to\infty} E_t\left[\frac{1}{(1+r)^T}p_{t+T}\right] = 0$, holds. Without imposing the transversality condition, $p_t = v_t$ is only one of many possible prices that solve the above expectational difference equation. Any price $p_t = v_t + b_t$, decomposed in the fundamental value, v_t , and a bubble component, b_t , such that

$$b_t = E_t \left[\frac{1}{(1+r)} b_{t+1} \right], \tag{2}$$

is also a solution. Equation (2) highlights that the bubble component b_t has to "grow" in expectations exactly at a rate of r. A nice example of these "rational bubbles" is provided in Blanchard and Watson (1982), where the bubble persists in each period only with probability π and bursts with probability $(1 - \pi)$. If the bubble continues, it has to grow in expectation by a factor $(1 + r)/\pi$. This faster bubble growth rate (conditional on not bursting) is necessary to achieve an expected growth rate of r. In general, the bubble component may be stochastic. A specific example of a stochastic bubble is an intrinsic bubble, where the bubble component is assumed to be deterministically related

to a stochastic dividend process.

US economy.

The fact that any bubble has to grow at an expected rate of r allows one to eliminate many potential rational bubbles. For example, a positive bubble cannot emerge if there is an upper limit on the size of the bubble. That is, for example, the case with potential bubbles on commodities with close substitutes. An ever-growing "commodity bubble" would make the commodity so expensive that it would be substituted with some other good. Similarly, a bubble on a non-zero supply asset cannot arise if the required return r exceeds the growth rate of the economy, since the bubble would outgrow the aggregate wealth in the economy. Hence, bubbles can only exist in a world in which the required return is lower than or equal to the growth rate of the economy. In addition, rational bubbles can persist if the pure existence of the bubble enables trading opportunities that lead to a different equilibrium allocation. Fiat money in an OLG model is probably the most famous example of such a bubble. The intrinsic value of fiat money is zero, yet it has a positive price. Moreover, only when the price is positive, does it allow wealth transfers across generations (that might not even be born yet). A negative bubble, $b_t < 0$, on a limited-liability asset cannot arise since the bubble would imply that the asset price has to become negative in expectation at some point in time. This result, together with ¹This can be the case in an overlapping generations (OLG) setting if there is an overaccumulation of private capital that makes the economy dynamically inefficient (Tirole (1985)). However, Abel, Mankiw, Summers, and Zeckhauser (1989) provide convincing empirical evidence that this is not the case for the

Equation (2), implies that if the bubble vanishes at any point in time, it has to remain zero from that point onwards. That is, rational bubbles can never emerge within an asset pricing model; they must already be present when the asset starts trading.

Empirically testing for bubbles is a challenging task. Initial tests proposed by Flood and Garber (1980) exploit the fact that bubbles cannot start within an rational asset pricing model and hence at any point in time the price must have a non-zero part that grows at an expected rate of r. However using this approach, inference is difficult due to an exploding regressor problem. That is, as time t increases, the regressor explodes and the coefficient estimate relies primarily on the most recent data points. More precisely, the ratio of the information content of the most recent data point to the information content of all previous observations never goes to zero. This implies that as time t increases, the time series sample remains essentially small and the central limit theorem does not apply. Diba and Grossman (1988) test for bubbles by checking whether the stock price is more explosive than the dividend process. Note that if the dividend process follows a linear unit-root process (e.g., a random walk), then the price process has a unit root as well. However the change in price, Δp_t , and discounted expected dividend stream, $p_t - d_t/r$, are stationary under the no-bubbles hypothesis. That is, p_t and d_t/r are cointegrated. Diba and Grossman test this hypothesis using a series of unit root tests, autocorrelation patterns, and co-integration tests. They conclude that the no-bubble hypothesis cannot be rejected. However, Evans (1991) shows that these standard linear econometric methods may fail to detect the explosive non-linear patterns of periodically collapsing bubbles. West (1987) proposes a different test that exploits the fact that one can estimate the parameters needed to calculate the expected discounted value of dividends in two different ways. One way of estimating them is not affected by the bubble, the other is. Note that the accounting identity (1) can be rewritten as $p_t = \frac{1}{1+r} (p_{t+1} + d_{t+1}) - \frac{1}{1+r} (p_{t+1} + d_{t+1} - E_t [p_{t+1} + d_{t+1}])$. Hence, in an instrumental variables regression of p_t on $(p_{t+1} + d_{t+1})$ – using e.g. d_t as an instrument – one obtains an estimate for r that is independent of the existence of a rational bubble. Second, if, for example, the dividend process follows a stationary AR(1) process, $d_{t+1} = \phi d_t + \eta_{t+1}$, with independent noise η_{t+1} , one can easily estimate ϕ . Furthermore, the expected discounted value of future dividends is $v_t = \frac{\phi}{1+r-\phi}d_t$. Hence, under the null-hypothesis of no bubble, i.e. $p_t = v_t$, the coefficient estimate of the regression of p_t on d_t provides a second estimate of $\frac{\phi}{1+r-\phi}$. In a final step, West uses a Hausman-specification test to test whether both estimates coincide. He finds that the U.S. stock market data usually reject the null hypothesis of no bubble.

Excessive volatility in the stock market seems to provide further evidence in favor of stock market bubbles. LeRoy and Porter (1981) and Shiller (1981) introduced variance bounds that indicate that the stock market is too volatile to be justified by the volatility of the discounted dividend stream. However, the variance bounds test is controversial (see e.g. Kleidon (1986)). Also, this test, as well as all the aforementioned bubble tests, assumes that the required expected returns, r, are constant over time. In a setting in which the required expected returns can be time–varying, the empirical evidence favoring excess

volatility is less clear-cut. Furthermore, time-varying expected returns can also rationalize the long-horizon predictability of stock returns. For example, a high price-dividend ratio predicts low subsequent stock returns with a high R^2 (Campbell and Shiller (1988)).

Finally, it is important to recall that the theoretical arguments that rule out rational bubbles as well as several empirical bubble tests rely heavily on backward inductions. Since a bubble cannot grow from time T onwards, there cannot be a bubble of this size at time T-1, which rules out this bubble at T-2, etc. However, there is ample experimental evidence that individuals violate the backward induction principle. Most convincing are experiments on the centipede game (Rosenthal (1981)). In this simple game, two players alternatively decide whether to continue or stop the game for a finite number of periods. On any move, a player is better off stopping the game than continuing if the other player stops immediately afterwards, but is worse off stopping than continuing if the other player continues afterwards. This game has only a single subgame perfect equilibrium that follows directly from backward induction reasoning. Each player's strategy is to stop the game whenever it is her turn to move. Hence, the first player should immediately stop the game and the game should never get off the ground. However, in experiments players initially continue to play the game – a violation of the backward induction principle (see

²At first sight, one might think that these reversing trends can also be due to the correction of bubbles instead of time-variation of expected returns or risk-premia. While the bubble correction hypothesis might hold for certain type of bubbles, it does not hold for the subclass of rational bubbles. From Equation (2) one can see that rational bubbles cannot exhibit reversing trends in expectations.

e.g. McKelvey and Palfrey (1992)). These experimental findings question the theoretical reasonings used to rule out rational bubbles under symmetric information.

In a rational bubble setting an investor only holds a bubble asset if the bubble grows in expectations ad infinitum. In contrast, in the following models an investor might hold an overpriced asset if he thinks he can resell it in the future to a less informed trader or someone who holds biased beliefs. In Kindleberger's (2000) terms, the investor thinks he can sell the asset to a greater fool.

Asymmetric information bubbles can occur in a setting in which investors have different information, but still share a common prior distribution. In these models prices have a dual role: they are an index of scarcity and informative signals, since they aggregate and partially reveal other traders' aggregate information (see e.g. Brunnermeier (2001) for an overview). Unlike in the symmetric information case, the presence of a bubble need not be commonly known. For example, it might be the case that everybody knows the price exceeds the value of any possible dividend stream, but it is not the case that everybody knows that all the other investors also know this fact. It is this lack of higher-order mutual knowledge that makes it possible for finite bubbles to exist under certain necessary conditions (Allen, Morris, and Postlewaite (1993)). First, it is crucial that investors remain asymmetrically informed even after inferring information from prices and net trades. This implies that prices cannot be fully revealing. Second, investors must be constrained from (short) selling their desired number of shares in at least one

future contingency for finite bubbles to persist. Third, it cannot be common knowledge that the initial allocation is interim Pareto efficient, since then it would be commonly known that there are no gains from trade and hence, the buyer of an overpriced "bubble asset" would be aware that the rational seller gains at his expense (Tirole (1982)). In other words, there have to be gains from trade or at least some investors have to think that there might be gains from trade. There are various mechanisms that lead to gains from trade. For example, fund managers who invest on behalf of their clients can gain from buying overpriced bubble assets, since trading allows them to fool their clients into believing that they have superior trading information. A fund manager that does not trade would reveal that he does not have private information. Consequently, bad fund managers churn bubbles at the expense of their uninformed client investors (Allen and Gorton (1993)). Furthermore, fund managers with limited liability might trade bubble assets due to classic risk-shifting incentives, since they participate on the potential upside of a trade but not on the downside risk.

Bubbles due to limited arbitrage arise in models in which rational, well-informed and sophisticated investors interact with behavioral market participants whose trading motives are influenced by psychological biases. Proponents of the "efficient market hypothesis" argue that bubbles cannot persist since well-informed sophisticated investors will undo the price impact of behavioral non-rational traders. Thus, rational investors should go against the bubble even before it emerges. The literature on limits to arbitrage

challenges this view. It argues that bubbles can persist, and provides three channels that prevent rational arbitrageurs from fully correcting the mispricing. First, fundamental risk makes it risky to short a bubble asset since a subsequent positive shift in fundamentals might ex-post undo the initial overpricing. Risk aversion limits the aggressiveness of rational traders if close substitutes and close hedges are unavailable. Second, rational traders also face noise trader risk (DeLong, Shleifer, Summers, and Waldmann (1990)). Leaning against the bubble is risky even without fundamental risk, since irrational noise traders might push up the price even further in the future and temporarily widen the mispricing. Rational traders with short-horizons care about prices in the near future in addition to the long-run fundamental value and only partially correct the mispricing. For example, in a world with delegated portfolio management, fund managers are often concerned about short-run price movements, because temporary losses instigate fund outflows (Shleifer and Vishny (1997)). A temporary widening of the mispricing and the subsequent outflow of funds forces fund managers to unwind their positions exactly when the mispricing is the largest. Anticipating this possible scenario, mutual fund managers trade less aggressively against the mispricing. Similarly, hedge funds face a high flow-performance sensitivity, despite some arrangements designed to prevent outflows (e.g. lock-up provisions). Third, rational traders face synchronization risk (Abreu and Brunnermeier (2003)). Since a single trader alone can typically not bring the market down by himself, coordination among rational traders is required and a synchronization problem arises. Each rational trader faces the following trade-off: if he attacks the bubble too early, he forgoes profits from

the subsequent run-up caused by behavioral momentum traders; if he attacks too late and remains invested in the bubble asset, he will suffer from the subsequent crash. Each trader tries to forecast when other rational traders will go against the bubble. Timing other traders' moves is difficult because traders become sequentially aware of the bubble, and they do not know where in the queue they are. Because of this "sequential awareness", it is never common knowledge that a bubble has emerged. It is precisely this lack of common knowledge which removes the bite of the standard backward induction argument. Since there is no commonly known point in time from which one could start backward induction, even finite horizon bubbles can persist. The other important message of the theoretical work on synchronization risk is that relatively insignificant news events can trigger large price movements, because even unimportant news events allow traders to synchronize their sell strategies. Unlike in the earlier limits to arbitrage models, in which rational traders do not trade aggressively enough to completely eradicate the bubble but still short an overpriced bubble asset, in Abreu and Brunnermeier (2003) rational traders prefer to ride the bubble rather than attack it. The incentive to ride the bubble stems from a predictable "sentiment" in the form of continuing bubble growth.

Empirically, there is supportive evidence in favor of the "bubble riding hypothesis." For example, between 1998 and 2000 hedge funds were heavily tilted towards highly priced technology stocks (Brunnermeier and Nagel (2004)). Contrary to the efficient market hypothesis, hedge funds were not a price-correcting force even though they are among the most sophisticated investors and are arguably closer to the ideal of "rational arbitrageurs"

than any other class of investors. Similarly, Temin and Voth (2004) document that Hoares Bank was profitably riding the South Sea bubble in 1719-1720, despite giving numerous indications that it believed the stock to be overvalued. Many other investors, including Isaac Newton, also tried to ride the South Sea bubble but with less success. Frustrated with his trading experience, Isaac Newton concluded "I can calculate the motions of the heavenly bodies, but not the madness of people."

Heterogeneous beliefs bubbles: Bubbles can also emerge when investors have heterogeneous beliefs and face short-sale constraints. Investors' beliefs are heterogeneous if they start with different prior belief distributions which could possibly be due to psychological biases. For example, if investors are overconfident about their own signals, they have a different prior distribution (with lower variance) about the signals' noise term. Investors with non-common priors can agree to disagree even after they share all their information. Also, in contrast to an asymmetric information setting, investors do not try to infer other traders' information from prices. Combining heterogeneous beliefs with short-sale constraints can result in overpricing since optimists push up the asset price, while pessimists cannot counterbalance it since they face short-sale constraints (Miller (1977)). Ofek and Richardson (2003) link this argument to the internet bubble of the late 1990s. In a dynamic model, the asset price can even exceed the valuation of the most optimistic investor in the economy. This is possible, since the currently optimistic investors – the current owners of the asset – have the option to resell the asset in the

future at a high price whenever they become less optimistic. At that point other traders will be more optimistic and, hence, be willing to buy the asset since optimism is assumed to oscillate across different investor groups (Harrison and Kreps (1978)). It is essential that less optimistic investors, who would like to short the asset, are prevented from doing so by the short-sale constraint. Heterogeneous belief bubbles are accompanied by large trading volume and high price volatility (Scheinkman and Xiong (2003)).

Experimental evidence: Many theoretical arguments in favor of or against bubbles are difficult to test with (confounded) field data. Laboratory experiments have the advantage that they allow the researcher to isolate and test specific mechanisms and theoretical arguments. For example, the aforementioned experimental evidence on centipede games questions the validity of backward induction. There is a large and growing literature that examines bubbles in a laboratory setting. For example, Smith, Suchanek, and Williams (1988) study a double-auction setting, in which a risky asset pays a uniformly distributed random dividend of $d \in \{0, d_1, d_2, d_3\}$ in each of the 15 periods. Hence, the fundamental value for a risk-neutral trader is initially $15 \sum_i \frac{1}{4} d_i$ and declines by $\sum_i \frac{1}{4} d_i$ in each period. Even though there is no asymmetric information and the probability distribution is commonly known, there is vigorous trading, and prices initially rise despite the fact that the fundamental value steadily declines. More specifically, the time-series of asset prices in the experiments are characterized by three phases. An initial boom phase is followed by a period during which the price exceeds the fundamental value, before the price col-

lapses towards the end. These findings are in sharp contrast to any theoretical prediction and seem very robust across various treatments. A string of subsequent articles show that bubbles still emerge after allowing for short-sales, after introducing trading fees, and when using professional business people as subjects. Only the introduction of futures markets and the repeated experience of a bubble reduce the size of the bubble. Researchers have speculated that bubbles emerge because each trader hopes to outsmart others and to pass the asset on to some less rational trader in the final trading rounds. However, subsequent research has revealed that the lack of common knowledge of rationality is not the cause of bubbles. Even when investors have no resale option and are forced to hold the asset until the end, bubbles still emerge (Lei, Noussair, and Plott (2001)).

In summary, the literature on bubbles has taken giant strides in the last three decades that led to several classes of models with distinct empirical tests. However, many questions remain unresolved. For example, we do not have many convincing models that explain when and why bubbles start. Also, in most models bubbles burst, while in reality bubbles seem to deflate over several weeks or even months. While we have a much better idea of why rational traders are unable to eradicate the mispricing introduced by behavioral traders, our understanding of behavioral biases and belief distortions is less advanced. From a policy perspective, it is interesting to answer the question whether central banks actively try to burst bubbles. I suspect that future research will place greater emphasis on these open issues.

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