On the Performance of Shared-Channel Multihop Lightwave Networks

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Abstract

Channel sharing is an attractive approach to improve the performance of multihop optical networks. It can be used to implement virtual topologies with higher connectivity, to reduce the number of hops taken by a packet, and, therefore, to reduce delay and increase throughput. In this paper we study the effect of channel sharing on the performance of multihop networks when channel sharing is achieved by using a time-division multiple-access (TDMA) technique. If TDMA is used, the same wavelength channel is shared by more than one virtual link, which reduces capacities of the virtual links that share the wavelength. As a result of this, the throughput may decrease with the increase in channel sharing. We present a result which gives an upper bound on the throughput that can be achieved with any virtual topology that can be established with N stations assuming that the traffic distribution is uniform and that all virtual links have the same capacity. Using this result we determine the optimal degree of channel sharing which maximizes throughput. We also determine the optimal degree of channel sharing when the criterion is to maximize the network power, which is defined as a ratio of throughput and delay.

1 Introduction

Lightwave networks are becoming increasingly popular on account of their very broad bandwidth, which is unmatched by any other transmission media. The main challenge now is

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how to exploit this enormous potential, since each user is able to use only a small fraction of the total bandwidth, as determined by the access speed of the electrooptic interface. Wavelength-division multiplexing (WDM) over a passive broadcast medium (e.g., optical star, tree or bus), investigated by many researchers recently, appears to be a natural solution to this problem. Most proposed WDM network architectures can be classified into two broad categories: single-hop and multihop networks [1, 2].

Single-hop networks can be defined as networks where a direct transmission can be achieved between each pair of stations. These networks typically require that stations have tunable transmitters or receivers [unless only a single wavelength channel is used and shared using a time-division multiple-access (TDMA) technique]. If the single-hop network is used for packet-switching, then these devices have to be rapidly tunable. The problem is, however, that the current optical technology is not mature enough to provide such devices.

Multihop networks, on the other hand, can be realized with fixed transmitters and receivers. Instead of using a direct path from source to destination, multihop networks require some packets to travel across several hops. Basically, the multihop lightwave network is a store-and-forward network embedded in a passive optical network. The switches of the multihop lightwave network are represented by the user stations (thus, they are located at the periphery of the passive broadcast medium), and the links consist of dedicated wavelength channels established between pairs of stations [3]. Thus, over the physical broadcast topology, there is a virtual topology that determines the logical connectivity between the stations of the network.

The most prominent example of a virtual multihop network is ShuffleNet, proposed in [3]. ShuffleNet exploits WDM to embed a perfect shuffle interconnection within a fully broadcast physical topology. The virtual topology can be modeled as a directed graph (digraph) in which the existence of an arc from one node to another implies the cotuning of the corresponding stations' transmitter and receiver. Each node of the digraph corresponds to a station in the network. A (p, k) ShuffleNet can be constructed with $N = kp^k$ nodes where p and k are positive integers. The nodes are arranged in k columns of p^k nodes each, and the nodes of column i are connected to the nodes of column $i+1 \pmod{k}$ in a p-way shuffle.

Figure 1 shows an example of an 8 station ShuffleNet virtual topology implemented on a physical tree. In this example each station has two pairs of "fixed" transmitters and receivers (i.e., $p = 2$). (In fact, these transmitters and receivers are not really fixed, but slowly tunable in order to make it possible to change the virtual topology.) Each wavelength is dedicated to a transmitter-receiver pair. Thus, the number of wavelengths required in this case is twice the number of stations. The logical interconnection between stations is more intuitively drawn in Figure 2. The maximum hop distance between two nodes in the ShuffleNet is $2k-1$. Since k grows logarithmically with N, the maximum as well as the average hop distance grows logarithmically as well.

Other regular virtual topologies that have been proposed for the multihop optical network are the de Bruijn graph, the torus (Manhattan Street Network), the hypercube, the ring, and the dual bus [2].

Clearly, each hop incurs the penalty (in terms of additional packet delay and processing overhead) of an electrooptical conversion. Also, multihopping reduces network throughput, since the effective network capacity is inversely proportional to the number of hops for packet transmissions. Thus, the virtual topology should be designed to minimize the number of hops. By choosing an appropriate virtual topology it is possible to minimize the number of hops and to achieve very high throughputs. The network capacity grows with $O(N/\log N)$ if the ShuffleNet or de Bruijn virtual topology is used.

In order to utilize wavelengths better and to reduce the number of required wavelengths, more than one transmitter and receiver can be allowed to access the same wavelength channel. Channel sharing also makes it possible to implement the same virtual topology by using fewer transceivers per station or to implement virtual topologies with higher connectivity using the same number of transceivers. It can also provide additional flexibility in routing. The channels are typically shared using TDMA [4, 5, 6]. Hluchyj and Karol [4] studied channel sharing for ShuffleNet and presented a routing algorithm for the shared-channel ShuffleNet that routes traffic along shortest paths in such a way that the traffic load on all channels is perfectly balanced when traffic is uniform. Bannister et al. [5] studied optimal shared-channel virtual topologies for nonuniform traffic. A perfect-shuffle virtual topology is modified by using a genetic algorithm. In [6] channel sharing in the Manhattan Street Network (MSN) is studied. In this network each row and each column represents a shared channel. It is shown that for the uniform-traffic case the shared-channel MSN can support higher aggregate network throughput than the original MSN. In [7] channel sharing using subcarrier multiplexing is studied. A methodology for building a shared-channel virtual topology is developed and it is shown that the ShuffleNet and de Bruijn graph belong to the class of topologies that permit channel sharing.

In this paper we study the effect of TDMA channel sharing on the performance of multihop networks. The paper is organized as follows. In the second section we analyze how channel sharing affects network performance when each station has only one transmitter-receiver pair. In the third section we extend the performance analysis for the multiple transmitter-receiver case. The fourth section concludes the paper.

2 Channel sharing with one transceiver per station

We consider first virtual topologies for the multihop network with minimal hardware requirements: one fixed transmitter and receiver per station. If no channel sharing is permitted, each station can receive from or transmit to only one other station in a single hop. In such a case, the only virtual topology that can provide full connectivity among stations is the ring. Such a multihop network is used in STARNET [8].

We define the station's connectivity factor p as the maximum number of neighbors the station can receive from or transmit to in a single hop (i.e., the maximum indegree or outdegree of each station). Clearly, without channel sharing the station's connectivity factor p is 1. We can increase p by allowing channel sharing using the TDMA technique. Figure 3 shows a ShuffleNet [4] virtual topology with $p = 2$ implemented with a single transmitter-receiver pair. In TDMA, time is divided into slots, i.e., time intervals of fixed length in which a single station is allowed to transmit one data block. A number of consecutive blocks is grouped into a frame. The slots of the frame are assigned to stations statically. In this example two transmitting stations and four virtual links share a wavelength channel. We may assign one TDM slot per frame to each virtual link as illustrated in Figure 3.

The TDMA channel sharing requires synchronization of stations transmitting on the same wavelength. This introduces additional complexity compared to the network with dedicated channels. In the HONET architecture [9, 10], which combines a single-hop and a multihop network, the problem of stations' synchronization is elegantly solved by using its single-hop network for broadcasting global synchronization signals. The synchronization in a TDMA shared-channel multihop network could also be achieved with an additional receiver per each station used for this purpose. Alternatively, the global synchronization requirement could be eliminated by assigning a single wavelength per station, and by using TDMA to establish p links to p different neighbors. We note, however, that in this case each station must receive on p different wavelengths, and thus p receivers per station are necessary.

The increase in p reduces the average number of hops which in turn increases the utilization of virtual links. On the other hand, the increase in p reduces the capacity of virtual links, since more virtual links share the same wavelength channel.

In this analysis we want to estimate the optimal value for p which maximizes throughput per station. We make the following assumptions:

- Uniform traffic distribution. Each station generates an equal amount of traffic destined to all other stations.
- All virtual links have the same capacity. We assume that the capacity of each virtual link in a virtual topology of degree p is $1/p^2$ of the capacity of a wavelength channel since at most p transmitters and p receivers share the same channel. We define a virtual

topology to be of degree p if the maximum connectivity factor of its stations is p.

Let γ be the arrival rate of traffic entering (leaving) the network (i.e., the network throughput) and let E be the average number of hops on a path from a source to a destination station. If the traffic distribution is uniform, each packet entering the network will require E transmissions on the average. Thus, the total arrival rate to stations in the network, which includes new and forwarded packets, is

$$\lambda = \gamma E \tag{1}$$

If the capacity of a wavelength channel is C, the capacity of each virtual link in the network with a virtual topology of degree p is

$$C_{vl} = \frac{C}{p^2}$$

The maximum number of virtual links in the network is

$$n = Np$$

and the maximum total network capacity

$$C_{tot} = nC_{vl} = \frac{NC}{p} \tag{2}$$

The total traffic in the network cannot exceed the total capacity. Thus,

$$\lambda < C_{tot} \tag{3}$$

which gives, after combining (1), (2) and (3)

$$\gamma < \frac{NC}{pE} \tag{4}$$

In order to find E, let us first determine D, the diameter of the digraph. The diameter is defined as the maximum number of hops on any shortest path of the digraph. It is well known that the number of nodes N in any digraph of maximum degree p and diameter D satisfies the so-called Moore bound [11]:

$$N \leq \begin{cases} \frac{p^{D+1}-1}{p-1} & p > 1 \\ D+1 & p = 1 \end{cases}$$

or, equivalently:

$$D \geq H = \begin{cases} \lceil \log_p(1 + N(p-1)) \rceil - 1 & p > 1 \\ N - 1 & p = 1 \end{cases}$$

Let us now determine the value of E. From [4] we have that the lower bound for the expected number of hops for a digraph of maximum degree p is

$$E_{lb} = \begin{cases} \frac{p - p^{H+1} + NH(p-1)^2 + H(p-1)}{(N-1)(p-1)^2} & p > 1 \\ \frac{N}{2} & p = 1 \end{cases}$$

Figure 4 shows how the lower bound on the average number of hops E_{lb} changes with p. Assuming that propagation delays dominate over transmission and processing delays at intermediate stations and that the propagation delay for any pair of stations is the same (e.g., star with equidistant nodes), the average network delay under light load is proportional to the average number of hops.

From (4) we see that the upper bound for the aggregate throughput is

$$\gamma_{ub} = \frac{NC}{pE_{lb}}$$

and the upper bound for the mean throughput per station is

$$\sigma_{ub} = \frac{\gamma_{ub}}{N} = \frac{C}{pE_{lb}} \qquad (5)$$

Figures 5 and 6 show how γ_{ub} and σ_{ub} depend on the (maximum) connectivity factor p (i.e., the degree of virtual topology) when C is normalized to 1. Parameter N represents the number of stations in the network. We see a significant improvement in throughput with the increase in p from 1 to 2 for all cases of N. In fact, the improvement is higher when N is larger. This is an expected result, since for $p = 1$ (i.e., the ring) the number of hops grows linearly with N, while for $p > 1$ it grows logarithmically. We also see that when the number of stations is 10, 100 and 1000, σ_{ub} decreases for $p > 2$. The decrease is slower as the number of stations is larger. When the number of stations is 10000 and 1000000, σ_{ub} is almost the same for $p = 2$ and $p = 3$. In fact, σ_{ub} is slightly higher for $p = 3$ in those cases. For $p > 3$, σ_{ub} slowly decreases. Thus, we see that the choice of $p = 2$ or $p = 3$ gives the optimal value for σ_{ub} in most cases of practical interest.

Let us consider the ShuffleNet topology (which requires that the number of stations is $N = kp^k$, where k is a positive integer). It is shown in [4] that the average number of hops in ShuffleNet is

$$E_{sh} = \frac{kp^k(p-1)(3k-1) - 2k(p^k-1)}{2(p-1)(kp^k-1)}$$

Using the routing algorithm for the shared-channel ShuffleNet developed in [4] it is possible to route traffic along shortest paths in such a way that the traffic load on all channels is perfectly balanced. The maximum throughput that can be achieved in ShuffleNet is thus

$$\gamma_{max} = \frac{NC}{pE_{sh}}$$

and the maximum throughput per station is

$$\sigma_{max} = \frac{C}{pE_{sh}}$$

Figure 7 shows the maximum throughput per station in ShuffleNet for station connectivity $p = 2$ and $p = 3$. We see that the maximum throughput that can be achieved using the ShuffleNet virtual topology is very close to the upper bound.

As we already pointed out, the throughput decreases very slowly with the increase in p when the number of stations is large. In such a case, it may be more beneficial to use higher p, since higher p reduces the number of hops, and therefore the delay. In addition, the increase in p reduces the number of wavelength channels since the required number of channels is equal to N/p. Thus, by increasing connectivity we can allow more stations to be connected to the network if the number of wavelengths is limited.

In order to optimize both throughput and delay we use the network power function P, which is defined as a ratio of average throughput per station and average number of hops. Thus

$$P = \frac{\sigma}{E}$$

We have that

$$P \leq P_{ub} = \frac{\sigma_{ub}}{E_{lb}} = \frac{C}{pE_{lb}^2} \tag{6}$$

Figure 8 shows the upper bound for the power function versus p for various numbers of stations N. We see that the optimal value p is higher than the one when the criterion was to maximize the throughput only.

3 Channel sharing with multiple transmitters and receivers per station

We have up to now assumed that each station possesses only one transmitter and receiver. Let us now consider the effects of channel sharing in the multihop network when a station can have d_r receivers and d_t transmitters. In general, we permit $d_r \neq d_t$, referring to such stations as asymmetric. Figure 9 shows an example of an asymmetric station with d_r receivers and d_t transmitters used to realize a degree-p virtual topology. Let s_r and s_t be relatively prime positive integers such that $s_r d_r = s_t d_t$. If α is a positive integer, then the ith receiver is tuned to wavelength channel λ_i, which can be shared by αs_r other transmitters, and the jth transmitter is tuned to wavelength channel ω_j, which can be shared by αs_t other receivers. The integer α is called the channel-sharing factor. When $\alpha = 1$ the network has the least degree of channel sharing possible; thus α represents the relative amount of channel sharing used beyond the minimum configuration. In a dedicated-channel multihop network, αs_t and αs_r are equal to 1; such a configuration is possible only if $d_r = d_t$. So that the underlying virtual topology is a regular digraph of degree p, it is required that

$$\alpha s_r d_r = \alpha s_t d_t = p \tag{7}$$

Note that it is not always possible to establish a regular digraph of degree p when the number of stations is arbitrary. When $d_r \neq d_t$, then some form of channel sharing *must* be employed.

Since each transmitter supports up to αs_t virtual links and each receiver supports up to αs_r virtual links, the wavelength channel between any transmitter-receiver pair is allocated up to $\alpha^2 s_r s_t$ virtual links. Thus, the capacity of a virtual link is

$$C_{vl} = \frac{C}{\alpha^2 s_r s_t}$$

the maximum number of virtual links is

$$n = Np = N\alpha s_t d_t$$

and the total network capacity is

$$C_{tot} = nC_{vl} = \frac{N d_t C}{\alpha s_r} = \gcd(d_r, d_t) NC/\alpha \tag{8}$$

where $\gcd(d_r, d_t)$ represents the greatest common denominator of d_r and d_t. If we repeat the analysis from Section 2 using expression (8) for the total network capacity and also using expression (7) we find that the station throughput is bounded above by

$$\sigma_{ub} = \frac{d_t C}{\alpha s_r E_{lb}} = \frac{d_r C}{\alpha s_t E_{lb}} = \frac{d_r d_t C}{p E_{lb}} \tag{9}$$

and an upper bound on network power is given by the following:

$$P_{ub} = \frac{d_t C}{\alpha s_r E_{lb}^2} = \frac{d_r C}{\alpha s_t E_{lb}^2} = \frac{d_r d_t C}{p E_{lb}^2} \tag{10}$$

When the number of transmitters and receivers per station is the same we see that $d_r = d_t = d$ and $s_r = s_t = 1$. In such a case expression (9) reduces to

$$\sigma_{ub} = \frac{dC}{\alpha E_{lb}} = \frac{d^2 C}{p E_{lb}} \tag{11}$$

and expression (10) to

$$P_{ub} = \frac{dC}{\alpha E_{lb}^2} = \frac{d^2 C}{p E_{lb}^2} \tag{12}$$

If we compare expressions (11) and (12) with expressions (5) and (6) for the single transmitter-receiver case, we notice that for the same connectivity p the increase in the number of devices by a factor of d increases throughput and power by a factor of d^2.

Another interesting special case is when the number of transmitters per station d_t is 1, and the number of receivers per station d_r is p. As we mentioned in the previous section, such a configuration allows us to implement the TDMA channel sharing without need to synchronize the stations. From (9) and (10) we get the following expressions for throughput and network power:

$$\sigma_{ub} = \frac{C}{E_{lb}}$$

$$P_{ub} = \frac{C}{E_{lb}^2}$$

We see that in this case the throughput and power increase by a factor of p compared to the single transmitter-receiver case.

Let us now analyze how channel sharing affects the performance. We consider first the symmetric case, in which the numbers of transmitters and receivers per station are equal. Figure 10 shows how σ_{ub} changes with the increase in channel sharing for different values of parameter d. We see that when $d > 1$ the best choice is to have $\alpha = 1$. Thus, for multiple transmitter-receiver pairs, the limiting throughput σ_{ub} is highest when no channel sharing is employed (i.e., each wavelength channel is dedicated to one transmitter and one receiver).

Figure 11 shows how P_{ub} changes with the increase in channel sharing in a symmetric network when $N = 1000$. We see from the figure that for $d = 1$, 2 and 3, channel sharing can improve the value of P_{ub}. When the number of transmitter–receiver pairs per station is 2, channel sharing with $\alpha = 2$ increases the upper bound for network power more than 50%, compared to the case when no channel sharing is employed (i.e., when $\alpha = 1$).

Given that the upper bound is tight, as evidenced by Figure 7, the graphs of per-station throughput and network power in Figures 10 and 11 are good approximations of the maximum performance that can be achieved with a specified network configuration and channel-sharing factor.

In Figure 12 we analyze the performance of asymmetric networks for d_t, d_r pairs of (1,3), (1,4), (2,3), and (4,3). These configurations all have the same total capacity of $C_{tot} = NC/\alpha$. We plot the upper bounds on throughput per station σ_{ub} as a function of the channel-sharing factor α for specific transmitter-receiver configurations in a 1000-node multihop network. As in the symmetric case, we see that channel sharing does not improve the value of σ_{ub} in these asymmetric configurations. The limiting value of network power, as a function of the channel-sharing factor, is shown in Figure 13. In all but one configuration the value of P_{ub} is highest when the channel-sharing factor $\alpha = 1$, and in the exceptional case P_{ub} achieves its maximum when $\alpha = 2$, but the improvement over the value at $\alpha = 1$ is marginal.

As expected, the symmetric configuration gives the better performance than asymmetric networks with the same total number of receivers and transmitters [the product $d_t d_r$ in expressions (9) and (10) has maximum when $d_t = d_r$]. Indeed, a symmetric network with $d_t + d_r = 2d$ uses dN/α physical channels, whereas an asymmetric network with $d_t + d_r = 2d$ uses only $\gcd(d_t, d_r)N/\alpha$ physical channels.

4 Conclusion

In this paper we have analyzed the performance of TDMA channel sharing in multihop lightwave networks. We have shown that when the number of transmitters and receivers per station is 1, channel sharing significantly improves network throughput. However, the best throughput can be achieved when each wavelength channel is shared by only a few stations. When the number of transmitter-receiver pairs per station is greater than 1, channel sharing does not improve throughput. However, if the criterion is to optimize both throughput and delay, using, for instance a "power" function, we show that channel sharing can improve performance even when multiple transmitter-receiver pairs are used.

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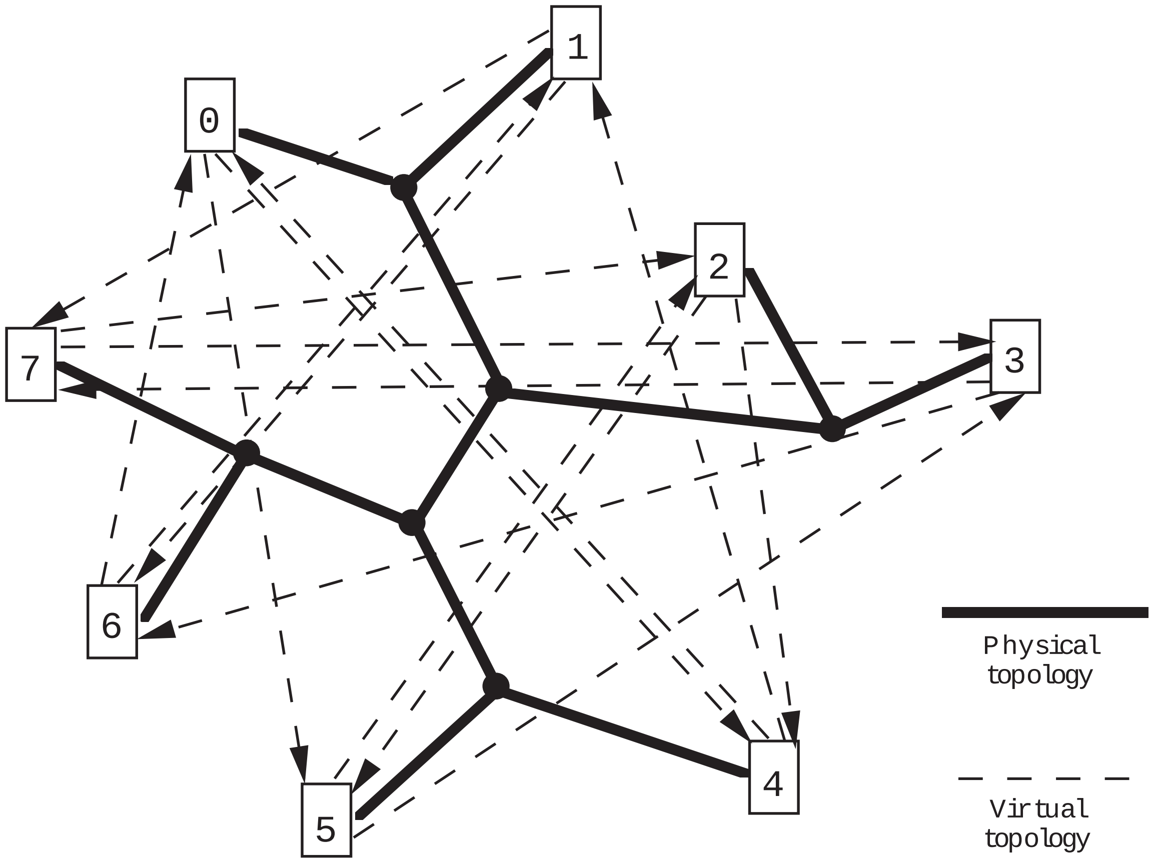

Figure 1: ShuffleNet virtual topology implemented on a physical tree

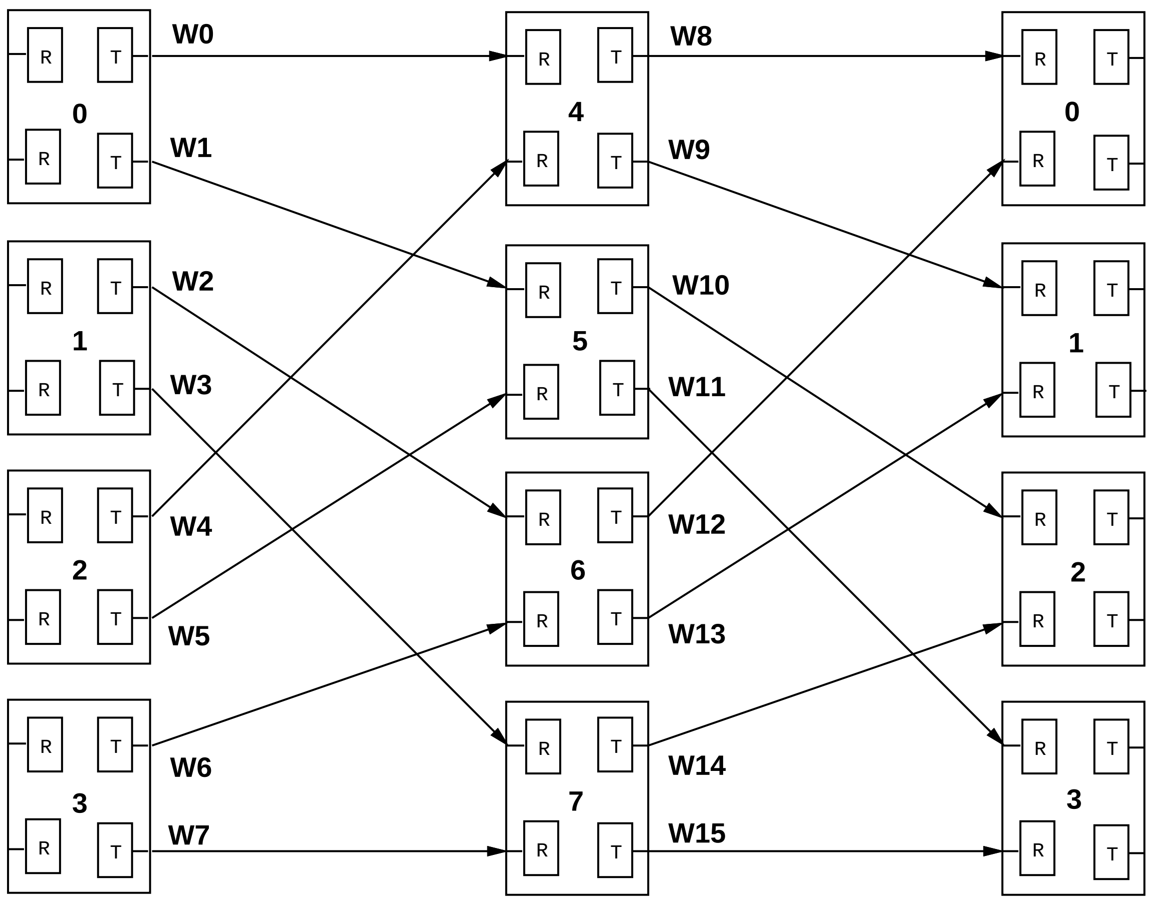

Figure 2: Virtual Topology of the Eight Station ShuffleNet

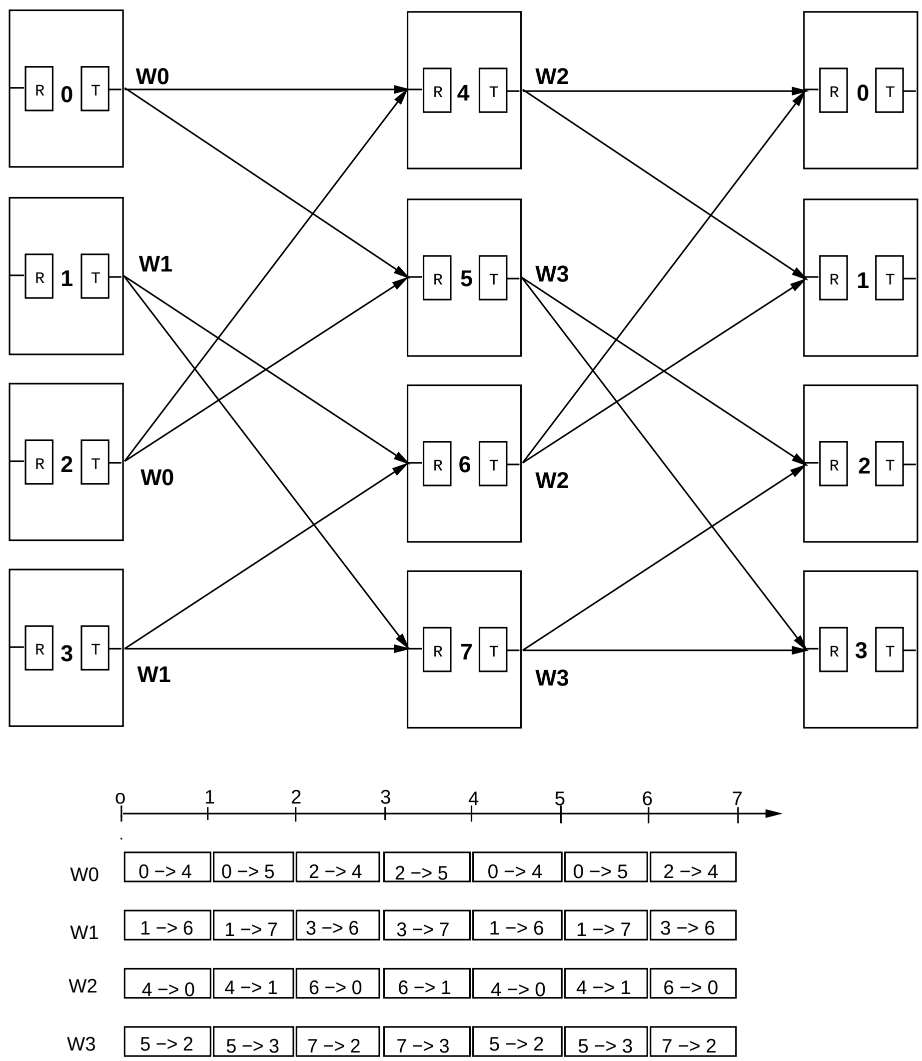

Figure 3: ShuffleNet virtual topology implemented using one transmitter and one receiver per station and TDMA

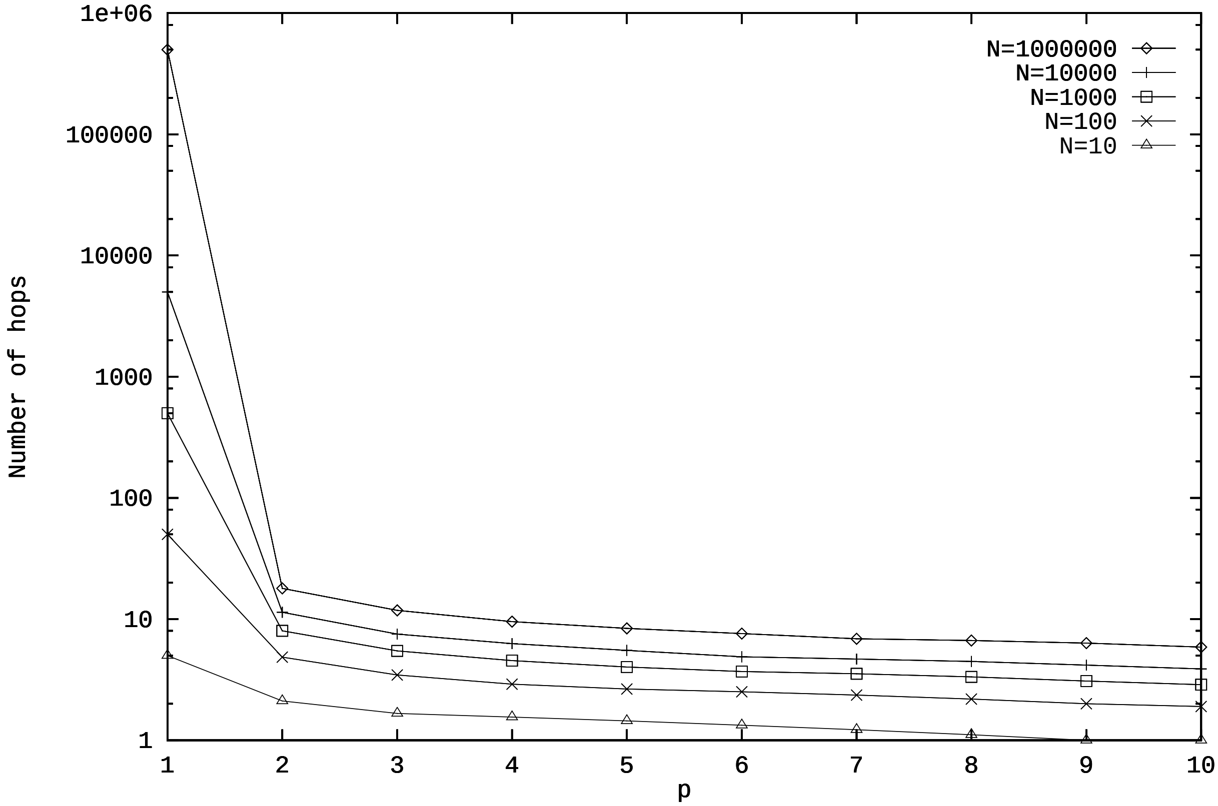

Figure 4: Lower bound for the average number of hops E_{lb} versus connectivity factor p in networks of single-transceiver stations when the number of stations N is 10, 100, 1000, 10000 and 1000000

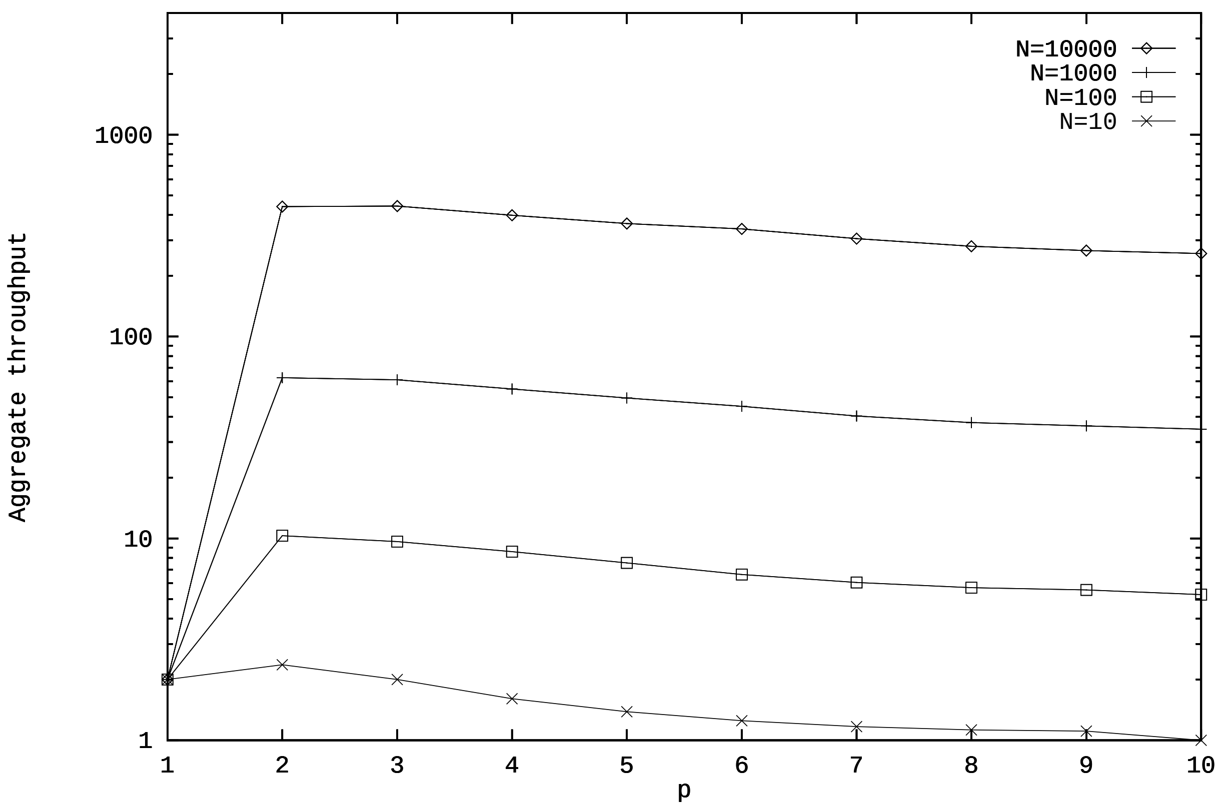

Figure 5: Upper bound on the aggregate network throughput γ_{ub} versus connectivity factor p in networks of single-transceiver stations when the number of stations N is 10, 100, 1000 and 10000

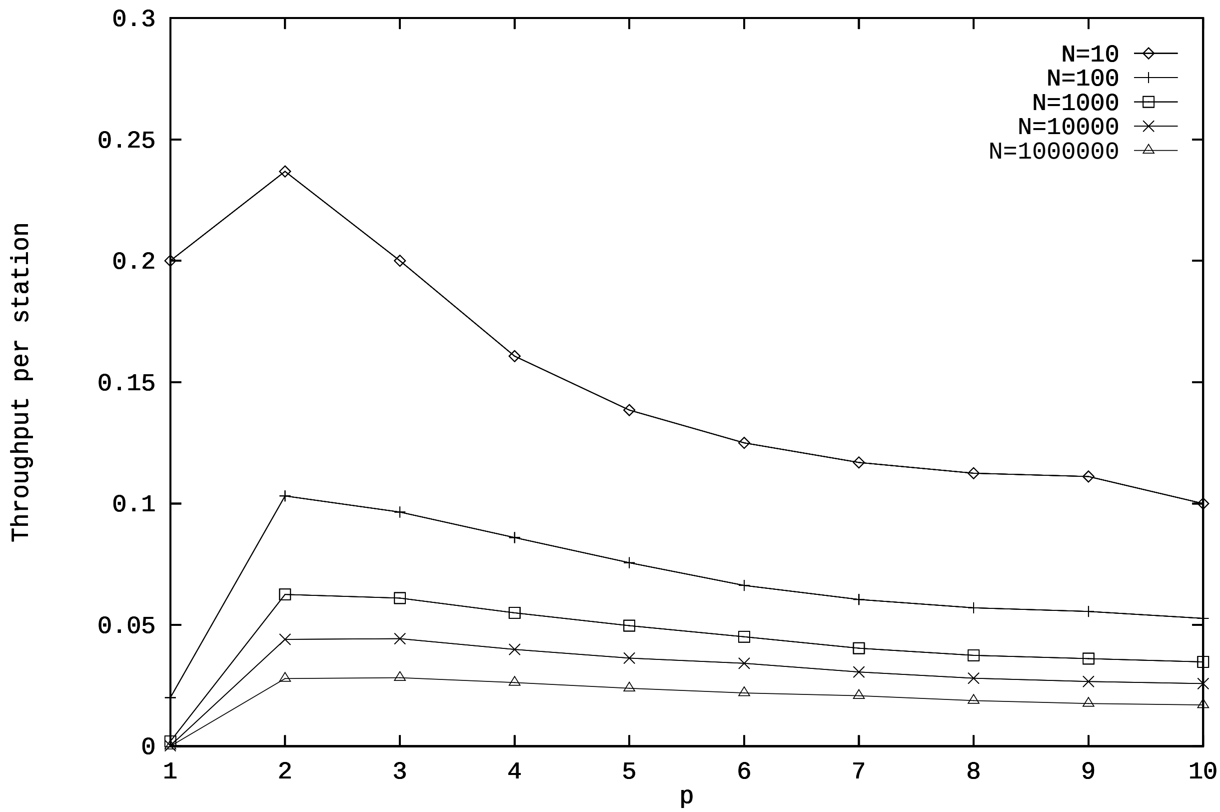

Figure 6: Upper bound for the average throughput per station σ_{ub} versus connectivity factor p in networks of single-transceiver stations when the number of stations N is 10, 100, 1000, 10000 and 1000000

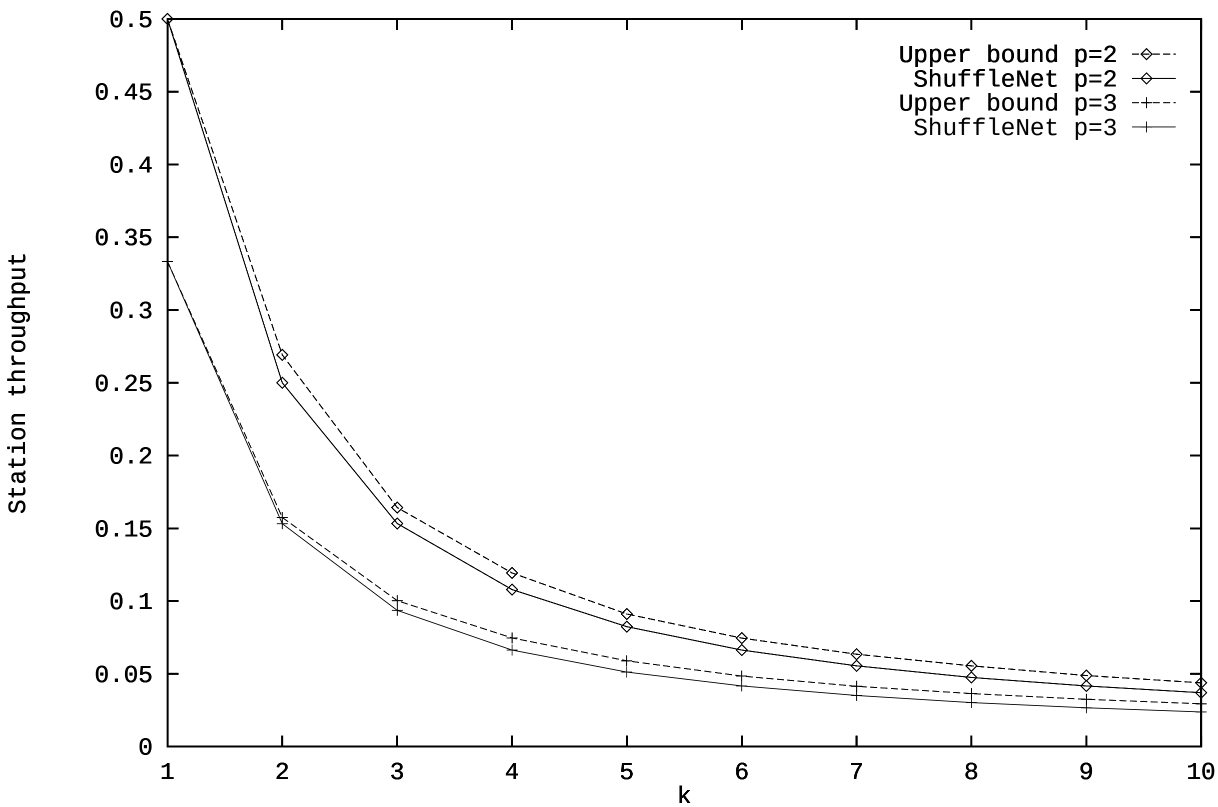

Figure 7: The maximum average throughput per station σ_{max} versus parameter k for ShuffleNet with single-transceiver stations and connectivity factors $p = 2$ and $p = 3$

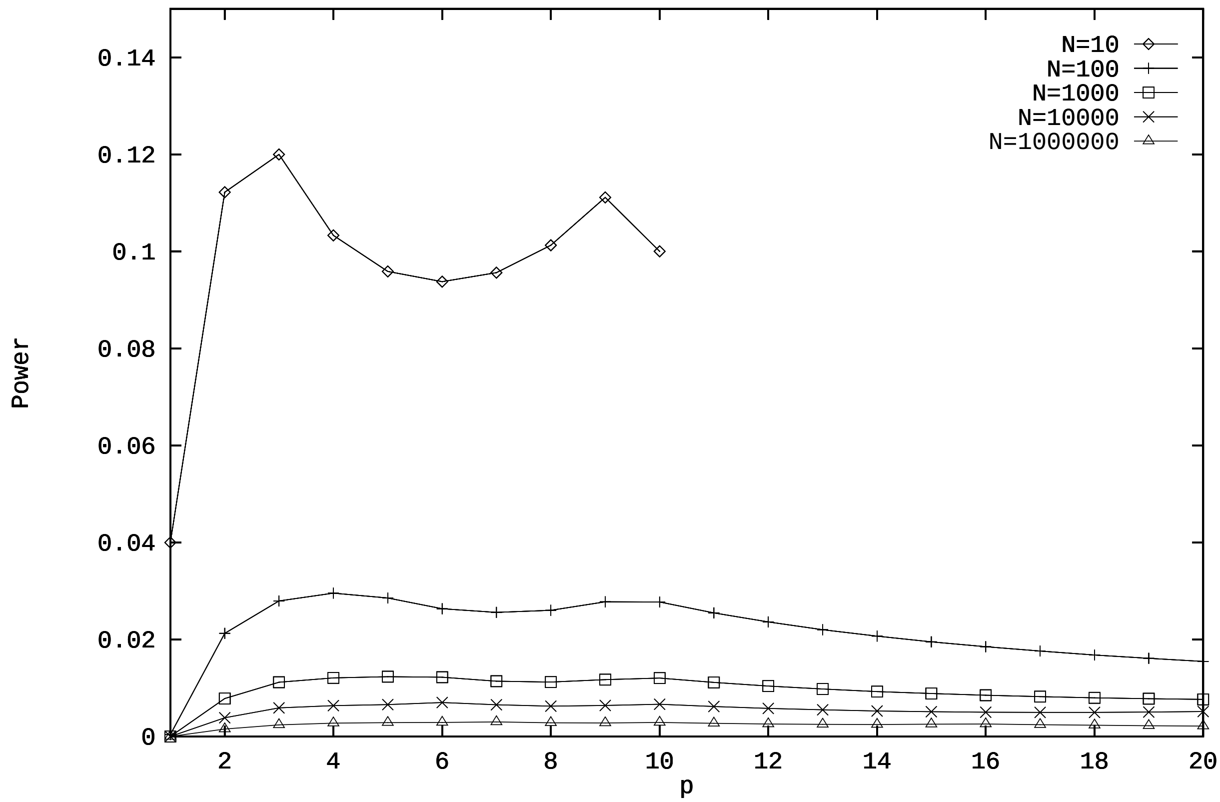

Figure 8: The upper bound on the network power P_{ub} versus connectivity factor p in networks of single-transceiver stations

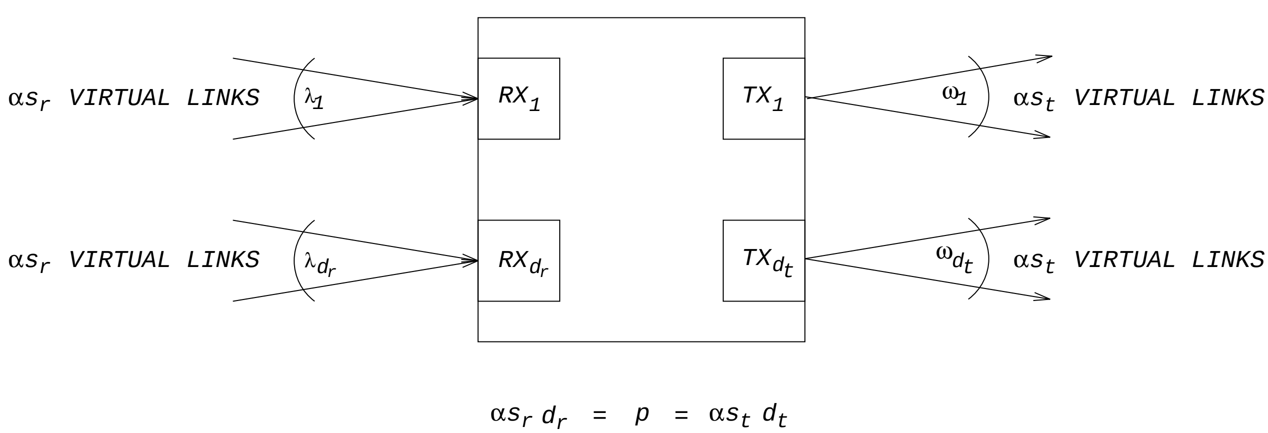

Figure 9: An asymmetric station with d_r receivers and d_t transmitters used to realize degree-p virtual topologies.

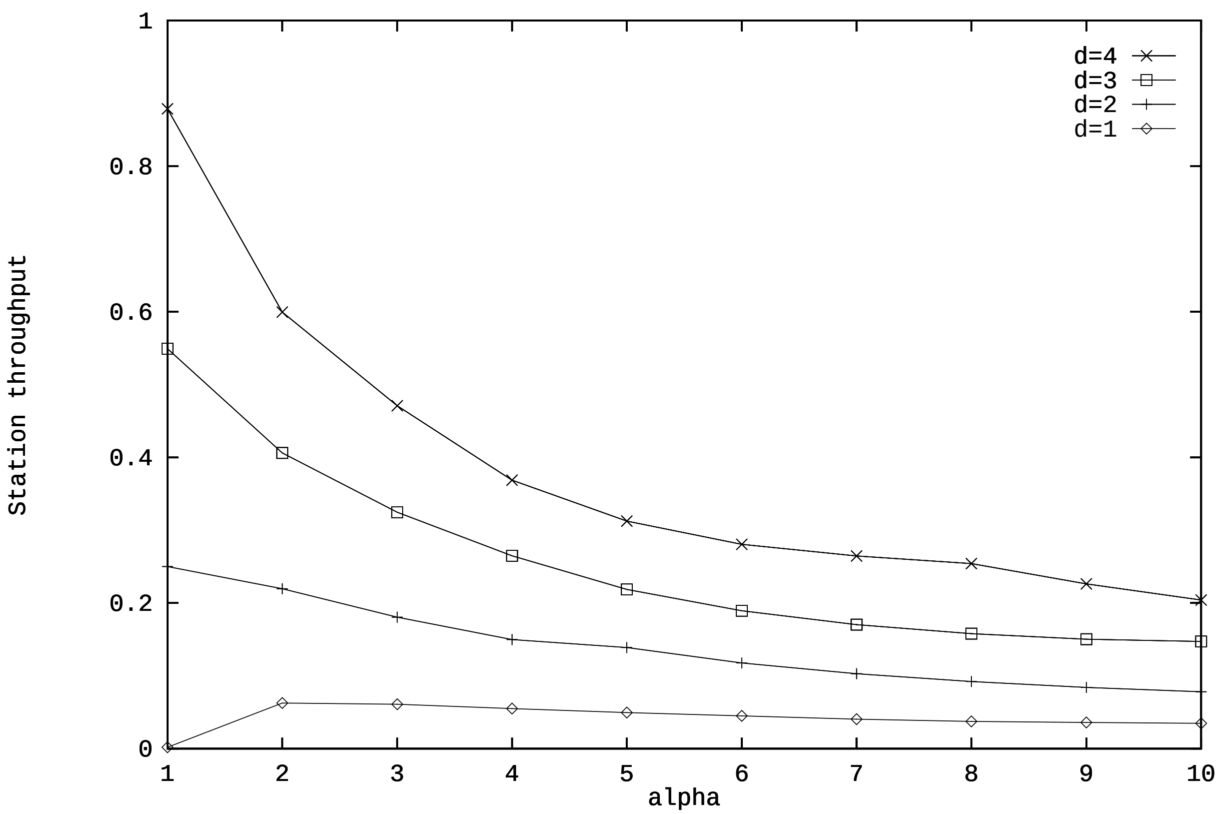

Figure 10: Upper bound on the average throughput per station σ_{ub} versus channel-sharing factor α when the number of stations is 1000 and the number of transmitter–receiver pairs per station d is 1, 2, 3, and 4.

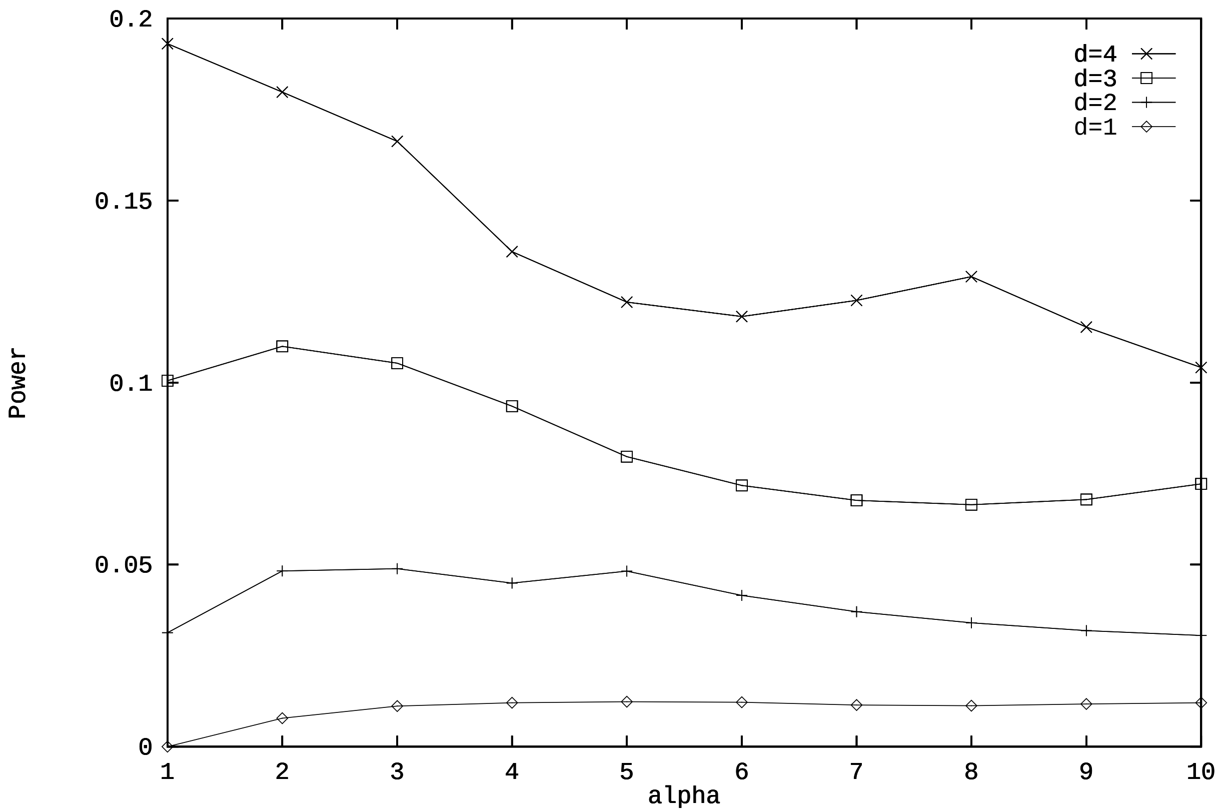

Figure 11: Upper bound on the power function P_{ub} versus channel-sharing factor α when the number of stations is 1000 and the number of transmitter–receiver pairs per station d is 1, 2, 3, and 4.

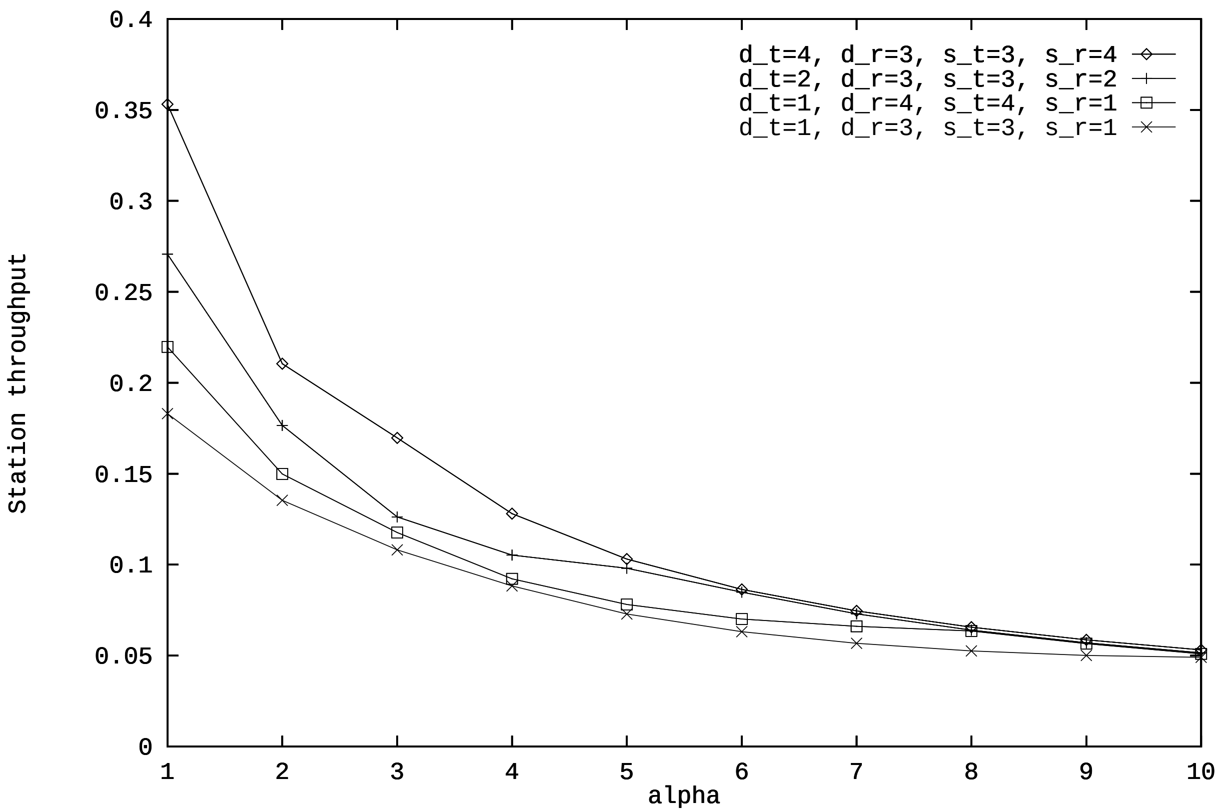

Figure 12: Upper bound on the throughput per station σ_{ub} versus the channel-sharing factor α for several transmitter–receiver configurations when the number of stations is 1000.

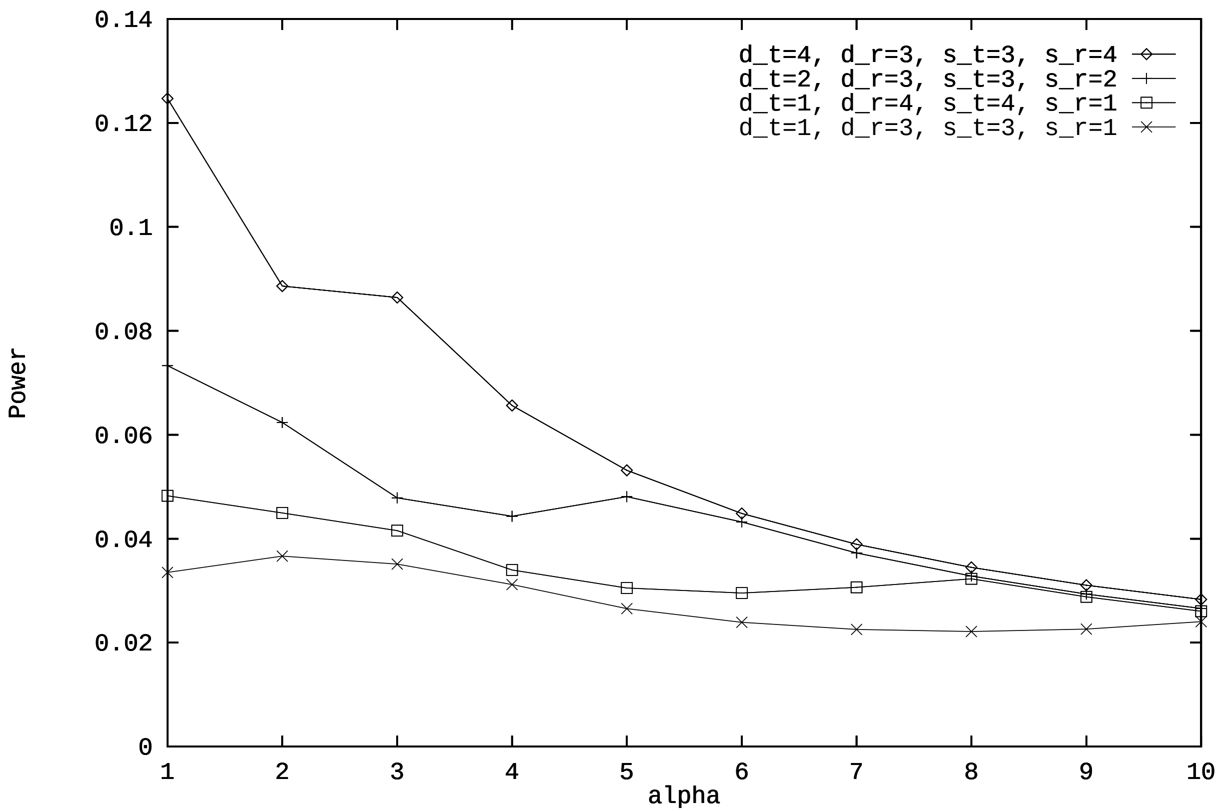

Figure 13: Upper bound on the power function P_{ub} versus the channel-sharing factor α for several transmitter–receiver configurations when the number of stations is 1000.