Electric Load Model Synthesis by Diffusion Approximation of a High-Order Hybrid-State Stochastic System

Written by Roland Malhamé and Chee-Yee Chong, Member, IEEE

Presented by Mark Rosenberg

advised by

Professor Sean Meyn

Lab for Cognition & Control in Complex Systems, University of Florida

August 6, 2014

Abstract (1 of 2)

A statistical approach is used to model the dynamics of the electric demand of large aggregates of electric space heaters or air conditioners.

Abstract (1 of 2)

A statistical approach is used to model the dynamics of the electric demand of large aggregates of electric space heaters or air conditioners.

Why are these choices of loads important?

Abstract (1 of 2)

A statistical approach is used to model the dynamics of the electric demand of large aggregates of electric space heaters or air conditioners.

Why are these choices of loads important?

1. They account for a significant portion of power system dynamics following a power outage.

Abstract (1 of 2)

A statistical approach is used to model the dynamics of the electric demand of large aggregates of electric space heaters or air conditioners.

Why are these choices of loads important?

1. They account for a significant portion of power system dynamics following a power outage.
2. Because they are associated with energy storage, they are often selected for load shedding within a load management program.

Abstract (2 of 2)

- The aggregate electrical dynamics are derived for both homogeneous and nonhomogenous groups of devices.

Abstract (2 of 2)

- The aggregate electrical dynamics are derived for both homogeneous and nonhomogenous groups of devices.
- The homogeneous group aggregate load model is the Fokker-Planck equations.

Abstract (2 of 2)

- The aggregate electrical dynamics are derived for both homogeneous and nonhomogenous groups of devices.
- The homogeneous group aggregate load model is the Fokker-Planck equations. To obtain it:
    1. Model the switching dynamics of individual devices as occurring on a hybrid state Markov system

Abstract (2 of 2)

- The aggregate electrical dynamics are derived for both homogeneous and nonhomogenous groups of devices.
- The homogeneous group aggregate load model is the Fokker-Planck equations. To obtain it:
  1. Model the switching dynamics of individual devices as occurring on a hybrid state Markov system
  2. Use equations describing probability density evolution.

Abstract (2 of 2)

- The aggregate electrical dynamics are derived for both homogeneous and nonhomogenous groups of devices.
- The homogeneous group aggregate load model is the Fokker-Planck equations. To obtain it:
  1. Model the switching dynamics of individual devices as occurring on a hybrid state Markov system
  2. Use equations describing probability density evolution.
- The result might be applicable to general hybrid-state stochastic systems, which occur frequently in relay control and power system reliability.

Abstract (2 of 2)

- The aggregate electrical dynamics are derived for both homogeneous and nonhomogenous groups of devices.
- The homogeneous group aggregate load model is the Fokker-Planck equations. To obtain it:
  1. Model the switching dynamics of individual devices as occurring on a hybrid state Markov system
  2. Use equations describing probability density evolution.
- The result might be applicable to general hybrid-state stochastic systems, which occur frequently in relay control and power system reliability.
- Simulation results illustrate dynamical properties of this model.

Introduction (1 of 1)

- The goal of a power system is to provide electric power reliably, economically, and within certain voltage and frequency constraints, thus modeling load dynamics is important.

Introduction (1 of 1)

- The goal of a power system is to provide electric power reliably, economically, and within certain voltage and frequency constraints, thus modeling load dynamics is important.
- Common models of load dynamics:
  1. Empirical load modeling using regression analysis on past load data at the system level.
  2. Physically based load modeling using aggregation of individual component demands to obtain load at given point in system. (More costly, complex, but justifiable for load management).

Introduction (1 of 1)

- The goal of a power system is to provide electric power reliably, economically, and within certain voltage and frequency constraints, thus modeling load dynamics is important.
- Common models of load dynamics:
  1. Empirical load modeling using regression analysis on past load data at the system level.
  2. Physically based load modeling using aggregation of individual component demands to obtain load at given point in system. (More costly, complex, but justifiable for load management).
- This paper's approach is the latter, where the switching dynamics depend on the on/off load management control. The goal is to evaluate whether load management results in delayed, larger demand peaks.

Model (1 of 2)

- Hybrid-state model has continuous state for temperature and discrete state for switching mechanism.
  - Continuous State

$$Cdx(t) = -a'(x(t) - x_a(t))dt + R'm(t)b(t)dt + dv'(t) \quad (1)$$

Where,

- C is average thermal capacity of house
- a' is average heat loss rate
- $x(t)$ is average temperature of house
- $x_a(t)$ is ambient temperature
- R' is rate of heat gain
- $m(t)$ is operating state (1 for on, 0 for off)
- $v'(t)$ is a noise process
- $b(t)$ is the control state of power supply (1 for on, 0 for off)

Model (2 of 2)

- Discrete State

$$m(t+\Delta t) = m(t) + \pi(x(t); x_+, x_-) \quad (2)$$

where,

$$\pi(x, m; x_+, x_-) = \begin{cases} 0 & : x_- < x < x_+ \\ -m & : x \geq x_+ \\ 1-m & : x \leq x_- \end{cases} \quad (3)$$

Aggregation

Two assumptions

1. Geographical proximity: $x_a(t)$ is the same for every load.

Aggregation

Two assumptions

1. Geographical proximity: $x_a(t)$ is the same for every load.
2. Elemental independence: The loads evolve independently from each other.

Aggregation

Two assumptions

1. Geographical proximity: $x_a(t)$ is the same for every load.
2. Elemental independence: The loads evolve independently from each other.

Homogeneous Case

- Same storage, switching dynamics, and control, $b(t)$

$$P(t) = R' b(t) \sum_{i=1}^{N} m_i(t) = R' b(t) N \tilde{m}(t) \tag{4}$$

Aggregation

Two assumptions

1. Geographical proximity: $x_a(t)$ is the same for every load.
2. Elemental independence: The loads evolve independently from each other.

Homogeneous Case

- Same storage, switching dynamics, and control, $b(t)$

$$P(t) = R'b(t)\sum_{i=1}^{N} m_i(t) = R'b(t)N\tilde{m}(t) \tag{4}$$

From Kolgomorov's strong law of large numbers

$$\tilde{m}(t) \triangleq \frac{1}{N}\sum_{i=1}^{N} m_i(t) \to \mathbb{E}_w(m(t)|\zeta), a.e. \tag{5}$$

where $\tilde{m}(t)$ is the aggregate operating state, w is the weather information, and ζ contains all parameters that characterize the device dynamics in the homogenous group.

Main Result (1 of 3)

Two hybrid densities are defined:

$$f_1(\lambda, t)d\lambda = \Pr\left[(\lambda < x(t) \leq \lambda + d\lambda) \cap (m(t) = 1)\right] \tag{6}$$

$$f_0(\lambda, t)d\lambda = \Pr\left[(\lambda < x(t) \leq \lambda + d\lambda) \cap (m(t) = 0)\right] \tag{7}$$

Main Result (1 of 3)

Two hybrid densities are defined:

$$f_1(\lambda, t)d\lambda = \Pr\left[(\lambda < x(t) \leq \lambda + d\lambda) \cap (m(t) = 1)\right] \tag{6}$$

$$f_0(\lambda, t)d\lambda = \Pr\left[(\lambda < x(t) \leq \lambda + d\lambda) \cap (m(t) = 0)\right] \tag{7}$$

Theorem (The Coupled Fokker-Planck Equations):

- The hybrid-state probability densities $f_1(\lambda, t)$, $f_0(\lambda, t)$ for this system satisfy the following system of Fokker-Planck equations

$$T^1_{\lambda,t}\left[f_1(\lambda, t)\right] = 0 \tag{8}$$

in regions a, b of Fig.1 and

$$T^0_{\lambda,t}\left[f_0(\lambda, t)\right] = 0 \tag{9}$$

in regions b, c of Fig.1, where

$$T^k_{\lambda,t}\left[f\right] = \frac{\partial f}{\partial t} - \frac{\partial}{\partial \lambda}\left[(a(\lambda - x_a(t)) - kb(t)R)f\right] - \frac{\sigma^2}{2}\frac{\partial^2}{\partial \lambda^2}f, k = 0, 1. \tag{10}$$

Main Result (2 of 3)

The state space:

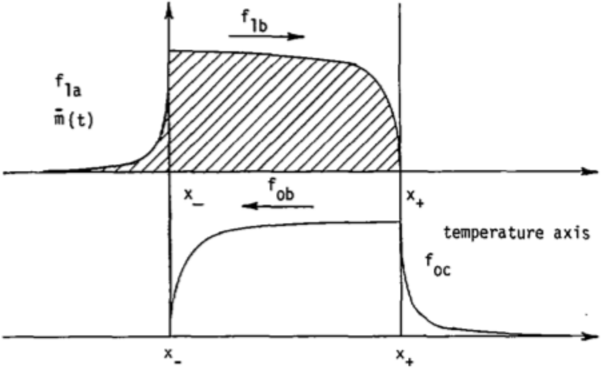

Fig. 1.

Main Result (3 of 3)

Boundary Equations

Absorbing Boundaries:

$$f_{1b}(x_+,\ t) = f_{0b}(x_-,\ t) = 0.$$

Conditions at Infinity:

$$f_{1a}(-\infty,\ t) = f_{0c}(+\infty,\ t) = 0.$$

Continuity Conditions:

$$f_{1a}(x_-,\ t) = f_{1b}(x_-,\ t)$$

$$f_{0b}(x_+,\ t) = f_{0c}(x_+,\ t).$$

Probability Conservation:

$$-\frac{\partial}{\partial\lambda} f_{1a}(x_-,\ t) + \frac{\partial}{\partial\lambda} f_{1b}(x_-,\ t) + \frac{\partial}{\partial\lambda} f_{0b}(x_-,\ t) = 0$$

$$\frac{\partial}{\partial\lambda} f_{0c}(x_+,\ t) - \frac{\partial}{\partial\lambda} f_{0b}(x_+,\ t) - \frac{\partial}{\partial\lambda} f_{1b}(x_+,\ t) = 0.$$

Proof of Fokker-Planck Equations (1 of 3)

Begin with Chapman-Komogorov Equations as in the original derivation of Fokker-Planck Equations for Markov diffusion processes

$$f_{ij}(\lambda', t', \lambda, t) = \sum_{k=0}^{1} \int_{-\infty}^{+\infty} f_{ik}(\lambda', t', z, \tau) f_{kj}(z, \tau, \lambda, t) dz \qquad (11)$$

for $i = 0, 1, j = 0, 1$ and any $\tau \in (t', t)$, where transition probability density functions are

$$\begin{aligned} f_{ij}(\lambda', t', \lambda, t) d\lambda = \Pr \big[&(\lambda < x(t) \leq \lambda + d\lambda) \cap (m(t) = j) | x(t') \\ &= \lambda', m(t') = i \big] \end{aligned} \qquad (12)$$

Proof of Fokker-Planck Equations (2 of 3)

- The partial time derivative satisfies:

$$\lim_{h \to 0} \int_{x_- + \epsilon}^{x_+} \frac{f_{11}(\lambda', t', \lambda, t+h) - f_{11}(\lambda', t', \lambda, t)}{h} R(\lambda)\, d\lambda$$

$$= \int_{x_- + \epsilon}^{x_+} \frac{\partial}{\partial t} f_{11}(\lambda', t', \lambda, t+h) R(\lambda)\, d\lambda.$$

- Given that τ is a minimum temperature transition time,

$$\tau_{11r}^{\lambda,\lambda'} = \inf \{(t' - t) : x(t') = \lambda',\ m(t') = 1 | x(t) = \lambda,\ m(t) = 1\}$$

it's more likely the load will have a temperature transition to the same discrete state without switching back and forth in the discrete states.

$$\int_{x_- + \epsilon}^{x_+} \int_{-\infty}^{+\infty} f_{10}(\lambda', t', z, t) f_{01}(z, t, \lambda, t+h)\, dz\, d\lambda$$

$$\leq \Pr [\tau_{11r}^{x_-, x_- + \epsilon} \leq h]$$

Proof of Fokker-Planck Equations (3 of 3)

- The limiting case resembles a one-dimensional Markov diffusion process.

$$\lim_{h \to 0} \frac{1}{h} \int_{x_- + \epsilon}^{x_-} \int_{-\infty}^{+\infty} f_{10}(\lambda' t', z, t) \cdot f_{01}(z, t, \lambda, t+h) R(\lambda) \, dz \, d\lambda = 0.$$

Therefore, the remaining steps follow the standard derivation of the Fokker-Planck equations of a one-dimensional Markov diffusion process, and are omitted. The result is

$$T^{1}_{\lambda,t} \left[f_{11}(\lambda', t', \lambda, t) \right] = 0. \quad (13)$$

Proof of Fokker-Planck Equations (3 of 3)

- The limiting case resembles a one-dimensional Markov diffusion process.

$$\lim_{h \to 0} \frac{1}{h} \int_{x-+\epsilon}^{x-} \int_{-\infty}^{+\infty} f_{10}(\lambda' t', z, t) \cdot f_{01}(z, t, \lambda, t+h) R(\lambda) \, dz \, d\lambda = 0.$$

Therefore, the remaining steps follow the standard derivation of the Fokker-Planck equations of a one-dimensional Markov diffusion process, and are omitted. The result is

$$T^1_{\lambda,t} \left[f_{11}(\lambda', t', \lambda, t) \right] = 0. \tag{13}$$

Similarly, by starting with f_{01}, we obtain

$$T^1_{\lambda,t} \left[f_{01}(\lambda', t', \lambda, t) \right] = 0. \tag{14}$$

Proof of Fokker-Planck Equations (3 of 3)

- The limiting case resembles a one-dimensional Markov diffusion process.

$$\lim_{h \to 0} \frac{1}{h} \int_{x-+\epsilon}^{x_-} \int_{-\infty}^{+\infty} f_{10}(\lambda' t', z, t) \cdot f_{01}(z, t, \lambda, t+h) R(\lambda) \, dz \, d\lambda = 0.$$

Therefore, the remaining steps follow the standard derivation of the Fokker-Planck equations of a one-dimensional Markov diffusion process, and are omitted. The result is

$$T^1_{\lambda,t}\left[f_{11}(\lambda', t', \lambda, t)\right] = 0. \tag{13}$$

Similarly, by starting with f_{01}, we obtain

$$T^1_{\lambda,t}\left[f_{01}(\lambda', t', \lambda, t)\right] = 0. \tag{14}$$

Multiplying each by another function, adding, and integrating, yields the desired result, completing the proof.

Justification of Boundary Equations

- The first equation follows by definition of an absorbing state.

Justification of Boundary Equations

- The first equation follows by definition of an absorbing state.
- The second equation follows from integrability of a probability density.

Justification of Boundary Equations

- The first equation follows by definition of an absorbing state.
- The second equation follows from integrability of a probability density.
- The third and fourth equations follow from continuity.

Justification of Boundary Equations

- The first equation follows by definition of an absorbing state.
- The second equation follows from integrability of a probability density.
- The third and fourth equations follow from continuity.
- The final two boundary equations arise from using the Fokker-Planck equations and constraining the probability density rates at the boundaries such that the rate from one section to an adjacent section is the same on each side. (Conservation of probability).

Nonhomogeneous Case

- The nonhomogeneous case is characterized by devices with different energy storage and switching dynamics, but the same control signal.

Nonhomogeneous Case

- The nonhomogeneous case is characterized by devices with different energy storage and switching dynamics, but the same control signal.
- Now the parameter ζ is not a constant vector, but rather a random vector with a mean vector and covariance matrix.

Nonhomogeneous Case

- The nonhomogeneous case is characterized by devices with different energy storage and switching dynamics, but the same control signal.
- Now the parameter ζ is not a constant vector, but rather a random vector with a mean vector and covariance matrix.
- From a perturbation analysis, we generate the approximation:

$$\bar{m}(t) \approx \bar{m}(t, \underline{\zeta}_0) + \sum_{i=1}^{P} \sum_{j=1}^{P} \frac{1}{2} \frac{\partial^2}{\partial \zeta_i \partial \zeta_j} \bar{m}(t, \underline{\zeta}) \Big|_{\underline{\zeta}=\underline{\zeta}_0} \sigma_{ij}^2 .$$

Nonhomogeneous Case

- The nonhomogeneous case is characterized by devices with different energy storage and switching dynamics, but the same control signal.
- Now the parameter ζ is not a constant vector, but rather a random vector with a mean vector and covariance matrix.
- From a perturbation analysis, we generate the approximation:

$$\tilde{m}(t) \approx \tilde{m}(t, \underline{\zeta}_0) + \sum_{i=1}^{P} \sum_{j=1}^{P} \frac{1}{2} \frac{\partial^2}{\partial \zeta_i \partial \zeta_j} \tilde{m}(t, \underline{\zeta}) \Big|_{\zeta = \zeta_0}^{\sigma_{ij}^2} .$$

- If the approximation is poor, the nonhomogeneous group can be split into two or more groups with less variation.

Numerical Results (1 of 3)

The following parameters were used to run simulations.

$$\Delta = 1.1 \ \text{deg} \ c,$$

$$\frac{x_a}{\Delta}(t) = 15$$

$$\frac{a}{\Delta} = 0.01774 \ (\text{deg} \ c \ \text{min})^{-1}$$

$$\frac{R}{\Delta} = 0.4 \ (\text{min}^{-1})$$

$$\bar{\sigma} = \frac{\sigma}{\Delta} = 0.3 \ (\text{min})^{-1/2}.$$

Numerical Results (2 of 3)

After simulations with varying noise variances reached steady state, the control signal turned them off for a brief period. The behavior when the control was turned on is in Fig 4.

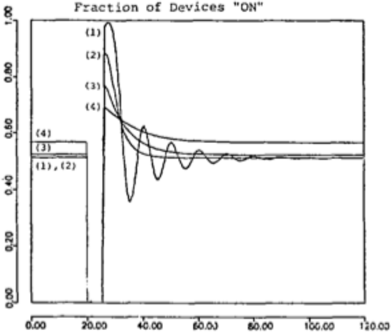

Fig. 4.

Numerical Results (3 of 3)

The behavior with increasing times where the control is off is in Fig 5. The peaks are larger with more delay.

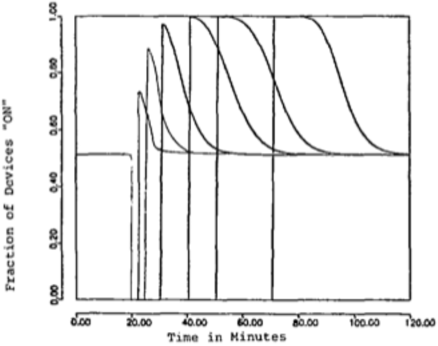

Fig. 5.

Conclusion

- Using the Fokker-Planck equations, a model for the switching dynamics of large aggregates of electric space heaters within a load management system has been presented.

Conclusion

- Using the Fokker-Planck equations, a model for the switching dynamics of large aggregates of electric space heaters within a load management system has been presented.
- The aggregate model is exact in the limiting statistical sense.

Conclusion

- Using the Fokker-Planck equations, a model for the switching dynamics of large aggregates of electric space heaters within a load management system has been presented.
- The aggregate model is exact in the limiting statistical sense.
- The result of the main theorem appears applicable to more general hybrid-state stochastic systems.

Any questions?