Idiosyncratic Risk and Security Returns*

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Abstract

The traditional CAPM approach argues that only market risk should be incorporated into asset prices and command a risk premium. This result may not hold, however, if some investors can not hold the market portfolio. For example, if one group of investors fails to hold the market portfolio for exogenous reasons, the remaining investors will also be unable to hold the market portfolio. Therefore, idiosyncratic risk could also be priced to compensate rational investors for an inability to hold the market portfolio. A variation of the CAPM model is derived to capture this observation as well as to draw testable implications. Under both the Fama and MacBeth (1973) and Fama and French (1992) testing frameworks, we find that idiosyncratic volatility is useful in explaining cross-sectional expected returns. We also discover that returns from constructed portfolios directly co-vary with idiosyncratic risk hedging portfolio returns.

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Abstract

The traditional CAPM approach argues that only market risk should be incorporated into asset prices and command a risk premium. This result may not hold, however, if some investors can not hold the market portfolio. For example, if one group of investors fails to hold the market portfolio for exogenous reasons, the remaining investors will also be unable to hold the market portfolio. Therefore, idiosyncratic risk could also be priced to compensate rational investors for an inability to hold the market portfolio. A variation of the CAPM model is derived to capture this observation as well as to draw testable implications. Under both the Fama and MacBeth (1973) and Fama and French (1992) testing frameworks, we find that idiosyncratic volatility is useful in explaining cross-sectional expected returns. We also discover that returns from constructed portfolios directly co-vary with idiosyncratic risk hedging portfolio returns.

Introduction

The usefulness of the well-celebrated CAPM theory of Sharpe, Lintner, and Black to predict cross-sectional security and portfolio returns has been challenged by researchers such as Fama and French (1992, 1993). It is still debatable, however, whether Fama and French's empirical approach has invalidated the CAPM (see, for example, Berk, 1995; Ferson and Harvey, 1991; Kothari, Shanken, and Sloan, 1995; Jagannathan and Wang, 1996; and Loughran, 1996). Moreover, as Roll (1977) has pointed out, it is difficult, if not impossible, to devise an adequate test of the theory. Nevertheless, financial economists have worked in several directions to improve the theory of asset pricing. The first route has involved relaxing the underlying assumptions of the model, including the introduction of a tax effect on dividends (e.g., Brennan (1970)), non-marketable assets (e.g., Mayers (1972)), as well as accounting for inflation and international assets (e.g., Stulz (1981)). A second route has been to extend the one period CAPM to an intertemporal setting (e.g., Merton, 1973; Lucas, 1978, Breeden, 1979, and Cox, Ingersoll and Ross, 1985) to connect factors that affect consumption growth to asset returns. Ross[1] (1976) has taken a different route by assuming that the stochastic properties of asset returns are consistent with a factor structure. We accept the CAPM modeling environment as a reasonable first order approximation, but find that idiosyncratic risk might play a role under certain plausible conditions.

Starting from mean variance analysis, the traditional CAPM theory predicts that only market risk should be priced in equilibrium; any role for idiosyncratic risk is completely excluded through diversification. CAPM must surely hold if investors are alike and can hold a combination of the market portfolio and a risk-free asset as the theory prescribes. In real-

[1] Also see, for example, Chamberlain and Rothschild, 1983; Chen, Roll, and Ross, 1986; Connor, 1984; Connor and Korajczyk, 1988; Dybvig, 1983; Lehmann and Modest, 1988; Shanken, 1982; and Shukla and Trzcinka 1990).

ity, however, institutional investors will often deliberately structure their portfolios to accept considerable idiosyncratic risk in an attempt to obtain extraordinary returns. Even in the absence of unusual cases such as Enron and WorldCom, these investors fully appreciate the importance of idiosyncratic factors in affecting the risk to which they are exposed. Therefore, as Merton (1987) wrote in his AFA presidential address, "... financial models based on frictionless markets and complete information are often inadequate to capture the complexity of rationality in action." When one group of investors – who we call "constrained" investors – is unable to hold the market portfolio for various reasons, such as transactions costs, incomplete information, and institutional constraints such as, taxes, liquidity needs, imperfect divisibility of securities, taxes, restrictions on short sales, and so on, the remaining investors – labeled as "free" or "unconstrained" investors – will also be unable to hold the market portfolio. This is so because it is the total holdings from the two groups of investors that make up the whole market. Since the relative per capita supply will be high for those stocks that the constrained investors only hold in very limited amounts, the prices of these stocks must be relatively low. In other words, an idiosyncratic risk premium can be rationalized to compensate investors for the "over supply" or "unbalanced supply" of some assets. An inability to hold the market portfolio will force investors to care about total risk to some degree in addition to the market risk.

Still another intuition can be gained in terms of diversification. Suppose the actual market portfolio consists of only tradable securities. In other words, the market portfolio is observable and measurable. If some investors are constrained from holding all securities, the "available market portfolio" that unconstrained investors can hold will be less diversified than the *actual* market portfolio. When individual investors use the *available* market portfolio to price individual securities, the corresponding risk premia tend to be higher than those under the CAPM where all investors are able to hold the *actual* market portfolio. This is because some of the systematic risk would be considered as idiosyncratic risk relative to the *actual* market portfolio. Hence, idiosyncratic risk would be priced in the market.

Based on the above intuition, we will build a simple CAPM type of model to see what kind of idiosyncratic risks might be priced in order to motivate our empirical investigation. Although the static CAPM is rejected in almost all the empirical studies, "the CAPM is wanted, dead or alive" (see Fama and French 1996). First, it is the CAPM that establishes the role of the market factor in asset pricing. In fact, the market factor captures the most variations in individual securities over time compared to other known factors or proxies. Second, despite the fact that other factors such as size and book-to-market are important in explaining security returns, they do not exist in the CAPM world. Therefore, by focusing on the CAPM environment, we are able to study under what condition idiosyncratic risk might play a role in asset pricing independent of these other factors. In fact, both the model and the empirical results suggest that it is the "undiversified" idiosyncratic risk that explains the cross-sectional difference in equity returns.

The role of idiosyncratic risk in asset pricing has been studied in the literature to some extent. Most theoretical papers have focused on the effect of idiosyncratic (or uninsurable) income risk on asset pricing (see for example, Heaton and Lucas (1996), Thaler (1994), Aiyagari (1994), Lucas (1994), Telmer (1993), Franke, Stapleton, and Subrahmanyam (1992), and Kahn (1990)). Based on assumptions similar to ours, Levy (1978) derived a modified CAPM that revealed a possible bias in the beta estimator as well as a possible role for idiosyncratic risk. In contrast, our model demonstrates an explicit role of idiosyncratic risk in asset pricing. Furthermore, we show that the beta estimator will be unbiased if idiosyncratic risk is appropriately account for. Perhaps the paper most relevant to our study is Merton (1987). Starting from a single factor structure of returns, he assumes that investors can only invest in securities where they have exact information about the expected returns, beta loadings, and volatilities. This assumption seems to be unduly restrictive. Although both the Merton model and ours yield simular pricing implications, our model is less restrictive and more general in two respects. First, we do not require that idiosyncratic returns are uncorrelated across individual stocks. If this condition is imposed, our model will reduce to that of Merton (1987). Second, we demonstrate that the price of idiosyncratic risk for an individual stock depends on its correlation with the aggregated *undiversified* idiosyncratic return. This motivates the construction of the return proxy for the idiosyncratic risk used in our time series study.

On the empirical front, Douglas (1969) is perhaps the first study that considers the role of idiosyncratic risk. He concluded that residual variance was also priced based on a single cross-sectional regression using average returns.[2] Fama and MacBeth's (1973) important study both rejected the role of idiosyncratic risk in the CAPM and provided a more powerful cross-sectional test. Lehmann (1990), however, studied the significance of residual risk in the context of statistical testing methodology. Some indirect evidence regarding the role of idiosyncratic risk has also surfaced. Falkenstein (1996) found some evidence that the equity holdings of mutual fund managers appeared to be related to idiosyncratic volatility. Using Swedish government lottery bonds where the underlying risk is idiosyncratic by construction, Green and Rydqvist (1997), find that bond prices appear to reflect aversion to idiosyncratic risk. Bessembinder (1992) finds strong evidence that idiosyncratic risk was priced, looking at a cross-section of foreign currency and agricultural futures. In studying the volatility linkage between national stock markets, King, Sentana, and Wadhwani (1994) have provided evidence that idiosyncratic economic shocks are priced and that the 'the price of risk' is different across stock markets. The time series pricing implications of idiosyncratic volatility have also been

[2]Miller and Scholes (1972) suggested that several sources of bias may exist including omitting the risk-free rate, errors in beta measures, and correlation between betas and residual variances. They claimed that the errors-in-variables issue is especially important.

studied by Goyal and Santa-Clara (2003). They find a significant positive relationship between average idiosyncratic volatility and the return on the market.

Since most empirical evidence supporting the role of idiosyncratic risk from early studies in asset pricing was disregarded after the comprehensive study by Fama and MacBeth (1973), we start our empirical study by replicating the Fama and MacBeth study and extending it to different settings and sample periods. In addition, we also consider Fama and French's (1992) framework. The empirical results support our model by showing that (1) idiosyncratic volatility by itself is important in explaining cross-sectional expected return differences; (2) its explanatory power does not seem to be taken away by other variables, such as size, book-to-market, and liquidity; and (3) the findings are robust to Japanese stock return data. Recently, Ang, Hodrick, Xing, and Zhang (2003) provide empirical evidence suggesting that individual stock returns are negatively related to idiosyncratic volatility estimated using daily returns over a short period of time.[3]

The paper is organized as follows: A simple CAPM type of model with some constrained investors is constructed in the first section. After studying the implications of the model, we discuss issues related to empirical testing and data construction in section 2. Section 3 presents cross-sectional evidence in the spirit of Fama and MacBeth (1973) and Fama and French (1992). Times series evidence concerning the role of idiosyncratic volatility is briefly discussed in Section 4. Section 5 presents concluding comments.

[3]The results might subject to "errors-in-variables problem" when fitting a market model to a short sample of individual daily stock returns. This is one of the major concerns in Fama and French (1992) even when monthly returns are used, which leads to the use of portfolio estimates. In contrast, we use "undiversified idiosyncratic" volatility, which not only reduces the "error-in-variables" problem but also bears theoretical support.

1 The basic model and its implications

The Capital Asset Pricing Model is an equilibrium model in which the demand for equity securities is determined under a mean-variance optimization framework. The market clearing condition then equates demand and the exogenous supply to achieve equilibrium. Since it is assumed that investors are homogenous and are able to hold every asset in the market portfolio, their holdings will be similar in equilibrium. As a result, investors' holdings of risky stocks will comprise shares held in proportion to the market portfolio, which is a value-weighted portfolio of all the securities available for investment. In other words, the market portfolio is always feasible and will be the only portfolio held in equilibrium. Such an available market portfolio will be altered, however, whenever a group of investors does not or cannot hold every stock for the following reasons.

First, transactions costs are likely to prevent individual investors from holding large numbers of individual stocks in their portfolios. In fact, Hirshleifer (1988) has predicted that trading costs limit the participation of some classes of traders in commodity futures markets and that idiosyncratic risk will be priced cross-sectionally. Furthermore, more than half of the U.S. households have accounts with brokerage firms. Because of limited resources or their desires to exploit the unique characteristics of individual stocks, these investors normally only hold a handful of stocks.[4] In addition, as Hirshleifer (2001) has pointed out that "there is also experimental evidence that investors sometimes fail to form efficient portfolios and violate two-fund separation." Also, in order to provide financial incentives for their employees, many companies now grant stock options or restricted stock to match the employee contributions to 401K retirement plans with company stock. In general, such employees are constrained

[4] One may argue that the idiosyncratic volatility of a portfolio is close to zero when there are more than 20 stocks. However, this conclusion is based on a random sampling. In reality, investors do not randomly select their stocks. In addition, Campbell, Lettau, Malkiel, and Xu (2000) have shown that a well-diversified portfolio must have 50 or more stocks in recent decades because idiosyncratic volatility has increased.

from liquidating their positions or to hedge the stocks of their own firms, hence they tend to hold very unbalanced portfolios.[5] Moreover, some stock traders and market makers hold large positions in individual stocks.

Finally, despite the fact that there are several thousand actively managed mutual funds and pension funds, these funds typically do not hold a market portfolio even though they are able to do so.[6] Moreover, Day, Wang, and Xu (2000) have demonstrated that the portfolios of equity mutual funds are not even mean-variance efficient with respect to their holdings. These "active" portfolio managers are able to obtain large management fees because they claim to be able to find "undervalued" securities and hence offer investors the possibility of risk-adjusted returns superior to the market averages. While there is no evidence that they can achieve this goal even before expenses (see Jensen (1968) and Malkiel (1995)), they do affect the relative supply of stocks available for other investors. Equity mutual funds hold portfolios comprising almost one-third of the total capitalization of the U.S. stock market, and thus they have the potential to alter the supply of securities available to other investors in an important way. The fact that investors are willing to pay the high costs to invest in non-indexed mutual funds indicates that they do not choose to allocate their portfolios between a market portfolio and a risk-free asset as the CAPM theory assumes.

Behavioral finance provides additional insights that help to explain why institutional investors may be sensitive to idiosyncratic risk of individual securities even though such volatility can be diversified away. The prospect theory of Kahneman and Tversky (1979) makes clear that the major force influencing the decisions of investors is loss aversion. Mutual fund man-

[5] This practice is particularly prevalent in high technology industries.

[6] Institutional investors are more likely to purchase index funds than are individual investors. A proximately 10 percent of the mutual funds held by individuals were indexed in 2003 while about one third of institutional funds were indexed. Thus, the vast majority of investors do not hold the market portfolio. According to one survey, about 15% of individual investors only held one stock in their account and an average investor only owned about three stocks.

agers, pension fund managers and other institutional investors are usually required to report quarterly to their directors, trustees, etc., on their recent investment performance. That report typically includes a discussion of the best and worst performing stocks in their portfolios. Even if balanced by favorable performance in other parts of the portfolio, it is extraordinarily difficult to explain why the manager bought and held those stocks, that declined sharply in value. Trustees and directors are quite likely to ask the indelicate question of the manager, “How could you have held WorldCom or Enron as these common stocks had lost essentially all of their value?” Trustees and directors are unlikely to be sympathetic to arguments that idiosyncratic factors (accounting fraud, unexpected industry overcapacity, etc.) are valid excuses for such investment errors. It is reasonable, therefore, to believe that stocks that are sensitive to substantial idiosyncratic risks may be subject to additional risk premiums.

There is no doubt, therefore, that not every investor is willing or able to hold the market portfolio. Indeed, even index funds that attempt to replicate the very broad market indexes, such as the Wilshire 5000 and Russell 3000, do not hold all the stocks in the index in order to minimize transactions costs. To what extent this distortion will affect the CAPM is purely an empirical question. Our approach is not to conduct a direct investigation of the portfolio holdings of investors. Instead, we will take as given that investors are unable to hold the market portfolio. Starting from there, we investigate the consequences.

1.1 Asset returns in a traditional CAPM world

For ease of exposition, we assume that there are three risky stocks denoted a, b, and c that generate a return vector $\mathbf{R} = [R_a, R_b, R_c]'$ and one riskless bond that pays interest rate r. Not all of the stocks are necessarily on the traditional mean-variance efficient frontier. The final result will not depend on the number of stocks assumed since we use vector notations.

The risk structure for the three stocks is represented by their variance-covariance matrix of returns, $\mathbf{V} = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} & \sigma_{bc} \\ \sigma_{ab} & \sigma_b^2 & \sigma_{ca} \\ \sigma_{ca} & \sigma_{bc} & \sigma_c^2 \end{bmatrix}$. Each investor has the following utility function,

$$u(W) = E(W) - \frac{1}{2\tau} Var(W), \tag{1}$$

where W represents future wealth and τ is the coefficient of risk tolerance. This particular utility function is consistent with the family of exponential utility functions when future wealth has a normal distribution. If we denote $\mathbf{X}_j = [x_{a,j}, x_{b,j}, x_{c,j}]$ as investor j's dollar amount invested in the three stocks, utility maximization of equation (1) subject to a budget constraint leads to the well-known demand function,

$$\mathbf{X}_j = \tau \mathbf{V}^{-1}(\boldsymbol{\mu} - r\underline{\mathbf{1}}), \tag{2}$$

where $\boldsymbol{\mu} = E(\mathbf{R})$ is the vector of expected returns for individual stocks and r is the risk-free rate. Although the risk structure (variance-covariance matrix) of individual stock returns is given exogenously, the expected returns should be determined in equilibrium by total supply. In other words, the equity market clears with the condition of $\sum_j^n \mathbf{X}_j = \mathbf{S}$, where $\mathbf{S} = [S_a, S_b, S_c]$ is the total supply of individual stocks and n is the total number of investors. The equilibrium expected returns in this unrestricted world can thus be written as,

$$\boldsymbol{\mu} - r\underline{\mathbf{1}} = \frac{1}{n\tau} \mathbf{VS}. \tag{3}$$

Following convention, we define the market portfolio as $\boldsymbol{\alpha} = \frac{1}{M}\mathbf{S}$, where $M = \mathbf{S}\underline{\mathbf{1}}' = S_a + S_b + S_c$. Under this notation, the expected market return and market volatility can be expressed as $\mu_m = \boldsymbol{\alpha}'\boldsymbol{\mu}$ and $\sigma_m^2 = \boldsymbol{\alpha}'\mathbf{V}\boldsymbol{\alpha}$. Equation (3) can thus be converted into the traditional CAPM,

$$\boldsymbol{\mu} - r\underline{\mathbf{1}} = \boldsymbol{\beta}(\mu_m - r), \tag{4}$$

where $\boldsymbol{\beta} = [\beta_a, \beta_b, \beta_c] = \frac{1}{\sigma_m^2}\mathbf{V}\boldsymbol{\alpha}$ is the conventional measure of systematic risk. What makes this single factor model a truly equilibrium model is the existence of an equilibrium market portfolio, which is determined by the aggregate supply. Since the variance-covariance structure and the total supply of stocks are common knowledge, this market portfolio can be constructed by an econometrician even when there are limited investment opportunities.[7] The important implication of equation (4) is that only systematic risk, represented by the scaled covariance between individual stock returns and the market return, matters for valuation. Idiosyncratic risk can be diversified away in this framework, and will not command a risk premium. In the discussion that follows, we will use idiosyncratic volatility to measure idiosyncratic risk. In light of equation (4), idiosyncratic volatility $\mathbf{V}_\epsilon$ can simply be defined as the difference between the total volatility and the market volatility,

$$\mathbf{V}_\epsilon = \mathbf{V} - \sigma_m^2 \boldsymbol{\beta}\boldsymbol{\beta}'. \tag{5}$$

1.2 Asset returns under an imperfect market portfolio

When some investors cannot or do not hold every security for the reasons discussed at the beginning of the section, the CAPM will fail to hold. For ease of exposition, we assume that there are three groups of investors. While the "free" investors in the second group have full investment opportunities and can hold all securities, the first and the third groups of investors are assumed to be constrained from holding the first and the third stocks, respectively. Following the same steps, we can derive demand equations similar to equation (2) for representative investors in each group as,

$$\begin{aligned}
\mathbf{X}_{(1)} &= \tau \begin{bmatrix} 0 & \underline{\mathbf{0}}' \\ \underline{\mathbf{0}} & \Sigma_{bc}^{-1} \end{bmatrix} (\boldsymbol{\mu} - r\underline{\mathbf{1}}), \\
\mathbf{X}_{(2)} &= \tau \mathbf{V}^{-1} (\boldsymbol{\mu} - r\underline{\mathbf{1}}),
\end{aligned}$$

[7] For illustrative purposes, if we assume that only the NYSE/AMEX/NASDAQ listed stocks form the entire universe of investment assets, the conventional market index portfolio, such as the value weighted NYSE/AMEX/NASDAQ index, is indeed the market portfolio and is observable to everyone. Since we will study the effect of limited investment opportunities under such a scenario where we know the market portfolio, Roll's (1977) critique is inapplicable.

$$\mathbf{X}_{(3)} = \tau \begin{bmatrix} \Sigma_{ab}^{-1} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}}' & 0 \end{bmatrix} (\boldsymbol{\mu} - r\underline{\mathbf{1}}),$$

where

$$\Sigma_{ab} = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix}, \quad and \quad \Sigma_{bc} = \begin{bmatrix} \sigma_b^2 & \sigma_{bc} \\ \sigma_{bc} & \sigma_c^2 \end{bmatrix}.$$

If there are n_1, n_2, and n_3 number of investors in the first, the second, and the third groups, respectively, the market clearing condition leads to the following,

$$\mathbf{S} = n_1\mathbf{X}_{(1)} + n_2\mathbf{X}_{(2)} + n_3\mathbf{X}_{(3)} = n\tau[\eta_{1.3}(\mathbf{V}_*)^{-1} + \eta_2\mathbf{V}^{-1}](\boldsymbol{\mu}^c - r\underline{\mathbf{1}}), \tag{6}$$

where $\boldsymbol{\mu}^c$ is the vector of expected equilibrium stock returns in this constrained world, and $\mathbf{V}_* = (\frac{n_1}{n_1+n_3}\begin{bmatrix} 0 & \underline{\mathbf{0}}' \\ \underline{\mathbf{0}} & \Sigma_{bc}^{-1} \end{bmatrix} + \frac{n_3}{n_1+n_3}\begin{bmatrix} \Sigma_{ab}^{-1} & \underline{\mathbf{0}} \\ \underline{\mathbf{0}}' & 0 \end{bmatrix})^{-1}$ is the aggregate variance-covariance matrix *perceived* by constrained investors. $\eta_{1.3} = (n_1+n_3)/n$ is the proportion of constrained investors and $\eta_2 = n_2/n = 1-\eta_{1.3}$ is the proportion of "free" investors. The expected equilibrium return vector $\boldsymbol{\mu}^c$ is then determined by the following equation,

$$\boldsymbol{\mu}^c - r\underline{\mathbf{1}} = \frac{1}{n\tau}[\eta_{1.3}(\mathbf{V}_*)^{-1} + \eta_2\mathbf{V}^{-1}]^{-1}\mathbf{S}. \tag{7}$$

Equation (7) says that in a constrained market, investors apply an altered variance-covariance matrix $[\eta_{1.3}(\mathbf{V}_*)^{-1} + \eta_2\mathbf{V}^{-1}]^{-1}$ to price stocks instead of using the true variance-covariance matrix $\mathbf{V}$ of individual stock returns. In other words, a CAPM would have prevailed, had the altered variance-covariance matrix represented the true risk structure. However, since the risk structure is given, we should rewrite equation (7) alternatively as,

$$\boldsymbol{\mu}^c - r\underline{\mathbf{1}} = \frac{1}{n\tau}\mathbf{V}[\eta_{1.3}(\mathbf{V}_*)^{-1}\mathbf{V} + \eta_2\mathbf{I}]^{-1}\mathbf{S} = \frac{1}{n\tau}\mathbf{V}\mathbf{S}_*. \tag{8}$$

where $\mathbf{S}_*$ is the effective supply. Therefore, equation (8) can be interpreted as if investors are subject to an altered market portfolio $\boldsymbol{\alpha}_*(= \frac{\mathbf{S}_*}{\mathbf{S}_*'\underline{\mathbf{1}}})$. *In other words, a CAPM type of relationship continues to hold with regard to the altered market portfolio in equilibrium. But the CAPM relationship will not hold with respect to the actual total market portfolio.*

Equation (8) also suggests that the portfolio $\boldsymbol{\alpha}_*$ is a tangency portfolio in the space of $(\boldsymbol{\mu}^c, V)$. However, since we (the econometricians) do not know the distribution of investors among different groups, it is impossible to construct such an altered market portfolio in empirical studies. When investors price individual stocks with respect to the *altered* market portfolio ($\boldsymbol{\alpha}_*$) available to them, the econometricians tend to find an imperfect CAPM. This is because econometricians can only use the market return ${R_m}^\dagger$ derived from the actual observable market portfolio weights $\boldsymbol{\alpha}$ constructed from all of the outstanding shares of stocks, that is ${R_m}^\dagger = \boldsymbol{\alpha}'\mathbf{R}$. The net effect is that some of the idiosyncratic risk with respect to the actual *observed* market portfolio will be priced.

In order to illustrate the point, we rewrite equation (7) in the following way;

$$\begin{aligned}\boldsymbol{\mu}^c - r\underline{\mathbf{1}} &= \frac{1}{n\tau}[\mathbf{V}^{-1} - \eta_{1.3}(\mathbf{V}^{-1} - \mathbf{V}_*^{-1})]^{-1}\mathbf{S} \\ &= \frac{1}{n\tau}\mathbf{V}\mathbf{S} + \frac{\eta_{1.3}}{n\tau}\mathbf{V}\boldsymbol{\omega}, \qquad (9)\end{aligned}$$

where $\boldsymbol{\omega}$ $(= [(\mathbf{I} - \mathbf{V}_*^{-1}\mathbf{V})^{-1} - \eta_{1.3}\mathbf{I}]^{-1}\mathbf{S})$ is the supply adjustment. Equation (9) reveals that the equilibrium expected returns will adjust both to the actual total supply, as in the traditional CAPM, and to the supply adjustment from constrained investors. When the aggregate demand from the constrained investors is large, as we suggest it is, substantial adjustments will be required. Next, we multiply both sides of equation (9) by the actual market portfolio weights $\boldsymbol{\alpha}$, that is,

$${\mu_m}^\dagger - r = \frac{M}{n\tau}\boldsymbol{\alpha}'\mathbf{V}\boldsymbol{\alpha} + \frac{M}{n\tau}\eta_{1.3}\boldsymbol{\alpha}'\mathbf{V}\boldsymbol{\omega}_* = \frac{M}{n\tau}\sigma_m^2 + \frac{M}{n\tau}\eta_{1.3}\sigma_m^2\boldsymbol{\beta}'\boldsymbol{\omega}_*, \qquad (10)$$

where ${\mu_m}^\dagger = \boldsymbol{\alpha}'\boldsymbol{\mu}_c$ is the observed expected market return, and $\boldsymbol{\omega}_* = \frac{1}{M}\boldsymbol{\omega}$ is the relative supply adjustment. Substituting equation (10) back into equation (9) and applying equation (5), we have the following result,

$$\boldsymbol{\mu}^c - r\underline{\mathbf{1}} = \boldsymbol{\beta}\frac{{\mu_m}^\dagger - r}{1 + \eta_{1.3}\boldsymbol{\omega}_*'\boldsymbol{\beta}} + \frac{({\mu_m}^\dagger - r)/\sigma_m^2}{1 + \eta_{1.3}\boldsymbol{\omega}_*'\boldsymbol{\beta}}\eta_{1.3}\mathbf{V}\boldsymbol{\omega}_*$$

$$
\begin{aligned}
&= \boldsymbol{\beta}({\mu_m}^{\dagger} - r) + \frac{({\mu_m}^{\dagger} - r)/\sigma_m^2}{1 + \eta_{1.3}\boldsymbol{\omega}_*'\boldsymbol{\beta}}\eta_{1.3}[\mathbf{V}\boldsymbol{\omega}_* - \boldsymbol{\beta}\sigma_m^2\boldsymbol{\beta}'\boldsymbol{\omega}_*] \\
&= \boldsymbol{\beta}({\mu_m}^{\dagger} - r) + k\delta_{SR}\mathbf{V}_{\epsilon}\boldsymbol{\omega}_*, \qquad (11)
\end{aligned}
$$

where $\mathbf{V}_{\epsilon}$ is the idiosyncratic volatility defined in equation (5), $k = \frac{\eta_{1.3}}{1+\eta_{1.3}\boldsymbol{\omega}_*'\boldsymbol{\beta}}$ and $\delta_{SR} = \frac{{\mu_m}^{\dagger}-r}{\sigma_m^2}$ are constant and the market Sharpe Ratio, respectively.

If we define the *undiversified* market wide idiosyncratic return with respect to equation (5) as $\epsilon_m^I = \boldsymbol{\epsilon}'\boldsymbol{\omega}_*$, equation (11) can be rewritten as,

$$
\mu_i^c - r = \beta_i({\mu_m}^{\dagger} - r) + \beta_{I,i}\mu_{\epsilon}, \qquad (12)
$$

where $\beta_{I,i} = \frac{Cov(R_i,\epsilon_m^I)}{Var(\epsilon_m^I)}$ represents the sensitivity coefficient of the market wide undiversified idiosyncratic risk factor, and $\mu_{\epsilon} = kVar(\epsilon_m^I)\delta_{SR}$ is the market wide undiversified idiosyncratic risk premium that arises in our model because of the constrained investors. Similar to the implication of Shanken's (1982) model, what matters here is the covariance risk between individual stocks return and the market wide *undiversified* idiosyncratic risk.

Equation (12) says that, if not all investors can hold a market portfolio, the expected return of a stock will be determined not only by the observed market expected return through the conventional beta measure, but also by an extra risk premium because some undiversified idiosyncratic risk will be forced on investors by the constraints imposed. Of course, the portfolio $\boldsymbol{\omega}_*$ is unobservable to an econometrician, but we are able to construct an idiosyncratic risk hedging portfolio in the spirit of Fama and French (1993) to approximate portfolio $\boldsymbol{\omega}_*$ and will briefly discuss our empirical findings in section 4.

Similar to the CAPM model, our model offers cross-sectional implications that can be tested in an empirical study. Under the assumption of zero or close to zero pairwise correlations among the idiosyncratic returns for individual securities (see for example, Dybvig, 1983; and

Grinblatt and Titman, 1983), i.e. $Cov(\epsilon_i, \epsilon_j) \approx 0$, equation (11) can be further simplified as,

$$\mu_i^c - r \approx \beta_i({\mu_m}^{\dagger} - r) + k\delta_{SR}\hat{\sigma}^2_{I,i}, \tag{13}$$

where $\hat{\sigma}^2_{I,i}(= w_{*,i}\sigma^2_{I,i})$ can be interpreted as stock i's undiversified idiosyncratic volatility and $\sigma^2_{I,i}$ is the conventional measure of idiosyncratic volatility. The appearance of the Sharpe Ratio, δ_{SR}, in addition to the idiosyncratic volatility makes perfect sense in this context. It translates idiosyncratic risk into a comparable risk premium.

Equation (13) is useful in understanding the cross-sectional implications of the pricing of idiosyncratic risk. It suggests that the differences among individual stocks' expected returns will be related not only to their firms' systematic volatilities (β), but also to the firms' *undiversified* idiosyncratic volatilities. In other words, firms that are subject to large idiosyncratic shocks will tend to have high expected returns. There are two difficulties in implementation of equation (13). First, undiversified idiosyncratic volatility cannot be estimated directly. Second, residuals of individual stocks with similar characteristics are correlated to some degree.[8] However, if the pairwise residual correlations are similar within a group of stocks with similar characteristics, equation (11) suggests that the equal weighted portfolio residual variance, which represents the undiversified idiosyncratic risk, can be used in (13) instead. This is the approach that we will adopt in our empirical study. Our methodology has another advantage. Since individual stocks' betas are poorly estimated, as argued by Fama and French (1992), one should use the portfolio beta assigned to individual stocks within the portfolio in order to reduce the "errors-in-variables" problem. A similar argument can be made for idiosyncratic volatility estimates since they depend on the beta estimates. Therefore, using portfolio residual volatility not only is consistent with the model implication but also helps to reduce potential bias.

[8]The average absolute value of correlations among residuals of individual stocks is about 0.33.

2 The data and idiosyncratic risk proxies

Three data sets are employed in this study. The first data set is the COMPUSTAT tape, which is primarily used to obtain the book values for individual stocks in the later part of the study. The second data set is from the CRSP (Center for Research in Security Prices) tape, which includes NYSE, AMEX, and NASDAQ stock returns. Since so many papers have been written on testing the CAPM and most researchers rely on the CRSP tapes, there might be a data snooping concern (Leamer, 1983, and Lo and MacKinlay, 1990). In order to address this issue, we also examine the Japanese stock market for all stocks listed on the First and Second Sections of the Tokyo Stock Exchange (TSE). The monthly individual stock returns and annual financial statements (for book value of equity) data are from the PACAP Japan database.[9] The period covered in this study is from 1975 to 2000. The total number of stocks available varies from 1174 to 1607.

Our study covers both the Fama and MacBeth (1973) sample period from January 1935 to June 1968 and the extended Fama and French (1992) sample period from July 1963 to June 2000. Since the Fama and MacBeth (1973) study was influential in dismissing the role of idiosyncratic risk, it is important to know why the rejection occured. We begin our investigation by replicating their study using their choice of sample of NYSE stocks only. Like Fama and MacBeth (1973), the whole sample period under consideration is divided into portfolio formation, estimation, and testing periods. Empirical tests are performed on portfolios by aggregating individual estimates into the corresponding portfolio estimates using equal weights.[10] According to Fama and MacBeth (1973), 20 portfolios are constructed based on

[9] We are grateful to Yasushi Hamao for providing the data.

[10] As Miller and Scholes (1972) pointed out, the residual variance will bias the coefficient estimate of the beta variable in an cross-sectional regression. Therefore, we only use the orthogonal part of the residual variance to the beta variable in the cross-sectional regressions of Tables 5 and 6. However, we still use the original residual variance in Table 4 in order to make our results comparable to the Fama and MacBeth (1973) study.

each stock's beta obtained from the seven-year portfolio formation period prior to the corresponding estimation period.[11] Portfolio betas and idiosyncratic volatilities are time varying in the same way as in the Fama and MacBeth (1973) study. Despite the fact that we have had tremendous advances in computing since Fama and MacBeth did their original study at the beginning of 1970s and CRSP has made numerous changes and updates for their stock files, we are able to replicate Fama and MacBeth's (1973) 20 portfolios closely in terms of average beta and idiosyncratic volatility estimates. (available upon request). Since the early version of the CRSP tape is no longer available, it is impossible to trace the exact differences but they are small. Given the fact that we are able to replicate their study, we are able to ask if the number of portfolios used in the study matters in the next section.

The size variable is one of the most important variables studied in Fama and French (1992) and has been widely used in recent research. We have also incorporated the size variable in the Fama and MacBeth (1973) framework in the following way. Stocks are first sorted into five size groups according to their market capitalization in the month prior to each testing period. There is no particular reason to choose five size groups except for insuring that the portfolios have sufficient numbers of stocks. Within each size group, stocks are then sorted into ten beta portfolios as in the original study.

The essential characteristics for the 50 portfolios over the sample period from 1935 to 1968 are shown in Table 1. We report the average monthly returns for each of the 50 portfolios sorted on both size and beta computed from a market model. In general, portfolio returns decrease with the portfolio sizes except for the portfolio with the lowest beta and smallest size. Portfolio returns also increase with portfolio betas. But this relation weakens when portfolio

[11] The first portfolio formulation period has only four years from 1926 to 1929. According to Fama and MacBeth (1973), a security available in the first month of a testing period must also have data for all five years of the preceding estimation period and for at least four years of the portfolio formation period. In order to have comparable numbers of stocks selected for the first period, we require that each stock has at least three years of data in the portfolio formation period.

size increases and when betas are large. At the same time portfolio betas are monotonic over both the beta group and the size group. Although these betas range from 0.64 to 1.72, only one-quarter of the portfolios have betas less than one. It is also interesting to note that portfolio size does not vary much across beta groups. In contrast, portfolio idiosyncratic volatilities aggregated from the root mean squared residuals of individual stocks computed from a market model, vary considerably both across the size groups and beta deciles. This suggests that the idiosyncratic volatility variable may be more useful in explaining the cross-sectional return difference than the size variable.

Insert Table 1

For the sample period from 1963 to 2000, as show in Table 2, the average log market capitalizations of the 50 portfolios have gone up between 40% to 60%. For example, the overall average portfolio size for Fama and MacBeth period is 3.4 while that for the more current sample period is 5.2 (not shown in the table). The increases in each portfolio size are uniform across portfolios. However, portfolio returns seem to vary much less across both the size groups and the beta deciles than those in the previous sample period. This suggests that cross-sectional test results might be weaker for the recent sample period. Similarly, variations in portfolio betas are also much smaller from 0.64 to 1.37 and are more symmetrical around 1. In contrast, variations in the portfolio aggregate idiosyncratic volatilities increase across beta deciles but decrease across size quintiles.

Insert Table 2

A more powerful test can be undertaken by running cross-sectional regressions on *individuals* stocks in the spirit of Fama and French (1992). Similar to their study, portfolio betas of the 100 portfolios are assigned to each individual stock within each portfolio in order to reduce errors-in-variables problems. In particular, since there are so many small NASDAQ stocks in terms of market capitalization, portfolio breakdowns are determined using only NYSE stocks to avoid the small size portfolios from being too small. Each year, all NYSE stocks on the CRSP tapes are sorted in groups according to their size. The ten NYSE size deciles are then used to split the whole sample. At the same time, the beta of each stock is estimated from a market model using the previous 24 to 60 months of sample returns. Within each size group for NYSE stocks only, stocks are sorted again by their betas into ten equal number groups. Similarly, the break points thus obtained are used to sort all the stocks in our sample. All portfolios are rebalanced on June each year. The 100 portfolios thus constructed are very close to those used in Fama and French (1992), except that we have extended the sample period to June 2000. Using the whole sample period returns, we then estimate individual portfolio betas from the sum of the beta coefficients from regressions of individual portfolio returns on market and lagged market returns (see Fama and French, 1992). These portfolio beta estimates are then assigned to individual stocks in the corresponding portfolios. Individual stocks used in this part of the study should also have book values identifiable from the COMPUSTAT tape. One average, we have 2537 stocks per month for the extended sample period. Finally, we use NYSE/AMEX/NASDAQ index returns. The 3-month treasury-bill rates from Ibbotson Associates (2001) are used as the risk-free rates.

In this part of the investigation, we measure idiosyncratic risk using portfolio residual volatilities since we can consider the residual risk of a none randomly selected portfolio as the undiversified idiosyncratic risk. In other words, the size-beta sorted portfolio idiosyncratic volatility measures can also be used in the cross-sectional regression of individual stocks. This is legitimate since our model suggests that it is the *undiversified* idiosyncratic volatilities that affect returns of individual stocks when residuals are correlated to some degree. Moreover, by assigning portfolio idiosyncratic volatility to each individual stock within each portfolio, we will also reduce "errors-in-variables" problems if individual betas cannot be accurately estimated. At the same time, we recognize that, due to the diversification effect, a portfolio's idiosyncratic volatility will not be representative of individual stocks' undiversified idiosyncratic volatilities in the portfolio when there are too many stocks in a portfolio. Therefore, in order to balance the benefit of accurate estimates and the diversification effect, we use 200 portfolios' idiosyncratic volatilities to approximate those of the individual stocks within each portfolio.[12]

Since volatilities, especially idiosyncratic volatilities, are unobservable, most empirical studies estimate them using residuals from fitting a market model. Empirically, however, it is very difficult to interpret the residuals from the CAPM or even a multi-factor model as solely reflecting idiosyncratic risk. One can always argue that these residuals simply represent omitted factors. Therefore, we can only assert that the residuals from a market model measure idiosyncratic risk in the context of that model. In fact, this is the approach used in Fama and MacBeth (1973). Empirically, we should control for other empirically known factors in the tests. Alternatively, since the current literature has leaned toward a three-factor model of Fama and French (1993), we can compute residuals from equation (14) below,

$$R_{i,t} = \beta_{m,i} R_{m,t} + \beta_{smb,i} R_{smb,t} + \beta_{hml,i} R_{hml,t} + \epsilon_{i,t}, \qquad (14)$$

where $R_{m,t}$ is the market return, with R_{smb} and R_{hml} respectively representing the returns on portfolios formed to capture the size effect and the book-to-market equity effect.[13] Therefore, in this part of the investigation, we use idiosyncratic volatility estimates both from a market model and from the above Fama-French three-factor model. Since residual volatility is a second moment, we view this approach as an indirect control for other factors.

[12]We first sort all stocks into 20 portfolios according to midyear market capitalization and the sort each size portfolio into 10 beta portfolios.

[13]We are grateful to Eugene Fama for making these data available to us.

3 The cross-sectional evidence

Ever since doubts were raised about the CAPM model by Fama and French (1992), considerable attention has been devoted to risk measurement. For example, Jagannathan and Wang (1996) have argued that a conditional CAPM behaves well. Some have argued that the variables used by Fama and French are not robust (see for example Loughran (1996), and Kothari, Shanken, and Sloan (1995)). Others have suggested that a multifactor model in the spirit of the Merton's (1973) ICAPM model provides a better explanation of returns than a single factor CAPM model.[14] As noted by Fama and French (1992) and others, the most significant factors in "explaining" cross-sectional returns appear to be the market capitalization (size) and book to market ratios. These are largely empirical findings rather than equilibrium implications. Therefore, it is difficult to understand why these factors should matter in determining expected returns unless they are proxies for other (systematic) risk factors. Moreover, combining time series evidence of return predictability and cross-sectional testing in a conditional framework, Ferson and Harvey (1999) have rejected the three-factor model advocated by Fama and French (1993) as a conditional asset pricing model. Meanwhile, Malkiel and Xu (1997) have found that size and idiosyncratic volatility are highly correlated. Therefore, the so-called size effect may just as well be attributed to idiosyncratic risk. Guided by our theoretical model, we will study the empirical significance of idiosyncratic risk in addition to other factors from both cross-sectional and time series perspectives.

As suggested by our model (13), idiosyncratic volatilities for individual securities and their expected returns will be related. In other words, we need cross-sectional evidence to conclude that return differences among securities can be partially explained by differences in their idiosyncratic volatilities. In order to provide an overview of the relationship, we first

[14] This does not mean that the market factor is unimportant, only that other factors are important as well.

plot the average monthly returns versus the average idiosyncratic volatility calculated from the residuals to the three-factor model for the ten-decile portfolios in Figure 1. Clearly there is a positive association between idiosyncratic volatility and average returns. The significance of such a relationship is further demonstrated in the following cross-sectional tests.

Insert Figure 1

3.1 The Fama and MacBeth (1973) study revisited

The Fama and MacBeth (1973) study is an important one both in terms of its influential testing methodology, and also in terms of its empirical support of the CAPM model. In addition, this study also reversed earlier findings on the role of idiosyncratic risk. As a natural starting point in investigating the cross-sectional implications of idiosyncratic risk, we replicate Fama and MacBeth's (1973) Table 3 in our Table 3. In particular, we report the time series averages of the gamma estimates from cross-sectional regressions for each time t. For example, we calculate the time series average $\bar{\gamma}_x = \frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{x,t}$ for the cross-sectional estimates $\hat{\gamma}_{x,t}$. The corresponding t ratio in Fama and MacBeth (1973) and Fama and French (1992) is defined as $t_\gamma = \sqrt{T}\bar{\gamma}_x/std(\hat{\gamma}_{x,t})$. Since the betas used in the cross-sectional regression are themselves generated regressors, starting with Table 4[15], we use the Shanken (1992) correction factor $(1 + \hat{\mu}_m^2/\hat{\sigma}_m^2)$. In addition, since there might be autocorrelation in the estimates $\hat{\gamma}_{x,t}$, we use Newey and West (1987) estimator,[16]

Insert Table 3

[15]The purpose of Table 3 is to replicate Fama and MacBeth (1973). We do not adjust the t ratios in the table.

[16]Note that the t ratios in Table 3 are computed using $std(\hat{\gamma}_{x,t})$ in order to faithfully replicate Fama and MacBeth's (1973) results.

When the portfolio beta variable $\hat{\beta}_{p,t-1}$ is used alone in the cross-sectional regressions, the gamma estimates and the corresponding t-ratios closely match those of Fama and MacBeth over both the whole sample period from 1935 to June 1968 and the six five-year subsample periods. After introducing the additional variable of portfolio beta squared, $\hat{\beta}^2_{p,t-1}$, our estimates suggest that the beta variable is very significant compared with only marginal significance in Fama and MacBeth study. The main difference arises in the first two subsample periods. In fact, two of the subsample periods have matched almost exactly. When the additional variable introduced in the cross-sectional regressions is idiosyncratic volatility (residual standard deviation) $\bar{s}_{p,t-1}(\epsilon)$ instead, Fama and MacBeth's result continue to hold except that the gamma estimate is much smaller and is statistically insignificant for the whole sample period. Examining the gamma estimates from each subsample period, we have a reasonable match except for the first subsample period.

Finally, we include both the $\hat{\beta}^2_{p,t-1}$ variable and the $\bar{s}_{p,t-1}(\epsilon)$ variable in addition to the beta variable in the cross-sectional regressions. In contrast to Fama and MacBeth's finding that only the beta variable is marginally significant, we find that both the beta variable and the beta-square variable are statistically significant at a 5% level for the whole sample period. Therefore, Fama and MacBeth would have concluded that the asset pricing relationship is non-linear if the current version of the CRSP tape is used. It is also interesting to note that, despite the fact that the idiosyncratic risk variable is still insignificant, it has made the squared beta variable significant. This suggests that the two variables might be correlated with each other.

One caveat is that the significance of idiosyncratic volatility variable could be due to a bias created when beta and the residual variable are correlated, as has been pointed out by Miller and Scholes (1972). Since the Fama and MacBeth procedure is more powerful than the simple cross-sectional regression using average returns that was popular in the early studies, and the beta variable itself is not significant when used alone except for the case of early sample period, we do not believe that the above mentioned bias exists here. Nevertheless, we only use the orthogonal part of the idiosyncratic volatility to the corresponding beta variable in the cross-sectional regressions in the rest of the paper.

Three issues might arise in the Fama and MacBeth's (1973) study. First, an equally weighted index was used to estimate both the beta and the idiosyncratic volatility measures in the original study. According to the CAPM theory, it is the value-weighted index that should more closely resemble the market index. In Table 4, we will redo the estimation using a value-weighted index instead. Second, forming large portfolios not only reduces the errors-in-variables problem in the beta estimates, but also makes the residual variance estimates more accurate. At the same time, due to diversification effects, idiosyncratic volatilities do not have much variability across portfolios when there are too many stocks in each portfolio. In fact, there is no particular reason for Fama and MacBeth to choose 20 portfolios except for combatting the "errors-in-variables" problem. In order to increase the power of the tests, we have also investigated a different specification with 50 portfolios in Table 4.[17] Finally, it is also important to see if the results are robust different sample period. Therefore, we extend the Fama and MacBeth study to the sample period from 1963 to 2000 for NYSE/AMEX stocks.

Insert Table 4

To ensure the robustness of our results, we use Newy-West (1987) robust t-statistics from now on. When a value-weighted index is used in the market model that estimates both beta

[17] As a result the average standard error of the 50 portfolio betas will be in the order of one half to one-eleventh that of individual stocks. Fama and MacBeth (1973) reported the average standard error of the twenty portfolio betas is of the order of one-third to one-seventh that of individual stocks. When 50 portfolios are used instead, the relative increase in the standard error is about $\sqrt{50/20} = 1.58$. In fact, the empirical results suggests that the significance of beta variable is not affected by this grouping approach.

and idiosyncratic volatility, the beta variable is almost significant at a 5% level, and the beta-squared variable and the idiosyncratic volatility are only significant at a 10% level (see Panel A of Table 4). When all three variables are used in the regression, only the beta and the beta-squared variables are significant at a 1% level. Therefore, the significance of beta in explaining differences in portfolio returns continue to hold in the early sample period. But the relationship seems to be nonlinear. The positive relationship between beta and return is weaker for large beta portfolios than that for small beta portfolio. If we increase the number of portfolios to 50,[18] the cross-sectional regression results are very similar except that the idiosyncratic volatility variable is now statistically significant at a 5% level in both univariate and multivariate regressions. Therefore, we conclude that both beta and idiosyncratic volatility appear to be important in explaining the cross-sectional return differences for the early sample period.

The Fama and French (1992) study and the Fama and MacBeth (1973) study conducted under a different framework including sample periods, came to different conclusions regarding the importance of the beta variable. For consistency, it is important to study the same issue within the same framework. Since AMEX stocks were introduced into CRSP tape after July 1962, we examine NYSE/AMEX stocks in this part of the study over the extended Fama and French (1992) sample period from 1963 to 2000 using both 20 and 50 portfolio groupings. When a value-weighted index is used in the market model that estimates both beta and idiosyncratic volatility, the idiosyncratic volatility variable is again statistically significant at a 1% level while the beta variable is insignificant in the multivariate regressions for both groupings. The insignificance of the beta variable is consistent with Fama and French's (1992) finding using individual stocks. Therefore, the difference in the significance of beta found in the two studies is largely due to differences in sample periods. In contrast, the idiosyncratic

[18] Similar results hold when grouping into 30 or 40 portfolios.

volatility variable is significant in both sample periods. One may wonder why idiosyncratic volatility variable is significant in the multivariate regression when none of the variables is significant individually as in the left block of Panel B in Table 4.[19] One explanation is that there is substantial and correlated noise in all the three variables. If the true beta variable does not have much explanatory power to begin with, noise in the beta variable may help to cancel out noise in the idiosyncratic volatility variable when used jointly. The evidence from portfolios thus suggests that the idiosyncratic risk factor may play some role in valuation.

3.2 The role of the size and the idiosyncratic volatility variables in the framework of Fama and MacBeth (1973)

As suggested by Malkiel and Xu (1997), portfolio size and idiosyncratic volatility are highly correlated, therefore, one could argue that the significance of the idiosyncratic volatility simply captures the well-documented size effect. It is necessary, therefore, to consider the size and idiosyncratic volatility variables simultaneously. Thus, we extend the basic Fama-MacBeth sorting procedure by first sorting stocks into five size groups.[20] Stocks in each size group are then sorted into ten beta portfolios using exact procedure used in Fama and MacBeth (1973). The details of this procedure and the characteristics of the 50 portfolios were described in the previous section. For robustness, we apply both the market model and the Fama-French three-factor model to estimate the idiosyncratic volatilities used in estimation. The cross-sectional regression results for the 50 size-beta sorted portfolios are reported in Table 5.

Insert Table 5

[19] The beta variable and the idiosyncratic volatility variable are orthogonal to each other at the individual stock level.

[20] The reason for five size groups instead of ten groups as in Fama and French (1992) is to have 50 portfolios that are consistent with Table 4 and to allow for enough stocks in each portfolio for the early sample period to reduce the "errors-in-variables" problem.

For the early sample period of Fama and MacBeth (1973), using a market model to obtain beta and idiosyncratic volatility, results in the first block of Panel A in Table 5 show that all the four individual variables ($\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, $ME_{p,t-1}$, and $\bar{s}_{p,t-1}(\epsilon)$) are statistically significant at a 1% level with correct signs when used alone in cross-sectional regressions.[21] Note that $ME_{p,t-1}$ denotes the log market capitalization of the $p-th$ portfolio. Under the original specification of Fama-MacBeth, the three variables–$\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, and $\bar{s}_{p,t-1}(\epsilon)$ are simultaneously significant at a 1% level as shown in equation 6 of Panel A of Table 5, which not only confirms our finding from Table 4 using a different sorting approach but also supports our model implication for the role for idiosyncratic risk. If we replace the idiosyncratic volatility measure with the size measure of $ME_{p,t-1}$, a very similar result holds. This also suggests that idiosyncratic volatility and size are likely to be highly correlated. Therefore, we examine the cross-sectional regressions including both the size and the idiosyncratic volatility variables. The result shown in equation 7 of Panel A of Table 5 indicates that idiosyncratic volatility variable is still significant at a 5% level while the size variable is insignificant. If the size variable were the primary driving force, the opposite result should have occurred. When all the variables are used simultaneously, each variable, except for the size variable, continues to be significant. This means that the size variable does not replace the role of idiosyncratic risk in the cross-sectional regression. In addition, there is a decreasing positive relationship between the expected return and the beta in the early sample period.

Results are little different when idiosyncratic volatilities are estimated from the residuals of the Fama-French three-factor model. From the second block of Panel A in Table 5 we see that the significance of the idiosyncratic volatility variable is virtually unchanged. When both the size variable and the idiosyncratic volatility variable are used together in the cross-sectional

[21] Results are different from those reported in Table 4 since the 50 portfolio used in Table 4 are sorted according to beta only.

regression, only the latter variable is statistically significant at a 5% level. In other words, the idiosyncratic volatility variable takes away all the explanatory power of the size variable despite the fact that we have "double" counted the effect of the size variable. When all the variables are included in the last equation of the right block of Panel A, the size variable is again statistically insignificant while the idiosyncratic volatility variable continues to be very significant. Therefore, the results are robust to different estimates of idiosyncratic risk.

For the most recent sample period of 1963-2000, results are not as strong as those of the previous sample period in general. This is partly due to the fact that portfolio returns are not as variable as before (see Table 2). Although the size variable and the idiosyncratic variable are statistically significant at 5% and 7% levels, respectively when used alone, only the idiosyncratic volatility variable continue to be significant at a 6% level in the multiple cross-sectional regression while the size variable is insignificant (see the left block of Panel B in Table 6). Despite a different specification from that of the Fama and French (1992), which was based on individual stocks, the insignificance of the beta variable is confirmed here. More interestingly, the noise in the beta measure seems to have helped in cancelling the noise in idiosyncratic volatility measure. As a result, in a multiple cross-sectional regression without the size variable shown in equation 6 of Panel B of Table 5, the idiosyncratic volatility variable is significant at a 3% level. Therefore, our hypothesis that the size variable may have served as a proxy for the idiosyncratic volatility variable, is again confirmed from evidence for the most recent sample period. When idiosyncratic volatility is estimated from the Fama-French three-factor model, results are similar.

We, therefore, conclude that (1) beta estimated from a market model is important in explaining return differences for the early sample period but its role has substantially weakened in the recent sample period; (2) the idiosyncratic volatility variable is very important especially in the previous sample period no matter how it is measured; (3) the size effect is dominated by the idiosyncratic risk factor in both sample periods; and (4) the beta variable seems to help reduce the noise in the idiosyncratic volatility variable, especially in the recent sample period.

3.3 The cross-sectional expected returns of individual stocks and idiosyncratic volatilities in the framework of Fama and French (1992)

The fundamental differences between the Fama and French (1992) and Fama and MacBeth (1973) studies are that in the later study (1) the cross-sectional regressions are run for individual stocks; (2) new variables such as size and book-to-market are considered; and (3) portfolio betas (β_p) are assigned to individual stocks within the portfolio. We extend the Fama and French (1992) study by extending their sample period to June 2000 and introducing the idiosyncratic volatility variable in order to show its importance in explaining the cross-sectional return differences. Note that we have dropped the beta-squared variable (β_p^2) introduced in Fama and MacBeth's (1973) study since it is insignificant in the current sample period. While the log size variable ($ME_{i,t-1}$) can be specified accurately, idiosyncratic volatility is unobservable and has to be estimated. Just like beta estimates for individual stocks that suffer from errors-in-variables problems, idiosyncratic volatility estimates for individual stocks face the same challenge since they are estimated from the same models. In order to mitigate the problems, we apply the same approach as Fama and French (1992) by assigning the portfolio idiosyncratic volatility (s_p) to each stock within the portfolio.[22] As pointed out earlier, we have a diversification challenge when substituting a portfolio idiosyncratic volatility for those of individual stocks within the portfolio. To mitigate the problem, we increase the number of portfolios in computing the idiosyncratic volatilities to 200. Using portfolios' idiosyncratic

[22] This is the major difference between this study and Ang, Hodrick, Xing, and Zhang (2003), where individual stocks' volatilities are used.

volatilities is also justified by the fact that it is the *undiversified* idiosyncratic risk that matters in determining the asset prices implied by our model. As documented in Malkiel and Xu (1997), size and idiosyncratic volatility are highly correlated. Thus, using a size portfolio's idiosyncratic volatility measure as an estimate of undiversified idiosyncratic volatility of individual stocks in the portfolio makes sense. Similar to Fama and French (1992), we use the book value from the last fiscal year to compute the log book-to-market value ($B/M_{i,t-1}$). The results reported in Table 6 are for all the NYSE/AMEX/NASDAQ stocks over the sample period from July 1963 to June 2000.

Insert Table 6

Comparable to Fama and French's (1992) results, the first equation in Panel A of Table 6 shows a statistically insignificant estimate for the beta variable when used alone although it is a little stronger now. This result provides supplemental evidence that beta does not appear to be useful in explaining the cross-sectional stock return differences. However, as noted by Fama and French (1992), the size variable does appear to explain the cross sectional variability of returns as shown in the third equation with an estimate of $-.0019$ and a $t-$value of -3.50. This is much stronger than Fama and French's (1992) estimates, where a shorter sample period is used. The size variable is even strong when both the beta variable β_p and the size variable $ME_{i,t-1}$ are added shown in equation 6. In addition, the book-to-market variable produces a similar estimate as that in Fama and French (1992), no matter whether it is used alone or with other variables. As suggested by equation (13), high undiversified idiosyncratic risk, measured by idiosyncratic volatility, is associated with high returns on average. This is exactly the case as shown in the fourth equation for s_p of Panel A. The significance of the idiosyncratic volatility variable is also even stronger when the beta variable is also added shown in equation 7 of Table 6, suggesting that the beta variable is useful in reducing the noise in the volatility measure. The idiosyncratic volatility measure (s_p), however, again takes away the explanatory power of the size variable shown in equation 8.

More importantly, including the B/M_i variable does not alter the coefficient estimate of the idiosyncratic volatility variable very much as shown in equation 10. Therefore, controlling for the book-to-market variable does not have a large impact on the significance of the idiosyncratic volatility variable. Finally, when all the four variables are used, both the book-to-market and the idiosyncratic volatility variables continue to be statistically significant, while the size variable is insignificant with the wrong sign in the regression. It is interesting to note that the beta variable is also significant with the right sign. This could due to the fact that correlation between size and idiosyncratic volatility variables helps to reduce the noise in the beta measure.

As noted above, portfolio size is strongly related to idiosyncratic volatility. Stocks of smaller size tend to have larger idiosyncratic risks than stocks of larger size. As Berk (1995) and Cochrane (2001) have forcefully argued, characteristic based variables, such as size, cannot be considered risk factors. Our empirical evidence suggests that the size variable can effectively be interpreted as a proxy for idiosyncratic risk. Arguably, large companies tend to have more diversified lines of business which will have low idiosyncratic volatility, while small firms tend to have more focused business which are more susceptible to exogenous shocks. Thus, this study provides an alternative way to understand the role of the size effect in Fama and French.

3.4 The Robustness of Our Results

When the idiosyncratic volatility measure s_p is computed from the residuals of a three-factor model, the fourth equation from Panel B of Table 6 shows that the positive relationship between return and idiosyncratic risk continues to be significant at the 1%. In fact, the results for either of the idiosyncratic volatility measures we used change only slightly relative to the results using CAPM residuals as a measure of idiosyncratic risk. Therefore, our results appear to be robust whatever definition we use for idiosyncratic volatility.

Since the size effect may be concentrated on very small stocks as Loughran (1996) has noted, we need to study the robustness of the idiosyncratic risk factor relative to the smallest stocks. NASDAQ stocks were added to the CRSP tape in July, 1973 and the additions mostly consisted of small stocks. A natural empirical design is to exclude these NASDAQ stocks from our study. Results for NYSE/AMEX stocks only are reported in Table 7.

Insert Table 7

Comparing estimates with those shown in Table 6, the overall results are a little weaker. Nevertheless, the idiosyncratic volatility measure of s_p is still very significant at a 1% level when estimated from the CAPM residuals shown in Panel A of Table 7. When the book-to-market variable is added, the idiosyncratic volatility measure continue to be significant at a conventional level as shown in equations (10) of Panel A. The size variable is insignificant when all the variables are used in the cross-sectional regression. However, the s_p variable is again significant at a 1% level.

When idiosyncratic volatilities are estimated from the Fama and French three-factor model, results are again very similar as shown in Panel B of Table 7. Therefore, we conclude that the usefulness of idiosyncratic volatility variables is also robust when the sample includes only large stocks. At the same time, the even stronger results for the idiosyncratic volatility measure, s_p, versus $s_{p,t-1}$ used in Fama and MacBeth (1973) study, suggests that it is undiversified idiosyncratic risk that is more relevant in explaining the cross-sectional return difference of individual stocks.

Therefore, additional tests continue to support our hypothesis that undiversified idiosyncratic volatility helps to explain the cross-sectional variability in average returns in an important way. It is also reasonable to conclude that the idiosyncratic risk factor is more robust than the size variable in explaining the cross-sectional difference of asset returns over the different sample periods considered here.

3.5 The Liquidity Effect

Liquidity may also play an important role in affecting asset prices. Although different researchers have offered different definitions, it is generally believed that "liquidity" should measure how easy it is to trade a large number of shares without altering the share prices. There are many theoretical papers including Amihud and Mendelson (1986), Constantinides (1986), Heaton and Lucas (1996), and Huang (2002), that tie liquidity to asset prices. Intuitively, assets that are "difficult to trade" should have lower prices, other things being equal, in order to compensate investors for the inability to trade them quickly or for the increased cost of trading. Indeed, many empirical studies, such as Amihud and Mendelson (1986), Brennan, Chordia, and Subrahmanyam (1998), and Datar, Naik, and Radcliffe (1998) have generally found a negative relationship between liquidity and expected stock returns. Alternatively, using market wide liquidity as a state variable (or factor), Pastor and Stambaugh (2003) and Jones (2002) find stocks with high covariance with the market liquidity generally offers high expected return. This is reasonable since those stocks will face severe liquidity problems when the market liquidity is low.

If liquidity is indeed priced, residuals from any asset pricing model that excludes liquidity factor will reflect it. However, since idiosyncratic volatility is a second moment, it can only indirectly capture some of the liquidity effect. Nevertheless, in this section, we will attempt to control for liquidity. The bid-ask spread is often used in the literature as a measure of liquidity. We will use trading volume in this study as the liquidity measure since it is the most important determinant of the bid-ask spread (see Stoll, 1978) and because data are readily available. Unlike Brennan, Chordia, and Subrahmanyam (1998), we use relative volume defined as the ratio between share volume and shares outstanding instead of log dollar volume. When both log size and log dollar volume are used in the cross-sectional regressions, they share log price as the common component. Therefore, we believe it is better to use a relative volume variable here. In particular, we use last month's relative volume in the cross-sectional regression of current month.[23] In this part of the study, we use the s_p constructed from the CAPM residuals as the idiosyncratic risk measure. The results are reported in Table 8.

Insert Table 8

For all stocks, the volume variable is statistically significant at a 1% level when used alone. Surprisingly, however, the sign is "wrong".[24] Potentially, the size variable could be correlated with the volume variable. One could argue that large stocks are more actively traded than small stocks. Therefore, the positive sign on the relative volume variable could simply due to the size effect. This is confirmed in the multiple regression with the size variable as shown in the fifth equation of Panel A of Table 8. The size variable virtually takes away all the explanatory power of the volume variable. When idiosyncratic volatility and the volume variable are use simultaneously, the significance of the idiosyncratic volatility

[23] Alternatively, one can use Pastor and Stambaugh's (2003) indirect measure of liquidity. However, this measure essentially is a slope coefficient estimate from regressing an individual stock's return on past volume, which will subject to large "errors-in-variable" problem in our context since we are dealing with individual stocks.

[24] We obtain similar results when the log dollar volume is used instead. Since Brennan, Chordia, and Subrahmanyam (1998) did not report univariate regression result, we do not know if similar results would occur in their framework.

is unchanged. This suggests that our idiosyncratic volatility measure does not represent the liquidity effect. More importantly, the volume variable now has the "correct" sign although it is still insignificant. From our previous results, This could be due to the fact that the idiosyncratic volatility variable has taken away some of the "size effect" in the volume variable. Equation 7 is similar to Brennan, Chordia, and Subrahmanyam's (1998) specification except for the lagged return variables. The liquidity proxy is again statistically insignificant with the "right" sign. When all the variables are used in the cross-sectional regression as shown in the last equation, both the magnitude and the significance of the idiosyncratic volatility variable is virtually unchanged. Very similar results continue to hold when only the NYSE/AMEX stocks are used in the cross-sectional regression shown in the Panel B of Table 9. Therefore, we conclude that controlling for liquidity will not have much impact on the significance of the idiosyncratic volatility variable.

3.6 Idiosyncratic Risk in a Different Market

Considerable recent attention has been paid to data snooping problems in empirical studies. Since CAPM studies have mostly been based on the same data source, i.e., the CRSP tape for U.S. stock data, such a problem is inevitable at least conceptually. In order to obtain a different perspective, we also study Japanese stock returns. The available data set runs from 1975 to 1999, and we use the first five years of data to do the pre-sorting in constructing our portfolios. Therefore, the actual sample used in the study runs from 1980 to 1999. During this particular period, the Japanese stock market went through dramatic changes–from the pre bubble period (1980-1984) to the bubble period (1985-1989), and followed by a decade of post-crash market slump (1990-1999). Therefore, we also study the two subsample periods: 1980-1989 and 1990-1999. The average numbers of stocks in each sub-period are 1321 and 1564, respectively. We followed the exact procedures employed with U.S. data to construct portfolios and to perform cross-sectional regressions. The results are shown in Table 9.

Insert Table 9

First, we find that the beta variable is insignificant no matter whether it is used alone or with other variables in the cross-sectional regressions. The same results hold for different subsample periods. This reconfirms our results from the U.S. data. In contrast to the U.S. experience, however, the size variable is insignificant in all of the cross-sectional regressions. The book-to-market variable is significant for the whole sample period both when used alone and together with other variables. It is interesting to note, however, that the significance arises mainly from the effect in the post crash period. As for the idiosyncratic volatility variable, it is significant at a 6% level for the whole sample period when used alone. It is very significant during the first subsample period but not in the second subsample period when used alone. What is most interesting is that the idiosyncratic volatility variable is always very significant when used with the size measure, as shown in the last equation of all the three blocks of Table 9. Apparently, the size variable helps to reduce the measurement error in the idiosyncratic volatility variable. Japanese stock market data again provide evidence that the idiosyncratic volatility variable is much stronger than the size variable, and support the role of idiosyncratic volatility in explaining cross-sectional returns.

While beta measures do appear to play some role in explaining the cross-sectional pattern of returns in accordance with the CAPM in the early sample period, the general conclusion is that book value/market value and especially idiosyncratic volatility are the only variables that show a consistently strong relationship with returns in the recent sample period. Of course, it is possible to interpret our measures of idiosyncratic volatility as simply an approximation for some omitted systematic risk factor(s). However, this does not seem to be the case after controlling for size, book-to-market, and liquidity. At the very least, our model seems to offer a consistent and empirically supported explanation for some of the deficiencies of popular asset pricing models.

4 Preliminary Time Series Evidence on Idiosyncratic Risk

Generally speaking, two approaches have been applied in testing the CAPM. Cross-sectional studies of the return and risk implications of the CAPM have received the most attention. This is what we have undertaken in the previous section by showing that idiosyncratic volatility appears to be an independent pricing factor. It is also important to examine an alternative time-series approach that utilizes the constraint on the intercept of a market model as our model predicts. Here we present some preliminary results on the explanatory power of idiosyncratic volatility from an *ex post* perspective.

We study the time series behavior for the same 100 portfolios constructed in the cross-sectional studies. In order to perform time series tests on equation (12), we need a proxy for the market wide undiversified idiosyncratic risk factor. The proxy $R_{I,t}$ that we propose to use here is the idiosyncratic risk hedging portfolio. In order to motivate it, we rewrite the time-series CAPM and our model (equation (12)) as the following two equations:

$$\begin{aligned} I: \quad \tilde{R}_{i,t} - R_{f,t} &= \beta_i(R_{M,t} - R_{f,t}) + \epsilon_{i,t}, \quad \textit{and} \\ II: \quad \tilde{R}_{i,t} - R_{f,t} &= \beta_i(R_{M,t} - R_{f,t}) + \beta_{I,i}R_{I,t} + \xi_{i,t}, \end{aligned} \tag{15}$$

where $\tilde{R}_i$, $R_{f,t}$, and $\tilde{R}_M$ are the returns for security i, the risk-free rate, and the market portfolio respectively. If model II is indeed true, the part of $\beta_{I,i}R_{I,t} + \xi_{i,t}$ will be specified as $\epsilon_{i,t}$ in the CAPM specification of model I. Therefore, firms that are more sensitive to $R_{I,t}$ will be likely to have large $\sigma(\epsilon_{i,t})$. Since $R_{M,t}$ and $R_{I,t}$ are independent, the two portfolios constructed by sorting stocks according to their $\sigma(\epsilon_{i,t})$ will have similar beta sensitivities to $R_{M,t}$ and very different beta sensitivities to $R_{I,t}$.

In our implementation, we follow the logic of Fama and French (1993) by constructing six size-idiosyncratic volatility sorted portfolios. We split the sample into two size groups. Within each size group, stocks are sorted again by their idiosyncratic volatilities into three groups of equal numbers of securities.[25] The idiosyncratic volatility measure for each stock is estimated using the mean squared residuals from the CAPM model over the previous 24 to 60 month period. We denote B as *big* size, S as *small* size, H as *high* idiosyncratic volatility, M as *median* idiosyncratic volatility, and L as *low* idiosyncratic volatility. The six portfolios can be characterized as the B/L portfolio, the B/M portfolio, the B/H portfolio, the S/L portfolio, the S/M portfolio, and the S/H portfolio. The return proxy for the market wide idiosyncratic risk factor is then calculated as the difference between the average returns for portfolios B/L and S/L and the average returns for portfolios B/H and S/H. Therefore, our method of constructing the idiosyncratic risk hedging portfolio is consistent with the model presented in the first section.

Our empirical results in this section are based on the portfolio returns of the 100 size-beta sorted portfolios obtained by equally weighting each security's returns in the portfolio. As a first step, we report the average and the standard deviation of coefficient estimates for individual portfolios in Table 10. For comparison, we report estimates for both the Fama and MacBeth sample period (1935.1-1968.6) and the extended Fama and French sample period (1963.7-2000.6). Although the average R^2s across all the portfolios are relatively high with 69.9% and 67.10% for each of the sample periods respectively, the CAPM fails to hold because most of the individual regressions have significant intercepts, α. In general, adding additional variables to the CAPM specification does not reduce either the magnitude or the significance of alpha. However, the idiosyncratic risk factor is very significant in both sample periods. In particular, the distribution of the beta estimates on $R_{I,t}$ is on the positive side and far from zero. On average, each percentage increase in the idiosyncratic hedging proxy $R_{I,t}$ is associated with .57% increase in return, over the early sample period and .31% in the recent sample period. For individual portfolios, the idiosyncratic hedging proxy is significant at a

[25] The actual breakdowns are based on NYSE stocks only for the same reason discussed previously.

5% level for more than 80% of these portfolios in both sample periods. At the same time, the average adjusted R^2 across all the portfolios jumps to 77.1% for the early sample period and 73.3% for the recent sample period. Therefore, these preliminary results support our model in equation (12).

Insert Table 10

Fama and French (1993) have argued that a size proxy and a book-to-market proxy are also very important in explaining return fluctuations over time. Their three-factor model is reconfirmed in the third equation of Table 10. As expected, the average adjusted R^2 has been improved to 79.5% and 81.1% for the previous and recent sample periods, respectively. At the same time, as we suggested in discussing the cross-sectional evidence, the size factor may very well be measuring idiosyncratic risk. In this case, the idiosyncratic risk factor will be more appropriate, which is what we represent in the fourth equations of Table 10. For both sample periods, we see that the corresponding coefficient estimates of both the idiosyncratic hedging factor and the book-to-market factor are distributed far from zero. The number of individual portfolios showing statistical significant estimates on the two factors at a 5% level also resembles that of the three-factor model in the recent sample period. Similar results hold for the early sample period. Therefore, idiosyncratic volatility appears to capture the size effect in the time series as well as the cross-sectional framework.

One might argue that our idiosyncratic risk proxies simply capture the size and book-to-market factors. Thus, it is important to examine idiosyncratic risk after controlling for both the size and book-to-market effect. It is evident that, for the Fama-MacBeth sample period, the majority of the portfolios have statistically significant estimates on the idiosyncratic risk factor. For example, after controlling for the size and the book-to-market factors, there are still 54% of the portfolios showing statistically significant estimates at the 5% level on the idiosyncratic risk factor. In other words, there is a strong connection between idiosyncratic risk and individual portfolio returns, even after taking account of size and book-to-market effects. For the Fama and French sample period, the percentage of portfolios with statistically significant estimates on the idiosyncratic risk factor increases to 77%.

While a more detailed time series analysis is required for conclusive results, the preliminary evidence from individual portfolios supports our theoretical prediction that idiosyncratic volatility appears to be important in explaining asset return fluctuations over time. Note also that the significant beta estimates from the time series regressions do not contradict the Fama and French's (1992) findings that were discussed in our cross-sectional study.

One may argue that the role we ascribe to idiosyncratic volatility may result from our inability to define the market portfolio. Perhaps idiosyncratic risk is just an artifact of approximation error. If so, we view our argument here as deepening the critique of Roll. We have argued, however, that even if investors are shortsighted and pay attention to only tradable financial assets, in which case Roll's critique is irrelevant, we may still find an imperfect CAPM from an econometrician's point of view. Such an imperfection can be ameliorated by including an idiosyncratic risk measure.

5 Concluding comments

In this paper, we have made some progress in understanding the role of idiosyncratic risk in asset pricing. Other things being equal, idiosyncratic risk will affect asset returns when not every investor is able to hold the market portfolio. Even after controlling for factors such as size, book-to-market, and liquidity, evidence from both individual U.S. stocks and a sample of Japanese equities supports the predictions of our model. Most importantly, the cross-sectional results demonstrate that idiosyncratic volatility is more powerful than either beta or size measures in explaining the cross section of returns. At the very least, our results provide a unique perspective in understanding the possible role of idiosyncratic risk in asset pricing.

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Table 1: **Size-Beta Portfolio Characteristics (Sample Period: January 1935-June 1968)**

This table reports the average return, beta, log size, and residual standard deviation for the 50 size-beta sorted portfolios over the Fama and MacBeth (1973) sample period of January 1935 to June 1968. Stocks from the NYSE/AMEX are first sorted into five size groups according their market capitalization in December of the last year of the estimation period. Each group of stocks is then sorted again into decile portfolios according to the selection period betas. Detailed information on portfolio selection, estimation, and testing periods are available in Fama and MacBeth (1973). Both the reported betas and the idiosyncratic volatilities, as well as betas used in sorting, are estimated from a market model with a value-weighted index.

	Low-β	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	High-β
	Average Monthly Returns (in Percent)									
Small-ME	1.08	1.89	1.83	2.00	1.82	1.96	2.03	2.38	2.50	1.89
ME-2	2.14	1.18	1.23	1.55	1.69	1.52	1.87	1.72	1.81	1.87
ME-3	1.73	1.30	1.39	1.22	1.27	1.46	1.41	1.51	1.36	1.52
ME-4	1.32	.889	1.08	1.04	1.24	1.34	1.27	1.25	1.21	1.22
Large-ME	1.44	.806	.896	1.03	1.01	1.16	1.27	1.16	1.02	1.12
	Average portfolio beta									
Small-ME	.990	1.11	1.24	1.41	1.31	1.31	1.45	1.53	1.71	1.72
ME-2	.806	.901	1.07	1.22	1.31	1.39	1.49	1.50	1.59	1.68
ME-3	.719	.909	1.06	1.17	1.26	1.35	1.35	1.44	1.54	1.69
ME-4	.644	.841	.991	1.10	1.11	1.20	1.27	1.31	1.52	1.57
Large-ME	.643	.723	.845	0.911	1.00	1.11	1.15	1.19	1.28	1.41
	Average portfolio log size									
Small-ME	1.66	1.68	1.61	1.67	1.61	1.61	1.64	1.62	1.61	1.63
ME-2	2.59	2.49	2.51	2.56	2.52	2.60	2.58	2.58	2.56	2.54
ME-3	3.40	3.36	3.33	3.31	3.38	3.39	3.37	3.34	3.34	3.28
ME-4	4.28	4.28	4.26	4.32	4.26	4.27	4.24	4.21	4.25	4.25
Large-ME	5.85	5.82	5.82	6.00	5.80	5.96	5.76	5.83	5.60	5.58
	Average portfolio idiosyncratic volatility (in Percent)									
Small-ME	12.2	11.8	13.2	13.1	12.3	13.2	14.2	13.7	15.0	14.7
ME-2	7.76	8.12	9.13	9.52	9.80	10.3	10.5	10.2	10.6	11.8
ME-3	6.77	7.09	7.64	8.11	7.86	8.24	8.46	8.10	8.80	9.36
ME-4	5.64	6.29	6.46	7.02	7.06	7.07	7.33	7.31	8.97	8.43
Large-ME	4.70	4.78	5.17	5.04	5.36	5.73	6.05	5.73	6.39	6.53

Table 2: **Size-Beta Portfolio Characteristics (Sample Period: 1963-2000)**

This table reports the average return, beta, log size, and residual standard deviation for the 50 size-beta sorted portfolios over the extended Fama and French (1992) sample period of 1963-2000. Stocks from the NYSE/AMEX are first sorted into five size groups according their market capitalization in December of the last year of the estimation period. Each group of stocks is then sorted again into decile portfolios according to the selection period betas. Detailed information on portfolio selection, estimation, and testing periods are available in Fama and MacBeth (1973). Both the reported betas and the idiosyncratic volatilities, as well as, betas used in sorting are estimated from a market model with a value-weighted index.

	Low-β	β_2	β_3	β_4	β_5	β_6	β_7	β_8	β_9	High-β
	Average Monthly Returns (in Percent)									
Small-ME	1.04	1.52	1.31	1.45	1.58	1.22	1.63	1.69	1.75	1.67
ME-2	1.78	1.15	1.16	1.35	1.33	1.48	1.44	1.25	1.32	1.30
ME-3	1.43	1.04	1.12	1.28	1.16	1.43	1.26	1.27	1.19	1.29
ME-4	1.43	1.01	1.13	1.09	1.15	1.17	1.36	1.17	1.20	1.20
Large-ME	1.09	1.03	1.09	1.09	1.13	1.22	1.03	1.10	1.10	.969
	Average portfolio beta									
Small-ME	.783	.932	.939	1.04	1.05	1.10	1.19	1.18	1.28	1.37
ME-2	.576	.798	.928	1.00	1.10	1.19	1.21	1.26	1.37	1.44
ME-3	.635	.805	.945	1.02	1.10	1.15	1.20	1.28	1.36	1.51
ME-4	.628	.735	.934	1.00	1.05	1.09	1.14	1.18	1.30	1.45
Large-ME	.643	.786	.860	.908	.999	1.03	1.06	1.07	1.16	1.25
	Average portfolio log size									
Small-ME	2.63	2.75	2.78	2.70	2.80	2.81	2.73	2.80	2.67	2.77
ME-2	4.10	4.08	4.11	4.14	4.13	4.17	4.17	4.22	4.18	4.10
ME-3	5.19	5.25	5.25	5.23	5.28	5.22	5.21	5.17	5.23	5.27
ME-4	6.27	6.33	6.32	6.38	6.35	6.34	6.41	6.32	6.35	6.32
Large-ME	7.65	7.94	7.88	8.06	7.82	7.88	7.83	7.90	7.64	7.55
	Average portfolio idiosyncratic volatility (in Percent)									
Small-ME	10.2	11.5	11.3	12.1	11.9	12.4	13.1	12.8	14.2	14.6
ME-2	6.41	7.54	8.51	8.99	9.11	9.93	10.2	10.2	10.8	11.6
ME-3	6.32	6.58	7.38	7.64	8.00	8.14	8.60	8.89	9.73	10.1
ME-4	5.38	5.80	6.50	6.56	6.92	7.01	7.17	7.74	8.10	8.94
Large-ME	5.11	5.26	5.71	5.71	5.84	5.93	6.09	6.18	6.63	7.31

Table 3: **Replicating Fama-MacBeth Cross-sectional Regression on Portfolios**

This table is intended to replicate the cross-sectional regression results reported in Fama-MacBeth (1973) Table 3. In general, stocks from the NYSE are selected into one of 20 portfolios according to the selection period (seven years) with betas estimated from a market model. Details are available in Fama-MacBeth (1973). Both the beta estimates and idiosyncratic volatility estimates used in cross-sectional regressions of the testing periods are from the estimation period (next five years). In particular, the independent variables $\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, and $\bar{s}_{p,t-1}(\epsilon)$ are equal-weighted averages of individual stock's beta, beta squared, and residual standard deviation, respectively, estimated using a market model with an equal-weighted index.

	Replicating Results					Fama-MacBeth Table 3				
	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\bar{R}^2$	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\bar{R}^2$
1935-6/68	.0061	.0087			.30	.0061	.0085			.29
t Ratio	3.26	2.69				3.24	2.57			
1935-40	.0012	.0125			.22	.0024	.0109			.23
t Ratio	.155	.950				.320	.790			
1941-45	.0071	.0212			.36	.0056	.0229			.37
t Ratio	1.72	2.46				1.27	2.55			
1946-50	.0048	.0031			.39	.0050	.0029			.39
t Ratio	1.22	.511				1.27	.480			
1951-55	.0122	.0026			.24	.0123	.0024			.24
t Ratio	4.98	0.576				5.06	.530			
1956-60	.0148	-.0058			.23	.0148	-.0059			.22
t Ratio	5.62	-1.34				5.68	-1.37			
1961-6/68	.0013	.0141			.33	.0001	.0143			.32
t Ratio	.351	2.73				.030	2.81			
1935-6/68	.0027	.0165	-.0037		.33	.0049	.0105	-.0008		.32
t Ratio	1.04	2.72	-1.46			1.92	1.79	-.290		
1935-40	-.0056	.0302	-.0091		.23	.0013	.0141	-.0017		.24
t Ratio	-.669	1.44	-1.21			.160	.75	.190		
1941-45	.0094	.0166	.0018		.38	.0148	.0004	.0108		.39
t Ratio	1.63	1.72	.277			2.28	.030	1.15		
1946-50	-.0004	.0143	-.0047		.45	-.0008	.0152	-.0051		.44
t Ratio	-.092	1.09	-1.14			-.180	1.14	-1.24		
1951-55	.0004	.0280	-.0120		.28	.0004	.0281	-.0122		.28
t Ratio	.089	2.53	-2.63			.100	2.55	-2.72		
1956-60	.0124	-.0003	-.0025		.26	.0128	-.0015	-.0020		.25
t Ratio	3.18	-.028	-.676			3.38	-.160	-.540		
1961-6/68	.0030	.0101	.0021		.35	.0029	.0077	.0034		.34
t Ratio	.429	.665	.299			.420	.530	.510		

Table 3–**Continued**

	Replicating Results					Fama-MacBeth Table 3				
	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\bar{R}^2$	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\bar{R}^2$
1935-6/68	.0058	.0083		.0037	.33	.0054	.0072		.0198	.32
t Ratio	2.45	2.36		.085		2.10	2.20		.460	
1935-40	.0027	.0152		-.0450	.25	.0036	.0119		-.0170	.25
t Ratio	.324	1.09		-.653		.370	.970		-.190	
1941-45	.0021	.0102		.1610	.39	-.0006	.0085		.2053	.41
t Ratio	.309	1.65		1.29		-.080	1.25		1.46	
1946-50	.0076	.0095		-.1230	.44	.0069	.0081		-.0920	.42
t Ratio	1.46	1.07		-1.33		1.56	.950		-1.41	
1951-55	.0149	.0072		-.1230	.28	.0150	.0069		-.1185	.27
t Ratio	3.95	1.29		-1.33		4.05	1.24		-1.31	
1956-60	.0122	-.0086		.0925	.26	.0127	-.0081		.0728	.26
t Ratio	2.54	-1.41		.569		2.68	-1.40		.480	
1961-6/68	.0000	.0123		.0483	.34	-.0014	.0122		.0570	.33
t Ratio	.001	1.97		.523		-.320	2.12		.640	
1935-6/68	-.0011	.0175	-.0061	.0630	.34	.0020	.0114	-.0026	.0516	.34
t Ratio	-.315	2.85	-2.21	1.32		-.550	1.85	-.860	1.11	
1935-40	-.0083	.0320	-.0117	.0224	.26	.0009	.0156	-.0029	.0025	.26
t Ratio	-.760	1.51	-1.35	.300		.070	.780	-.290	.030	
1941-45	-.0076	.0235	-.009	.2370	.40	.0015	.0073	.014	.1767	.43
t Ratio	-.684	2.46	-1.43	1.63		.120	.520	.150	1.16	
1946-50	.0032	.0136	-.0029	-.0719	.44	.0011	.0141	-.0040	-.0313	.44
t Ratio	.576	1.05	-.749	-.984		.180	1.03	-.730	-.410	
1951-55	.0034	.0275	-.0104	-.0773	.30	.0023	.0277	-.0112	-.0443	.29
t Ratio	.691	2.50	-2.33	-.860		.480	2.53	-2.54	-.530	
1956-60	.0087	-.0042	-.0028	.1370	.28	.0103	-.0047	-.0020	.0979	.28
t Ratio	1.21	-.430	-.649	.717		1.63	-.470	-.490	.590	
1961-6/68	-.0025	.0120	-.0008	.1130	.37	-.0017	.0088	.0013	.0957	.35
t Ratio	-.310	.767	-.110	1.12		-.210	.580	.190	1.02	

Table 4: **Fama-MacBeth Cross-sectional Regressions on Portfolios**

This table reports the cross-sectional regression results for portfolios over both the Fama-MacBeth (1973) sample period of January 1935 to June 1968 and the extended Fama-French (1992) sample period of January 1963 to December 2000. In general, stocks from the NYSE/AMEX are selected into one of 20 portfolios according to the selection period (seven years) betas estimated from a market model. Details are available in Fama-MacBeth (1973). Both the beta estimates and idiosyncratic volatility estimates used in the cross-sectional regressions of the testing periods are from the estimation period (next five years). In particular, the independent variables $\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, and $\bar{s}_{p,t-1}(\epsilon)$ are equal-weighted averages of individual stock's beta, beta squared, and residual standard deviation, respectively, estimated using a market model.

		With 20 Portfolios				With 50 Portfolios			
Eq. #		Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	Const.	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$
		Panel A: 1935-1968/6							
1	$\bar{\gamma}$	.0073	.0057			.0076	.0054		
	$t(\bar{\gamma})$	3.12	1.91			3.41	1.86		
2	$\bar{\gamma}$	.0106		.0020		.0110		.0018	
	$t(\bar{\gamma})$	4.26		1.73		4.33		1.64	
3	$\bar{\gamma}$	-.0022			.0574	-.0019			.0560
	$t(\bar{\gamma})$	-0.34			1.77	-0.34			1.93
4	$\bar{\gamma}$	.0014	.0187	-.0056	-.0070	-.0084	.0136	-.0049	.0475
	$t(\bar{\gamma})$	0.21	2.85	-2.52	-0.26	-1.55	2.74	-2.94	2.17
		Panel B: 1963-2000							
1	$\bar{\gamma}$	.0079	.0043			.0082	.0040		
	$t(\bar{\gamma})$	3.51	1.43			3.73	1.37		
2	$\bar{\gamma}$	.0101		.0018		.0104		.0016	
	$t(\bar{\gamma})$	5.08		1.32		5.15		1.24	
3	$\bar{\gamma}$	.0018			.0387	.0028			.0348
	$t(\bar{\gamma})$	0.32			1.47	0.53			1.44
4	$\bar{\gamma}$	-.0100	.0107	-.0069	.0666	-.0043	.0057	-.0035	.0529
	$t(\bar{\gamma})$	-1.41	1.36	-1.97	2.48	-.791	0.96	-1.36	2.59

Table 5: **Cross-sectional Regressions for Size-Beta Portfolios**

This table reports the cross-sectional regression results on the 50 size-beta sorted portfolios over both the Fama-MacBeth (1973) sample period of January 1935 to June 1968 and the extended Fama-French (1992) sample period of January 1963 to December 2000. Stocks from the NYSE/AMEX are first sorted into five size groups according their market capitalization in December of the last year of the estimation period. Each group of stocks is then sorted again into decile portfolios according to the selection period betas. Detailed information on the portfolio selection, estimation, and testing periods are available in Fama and MacBeth (1973). Under the column "Based on CAPM", both the reported betas and the idiosyncratic volatilities as well as betas used in sorting are estimated from a market model with a value-weighted index. Similarly, under the column 'Based on FF3 Model', the idiosyncratic volatilities are estimated from the Fama and French (1993) three-factor model instead. Both the beta estimates and idiosyncratic volatility estimates used in cross-sectional regressions of the testing periods are from the estimation period. In particular, the independent variables $\hat{\beta}_{p,t-1}$, $\hat{\beta}^2_{p,t-1}$, $ME_{p,t-1}$, and $\bar{s}_{p,t-1}(\epsilon)$ are equal-weighted averages of individual stock's beta, beta squared, log size, and residual standard deviation, respectively.

Eq.		Based on CAPM				Based on FF3 Model			
#		$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$ME_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$	$\hat{\beta}_{p,t-1}$	$\hat{\beta}^2_{p,t-1}$	$ME_{p,t-1}$	$\bar{s}_{p,t-1}(\epsilon)$
		Panel A: For sample period of 1935-1968/6							
1	$\bar{\gamma}$	.0077				.0077			
	$t(\bar{\gamma})$	2.24				2024			
2	$\bar{\gamma}$		.0027				.0027		
	$t(\bar{\gamma})$		2.13				2.13		
3	$\bar{\gamma}$			-.0022				-.0022	
	$t(\bar{\gamma})$			-2.74				-2.74	
4	$\bar{\gamma}$				0.070				0.077
	$t(\bar{\gamma})$				2.61				2.61
5	$\bar{\gamma}$	.0141	-.0045	-.0018		.0141	-.0045	-.0018	
	$t(\bar{\gamma})$	2.46	-2.29	-2.73		2.46	2.29	-2.73	
6	$\bar{\gamma}$	.0134	-.0055		0.073	.0124	-.0049		0.077
	$t(\bar{\gamma})$	2.55	-3.10		2.69	2.36	-2.76		2.70
7	$\bar{\gamma}$			-.0004	0.062			-.0004	0.067
	$t(\bar{\gamma})$			-0.77	2.03			-0.76	2.03
8	$\bar{\gamma}$	.0163	-.0063	-.0004	0.053	.0157	-.0058	-.0005	0.051
	$t(\bar{\gamma})$	2.95	-3.36	-0.96	2.16	2.83	-3.12	-1.13	2.09
		Panel B: For sample period of 1963-2000							
1	$\bar{\gamma}$	.0044				.0044			
	$t(\bar{\gamma})$	1.57				1.57			
2	$\bar{\gamma}$		.0018				.0018		
	$t(\bar{\gamma})$		1.48				1.48		
3	$\bar{\gamma}$			-.0009				-.0009	
	$t(\bar{\gamma})$			-1.94				-1.94	
4	$\bar{\gamma}$				0.043				0.048
	$t(\bar{\gamma})$				1.81				1.88
5	$\bar{\gamma}$	.0089	-.0029	-.0009		.0089	-.0029	-.0009	
	$t(\bar{\gamma})$	1.30	-1.13	-2.05		1.30	-1.13	-2.05	
6	$\bar{\gamma}$	.0059	-.0033		0.054	.0055	-.0031		0.059
	$t(\bar{\gamma})$	0.97	-1.33		2.16	0.92	-1.26		2.19
7	$\bar{\gamma}$			-.0003	0.034			-.0003	0.036
	$t(\bar{\gamma})$			-0.69	1.36			-0.83	1.36
8	$\bar{\gamma}$	.0072	-.0036	-.0003	0.041	.0071	-.0035	-.0003	0.040
	$t(\bar{\gamma})$	1.08	-1.39	-0.69	1.86	1.06	-1.37	-0.82	1.76

Table 6: **Cross-sectional Fama and French Regressions for Individual NYSE/AMEX/NASDAQ Stocks (1963-2000)**

This table reports the cross-sectional regression results for all NYSE/AMEX/NASDAQ individual stocks over the extended Fama-French (1992) sample period of July 1963 to June 2000. The beta estimates (β_p) used in the cross-sectional regressions are estimated from the 100 size-beta sorted portfolios and then assigned to individual stocks. Portfolio breakdowns in each year are determined by first sorting the NYSE stocks only into 10 size groups according their market capitalization in June of each year. Each group of the NYSE stocks is then sorted again into decile portfolios according to their beta estimates computed using the sum of the betas from a market model (with lagged market returns) of the previous 24 to 60 monthly returns. Similarly, the idiosyncratic volatility measure $s_p(\epsilon)$ is the root mean square error of either the market model or the Fama-French (1993) three-factor model for each of the 20(size) $\times$ 10(beta) portfolios and then assigned to individual stocks. $ME_{i,t-1}$ and $B/M_{i,t-1}$ are log market capitalization as of June and the last fiscal year's log book to market measure, respectively. In Panel A, both idiosyncratic volatility measures are estimated from the market model. While in Panel B, both idiosyncratic volatility measures are estimated from the Fama and French (1993) three-factor model.

		Panel A: Based on CAPM				Panel B: Based on FF3 Model			
Eq#		β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$
1	$\bar{\gamma}$	.0047				.0047			
	$t(\bar{\gamma})$	1.37				1.37			
2	$\bar{\gamma}$		.0042				.0042		
	$t(\bar{\gamma})$		5.43				5.43		
3	$\bar{\gamma}$			-.0019				-.0019	
	$t(\bar{\gamma})$			-3.50				-3.50	
4	$\bar{\gamma}$				0.315				0.318
	$t(\bar{\gamma})$				4.75				3.59
5	$\bar{\gamma}$	.0056	.0041			.0056	.0041		
	$t(\bar{\gamma})$	1.66	5.53			1.66	5.53		
6	$\bar{\gamma}$	-.0003		-.0020		-.0003		-.0020	
	$t(\bar{\gamma})$	-0.11		-4.34		-0.11		-4.34	
7	$\bar{\gamma}$	.0054			0.327	-.0090			0.460
	$t(\bar{\gamma})$	1.54			4.83	-3.06			5.16
8	$\bar{\gamma}$			-.0007	0.234			-.0005	0.265
	$t(\bar{\gamma})$			-1.09	3.62			-1.46	3.00
9	$\bar{\gamma}$	.0016	.0030	-.0016		.0016	.0030	-.0016	
	$t(\bar{\gamma})$	0.58	4.08	-3.23		0.58	4.08	-3.23	
10	$\bar{\gamma}$	.0061	.0030		0.280	-.0066	.0030		0.405
	$t(\bar{\gamma})$	1.77	4.15		4.05	-2.51	4.22		4.51
11	$\bar{\gamma}$	.0081	.0031	.0006	0.360	-.0070	.0030	.0003	0.446
	$t(\bar{\gamma})$	2.44	4.16	1.65	4.87	-2.62	4.09	0.74	5.43

Table 7: **Cross-sectional Fama and French Regressions for Individual NYSE/AMEX Stocks (1963-2000)**

This table reports the cross-sectional regression results for NYSE/AMEX individual stocks only over the extended Fama-French (1992) sample period of July 1963 to June 2000. The beta estimates (β_p) used in the cross-sectional regressions are estimated from the 100 size-beta sorted portfolios and then assigned to individual stocks. Portfolio breakdowns in each year are determined by first sorting NYSE stocks only into 10 size groups according their market capitalization in June of each year. Each group of NYSE stocks is then sorted again into decile portfolios according to their beta estimates computed using the sum of the betas from a market model (with lagged market returns) of the previous 24 to 60 monthly returns. Similarly, the idiosyncratic volatility measure $s_p(\epsilon)$ is the root mean square error of either the market model or the Fama-French (1993) three-factor model for each of the 20(size) $\times$ 10(beta) portfolios and then assigned to individual stocks. $ME_{i,t-1}$ and $B/M_{i,t-1}$ are log market capitalization as of June and the last fiscal year's log book to market measure, respectively. In Panel A, both idiosyncratic volatility measures are estimated from the market model. While in Panel B, both idiosyncratic volatility measures are estimated from the Fama and French (1993) three-factor model.

		Panel A: Based on CAPM				Panel B: Based on FF3 Model			
Eq#		β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$
1	$\bar{\gamma}$	.0039				.0039			
	$t(\bar{\gamma})$	1.19				1.19			
2	$\bar{\gamma}$		.0039				.0039		
	$t(\bar{\gamma})$		5.22				5.22		
3	$\bar{\gamma}$			-.0013				-.0013	
	$t(\bar{\gamma})$			-2.47				-2.47	
4	$\bar{\gamma}$				0.199				0.244
	$t(\bar{\gamma})$				3.53				2.89
5	$\bar{\gamma}$	.0035	.0036			.0035	.0036		
	$t(\bar{\gamma})$	1.10	5.10			1.10	5.10		
6	$\bar{\gamma}$	-.0002		-.0014		-.0002		-.0014	
	$t(\bar{\gamma})$	-0.09		-3.19		-0.09		-3.19	
7	$\bar{\gamma}$	.0043			0.213	-.0030			0.293
	$t(\bar{\gamma})$	1.29			3.61	-1.13			3.99
8	$\bar{\gamma}$			-.0007	0.119			.0003	0.299
	$t(\bar{\gamma})$			-1.11	1.95			0.88	3.59
9	$\bar{\gamma}$	.0010	.0028	-.0009		.0010	.0028	-.0009	
	$t(\bar{\gamma})$	0.38	4.35	-2.01		0.38	4.35	-2.01	
10	$\bar{\gamma}$	.0040	.0028		0.156	-.0017	.0027		0.227
	$t(\bar{\gamma})$	1.19	4.49		2.73	-0.66	4.26		3.18
11	$\bar{\gamma}$	.0050	.0028	.0003	0.195	-.0021	.0028	.0008	0.342
	$t(\bar{\gamma})$	1.72	4.36	0.73	3.85	-0.83	4.30	1.94	4.83

Table 8: **Investigating the Liquidity Effect for Individual Stocks (1963-2000)**

This table reports the cross-sectional regression results for individual stocks over the extended Fama-French (1992) sample period of July 1963 to June 2000. The beta estimates (β_p) used in the cross-sectional regressions are estimated from the 100 size-beta sorted portfolios and then assigned to individual stocks. Portfolio breakdowns in each year are determined by first sorting NYSE stocks only into 10 size groups according their market capitalization in June of each year. Each group of NYSE stocks are then sorted again into decile portfolios according to their beta estimates computed using the sum of the betas from a market model (with lagged market returns) of the previous 24 to 60 monthly returns. Similarly, the idiosyncratic volatility measure $s_p(\epsilon)$ is the root mean square error of the market model for each of the 20(size) $\times$ 10(beta) portfolios and then assigned to individual stocks. $Vlm_{i,t-1}$ is the relative volume of the previous month. $ME_{i,t-1}$ and $B/M_{i,t-1}$ are log market capitalization as of June and the last fiscal year's log book to market measure, respectively. In Panel A, all the NYSE/AMEX/NASDAQ individual stocks are used in the cross-sectional regression. In Panel B, only NYSE/AMEX stocks are used in estimation.

Eq.		Panel A: NYSE/AMEX/NASDAQ Stocks					Panel B: NYSE/AMEX Stocks				
No.		β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	$Vlm_{i,t-1}$	β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	$Vlm_{i,t-1}$
1	$\bar{\gamma}$				0.315					0.199	
	$t(\bar{\gamma})$				4.75					3.53	
2	$\bar{\gamma}$					.0065					.0066
	$t(\bar{\gamma})$					2.68					2.80
3	$\bar{\gamma}$	.0048				.0068	.0037				.0066
	$t(\bar{\gamma})$	1.39				2.92	1.16				3.04
4	$\bar{\gamma}$		.0042			.0039		.0038			.0031
	$t(\bar{\gamma})$		5.37			1.72		5.16			1.48
5	$\bar{\gamma}$			-.0019		.0001			-.0012		.0013
	$t(\bar{\gamma})$			-3.40		0.04			-2.31		0.78
6	$\bar{\gamma}$				0.317	-.0015				0.194	-.0011
	$t(\bar{\gamma})$				4.79	-0.85				3.57	-0.79
7	$\bar{\gamma}$	.0013	.0030	-.0016		-.0018	.0009	.0028	-.0009		-.0007
	$t(\bar{\gamma})$	0.49	4.08	-3.32		-1.35	0.33	4.36	-2.04		-0.69
8	$\bar{\gamma}$	.0060	.0030		0.284	-.0016	.0039	.0028		0.157	-.0004
	$t(\bar{\gamma})$	1.72	4.16		4.15	-1.14	1.18	4.49		2.81	-0.37
9	$\bar{\gamma}$	.0057		.0008	0.352	-.0021	.0036		-.0002	0.199	-.0005
	$t(\bar{\gamma})$	1.60		0.21	4.76	-1.59	1.23		-0.60	3.93	-0.50
10	$\bar{\gamma}$	.0078	.0031	.0006	0.360	-.0015	.0048	.0028	.0003	0.195	-.0011
	$t(\bar{\gamma})$	2.37	4.16	1.55	4.86	-1.19	1.67	4.37	0.67	3.87	-1.03

Table 9: **Cross-sectional Regressions for Individual Japanese Stocks (Jan. 1980-Dec. 1999)**

Both the beta estimates and idiosyncratic volatility estimates used in cross-sectional regressions are estimated from 100 size-beta sorted portfolios and then assigned to individual stocks. In each year, stocks are first sorted into 10 size groups according their market capitalization in December of the previous year. Each group of stocks are then sorted again into decile portfolios according to their beta estimates computed using the sum of the betas from a market model of previous 24 to 60 monthly returns. We use all the listed stocks that also have book-to-market information over the whole sample period. The idiosyncratic volatilities are estimated from the Fama and French (1993) three-factor model. In particular, $\hat{\beta}_{p,t-1}$, $ME_{i,t-1}$, $B/M_{i,t-1}$ and $\bar{s}_{p,t-1}(\epsilon)$ are the independent variables of beta, log size, book-to-market equity, and residual standard deviation, respectively.

	Const.	β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$	Const.	β_p	$B/M_{i,t-1}$	$ME_{i,t-1}$	$s_p(\epsilon)$
	Panel A: The Whole Sample									
$\bar{\gamma}$	-.00367	.0121								
$t(\bar{\gamma})$	-0.555	1.362								
$\bar{\gamma}$	0.00523		.0081							
$t(\bar{\gamma})$	1.163		2.569							
$\bar{\gamma}$	0.02518			-.0016						
$t(\bar{\gamma})$	1.651			-1.470						
$\bar{\gamma}$	0.00332				1.358					
$t(\bar{\gamma})$	0.935				1.867					
$\bar{\gamma}$	0.01445	.0052	.0070	-.0013						
$t(\bar{\gamma})$	0.991	0.707	2.375	-1.285						
$\bar{\gamma}$	0.00076	.0003	.0071		1.306					
$t(\bar{\gamma})$	0.123	0.037	2.312		1.871					
$\bar{\gamma}$	-.00632	-.0003	.0073	.0006	1.686					
$t(\bar{\gamma})$	-0.457	-0.041	2.466	0.701	3.354					
	Panel B: The First Subsample					Panel C: The Second Subsample				
$\bar{\gamma}$	0.00843	.0132				-.01580	.0110			
$t(\bar{\gamma})$	0.893	1.206				-1.697	0.773			
$\bar{\gamma}$	0.01922		.0077			-.00877		.0085		
$t(\bar{\gamma})$	3.268		1.614			-1.273		2.028		
$\bar{\gamma}$	0.04748			-.0024		0.00289			-.0007	
$t(\bar{\gamma})$	2.499			-1.773		0.124			-0.455	
$\bar{\gamma}$	0.01470				1.835	-.00805				0.881
$t(\bar{\gamma})$	3.083				1.990	-1.501				0.801
$\bar{\gamma}$	0.04366	.0020	.0060	-.0024		-.01476	.0084	.0080	-.0002	
$t(\bar{\gamma})$	1.861	0.186	1.343	-1.647		-0.891	0.782	2.017	-0.131	
$\bar{\gamma}$	0.01603	-.0038	.0065		2.031	-.01451	.0043	.0078		0.581
$t(\bar{\gamma})$	1.673	-0.344	1.386		1.937	-1.847	0.419	1.891		0.654
$\bar{\gamma}$	0.02501	-.0034	.0062	-.0006	1.625	-.03765	.0028	.0085	.0019	1.747
$t(\bar{\gamma})$	1.152	-0.329	1.389	-0.510	2.435	-2.262	0.270	2.100	1.600	2.322

Table 10: **Summary Statistics for Individual Times Series Regressions of 100 Portfolios**

This table reports the average and standard deviation of regression coefficients of the 100 size-beta sorted portfolios using different time-series models. $R_{VW,t}$ and $R_{i,t}$ denote returns from a value-weighted index and the $i-th$ portfolio respectively; $R_{I,t}$ denotes the return proxies for idiosyncratic volatility from a hedging portfolio; $R_{SMB,t}$ and $R_{HML,t}$ denote proxies for the size and book-to-market factors respectively. The dependent variable is $R_{i,t}$. "Ind." counts the percentage of individual regressions for each portfolio with significant coefficients at 5% level.

	α	$R_{VW,t}$	$R_{SML,t}$	$R_{HML,t}$	$R_{I,t}$	R^2(%)
	Panel A: 1935.1-1968.6					
Mean	0.105	1.279				69.94
St.D	(.195)	(.088)				(13.48)
Ind.(%)	26.00	100.0				
Mean	0.166	0.957			0.574	77.09
St.D	(.166)	(.082)			(.141)	(7.66)
Ind.(%)	39.00	97.00			82.00	
Mean	-0.002	1.021	0.679	0.347		79.46
St.D	(.147)	(.061)	(.116)	(.123)		(7.10)
Ind.(%)	34.00	100.0	87.00	71.00		
Mean	0.106	0.961		0.225	0.462	77.62
	(.163)	(.080)		(.110)	(.132)	(7.47)
Ind.(%)	30.00	98.00		56.00	73.00	
Mean	0.021	0.997	0.609	0.303	0.094	80.20
St.D	(.155)	(.079)	(.125)	(.105)	(.160)	(6.52)
Ind.(%)	27.00	99.00	91.00	77.00	54.00	
	Panel B: 1963.7-2000.6					
Mean	0.079	1.057				67.10
St.D	(.161)	(.051)				(11.96)
Ind.(%)	14.00	100.0				
Mean	0.087	0.889			0.314	73.25
St.D	(.137)	(.052)			(.073)	(9.76)
Ind.(%)	16.00	100.0			80.00	
Mean	-0.091	1.019	0.577	0.311		81.13
St.D	(.117)	(.037)	(.055)	(.066)		(7.08)
Ind.(%)	35.00	100.0	91.00	85.00		
Mean	-0.064	0.920		0.369	0.439	77.26
St.D	(.130)	(.049)		(.076)	(.055)	(7.45)
Ind.(%)	27.00	100.0		88.00	87.00	
Mean	-0.091	1.019	0.576	0.311	0.000	82.19
St.D	(.111)	(.044)	(.081)	(.064)	(.073)	(6.46)
Ind.(%)	37.00	100.0	92.00	88.00	77.00	

Figure 1 The Relationship between Average Return and Idiosyncratic Volatility (1963-2000)