Pursuing Problems in Growth

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Abstract

This paper explores the theoretical implications for Schmookler's (1966) argument that a key determinant of technological change is the usefulness of new technologies. There is both historical and empirical support for his argument. The analysis implies that on-going growth depends delicately on a tension between uses for solutions to technological problems and the allocation of resources toward pursuing those solutions. Even alongside an endogenously increasing number of problems pursued, increasing research labor need not increase technology growth or per capita income growth. The results provide reconciliation of stylized facts regarding technological change and growth in the US and Western Europe.

Keywords: Technological Change, Growth, Schmookler

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1 Introduction

In *Invention and Economic Growth*, Schmookler (1966) proposed that the usefulness of new solutions to technological problems will determine the extent to which those problems are both identified and solutions are pursued. Schmookler's insights have influenced the literature on the economic history of technological change. An empirical literature attempts to clarify and determine empirical support for Schmookler's argument. Significantly, one literature where Schmookler's thesis has not been explored is that of theoretical growth. Rather, the focus of that literature has been on theories of supply-induced technological change[1]. This paper explores the theoretical implications of Schmookler's thesis of demand side-induced technological change.

Schmookler's theory is important to consider not only because it is a viable argument, but also because it is quite distinct from the thesis that the supply of previous innovations is a key input into the production of future innovations. At the center of the model's results is an equilibrium tension between uses for new technologies and the allocation of resources to invent the new technologies. Formalizing Schmookler's argument allows for explanation of how positive population growth and the increasing proportion of labor that is allocated toward research can occur without corresponding increases in technology growth and per capita income growth.

Schmookler begins by proposing that the pursuit of solutions to technological problems is not stimulated primarily by previous knowledge or previous inventions. He uses as evidence 934 inventions between 1800-1957 in agriculture, petroleum refining, paper making, and railroads. He concludes that there was not an evident relationship between the existence of knowledge, or old science, and the production of those inventions. A related study of the relationship between major inventions and minor inventions shows little support for the hypothesis that major inventions induce minor ones.

Schmookler therefore examines the argument that economic considerations constrain the number of technological problems deemed useful and thus pursued. The number of technological problems pursued may be driven by the usefulness or cost of innovation. If the number is driven by cost, then the consideration is still essentially supply-side. Only if the number is primarily driven by the usefulness of innovation is the demand side going to play a key role in the mechanism determining the technological problems to try to solve. Schmookler argues that it is usefulness, rather than cost, that plays the key role. Several case studies provide support for this argument, including one of railroad inventions[2] (measured by patents) from 1845-1950 in the US. Railroad inventions were on an upward trend over the period until about 1910. Inventions peaked at that time and then the trend was sharply downward. Schmookler points out that 1910 also marks the point when construction of the railroad network in the US was completed, and argues that it was the change in railroad capital investment that led to a change in inventive activity. More generally, Schmookler concludes that the number of problems pursued will be an increasing function of the level of physical capital investment in the sector that will use the solutions, or new technologies.

It is exactly the frameworks that Schmookler rejects - that past knowledge induces further invention and that cost of invention determines the extent of innovative activity - that have been the focus of R&D-based models of endogenous growth, such as those developed by Kortum (1997), Romer (1990), Olsson (2000), and Weitzman (1998). In each of these models, the existing supply of knowledge ultimately determines the rate at which new technologies can be produced.

In Kortum (1997), solutions to technological problems occur with some probability, but, in order to be patentable, they must surpass the quality of existing solutions. In this way, the derived rate of patenting solutions is decreasing in the stock of existing knowledge. How those problems are identified is exogenous to the model; it is generation of solutions that is the focus of that analysis.

A similar focus exists in Romer (1990). In that model, existing knowledge is used as an input into the generation of new technologies. Existing knowledge enters linearly into the technology production function of Romer (1990), and this assumption is reconsidered in Jones (2002) by allowing decreasing returns.

Olsson's (2000) and Weitzman's (1998) models, like the one developed in this paper, describe a process determining the number of new technological problems pursued. The process is supply-induced and appeals to a recombination of old solutions to generate new problems. This process can be interpreted as a more detailed specification of the mechanism in Romer (1990). The re-combinatory mechanism can lead to an increasing flow of new problems. This may not lead to increasing growth in Weitzman's model because the *cost* of transforming those problems into new innovations is increasing in the number of new problems. Thus, as in Kortum (1997), finding patentable solutions to identified problems endogenously becomes more difficult over time.

This paper's model, in contrast to others, describes a demand side-induced process for the choice of technological problems in the spirit of Schmookler (1966). In this paper, as well as in Kortum (1997) and Weitzman (1998), the research sector is modeled using the notion that an idea is turned into an innovation with some probability less than one. An idea is defined as a well-defined problem to which researchers have a potential solution. It is argued that there is a real and important distinction between ideas and the stock of technology. Problems to be solved are not the same as the stock of technology. What determines whether a problem is solved is whether it is useful or not in market production, and then whether it is profitable to solve. What will constrain growth in this model will not be the probability of solving the problem. Rather, it will be the endogenous determination of the number of technological problems pursued in the general equilibrium. The usefulness of ideas is critical for on-going growth.

Schmookler and other authors have called this approach a 'demand-pull' theory of technological change, as opposed to a 'supply-push' theory. The term 'demand' must be used carefully, however, because the demand for technology ought, in a general equilibrium setting, to determine its price. In this paper's model, there is a clear and consistent distinction between the usefulness of new technology and the demand for technology. The usefulness of new technology is determined by capital investment levels in the sector that will consume the technology. Demand for technology will imply a price of new technology, and that price will determine the allocation of labor resources to research. The usefulness of innovation does not determine resource allocation, but is a mechanism for generating a set of technological problems to be pursued. Although both are integral to the analysis of paper's model, usefulness of innovation is the focus of Schmookler's theory and that of this paper.

To be clear, the argument here is *not* that supply-side influences on technology production are not important. Rather, this author's view is that arguments for supply and demand side influences on technological change are complementary, and both deserve serious attention. This view is consistent with Rosenberg's [27](1974) critique of Schmookler's hypothesis. The reality of how technological problems are identified, chosen to be pursued, and are solved by the creativity and toil of individuals is extremely complicated. This paper does not try to unravel all of these issues. It instead focuses on an element of technology production that has not been examined by the literature of R&D-based models of endogenous growth: the usefulness of new technologies is determined by the sector demanding them. The results of this model are compared with those of others, to see how robust earlier conclusions are and to provide additional insight into the process of technological change and its effect on income growth.

Mokyr (1990) cautions against trying to explain technological change using exogenous changes in demand or using explanations that imply a symmetry between demand and supply of technology. These types of explanations are simplistic and not a part of this paper. The effect of usefulness of technology on the supply of technology is treated in a dynamic general equilibrium model with endogenous capital accumulation and endogenous technological change. The relationship between capital investment and technological change is a delicate one and evolves in the model. There is asymmetry between demand and supply of technology in the model. Necessary for technological change will be not only the identification and pursuance of technological problems, but also researchers who work on these problems, and their ability to do so. The number of researchers and their experience will be determined endogenously.

A key implication of this paper's analysis is that at the heart of the determination of the rate of technological change is the tension between allocating labor toward research and labor toward manufacturing. An increase in the size of the research sector, though it increases the probability that any given technological problem is solved, has an impact on the future productivity of the research sector via its impact on the level of manufacturing labor. The research sector exists ultimately in order to increase the productivity of other sectors of the economy. If the equilibrium induces an increase in the size of the research sector, this increase is at the expense of the rest of the economy. While the increasing resources toward research can lead to on-going - even increasing - technological change, eventually technology growth and per capita income growth will increase more slowly and then decline.

2 Support for Technological Problems as a Source of Growth

Empirical work that examines Schmookler's thesis is briefly reviewed next. It is then argued, using several examples, that support for his insight is found in subsequent analyses of the economic history of technological change.

2.1 Empirical Support

To examine the empirical relevance of his proposed relationship between the demand for technology and technological change, Schmookler tried formalizing his idea in three related ways. Attention is given throughout the analysis in the paper to the three formulations to determine how their implications differ.

Schmookler's first exercise was to regress the log of patents over a three year period (1940-42 or 1948-50) on the log of past capital investment (1939 or 1947). The data is across manufacturing industries, and patents are classified by the industry in which they were used. To repeat, a capital investment measure is meant to capture the demand for pursuance of technological problems. For the second and third formulations, he argues that industry size may affect the level of patenting (and investment) across industries. In the second formulation he controls for it by including a variable for the log of manufacturing employment (1939 or 1947), and in the third formulation he controls for it using the log of patenting from previous periods[3] (1937-39 or 1945-47). The relationship between investment and patenting is statistically significant in all three regressions. In all three, the sum of the estimated coefficients is almost exactly one and all are of the expected positive sign[4]. These results are meant to provide support for Schmookler's hypothesis regarding the role of technology demand on the supply of technology.

There is a small body of follow-on research that re-examines Schmookler's hypothesis using more recent data, as well as more sophisticated models and econometric techniques. The finding of these studies is that the spirit of Schmookler's hypothesis survives, in that capital investment appears to be a significant predictor of innovative activity. This relationship appears to be subject to diminishing returns.

The study that most closely follows Schmookler is that of Scherer (1982). Using a sample of 443 large US corporations from the mid-1970's, Scherer finds that there is a statistically significant relationship between the log of patents and the log of lagged investment, but he finds the estimated coefficient is less than one[5], in the range of 0.44-0.70. Scherer concludes that "the main thrust of his theory survives." Studies by Jaffe (1988) and Wyatt (1986) also report results consistent with Schmookler's hypothesis.

Schmookler's argument is concerned with long-run investment trends and the effect on innovation activity. A related, though distinct, issue is the effect of productivity shocks on growth and innovation. There are theoretical arguments in favor of both pro-cyclical and counter-cyclical movements in productivity growth (see Aghion and Howitt (1998) Chapter 8 and Geroski and Walters (1995)). However, focusing particularly on how a productivity shock would affect R&D investment, theory suggests that R&D would vary pro-cyclically (Aghion and Howitt (1993)). An argument laid out by Stiglitz (1993) is that capital market imperfections lead credit-constrained firms to vary R&D procyclically.

In support of such an argument is an empirical study by Geroski and Walters (1995). They examine the relationship between changes in economic activity and changes in innovation activity over 1948-1983 in the UK. Granger causality tests suggest that variations in economic activity (measured by output) cause changes in innovative activity. They also find evidence of a longer term secular relationship between the two variables.

A study by Kleinknecht and Verspagen (1990) attempts to examine Schmookler's hypothesis[6]. They do not reject Schmookler's hypothesis, and, similar to Scherer, suggest that the relationship between economic activity and inventive activity is concave. Their results lead them to emphasize the endogeneity of both innovative and other investment activities. This matter is dealt with naturally in the dynamic general equilibrium setting with an R&D sector of this paper.

The data, model, and results of these papers are summarized in Figure 1.

[Figure 1 here]

2.2 Historical Support

Although Schmookler's thesis is well-known, direct references to Schmookler's insights are not frequent in the literature on the economic history of technological change[7]. Nevertheless, numerous analyses in support of his argument are found in the literature, three of which are presented here. Their common thread is that they argue the decision to pursue technological problems is based on consideration of the usefulness of solutions to those problems.

Nathan Rosenberg and co-authors have developed an argument, through several pieces of work, that technological change is responsive to the economic environment in which the inventions occur and that it is fruitful to consider technological change in this way (Landau and Rosenberg (1992), Mowery and Rosenberg (1990), Rosenberg (1994)). One illustration of this argument comes from the chemicals industry in the United States in the twentieth century. Their argument can be interpreted as follows: the United States had a natural resource base that provided stimulus for considering uses for these resources[8]. Moreover, the United States constituted a market of a size and affluence to use innovations for the petrochemical industry, particularly with the diffusion of the automobile into consumer society after World War I. The result was American dominance in the chemicals industry, and particularly chemicals research, over much of this period[9].

Many of the important inventions of the industrial revolution were not based on scientific knowledge, but were largely the result of continual effort by craftsmen and artisans who worked to solve technological problems. These problems had sometimes been well-known for most of human history, such as the problem of spinning in the production of textiles (Mokyr (1990)). So the crucial issue here is not how these problems were identified. It is not even how previous knowledge was used in inventions such as those related to the spinning wheel, although certainly dissemination of previous inventions contributed to the know-how for follow-on inventions. Rather, the question whose answer is most elusive is why these spinning wheel inventions occurred when and where they did - eighteenth century Britain? Certainly the existence of craftsmen and artisans who had the skill and standard of living to work on such problems was necessary. But another element was the existence of a market for such inventions; this market consisted of the producers of textiles who would invest in these spinning wheels.

Moving back even further in time, consider a final example of the invention of movable type printing. Printing was invented in China in the ninth century but did not lead to the enormous output of books that occurred in Europe when printing was reinvented in the fifteenth century (Landes (1998)). Treating these two inventions as separate[10], an important question is why printing led to a book industry in Europe but not in China. The answer provided by Landes (1998) lies in what happened *before* Gutenberg's bible was published in Europe. During the Middle Ages demand for written documents increased with the establishment of government administration and bureaucracy. Authority was fragmented, and the documents produced for these authorities tended to be in the vernacular rather than in Latin. The production of these documents opened the door to interest in other sorts of reading as well, such as the bible, so that the notion of a book, and interest in producing reading material generally, was established before the invention of movable type. Other methods of producing reading material had been tried in order to allow more than one person to read a document at a time, such as binding manuscripts so that they could be split into several pieces. Production of books was insufficient to respond to the end user demand. The previous investment in manuscripts and books, spurred on by the end user demand for such items, provided incentive to look for a way of printing that would allow many copies of a manuscript to be printed in a way less laborious than copying them by hand or using block printing. There was pent-up demand for the technology that Gutenberg finally invented to more easily produce multiple copies of reading material.

3 A Model of Innovation

As described in the Introduction, the number of technological problems $H(t)$, hereafter called *ideas*, that are identified and pursued will depend on the usefulness of innovation.

Following Schmookler's (1966) empirical framework described in Section 2.1, three formulations for $H(t)$ are considered. For simplicity, $H(t)$ will be assumed to exhibit constant returns to scale in its inputs in all three cases[11]. In all three formulations, $H(t)$ will depend on physical capital investment, $\dot{K}(t)$. In the first formulation, $H(t)$ will depend only on $\dot{K}(t)$. In the second case $H(t)$ will depend on $\dot{K}(t)$ and the level of technology, $A(t)$. In the third formulation $H(t)$ will depend on $\dot{K}(t)$ and $L_Y(t)$. Formally the first formulation is:

$$H(t) = \Omega \dot{K}(t) \tag{F1}$$

the second formulation is

$$H(t) = \Omega \dot{K}(t)^{\Upsilon} A(t)^{1-\Upsilon}, \ \Upsilon \in (0,1) \tag{F2}$$

and the third formulation[12] is:

$$H(t) = \Omega \dot{K}(t)^{\Gamma} L_Y(t)^{1-\Gamma} \ \ \Gamma \in (0,1) \tag{F3}$$

where $L_Y(t)$ is labor input into production of the final good, $Y(t)$.

Ideas $H(t)$ will be treated by firms as an externality; each firm's input is negligibly small and so will not affect any decisions in the model except for research labor allocation. In particular, it will not affect production of capital goods.

This theoretical framework departs from Schmookler's empirical framework in two ways but remains consistent with Schmookler's theoretical discussion. First, Schmookler's theoretical hypothesis is that the number of *technological problems* to which solutions are sought is dependent on the usefulness of innovations; this will be the case in the model here. Note that this link is distinct from a dependence of realized innovations - or solutions - on use for innovations. Such a distinction is important in the context of the model - as will be seen in the expression for innovations (1) below. Schmookler, however, did use realized innovation data - patents - as the dependent variable in his regressions. Second, Schmookler's data uses each industry as an observation. In the Romer (1990) model innovations are interpreted as the creation of a new industry, and formally it will be overall capital investment that is the determinant of the number of pursued technological problems. Schmookler's hypothesis can be interpreted more broadly to include the exact structure of the model[13].

The dynamic general equilibrium model used is close to Romer (1990), whereby innovation is conceptualized as an increase in the variety of capital goods. The distinguishing feature here is the formalization of the research sector. Research labor will attempt to innovate, using ideas that have been identified and are being pursued. Those ideas will be turned into innovations with some probability, depending on the number of persons working in the research sector, as well as the time per idea and experience they use in the innovation effort. Innovation at time t is formalized as[14]:

$$\begin{aligned} \dot{A}(t) &= H(t)\left[1-(1-v(t))^{L_R(t)}\right] \qquad (1) \\ v(t) &= \theta\left[\frac{1}{H(t)}\right]^{\frac{1}{S(t)}}. \end{aligned}$$

The intuition for (1) is as follows. There are $H(t)$ ideas pursued in the economy at time t, $H(t) \in (0, \infty)$. All ideas are *ex ante* considered to be of the same quality. Each worker in the research sector at time t, who together number $L_R(t)$, may achieve successful innovation with respect to a single idea with probability $v(t)$. Researchers are assumed symmetric. The probability of innovation $v(t)$ will depend positively on some exogenous productivity parameter $\theta < 1$. The probability will also depend positively on the overall economy's research experience $S(t)$, where as $S(t) \to 0$ then $v(t) \to 0$, and as $S(t) \to \infty$ then $v(t) \to \theta$. It is through this research experience that spillovers between researchers would occur, reflecting the overall level of technical expertise in the economy.

The probability $v(t)$ also depends on the portion of a research laborer's time in period t spent on trying to turn each idea into an innovation. It is possible for researchers to work on more than one idea at a time. From the point of view of maximizing the level of innovation, it is best for researchers to each work on all available ideas generated[15]. Therefore, this is what is assumed here. In particular, each researcher is assumed to devote the fraction $\frac{1}{H(t)}$ of her time to each idea in time t. If $v(t)$ were to be adjusted to not include the variable $H(t)$, the main results of the paper would remain. However, it seems appropriate to include the fact that someone engaged in innovation may choose to be involved with several ideas at once, and this necessarily limits the time and attention given to any one idea[16].

The economy's overall level of research experience $S(t)$ is a function of the previous level of research, subject to some depreciation. That is,

$$\dot{S}(t) = -\varepsilon S(t) + \delta L_R(t) \tag{2}$$

where ε is the rate of depreciation and δ is the rate at which the existence of research labor contributes to research experience. The expression (2) captures two features about the accumulation of experience. First, experience is subject to obsolescence as the technology frontier advances. Second, the existence of technical expertise is dependent for its existence on research labor. Expertise can be interpreted as akin to learning by doing, which is normally assumed in the literature to be accidental, non-rival, and non-excludable[17].

The overall population $L(t)$ will consist of workers in the research sector, $L_R(t)$, and workers in the production of final output, $L_Y(t)$. Overall population grows at some rate $l \geq 0$. Labor-market clearing requires that $L(t) = L_R(t) + L_Y(t)$. This condition implies that wages will be equal across the two sectors in equilibrium at each point in time.

If the probability of each research laborer successfully turning an idea into an innovation is $v(t)$, then the overall probability of that idea being turned into an innovation at least once at time t is $\left[1 - (1 - v(t))^{L_R(t)}\right]$. This overall probability per idea times the number of ideas $H(t)$ yields innovation.

The remainder of the model closely follows that of Romer (1990), and is therefore placed in an appendix. The general equilibrium model implies two expressions that will be used in following analysis. Algebraic manipulation of wage equalization across sectors yields:

$$\frac{H(t)}{A(t)}\left[1 - (1 - v(t))^{L_R(t)}\right] = \frac{\xi\Phi}{P_A(t)}\frac{L_R(t)}{L_Y(t)}\left[L_Y(t)\right]^{\Phi}\bar{x}(t)^{1-\Phi} \tag{3}$$

where ξ is an exogenous productivity parameter in final good production, Φ is the share of labor in final good production, $P_A(t)$ is the price of an innovation paid to the research sector, and $\bar{x}(t)$ is the equilibrium production of each variety of the capital good in period t. The equilibrium capital stock is given by:

$$K(t) = \eta A(t)\bar{x}(t)$$

where η is the units of foregone consumption it takes to produce one unit of the capital good.

4 Growth

The main objective is to analyze how the innovation growth rate is determined and evolves. Steady state growth will be examined first. The results of simulations of equilibrium dynamics are then described.

4.1 Examining Steady State Growth

The analysis considers a baseline case of positive population growth and a steady state balanced growth equilibrium. It is established whether such an equilibrium is consistent with the model for each of the formulations of $H(t)$, $(F1)$, $(F2)$, and $(F3)$. Where appropriate, parallels are drawn between these results and those of other models.

A balanced growth equilibrium implies that $g = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} = \frac{\dot{C}(t)}{C(t)} = \frac{\dot{A}(t)}{A(t)} + \frac{\dot{L}_Y(t)}{L_Y(t)}$, where $\frac{\dot{Y}(t)}{Y(t)}$ is income growth, $\frac{\dot{K}(t)}{K(t)}$ is capital stock growth, $\frac{\dot{C}(t)}{C(t)}$ is consumption growth, and $\frac{\dot{A}(t)}{A(t)}$ is technology growth. This income growth rate g must be constant, and thus an equilibrium outcome is examined where the growth rate of manufacturing labor, $\frac{\dot{L}_Y}{L_Y} = l_Y$, is constant if and only if there is a constant growth rate of innovation g_A under wage equalization so that $g = g_A + l_Y$.

From the wage equalization expression (3) it is possible to deduce that $\frac{\dot{A}}{A}$ must be constant under the balanced growth assumption. From the right hand side of (3), $\Phi \frac{L_R(t)}{L_Y(t)} \left(\frac{L_Y(t)}{\bar{x}(t)}\right)^{\Phi}$ will be constant. Thus the growth rate of $L_Y(t)$ equals that of $L_R(t)$: $\frac{\dot{L}_Y}{L_Y} = \frac{\dot{L}_R}{L_R} = l$. The expression $\frac{\bar{x}(t)}{P_A(t)}$ can also be shown to be constant if l_Y is constant. Thus,

$$g_A = \xi \Phi \frac{L_R(t)}{L_Y(t)} \left[\frac{L_Y(t)}{\bar{x}(t)}\right]^{\Phi} \frac{\bar{x}(t)}{P_A(t)}$$

will be constant.

It is now necessary to examine conditions under which the innovation process is consistent with a constant growth rate:

$$g_A = \frac{H(t)}{A(t)}\left[1-(1-v(t))^{L_R(t)}\right]. \tag{4}$$

Note first the probability term, $\left[1-(1-v(t))^{L_R(t)}\right]$. Because it has been determined that $L(t)$ is increasing at rate l, it will either be the case that the overall probability converges toward zero, if $v(t)$ converges toward zero, or the overall probability will converge toward one, if $v(t)$ remains positive or converges toward θ. It turns out that in the case where $L_R(t)$ grows at rate l, then it will always be the case that $v(t) \rightarrow \theta$. This is proven as a Lemma below.

Lemma 1 *The probability of a single research innovation converges to its maximum value* θ *:* $v(t) \rightarrow \theta$ *for* $L_R(t)$ *growth at rate* $l > 0$ *and* $H(t) \in (0, \infty)$.

Proof It is noted that $v(t) \rightarrow 0$ only if $S(t) \rightarrow 0$ under the modeling assumption that $H(t) \in (0, \infty)$. Therefore, it is sufficient to show that $S(t)$ is increasing under the conditions of the Lemma.

By (2), the growth of research experience $\frac{\dot{S}(t)}{S(t)}$ is

$$\frac{\dot{S}(t)}{S(t)} = -\varepsilon + \frac{\delta L_R(t)}{S(t)}$$

and if it is positive (negative) that implies $\frac{\delta L_R(t)}{S(t)}$ is declining (increasing). Thus, $\frac{\dot{S}(t)}{S(t)}$ will settle at l, with $\frac{\delta L_R(t)}{S(t)}$ constant.■

Lemma 1 establishes that $\left[1-(1-v(t))^{L_R(t)}\right] \rightarrow 1$ as $L_R(t)$ increases without bound at rate l. The expressions for steady state balanced growth in each formulation are now examined.

Formulation 1.

$$H(t) = \Omega \dot{K}(t)$$

The expression for physical capital investment $\dot{K}(t)$ used in the growth analysis is derived in the appendix:

$$\dot{K}(t) = K(t)[g_A(t) + l_Y] = \eta\Psi_0 A(t) L_Y(t)[g_A(t) + l_Y] \tag{5}$$

where $\Psi_0 = \left[\frac{r\eta}{(1-\Phi)^2}\right]^{-\frac{1}{\Phi}}$. In a steady state equilibrium, the constant growth rate of technology g_A would be

$$g_A = \left[(\Omega\eta\Psi_0 L_Y(t))^{-1} - 1\right]^{-1} l_Y. \tag{6}$$

By (6), g_A and l_Y cannot be constant at the same time unless both are equal to zero since g_A is increasing in $L_Y(t)$. Therefore, a steady state equilibrium with positive population and technology growth rates, and finite population, is not consistent with this formulation of the model.

Formulation 2.

$$H(t) = \Omega\dot{K}(t)^{\Upsilon} A(t)^{1-\Upsilon}$$

In the second formulation, the steady state growth of technology g_A with positive population growth would be given by the expression:

$$g_A\left[1 - \Omega(\eta\Psi_0 l_Y(t))^{\Upsilon} g_A^{\Upsilon-1}\right] = \Omega(\eta\Psi_0 L_Y(t) l_Y)^{\Upsilon}.$$

Again, as in formulation 1, positive constant technology growth is not consistent with positive population growth, and finite population. Also, zero population growth implies a steady state technology growth of zero.

Because of the Schmooklerian idea use mechanism in the model, g_A here depends on manufacturing labor $L_Y(t)$. In general, the rate of technology growth is not necessarily increasing as research labor $L_R(t)$ increases. Indeed, in the steady state above, using the substitution $L_Y(t) = L(t) - L_R(t)$, it is straightforward to show that technology growth is decreasing in $L_R(t)$. This result highlights a tension fundamental to the model: as research labor increases, labor is diverted from manufacturing. Manufacturing, which provides the resources for investment, is the source of technological problems. Thus, even outside of the steady state, an increase in research labor does not unambiguously imply an increase in technology growth, holding all else equal. Further, it will also be seen below that, outside of the steady state, the endogenous level of $L_Y(t)$ is not necessarily increasing, even as the population $L(t)$ increases. In this model, dependence of technology growth on the level of manufacturing labor need not induce explosive growth in equilibrium if $l_Y \neq l_R$.

Formulation 3.

$$H(t) = \Omega \dot{K}(t)^{\Gamma} L_Y(t)^{1-\Gamma}.$$

In the third formulation, steady state growth of technology equals

$$g_A = \frac{l_Y}{1-\Gamma}. \tag{7}$$

The steady state growth rate of technology implied by the third formulation, (7), is a function of the growth rate of manufacturing labor, which in the steady state will equal population growth. This result, on the surface, is similar to that of Jones (2002) and Kortum (1997). The growth rate of technology in the Jones model is a function of the growth rate of research labor rather than manufacturing labor[18]. This type of steady state outcome is not specific to the constant returns setup in the innovation production function ($F3$).

From the foregoing analysis, two key differences between the technology production in this model and those of other R&D-based growth models become clear. First, necessary for the growth rate of technology to be positive is not only the allocation of labor to research, via the probability term in (1), but also the level of manufacturing labor, via $H(t)$. Second, the dependence of technological problems pursued, $H(t)$, on capital investment $\dot{K}(t)$ implies an equilibrium dependence of technological change on previous growth of manufacturing labor and technology itself.

Nevertheless, the balanced growth results of this paper's framework have parallels in the results of the growth models discussed in the introduction. The reason why these parallels exist is that in the balanced growth steady state, the two differences described above are washed away. First, the long run results are not affected by the probability of innovation. The probability of innovation can play an important role in the innovation process except in the very long run. In particular, in the steady state the role for research labor $L_R(t)$ is asymptotically removed. However, because the steady state imposes that $l_Y = l = l_R$, the impact of the level of manufacturing labor L_Y corresponds closely to that of research labor in other models of R&D-based growth. Second, the steady state - when it exists - imposes that l_Y and g_A are constant, so that technology production $\dot{A}(t)$ effectively varies only with levels of A and L_Y, not their growth rates.

A positive steady state growth rate with positive population growth, and finite population, does not exist in two of the three formulations. The discussion of growth is thus incomplete. The purpose of this analysis was to facilitate comparison with existing models. It is necessary to determine the growth process in the general equilibrium without imposing balanced growth. This is done next.

4.2 Equilibrium Dynamics

The equilibrium of each of the three formulations is simulated to examine the path of technology: A; the paths of labor: L, L_R and L_Y; the path of technological problems H; the path of the cost of capital r; and the paths of income, capital, and consumption: Y, K, and C. The details of the structure of the simulations are provided in the appendix. The results are summarized and interpreted here. The paths of key variables from the simulation using the third formulation ($F3$) are plotted in Figures 2-5.

[Figures 2-5 here]

In the figures, it is the range of the transition dynamics in which $\frac{L_R}{L}$ is rising that will be of interest, and the figures focus on that range of the dynamics. If the simulation is continued, the directions in which the variables are moving are sustained, as g_A declines toward a positive steady state with positive population growth[19]. Over the range where $\frac{L_R}{L}$ is rising, the stock of technology A is rising, and g_A is rising at a decreasing rate and then begins to decline. The ratio $\frac{L_R}{L}$ is increasing at a decreasing rate, while the level of manufacturing labor L_Y decreases and then begins to increase. Even after $\frac{L_R}{L}$ begins to decline, L_R still increases if population growth is positive.

The reason why g_A begins to decline is twofold. First, the number of researchers is rising, as is the probability that any one problem is successfully turned into an innovation. However, the probability is increasing at a decreasing rate. Second, the number of technological problems pursued H is increasing[20], but the ratio $\frac{H}{A}$ begins to decline. To see why, note that the endogenous growth rate of manufacturing labor l_Y is the variable on which the analysis turns and is determined by the wage equalization condition, which is non-linear in l_Y. Equilibrium capital investment, which determines H, is a function of the growth rate of technology and the growth rate of manufacturing labor in equilibrium as in (5). While l_Y is positive in this range, there is intensive growth of capital. However, as g_A is declining, extensive growth of capital declines. Overall, the number of problems pursued declines relative to the stock of technology.

The three models produce similar patterns in their transition dynamics, which may initially seem surprising. The only difference in the three models is in the function determining the production of ideas; in formulations 2 and 3 H depends on the scale of the economy via either the technology stock A (formulation 2) or the size of manufacturing labor L_Y (formulation 3). These differences do not qualitatively affect the dynamics of the model because they do not significantly affect the growth rate of manufacturing labor via the wage equalization expression (3) . To see why this is the case, note that in equilibrium capital investment, and thus H, is increasing in A and L_Y. The number of technological problems pursued, H, is either linear or concave in A and L_Y. The equilibrium growth rate of technology is thus either constant or increasing in L_Y and either constant or decreasing in A in all three models. Even in formulation 3, where technological change is increasing in L_Y, and in the range of transition dynamics where L_Y is increasing, the equilibrium growth of technology will still decline.

These simulations highlight that on-going growth is not sustained solely by continual allocation of labor into the research sector. The profit from innovation, for a given number of technological problems, is a determinant of the level of research labor L_R. Profit is clearly necessary to provide reward for innovation. Also necessary is the usefulness of innovation, for this determines whether and how many problems are pursued in the first place. Capital investment, which determines the usefulness of innovation, is a function of the growth of production labor as well as the growth of technology. Both useful ideas and research labor are necessary for on-going innovation and the decline of useful ideas relative to the stock of technology, even if there is a rise in the research labor in general equilibrium, can reduce the growth rate of technology. In the simulations described above, though research labor L_R is rising, technology growth g_A is not rising at the same pace and even declines.

4.3 The Reconciliation of Three Stylized Facts

The simulation results are now applied to resolving an empirical puzzle that consists of three stylized facts. These are stylized facts that have been recognized and considered in other R&D-

based models of growth, but how to reconcile these related facts is not well-understood[21]. The findings refer to North America and Europe over the past four decades.

1. While the overall rate of population growth has been low, there has been a reallocation of labor toward the research sector in the form of scientists and engineers (S&E) (see Jones (2002) and Segerstrom (1998)).

2. The growth rate of technology is on average not increasing over time, or at least not at the same rate as the level of research labor has increased (see Jones (2002), Kortum (1997), and Segerstrom (1998)).

3. The growth rate of per capita income has been roughly constant[22] (Jones (2002)).

The first fact, that the growth of research labor has exceeded population growth, is not interpreted as due to the initial establishment of R&D as a distinct market-driven sector of economic activity. Since at least the mid-19th century in the US, the growth of trained labor dedicated to R&D activity has outstripped overall population growth. The transition of scientific activity from being the domain of artisans and craftsmen, whose innovation came in spare time or as part of a production process, to professional scientists and engineers during the 19th century has been documented by Cowan (1997) and Lebergott (1984), and is discussed by Rosenberg (1982). The growth of the research sector in the prewar 20th century, after the transition, is detailed by Mowery and Rosenberg (2000). Therefore, the late-20th century trend of R&D labor growth that outstripped population growth is a phenomenon that cannot be explained by the transition from an amateur to professional scientific community, as that transition was completed by the early part of the 20th century[23]. The first fact is thus recognizable as something to be understood in the context of models of technological change and growth.

On the second fact, there is some potential for debate over the measure of the rate of growth in the knowledge stock. One way of measuring the rate of change in the stock of knowledge is to measure the rate of change in patenting. Looking first at the *flow* of patents, Segerstrom (1998) and Kortum (1997) describe patenting as roughly constant over time. Patenting over time is, however, quite volatile. Recent data on patenting shows a resurgence of patenting in the US (see Kortum and Lerner (1998)). The evidence suggests that the discounted *stock* of previous patents is increasing over time, as described by Kortum (1997). Thus, even if patenting is increasing, the rate of change in patenting could be on average decreasing over time[24].

The use of patents as the measure of technology is open to criticism. Aside from the variability in the level of patenting over time, it has also been widely recognized in the empirical literature on patents that patents cover only a portion of innovations, and a small one at that (see Keely (2001)[13] and references cited therein). Therefore, the use of patents as a measure of the technology stock and flow may be misleading. An alternative measure of the rate of growth in the stock of knowledge is TFP growth. Jones (2002) points out that TFP growth has exhibited a small or no persistent increase in countries with a large increase in scientists and engineers (over 1960-1988). Experiences with TFP growth vary greatly across countries and periods of time. As Edwards (1998) and Wolff (1996) document, TFP growth from 1960-1990 over a set of OECD countries was on average decreasing, though the variance in TFP growth was increasing, with the United States and Japan actually experiencing an increase in average TFP growth over the 1980's relative to the previous decade. Even if one interprets the data to indicate that the rate of growth of the technology stock is increasing, it is reasonable to conclude from the data that it is smaller than the rate of growth of research labor.

Jones (2002), Kortum (1997), and Segerstrom (1998) develop models for understanding the second fact: the disconnect between the rate of growth of research labor and the rate of growth of the knowledge stock. In each of these models, innovation becomes more difficult over time by assumption, or directly due to the accumulation of technology, as opposed to some distinct mechanism that generates the number of ideas pursued. In Jones (2002), the technology stock exhibits decreasing returns in the technology production function. In Kortum (1997) and Segerstrom (1998) the probability of achieving a research goal is decreasing over time. In these models, the increase in research labor is balanced against the increased difficulty of innovation, to yield a constant balanced growth equilibrium technology growth rate. This decreasing probability is also the basis for Weitzman's (1998) result that despite an increase in number of research ideas (explicitly held constant in Kortum (1997)), the rate of technology growth may not be increasing over time. Weitzman's model assumes a fixed labor supply and so there is no mechanism for considering positive population growth or a change in the allocation of labor between sectors.

Finally, regarding the third fact, Jones (2002) presents evidence that income per capita growth has been roughly constant over the past century in the United States. He analyzes the transition dynamics of a model with human capital accumulation to show that per capita income growth can be constant at some point along the transition path, and to reconcile this evidence with the reallocation of labor toward R&D-related activities, the first stylized fact noted above[25].

Turning to the simulations themselves, in a range of the transition, the equilibrium mimics the three stylized facts. As the ratio $\frac{L_R}{L}$ continues to rise but nears leveling off, the growth rate of technology g_A also slows to become roughly constant, and then begins to decline. The number of problems pursued, H, continues to increase, but not at a fast enough pace to cause technology growth to rise. Finally, in this range of the simulation per capita income is rising but the growth rate is fluctuating around a constant, is thus approximately constant.

In the simulation of the equilibrium in each formulation, the growth rate of research labor exceeds the growth rate of total labor, consistent with the first fact. The growth rate of technology is increasing, at least over some time range, but is smaller than research labor growth, consistent with the second fact. An ongoing increase of research labor does not ensure an ongoing increase in technology growth. This is because there must be an ongoing increase in technological problems relative to the technology stock in order for technology growth to increase, and as resources are allocated to research labor and not to manufacturing, overall investment is diminished.

4.4 Is Growth Sustainable?

Formally, the key distinction between this model's formulation and the models described above is the source of the increased difficulty of innovation. Here, the increased difficulty is endogenous and comes from the equilibrium trade-off between allocating labor to research or to manufacturing via the level of capital investment. The number of technological problems pursued relative to the stock of technology is falling. The probability of turning any given idea into an innovation is actually increasing, in contrast to those models described above. An unbounded increase in research labor would *not* lead to an unbounded increase in technological change. In the other models it would.

As stated above, in the model developed here it is possible to understand why three stylized facts can occur alongside each other. In much of the endogenous growth literature, the equilibrium analyzed is a balanced growth equilibrium in which the growth rate of research labor equals the growth rate of total labor. Therefore, the first puzzle cannot be explained by those models. In both this paper and Jones (2002), transition dynamics are examined instead. Jones (2002) provides an an alternative, though complementary, explanation that appeals to the existence of human capital in production of output and technology. As Jones points out in discussion of the model, the determinants of the inputs other than labor into technology production are not yet well understood. This paper delves into that part of the technology production function, to provide an alternative explanation for the phenomena.

As Jones discusses in the context of his model, one feature of this paper's model that is potentially troubling is the zero steady state per capita income growth. While capital accumulation was an important source of growth in the nineteenth century (David (1977)), and technological change was an important source in the twentieth century, one might be troubled by the prospect that a source of growth is not sustainable. This model and that of Jones (2002), on their own, reinforce such fears. If one takes seriously models such as ($F3$) above and Jones (2002), where the steady state growth rate of per capita income depends on population growth, the recent low fertility rates of the U.S and Western Europe are concerning.

However, at least two models suggest that such fears may not be realized. First, Weitzman's (1998) model relies upon increasing cost of innovation to generate non-increasing growth. In his model, new technological problems are derived from the solutions to old technological problems, and thus the ratio of technological problems relative to the stock of technology need not be decreasing. If technological problems are identified via other sources in addition to that of the capital sector, then the level of technological problems relative to the stock of technology may not be decreasing in equilibrium.

Second, a recent study by Dalgaard and Kreiner (2001) models the endogenous formation of human capital accumulation in a model of growth. They show that, even in the absence of population growth, per capita income growth in their model can still be positive in the steady state. Therefore, the increase in human capital accumulation, through education and experience, may provide an avenue for sustained growth[26].

5 Conclusion

Much of the literature on technological change and growth has focused on the supply of science or knowledge that is used in producing new technologies. By contrast, Schmookler (1966) directed his attention to the equally important consideration of the intended uses of new technologies as a determinant of the production of new technologies. This paper has explored the theoretical implications of Schmookler's argument. The general equilibrium trade-off between generating uses for new technologies and committing resources to invent the new technologies is at the crux of the theory's implications. Bounded technological change results with an endogenous allocation of resources toward the research sector, and away from the sectors that will use the research sector's output.

6 Appendix

6.1 Development of the Model

The Model

Final output $Y(t)$ is a function of labor in that sector and the use of $A(t)$ existing varieties of capital goods. The variety of capital goods can also be thought of as the stock of knowledge. Each capital good is indexed by i. Output of the final good is:

$$\begin{aligned} Y(t) &= \xi L_Y(t)^{\Phi} \int_0^{\infty} x(i,t)^{(1-\Phi)}\, di \\ &= \xi L_Y(t)^{\Phi} \int_0^{A(t)} x(i,t)^{(1-\Phi)}\, di. \end{aligned}$$

The profit maximization problem of the representative final good producer is standard. The price of the final good is normalized to one. Profit maximization is used to determine the price $p(i,t)$ of $x(i,t)$:

$$p(i,t) = \xi(1-\Phi) L_Y(t)^{\Phi} x(i,t)^{-\Phi}.$$

The capital stock of the economy is a proportional sum of the capital of each variety:

$$K(t) = \eta \int_0^{A(t)} x(i,t)\, di. \tag{8}$$

Capital accumulation is the result of foregone consumption $C(t)$:

$$\dot{K}(t) = Y(t) - C(t).$$

For each variety of capital good associated with an existing innovation, a monopolist produces that capital good. The monopolist solves its profit maximization problem, with an interest cost of r on ηx units in order to produce x units of its capital good. The resulting quantity of capital good i produced at time t is:

$$x(i,t) = \left[\frac{r(t)\eta}{\xi(1-\Phi)^2 L_Y(t)^{\Phi}}\right]^{-\frac{1}{\Phi}} = \bar{x}(t) \;\; \forall i. \tag{9}$$

The price of each variety, $p(i,t)$, is:

$$p(i,t) = \frac{r(t)\eta}{(1-\Phi)} = \bar{p}(t) \;\; \forall i.$$

The resulting profit of each variety producer $\pi(i,t)$, is:

$$\pi(i,t) = \Phi \bar{p}(t) \bar{x}(t) = \bar{\pi}(t) \;\; \forall i.$$

The price of each innovation, $P_A(t)$ is the present discounted value of the expected future profits from the capital good sector:

$$P_A(t) = \int_t^{\infty} e^{-R(\tau)} \bar{\pi}(\tau)\, d\tau \tag{10}$$

where $R(\tau) = \int_t^\tau -r(s)\,ds$. The innovation price evolves according to:

$$\frac{\dot{P}_A(t)}{P_A(t)} = \frac{-\bar{\pi}(t)}{P_A(t)} + r(t).$$

Consumers maximize intertemporal utility by their choice of consumption path. In particular, their problem is maximize

$$\int_0^\infty e^{-\rho t} U(C(t))\,dt$$

$$U(C(t)) = \ln C(t)$$

subject to the budget constraint:

$$\dot{K}(t) = r(t)K(t) + w(t)L_Y(t) + w(t)L_R(t) - P_A(t)\dot{A}(t) + A(t)\bar{\pi}(t) - C(t).$$

Equilibrium consumption growth is therefore:

$$\frac{\dot{C}(t)}{C(t)} = r(t) - \rho. \tag{11}$$

To determine the effect of equilibrium wage equalization across sectors, note first that in the research sector, all income from innovations will be paid out as wages:

$$P_A(t)H(t)\left[1 - (1 - v(t))^{L_R(t)}\right] = w(t)L_R(t).$$

In the final output sector labor is paid its marginal product:

$$w(t) = \xi\Phi L_Y(t)^{\Phi-1}\bar{x}(t)^{1-\Phi}.$$

These two expressions imply a wage equalization expression:

$$\frac{H(t)}{A(t)}\left[1 - (1 - v(t))^{L_R(t)}\right] = \frac{\xi\Phi}{P_A(t)}\frac{L_R(t)}{L_Y(t)}L_Y(t)^{\Phi}\bar{x}(t)^{1-\Phi} \tag{12}$$

6.2 Simulation Structure

Rewriting the innovation price and wage equalization conditions and using a discrete-time approximation:

$$g_A(t+1) = \frac{\Psi_1}{P_A(t)} [L(t) - L_Y(t)] [r(t)]^{1-\frac{1}{\Phi}}$$

where

$$\Psi_1 = \xi\Phi \left[\frac{\eta}{\xi(1-\Phi)^2}\right]^{1-\frac{1}{\Phi}}$$

and

$$P_A(t+1) = -\Psi_2 L_Y(t)\, r(t)^{1-\frac{1}{\Phi}} + [1 + r(t)]\, P_A(t)$$

where

$$\Psi_2 = \frac{\Phi\eta}{(1-\Phi)} \left[\frac{\eta}{\xi(1-\Phi)^2}\right]^{-\frac{1}{\Phi}}$$

The exogenous evolution of population is given by

$$L(t+1) = L(t)(1+l).$$

Research labor is determined by

$$L_R(t) = L(t) - L_Y(t)$$

where $L_Y(t)$ is determined by an expression below. The evolution of research experience is given by

$$S(t+1) = (1-\varepsilon) S(t) + \delta [L(t) - L_Y(t)].$$

The expressions determining consumption, income, and the capital stock are:

$$Y(t) = \xi A(t) L_Y(t) \left[\frac{\eta}{\xi(1-\Phi)^2}\right]^{1-\frac{1}{\Phi}} r(t)^{1-\frac{1}{\Phi}}$$

$$K(t) = \eta A(t) L_Y(t) \left[\frac{r(t)\eta}{\xi(1-\Phi)^2}\right]^{-\frac{1}{\Phi}}$$

$$C(t+1) = C(t)\left[r(t) - \rho + 1\right]$$

The wage equalization condition is

$$\frac{H(t+1)}{A(t+1)}\left[1 - (1 - v(t+1))^{L(t+1)-L_Y(t+1)}\right] = g_A(t+2)$$

where

$$g_A(t+2) = \frac{\Psi_1}{P_A(t+1)}\left[L(t+1) - L_Y(t+1)\right]\left[r(t+1)\right]^{1-\frac{1}{\Phi}}$$

$$r(t+1) = \left[\frac{K(t) + Y(t) - C(t)}{\eta A(t+1)}\right]^{-\Phi}\left[\frac{\xi(1-\Phi)^2}{\eta}\right]L_Y(t+1)^{\Phi}$$

$$v(t+1) = \theta\left[\frac{1}{H(t+1)}\right]^{\frac{1}{S(t+1)}}$$

and the generation of technological problems will be different for each model. For formulation 1, substituting in for $\dot{K}(t)$ yields:

$$H(t+1) = \Omega\left[Y(t) - C(t)\right]^{\Lambda},\ \Lambda \in (0,1].$$

The exponent Λ is added for generality.

For formulation 2, it is:

$$H(t+1) = \Omega\left[Y(t) - C(t)\right]^{\Upsilon} A(t)^{\Sigma}$$

where $\Sigma \in (0,1)$ is added for flexibility. For formulation 3, substituting in for $\dot{K}(t)$ yields:

$$H(t+1) = \Omega\left[Y(t) - C(t)\right]^{\Gamma} L_Y(t)^{\Pi}$$

where $\Pi \in (0,1)$ is added also for flexibility.

Initial conditions are assumed as follows:

$$C(0) = 50$$

$$r(0) = .03$$

$$
\begin{aligned}
L(0) &= 100 \\
L_Y(0) &= 90 \\
S(0) &= 1 \\
A(0) &= 1000 \\
P_A(0) &= 500
\end{aligned}
$$

Exogenous parameters are assumed as follows:

$$
\begin{aligned}
\varepsilon &= 0.01 \\
\delta &= 0.01 \\
\eta &= 0.1 \\
\Omega &= 5 \\
\Phi &= 0.66 \\
\xi &= 0.5 \\
\Lambda &= 0.2 \\
\Upsilon &= 0.2 \\
\Gamma &= 0.2 \\
\Sigma &= 0.2 \\
\Pi &= 0.2 \\
\rho &= 0.01.
\end{aligned}
$$

7 Notes

[1]See Aghion and Howitt (1998), Grossman and Helpman (1991), Romer (1990), and the following literature. The term 'supply induced' refers to the technology production function, and not to the rate of technological change resulting in the general equilibrium. The rate of technological change in these models, is, of course, determined in part by the demand for technology through the price of an innovation.

[2]Others are inventions in shoe manufacturing, agriculture, construction, and the horseshoe.

[3]Given that the estimated depreciation rate for innovations is about 15% per year, previous patenting or R&D may be appropriately interpreted as a proxy for the stock of patents (see Hall (1993)).

[4]Schmookler then goes on to explore the hypothesis that technological opportunity, distinct from the role of usefulness, varies across industries. He does so by using patent data that is classified by the industry of origin rather than industry of use. In these regressions, the log of lagged sales, assets or employment (in 1955) is used as the independent variable instead of investment. The dependent variable is log patents for one year (in 1959). The estimated coefficient on the independent variable is still not statistically significantly different from one. The R^2 on these regressions is much lower than on the ones with patents classified by industry of use. Schmookler interprets the unexplained part of these regressions as capturing technological opportunity. Section 5 further discusses the role of technological opportunity.

[5]Scherer also reports that the R^2 for his regression is lower than what Schmookler reported (about 0.38 versus about 0.9).

[6]Their paper suffers from significant flaws in attempting to evaluate Schmookler's thesis. These flaws have largely to do with their choice of variables. The choices are not well-motivated and it is

thus not clear how to interpret their results.

[7]An example of explicit discussion of Schmookler's insights is Bairoch (1991).

[8]Mokyr (1990) makes a related general argument supporting the role of physical environment and natural resources on innovative activity.

[9]This American importance is despite earlier British and German preeminance before and in the early part of the twentieth century, and a late-twentieth century rejuvenation of the European chemicals industry.

[10]The evidence is not undisputed but it seems that Gutenberg was not aware that his invention of the technique of movable type had already been invented by the Chinese (Mokyr (1990)).

[11]The later empirical work by Scherer (1982) and that presented in this paper suggests that decreasing returns is more consistent with the data. Maintaining constant returns to scale allows for more tractable analysis, and in either case the spirit of Schmookler's hypothesis is maintained. Constant and decreasing returns were considered when simulating the model's transition dynamics (see Section 4.2 and results of Section 5) and led to similar results.

[12]Schmookler's empirical tests do not support the inclusion of manufacturing employment. This formulation is included for the sake of completeness. In Section 5, a variant of this formulation is tested, using real GDP as a proxy for the size of a country's economy, and the coefficient on that variable is significantly different from zero.

[13]Alternatively, one could respecify the model. An option would be the Aghion and Howitt (1998) model of creative destruction. In some sense, which model of the two models is used is not very important since the formal structures are so similar, even if the interpretations are quite different. The advantage of the Romer framework is increased simplicity of analysis.

[14]This expression is an example of a more general functional form in which $\frac{\partial \dot{A}}{\partial L_R} > 0, \frac{\partial^2 \dot{A}}{\partial L_R^2} < 0$;

$\frac{\partial \dot{A}}{\partial H} > 0$; $\frac{\partial v}{\partial H} < 0, \frac{\partial^2 v}{\partial H^2} > 0$; $\frac{\partial v}{\partial S} > 0, \frac{\partial^2 v}{\partial S^2} < 0$.

[15]This result is described in a previous version of the paper. In that version, a single research firm employing all researchers was able to choose to work on only a fraction of available ideas. It was also allowed to split its researcher between ideas, so that each researcher would work on fewer than the total number of used ideas. However, it was found that if the research firm took the innovation reward as given and so maximized the quantity of expected innovation, then the firm would choose to use all available ideas and to have each researcher work on all of those ideas. The price-taking assumption may not be innocuous. Nevertheless, it seems appropriate to use this result as an approximation for what the individual researcher will choose.

If we take each researcher to be working independently, then the price-taking assumption seems a more appropriate assumption. Each researcher faces a probability of success of $H(t)\,v(t)$, taking all else given. In that case, the worker will choose to work on all available ideas to maximize her individual expected innovation.

[16]The probability term in Weitzman's (1998) model is dependent on a fraction of $\frac{Y}{H}$, where Y is output, as the probability in that model does not have the interpretation of being attached to a single researcher, as it does here.

[17]I thank a referee for suggesting this interpretation. Learning is non-rival by nature, as a type of knowledge (see Romer (1994)). It is frequently assumed to be non-excludable and a by-product of investment or sales (for example, see Aghion and Howitt (1998), Jovanovic (1995), and Young (1991)).

[18]Also, g_A in the Jones model is increasing in the exponent on research labor in the technology production function. Here, g_A is decreasing in the exponent on manufacturing labor.

[19]In formulations 1 and 2, g_A declines toward a zero growth rate.

[20]This result is consistent with the finding that the set of technological problems, or ideas, is not declining (see Klevorick, Levin, Nelson and Winter (1995)).

[21]An alternative framework that focuses on the US experience, using human capital accumulation, is developed in Jones (2002).

[22]Jones (2002) presents and focuses on U.S. data. A check of the 15 EU countries' aggregate per capita income growth for 1950-1991, using the Summers-Heston data (1991), yields a picture similar to that for the U.S.

[23]Although this evidence relates to the US, the interaction between the two continents and the human capital investment in Western Europe allows one to reasonably infer similar conclusions regarding Western Europe.

[24]The flow of patents per researcher would also be decreasing, see Kortum (1997).

[25]His model is also consistent with the second stylized fact, though this is not focused upon in the paper.

[26]The Dalgaard and Kreiner modelling of human capital accumulation differs from that of Jones (2002) in the following, crucial, way. Jones models each persons human capital as:

$$h = \exp\left(\Psi l_h\right)$$

where l_h is the share of labor devoted to accumulating human capital. Human capital is dependent on, for instance, the number of years spent in education. Dalgaard and Kreiner model human capital accumulation as:

$$\dot{h} = \frac{\sigma_H Y}{L} - nh$$

where σ_H is the share of overall output used to produce human capital and n is population growth. Thus, an increase in the quality of education, without the number of years of education increasing, can yield an increase in human capital accumulation. The question of which model of human capital accumulation is more appropriate is an empirical one. Human capital accumulation offers an important potential research avenue, though it is beyond this paper's scope.

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Author	Data	Model	Results
Scherer (1982)	Industry-level data (245 industries) based on 443 large US firms, divided into material goods and capital goods	Dependent variable: 1976-77 Patents Level or log Independent Variable: 1974 Investment Level or log	Coefficient on log variables regressions between .443-.686 and significantly different from zero.
Jaffe (1988)	537 US firms with positive R&D in 1976	Dependent variable: 1976 log of R&D Independent Variables: Log of Sales, Log of Capital Stock, Market share, Log of Industry R&D	Coefficient on Sales is between .877-.98 and significantly different from zero; coefficient on capital stock is not statistically significantly different from zero.
Kleinknecht and Verspagen (1990)	Industry-level data (46 industires) based on Dutch firms with positive R&D	Dependent variable: 1983 R&D man years as a percentage of total manpower per industry Independent variable: Sales growth 1981-83 or 1982-83	Coefficient on Sales growth is .11-.14 and statistically significantly different from zero.
Geroski and Walters (1995)	1948-83 United Kingdom data on firms with quality-weighted innovations and patents in US.	Dependent variable: time-differenced log of innovations Independent variables: time-differenced lagged log of innovations, of output, and of patents	Coefficient on the change in the log of output is 1.2 but statistically insignificant. The other coefficients are less than one.

Figure 1: Summary of Studies of Schmookler Demand-pull hypothesis

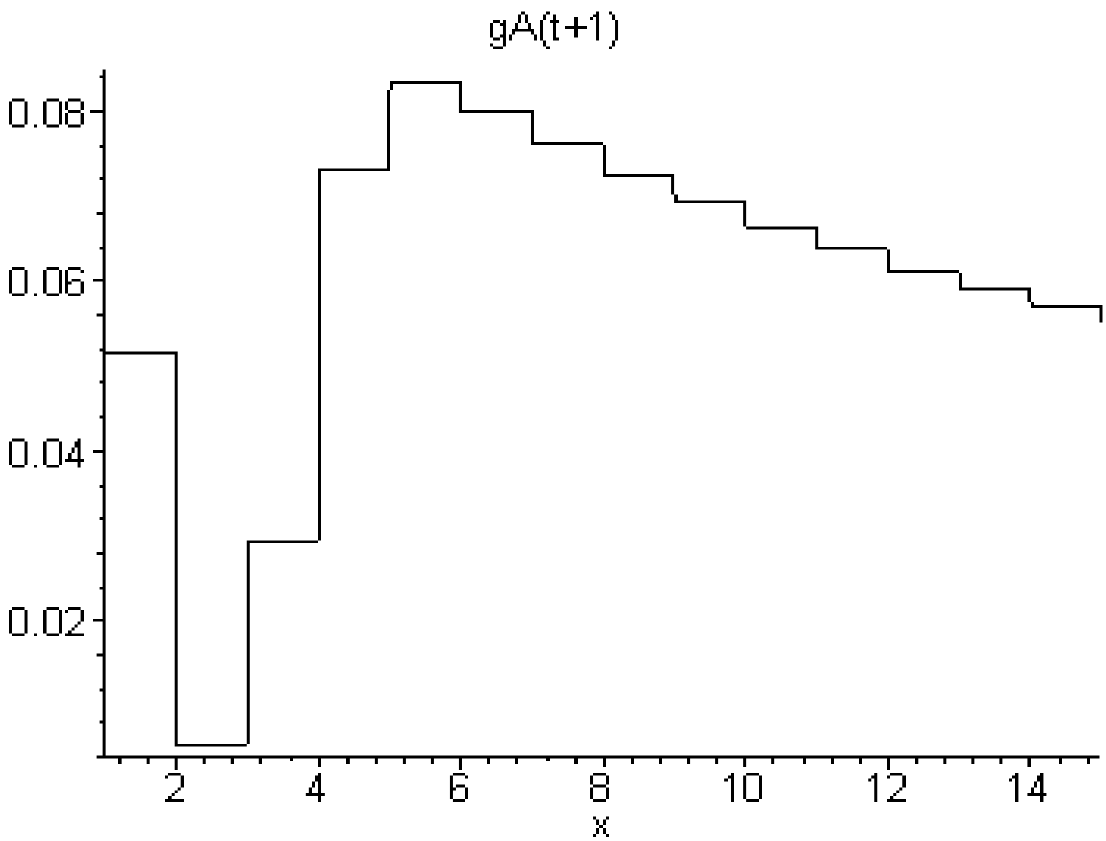

Figure 2: Growth of Technology Stock ; x=time

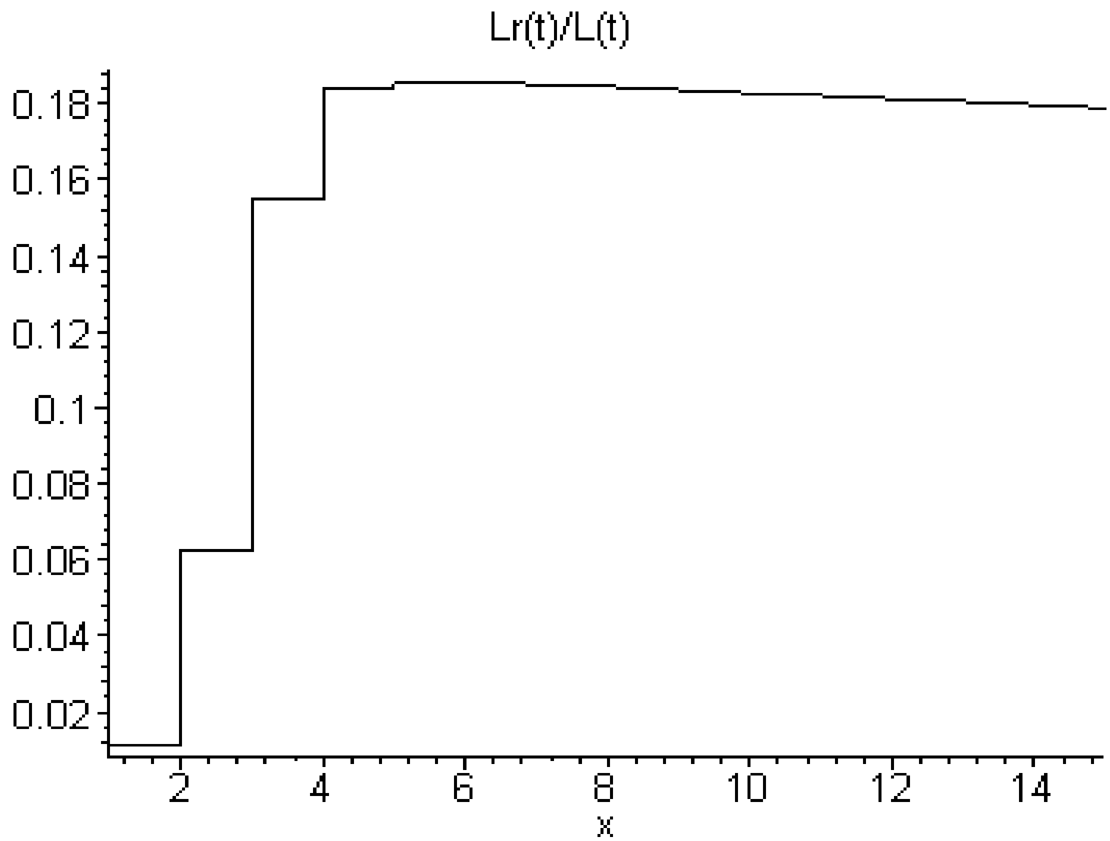

Figure 3: Ratio of Research Labor to Total Labor ; x=time

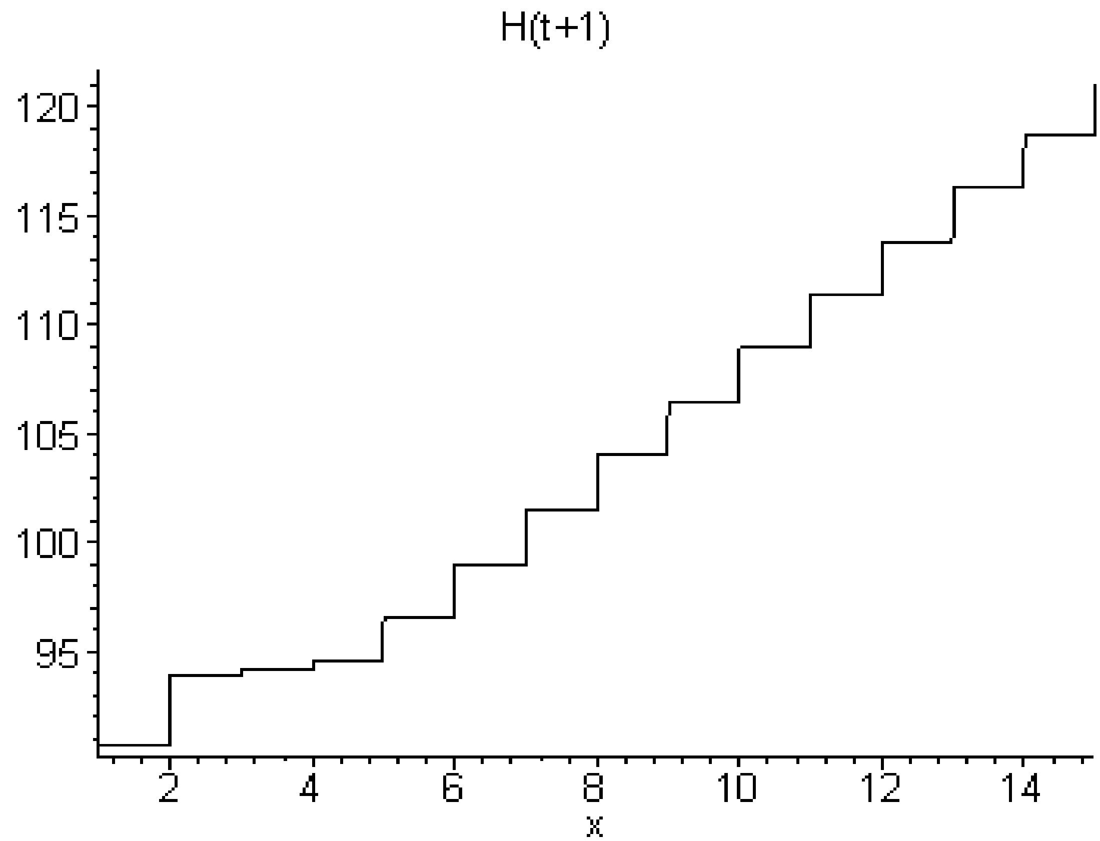

Figure 4: Technological Problems ; x=time

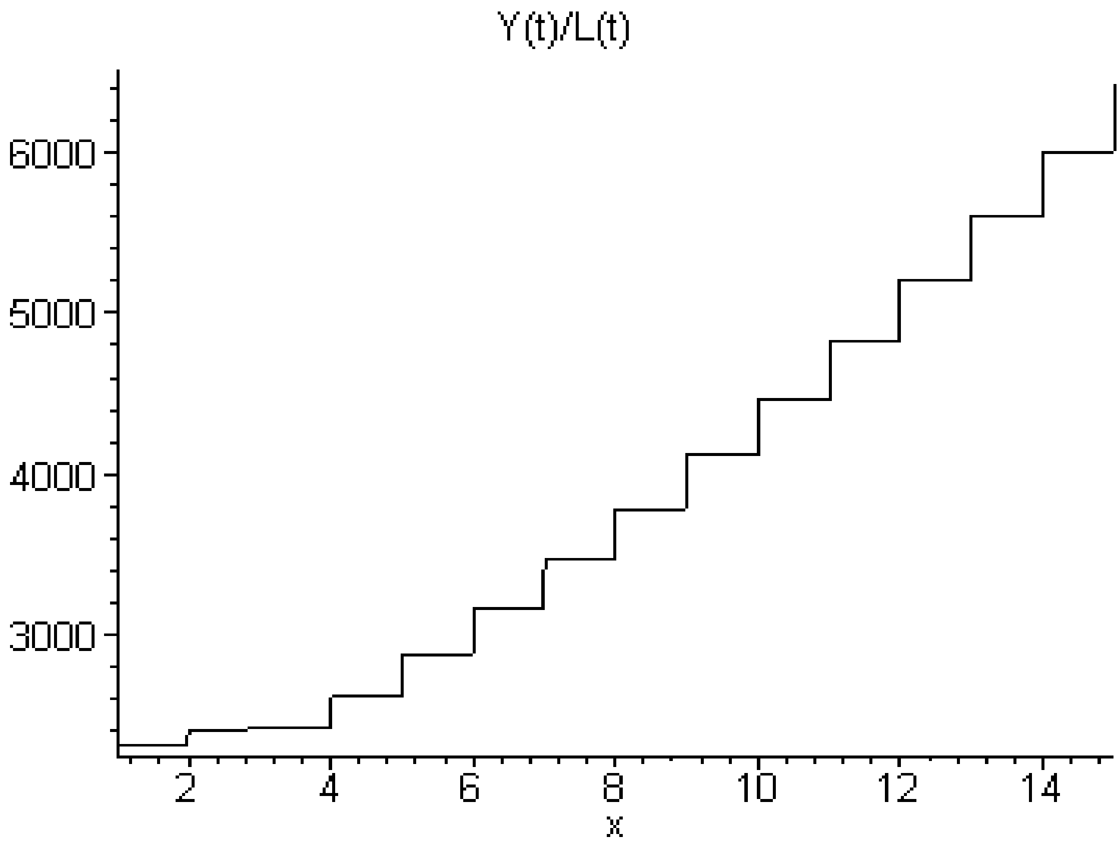

Figure 5: Per Capita Income ; x=time