Sustaining vs. Disruptive Technology: Industry Equilibrium under Technology Evolution

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Abstract

We study a special type of technology evolution, referred to in the literature as *disruptive* technology vs. *sustaining* technology. In general, "old" products based on sustaining technology are perceived to be superior to the "new" ones based on disruptive technology. However, the latter have distinctive features that allow them to attract an exclusive set of customers. Examples include laser printers vs. inkjet printers, landline services vs. VoIP phones, etc. Our study analyzes industry equilibria in a model with an incumbent and an entrant that have heterogeneous product-offering capabilities: the incumbent can offer either or both types of products, while the entrant can only offer new products. Firms make capacity, pricing, or quantity decisions that maximize their ex-ante profit. Within this framework, we analyze deterministic games with perfect information and stochastic games with uncertain valuation of the disruptive technology.

Our results provide threshold conditions that assist the incumbent firm in identifying the right product portfolio. We show that a failure to recognize the opportunity for adopting disruptive technology-based products may lead to an unnecessary price war between the two types of products. In addition, we show that old products exit the market when their prices become too high for their customers, rather than when they become too low for their manufacturers.

Our analysis of the effects of uncertainty implies that capacity usage decreases with the degree of uncertainty when firms adopt ex-post pricing (Bertrand-type competition). In addition, over-investment and over-production are likely to occur when the level of uncertainty is high and/or marginal costs are low. However, if the firms postpone decisions on product types (Cournot-type competition), the total production quantity will not be affected by the degree of uncertainty.

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1. Introduction

For many industries, attributes of the products and/or services offered largely depend upon the type of technology on which they are built. Consequently, manufacturers and service providers are constantly faced with the challenge of adopting the right technology. As an example, consider a firm that offers voice communication service. It may either use the public switched telephone network (PSTN), as landline services do, or it may transmit the voice over a data network, the way Voice-over-Internet Protocol (VoIP) phones work. Services built on PSTN may provide better voice quality and stable connection, and they can be used even when the power is down, which are properties that are not usually the main characteristics of VoIP phones. However, as the Internet is becoming ubiquitous part of both personal and corporate life today, using a voice service based on a network to which customers are already subscribing may provide huge cost savings. While voice transmitted over a data network underperforms over several dimensions when compared with PSTN, it nevertheless brings distinct values (e.g., cost savings, network integration, etc.), and it is likely to improve its performance over the dimensions mentioned above as time goes on.

In recent decades, many industries have experienced similar technology evolutions, in which a new technology traded off *core-function* performance for evolutionary *side-function* improvements. For instance, in the mid-'1980s, when the 3.5-inch hard disk drive (HDD) was introduced in the market that had previously been dominated by the 5.25-inch HDD, it sacrificed capacity to achieve greater portability, and more recently solid-state-drives (SSDs) were introduced in the market dominated by HDDs, and they traded off capacity, writing speed, etc. to achieve less noise and greater temperature tolerance. One can think of numerous similar examples—laser printers vs. ink-jet printers, mortar-and-brick retailing vs. online retailing, to mention just two. Christensen (1992, 2003) introduced the terms *sustaining technology* and *disruptive technology* to describe such phenomena:

> Most new technologies foster improved product performance. I call these *sustaining technologies*. (...) What all sustaining technologies have in common is that they improve the performance that mainstream customers in major markets have historically valued. ... *disruptive technologies* emerge: innovations that result in *worse* product performance, at least in the near-term. (...) But they have other features that a few fringe (and generally new) customers value. Products based on disruptive technologies are typically cheaper, simpler, smaller, and frequently, more convenient to use. (Christensen, 2003, p. xviii).

Christensen and Raynor (2003) and Christensen et al. (2004) documented the many ups and downs faced by the firms trying to correctly identify the role of disruptive technology, and they provided enterprise-level strategies for decision makers and an overview of the changes that disruptive technology may bring to an entire industry. Their work triggered a stream of research focused on the various problems that may arise for this particular type of technology evolution. Schmidt and Porteus (2000a) studied incomplete substitution between products based on disruptive and sustaining technology, and compared the equilibria obtained when each product is offered by a different firm with equilibria obtained when both products are offered by a monopolistic firm. Adner and Zemsky (2005) developed a model that explains the effects of disruptive technology on different aspects of the market, and Van der Rhee et al. (2007) examined how firms in a duopoly make their technology adoption decisions under different market conditions. Druehl and Schmidt (2008) compared the market impact for the case in which the new products target high-end new customers vs. the case in which they target low-end existing customers. While all of these papers provided important insights on issues that arise in the presence of disruptive technology, they all have one assumption in common—when there are multiple firms, each of them can adopt exactly one type of technology.

There are a few papers that allow a firm to adopt either or both types of technologies. Schmidt and Porteus (2000b) modeled a game between an incumbent and an entrant, in which they examined how capability for cost reduction and innovation affect the equilibrium investment decisions. Schmidt and Porteus (2007) analyzed additional factors that let the incumbent stand out from the entrant.

On the empirical side, Agarwal and Bayus (2002) provided statistical evidence that the entrance of many new firms into the market contributes to the take-off in sales of the new product. De Figueiredo and Silverman (2007) provided data that supports the claim that the entry of dominant firms into the market for new products drives down the price and causes the increase in the number of firms in the market segment.

Our paper differs from the existing literature in several respects. First, we look at firms with heterogeneous product-offering capabilities, by allowing the incumbent to adopt either or both technologies, while restricting the entrant to work with disruptive technology only. As mentioned above, the assumption that a firm can only adopt one type of technology holds in most of the existing research (for exceptions, see Schmidt and Porteus, 2000b, 2007). We believe that it is important to recognize the difference between an incumbent and an entrant in terms of technology adoption. Because sustaining technologies are well-established and their market is usually dominated by strong players, an entrant may simply choose to start with alternative technologies. On the other hand, disruptive technologies are likely to have lower entry barriers, which makes them easier to implement by either firm.

Second, we address capacity decisions, which are critical from the operations-management perspective, but have not yet been explored within the framework of disruptive technology. Capacity constraints may have a great impact on equilibrium outcomes. As noticed by Kreps and Scheinkman (1983), a capacity-constrained Bertrand game may yield the same equilibrium as a corresponding Cournot game (unlike the uncapacitated game, which may leave both firms with zero profit). We apply a similar idea in our game with a *two*-product duopoly market, and we identify the corresponding equilibria. In our model, the equilibrium may consist of both firms simultaneously offering products based on disruptive technology, at the same price level. This is in contrast to Schmidt and Porteus (2000b), in which the incumbent, due to Bertrand competition, does not adopt the disruptive technology when the entrant adopts the same.

Finally, we assume that the firms face uncertainty in the potential valuation of the disruptive technology at the time when they have to make their capacity commitment. However, once they observe the actual valuation, they have an opportunity to readjust their initial decisions by using two types of postponement: price postponement and capacity-type postponement. We characterize equilibria under both assumptions, and discuss how the degree of uncertainty and marginal costs may affect the quantity of products offered in the market.

The paper is organized as follows. In §2 we introduce the basic elements of the model, and §3 studies two deterministic games: capacity-constrained Bertrand competition and Cournot competition. We characterize equilibrium capacity and pricing decisions and derive the threshold conditions that help the incumbent in identifying the optimal product portfolio. In §4 we discuss the impact of model parameters on equilibrium outcomes and illustrate analytical findings with numerical examples, and §5 extends the analysis from §3 by introducing uncertainty into the valuation of disruptive technology. We conclude in §6. All longer proofs are given in the technical appendix.

2. Model Description

We study a market with two kinds of products—products based on sustaining technology, which we denote by S, and products based on disruptive technology, which we denote by D. Our model consists of two firms, which we call Firm 1 and Firm 2. Firm 1 is an established firm that dominates the market for product S, and can decide to include product D in its portfolio. Firm 2 is a start-up firm that can manufacture and sell only product D. There are several real-life examples that fall into this framework. For instance, AT&T has been a part of the telecommunications industry since 1984, and through several mergers and acquisitions it has increased its product and services portfolio, which now offers a full line of telecommunication services (landlines, wireless, Internet, television, etc.). As an example of Firm 1, AT&T has the option to decide if it wants to include a new type of product/service into its portfolio. Indeed, it did not plan to include Internet-based phone service (VoIP) for residential customers until 2004. One the other hand, Vonage, an example of Firm 2, was among the first to offer residential VoIP services, starting as early as 2001.[1]

We denote by v_i and p_i, $i \in \{S, D\}$, the customer's valuation for product i and the price of product i, respectively. Thus, the customer's utility from purchasing product i equals $v_i - p_i$. Customers will buy the product that gives them the highest non-negative utility, $\max\{v_S - p_S,\, v_D - p_D,\, 0\}$, and if both utilities are negative they will not buy any product. Notice that we allow customers to choose at most one of the products; the case in which a customer may choose to have both products at the same time (e.g., a VoIP to make long-distance calls and a landline for local calls) could be a possible extension of this model, but it is not within the scope of this paper.

We assume that there is a *flexible* market with size M in which customers consider buying both products, S and D, and a *dedicated* market sized m in which only product D is considered. Specifically, let us denote by θ the customers' type, with a small θ implying a high-end customer. In addition, let s and d, which we call *value factors*, denote the rate of increase in valuations as the customer type increases from low-end to high-end. We assume $s > d$ to rule out some trivial cases. Then, for product S, we have

$$v_S = s(M - \theta), \qquad \theta \in [0,\, M],$$

and for product D,

$$v_D = d(M + m - \theta), \qquad \theta \in [0,\, M + m].$$

The linear valuation is similar to the one adopted in Schmidt and Porteus (2000). Figure 1 depicts some possible realizations of the valuations as s and d take different values. One may observe that both products have a similar set of high-end customers. Thus, the competition is quite intense in the core functions of the products (e.g., product quality, reliability, etc.), and the distinctive features provided by product D (e.g., compact size, ease-of-use, etc.) attract the low-end customers.[2]

The valuation function can also be viewed as an inverse demand function when a single product exists in the market. For example, when the price of product S satisfies $p = v_S(\theta)$ for some $\theta \in [0,\, M]$, customers within $[0,\, \theta]$ are willing to buy S in the absence of D, and we can write the corresponding

[1] It is worthwhile to note that today, seven years later, Vonage still concentrates on Internet-based products and services. It would be hard for such a start-up to get a share (technically or financially) in the traditional (mature) landline or wireless markets.

[2] For a model in which the products are designed exactly for the opposite ends of the market, we refer the reader to Druehl and Schmidt (2008).

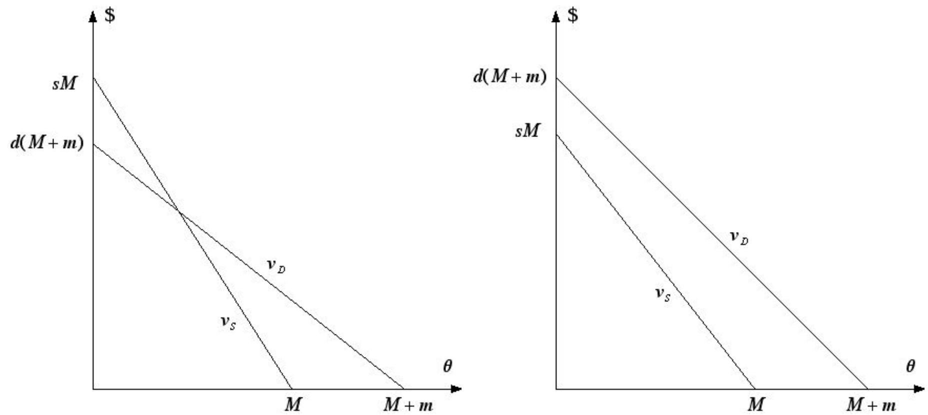

Figure 1: Valuation under different scenarios.

demand function as

$$Q_S(p) = M - \frac{p}{s}, \qquad p \in [0,\, sM].$$

Similarly, we have

$$Q_D(p) = M + m - \frac{p}{d}, \qquad p \in [0,\, d(M+m)].$$

Finally, we use c_S and c_D to denote the unit capacity costs.[3] We assume that marginal capacity costs for products based on sustaining technology exceed those of products based on disruptive technology, $c_S \geq c_D$. This has been commonly observed in real life, as disruptive technology aims to capture the low-end market.

3. Game Formulation and Results

In this section, we describe two different kinds of competition in which the two firms may engage—Bertrand and Cournot games.

We first consider a three-stage Bertrand game. In the first stage, the firms make simultaneous capacity decisions, $\vec{y}_1 = (y_S,\, y_{D_1})$ and $\vec{y}_2 = (0,\, y_{D_2})$. In the second stage, the firms simultaneously determine the prices of their products, $\vec{p}_1 = (p_S,\, p_{D_1})$ and $\vec{p}_2 = (0,\, p_{D_2})$. Finally, demand is allocated to the two firms based on their capacity and pricing decisions. Thus, the customers will first buy products (if any) that give them the highest non-negative utility. If customers receive equal utility from both products, they are equally likely to buy either of them, and if the installed capacity within a firm cannot meet the allocated demand for a given product, the unmet demand is passed to the

[3]To simplify the analysis, especially for the model with uncertainty, we assume that both firms incur equal marginal capacity costs for products based on disruptive technology. We note, however, that the analysis conducted with $c_{D_1} \neq c_{D_2}$ for the deterministic case generated results similar to those presented here. We discuss this further in §4.

other product, if there is any idle capacity. To simplify the analysis, we assume that no production cost is incurred after the capacity investment has been made.

The second game is in the form of Cournot competition. Under this scenario, the firms simply make quantity decisions $\vec{q}_1 = (q_S, q_{D_1})$ and $\vec{q}_2 = (0, q_{D_2})$, which then determine the price for each product in the market.

At the end of this section, we show that these approaches generate the same equilibrium outcome, which coincides with the results in Kreps and Scheinkman (1983). However, the second formulation is more convenient for both the analysis and the implementation.

3.1 Bertrand Competition

For given p_S and p_{Di}, we define θ_i as the customer that is indifferent between buying product S at price p_S and product D at price p_{Di}:

$$\theta_i \quad = \quad M - \frac{dm - p_{D_i}}{s-d} - \frac{1}{s-d} p_S, \qquad i = 1, 2. \tag{1}$$

Then, customers within $[0, \min\{\max\{\theta_i, 0\}, Q_D(p_{D_i})\}]$ prefer product S, while those within $[\min\{\max\{q_i, 0\}, Q_D(p_{D_i})\}, Q_D(p_{D_i})]$ prefer product D. Figure 2 illustrates θ_1 and θ_2 under two

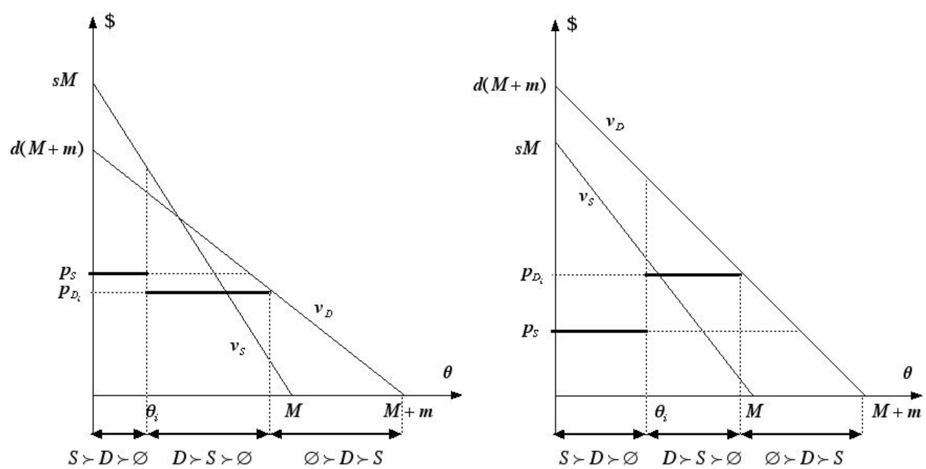

Figure 2: An illustration of θ_i under different scenarios.

different scenarios. In the left figure, d is small, so that $d(M+m) < sM$, and the price is such that $p_{D_2} < p_S < p_{D_1}$. We can use (1) to determine the customer type that is indifferent between buying product S at p_S and product D at p_{D_i}, θ_1 and θ_2. The arrows in the figure show the market's preference between product S and product D at different prices. The portion to the left of θ_i shows the market segment that prefers product S at p_S to product D at p_{D_i}, while the portion to the right of θ_i represents the market that prefers the opposite. The fact that high-end customers prefer product S and low-end customers prefer D also holds in the second scenario, on the right, where $d(M+m) \geq sM$. Notice that this is only a comparison for a given set of prices, which does not consider product D offered at a different price. The equilibrium prices should lead to

$$\theta_i \in [0, \min\{Q_S(p_S), Q_D(p_{D_i})\}]; \tag{2}$$

if (2) does not hold, one of the products is not preferred by any customer under any circumstance, which can be changed by modifying its price. Thus, hereinafter we assume that (2) holds, which leads to the following constraints:

$$p_{D_i} - p_S > dm - (s-d)M, \qquad m > \frac{p_{D_i}}{d} - \frac{p_S}{s}.$$

Lemma A1 in the Appendix describes the allocation of demand among the products in the third stage, given their prices. In the second stage, we consider the subgame in which the capacity $\vec{y} = (y_S, y_{D_1}, y_{D_2})$ has been installed and revealed to each firm. The subgame equilibrium for prices is denoted by $\vec{p} = (p_S^*, p_{D_1}^*, p_{D_2}^*)$. For the given set of prices, Lemma A2 in the Appendix describes conditions under which only one product (S or D) is offered in the market, and under which both products simultaneously exist. In the first stage, we first determine the capacity and pricing equilibria given the product portfolio of Firm 1 (see Proposition 1), and then we establish intervals over which a particular portfolio is offered (see Theorem 1). Longer proofs can be found in the Appendix. Before we state our results, we introduce some notation that will be used in the reminder of the paper:

1. *Technology factors*, γ and $\bar{\gamma}$:

   $$\gamma = \frac{d}{s-d}, \quad \bar{\gamma} = 2 + 3\gamma = \frac{2s+d}{s-d}.$$

   Both factors are increasing in d and decreasing in s, with $0 < \gamma < \bar{\gamma}$.

2. *Standardized marginal costs*, $\bar{c}_S$ and $\bar{c}_D$: $\bar{c}_S = \frac{c_S}{s}, \quad \bar{c}_D = \frac{c_D}{d}$.

3. *Market ratio*, R:

   $$R = \frac{M - \bar{c}_S}{m + \bar{c}_S - \bar{c}_D}.$$

We first present equilibrium capacity and pricing decisions given the product portfolio of Firm 1.

Proposition 1.

1. *If Firm 1 offers only product D, the equilibrium capacity investment and prices are given by*

   $$y_{D_i}^{(1)} = \frac{M + m - \bar{c}_D}{3}, \qquad p_{D_i}^{(1)} = \frac{d(M + m + 2\bar{c}_D)}{3}.$$

2. If Firm 1 offers only product S, and Firm 2 offers D, the equilibrium is

$$y_S^{(2)} = \frac{2(\gamma+1)(M-\bar{c}_S) - \gamma(M+m-\bar{c}_D)}{3\gamma+4}, \qquad p_S^{(2)} = \frac{s[2(\gamma+1)(M-\bar{c}_S) - \gamma(M+m-\bar{c}_D)]}{3\gamma+4} + c_S,$$
$$y_D^{(2)} = \frac{(\gamma+1)[2(M+m-\bar{c}_D) - (M-\bar{c}_S)]}{3\gamma+4}, \qquad p_D^{(2)} = \frac{d(\gamma+1)[2(M+m-\bar{c}_D) - (M-\bar{c}_S)]}{3\gamma+4} + c_D.$$

3. If Firm 1 offers both products, and Firm 2 offers D, the equilibrium capacity investment is

$$y_S^{(3)} = \frac{(M-\bar{c}_S) - \gamma(m+\bar{c}_S-\bar{c}_D)}{2}, \ y_{D_1}^{(3)} = \frac{(1+\gamma)(m+\bar{c}_S-\bar{c}_D)}{2} - \frac{M+m-\bar{c}_D}{6}, \ y_{D_2}^{(3)} = \frac{M+m-\bar{c}_D}{3},$$

and the corresponding equilibrium prices are

$$p_S^{(3)} = \frac{s}{2}(M+\bar{c}_S) - \frac{d}{6}(M+m-\bar{c}_D), \qquad p_{D_i}^{(3)} = \frac{d}{3}(M+m+2\bar{c}_D).$$

Observe that the expressions above simplify significantly when $\bar{c}_S = \bar{c}_D = 0$. Because equilibrium decisions in our game depend upon $m, M, \bar{c}_S$, and $\bar{c}_D$, let us denote a three-stage Bertrand game described in this section by $B(m, M, \bar{c}_S, \bar{c}_D)$. Then, we have the following result.

Corollary 1. *Bertrand games $B(m, M, \bar{c}_S, \bar{c}_D)$ and $B(m+\bar{c}_S-\bar{c}_D, M-\bar{c}_S, 0, 0)$ yield the same equilibrium capacity and pricing decisions.*

The proof follows immediately by letting $c_S = c_D = 0$ in Proposition 1. For the reminder of the paper, we will denote the standardized market sizes by

$$\overline{M} = M - \bar{c}_S, \quad \overline{m} = m + \bar{c}_S - \bar{c}_D, \quad \text{and} \quad \overline{M+m} = \overline{M} + \overline{m} = M + m - \bar{c}_D.$$

Corollary 1 implies that greater marginal costs lead to smaller potential markets. In addition, together with Proposition 1, this result implies that the total capacity decreases as the marginal costs increase. However, it is unclear whether the size of the dedicated market, $\overline{m}$, decreases or increases with respect to m, and hence the change of production quantity for a single product/firm is indeterminate. For example, when Firm 1 offers only product S and $\bar{c}_S > 2\bar{c}_D$, the capacity for product D, y_D, will be greater than when $c_S = c_D = 0$, as product S faces greater cost disadvantage.

Notice that Proposition 1 does not imply positivity of equilibrium capacities and/or prices. A negative number in an equilibrium solution (for a given product portfolio) may indicate that Firm 1 should add/drop product(s) to/from the current portfolio. Our next theorem fully characterizes the equilibrium of this game.

Theorem 1. *In the three-stage Bertrand game with $M > \bar{c}_S$ and $M + m > \bar{c}_D$, both firms will have a non-empty product portfolio:*

1. *if $R \leq \gamma$, Firm 1 offers only product D, with $\vec{y}^B = \vec{y}^{(1)}$ and $\vec{p}^B = \vec{p}^{(1)}$;*

2. *if $\bar{\gamma} \leq R$, Firm 1 offers only product S, with $\vec{y}^B = \vec{y}^{(2)}$ and $\vec{p}^B = \vec{p}^{(2)}$;*

3. *if $\gamma < R < \bar{\gamma}$, Firm 1 offers both products, with $\vec{y}^B = \vec{p}^{(3)}$ and $\vec{p}^B = \vec{p}^{(3)}$.*

Proof. Follows from Proposition 1: If $R < \gamma$, then $y_S^{(3)} < 0$ and Firm 1 should offer only D, with $\vec{y}^{(1)}$. If $R > \bar{\gamma} = 3\gamma + 2$, then $y_{D_1}^{(3)} < 0$, and Firm 1 should offer only S, with $\vec{y}^{(2)}$. □

Theorem 1 is our main result so far, and it merits some additional discussion. The first case, $R \leq \gamma$, describes a situation in which the disruptive technology receives high customer valuation. Thus, it may generate a larger dedicated market, which increases the value of the technology factor, γ, and decreases the value of the market ratio, R. It is then likely that Firm 1 may drop product S from its portfolio and offer product D only.

The second case, $\bar{\gamma} \leq R$, describes a situation in which disruptive technology is not overly successful. Consequently, both γ and $\bar{\gamma}$ are small, and the market ratio may be rather large. Under such a scenario, Firm 1 benefits by keeping D out of its portfolio and focusing on S only.

Finally, if the disruptive technology achieves a moderate success, the market ratio is within a closed interval determined by the technology factors, $\gamma < R < \bar{\gamma}$. By comparing $p_i^{(2)}$ and $p_i^{(3)}$, $i \in \{S, D\}$, we can conclude that exclusion of product D from Firm 1's portfolio leads to underpricing of S and overpricing of D. Thus, Firm 1 should offer both products.

So far, we have established equilibrium decisions for a Bertrand game under different values of its parameters. We further discuss the impact of various parameters on equilibrium decisions in §4. In the following subsection, we develop equilibrium decisions under Cournot competition.

3.2 Cournot Competition

While in a Bertrand game firms compete on prices, which determines the quantity of products that will be sold, in this section we analyze a game in which firms compete on production quantities, and the prices are determined by the product quantity in the market. It is well-known that in a one-product market, price competition and quantity competition may yield very different outcomes. However, as shown in Kreps and Scheinkman (1983), this may not happen in a one-product duopoly model if the quantity competition occurs under a capacity constraint determined in an earlier stage of the game. In our problem, however, Firm 1 may choose to offer two products, and in a Bertrand game the two firms can select different prices for product D. However, as we showed in the previous subsection, this does not happen in an equilibrium because both firms set the same price for product

D. Thus, a natural question is whether the equilibrium of this Cournot game will correspond to the equilibrium of the Bertrand game. As we show in this subsection, this will indeed be the case. All longer proofs can be found in the Appendix.

We again analyze the problem backwardly. Let us denote the total quantity produced by Firm 1 by $q_1 := q_S + q_{D_1}$, the total quantity produced by Firm 2 by $q_2 := q_{D_2}$, and the total quantity produced of product D by $q_D := q_{D_1} + q_{D_2}$. If we assume that the quantity decisions, $\vec{q}_1 = \{q_S, q_{D_1}\}$ and $\vec{q}_2 = \{0, q_{D_2}\}$, are given, then the following proposition describes the corresponding prices.

Proposition 2. *Given the production quantities, the prices in the Cournot game are*

$$p_D = d(M + m - q_S - q_D), \qquad p_S = sM - sq_S - dq_D. \tag{3}$$

Proof. As the firms compete in quantities, there is only one price for product D, $p_{D_1} = p_{D_2}$. Then, equation (1) and Lemma A2 imply that

$$q_S = M - \frac{d}{s-d}m + \frac{1}{s-d}p_D - \frac{1}{s-d}p_S, \qquad q_D = q_{D_1} + q_2 = \frac{s}{s-d}m - \frac{s}{d(s-d)}p_D + \frac{1}{s-d}p_S.$$

Solving these for p_S and p_D gives the result. □

Because this game does not directly involve any capacity decisions, we can interpret c_S and c_D as the marginal production cost, or assume that the capacities in this game are equivalent to the quantity decisions. As a result, the reaction functions are given by the following proposition.

Proposition 3. *The production reaction functions are given by*

$$q_2(q_1) = \frac{\overline{M+m} - q_1}{2}, \; q_S(q_2) = \max\left\{\min\left\{\frac{\overline{M} - \gamma m}{2}, \frac{\overline{M} - \frac{d}{s}q_2}{2}\right\}, 0\right\}, \; q_1(q_2) = \max\left\{\frac{\overline{M+m} - q_2}{2}, q_S\right\}. \tag{4}$$

Equation (4) implies that the total production of each firm depends only upon the total production of its rival. That is, Firm 2 only cares about the total amount produced by Firm 1, q_1, and not about the allocation of this quantity among the products in the portfolio of Firm 1, q_S vs. q_{D_1}—the actual allocation only matters to Firm 1. Let us define the *production limit* for product S as

$$L = \frac{\overline{M} - \gamma\overline{m}}{2}. \tag{5}$$

If $\frac{\overline{M}}{\overline{m}} \leq \gamma$, the production limit is non-positive, and hence $q_S = 0$ and Firm 1 does not include S in its portfolio. As we can see from Figure 3, in equilibrium,

$$q_1^* = q_{D_1}^* = q_2^* = q_{D_2}^* = \frac{\overline{M+m}}{3}. \tag{6}$$

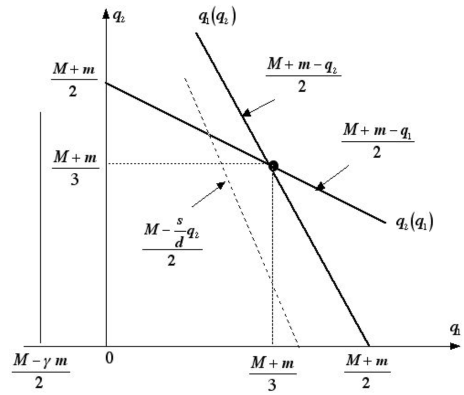

Figure 3: When $\frac{\overline{M}}{\overline{m}} \leq \gamma$.

Now, if $\gamma < \frac{\overline{M}}{\overline{m}} < \bar{\gamma}$, we have a strictly positive production limit. As Figure 4 shows, whenever q_2 is "moderate" ($\frac{\overline{M+m}-q_2}{2} > L$), Firm 1 will make L units of S and use the rest of $\frac{\overline{M+m}-q_2}{2}$ for D. If, on the other hand, $L > \frac{\overline{M+m}-q_2}{2}$, Firm 1 will make only $\frac{\overline{M}-\frac{d}{s}q_2}{2}$ units of S. In equilibrium, we have

$$q_1^* = q_2^* = \frac{\overline{M+m}}{3}, \qquad p_S^* = L = \frac{\overline{M} - \gamma\overline{m}}{2}. \tag{7}$$

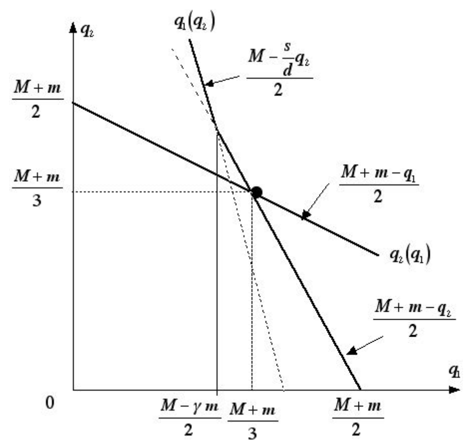

Figure 4: When $\gamma < \frac{\overline{M}}{\overline{m}} < \bar{\gamma}$.

Finally, if $\bar{\gamma} \leq \frac{\overline{M}}{\overline{m}}$, Firm 1 will not include D in its portfolio (see Figure 5), and the equilibrium is

$$q_1^* = q_S^* \;=\; \overline{M+m} - \frac{2s(\overline{M}+2\overline{m})}{4s-d}, \qquad q_2^* = \frac{s(\overline{M}+2\overline{m})}{4s-d}. \tag{8}$$

Notice that the thresholds above depend upon $\frac{\overline{M}}{\overline{m}}$ and upon the technology factors defined in the previous subsection. By comparing (6) with $\vec{y}^{(1)}$, (7) with $\vec{y}^{(3)}$, and (8) with $\vec{y}^{(2)}$, it is easy to

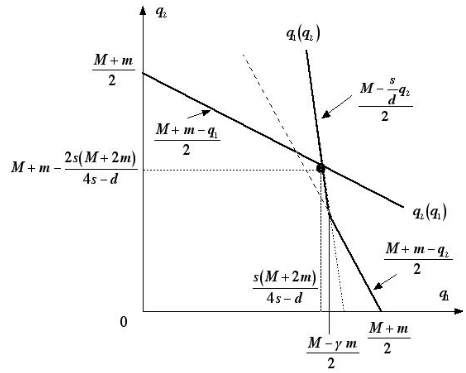

Figure 5: When $\bar{\gamma} \leq \frac{M}{m}$.

verify that the quantity decisions in the Cournot game correspond to the capacity decisions in the Bertrand game. This is formalized in our next result.

Theorem 2. *The two-stage Cournot game yields the same equilibrium as the three-stage Bertrand game,* $\vec{q}^C = \vec{y}^B, \quad \vec{p}^C = \vec{p}^B$.

Thus, while the two games have the same equilibria and use the same variables—market ratio and technology factors—to identify the optimal product portfolio, we note that the Cournot game is easier to solve, because it has fewer stages and decision variables. Additionally, the Cournot game is easier to implement, because the value of the production limit, L, helps decision-makers to easily decide what products to offer and how much to produce.

4. Discussion of the Equilibrium Decisions

In this section, we discuss how different model parameters affect the equilibrium outcomes. Specifically, we analyze the impact of market sizes and value factors on the equilibrium capacities and prices. Since we have already discussed the role of marginal capacity costs in §3.1, in this section we focus on games with zero marginal costs. We will illustrate our findings with numerical examples, and comment on the assumption of zero marginal costs at the end of the section.

4.1 General Observations

If the marginal costs are zero, the market ratio R is simply $\frac{M}{m}$. Equilibrium decisions from Proposition 1 can then be simplified as shown in Table 1.

We would first like to mention that the analysis in §3 shows that *there is a single price for product D.*[4] Thus, there is no "brand" effect by which equal products can be sold at different prices simply

[4] As mentioned earlier, we conducted some analysis for the case with $c_{D_1} \neq c_{D_2}$ and found that both firms charge

	y_S	y_{D_1}	y_{D_2}	p_S	p_D
$R \leq \gamma$	0	$\frac{M+m}{3}$	$\frac{M+m}{3}$	0	$\frac{d(M+m)}{3}$
$\gamma < R < \bar{\gamma}$	$\frac{M}{2} - \frac{dm}{2(s-d)}$	$\frac{sm}{2(s-d)} - \frac{M+m}{6}$	$\frac{M+m}{3}$	$\frac{(3s-d)M-dm}{6}$	$\frac{d(M+m)}{3}$
$\bar{\gamma} \leq R$	$\frac{(2s-d)M-dm}{4s-d}$	0	$\frac{s(M+2m)}{4s-d}$	$\frac{s(2s-d)M-sdm}{4s-d}$	$\frac{ds(M+2m)}{4s-d}$

Table 1: Equilibrium Decisions when $c_S = c_D = 0$

because they come from different manufacturers. Firm 1 uses the fact that it can manufacture both products to leverage the market share, and consequently to maximize its profit.

Next, we note that p_D and y_{D_2} obtained when $R \leq \bar{\gamma}$ correspond to solutions in a one-product duopoly market. In other words, once Firm 1 has product D in its portfolio, neither p_D nor y_{D_2} are further influenced by the production of S. If Firm 1 produces D and decides to add S, the effect is the same as if it shifts some of its total capacity toward product S. The price of S will determine which portion of the capacity will be allocated to product S, and the price of D will not change. As the total capacities for each firm and the price for product D corresponds to the solution of a duopoly market with a single product, we call the scenario in which Firm 1 includes product D in its portfolio a *semi-duopoly*. Some additional results are given in the following proposition.

Proposition 4.

(i) $y_1 \geq y_2$; $y_1 = y_2$ if and only if Firm 1 has product D in its portfolio (that is, $R \leq \bar{\gamma}$).

(ii) $p_S > p_D$ whenever Firm 1 includes product S in its portfolio (that is, $R > \gamma$).

Proof. (i) Theorem 1 and Proposition 1 imply that $y_1^{(1)} = y_2^{(1)} = y_1^{(3)} = y_2^{(3)} = \frac{\overline{M+m}}{3}$. $y_1^{(2)} - y_2^{(2)} = \frac{(3\gamma+3)\overline{M}-(3\gamma+2)\overline{M+m}}{3\gamma+4} = \frac{M-\bar{\gamma}m}{3\gamma+4} > 0$ when $R > \bar{\gamma}$.

(ii) $p_S^{(2)} - p_D^{(2)} = \frac{s(2\overline{M}-3\gamma\overline{m})}{3\gamma+4} + c_S - c_D > 0$ when $R \geq \bar{\gamma}$, and $p_S^{(3)} - p_D^{(3)} = \frac{(s-d)}{2}\overline{M} - \frac{d}{2}\overline{m} + c_S - c_D > 0$ when $\gamma < R < \bar{\gamma}$. □

The first result states that Firm 1, as an established firm with monopoly power over product S, always builds higher capacity than its rival firm. If the disruptive technology is not well received ($R \geq \bar{\gamma}$), Firm 1 produces and sells higher total quantity than Firm 2. Whenever $R < \bar{\gamma}$, the total production quantity of the two firms becomes equal, and Firm 1 offers product D. Recall that such equivalence in production quantity corresponds to the outcome of the single-product Cournot game.

the same price for product D, given that it is included in their portfolio. However, if $c_{D_1} > c_{D_2}$, Firm 1 may decide to drop product D from its portfolio earlier.

In our case, it holds when S is not offered ($R \leq \gamma$), but it also carries over to the model with two products ($\gamma < R < \bar{\gamma}$). We can think of the situation when products S and D co-exist in the market as a semi-duopoly case. In this scenario, Firm 1 loses its dominant status—when the technology of product D is improved or the dedicated market for product D is enlarged, Firm 1 opts to include D in its portfolio, while at the same time it keeps product S in order to serve the high-end market.

The second result states that, as long as product S stays in the portfolio, Firm 1 always charges more for it than for product D. A number of real-life examples support this observation—laser vs. inkjet printers, acoustic vs. digital pianos, etc. The underlying reason can be explained as follows. As we discussed in §2, if there is a set of customers that prefers product S to D , they are at a higher end of the market than those preferring D. Therefore, we should not reduce the price of product S below that of product D. On the other hand, as disruptive technology becomes more competitive, selling product S at a lower price than product D would not lead to revenue maximization—Firm 1 could either raise the price of S while keeping the same set of customers, or offer product D instead of S to the same set of customers. Therefore, as observed in some real-world examples (e.g., minicomputers), products based on sustaining technology are likely to exit as "*too expensive to be preferred*" rather than "*too cheap because no one likes them.*"

4.2 Impact of Market Sizes (M and m)

We now look at the impact of market sizes, M and m, on the capacity decisions. Our results follow directly from Proposition 1, and hence we omit the proofs.

Proposition 5. (Impact of Dedicated Market on Capacities)

(i) (Product capacity) y_S *decreases, while* y_{D_1} *and* y_{D_2} *increase with* m.

(ii) (Firm capacity) y_1 *decreases (resp., increases) with* m *when* $R \geq \bar{\gamma}$ *(resp.,* $R < \bar{\gamma}$*), while* y_2 *increases with* m.

Intuitively, a larger dedicated market encourages Firm 2 to build larger capacity. In the semi-duopoly case in which Firm 1 has product D in its portfolio, Firm 1's total capacity increases as well. However, when the disruptive technology fails to attract Firm 1 ($R \geq \bar{\gamma}$), its total capacity ($y_1 = y_S$) decreases. Overall, *an increase in* m *induces both firms to put more weight on product D.* We illustrate this with the following example.

Example 1. *Suppose that* $M = 120$ *and* $s = 4$ *and* $d = 2$*, which implies that* $\gamma = \frac{d}{s-d} = 1$ *and* $\bar{\gamma} = 2 + 3\gamma = 5$*. Because we assumed that* $c_S = c_D = 0$*, we have* $R = \frac{120}{m}$*. Figure 6 shows how the*

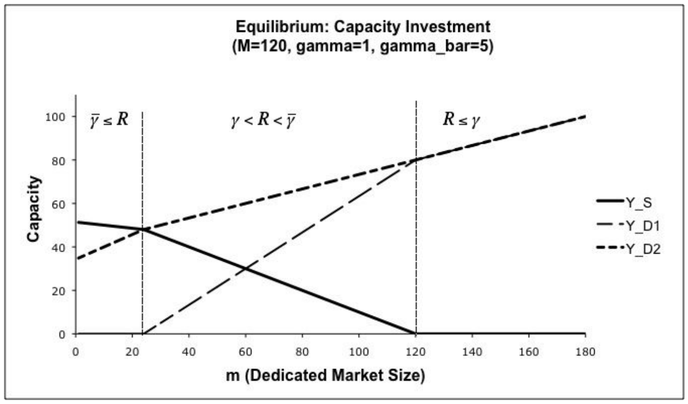

Figure 6: Equilibrium in capacity when the loyal market size varies.

equilibrium capacities change when m varies within $[0, 180]$. *Note that* $m \leq \frac{M}{\bar{\gamma}} = 24$ $(R \leq \bar{\gamma})$ *implies that* $y_{D_1} = 0$ *and that Firm 1 does not offer product* D, *while* $m \geq \frac{M}{\gamma} = 120$ $(R \leq \gamma)$ *implies that* $y_S = 0$ *and that Firm 1 does not offer product* S. *As the size of the market dedicated to product* D *grows, the capacity and the price for that product increase as well. At the same time, the capacity and the price for product* S *decrease until* m *reaches the threshold* $m = 120$, *when they both drop to zero.*

Proposition 6. (Impact of Flexible Market on Capacities)

(i) (Product capacity) y_S *and* y_{D_2} *increase with* M, *while* y_{D_1} *decreases with* M.

(ii) (Firm capacity) *Both* y_1 *and* y_2 *increase with* M.

As the customers in the flexible market value both products S and D, the total production of each firm increases with M. Note that Firm 1 puts more weight on product S—that is, *each firm focuses more on its "core" products.* Thus, Firm 1 serves as the "S-provider" and may carry product D as a side-offering, while Firm 2 is the main "D-provider" and serves the low-end market. As we can see, while the increase in the size of the dedicated market leads to a higher degree of competition, an increase in the size of the flexible market has the opposite effect—each firm ends up occupying one end of the market and offering a different product. We illustrate this in our next example.

Example 2. *Suppose that* $m = 30$, $s = 4$, *and* $d = 2$, *which implies that* $\gamma = 1$, $\bar{\gamma} = 5$, *and* $R = \frac{M}{30}$. *Figure 7 shows how the equilibrium capacity changes when* M *varies within* $[0, 180]$. *When* $M < 30$ $(R \leq \gamma)$, *Firm 1 does not offer product* S, *while both firms offer the same quantity of product* D *(which is increasing with* M*). However, as the size of the flexible market grows above 30, Firm 1 decreases the capacity for product* D *and increases the capacity for product* S *until* M *hits 150 (when*

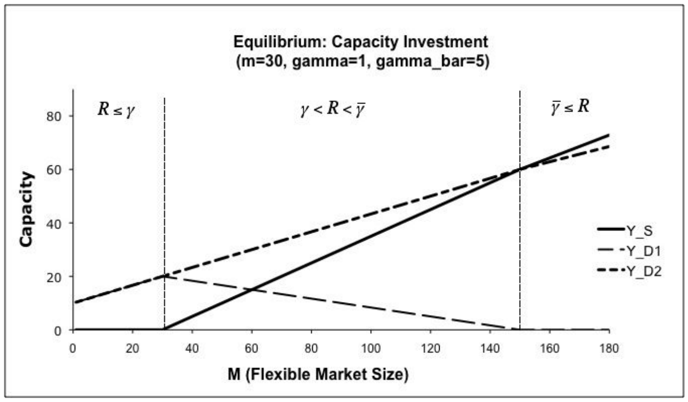

Figure 7: Equilibrium in capacity when the flexible market size varies.

it becomes optimal for Firm 1 to offer product S only). At the same time, Firm 2 increases the capacity of D as M increases.

Recall that our model assumes that the highest valuations are sM (for product S) and $d(M+m)$ (for product D), and that valuations linearly decrease as the customer type increases. As a result, any increase/decrease in market sizes triggers a corresponding change in product prices. However, if we assume that the highest valuations stay the same as market sizes change (that is, the value factors change along with market sizes to keep the highest valuations unchanged), we can state the following result.

Proposition 7. (Impact of Market Sizes on Prices) *Suppose that the highest valuations for both products remain fixed as the market size changes (i.e., $s \sim M^{-1}$ and $d \sim (M+m)^{-1}$). Then,*

(i) if $R < \bar{\gamma}$ (Firm 1 offers D), the prices are not affected by the change in the market size;

(ii) if $R \geq \bar{\gamma}$ (Firm 1 offers S only), p_S (resp., p_D) increases (resp., decreases) with M and decreases (resp., increases) with m.

In the semi-duopoly case (Firm 1 offers product D), the market sizes do not affect the prices, which corresponds to the single-product duopoly market. It may be less intuitive, however, that the prices are sensitive to the market sizes in the remaining case. Notice that when Firm 1 does not offer product D, it has (through product S) something similar to monopoly power over the high-end market. As M increases, customer valuation decreases at a slower rate with type, θ, and a larger portion of customers values product S higher than D. In some sense, Firm 1 is approaching (but may never reach) the *real* monopoly over the flexible market. Notice also that p_S always remains below the monopoly price, $\frac{sM}{2}$. *A larger M, therefore, enables Firm 1 to get closer to the monopoly price.* Other effects could be explained in a similar fashion.

4.3 Impact of Value Factors (s and d)

We next examine the impact of value factors, s and d, on the equilibrium decisions. When D is the only product offered in the market (that is, $R \leq \gamma$), each firm serves one third of the total market, $M + m$, and value factors do not have any effect on the capacities. Thus, we restrict our analysis to scenarios in which both products co-exist in the market. Our results follow directly from Proposition 1, and hence we omit the proofs.

Proposition 8. (Impact of Value Factors on Capacities) *When both products, S and D, are in the market ($R > \gamma$), the following holds.*

(i) (Product capacity) y_S *increases (resp., decreases) with s (resp., d),* y_{D_1} *decreases (resp., increases) with s (resp., d), while* y_{D_2} *is not affected by changes in s or d.*

(ii) (Firm capacity) y_1 *does not change with s (resp., d) when $s < \frac{R+1}{R-2}d$ (resp., $d > \frac{R-2}{R+1}s$), and it increases (resp., decreases) with s (resp., d) otherwise;* y_2 *is not affected by changes in either s or d.*

It is natural that capacity for a product increases when the competing product is not popular. In addition, as mentioned at the beginning of this section, the *value factors have no impact on the total capacity of either firm in* semi-duopoly. Thus, for instance, Firm 1's total capacity decreases with d when it offers only product S (until it adds product D to its portfolio), and it increases with s after it drops product D from its portfolio.

Proposition 9. (Impact of Value Factors on Prices)

(i) p_S *(resp., p_D) increases with s (resp., d).*

(ii) Given that the product is included in the portfolio, p_S *decreases with d, while* p_D *decreases with s for $R > \bar{\gamma}$ and is not affected by s when $R \leq \bar{\gamma}$.*

A higher value of d makes product D more "threatening" to product S (i.e., reduces the advantage of Firm 1 over Firm 2), and hence Firm 1 reduces the price of product S in response to the change in market valuations (note that p_S always remains higher than p_D, as shown in Proposition 4). When the market conditions cause $p_S = p_D$, Firm 1 benefits by removing product S from its portfolio, and we denote its price as $p_S = 0$.

A higher value of s, however, will not cause a reduction in the price of product D under *semi-duopoly* ($R \leq \bar{\gamma}$). To see this, assume that Firm 1 cannot manufacture product D. Then, an increase in s may reduce p_D. When Firm 1 has the option to include product D into its portfolio, the competition in prices becomes less intense as Firm 1 wants to obtain a portion of the market for product D as well. However, one should not assume that Firm 2 becomes better off (because p_D can remain unchanged) when Firm 1 can manufacture product D, because it will lose a part of its market share.

The discussion above implies that the *development of disruptive technology increases competition more than the development of sustaining technology.* This happens because any progress in sustaining technology only enhances the existing difference between the products, while progress in disruptive technology reduces this difference, which may eventually reverse the position of the two technologies. We also notice that *Firm 1 is more sensitive to value factors than Firm 2*, in both the capacity and the pricing decisions, because Firm 1 has the option to include product D in its portfolio or leave it out, while Firm 2 has no such choices in its product offering. We illustrate our results with a couple of examples.

Example 3. *Suppose that $M = 120$, $m = 40$, and $s = 9$, which implies that $R = \frac{M}{m} = 3$ and $\gamma = \frac{d}{9-d}$. Figures 8 and 9 show how the equilibrium capacity and price change when d increases*

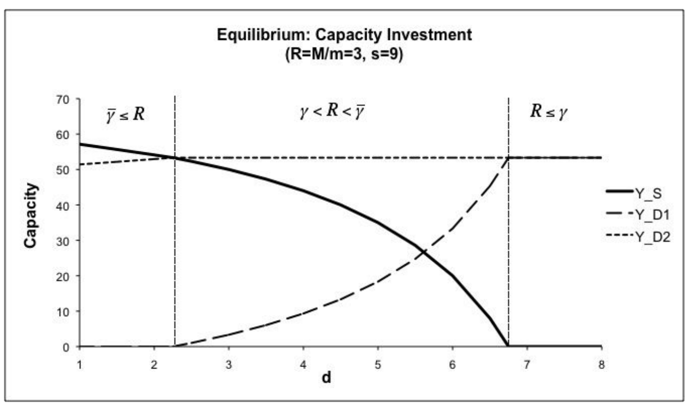

Figure 8: Equilibrium in capacity when the value factor d varies.

from 1 to 8. When d is small, $\bar{\gamma} < R = 3$, and Firm 1 does not offer product D. As d increases, $\gamma < 3 < \bar{\gamma}$, Firm 1 increases the capacity for product D. During this time, the price of product D increases with d, while the price of S decreases with d, until both of them reach \$360 (which happens at $\gamma = R = 3$). When $\gamma \geq 3$, both firms offer only product D.

Example 4. *Suppose that $M = 120$, $m = 40$, and $d = 3$, which implies that $R = \frac{M}{m} = 3$ and $\gamma = \frac{3}{s-3}$. Figures 10 and 11 show how the equilibrium capacity and price change when d increases from 3 to 15. Note that the graph in Figure 10 represents (in general) a horizontal inverse of that*

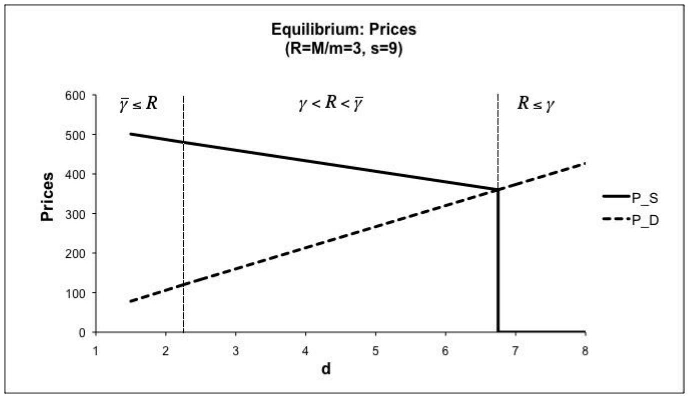

Figure 9: Equilibrium in price when the value factor d varies.

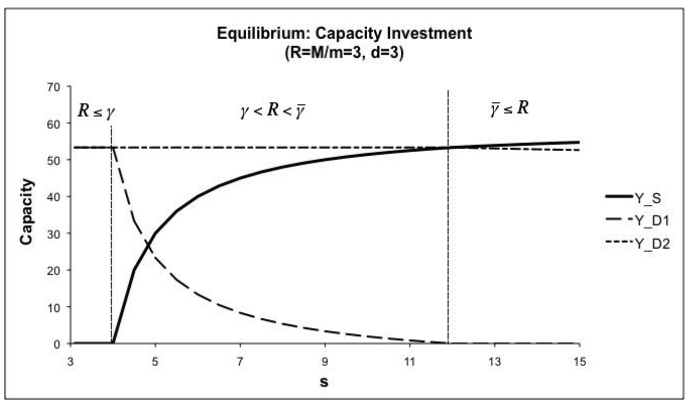

Figure 10: Equilibrium in capacity when the value factor s varies.

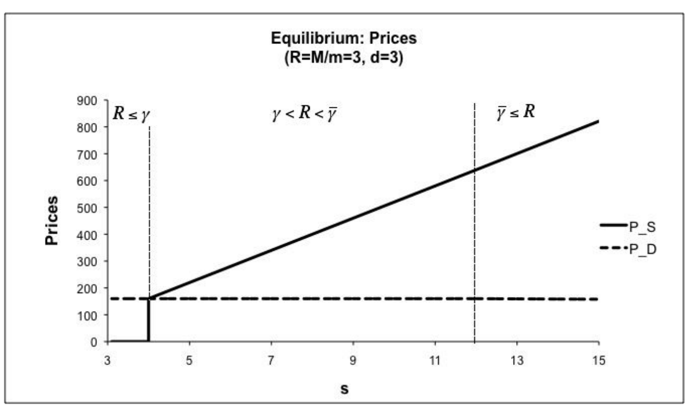

Figure 11: Equilibrium in price when the value factor s varies.

in Figure 8. The graph in Figure 11, on the other hand, is quite different from that in Figure 9. As we have discussed before, p_D is not affected by changes in s when $s < 12$ ($R < \bar{\gamma}$). When $s \geq 12$, p_D has an insignificant downward trend, which may not be captured in the current scale of the graph.

4.4 Effects of Positive Marginal Capacity Costs (c_S and c_D)

Throughout this section, we have assumed zero marginal capacity costs, $c_S = c_D = 0$. The impact of positive capacity costs on capacity decision is obvious—Corollary 1 implies that we should replace M with $M - \bar{c}_S$ and m with $m + \bar{c}_S - \bar{c}_d$ in our analysis. Thus, positive marginal cost of product $i, i \in \{S, D\}$, will have a *negative* impact on the capacity of product i. However, note that positive c_S may have a *positive* impact on the capacity of product D.

The analysis of the impact of positive capacity costs on prices is not that straightforward. When we compare games $B(m, M, \bar{c}_S, \bar{c}_D)$ and $B(m + \bar{c}_S - \bar{c}_D, M - \bar{c}_S, 0, 0)$, the equilibrium unit *prices* for the later game represent the equilibrium marginal *profit* for the original game. In other words, if the equilibrium prices in $B(m + \bar{c}_S - \bar{c}_D, M - \bar{c}_S, 0, 0)$ are p_S and p_D, then the equilibrium prices in $B(m, M, \bar{c}_S, \bar{c}_D)$ should be $p_S + c_S$ and $p_D + c_D$. Thus, our observation that $p_S > p_D$ when the two products co-exist in the market should be revised, in scenarios with positive marginal costs, to "product S is more profitable than product D."

5. Uncertain Disruptive Technology

The model that we have analyzed in §3 and §4 is deterministic—we assume that both firms know the exact values of market sizes and value factors before making their decisions. In this section, we analyze how uncertainty in actual realization of the properties of disruptive technology may affect the equilibrium outcomes. More specifically, we assume that the value factor d can take two values, $\{d_h, d_l\}$, with respective probabilities $\{\alpha, 1 - \alpha\}$. We assume that $d_h > d_l$, which implies $\gamma_h > \gamma_l$ and $\bar{\gamma}_h > \bar{\gamma}_l$, and we restrict our analysis in this section to the semi-duopoly cases, which means that under either realization of d Firm 1 offers both products in the ex-post optimum. That is, we assume that $\{d_h, d_l\}$ satisfy $\gamma_h < R < \bar{\gamma}_l$, where R is the standardized market ratio, $\overline{M}/\overline{m}$. Extreme cases in which Firm 1 offers only one product are not discussed here because we are more interested in analyzing the impact of uncertainty on firms' decisions in situations with diversified product portfolios. In §3, we analyzed a capacity-constrained Bertrand game and a Cournot game, and one of our key results in that section was that both games yielded the same equilibrium in capacities and prices. However, as we will show in this section, when uncertainty is introduced into the model, this result might change.

In our analysis of the Bertrand game, we assume that the true value of d is revealed after the capacity decisions are made, but before the pricing decisions. Thus, while firms may not be able to set their capacities according to the true d, they could enhance their profits through a strategic price setting. While in the model without uncertainty firms always utilized their entire capacity, they may now be left with some idle capacity. Van Mieghem and Dada (1999) refer to such strategies as *hold-back*.

In our analysis of the Cournot game, we assume that each firm determines its production quantities before the true value of d is revealed. However, Firm 1 can make some costly quantity conversion between the two products after observing the true value of d, while keeping the total quantity unchanged. In other words, if the committed quantity for Firm 1 is $\{q_S, q_{D_1}\}$, it may choose to modify it to $\{q_S + \Delta, q_{D_1} - \Delta\}$ and pay $c\Delta$ for the conversion, where c is the unit cost of such conversion. This may occur when resources, such as raw materials, labor, financial resources, etc., need to be secured long before the production actually takes place. In such scenarios, firms have to make commitments while uncertainty has not yet been resolved. Once the true values become known, a firm cannot breach the existing contracts, but it may increase its profits if the contracts have some built-in flexibility that allows them to take advantage of new information. For instance, as suggested in the uncertain Cournot game, Firm 1 may be allowed to pay a conversion fee and gain flexibility in determining the type of each committed unit.

5.1 Uncertain Bertrand Game (UBG): Flexible Quantity, Fixed Capacity

The timeline of the UBG is as follows:

Stage 1. Capacity decision: Firm 1 selects (y_S, y_{D_1}) and pays $c_S y_S + c_D y_{D_1}$; Firm 2 selects $y_2 = y_{D_2}$ and pays $c_D y_2$.

Stage 2. Value factor d is realized.

Stage 3. Pricing decision: Firm 1 determines (p_S, p_{D_1}), and Firm 2 determines p_{D_2}.

Note that after stage 2, in which the uncertainty is resolved, the game becomes deterministic. As we have shown in §3, there will be a single price for product D, $p_{D_1} = p_{D_2}$. Consequently, the last stage can be revised as:

Stage $3'$. Production decision: Firm 1 determines (q_S, q_{D_1}), and Firm 2 determines $q_2 = q_{D_2}$, where $q_i \le y_i$, $i \in \{S, D_1, D_2\}$.

We determine the equilibrium capacity decisions through backward induction. At stage 3, in which the capacities and the value factor d are known to both parties, the firms have to decide their optimal production quantity. Recall that we have assumed that in this game the capacity is costly, while the production is costless. Thus, the production decision serves as a tool in the selection of the right price. The optimal production strategies are given in Proposition A1 in the Appendix. The *reaction policy* for Firm 2 is simple—if there is enough capacity, *produce up to* $\frac{M+m-q_1}{2}$. For Firm 1, it could be described as follows:

1. Try to meet the optimal production levels for both products, and identify any leftover capacity.
2. If there is any leftover capacity for D, try to bring the total production up to $\frac{M+m-q_2}{2}$.
3. If there is any leftover capacity for S, try to bring the total production up to $\frac{M-\frac{d}{s}q_2}{2}+\frac{s-d}{s}y_{D_1}$.

In stage 1, firms select the capacities that maximize their ex-ante expected profit. The following result indicates one important feature of the equilibrium decisions.

Corollary 2. *In equilibrium, firms produce up to capacity for product D (resp., S) when $d = d_h$ (resp., $d = d_l$).*

This result is rather intuitive. On the one hand, the capacity for the product favored by the realization of the random factor (product D when $d = d_h$, product S when $d = d_l$) is fully utilized. On the other hand, it implies that a firm should not invest in capacity of each product that exceeds the maximum production under either realization of d.

Depending upon the game parameters, the equilibrium capacity decisions may take *five* different forms (denoted as types I, II, III, IV, and V). Their closed-form expressions can be found in the Appendix. We will briefly address all five types and then provide further details on types II, III, and IV, which occur for extreme cost values. Simulations in §5.3 illustrate how the equilibria may vary as parameters change.

We first analyze capacity utilization under different realizations of factor d. While Corollary 2 characterizes *product-wise* capacity usage, the following theorem describes the *firm-wise* capacity usage as the degree of uncertainty changes.

Theorem 3. *When all parameters in a UBG are fixed, there exist $0 \le \alpha_1 \le \alpha_3 \le \alpha_5 \le \alpha_6 \le \alpha_4 \le \alpha_2 \le 1$ and $0 \le \alpha_7 \le \alpha_8 \le 1$ such that the equilibrium type varies with α as shown in Table 2. More specifically, equilibrium of type V exists for $\alpha \in ((\alpha_1, \alpha_3] \cup [\alpha_4, \alpha_2)) \cap (\alpha_7, \alpha_8)$, and equilibrium of type I exists for $((\alpha_1, \alpha_3] \cup [\alpha_4, \alpha_2)) \cap ([0, \alpha_7] \cup [\alpha_8, 1])$.*

In addition, the capacity usage under each type of equilibrium is characterized in Table 3.

α	$[0,\, \alpha_1]$	$(\alpha_1,\, \alpha_3]$	$(\alpha_3,\, \alpha_5]$	$(\alpha_5,\, \alpha_6)$	$[\alpha_6,\, \alpha_4)$	$[\alpha_4,\, \alpha_2)$	$[\alpha_2,\, 1]$
NE Type	IV	I/V	II	III	II	I/V	IV

Table 2: Equilibrium Type Under Different Risk Exposures

Equilibrium Type	IV	I	V	II	III
Better Disruptive Technology: d_h	(Full, Full)	(Idle, Full)	(Idle, Full)	(Idle, Full)	(Idle, Full)
Worse Disruptive Technology: d_l	(Full, Full)	(Full, Full)	(Full, Idle)	(Idle, Full)	(Idle, Idle)

Table 3: Capacity for Each Type of Equilibrium: (Firm 1, Firm 2)

Table 2 shows how the equilibrium types change when the probability of a better disruptive technology increases. We note that some of the intervals may be degenerate (empty). In other words, we may only have selected types of equilibria (some examples are provided in §5.3).

Table 3 characterizes the capacity usage for all types of equilibria. A direct observation is that low uncertainty (a very high or very low value of α) leads to type IV equilibrium, which implies a full utilization of capacity for both firms, regardless of the realization of d. However, as uncertainty increases, some of the capacity may be left unused. Suppose, for instance, that the actual α leads to a type I equilibrium. As discussed earlier in this section, if product S has leftover capacity after meeting the optimal production level, the total production will be lower than when D has leftover capacity. Let us write $y_S = y_S^h + y_S^l$. One may argue that when product S is likely to capture a larger market share (that is, $d = d_l$, or $1 - \alpha > 1 - \alpha_3$, which implies type I), Firm 1 would invest in y_S^l to cover the possibility that $d = d_l$. This part of capacity will not be used if $d = d_h$, in which case Firm 1 will reduce the production quantity (so that only capacity y_S^h is used) and get a better price for S.

It can also be observed from Table 3 that a high degree of uncertainty ($\alpha \in [\alpha_3, \alpha_4]$; types II and III) implies that Firm 1 always has some idle capacity, regardless of the realization of d. Indeed, as the degree of uncertainty increases, the equilibrium changes as type IV → type I/V → type II/III, the corresponding capacity usage for $d = d_h$ changes as Full → Idle → Idle, and for $d = d_l$ it changes as Full → Full → Idle. Thus, *the capacity usage of Firm 1 decreases as the degree of uncertainty increases*. The direction of changes in the capacity usage of Firm 2, however, does not follow immediately from Tables 2 and 3. As we can see, the capacity utilization of Firm 2 can theoretically change when $d = d_l$ as Full → Idle → Full. However, numerical examples in §5.3 demonstrate strong degeneracy of some regions mentioned above (equality for some α_i's), which causes Firm 2 to observe the same trend in capacity utilization as Firm 1 when uncertainty increases.

So far, we have discussed the impact of the degree of uncertainty on the equilibrium types; the following figure illustrates how equilibrium types may vary with marginal costs.[5] The market parameters for this example are given in the title of the figure. The upper right area depicts the scenario in which both marginal costs are large; the capacity is then fully utilized, and we have an equilibrium of type IV. When both marginal costs are small, the equilibrium is of type III, and at least one of the firms has some idle capacity. The area in the middle is divided by equilibria of type I and V, and the corresponding capacity utilization is in the "middle" range. In this example, equilibrium of type II occurs only at $c_S = c_D = 0$.

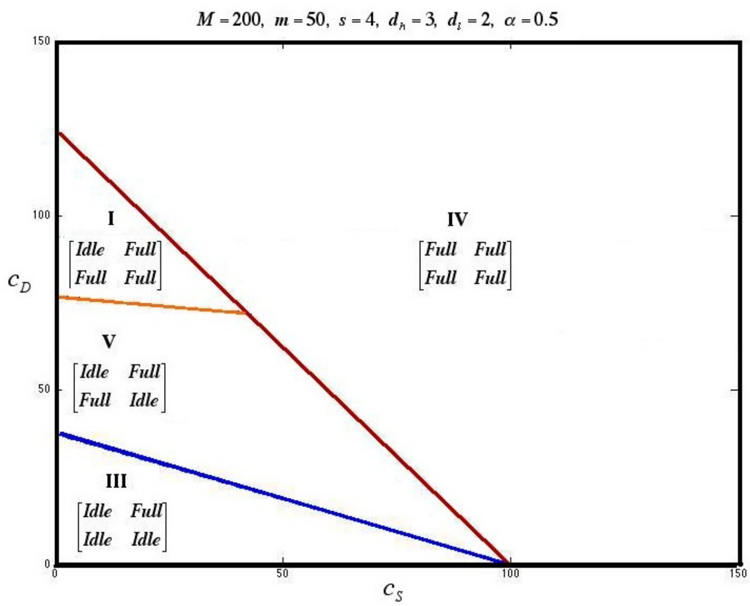

Figure 12: Equilibrium type varies with marginal costs.

We now provide some additional analysis of types IV, II, and III equilibria to illustrate how different parameters can lead to significantly different investment decisions.

Proposition 10. (Type IV Equilibrium) *In a UBG in which value factor d achieves d_h with probability α and d_l with probability $1-\alpha$, and c_S and c_D are large, both firms in equilibrium produce up to their capacity, regardless of the realization of d. The equilibrium capacity decisions are*

$$y_S^{IV} = \frac{M - \frac{c_S}{s}}{2} - \frac{E[d]\left(m + \frac{c_S}{s} - \frac{c_D}{E[d]}\right)}{2(s - E[d])}, \; y_{D_1}^{IV} = \frac{s\left(m + \frac{c_S}{s} - \frac{c_D}{E[d]}\right)}{2(s - E[d])} - \frac{M + m - \frac{c_D}{E[d]}}{6}, \; y_2^{IV} = \frac{M + m - \frac{c_D}{E[d]}}{3}.$$

Proposition 10 implies that high marginal capacity costs reduce the impact that the randomness in the value factor d has on both production and capacity decisions for either firm—the firms plan

[5]For full descriptions of conditions that have to be satisfied for different equilibrium types, please see the Appendix.

the capacity as in the deterministic case in which the value factor of the disruptive technology is $E[d]$.[6] In addition, there is no idle capacity under either realization of d. We can see that *the high marginal capacity costs result in full capacity utilization.*

Proposition 11. (Type II Equilibrium) *In a UBG in which value factor d achieves d_h with probability α and d_l with probability $1-\alpha$, and marginal costs are zero,*[7] *the incumbent over-invests in capacity, while both firms produce at the ex-post optimum. The equilibrium capacity decisions are*

$$y_S^{II} = \frac{M - \gamma_l m}{2},\ y_{D_1}^{II} = \frac{(1+\gamma_h)m}{2} - \frac{M+m}{6},\ y_2^{II} = \frac{M+m}{3},$$

and the production quantities are $q_{1_h}^{II} = q_{2_h}^{II} = q_{1_l}^{II} = q_{2_l}^{II} = \frac{M+m}{3}$.

Proposition 11 depicts a situation that is the opposite of the one described in Proposition 10. More specifically, when $c_S = c_D = 0$, each firm selects the capacity that enables production of the ex-post optimum.[8] Under either realization of d, Firm 2 produces at its capacity, while Firm 1 has some idle capacity.

Proposition 12. (Type III Equilibrium) *In a UBG in which marginal capacity costs are small or uncertainty is high, firms over-invest in capacity, which leads to over-production.*

It is an interesting observation that when costs are small but strictly positive, firms may behave in a way that might be considered irrational; that is, they invest *above* the ex-post optimum. While both firms select the capacity level above the one chosen in the deterministic case, Firm 1 will have idle capacity under either realization of d (albeit in one case for product S, while in the other for product D), while Firm 2 has idle capacity only when disruptive technology achieves its lower valuation. Thus, both firms will *over-invest* in their capacity. As a result of this over-investment, firms also produce higher total quantity than in the deterministic case. This further implies that the prices can be below the ones charged under the deterministic scenario and that a larger market share can be served. Therefore, the *customers benefit from uncertainty in disruptive technology when marginal capacity costs are low.*

5.2 Uncertain Cournot Game (UCG): Fixed Quantity, Flexible Type

We now analyze the UCG. Again, d is assumed to be d_h with probability α, and d_l otherwise. Both firms have to commit to a certain production quantity before d is realized. These quantities are

[6] We also note that, when the marginal costs are high enough, we may only have equilibria of type IV, regardless of the value of α (see §5.3).

[7] Type II equilibrium does not restrict the marginal costs to be zero, while zero marginal costs will result in a type II equilibrium.

[8] We also note that, in this case, we may only have equilibria of type II, regardless of the value of α (see §5.3).

usually not optimal after the true d is revealed. In this game, we require the total quantity to be fixed for each firm (this may occur, for instance, when raw materials, labor, and other resources have already been contracted, and the contracted amount would be difficult to change). However, Firm 1 has some flexibility in the design of its portfolio—it can adjust its production allocation given the fixed total quantity, q_1, and any reallocation of production quantities incurs a unit cost c. This could be viewed as delayed product differentiation or flexible production, in contrast to responsive pricing in the UBG.

The timeline of the UCG is as follows:

Stage 1. Capacity decision: Firm 1 selects $(q_S,\, q_{D_1})$; Firm 2 selects $q_2 = q_{D_2}$.

Stage 2. Value factor d is realized.

Stage 3. Pricing decision: Firm 1 selects Δ, produces $(q_S + \Delta,\, p_{D_1} - \Delta)$, and pays $c_S(q_S + \Delta) + c_D(q_{D_1} - \Delta) + c|\Delta|$; Firm 2 produces q_{D_2} and pays $c_D q_2$.

We can verify that the total equilibrium production of the UCG is the same as in the deterministic case. The initial selection of the committed quantity for product S falls between the two ex-post optima, L_h and L_l, where $L_u = \frac{\overline{M} - \gamma_u \overline{m}}{2}, \quad u \in \{h,\, l\}$.

Proposition 13. *Consider a UCG in which value factor d achieves d_h with probability α and d_l with probability $1-\alpha$, and Firm 1 is allowed to make quantity conversion at cost c per unit. Then, the equilibrium quantity decisions, $\vec{q}^{CU}$, satisfy*

$$q_1^{CU} = q_2^{CU} = \frac{\overline{M + m}}{3}, \quad L_h \leq q_S^{CU} \leq L_l.$$

Because we assume that either d_h or d_l makes the market a semi-duopoly, the reaction functions are not binding in the production of S. Therefore, the total quantities remain the same as when d is deterministic. In addition, Firm 1 would not benefit by setting the initial quantity too high or too low when it is aware of the upper and lower bound of the right quantity. We next characterize the initial choice of q_S.

Proposition 14. *Consider a UCG, and let $\widetilde{c} = \frac{c}{2(s-d)}$.*

1. *If $L_l - L_h \leq \widetilde{c}_l$, then*

$$q_S^{CU} = \frac{\overline{M} - \frac{E[d]}{s - E[d]}\overline{m}}{2}. \tag{9}$$

2. If $\widetilde{c}_l < L_l - L_h < \widetilde{c}_l + \widetilde{c}_h$*, then there exist* $0 \leq \alpha_1^{CU} \leq \alpha_2^{CU} \leq 1$ *s.t.*

$$q_S^{CU} = \begin{cases} L_l - \frac{\alpha}{1-\alpha}\widetilde{c}_l & \text{if} \quad \alpha \in [0,\, \alpha_1^{CU}), \\ L_h + \frac{\alpha}{1-\alpha}\widetilde{c}_h & \text{if} \quad \alpha \in (\alpha_2^{CU},\, 1], \\ \frac{\overline{M} - \frac{E[d]}{s-E[d]}\overline{m}}{2} & \text{if} \quad \alpha \in [\alpha_1^{CU},\, \alpha_2^{CU}]. \end{cases}$$

3. If $\widetilde{c}_l + \widetilde{c}_h \leq L_l - L_h$*, then*

$$q_S^{CU} = \begin{cases} L_l - \frac{\alpha}{1-\alpha}\widetilde{c}_l & \text{if} \quad \alpha \leq \frac{1}{2}, \\ L_h + \frac{\alpha}{1-\alpha}\widetilde{c}_h & \text{if} \quad \alpha > \frac{1}{2}. \end{cases}$$

The proof is given in the Appendix. The first case occurs when d_h and d_l are close—the selection of q_S only uses the expectation of d, and Firm 1 does not perform any conversion regardless of the realization of d. The last scenario occurs when d_h and d_l are far apart—the committed quantity is selected closer to the ex-post optimum with higher probability of occurrence. Finally, the second scenario depicts the case in which d_h and d_l are moderately different. When one of the outcomes occurs with a much higher probability, q_S is set closer to the corresponding ex-post optimum. However, when α is somewhere in the middle (that is, uncertainty in d is high), q_S will be set closer to the "middle", which is similar to the deterministic case in which d takes the value of its expectation.

5.3 Numerical Examples

Our analysis so far shows that introduction of uncertainty leads to different equilibrium outcomes under Bertrand and Cournot competitions. In the UBG with capacity constraints, firms may over-invest in the total capacity and not use up all of it in the final production. In the UCG, in which quantities are predetermined but Firm 1 may switch quantities between different product types, the total production quantity remains the same as in the deterministic game.

In this section, we explore through numerical examples how the expected profits and the equilibria vary as the game parameters change. We assume the flexible market size $M = 200$, the dedicated market size $m = 50$, the value factor for sustaining technology $s = 8$, and the value factor for disruptive technology that may take two values, $d_h = 6$ and $d_l = 4$. The marginal capacity costs in the UBG and marginal production/conversion costs in the UCG, $(c_S,\, c_D,\, c)$, vary through the following three sets: (200, 150, 120), (80, 40, 60), and (20, 15, 20). All examples satisfy our assumption that $\gamma_h \leq R \leq \bar{\gamma}_l$ (that is, Firm 1 will produce both types of products ex-post). We consider the changes in the expected profits for each firm in the two games when α, the probability that the value of disruptive technology achieves d_h, varies across [0.05, 0.95] with step 0.05.

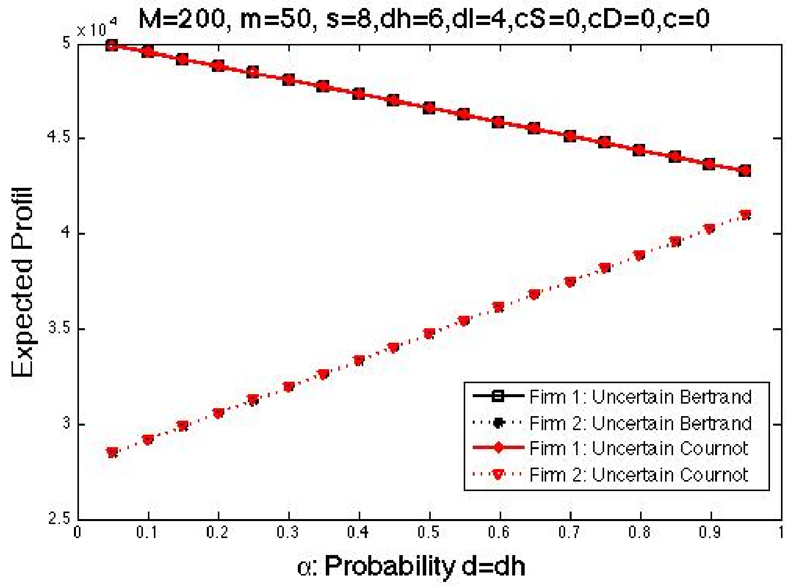

Figure 13: Uncertain Bertrand game with equilibrium of type II only.

Figure 13 is generated with (c_S, c_D, c) = (0, 0, 0), and this benchmark case verifies the feasibility of our model. When the capacity/production and quantity conversion are costless, the two uncertain games yield the same profit. For the UBG, the equilibrium is constantly of type II (see Proposition 11). The expected profit for Firm 1 (resp., Firm 2) decreases (resp., increases) as the disruptive technology is more likely to have a high value factor.

Figure 14 is generated with (c_S, c_D, c) = (200, 150, 120). For the UBG, the marginal capacity costs are high, and Proposition 10 implies that the equilibrium is of type IV.[9] For the UCG, the equilibrium belongs to the last category in Proposition 14, and this is the reason for a "kink" in the top line at $\alpha = 0.5$, where Firm 1 changes its quantity strategy from *close-to-*L_l to *close-to-*L_h. We can also observe that Firm 1 generates lower expected profit in the UBG than in the UCG, while the opposite holds for Firm 2. Due to the high investment costs, Firm 1 has to be extremely cautious in the UBG, but this is partially relaxed in the UCG, where Firm 1 can switch between product types.

The last graph, Figure 15, represents costs in the middle range: (c_S, c_D, c) = (80, 40, 60). This provides an illustration of the scenario in which an UBG may have multiple types of equilibria as α changes: for $\alpha \in [0.05, 0.15]$ the equilibrium is of type I, for $\alpha \in [0.45, 0.65]$ the equilibrium is of type III, for $\alpha \in [0.20, 0.40] \cup [0.70, 0.75]$ the equilibrium is of type V, and for $\alpha \in [0.80, 0.95]$ the equilibrium is of type IV. We note that for $\alpha \in [0.75, 0.85]$ we can also have an equilibrium of type V.

[9]Type IV remains to be the only type of NE if we scale the costs down for up to 20%.

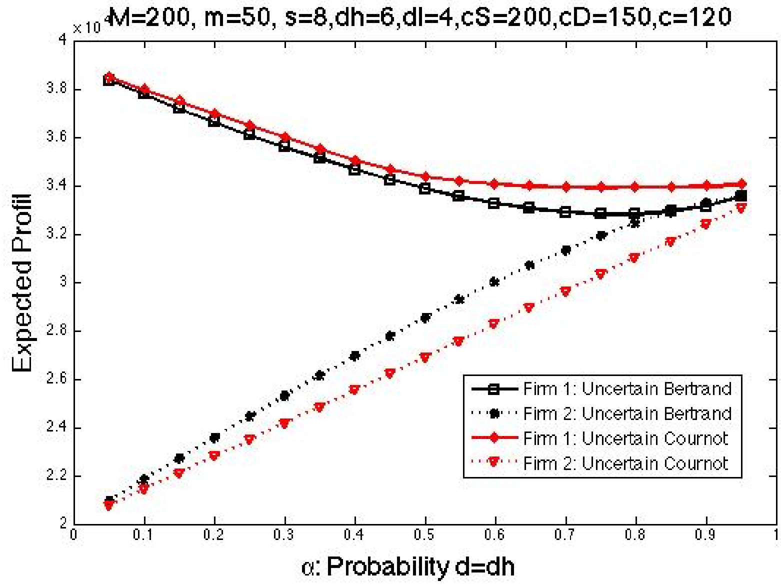

Figure 14: Uncertain Bertrand game with equilibrium of type IV only.

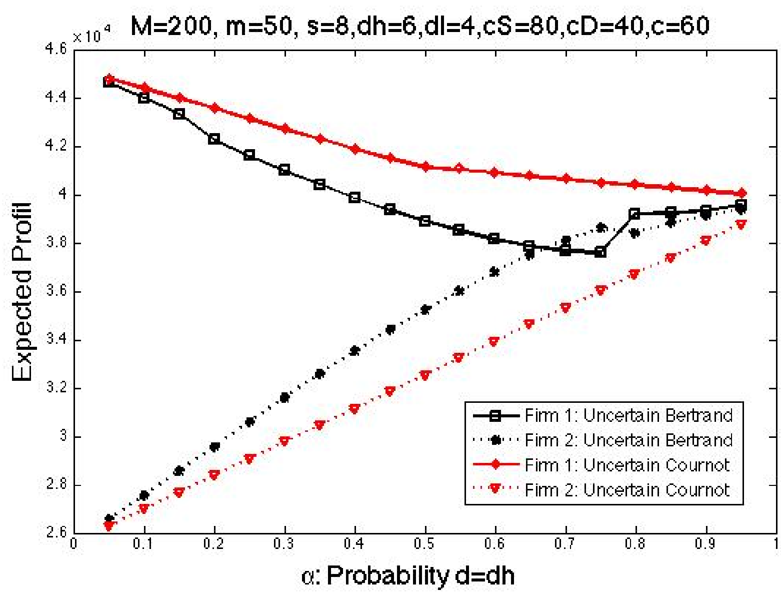

Figure 15: Uncertain Bertrand game with multiple types of equilibria.

The above figures provide some additional insights about the games with uncertainty:

- The UCG provides an upper (resp., lower) bound for the profit that Firm 1 (resp., Firm 2) can achieve in the UBG. Thus, the two firms have opposite preferences for the two games.
- The difference in expected profits achieved under the two games (UBG vs. UCG) is maximized when the degree of uncertainty is high, and minimized when the degree of uncertainty is low.
- A "promising" disruptive technology (that is, an increasing value of α) reflects positively on the expected profit of Firm 2. However, this might not be good news for Firm 1, unless the probability of a high valuation, α, is above some critical level. For example, Figure 14 illustrates that the expected profit for Firm 1 is decreasing until α reaches the value 0.8.

6. Concluding Remarks

Competition between products based on sustaining technology and those based on disruptive technology has always been intense. Today, the pace of innovation increases continuously (especially in high-tech industries), and the greatest threat for existing technologies comes from the disruptive ones. With its dramatically different cost, functionality, durability, product image, etc., disruptive technology places itself at the opposite side of the spectrum from the sustaining technology. This incomplete substitution between products could be looked upon as multi-dimensional vertical differentiation (which has been studied, for instance, by Moorthy, 1988, and Vandenbosch and Weinberg, 1995). However, this type of research generally imposes rather generic market assumptions, which may not fully reflect the setting in which the long-established sustaining technology has to face the emerging disruptive technology. In order to fully capture such an environment, the model has to accommodate the following characteristics: (1) disruptive technology can steal customers from the market for an existing product while at the same time creating a new market dedicated exclusively to the new product, (2) firms should not be restricted to adopting a single technology into their portfolio.

Our model aims to provide understanding of the impact that disruptive technology may have on an industry as a whole and uses a framework in which both of the above conditions are met. Unlike the existing literature, our results suggest that the incumbent may adopt disruptive technology simultaneously with the entrant and that the two will price the new product at the same level. Our model also helps the incumbent to identify the proper timing for entering the market for the new product (by analyzing the valuation of the new product and market conditions), and the proper timing and conditions for abandoning the old product (when its price is high, rather than engaging in a price war with the new product). Incorrect identification of these conditions may help to explain the numerous failures of companies dealing with disruptive technologies mentioned in Christensen (2003).

For many innovative products, the most critical issue is the chance of success. We model this uncertainty by assuming that the value factor of disruptive technology is random. This part of our analysis highlights the impact that the *degree of uncertainty* and *marginal investment costs* may have on investment, production, and marketing decisions. In contrast to the deterministic game in which firms always fully utilize their capacity, *over-investment, over-production*, and *low capacity usage* might occur when the new technology exhibits high uncertainty and/or low marginal costs. This result can be particularly useful for industries with high fixed costs and low marginal costs (e.g., telecommunications, software, power supplies, online retailing, online magazines, etc.).

There are many ways to extend our analysis. On the supply side, one might allow for the possibility of multiple established/entrant firms and analyze whether cooperation/collusion could take place between the incumbents and/or the entrants. On the demand side, one might allow the customers to have two products at the same time, or introduce a dedicated market for the sustaining technology and analyze whether it changes current results. Moreover, the analysis of uncertainty may include, for instance, randomness in market parameters. The model could also be modified to include the "brand effect" for the incumbent, which would lead to differentiated pricing of products based on the same technology. Finally, the game could be extended to a multi-period setting, to better analyze timing-related strategies on pricing and production (one may refer to Deng and Yano (2006) for single product case). Related topics under such setting may also include optimal market-entry time, first-mover advantage, etc.

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