

The law of perceptual stability: Abstract foundations*

(perception/vision/behavior/cognition/grammar)

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ABSTRACT Confronted with an object of perception, an individual will spontaneously try to identify unambiguously and consistently all its parts; except in rare instances of “illusory phenomena,” he will immediately succeed. This elementary fact is formalized in a law of visual perception. It is used to define sets of stable states for a sensory mode of a biological system. As characterized, stable states are to perception as quantum states are to atomic structure: they represent natural states of physical systems. They are shown to be observable and to have an exact mathematical representation.

A class of bounded open subsets of a two-dimensional Euclidean space, whose boundaries are piecewise compact analytic arcs, is used to construct a nontrivial mathematical model for stable states. The finitely many components of this mathematical model of a stable state (*image*) are mapped onto an object of perception (*icon*) by perceptual judgments. These judgments, which include the judgment of stability, have an exact interpretation in this model. They unify and make precise such traditional notions of psychology as “*Gestalt*,” “figure-ground,” and “(visual) boundary.”

Postulates for a general theory of perception are given. They are used to establish a formal relationship between biological and subjective studies of sensory phenomena and so provide a framework in which subjective studies can be used to analyze (their associated) biological processes. In applying these methods to cases, all icons are divided into two classes (the *static* and *dynamic* cases). The static case is treated.

1. Introduction

1.1. The law of perceptual stability, applied to human vision, has a simple intuitive form: perception occurs if and only if an individual stabilizes his field of vision or, equivalently, if he identifies all its parts in an unambiguous and mutually consistent way.^a There is a mathematical form for the stability law which is universally valid (see 3.6). This fact and its theoretical and practical consequences lead to the conjecture that the stability principle expresses a natural law (see ref. 1) that governs the dependency of a biological system and its sensory environment and which human vision can be shown to obey. Effects of the stability law can be directly observed in the common experience of perception. They account for ambiguous perception. And the stability law confirms the psychological hypothesis that there exist abstract conditions, defined for the entire field of vision, that constrain the independent perception of its parts.^b

To give an exact statement of the stability law, it was necessary to construct a new formal and conceptual framework for the mathematics and method that are used to investigate perception. This framework, (*stability theory*) unifies the study of perception. It rests on the discovery of a simple topological

characterization (see 2.6) of an observer's judgment of stability (see 3.3 and ref. 3) of his field of vision. The validity and weight of the stability law depend on the fact that the stability judgment is as strong, universal, and unambiguous as any perceptual judgment known.

1.2. The mathematics and method required to state and apply the stability law will be developed and verified in four stages.

(i) A new mathematical object is defined by using elementary properties of the Euclidean plane, which gives an *analytic model* for stability.

(ii) Six postulates for a theory of perception are stated; and from these postulates a *method of formal pairs* is described by means of which mathematical structures can be assigned to perceptual judgments.

(iii) It is demonstrated that the method is valid by showing that it applies exactly and universally to cases (objects of perception) and, thus, that the stability law is well defined.

(iv) Grounds for validity of the stability law are given, followed by some immediate effects of the law on the way we study both perception and the corresponding biology of sensation and cognition.

1.3. Cases to which stability theory can always be applied definitively and without qualification—cases to which perceptual judgments can in practice be applied unambiguously so that the model is well defined—are subject to several conditions. All perceptual judgments rendered by an observer of his visible surround are assumed to have been rendered by the observer of a reference object which, relative to the observer-frame, is a wholly contained, proper part (not equal to the whole) of the observer's field of vision and is fixed, static, flat, and without discernible holes. Eye movement, which is otherwise unrestricted, is defined relative to the observer-frame. An observer's head and body are always taken to define the fixed rigid spatiotemporal frame of reference. Physical orientation of the observer (with respect to visible body parts and the

Abbreviations: S(D), stable decomposition; OBD, oriented boundary decomposition.

* This work will appear in two parts; the second part will be in a subsequent issue.

^a As a simple illustration, the three obvious, distinct, stable, and mutually incompatible ways of viewing Necker's line drawing of a cube (two distinct readings in space and one in the plane) can be characterized by three distinct exact decompositions of the (flat) field of vision into independently and compatibly identified parts. The statement can be taken to define “perception.”

^b Stated in traditional terms, the stability phenomena obviously accounted for by this law include figure-ground relationships, multiple stability, *Gestalt* effects, and cognitive contours. The account is independent of metrical conditions, which are not discussed here. This work is self-contained and independent of standard accounts, which will be considered separately. See, for example, Gibson (2), pp. 310–318.

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physiological effects of gravity and motion) is defined relative to the observer-frame. These conditions define the *static theory*, which is the framework we now adopt. Unrestricted cases can be treated by refining the static theory and thus extending its scope.

The distinction of monocular, binocular, and cyclopean vision is irrelevant to the stability law for the static cases. It may be easily observed that the stability law holds as well for binocular as monocular perception, although it is somewhat easier and less abstractly understood by imagining a model for monocular perception. These distinctions can be interpreted in the extended theory.

2. The analytic model for stability

2.1. The mathematics of stability theory can be derived from several sources. We have chosen to use elementary properties of the Euclidean plane because of their inherent mathematical richness and intuitive accessibility and because they allow us to demonstrate the existence of a nontrivial model in a well-studied context, further justification for which depends on work in progress. In this model three classes of elementary objects are defined from the primitive structure of the plane, in whose composite forms we find mathematical counterparts to perceptual judgments.

Before giving the mathematical details, let us consider briefly a rough nontechnical description of the analytic model for stability. In simple terms, this model gives a particular way of decomposing an unbroken region of the plane into distinct subregions. The (topological) boundary of each subregion is itself broken into parts (arcs). And each of the arcs of a boundary is oriented. Orientation of an arc is marked by a single arrow pointing into one of the regions adjacent to the arc according to the rule: in going around a boundary, the arrow directions alternate.

In this model, our three classes of mathematical elements are a set of regions in the plane (which need not be unbroken), a set of oriented boundary arcs, and sets of isolated points, which are used to mark the places on boundaries where orientation changes. The exact decomposition of an unbroken region D in the plane into a (finite) collection of these elements is a *stable decomposition* $[S(D)]$. The set of oriented arcs of the boundary of a region is called an *oriented boundary decomposition* (OBD) of that region. Aided by this summary, the nonmathematical reader can now proceed directly to 2.7.

2.2. If we assume we are given the Euclidean plane together with the usual topology, then we can construct a model for stability theory by the proper choice of (i) a set of open sets, (ii) a set of arcs, and (iii) a set of points. It now will be shown how they can be chosen. All mathematical usage is standard unless noted otherwise.

The open sets of stability theory are called "allowable." The collection of all allowable open sets in the plane is represented by the symbol T . An open set u is *allowable* if two conditions are satisfied: (i) u is bounded, and (ii) the topological boundary of u is the union of a finite number of compact analytic arcs. A *compact analytic arc* is an image in the plane of the closed interval $[0,1]$ under a map that is analytic on a neighborhood of the interval and is 1:1 and has a nonvanishing derivative on the open interval $(0,1)$.

Three consequences of condition ii will be used: (a) we can define a relative-orientation, locally, on the boundary of an allowable open set u ; (b) the number of connected components of an allowable open set is finite; and (c) if two such sets intersect, then the number of components of their intersection is still finite.

2.3. An *arc* is a member of a special class of subsets of the boundary of an allowable open set u . We define an arc to be any subset of the boundary that is homeomorphic either with the open interval $(0,1)$ or a circle. We call attention to the fact that these arcs as defined are only piecewise-analytic.

Let us now suppose c is an arc of the boundary of u in the above sense. It is a fact that if p is a point in c and N is a sufficiently small neighborhood of p (topologically, a disc) then there is a subarc of c that divides N into exactly two disjoint open sets. Then c is *orientable relative to u* if, at every point p in c , it is possible to find one such set that lies wholly in u .

If c is an arc of the boundary of u , and c is orientable relative to u , then an *orientation of c relative to u* is a consistent choice, for each point p in c , of exactly one such set, such that each set chosen lies either wholly in u or wholly out of u . By a *consistent choice of sets* is meant that if, for each of two points in c a suitable neighborhood has been specified and one set chosen, and if the two points can be joined by a subarc of c that is wholly contained by the two neighborhoods, then the orientations agree wherever the neighborhoods have nonempty intersection. Any orientable arc has at least one such orientation.

2.4. A distinguished class of structures can now be defined relative to an arc decomposition of the boundary of an allowable open set u . The boundary of an allowable open set can always be exactly decomposed into a finite number of disjoint arcs and isolated points, such that each arc is orientable relative to u . A consequence of the definition of orientation is that we may not always be free to choose a particular orientation for a given arc; the orientation of an arc relative to a given decomposition of the boundary of an allowable open set may be unique.

A finite collection c of disjoint oriented arcs whose union, together with a finite set of isolated points, is exactly the topological boundary of u is an *oriented boundary decomposition* for u if the *alternation rule* holds. The alternation rule states: if c_i and c_j are two disjoint oriented arcs that belong to such a collection c , and if p is an isolated point for which $c_i \cup c_j \cup \{p\}$ is an arc, then c_i and c_j have opposite orientations.

All boundary structures that enter stability theory belong to the restricted class of sets of oriented arcs and isolated points that satisfy the alternation rule. The alternation rule guarantees that a boundary is broken into a set of oriented arcs whose associated set of points is exactly the set of points at which orientations reverse. This is a minimality condition that ensures the well-definedness of these structures in the model.

2.5. An allowable simply connected domain in the plane defines the base space in a fully developed model. The fundamental mathematical object of stability theory can be defined relative to such a subset of the plane.

Any allowable and simply connected domain in the plane can be decomposed into finitely many allowable open sets, each of which carries an *oriented boundary decomposition*. If D is such a domain, then a *stable decomposition* S , $S = S(D)$, is a finite collection of allowable open sets u_i in D together with an *oriented boundary decomposition* for each u_i that satisfy the following three conditions: (i) the open sets are pairwise disjoint, (ii) the collection of open sets together with their boundaries exactly cover the closure of D , and (iii) if two arcs that lie in D also lie in the boundary of open sets u_i and u_j of S , for some i and j , then their orientations agree wherever the arcs have a nonempty intersection. The associated collection of open sets is an *open decomposition* of D .

Condition iii reflects the fact that orientation of an arc is defined relative to a particular open set. Any arc in D that lies on the boundary of two distinct members of an open decomposition of D has an orientation relative to each. Condition iii

requires that they agree. (Technical refinement of the condition of "agreement" is possible; the practical consequences are neither obvious nor germane to this discussion.)

A stable decomposition S breaks D into a finite number of disjoint open sets, together with a finite set of unambiguously oriented arcs and a finite set of points. A stable decomposition specifies an exact decomposition of the subset of the plane consisting of the domain D and its boundary. Such a decomposition differs from $S(D)$ by a finite number of points. (The significance of this fact will not be discussed here.)

2.6. A stable decomposition is by construction a particular collection of open sets, arcs, and points lying in a domain D . But we do not make use of all the geometrical properties that such a collection has gotten from the plane. So we use an equivalence relationship on the set of all stable decompositions of a domain D to refine the definition of $S(D)$. Abusing notation, we then identify both the original object and this new one with the same symbol; we identify the class with its representative member. This makes sense because, although we have not yet specified a particular relation, any such relation chosen would be determined by exactly those applications of the model that we describe.

The equivalence relations that interest us are those given by sets of functions that are homeomorphisms of D with itself and also preserve stability. This means that if ϕ is such a function, and $S(D)$ is a stable decomposition of D , then $S(D)$ is mapped onto a stable decomposition $S'(D)$ by ϕ , and $S(D)$ and $S'(D)$ are indistinguishable as stable decompositions of D . All topological relations and arc orientations are preserved by ϕ , although $S(D)$ and $S'(D)$ will usually differ in other respects.

We will let Φ denote the set of all such mappings. Equivalence relations will be defined by subsets of Φ . One example of such a collection of functions is the set of real analytic homeomorphisms ϕ , where ϕ is a 1:1 real analytic map of D into itself whose inverse is also real analytic and whose differential is everywhere non-zero.

There is a well-defined procedure for associating one member of an equivalence class of stable decompositions with perceptual judgments. But it is the properties of these classes which the model can be said to represent. Their defining maps showed that it is possible to preserve stability, the structures on D defined by particular stable decompositions S , preserving all topological relations and the orientation of arcs while drastically altering both local and global geometry in the model. It is in this sense that we describe this model as being "essentially topological" and independent of geometry.

2.7. We can easily make drawings to represent properties of the abstract mathematical structures just defined. And we will assume that all such sketches are made in the way usual to mathematics. But, for us, a sketch is also an object of perception. To call attention to the potential ambiguities in their interpretation, we add three observations on the use of sketches to represent the mathematics of stability theory.

(i) An arc in an abstract model is represented in a sketch by a drawn line. But an abstract arc is one-dimensional (its breadth is zero) whereas its drawn representative has positive (nonzero) breadth if it is visible in a sketch.

(ii) Orientation of an (abstract) arc can be represented in a sketch by a short arrow, drawn at a point on the (sketched) arc and inward- or outward-directed relative to an adjacent region; the representation is unambiguous (*the arrow rule*).

(iii) The geometrical relationships of which an observer is aware in a sketch do not necessarily represent properties of the abstract model that the sketch records; they are to be taken as artifacts of the drawing, even though the sketch may be very

close in appearance to the original object of perception being modelled.

3. Method

3.1. The foundation for this discussion of perception is given by a set of six postulates.^c This set of postulates is sufficient to form a theoretical framework in which the complete relationship of a biological system χ to its sensory environment (surround) can be expressed. It is *prima facie* evident that these postulates apply compatibly to all sensory modes of which perceptual judgments can be rendered.^d

In the following, the notion "perceptual judgment" is primitive. The notion "state of χ " is also taken to be primitive, and its usage is adapted from physics.^e The notions "system" and "surround" are adapted from biology. We do not yet need to specify the mathematical structure of the space of states of χ .

The six postulates for a theory of perception are as follows:

(i) The *relationship of a system χ and its sensory surround* is a set of pairs of abstract states (x, y) , where x is a state of the system χ and y is a state of its sensory surround.

(ii) For a system χ , a *state y , of its surround* is an abstract model, constructed relative to χ , of a particular perceptual environment of χ .

(iii) A *state x of a system χ relative to a state y of the surround of χ* ^f can be formally characterized (in the study of human sensation) in two categorically independent ways: from subjective phenomena, and from biological phenomena. They are, respectively, the *subjective* and the *biological* characterizations of a state x of a system χ (relative to y).

(iv) If y is a state of the surround of a system χ , a *subjective characterization of a state x of the system χ relative to y* can be derived^g from y by one or more perceptual judgments that are rendered of y by an observer.

(v) If y is a state of the surround of a system χ , a *biological characterization of a state x , of the system χ relative to y* can be given in terms of anatomical and physiological^h characteristics of the system χ .

(vi) The (mathematical) properties of a subjective characterization of a state x of a system χ relative to y are faithfully representedⁱ in any valid complete biological model in which

^c A strong resemblance of this work to the theoretical basis for psychophysics advanced by Fechner (4) was noted in a personal communication from Hans Geissler. The general conceptions and methods are in accord; they differ fundamentally in their abstract organization and mathematics and in their dependence, respectively, on cognition and sensation.

^d The compatibility of method for vision and natural languages was shown by Shiman (5) and in unpublished work by L. Shiman and R. T. Oehrle.

^e A "state x of χ " is a member of a mathematically well-defined class of structures by which the nature of χ is characterized, as quantum states characterize atoms.

^f "Surround" has here the meaning of "environment"; it is not to be confused with "surround (of a figure)" or "ground (of a figure)."

^g By " x derived from y " we mean x , is assigned to y by analysis of y with respect to perceptual judgments. This accords with the principle of constraint.

^h That is, by physical, biophysical, electrophysiological, neuroanatomical, or other means. Notice that these results do not presume the well-definedness of any particular physical or biological descriptions of the system.

ⁱ The representation is required to preserve all (mathematical) properties of the original.

a biological characterization of the same state of the same system relative to the same state of its surround.^j

A subjective state x of a system χ is an *image*, and the corresponding state y of the surround of χ is an *icon*.^k An image x is *matched* to an icon y whenever the image x can be derived by perceptual judgements from the icon y . In this case the icon y *supports* the image x .

An image is well defined only if it has been matched to an icon by an observer. We will, therefore, assume that for every image a supporting icon and an observer can be specified.

3.2. In this study of perception, the word "icon" has three distinct but consistent usages. By definition, an icon can only be specified relative to a system χ . The general characterization given by the postulates is abstract. Concretely, an *icon* is:

(i) A mathematical object that models the complete visual-sensory (optical) relationship of a system χ to its surround.

(ii) An optical projection onto the system χ of its surround, that can be represented as a luminous image on an idealized sensing surface of the retina (ref. 8, pp. 345–346), represented topologically as a disc.

(iii) The effective, immediate, and complete visible source of the image, typically identified with physical reference objects whose location can be verified by touch—for example, a photograph.

The main effect of giving one (abstract) characterization of an icon whose formal properties are compatible with all three interpretations is to allow both the subjective and biological modes of the same system state to be defined relative to the same surround state. It is the case that physical differences in the visual systems of individuals (of whatever species) that affect the optical relationship of the individual (system) to its sensory surround will affect the properties of the icon. We assume that an observer can identify what he perceives with an actual source; in the same spirit, we will speak of breaking an icon into parts.

3.3. A *stable image* is the subjective (psychological) counterpart of an icon that is stably perceived. Stability of an image is established by a judgment of the entire icon that affirms both its coherence and the consistent and unambiguous identifiability of all its parts. The judgment itself is the *stability judgment*. Stability is a characteristic of an ordinary subjective state of an observer. In other words, the ordinary visual experience of a human observer is modeled by stable images. Given an icon, it is universally verifiable that an observer will always try to identify a stable image with it.

There are icons (such as Necker's crystallographic "cube") that support two or more distinct, incompatible, stable images. Because in such cases the corresponding icon is unchanged, these cases show that perception depends on properties both of the image and of the icon. Partial independence of perception from the icon is evident as well in cases of "mis-taken" identity, in which individual psychological factors, totally unrelated to the icon, are found to affect the images assigned by an observer to it. In restating this observation as a principle of perception, the *principle of constraint* affirms that: *an icon constrains but cannot uniquely determine an image perceived*.

The stability judgment of an icon is rendered as an unbroken

whole. It is fundamental and it is unambiguous. But the relationship that is verified by the judgment is global: no internal analysis of the image structure is provided. By introducing two additional judgments, by means of which stable images can be mathematically decomposed, stability theory establishes a structural relationship between image and icon that opens the way to the mathematical study of the subjective state itself and the dependence of perception on the structure of icons.

3.4. An image can be mathematically characterized by using the concept of a formal pair. Formal pairs are used to model perceptual judgment of *identification*. Identification matches a formal pair with a visible region (not necessarily connected) of an icon. The procedure is made more explicit in 4 of the subsequent paper. The two symbols that appear in these formal pairs represent (respectively) an *identity* and an allowable open set. This open set, defined in 2.2, is also called a "region" (of an image). We represent such a formal pair by the symbol (e, u) . A set of such pairs is represented by the symbol $\{(e_i, u_i) \mid i \in I\}$.

Usage of a symbol e to represent an identity is subject to two conditions:

(i) An identity is fixed once its symbol representative has been chosen.

(ii) Equality holds for two symbols if their corresponding identities are the same.

The symbol u represents an open set in the model of an image and is to be clearly distinguished from the corresponding region in the icon.

We can think of identification as giving a name to a specific region in an icon; the name corresponds to whatever meaning or characteristic significance we attach to, or identify with, and by which we can identify a region. The subjective counterpart of such a name is an *identity*. We represent an identity by an elementary symbol. This symbol is the only objective correlate of an identity.

An identity is defined (and is only claimed to exist) relative to a specific region in an icon. The second entry in a formal pair is used to symbolize the subjective counterpart of the surface extension of the identified visible region of the icon. We represent this region mathematically by an open set in T . We will always treat the formal pair as a primitive whole, and its component entries as derived.

3.5. The third judgment is *adjunction* and is applied according to the *adjunction rule*. The adjunction rule is a function defined on formal pairs (identity-region pairs) that assigns to a formal pair an oriented boundary decomposition for its component open set. An oriented boundary decomposition is a set of inward-oriented and a set of outward-oriented arcs that orient and decompose the topological boundary (except for a finite set of points) of an open set in T .

An inward-orientation (defining an adjoined boundary) is assigned to those subarcs of the boundary of a region along which, relative to its paired identity, the adjacent region is complete; in other words, an inward-orientation is assigned to an arc wherever, relative to the identity assigned to that region, the region is understood to terminate. An oriented boundary decomposition represents the dependence of an identity on the shape of the boundary of its paired region. An inward-oriented arc marks a part of a boundary where the visible shape of the region which that arc defines can be said to be dependent on the identity paired with that region. An outward-oriented arc represents a part of a boundary of a region along which the shape of the arc is independent of the paired identity.

3.6. A mathematical model of a stable image of an icon I is constructed by matching with I a finite set of formal pairs that

^j This is the bridge from subjective study of perception to biological study of behavior. See Tinbergen (6) pp. v-x and 1-14.

^k Usage of the terms "image" and "icon" is adapted from Panofsky (7), pp. 3-31. I am greatly indebted to Panofsky for his characterization of these terms.

define an exact decomposition for I . Such a matched set of formal pairs (F) is a *stable state of I* . A stable state gives a consistent and unambiguous account of all visible parts of I . If F is a stable state of I , I supports the stable state of F .

A finite set of formal pairs defines a stable state F of an icon I , relative to the analytic model, defined in 2, whenever:

(i) Each formal pair is matched under identification to a specific visible region in the icon and each visible part of the icon is accounted for exactly once by the set of formal pairs.

(ii) A simply connected domain D , $D \in T$, is assigned to the entire region of the icon (in effect, an additional formal pair identifying the icon as a whole).

(iii) Paired with each identity is an open set $u \in T$, $u \subseteq D$, chosen in such a way that the sets do not intersect.

(iv) The adjunction rule applied to each formal pair (e, u) assigns to its component open set u an oriented boundary decomposition.

(v) The collection of component open sets together with their boundary structures form a stable decomposition $S(D)$ of the domain D .

We identify a stable state with a stable image by the stability hypothesis. This identifies a mathematical model of a subjective state of an observer with a stably perceived icon. The following is called the "Stability Hypothesis":

If an icon I supports a stable image, then I supports a compatible stable state F , and conversely.

This means that, for any F , the F -decomposition of I supports a stable image and that, conversely, for any stable image there exists an F whose F -decomposition of I supports that image.

This hypothesis expresses in the form of a mathematical relationship the simple intuitions described in 1.1. It thus gives

an exact statement of the *Law of Perceptual Stability* for human vision.

The well-definedness and validity of the stability law will be treated in another communication that will consider in detail the immediate practical application and the further theoretical consequences of this principle.

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