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# Experimental Search for Quantum Gravity

**Bachelor Thesis**

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**SUPERVIZOR: doc. Franz Hinterleitner, Ph.D.** **BRNO 2011**

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# Declaration

I hereby declare that I wrote this bachelor thesis by myself using only the referenced sources. I agree that this thesis will be made available for study purposes.

Brno, 20th May 2011 …………………………………

Kateřina Klánová

# Abstract

There are many different theories that focus on combining quantum mechanics and general relativity. Recently, the technological progress made it possible to test some aspects of these theories experimentally. This thesis provides a summary of some quantum gravitational theories and presents several experimental methods. These methods include time-of-flight experiments in the Fermi Gamma-ray Space Telescope, studies of Planck scale by matter wave interferometry, experiments in neutrino telescopes, examining the effects of the hypothetical variable value of different physical constants (like the speed of light) on redshift, and searching for microscopic black holes at the Large Hadron Collider. To this date, there isn't any conclusive experimental evidence supporting any theory of quantum gravity, but new boundaries and limits were set on several of them.

# Abstrakt

Je mnoho různých teorií, které mají za cíl spojit kvantovou mechaniku s obecnou teorií relativity. V poslední době technický vývoj umožnil experimentálně ověřovat některé z aspektů těchto teorií. Tato práce nabízí přehled některých kvantově-gravitačních teorií a prezentuje i několik experimentálních metod. Mezi tyto metody patří experimenty pomocí satelitu Fermi Gamma-ray Space Telescope, zkoumání Planckovy stupnice pomocí interference De Broglieho vlny, experimenty v neutronových detektorech, pozorování vlivu hypotetické variabilní hodnoty některých fyzikálních konstant (například rychlosti světla) na červený posuv a hledání mikroskopických černých děr ve Velkém hadronovém urychlovači. Dodnes není žádné přesvědčivé experimentální potvrzení žádné kvantově-gravitační teorie, ale některým z nich byly stanoveny nové meze a hranice.

# Contents

# 1 Introduction

At the beginning of the 20th century, physics succeeded in explaining a wide range of phenomena by using two different apparatus: quantum mechanics and the general theory of relativity. Both quantum mechanics and general relativity have been acknowledged as the base of modern physics and haven’t been disproved since their creation. Combining these two theories has become probably the most essential task in today’s theoretical physics.

There are many different approaches to this problem. Most of them try to modify general relativity, so that it wouldn't contradict quantum field theory. In the last few years, technology in various fields got to the point, where we may soon be able to test some aspects of these theories experimentally. The research is expected to give these theories restrictions and to falsify some of their versions or dead ends.

The objective of this thesis was to outline some of the existing quantum gravitational theories and to present modern experiments that could test several of their implications. Many experiments still need to wait for technology to advance some more. But some of these experiments have already been performed and I provide the summary of their findings.

# 2 Theoretical background

## 2.1 Space-time

### 2.1.1 The different roles of space and time in history

Throughout history space-time had various meanings in science.

Descartes was the first one to introduce the relativity of motion [1]. He considered space and time fully relative. Location doesn't mean anything unless we choose a frame of reference. Space itself doesn't mean anything if there aren't any objects involved. *Any* kind of motion is completely relative.

Newton considered space-time to be a "stage", where all interactions take place and which exists whether there are any objects in it or not [1]. Location is the part of space where the objects are situated. Motion is a change of location. Because there is no preferred frame of reference, velocity is still relative, but all other aspects of motion are considered absolute, referring to space. This theory was later altered due to Newton's unnecessary use of absolute position. Nowadays, the main difference between Newtonian and Descartes' mechanics is the inclusion of absolute acceleration.

Einstein's special relativity keeps both the relative velocity and the absolute acceleration [1]. But it uses special relativistic space-time, which is a four-dimensional manifold with the Lorentzian metric $\eta_{\mu\nu}$. So it erases the border between space and time and assigns to each object a four-dimensional trajectory describing its existence in space-time—the world line. Acceleration is any change of the world line from a straight line. Another difference from Newtonian mechanics is the introduction of the field as a dynamical object in space-time.

General relativity brings a new type of field—the gravitational field and explains it as encoded in space-time itself; this way the gravitational field determines metric properties of space-time [1]. The energy and momentum of matter described by the stress-energy tensor $T_{\mu\nu}$, causes the curvature of space-time expressed by the Ricci curvature tensor $R_{\mu\nu}$ [2]. Einstein's field equations relate these tensors in the following way:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \tag{2-1}$$

where $g_{\mu\nu}$ is the general metric tensor (gravitational field tensor), $R$ is the scalar curvature, $G$ is Newton's gravitational constant, the speed of light is set $c = 1$. In all previous equations we can replace the Lorentzian metric tensor $\eta_{\mu\nu}$ with the general metric tensor $g_{\mu\nu}$ [1]. In conclusion, the motion of all dynamical objects, including acceleration, is fully relative to the gravitational field; free particles move on geodesic curves determined by the metric $g_{\mu\nu}$.

### 2.1.2 Time in general relativity and quantum mechanics

Unlike in Newtonian mechanics, time in general relativity isn't an absolute quantity [1]. Space-time isn't a stage anymore, but provides dynamical objects. The independence of any physical event of space and time is called the principle of general covariance (or diffeomorphism covariance) and as its consequence there is no preferred time.

The important quantity describing motion of objects is the *relation* between time and space variables, which can be expressed as lines in space-time [1]. We can parameterize any line in an infinite number of ways. If we confine ourselves to a 1+1-dimensional space-time, the motion of any object is defined by the relation between its space coordinate $q(\tau)$ and the conventional "clock" time $t(\tau)$, using any "arbitrary time parameter" $\tau$. The transformation between any two parameters (or sets of parameters) doesn't change the equations of motion. This leads us to believe that a theory of quantum gravity should be created without the exact definition of time.

Since time can't be a preferred parameter, we have a problem in defining phase space. The usual definition of phase space—the space of all states of the system at a given time—is, at first sight, useless in general relativity [1]. So we use the Lagrange definition—the space of solutions of the equations of motion for some chosen time. This equivalent definition turns out to be independent of the particular choice of time.

Now we take a look at how our assumptions reflect in quantum mechanics. In quantum theory, space-time is a static “classical” background—a reference frame with respect to which expectation values are formulated. A quantum state is a probability distribution in phase space. If we restrict ourselves to the time independent concept of states, we come to a surprising result—we need to get rid of the whole Schrödinger picture. Although the two pictures are usually considered formally equivalent, it has been proven that in quantum field theory the Heisenberg picture is gauge invariant and the Schrödinger picture is not [3]. Gauge invariance in quantum field theory corresponds to general covariance (arbitrary coordinate transformation invariance).

Quantum operators corresponding to gauge-invariant physical quantities are Heisenberg operators [1]. It seems that in quantum gravity, only the Heisenberg picture makes sense, it is more fundamental.

## 2.2 Defining the problem of quantum gravity

General relativity says that space-time (or the gravitational field) is a dynamical entity [1]. Quantum mechanics tells us that all dynamical entities have quantum properties. That means that we need to describe the gravitational field as a quantum object using states $\psi$ in a Hilbert space. These states must define space itself. Therefore, the search for quantum gravity can be formulated as a search for a quantum theory whose classical limit is general relativity.

In all quantum field theories a field has infinitely many independent degrees of freedom—they require a non-dynamical background metric [1]. But if we merge it with general relativity and replace the Minkowski space-time with a quantum field, all equations stop making sense.

We have two main directions to take. In the first option, we take a non-dynamical background space-time with the metric $\eta_{\mu\nu}$, expand the gravitational field as $g_{\mu\nu} = \eta_{\mu\nu}$ + fluctuations, quantize the fluctuations and at last try to retrieve general relativity. String theory, for example, takes this approach [1]. The other way is to search for a quantum field theory which does not require a static background space-time. This, on the other hand, is the path of loop quantum gravity.

# 2.3 Lorentz invariance

### 2.3.1 Introduction

Special relativity postulates that all laws of physics are invariant under Lorentz transformations—under changes of the reference frame [4]. Quantum field theories follow special relativity and keep Lorentz invariance in their structure. This makes Lorentz invariance one of the fundamental symmetries of relativity.

### 2.3.2 Lorentz transformation

When talking about the Lorentz transformation, usually it is about *passive (observer) transformation*. In special relativity, an observer Lorentz transformation considers two reference frames with different velocities and orientations [5]. The coordinates in the one system are related to those in the other by an observer Lorentz transformation—a combination of rotations and boosts. The velocity four-vector is not transformed, but its components are, because they are given relative to a transformed coordinate system. Action takes the form of a scalar, so the equations of motion are independent of the reference frame—both observers agree on the laws of physics.

A transformation in a given inertial frame relating two particles or fields with different momentum or spin orientation is called a *particle Lorentz transformation* [5]. Both objects (though being boosted and rotated to each other) are being studied by the same inertial observer. Both the matter and the fields in the experiment are physically transformed into a new configuration. In the case of free particles, this transformation is the inverse transformation of a passive transformation. But in the case of particles interacting with fixed background fields, it is not. If we (particle) transform an experiment, the background field remains unchanged and causes the breaking of Lorentz symmetry.

*Active Lorentz transformation* represents the transformation in a given reference frame, where all particles, fields and background fields are rotated or boosted [5]. Velocity four-vectors are transformed, but the reference system is not.

## 2.4 Large extra dimensions

### 2.4.1 Introduction

If there is a unifying physical theory of all physical interactions, it probably places gravity in a quantum framework [6]. The gravitational force is nearly negligible in the microscopic world, but it gains significance in larger scales. This is because it is much weaker than the other known forces, but it has only one pole (all particles have a positive mass). Thus the gravitational field surrounding large objects (such as planets) is much stronger than—for example—the electromagnetic field (having two polarities makes most objects macroscopically nearly neutral).

The scale, where quantum gravitational effects become strong, can be defined by Planck's constant $h$, Newton's gravitational constant $G$, and the speed of light $c$, and is often called the Planck scale [6]. From these constants we derive the Planck length $l_{Pl}$, Planck time $t_{Pl}$, Planck mass $m_{Pl}$, and Planck energy $E_{Pl}$:

$$l_{Pl} = \sqrt{\frac{\hbar G}{c^3}} \cong 1.61 \times 10^{-35}\,\mathrm{m} \tag{2-2}$$

$$t_{Pl} = \sqrt{\frac{\hbar G}{c^5}} \cong 5.39 \times 10^{-44}\,\mathrm{s} \tag{2-3}$$

$$m_{Pl} = \sqrt{\frac{\hbar c}{G}} \cong 10^{-5}\,\mathrm{g} \tag{2-4}$$

$$E_{Pl} = \sqrt{\frac{\hbar c^5}{G}} \cong 1.22 \times 10^{28}\,\mathrm{eV} \tag{2-5}$$

This gives gravity a natural energy scale $M_{Pl}$.

Large extra dimensions theory introduces $n$ extra dimensions in space, convolved on an $n$-dimensional torus or sphere with the radius $R$ [7]. Three of the fundamental interactions are confined to a four-dimensional membrane (3+1-dimensional space-time). Only gravity propagates in the remaining extra dimensions.

### 2.4.2 Schwarzschild radius

If we search the solution of Einstein's field equations for a spherically symmetric mass distribution, the metric outside the mass is described by the Schwarzschild solution [8]:

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\gamma(r)\mathrm{d}t^2 + \gamma(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 , \tag{2-6}$$

where

$$\gamma(r) = 1 - \frac{\hbar}{cm_{Pl}^2}\frac{2M}{r} , \tag{2-7}$$

$M$ is the total mass of the object, and $\mathrm{d}\Omega$ is the surface element of a 3-dimensional unit sphere. $\gamma(r)$ contains Newton's gravitational potential $GM/r$, which is obtained as a solution of the Poisson equation in 3 dimensions.

If the radius reaches the value:

$$r_s = 2\hbar M / cm_{Pl}^2 = 2GM/c^2 , \tag{2-8}$$

the so-called Schwarzschild radius, the component $\gamma(r)$ vanishes [8] and the space-time metric reaches a singularity. Objects with mass density so high, that its radius is smaller than the Schwarzschild radius (also called the event horizon), collapse into a black hole.

The general solution of the Poisson equation for a particle with mass $M$ in $n$+1-dimensional space-time is the potential [8]:

$$\Phi(r) = \frac{\hbar^{n+1}}{c^{n-1}}\frac{1}{M_f^{n+2}}\frac{M}{r^{n+d}} \tag{2-9}$$

where $r$ is the distance from the source and $M_f$ is a fundamental mass-scale. Since $n$ additional space dimensions are convolved on a torus with the radius $R$, the extra dimensions should be hidden at distances $r \gg R$, resulting in the standard $1/R$ potential [8]:

$$\Phi(r) \overset{r \gg R}{=} \frac{\hbar^{n+1}}{c^{n-1}} \frac{1}{M_f^{n+2}} \frac{1}{R^n} \frac{M}{r} \equiv \hbar c \frac{1}{m_{Pl}^2} \frac{M}{r} \tag{2-10}$$

The relation between the fundamental mass-scale and the Planck mass at long distances is therefore [8]:

$$m_{Pl} = \sqrt{\frac{c^n}{\hbar^n} M_f^{n+2} R^n} \ , \tag{2-11}$$

There are two main consequences of these calculations: Extra dimensions can be large compared to the Planck length—as much as 0.1 mm without contradicting the theory or experimental results [9]. And the gravitational interaction increases strongly on distances below the extension of these extra dimensions (by factors of the order of $10^{32}$) [8].

# 2.5 Neutrino oscillations

### 2.5.1 Neutrino

Neutrinos are electrically neutral elementary particles with a small but non-zero mass. They propagate nearly with the speed of light. Neutrinos interact with other mass mainly just through weak interaction, this makes them very difficult to detect.

Neutrinos are divided into three types based on the three different flavors they can have. There are electron neutrinos (flavor $e$), muon neutrinos (flavor $\mu$) and tau neutrinos (flavor tau $\tau$). The name of the flavor comes from the particle involved in the process creating the particular neutrino. Electron neutrinos come from beta decay, muon neutrinos from muon decay and tau neutrinos from tau decay.

### 2.5.2 Flavor oscillations

Neutrino oscillations are a quantum mechanical phenomenon where a neutrino created with a certain flavor state $\left|\nu_\alpha\right\rangle$ (where $\alpha = e, \mu$ or $\tau$) can later be measured to have a different one $\left|\nu_\beta\right\rangle$ (where $\beta = e, \mu$ or $\tau$). The probability of measuring a particular flavor for a neutrino changes periodically as it propagates.

The mass state of a neutrino of the definite flavor state $\left|\nu_\alpha\right\rangle$ is a superposition of definite mass states $\left|\nu_j\right\rangle$ (where j = 1, 2 or 3) [10]:

$$\left|\nu_\alpha\right\rangle = \sum_j U_{\alpha j}^* \left|\nu_j\right\rangle \tag{2-12}$$

and the flavor state of a neutrino of the definite mass state $\left|\nu_j\right\rangle$ is the superposition of definite flavor states $\left|\nu_\alpha\right\rangle$:

$$\left|\nu_j\right\rangle = \sum_\beta U_{\beta j} \left|\nu_\beta\right\rangle \tag{2-13}$$

where $U$ is the so-called Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS matrix or leptonic mixing matrix). It contains information on quantum states of neutrinos propagating freely and taking part in the weak interactions. It is therefore a subject to the unitary constraint [10]:

$$\sum_j U_{\alpha j}^* U_{\beta j} = \delta_{\alpha\beta} \tag{2-14}$$

In matrix symbolism the equations (2-12) and (2-13) can be denoted:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1}^* & U_{e2}^* & U_{e3}^* \\ U_{\mu 1}^* & U_{\mu 2}^* & U_{\mu 3}^* \\ U_{\tau 1}^* & U_{\tau 2}^* & U_{\tau 3}^* \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{2-15}$$

where:

$$\begin{pmatrix} U_{e1}^* & U_{e2}^* & U_{e3}^* \\ U_{\mu 1}^* & U_{\mu 2}^* & U_{\mu 3}^* \\ U_{\tau 1}^* & U_{\tau 2}^* & U_{\tau 3}^* \end{pmatrix} = U^{-1} = \left(U^*\right)^T \tag{2-16}$$

The Schrödinger equation for a neutrino with the mass state $\left|\nu_j\right\rangle$ in the neutrino's reference frame is [10]:

$$i\frac{\partial}{\tau_j}\left|\nu_j(\tau_j)\right\rangle = m_j\left|\nu_j(\tau_j)\right\rangle, \tag{2-17}$$

where $\tau_j$ is proper time and $m_j$ is the eigenvalue of the mass eigenstate $\left|\nu_j(0)\right\rangle$. Quantities are expressed in natural units: $c = \hbar = 0$. The solution of this equation is:

$$\left|\nu_j(\tau_j)\right\rangle = e^{-im_j\tau_j}\left|\nu_j(0)\right\rangle \tag{2-18}$$

If we transform the reference frame to the laboratory reference frame by a Lorentz transformation, the phase vector $e^{-im_j\tau_j}$ becomes $e^{-i(E_jt-\vec{p}_j\vec{x})}$ [10], where $E_j$ is the energy and $\vec{p}_j$ is the momentum associated with the mass eigenstate $\left|\nu_j(0)\right\rangle$, $t$ is the time from the start of the propagation and $\vec{x}$ is the current position of the particle (in reference to its starting position). The speed of neutrinos is very close to the speed of light, which makes $\left|p_i^2\right| \gg m_i^2$ and $t \approx L$, where $L$ is the distance traveled. This allows the following approximation of the relativistic dispersion relation:

$$E_j = \sqrt{p_j^2 + m_j^2} \cong p_j + \frac{m_j^2}{2p_j} = E + \frac{m_j^2}{2E} \tag{2-19}$$

Now we insert that back into the equation and get [10]:

$$\left|\nu_j(L)\right\rangle = e^{-im_j^2L/2E}\left|\nu_j(0)\right\rangle \tag{2-20}$$

Neutrinos with different masses have different propagation speeds. The difference in speed also causes the interference between the corresponding flavor components of each mass eigenstate [10]. Constructive interference causes it to be possible to observe a neutrino created with a given flavor to change its flavor during its propagation. The probability for a neutrino to oscillate from the flavor $\alpha$ to the flavor $\beta$ is:

$$P = \left|\left\langle\nu_\beta\middle|\nu_\alpha(t)\right\rangle\right|^2 = \left|\sum_j U_{\alpha j}^* U_{\beta j} e^{-im_j^2L/2E}\right|^2 \tag{2-21}$$

# 3 Theories of quantum gravity

## 3.1 String theory

### 3.1.1 Introduction

String theory solves the problem of quantum gravity by postulating that the elementary building blocks of nature are not point particles, but fundamental strands of strings [6]. Electrons and quarks within an atom are not 0-dimensional objects, but 1-dimensional strings. These strings can oscillate, giving the observed particles their flavor, charge, mass and spin.

String theory emerged from the physics of strong interactions. In this theory, two quarks interacting strongly are connected by a stream of carriers of the strong force. The potential energy between the two quarks grows linearly with the distance between the quarks and as the constant of proportionality has the dimensions of mass per length, it is often called the tension of the string $T_{string}$ [6]. The model says that if the quarks are pulled far enough apart, another quark-antiquark pair would be created in between them.

### 3.1.2 Methods

One of the reasons that string theory became so popular, is the fact that any basic string theory contains particles with such properties that they could serve as gravitons. But there are more problems that string theory finds a solution to: Simple approaches to quantum gravity split the metric into a flat metric and a quantized position-dependent part. But this perturbative method turns out to fail at larger energies, where, for example, the sum of the probabilities for final states of a collision of two particles is greater than 100% [6]. This is one of problems that string theory manages to solve—

In high-energy processes, the size of a string grows with its energy. When two strings with a very high energy interact, some small segment of one string exchanges a fraction of its total energy with a small segment of the other string. The interaction of strings instead of point particles "smoothens out" the divergences so that it yields finite results and sustains the conservation of probability.

Most string theories seem to require 10 space-time dimensions [6]. But it does not contradict the basic physical theory of fundamental fields and their interactions. For example, Einstein's theory of general relativity has many non-physical solutions in addition to the cosmological solutions of our part of the universe.

## 3.2 Loop quantum gravity

### 3.2.1 Introduction

Loop quantum gravity is another theory of quantum space. As opposed to string theory, loop quantum gravity doesn't require higher dimensions. This makes it a popular theory in physics. It is the application of quantum mechanics to Hamiltonian general relativity [1]. It is a quantum theory for an infinite number of degrees of freedom. Quantum states $\psi$ are represented by spin networks and these are relational—space is defined in relation to them.

### 3.2.2 Spin networks

A spin network is a diagram that represents quantum states of geometry in terms of states of the gravitational field in loop quantum gravity. Originally, as described in 1971 by Roger Penrose, each line segment symbolized the world line of a unit. Intersections of these lines stood for an event where a single unit split into two or two units collided and joined into a single unit.

As of today, a spin network is a diagram, whose edges are associated with representations of a compact Lie group and whose vertices are associated with intertwines of the edge representations adjacent to it.

One way to characterize the geometry (or curvature) of space is to describe how vectors are carried along any path (parallel transport). In curved space, parallel transport around a loop will produce a vector that is rotated compared to the original, resulting in phenomena such as sum of angles of a triangle being different from 180°. If we parallel transport a particle with a certain spin along a loop, its spin in the end will be dependant on the curvature of space.

In loop quantum gravity, each edge of a spin network is labeled by a spin value and carries a "quantum of area", on the other hand, each node carries a "quantum of volume".

### 3.2.3 Space in loop quantum gravity

Physical quantities in general relativity, such as lengths, volumes or time intervals, are invariant under active diffeomorphism* [1]. Due to quantum mechanics, these are quantum observables represented by self-adjoint operators, some of which have a discrete spectrum. So it is only logical to expect a discrete geometry—this is what loop quantum theory does.

The operators of area and volume have been constructed by simply expressing the metric (in the classical definition of area and volume) in terms of the gravitational field [1]. Then we replace that with a quantum field operator. The spectrum of these operators turned out to be discrete, confirming our speculation.

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* *A passive diffeomorphism simply changes one coordinate system to another one. An active diffeomorphism transforms the whole manifold.*

### 3.2.4 Lorentz violation in loop quantum gravity

The discreteness of space-time acts as a dispersive medium for particles propagating on it and this reflects to the kinematics [5]. If we derive equations of motions from expectation values of a quantum Hamiltonian in a dispersive medium, we obtain modified dispersion relations. These relations reveal the breaking of Lorentz symmetry.

### 3.2.5 Physical results of loop quantum gravity

The most important result of loop quantum gravity is the discreteness of space-time at the Planck scale [11]. It also succeeded in finding exact solutions of the Hamiltonian constraint, calculating the solution of the cosmological singularity problem and computing the entropy of a black hole.

# 4 Modifications of special relativity

## 4.1 Violation of Lorentz invariance

### 4.1.1 Introduction

In the last few years there have been hypotheses that Lorentz invariance may not be an exact symmetry at all energies [12]. The possibility of a four-dimensional Lorentz invariance violation has been investigated in a few quantum gravity models—string theory, loop quantum theory and others, though these theories don't require the Lorentz violation to work. But there are some models of space-time that contain Lorentz violations explicitly—like the non-commutative field theory.

If Lorentz invariance is violated by quantum gravity, it is usually expected to show up strongly at the Planck energy (approximately $10^{16}$ TeV) [12]. Since this energy is so much higher than the energy of any known particle, we can't ever detect this violation directly. But there should always be an interpolation to all energies, even the low ones that we are able to observe.

### 4.1.2 The Planck length as a relativistic invariant

Due to the base of quantum gravitational theories, the Planck length is supposed to be a constant (observer-independent). But a constant length is a concept that is not compatible with special relativity. Of course we may give the Planck length just the role of a coupling constant (simply a constant derived from the gravitational constant *G*) [13]. If we use this definition, there is no reason why the value of *G* couldn't be the same for all inertial observers. In string theory, the length of a string is closely related to the Planck length and is also considered simply a coupling constant.

### 4.1.3 Violation of CPT Invariance

Lorentz symmetry is connected to other discrete symmetries: parity (changing the sign of all spatial coordinates, denoted *P)*, time reversal (*T*), and charge conjugation (changes the sign of all quantum charges—changes a particle to its antiparticle, denoted *C*) [5].

The violation of Lorentz invariance doesn't imply CPT invariance violation [5]. But Oscar Greenberg in the year 2002 proved that the other way round it does—violation of CPT invariance implies the violation of Lorentz invariance. This makes experiments testing CPT invariance equivalently important to the research.

## 4.2 Doubly special relativity

### 4.2.1 Introduction

Doubly special relativity (DSR) is a generalization of special relativity based on the deformation of Lorentz transformations; hence it is also called Deformed special relativity. It deals with the breaking of Lorentz symmetry on the Planck scale. The relativity principle still works in the same way as in special relativity, but Doubly special relativity postulates that the Planck energy $E_{pl}$ is independent on the choice of reference frame [14]. The limit of mass $M_{pl} \rightarrow \infty$ is special relativity. This theory applies to all four fundamental interactions.

Special relativity has only one observer-independent scale—the speed of light *c*—while DSR has two scales, adding Planck length $\hbar$, gravitational constant $G$ and the cosmological constant $\Lambda$ [14]. In the limit situation of a flat space-time where we can neglect gravitational and quantum effects, some trace of these three constants remains, creating the second observer-independent scale $\kappa$.

One version of the formalism of DSR is a five-dimensional one [14]. The deformed Minkowski space is embedded in a larger Riemannian manifold and energy is added as an additional fifth dimension.

### 4.2.2 Different DSR theories

The physical variables of an object are at first transformed using a non-linear transformation into so-called pseudo-variables, then the linear Lorentz transformations can be applied, and at last they are transformed non-linearly into the new reference frame, being again physical variables [15]. Of course there is a great number of possible non-linear transformations satisfying the conditions. Theories using these different transformations have been assigned numbers: DSR 1, DSR 2, etc…

In some versions of DSR the speed of light is assumed to be energy-dependent. That means that the constant $c$ is only the low-energy limit of the speed of massless particles.

## 4.3 Variable speed of light

### 4.3.1 Introduction

The variable speed of light is an approach to solve the problem of quantum gravity by postulating that the speed of light in vacuum $c$ might not be, in fact, a constant. In special relativity $c$ must by measured exactly the same, but Einstein himself stated that:

"The results of the special relativity hold only so long as we are able to disregard the influence of gravitational fields on the phenomena."

E. A. Milne in 1948 was the first one to suggest solving the problem of quantum gravity by proposing that some acknowledged constant might not be constant. In his case it was the Planck constant $h$ and gravitational constant $G$, while examining the red shift [16]. But it was a long time before someone contested the constant speed of light $c$.

### 4.3.2 Petit's cosmological model

Jean-Pierre Petit examined the effects of varying all these constants combined by space and time scale factor changes [16]. They are linked together in such a way that all physical equations are invariant and that the variations of these constants aren't measurable throughout the universe, the only observable effect being the redshift—that is due to their secular variation (long-term non-periodic variation) [17]. Energy is conserved, but mass isn't. He defines $R$ as characteristic length—a fundamental parameter, where the characteristic lengths like Planck's length, Schwarzschild length and Compton length follow the variation $R$ and all characteristic times (and Planck time) vary like time $t$. The energy of a photon $h\nu$ must be conserved; and because the frequency of the photon $\nu$ decreases in time, the Planck constant must increase—vary like $t$. As a result, the gravitational constant $G$ would vary like $1/R$, the gravitational force $F_G$ like $1/R$ and, finally, the speed of light $c$ like $1/\sqrt{R}$ .

With these conditions the field equations have a single solution where the universe is overall homogeneous and the curvature of space is negative [17]. The evolution of the universe is then described with the evolution law: $R \sim t^{2/3}$.

# 5 Experimental methods in quantum gravity

## 5.1 Time-of-flight experiments with DSR

### 5.1.1 Introduction

Several quantum gravitational theories propose that the photons with higher energy travel faster (or slower) than the low-energy ones. The time delay between these photons isn't observable at short distances, but for very distant objects the path is long enough to give measurable results. So, in the time-of-flight experiment, we measure the energy-dependence of the speed of light coming from a distant source using a satellite [14].

### 5.1.2 Theory

If the speed of light is energy dependent, one should find a possible time delay $\Delta T$ of:

$$\Delta T \approx T\left(\frac{E}{m_{Pl}}\right)^n, \qquad (5\text{-}1)$$

where $T$ is the time of flight, $E$ is the energy of the photon [19] and $n$ is a parameter that varies in different theories ($n$ = 1, 2, …). For typical energies and distances around $10^9$ light years, the time delay would be approximately $\Delta T \approx 10^{-2}\,\mathrm{s}$, which is measurable.

But S. Hossenfelder in her paper [19] calculates that not the energy of a single particle is the relevant quantity but rather its energy density that curves the space it propagates in. The highest peak of energy of a γ-ray burst is around a GeV, which will have the localization of roughly a femtometer and energy density of about:

$$\rho \approx 10^{-76} \frac{m_{Pl}}{l_{Pl}^3} . \tag{5-2}$$

This is several orders of magnitude lower than we can so far measure. If she is right, there is no way we can test DSR using today's time-of-flight experiments.

### 5.1.3 Experiment

Today there is a functioning satellite Fermi Gamma-ray Space Telescope (formerly named GLAST) that should conduct these experiments. It carries a "pair conversion telescope" LAT that detects high-energy γ-ray bursts from distant sources (like exploding stars and black holes) [20]. In the correct photon energy range, if the photons happen to pass very close to a heavy nucleus, they interact with matter primarily through the "pair conversion" process—when a photon hits LAT, it interacts with the strong electromagnetic field surrounding the nucleus and its energy is converted into a pair of particles, an electron and a positron.

The gamma ray converts in one of 16 wolfram foils, and the resulting electron and positron are tracked by layers of position-sensitive detectors. The direction from which the photon came is derived from the tracks. After that a calorimeter measures the total energy of these particles [20].

### 5.1.4 Results

The Fermi Gamma-ray Space Telescope satellite is running since 2008, but any quantum gravity-related results are yet to be published.

## 5.2 Decoherence of matter waves

### 5.2.1 Introduction

Einstein was the first one to explore the possibility of accessing a scale he couldn't study directly by finding an analogous experiment. He wanted to study thermal fluctuations of molecules and—instead—he examined the motion of small particles through Brownian motion [21]. Modern experiments use a method analogous to it. We want to study physics at the Planck scale, but it is very difficult (if not impossible) to observe it directly. There are ideas to conduct similar experiments with the decoherence of matter waves.

The theoretical framework states that the effects of ground-state gravitons on the geometry of space-time can lead to observable effects by causing quantum matter waves to lose coherence. That means that we can study fluctuations of space-time by analyzing their "blurring" effects on coherent matter waves.

The curvature of space-time changes the proper time* of objects [21]. For time intervals approaching the Planck time, proper time fluctuates strongly due to quantum fluctuations; for longer intervals it is steady. Proper time therefore consists of quantum fluctuations as well as the steady drift. This leads to a gravitational analogue of Brownian motion whose correlation length is given by the Planck length (up to a scaling factor $\lambda$). The border between the semiclassical and the fully quantum model of gravity is expected to be defined by Planck time multiplied by the parameter: $\lambda \cdot t_{Pl}$ as well as the Planck length multiplied by the same parameter $\lambda \cdot l_{Pl}$. The latter statement makes it possible to say that $\lambda < 10^2$ (for larger $\lambda$ we are move back into the classical picture). But experimental research might be able to specify the upper limits of the fluctuations.

### 5.2.2 Experiment

Matter wave interferometers are often used in measuring decoherence effects [21]. In an atom interferometer, an atomic wave packet is split into two coherent wave packets, following different paths, and at last recombining. The phase change of each wave packet is proportional to the proper time along its path, resulting in constructive or destructive interference when the wave packets recombine. The detection of the decoherence due to space-time fluctuations on the

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* *Proper time $\tau$ is the time between two events measured by a moving clock that occur at the location of this clock.*

Planck scale would provide experimental access to quantum gravitational effects analogous to the access to atomic scales provided by Brownian motion.

### 5.2.3 Results

No results from similar experiments have been published yet. But investigating Planck scale physics using matter wave interferometry might soon become reality and when (or if) they are, it will serve as an argument in favor of the inflation of the universe (dealing with the problem of the cosmological constant) and explain the origin of dark matter and energy [21].

## 5.3 Search for quantum gravity using neutrino telescopes

### 5.3.1 Introduction

Neutrinos are a topic of interest in the field of quantum gravity because of their unique physical properties—that they are unaffected by strong interaction and electromagnetism. They are also relatively stable—not being a subject to rapid decay like many other particles makes them easier to study. There are hopes, that we might be able to isolate neutrinos and measure their gravitational interaction at a quantum level.

Neutrino oscillations provide an interferometer sensitive to very small changes in energy [22]. The measurement of these oscillations implies that a neutrino has a non-zero mass and means a shift from the standard model. While conventional oscillations are well explained, quantum gravitational oscillations or other flavor changes at higher energies indicate the need of a non-standard approach, as seen in chapter {2.1}.

There are mainly two domains of QG that can be tested using existing neutrino telescopes—the violation of Lorentz invariance (VLI) and quantum decoherence [22].

### 5.3.2 Theory

A neutrino telescope is a term used to describe certain neutrino detectors. The word *telescope* indicates that these detectors focus on astrophysical observations.

The main problem in the detection of a neutrino is its extremely small cross section [22]. The only fundamental interactions between neutrinos and other particles are the weak and gravitational interactions and the weak sub-atomic force is of a much shorter range than the electromagnetic force [23]. The size of the cross section grows with energy, but still the detector must cover an enormous volume in order to detect a sufficient number of neutrinos. This is why natural bodies of water and layers of ice in the South Pole are a popular cost-efficient solution. Being deep underwater or below ice also isolates the telescope from background radiation such as cosmic rays.

When an interaction occurs near a detector, we detect the consequent charged particles by means of their Čerenkov radiation—the electromagnetic radiation emitted when the particles pass through the dielectric medium at a greater speed than the local phase velocity of light [22]. The charged particles polarize molecules of the medium, which then drop to their ground state, emitting radiation. Čerenkov neutrino detectors usually consist of photomultiplier tubes attached to vertical cables lowered in deep water or ice.

In a charged-current interaction with the surrounding matter, if the neutrino has enough energy, it transforms into a charged lepton—an electron (in the case of an electron neutrino), muon (muon neutrino), or tau (from a tau neutrino) [22]. Solar neutrinos have sufficient energy to create electrons, accelerator neutrinos can also usually create muons, and a very few neutrinos can create taus. If the detector tells apart the different types of leptons, we can find out the flavor of the original neutrino, possibly suggesting its source.

In a neutral-current interaction, the neutrino transfers part of its energy to a light charged particle (like an electron). This particle is thereby accelerated to a sufficiently high speed and emits Čerenkov radiation, which can then be detected.

This type of interaction doesn't provide the information about the neutrino flavor [23].

There are less than ten Čerenkov neutrino telescopes in the world and I will focus on two of them, both ice detectors deep under the geographic South Pole: AMANDA-II, running since 1998 and IceCube, running since 2010:

The Antarctic Muon And Neutrino Detector Array (AMANDA) is located nearly 2 km beneath the Amundsen-Scott South Pole Station using 19 optical cable [23]. It functioned between years 1996 and 2009. It detects very high energy neutrinos (from 0.1 to 10 TeV) passing south through Earth.

The IceCube Neutrino Observatory was created as a follower of AMANDA, it was part of its predecessor from the year 2005 and was finally completed in 2010 [24]. It sinks as deep as 2,5 km beneath ice, using 86 cables. IceCube is sensitive mostly to high energy neutrinos, in the range of 0.1 to about $10^4$ TeV, but it can detect even particles below 0.1 TeV.

### 5.3.3 Experiment

It is expected that Lorentz and CPT invariance are broken when approaching the Planck scale, due to the discrete structure of space-time [25]. The standard model of particle physics is assumed to be the low-energy limit of a more fundamental theory, which should combine quantum field theory and general relativity at the Planck scale and to provide a theory of quantum gravity. The standard model extension (SME) is one of the existing models; it provides numerous searches for signatures of Lorentz invariance and CPT violation.

Experiments were performed in IceCube to search for sidereal modulation in the flux of atmospheric muon neutrinos [25]. They use a two-neutrino model derived from the SME. This model predicts neutrino oscillations to depend on the direction of propagation, which would be the evidence of the breaking of Lorentz symmetry.

There are also several planned experiments that are yet to be performed in IceCube [26]: Neutrinos with energies approaching this Planck scale are expected to interact by gravity with large interaction cross sections and this cross section will create dramatic signatures in a neutrino telescope including, possibly, the production of black holes. IceCube will also be searching for particle emission from cosmic strings created in the young Universe [26]. It has been suggested that they may be the sources of very high-energy cosmic rays.

### 5.3.4 Results

In experiments conducted from 2000 to 2010 with the AMANDA-II detector the data was consistent with the standard model [25]. That means that upper limits were set on several quantum gravitational parameters [24],[27].

In IceCube, direction-dependent neutrino oscillations were studied but there wasn't any evidence found [25]. A discrete Fourier transform method was used to constrain the Lorentz and CPT-violating coefficients in one of these models. As a result, constraints on certain Lorentz and CPT-violating coefficients were improved by about three or four orders of magnitude.

There is still no experimental evidence of quantum gravity in using neutrino telescopes.

## 5.4 Search for variable speed of light with redshifts

### 5.4.1 Introduction

Conventionally, the red shift is explained by the Doppler effect modified by special relativity. But there are some attempts to explain it by variation of different physical constants, like the speed of light [18]. Jean-Pierre Petit in his article [28] tries to explain the theory of redshift with the variable speed of light.

### 5.4.2 Experiment

It has been experimented on the effects of redshift observed on the light coming from a set of 134 radio quasars [29]. Conventionally all experimental results were explained using the Einstein-de-Sitter model. This is a model using Euclidean geometry and postulating that the universe expands from an infinitely condensed state, continually decreasing its expansion rate. It also assumes that the curvature of space is zero.

Jean-Pierre Petit in his article [28] uses a cosmological model, where the redshifts come from the secular variation of elementary physical constants. The basic difference between this cosmological model and the standard one is in the absence of the singularity and the matter density at the initial moment of time.

### 5.4.3 Results

It was in fact found that the proposed gauge model with variable constants provided a better fit to these distributions than the Einstein-de-Sitter model [29]. Though there are still a great number of tests to be performed, this can be considered a success for the variable speed of light model.

## 5.5 Search for microscopic black holes at the LHC

### 5.5.1 Introduction

Several physical theories have introduced extra dimensions to the three spatial ones to explain various phenomena. They usually try to unify general relativity and quantum mechanics by postulating extra curled up dimensions. For example, string theory requires extra dimensions for mathematical consistency; the whole brane cosmology is based on this idea. Some theories, such as the large extra dimensions theory, use extra dimensions to explain why the gravitational field is so much weaker when compared to other fields [9].

These theories predict that when particles at extremely high energies collide, they are “sensitive” to these extra dimensions [9]. Then the gravitational force between these particles should be so strong, it would be comparable to the other three fundamental interactive forces. These particles would then collapse into a microscopic black hole.

The Large Hadron Collider (LHC) at CERN was expected to accelerate particles up to the energy required to form a microscopic black hole. If it succeeded, string theory (and the other theories relying on extra dimensions) would get a great experimental argument to its advantage, which is very rare in the whole search for quantum gravity.

### 5.5.2 Theory

When two particles collide in the accelerator, we expect our theory of quantum gravity to give us quantum mechanical probabilities for what the collision fragments will be [6]. Simple approaches to quantum gravity predict that the probability of any given outcome when two energetic particles collide with each other grows with the energy, $E$, of the collision at a rate controlled by the dimensionless ratio $(E/M_{Pl})^2$. As for the probability distribution of final states, it has been discussed in chapter {3.1.2}.

The relationship between the original Planck scale $M_D$ and the observer's Planck scale $M_{Pl}$ can be derived from Gauss's law as $M_{Pl}^2 = 8\pi M_D^{n+2} R^n$ [30]. This change in space-time structure and the increased strength of the gravitational field would allow a black hole to form by particle collisions at energies greater than $M_D$. That means that particles will collapse into a black hole if the impact parameter of the particles is smaller than Schwarzschild radius of black hole $r_s$ with energy $M_{BH}$ (total accessible energy of the collision). The minimum black hole mass $M_{min}^{BH}$ cannot be smaller than $M_D$.

### 5.5.3 Experiment

If a microscopic black hole is produced, it would immediately start to decay through Hawking radiation, emitting mainly quarks and gluons, but also leptons, photons, bosons (in theory maybe Higgs bosons) [9]. The final-state particles should be carrying hundreds of GeV of energy and consist mostly of hadrons.

All these particles were supposed to be detected using the Compact Muon Solenoid (CMS). This is a large built-in general-purpose particle detector. It is divided in several subsystems designed to measure the energy and momentum of photons, electrons, and other collision products. It contains a crystal electromagnetic calorimeter, brass-scintillator hadronic calorimeter, and a solenoid producing a magnetic field of nearly 4 T including a silicon-based tracker and other particle detectors.

### 5.5.4 Results

In the year 2010, many proton-proton collision experiments were conducted running at center-of-mass energy of 7 TeV (that is twice the energy of one proton beam). But no trace of a microscopic black hole was detected whatsoever [30]. This is considered a disproof of the existence of black holes up to a mass of 3.5–4.5 TeV for theories relying on the existence of extra dimensions.

These are the ever first limits on the minimum mass of a black hole ever set. This, of course, doesn't rule out the existence of microscopic black holes at higher energies, ones we will be able to access in the future, using an even larger collider. The closer we will be to Planck energy, the more theoretical quantum gravitational anomalies we might be able to verify in experiment. We are still 16 orders of magnitude below.

# 6 Conclusions

There is great hope in experimental testing of quantum gravity and in the last few years many of these experiments were conducted.

Time-of-flight experiments in the Fermi Gamma-ray Space Telescope are based on the hypothetical energy-dependence of the speed of light; they measure high-energy γ-ray bursts from distant cosmic sources [14]. There wasn't so far any positive experimental evidence for this phenomenon. Hossenfelder's calculations implicate that we do not have the sufficient technology to be able to conduct these experiments yet [19].

Some studies propose to access the Planck scale through an analogous experiment with matter wave interferometry [21], but these experiments haven't been performed yet. Results might resolve the inflation of the universe (dealing with the problem of the cosmological constant) and explain dark matter and energy [21].

Experiments in neutrino telescopes search for evidence of the breaking of Lorentz symmetry through measuring the possible direction of propagation-dependence of neutrino oscillations. All the experiments conducted with the AMANDA-II detector and in IceCube were so far consistent with the standard model [25], but upper limits were improved on several quantum gravitational parameters [24],[27]. There are plans for more experiments, such as searching for particle emission from cosmic strings created in the young Universe.

Jean-Pierre Petit examined the effects of varying constants such as the speed of light *c* combined by space and time scale factor changes [16]. He used these calculations to explain the redshift and tested these calculations by observing a set of radio quasars. His gauge model with variable constants provided a better fit to these distributions than the conventional Einstein-de-Sitter model.

There is a chance that the Large Hadron Collider would accelerate particles up to the energy required to form a microscopic black hole [9]. If it succeeded, string theory (and the other theories relying on extra dimensions) would get an experimental argument to its advantage. But no microscopic black hole was observed [30]. This is a disproof of the existence of black holes up to a mass of 3.5–4.5 TeV for theories relying on the existence of extra dimensions.

To this date, there isn't any conclusive experimental evidence supporting any theory of quantum gravity. But new boundaries and limits were set on several theories and this will narrow down the huge span of different possibilities. There are also many more plans for more experiments waiting, bounded only by technological progress. We will probably witness some very interesting experimental results in the next few years.

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