A Two-Warehouse Production Inventory Model with Variable Demand and Permissible Delay in Payment under Inflation

S, R. Singh, Pinky Saxena

***Abstract*— In this paper, a two-warehouse production inventory model is developed for deteriorating items with variable demand .The effect of permissible delay in payment is considered under inflation. Since, the capacity of any warehouse is limited, it has to rent warehouse (R.W) for storing the excess units over the fixed capacity of the own warehouse (O.W). On the basis of this fact, we have developed a two-warehouse production inventory model for deteriorating items under inflation & permissible delay in payment. The objective of this study is to derive the retailer's optimal replenishment policy that minimizes the total relevant inventory costs. The necessary and sufficient conditions for an optimal solution are characterized. An algorithm is developed to find the optimal solution. Finally, numerical examples are provided to illustrate the proposed model. Sensitivity analysis is made and some managerial inferences are presented.**

***Index Terms*— Inventory, Two-warehouse, Deterioration, Permissible delay in payment, Inflation**

I. INTRODUCTION

In many inventory models it is idealistically assumed that all items produced are of good quality. But, production of defective items is a natural process in a production cycle. So, we consider the model with imperfect quality. In today's competitive business world, a supplier frequently offers his retailers a delay of payment for settling the amount owed to him. The permissible delay in payments is a successful method of attracting new customers and increasing sales.

In today's business world, it is more and more common to see that retailer are allowed a fixed time period before they settle their account to the supplier . This period is known as trade credit period. Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest. A higher interest is charged if the payment is not settled by the end of the trade credit period. Stocking or storing plays an important role in the inventory management .Generally; every company has its own warehouse (O.W) with a fixed capacity. If the capacity increases then these quantities should be stored in another rented warehouse (R.W). Liang and Zhou [1] provided a two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment with constant demand. H.L. Yang [2] developed two-warehouse partial backlogging inventory models for deteriorating items under inflation , Goyal [3] was the first to establish an economic order quantity model with a constant demand rate under the condition of a permissible delay in payments.

Manuscript received November, 2013.

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Shah [4] considered a stochastic inventory model when delays in payments are permissible. Aggarwal and Jaggi [5] extended Goyal's model to consider the deteriorating items. Jamal et al. [6] further generalized Aggarwal and Jaggi's model to allow for shortages. Hwang and Shinn [7] added the pricing strategy to the model, and developed the optimal price and lot-sizing for a retailer under the condition of a permissible delay in payments. Liao et al. [8] proposed an inventory model with deteriorating items under inflation when a delay in payment is permissible. Chang and Dye [9] developed a partial backlogging inventory model for deteriorating items with Wei-bull distribution and permissible delay in payments. Chang et al. [10] presented an inventory model for deteriorating items with linear trend under the condition of permissible delay in payments. Shah [11] considered an inventory model for deteriorating items and time value of money under permissible delay in payments during a finite planning horizon. Soni et al. [12] discussed an EOQ model for progressive payment scheme under discounted cash flow (DCF) approach. Chang et al. [13] made a review on previous related literatures under trade credit since 1985. Chang et al. [14] proposed an optimal payment time for deteriorating items under inflation and permissible delay in payments during a finite planning horizon. Yang and Chang [15] provided a two warehouse partial backlogging inventory model for deteriorating items with permissible delay in payment under inflation. Go swami et.al. [16] developed a two-warehouse inventory model with increasing demand and time varying deterioration. Trade credit represents an important proportion of company finance. Also, from a financial point of view, inventory represents a capital investment and it is a substantial asset for a firm's limited capital fund. As a result, it is necessary to consider the effects of inflation on the inventory system. According to this fact, we developed two- warehouse production model with the permissible delay in payments. Liang & Zhou [1] considered a two-warehouse inventory models for deteriorating items under conditionally permissible delay in payment with constant demand. We assume that production rate is taken as the linear combination of on-hand inventory and demand, while demand rate is taken as the function of time. Also, we assume production rate as being dependent on the demand rate and the two warehouses have different deterioration rates. Based on the above conditions, the two-warehouse production inventory model for deteriorating items is developed with permissible delay in payment under inflation. The objective of this study is to derive the retailer's optimal replenishment policy that minimizes the total inventory costs. The necessary and sufficient conditions for an optimal solution are characterized. An algorithm is also developed to determine the optimal replenishment policy. Finally, some numerical examples for illustration are provided and sensitivity analysis is made on parameters.

II. ASSUMPTION AND NOTATIONS

The mathematical models of the two-warehouse inventory problems are based on the following assumptions:

1. Production rate is greater than demand rate. Also, it is linear combination of on-hand inventory and demand rate i.e. $P(t)=[I(t)+bD(t)](1-e^{-dt})$.
2. Demand rate is exponentially an increasing function of time .i.e. $D(t)=\mu e^{\lambda t}, 0\le\lambda\le 1$.
3. Deterioration is taken as time dependent for O.W, while, Wei-bull distribution for R.W.
4. Planning Horizon is finite.
5. Model is considered for imperfect items and inflation is also taken in this model.
6. Shortages are not permitted.
7. Lead time is zero, and no replenishment or repair of deteriorated items is made during a given cycle.
8. A single item is considered over the prescribed period T units of time, which is subject to variable deterioration rate.
9. The owned warehouse (O.W) has a fixed capacity of W units, and the rented warehouse (R.W) has unlimited capacity.
10. The goods of the O.W are consumed only after consuming the goods kept in R.W.
11. The unit inventory costs (including holding cost) per unit time in R.W are higher than those in O.W.
12. The supplier provides the retailer a permissible delay of payments. During the trade credit period the account is not settled, the revenue is deposited in an interest bearing account. At the end of the permissible delay, the retailer pays off the items ordered, and starts to pay the interest charged on the items in stock.

In addition, the following notations are used throughout this paper.

$D(t)=\mu e^{\lambda t}, 0\le\lambda\le 1$: Demand rate increases with time, where μ is the initial demand rate.

$P(t)=[I(t)+bD(t)](1-e^{-dt}), 0\le d\le 1, b\succ 1$: Production rate

C_D : Deterioration cost per unit time.

$I_{O_1}(t)$: Inventory level in O.W. at time t with t ∈ [0, t_1].

$I_{R_2}(t)$: Inventory level in R.W. at time t with t ∈ [t_1, t_2].

r : Infation rate

d : Rate of imperfect production.

W : Fixed capacity of O.W.

C_S : Set up cost per production run.

C_{RW} : Holding cost per unit inventory held in R.W. per unit time

C_{OW} : Holding cost per unit inventory held in O.W. per unit time

$I_{R_2}(t)$: Inventory level in R.W. at time t with t ∈ [t_2, t_3].

$I_{O_4}(t)$: Inventory level in O.W. at time t with t ∈ [t_3, T].

$I_{O_5}(t)$: Inventory level in O.W. at time t with t ∈ [t_1, t_3].

t_1, t_2 : Production period for O.W and R.W.

t_3, T : Non-Production period.

T : Total cycle time

M : Retailer's trade credit period offered by supplier in years.

s : Unit selling price.

c : Unit purchase cost.

I_e : Interest which can be earned per $ per year.

I_C : Interest. charges per $ in stocks per year by the supplier

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$TC_i, i=1,2,,3$: Total relevant costs , consisting (a) Setup Cost (b) Holding Cost (c) Deterioration Cost (d) Interest payable cost. (e)Interest earned.

Where T^*, t_3^* are the optimal solutions and TC^* is the minimum total relevant costs.

III. MATHEMATICAL MODEL

The model begins as follows: initially, the inventory level is zero. The production starts at time t = 0, and items accumulate from 0 up to W units in O.W .and in t_1 units of time. After time t_1 any production quantity exceeding W will be stored in R.W. After this production stopped and the inventory level in R.W. begins to decrease at t_2 and will reach 0 units at t_3 because of demand and deterioration. The inventory level in O.W. comes to decrease at t_1 and then falls below W at t_2+ t_3 due to deterioration. But, during [t_3, T], the inventory is depleted due to both demand and deterioration. Fig1. Depicts the behavior of inventory system.

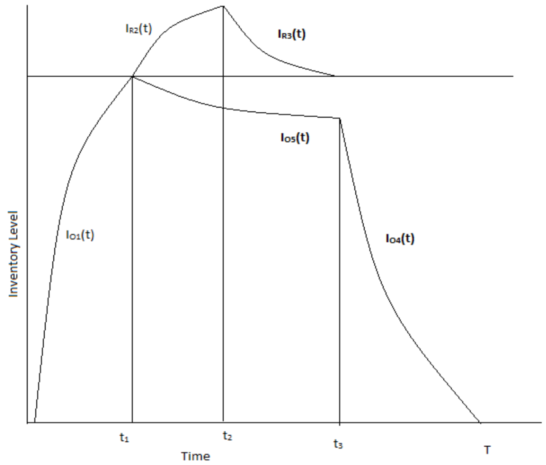

Fig1: Graphical Representation Of The Two-Warehouse Inventory System

The differential equations stating the inventory levels within the cycle are given as follows:

$$\frac{dI_{O_1}(t)}{dt}+\theta I_{O_1}(t)=P(t)-D(t),\quad 0\le t\le t_1 \text{.........(3.1)}$$

$$\frac{dI_{R_2}(t)}{dt}+\theta I_{R_2}(t)=P(t)-D(t),\quad t_1\le t\le t_2 \text{.......(3.2)}$$

$$\frac{dI_{R_3}(t)}{dt}+\theta I_{R_3}(t)=-D(t),\quad t_2\le t\le t_3 \text{.......(3.3)}$$

$$\frac{dI_{O_4}(t)}{dt}+\theta I_{O_4}(t)=-D(t),\quad t_3\le t\le T \text{......(3.4)}$$

$$\frac{dI_{O_5}(t)}{dt}+\theta I_{O_5}(t)=0,\quad t_1\le t\le t_3 \text{.............(3.5)}$$

With the boundary conditions

$$I_{O_1}(0)=0, I_{R_2}(t_1)=0, I_{R_3}(t_3)=0, I_{O_4}(T)=0, I_{O_5}(t_1)=W$$

The solutions to equations (3.1) - (3.5) are as follows:

$$I_{O_1}(t)=(b-1)\mu\left(\frac{dt^2}{2}+\frac{\lambda d t^3}{3}-\frac{(\theta-d)dt^4}{4}\right)-\mu\left(t+\frac{\lambda t^2}{2}-(\theta-d)\frac{t^3}{2}\right)$$

$$I_{R_2}(t)=(b-1)\mu\left(\frac{d(t^2-t_1^2)}{2}-\frac{\alpha d t^\beta(t^2-t_1^2)}{2}+\frac{(t^3-t_1^3)\lambda d}{3}\right)-\mu\left((t-t_1)-\alpha t^\beta(t-t_1)+\frac{\lambda(t^2-t_1^2)}{2}-(t^2-t_1^2)\frac{\lambda\alpha t^\beta}{2}\right)$$

$$I_{R_3}(t)=\mu\left((t_3-t)-\alpha t^\beta(t_3-t)+\frac{\lambda}{2}(t_3^2-t^2)-\frac{\lambda\alpha t^\beta}{2}(t_3^2-t^2)\right)$$

$$I_{O_4}(t)=\mu\left((T-t)-\frac{\theta t^2}{2}(T-t)+\frac{\lambda}{2}(T^2-t^2)-\frac{\lambda\theta t^2}{4}(T^2-t^2)\right)$$

$$I_{O_5}(t)=W e^{-\left(\frac{\theta}{2}\right)(t_1^2-t^2)}$$

Based on the assumptions and description of the model, the total annual costs, TC, include the following elements:

(1)The present value of setup cost = CS.

(2) The present values of the inventory holding costs in R.W. and O.W. are

$$I_{RW}=c_{RW}\left(\int_{t_1}^{t_2} I_{R_2}(t)e^{-rt}dt+\int_{t_2}^{t_3} I_{R_3}(t)e^{-rt}dt\right)$$

$$=c_{RW}\left(\mu\left((b-1)\left(\frac{d}{2}\left(\frac{t_2^3}{3}+\frac{2t_1^3}{3}\right)-\frac{dr}{3}\left(\frac{t_2^4}{4}+\frac{t_1^4}{2}\right)+\frac{d\lambda}{3}\left(\frac{t_2^4}{4}+\frac{3t_1^4}{4}\right)-\frac{rd\lambda}{3}\left(\frac{t_2^5}{5}+\frac{3t_1^5}{10}\right)\right)\right)\right.$$
$$\left.-\left(\left(\frac{t_1^2}{2}+\frac{t_3^2}{2}\right)-r\left(\frac{t_1^3}{6}+\frac{t_3^3}{6}\right)+\frac{\lambda}{2}\left(\frac{2t_1^3}{3}+\frac{2t_3^3}{3}\right)-\frac{\lambda r}{2}\left(\frac{t_1^4}{4}+\frac{t_3^4}{2}\right)\right)\right)$$

$$I_{OW}=c_{OW}\left(\int_0^{t_1} I_{O_1}(t)e^{-rt}dt+\int_{t_1}^{t_3} I_{O_5}(t)e^{-rt}dt+\int_{t_3}^{T} I_{O_4}(t)e^{-rt}dt\right)$$

$$=c_{OW}\left((b-1)\mu\left(\frac{dt_1^3}{6}-\frac{rdt_1^4}{8}+\frac{\lambda dt_1^4}{12}\right)-\mu\left(\frac{t_1^2}{2}-\frac{T^2}{2}-\frac{t_3^2}{2}-\frac{rt_1^3}{6}-\frac{rt_3^3}{3}-\frac{\lambda T^3}{6}\right)+W\left((t_3-t_1)-\frac{r}{2}(t_3^2-t_1^2)+\frac{\theta t_1^3}{2}\right)\right)$$

(3)The present value of the inventory deterioration cost is

$$I_D=c_D\left(\int_0^{t_1}\theta I_{O_1}(t)e^{-rt}dt+\int_{t_1}^{t_3}\theta I_{O_5}(t)e^{-rt}dt+\int_{t_3}^{T}\theta I_{O_4}(t)e^{-rt}dt+\int_{t_1}^{t_2}\alpha\beta t^{\beta-1}I_{R_2}(t)e^{-rt}dt\right.$$
$$\left.+\int_{t_2}^{t_3}\alpha\beta t^{\beta-1}I_{R_3}(t)e^{-rt}dt\right)$$

$$=c_D\left(\theta\mu\left((b-1)\left(\frac{dt_1^4}{8}+\frac{d\lambda t_1^5}{15}+\frac{drt_1^5}{10}\right)-\left(\frac{t_1^3}{3}+\frac{\lambda t_1^4}{8}+\frac{rt_1^4}{4}\right)\right)\right.$$
$$+W\theta\left(\frac{t_3^2}{2}-\frac{t_1^2}{2}+\frac{\theta t_3^4}{8}-\frac{\theta t_1^4}{8}-\frac{rt_3^3}{3}+\frac{rt_1^3}{3}\right)+\theta\mu\left(\left(\frac{-t_3^3}{6}-\frac{T^3}{3}\right)-r\left(\frac{-t_3^4}{12}-\frac{T^4}{4}\right)\right.$$
$$\left.+\frac{\lambda r}{2}\left(\frac{-2t_3^5}{15}-\frac{T^5}{5}\right)\right)+\mu^2\alpha^2\beta^2\left((b-1)d\left(\frac{t_2^{\beta+2}}{2(\beta+2)}+\frac{t_1^{\beta+2}}{2\beta(\beta+2)}\right)\right)-\left(\frac{t_2^{\beta+1}}{\beta+1}+\frac{t_1^{\beta+1}}{\beta(\beta+1)}\right)$$
$$\left.\left(\left(\frac{t_2^{\beta+1}}{(\beta+1)}+\frac{t_3^{\beta+1}}{\beta(\beta+1)}\right)-r\left(\frac{t_2^{\beta+1}}{(\beta+2)}+\frac{t_3^{\beta+2}}{(\beta+2)(\beta+1)}\right)\right)\right)$$

Next, Based on the parameter values t_3, M, T, there are three cases to be explored.

Case 1: $M \le t_3 \prec T$, In this case, interest payable is

$$IC_1=cI_C\left(\int_M^{t_3} I_{R_3}(t)e^{-rt}dt+\int_M^{t_3} I_{O_5}(t)e^{-rt}dt+\int_{t_3}^{T} I_{O_4}(t)e^{-rt}dt\right)$$

$$=cI_C\left(\mu\left(\frac{t_3^2}{2}-\frac{1}{6}rt_3^3-t_3M-rt_3\frac{M^2}{2}-\frac{M^2}{2}+\frac{rM^3}{3}\right)+\frac{\mu\lambda}{2}\left(\frac{2}{3}t_3^3-\frac{1}{4}rt_3^4-t_3^2M-rt_3\frac{M^2}{2}-\frac{M^3}{3}+\frac{rM^4}{4}\right)\right.$$
$$-\mu\alpha\left(\frac{t_3^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_3^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{t_3M^{\beta+1}}{\beta+1}-\frac{rt_3M^{\beta+2}}{\beta+2}-\frac{M^{\beta+2}}{\beta+2}+\frac{rM^{\beta+3}}{\beta+3}\right)$$

$$-\frac{\mu\lambda\alpha}{2}\left(\frac{2t_3^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{2rt_3^{\beta+4}}{(\beta+2)(\beta+4)}-\frac{t_3^2M^{\beta+1}}{\beta+1}-\frac{rt_3^2M^{\beta+2}}{\beta+2}-\frac{M^{\beta+3}}{\beta+3}+\frac{rM^{\beta+4}}{\beta+4}\right)+W\left((t_3-M)+\frac{r}{2}(M^2-t_3^2)+\frac{\theta}{2}t_1^2(M-t_3)\right)$$
$$+\frac{\theta}{2}rt_1^2\left(\frac{t_3^2}{2}-\frac{M^2}{2}\right)+\frac{\theta}{6}(t_3^3-M^3)+\frac{\theta r}{8}(M^4-t_3^4)\Big)+\mu\left(\frac{T^2}{2}+\frac{rT^3}{6}-Tt_3+\frac{rTt_3^2}{2}+\frac{t_3^2}{2}-\frac{rt_3^3}{3}\right)$$

$$-\frac{\mu\theta}{2}\left(\frac{T^4}{12}-\frac{rT^5}{20}-\frac{Tt_3^3}{3}+\frac{rTt_3^4}{4}+\frac{t_3^4}{4}-\frac{rt_3^5}{5}\right)+\frac{\mu\lambda}{2}\left(\frac{2T^3}{3}-\frac{rT^4}{4}-T^2t_3+\frac{rT^2t_3^3}{2}+\frac{t_3^3}{3}-\frac{rt_3^4}{4}\right)$$
$$-\frac{\mu\lambda\theta}{4}\left(\frac{2T^5}{15}-\frac{1}{12}rT^6-\frac{T^2t_3^3}{3}+\frac{rT^2t_3^4}{4}+\frac{t_3^5}{5}-\frac{rt_3^6}{6}\right)\Bigg)$$

Case 2: $t_3 \prec M \le T$,In this case, interest payable is

$$IC_2=cI_C\int_M^T I_{O_4}(t)e^{-rt}dt$$

$$=cI_C\mu\left(\left(\frac{T^2}{2}+\frac{rT^3}{6}-TM+\frac{rTM^2}{2}+\frac{M^2}{2}-\frac{rM^3}{3}\right)-\frac{\mu\theta}{2}\left(\frac{T^4}{12}-\frac{rT^5}{20}-\frac{TM^3}{3}+\frac{rTM^4}{4}+\frac{M^4}{4}-\frac{rM^5}{5}\right)\right.$$
$$\left.+\frac{\mu\lambda}{2}\left(\frac{2T^3}{3}-\frac{rT^4}{4}-T^2M+\frac{rT^2M^2}{2}+\frac{M^3}{3}-\frac{rM^4}{4}\right)-\frac{\mu\lambda\theta}{4}\left(\frac{2T^5}{15}-\frac{rT^6}{12}-\frac{T^2M^3}{3}+\frac{rT^2M^4}{4}+\frac{M^5}{5}-\frac{rM^6}{6}\right)\right)$$

Case 3: $M \succ T$, In this case, no interest charges are paid for the items $IC_3=0$

On the other hand, the retailer accumulates revenue in an account that earns I_e per dollar per year staring from t_3 to T. As a result, the interest earned is given as follows:

There are two cases as follows:

Case1: $M \le T$, In this case, the interest earned is

$$IE_1 = sI_e\int_0^M Dt\ e^{-rt}dt$$

$$=\mu sI_e\left(\frac{M^2}{2}-(r-\lambda)\frac{M^3}{3}+\frac{\lambda rM^4}{4}\right)$$

Case2: $M \succ T$, In this case, the interest earned is

$$IE_2=sI_e\left(\int_0^T Dt\,e^{-rt}dt+DT(M-T)\right)$$

$$=sI_e\mu\left(\frac{T^2}{2}-(r-\lambda)\frac{T^3}{3}+\lambda r\frac{T^4}{4}\right)+\mu e^{\lambda t}sI_eT(M-T)$$

Therefore, the annual total relevant costs for the retailer can be expressed as:

TC(t3,T) = ordering cost + stock holding cost in RW+ stock holding cost in OW + deteriorating cost +interest payable cost – interest earned.

That is,

$$TC(t3,T)=\begin{cases} TC_1, \text{if } M \le t_3 \prec T \\ TC_2, \text{if } t_3 \prec M \le T \\ TC_3, \text{if } M \succ T \end{cases}$$

IV SOLUTION TO THE TWO-WAREHOUSE INVENTORY MODELS

The present value of the total cost can be written as follows:

$$TC_1=\frac{1}{T}\left(c_S+c_{RW}\left(\mu\left((b-1)\left(\frac{d}{2}\left(\frac{t_2^3}{3}+\frac{2t_1^3}{3}\right)-\frac{dr}{3}\left(\frac{t_2^4}{4}+\frac{t_1^4}{2}\right)+\frac{d\lambda}{3}\left(\frac{t_2^4}{4}+\frac{3t_1^4}{4}\right)-\frac{rd\lambda}{3}\left(\frac{t_2^5}{5}+\frac{3t_1^5}{10}\right)\right)\right)\right)\right.$$
$$-\left(\left(\frac{t_1^2}{2}+\frac{t_3^2}{2}\right)-r\left(\frac{t_1^3}{6}+\frac{t_3^3}{6}\right)+\frac{\lambda}{2}\left(\frac{2t_1^3}{3}+\frac{2t_3^3}{3}\right)-\frac{\lambda r}{2}\left(\frac{t_1^4}{4}+\frac{t_3^4}{2}\right)\right)\Bigg)\Bigg)$$

$$+c_{OW}\Bigg((b-1)\mu\Big(\frac{dt_1^3}{6}-\frac{rdt_1^4}{8}+\frac{\lambda dt_1^4}{12}\Big)-\mu\Big(\frac{t_1^2}{2}-\frac{T^2}{2}-\frac{t_3^2}{2}-\frac{rt_1^3}{6}-\frac{rt_3^3}{3}-\frac{\lambda T^3}{6}\Big)$$

$$+W\Big((t_3-t_1)-\frac{r}{2}(t_3^2-t_1^2)+\frac{\theta t_1^3}{2}\Big)\Bigg)+c_D\Bigg(\Bigg(\theta\mu\Bigg((b-1)\Big(\frac{dt_1^4}{8}+\frac{d\lambda t_1^5}{15}+\frac{drt_1^5}{10}\Big)$$

$$-\Big(\frac{t_1^3}{3}+\frac{\lambda t_1^4}{8}+\frac{rt_1^4}{4}\Big)\Bigg)+W\theta\Big(\frac{t_3^2}{2}-\frac{t_1^2}{2}+\frac{\theta t_3^4}{8}-\frac{\theta t_1^4}{8}-\frac{rt_3^3}{3}+\frac{rt_1^3}{3}\Big)\Bigg)+\theta\mu\Bigg(\Big(\frac{-t_3^3}{6}-\frac{T^3}{3}\Big)$$

$$-r\Big(\frac{-t_3^4}{12}-\frac{T^4}{4}\Big)+\frac{\lambda r}{2}\Big(\frac{-2t_3^5}{15}-\frac{T^5}{5}\Big)\Bigg)+\mu^2\alpha^2\beta^2\Bigg(\Bigg((b-1)d\Big(\frac{t_2^{\beta+2}}{2(\beta+2)}+\frac{t_1^{\beta+2}}{2\beta(\beta+2)}\Big)$$

$$-\Big(\frac{t_2^{\beta+1}}{\beta+1}+\frac{t_1^{\beta+1}}{\beta(\beta+1)}\Big)\Bigg)-r\Big(\frac{t_2^{\beta+1}}{(\beta+2)}+\frac{t_3^{\beta+2}}{(\beta+1)(\beta+2)}\Big)\Bigg)\Bigg)\Bigg)+$$

$$cI_C\Bigg(\mu\Big(\frac{t_3^2}{2}-\frac{1}{6}rt_3^3-t_3M-rt_3\frac{M^2}{2}-\frac{M^2}{2}+\frac{rM^3}{3}\Big)+\frac{\mu\lambda}{2}\Big(\frac{2}{3}t_3^3-\frac{1}{4}rt_3^4-t_3^2M-rt_3\frac{M^2}{2}-\frac{M^3}{3}+\frac{rM^4}{4}\Big)$$

$$-\mu\alpha\Big(\frac{t_3^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rt_3^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{t_3M^{\beta+1}}{\beta+1}-\frac{rt_3M^{\beta+2}}{\beta+2}-\frac{M^{\beta+2}}{\beta+2}+\frac{rM^{\beta+3}}{\beta+3}\Big)$$

$$-\frac{\mu\lambda\alpha}{2}\Big(\frac{2t_3^{\beta+3}}{(\beta+1)(\beta+3)}-\frac{2rt_3^{\beta+4}}{(\beta+2)(\beta+4)}-\frac{t_3^2M^{\beta+1}}{\beta+1}-\frac{rt_3^2M^{\beta+2}}{\beta+2}-\frac{M^{\beta+3}}{\beta+3}+\frac{rM^{\beta+4}}{\beta+4}\Big)+W\Big((t_3-M)+\frac{r}{2}(M^2-t_3^2)+\frac{\theta}{2}t_1^2(M-t_3)$$

$$+\frac{\theta}{2}rt_1^2\Big(\frac{t_3^2}{2}-\frac{M^2}{2}\Big)+\frac{\theta}{6}(t_3^3-M^3)+\frac{\theta r}{8}(M^4-t_3^4)\Big)+\mu\Big(\frac{T^2}{2}+\frac{rT^3}{6}-Tt_3+\frac{rTt_3^2}{2}+\frac{t_3^2}{2}-\frac{rt_3^3}{3}\Big)$$

$$-\frac{\mu\theta}{2}\Big(\frac{T^4}{12}-\frac{rT^5}{20}-\frac{Tt_3^3}{3}+\frac{rTt_3^4}{4}+\frac{t_3^4}{4}-\frac{rt_3^5}{5}\Big)+\frac{\mu\lambda}{2}\Big(\frac{2T^3}{3}-\frac{rT^4}{4}-T^2t_3+\frac{rT^2t^3}{2}+\frac{t_3^3}{3}-\frac{rt_3^4}{4}\Big)$$

$$-\frac{\mu\lambda\theta}{4}\Big(\frac{2T^5}{15}-\frac{1}{12}rT^6-\frac{T^2t_3^3}{3}+\frac{rT^2t_3^4}{4}+\frac{t_3^5}{5}-\frac{rt_3^6}{6}\Big)\Bigg)$$

$$TC_2=\frac{1}{T}\Bigg(c_Se^{-rt}+c_{RW}\Bigg(\mu\Bigg((b-1)\Bigg(\frac{d}{2}\Big(\frac{t_2^3}{3}+\frac{2t_1^3}{3}\Big)-\frac{dr}{3}\Big(\frac{t_2^4}{4}+\frac{t_1^4}{2}\Big)+\frac{d\lambda}{3}\Big(\frac{t_2^4}{4}+\frac{3t_1^4}{4}\Big)-\frac{rd\lambda}{3}\Big(\frac{t_2^5}{5}+\frac{3t_1^5}{10}\Big)\Bigg)\Bigg)$$

$$-\Bigg(\Big(\frac{t_1^2}{2}+\frac{t_3^2}{2}\Big)-r\Big(\frac{t_1^3}{6}+\frac{t_3^3}{6}\Big)+\frac{\lambda}{2}\Big(\frac{2t_1^3}{3}+\frac{2t_3^3}{3}\Big)-\frac{\lambda r}{2}\Big(\frac{t_1^4}{4}+\frac{t_3^4}{2}\Big)\Bigg)\Bigg)\Bigg)$$

$$+c_{OW}\Bigg((b-1)\mu\Big(\frac{dt_1^3}{6}-\frac{rdt_1^4}{8}+\frac{\lambda dt_1^4}{12}\Big)-\mu\Big(\frac{t_1^2}{2}-\frac{T^2}{2}-\frac{t_3^2}{2}-\frac{rt_1^3}{6}-\frac{rt_3^3}{3}-\frac{\lambda T^3}{6}\Big)$$

$$+W\Big((t_3-t_1)-\frac{r}{2}(t_3^2-t_1^2)+\frac{\theta t_1^3}{2}\Big)\Bigg)+c_D\Bigg(\Bigg(\theta\mu\Bigg((b-1)\Big(\frac{dt_1^4}{8}+\frac{d\lambda t_1^5}{15}+\frac{drt_1^5}{10}\Big)$$

$$-\Big(\frac{t_1^3}{3}+\frac{\lambda t_1^4}{8}+\frac{rt_1^4}{4}\Big)\Bigg)+W\theta\Big(\frac{t_3^2}{2}-\frac{t_1^2}{2}+\frac{\theta t_3^4}{8}-\frac{\theta t_1^4}{8}-\frac{rt_3^3}{3}+\frac{rt_1^3}{3}\Big)\Bigg)+\theta\mu\Bigg(\Big(\frac{-t_3^3}{6}-\frac{T^3}{3}\Big)$$

$$-r\Big(\frac{-t_3^4}{12}-\frac{T^4}{4}\Big)+\frac{\lambda r}{2}\Big(\frac{-2t_3^5}{15}-\frac{T^5}{5}\Big)\Bigg)+\mu^2\alpha^2\beta^2\Bigg(\Bigg((b-1)d\Big(\frac{t_2^{\beta+2}}{2(\beta+2)}+\frac{t_1^{\beta+2}}{2\beta(\beta+2)}\Big)$$

$$-\Big(\frac{t_2^{\beta+1}}{\beta+1}+\frac{t_1^{\beta+1}}{\beta(\beta+1)}\Big)\Bigg)-r\Big(\frac{t_2^{\beta+1}}{(\beta+2)}+\frac{t_3^{\beta+2}}{(\beta+1)(\beta+2)}\Big)\Bigg)\Bigg)\Bigg)$$

$$+cI_C\mu\Bigg(\Big(\frac{T^2}{2}+\frac{rT^3}{6}-TM+\frac{rTM^2}{2}+\frac{M^2}{2}-\frac{rM^3}{3}\Big)-\frac{\mu\theta}{2}\Big(\frac{T^4}{12}-\frac{rT^5}{20}-\frac{TM^3}{3}+\frac{rTM^4}{4}+\frac{M^4}{4}-\frac{rM^5}{5}\Big)$$

$$+\frac{\mu\lambda}{2}\Big(\frac{2T^3}{3}-\frac{rT^4}{4}-T^2M+\frac{rT^2M^2}{2}+\frac{M^3}{3}-\frac{rM^4}{4}\Big)-\frac{\mu\lambda\theta}{4}\Big(\frac{2T^5}{15}-\frac{rT^6}{12}-\frac{T^2M^3}{3}+\frac{rT^2M^4}{4}+\frac{M^5}{5}-\frac{rM^6}{6}\Big)\Bigg)$$

$$-\mu sI_e\Big(\frac{M^2}{2}-(r-\lambda)\frac{M^3}{3}+\frac{\lambda rM^4}{4}\Big)$$

$$TC_3=\frac{1}{T}\Bigg(c_Se^{-rt}+c_{RW}\Bigg(\mu\Bigg((b-1)\Bigg(\frac{d}{2}\Big(\frac{t_2^3}{3}+\frac{2t_1^3}{3}\Big)-\frac{dr}{3}\Big(\frac{t_2^4}{4}+\frac{t_1^4}{2}\Big)+\frac{d\lambda}{3}\Big(\frac{t_2^4}{4}+\frac{3t_1^4}{4}\Big)-\frac{rd\lambda}{3}\Big(\frac{t_2^5}{5}+\frac{3t_1^5}{10}\Big)\Bigg)\Bigg)$$

$$-\Bigg(\Big(\frac{t_1^2}{2}+\frac{t_3^2}{2}\Big)-r\Big(\frac{t_1^3}{6}+\frac{t_3^3}{6}\Big)+\frac{\lambda}{2}\Big(\frac{2t_1^3}{3}+\frac{2t_3^3}{3}\Big)-\frac{\lambda r}{2}\Big(\frac{t_1^4}{4}+\frac{t_3^4}{2}\Big)\Bigg)\Bigg)\Bigg)$$

$$+c_{OW}\Bigg((b-1)\mu\Big(\frac{dt_1^3}{6}-\frac{rdt_1^4}{8}+\frac{\lambda dt_1^4}{12}\Big)-\mu\Big(\frac{t_1^2}{2}-\frac{T^2}{2}-\frac{t_3^2}{2}-\frac{rt_1^3}{6}-\frac{rt_3^3}{3}-\frac{\lambda T^3}{6}\Big)$$

$$+W\Big((t_3-t_1)-\frac{r}{2}(t_3^2-t_1^2)+\frac{\theta t_1^3}{2}\Big)\Bigg)+c_D\Bigg(\Bigg(\theta\mu\Bigg((b-1)\Big(\frac{dt_1^4}{8}+\frac{d\lambda t_1^5}{15}+\frac{drt_1^5}{10}\Big)$$

$$-\Big(\frac{t_1^3}{3}+\frac{\lambda t_1^4}{8}+\frac{rt_1^4}{4}\Big)\Bigg)+W\theta\Big(\frac{t_3^2}{2}-\frac{t_1^2}{2}+\frac{\theta t_3^4}{8}-\frac{\theta t_1^4}{8}-\frac{rt_3^3}{3}+\frac{rt_1^3}{3}\Big)\Bigg)+\theta\mu\Bigg(\Big(\frac{-t_3^3}{6}-\frac{T^3}{3}\Big)$$

$$-r\Big(\frac{-t_3^4}{12}-\frac{T^4}{4}\Big)+\frac{\lambda r}{2}\Big(\frac{-2t_3^5}{15}-\frac{T^5}{5}\Big)\Bigg)+\mu^2\alpha^2\beta^2\Bigg(\Bigg((b-1)d\Big(\frac{t_2^{\beta+2}}{2(\beta+2)}+\frac{t_1^{\beta+2}}{2\beta(\beta+2)}\Big)$$

$$-\Big(\frac{t_2^{\beta+1}}{\beta+1}+\frac{t_1^{\beta+1}}{\beta(\beta+1)}\Big)\Bigg)-r\Big(\frac{t_2^{\beta+1}}{(\beta+2)}+\frac{t_3^{\beta+2}}{(\beta+1)(\beta+2)}\Big)\Bigg)\Bigg)\Bigg)$$

$$-sI_e\mu\Big(\frac{T^2}{2}-(r-\lambda)\frac{T^3}{3}+\lambda r\frac{T^4}{4}\Big)+\mu e^{\lambda t}sI_eT(M-T)$$

In the next, our object is to determine the optimal values of t_3^*,T^* such that $TC(t_3,T)$ is minimum. The necessary conditions for TC_1 to be minimized are:

TC_1 to be minimized are:

$$\frac{\partial TC_1}{\partial t_3}=0,\frac{\partial TC_1}{\partial T}=0$$

Let t_3^{1*}, T_1^* be the solution of above equation s, $H_1(t_3^*,T^*)$ be the Hessian matrix of TC_1evaluated at $t_3^{1*}$and T_1^*.It is known , if this matrix is positive definite, then the solution of (t_3^{1*}, T_1^*) is an optimal solution. Since

$$\frac{\partial^2TC_1}{\partial t_3^2}\succ 0,\frac{\partial^2TC_1}{\partial T^2}\succ 0,\frac{\partial^2TC_1}{\partial t_3^2}\frac{\partial^2TC_1}{\partial T^2}-\frac{\partial^2TC_1}{\partial t_3\partial T}\frac{\partial^2TC_1}{\partial T\partial t_3}\succ 0$$

It was noted that the matrix $H_1(t_3^{1*},T_1^*)$ is positive definite and (t_3^{1*},T_1^*) is the optimal solution of TC_1.Similarly,it was noted that (t_3^{2*},T_2^*)band (t_3^{3*},T_3^*) is the required optimal solution of TC_2 and TC_3 respectively. So, numerical solution of t_3^i (i=1, 2, 3).In the next, an optimization algorithm is presented to find the optimal solution of our model.

Algorithm

Step 0. Input initial parameters, 0. Input initial parameters.

Step 1.Find the optimal solution for Case 1, getting the optimal solution t31* and T1* .If $M\le t_3^{1*}\prec T_1^*$,
Let t3*= t31*, T* = T1*, TC*= TC1(t31*, T1*) ; Otherwise, go to Step 2.

Step 2. Find the optimal solution for Case 2, getting the optimal solution t32* and T2* .If $t_3^{2*}\prec M\le T_2^*$, Let t3*= t32*, T*=T2*, TC* = TC2(t32*, T2*) ; Otherwise, go to Step 3.

Step 3. Find the optimal solution of Case 3, getting the optimal solution t33* and T3* .If $t_3^{3*}\prec M\le T_3^*$,Let t3*= t33*, T *= T3*, TC* = TC3 (t33*, T3*); Otherwise, go to Step 4.

Step 4. Let (t*3,T*) = arg min{ TC1(t31*, T1*), TC2(t32*, T2*), TC3(t33*, T3*)},output the optimal t3*, T*, TC*.

Numerical example

To illustrate the above model described, we considered the following data on the basis of the previous study.

Let c_S =10, θ=0.01, r =.01, λ = 2, μ=50, CD =2, IC = 0.15, M = 1.45, α = 0.06, β = 1, d = 0.10, t1= 0.5, t2 = 0.6, b = 0.5, cRW = 5, W =500, cOW =4, c =1000, s=100, Ie = 0.13 in appropriate unit. According to the above algorithm, it can be found the optimal solutions t3*= 0.8, T^*=1.40162 and TC^*=184.827.

A simple economic interpretation is that if the permissible delay period increases, then the retailer receives a higher value of benefit from the permissible delay and shortens the cycle which minimizes the total cost.

Sensitivity analysis

To study the effect of change of the parameter, sensitivity analysis is performed considering the numerical example given above .Sensitivity analysis is performed by changing the parameters and taking one parameter at a time, taking the remaining parameter at original value on the basis of data given in example. The results are shown in Table for permissible delay in payment (Trade credit) by using software Mathematica 5.2.

Sensitivity Analysis Table

Parameter	% change	T^*	TC^*
M	1.46	**1.40669**	178.327
	1.47	**1.41179**	171.827
	1.48	**1.41694**	165.327
	1.49	**1.42212**	158.827
α	0.08	**1.40162**	181.901
	0.12	**1.40162**	173.571
	0.18	**1.40161**	154.811
	0.20	**1.40161**	146.89
β	4	**1.40162**	188.255
	8	**1.40162**	188.56
	10	**1.40162**	188.574
	12	**1.40162**	190.574
C_S	12	**1.40442**	186.827
	13	**1.40582**	187.827
	14	**1.40722**	188.827
	15	**1.41556**	189.827
μ	51	**1.07986**	168.435
	52	**1.27592**	152.04
	53	**1.23003**	135.641
	54	**1.21806**	119.24
C_{OW}	4.1	**1.422**	215.723
	4.2	**1.46177**	246.618
	4.3	**1.5003**	277.513
	4.4	**1.60932**	524.676
C_{RW}	5.1	**1.40075**	184.658
	5.2	**1.40031**	184.489
	5.3	**1.39987**	184.519
	5.4	**1.399z**	184.601
W	550	**1.48298**	284.573
	600	**1.6321**	384.319
	650	**1.69823**	484.065
	700	**1.76412**	583.81

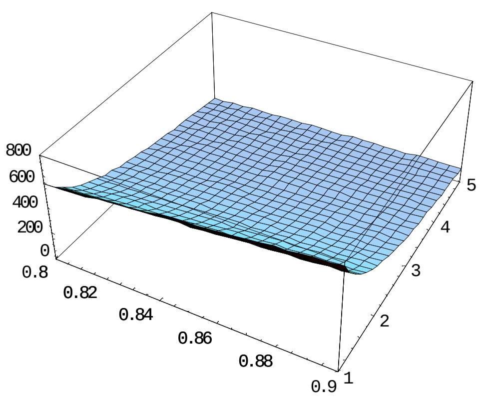

Fig 2 : Convexity of total cost with respect to t3 * and T*

From the above table, following inferences can be observed:

(1) When retailer's warehouse capacity W is increasing, the optimal replenishment cycle time T* and relevant total costs TC* increases. This implies that the retailer can order quantity less frequent to reduce costs when the retailer owns larger storage space.
(2) As the order cost cS is increasing, both the optimal replenishment cycle time T* and the relevant total costs TC* increases. It means that the retailer may order more quantity to reduce the average total relevant costs.
(3) It can be found that the optimal cycle time will decrease when the initial demand rate μ is increasing. It means that the retailer will order less quantity to take the benefits of the trade credit more frequently.
(4) On increasing the deterioration parameter for O.W, then T* increases, TC* decreases.
(5) On increasing the deterioration parameter for R.W, total cost decreases. So, the total cost is minimum when the rate of deterioration for O.W. is less than the deterioration rate for R.W.

V. CONCLUSION

In this paper, a two-warehouse imperfect production inventory model is developed for a manufacturing system with deteriorating items having time varying demand patterns .We have considered Wei-bull distribution deterioration .The effect of permissible delay in payment ,inflation and time value of money is also considered. In order to reduce the inventory costs, it will be economical to consume the goods of R.W at the earliest. Consequently, the firm should store goods in O.W before R.W, but clear the stocks in R.W before O.W. Our aim is to find the optimal replenishment policies for minimizing the total cost. Numerical examples are also provided to illustrate the proposed model. Moreover, sensitivity analysis of the optimal solutions with respect to major parameters is carried out. The proposed model can be extended in several ways, firstly, we may extend the model by allowing shortages and partial backlogging with time-dependent backlogging rate, and secondly, we could extend the model by applying fuzzy approach, which eases the difficulties in searching for suitable probability distribution function. Thirdly, we could generalize the model under two-level credit period strategy.

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