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Some Identities of Subjective Analysis Derived on the Basis of the Subjective Entropy Extremization Principle by Professor V.A. Kasianov

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Abstract Herein it has been made an attempt to find a theoretical explanation to the responsible person controlling behavior. On the basis of the subjective entropy of individual preferences extremization principle developed by Professor V.A. Kasianov we can derive some identities. Using the necessary conditions for extremums of a functional to exist in the view of the system of the Euler-Lagrange equations we get the widely known fundamental laws, namely, the law of subjective value by Jakob Bernoulli, as well as the main law of psychophysiology: the Weber-Fechner law in application to problems of optimal control in active systems. The discussed approach allows finding optimal paths as well as has an intrinsic universal value. The derived dependences have the significance of the conservative values at solving optimization problems. The corresponding modeling performed is illustrated with the necessary diagrams.

Keywords: *optimal control, individual preferences, subjective entropy, objective functional, variational principle, subjective conservatism, modeling*

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1. Introduction

Comprehensive analysis of researches that applied the notion of entropy was performed in paper [1]. The all-embracing investigated trend of the number of publications in the science fields implying entropy paradigm illustrates the usefulness of this term.

According to [[1], P. 239, Figure 1] the number of scientific publications used the one of the most important notions – entropy had grown 3 to 4 times since 1991 up to 2010.

In this paper we also apply that universal measure of uncertainty – entropy in an original method.

An engineering system which has an individual (active element, responsible person (RP)) who controls or managing decisions the RP happens to be in the situations of uncertainty due to his/her preferences are distributed on a certain set of achievable for his/her goals alternatives. In order to measure this uncertainty and modeling the processes of optimal control more accurately, we have introduced and are still researching the notion of subjective entropy of individual preferences [2-8].

There is a wide variety of interpretations of active systems and entropy approaches applications, and all that leads to an ambiguity in the definitions and excessive polemics on the given topic. We would like to emphasize at once the properties of an active system for this consideration, so that to avoid unneeded theoretic contradictions.

From our point of view, an active system [[3], P. 58, § 3.1]:

Being closed, is able to decrease its own entropy;

Requires the presence of an active element, the bearer of the subjective preferences;

The extremizing functional comprises the subjective entropy;

The behavior of the active element is dictated by the postulated variational principle and is directed into optimization of the solution to a problem-resource situation with the extremization of the entropy;

Has the ability to aggregate preferences.

All true theories that have finished their processes of formation (mechanics, geometrical optics, thermodynamics) involve at their cores some principles of optimality [9]. We hope this situation exists, and not occasionally, also in other sciences: Jaynes formalism [10,11] in statistical physics; information theory [12]; biology [13]; theories of evolution [9,13,14]; light and shadow economics [[3], P. 43-47, § 2.2]. Concerning optimal control in active systems, for this study we will be considering the postulated in subjective analysis [2-8] variational principle and concentrate our attention on some identities that are being derived from it. Also, it will be performed, an example of the calculation experimental modeling.

The significance of the presented research is that the derivations relate to psychology, the essential lever of control and managing. Up until now, the enormous number of regularities and facts revealed by psychologists is just a collection (group, combination, set, complex) of empirical knowledge, not a full scale theory, even having forms of mathematical expressions, though [9].

2. Methods

The mentioned above Jaynes formalism [10,11] is the initial point for the methods applied herein in order to get a few relationships of subjective analysis on the basis of the postulated subjective entropy exptemization principle.

2.1. Subjective Entropy Extremization Principle by Professor V.A. Kasianov

The functional postulated in subjective analysis has the view of a linear combination [[2], P. 119, (3.38)]:

$$\Phi_\pi = \alpha H_\pi + \beta \varepsilon + \gamma \mathrm{N} \tag{1}$$

where π – function of the individual's subjective preferences distributed on the set of achievable for the RP's goals alternatives; α, β, γ – structural parameters, they can be considered in different situations as Lagrange coefficients, weight coefficients or endogenous parameters which represent certain psychic properties of the RP; H_π – subjective entropy; $\varepsilon = \varepsilon(\pi, U, \ldots)$ – function of subjective effectiveness, where U – utility function; N – normalizing condition.

A generalization of the functional (1) is the following functional [[3], P. 42, (A)], [[4], P. 7, (1)]:

$$\Phi_\pi = \int_{t_0}^{t_1} \left\{ \begin{aligned} & -\sum_{i=1}^{N} \pi_i(t) \ln \pi_i(t) \\ & + \beta \sum_{i=1}^{N} \pi_i(t) F_i + \gamma \left[\sum_{i=1}^{N} \pi_i(t) - 1 \right] \end{aligned} \right\} \mathrm{d}t \tag{2}$$

where t – time; N – number of the considered alternatives; F_i – function, related with the corresponding alternative.

The functional (2) in the simplest particular case, for example, can be represented by the expression [[4], P. 57, (2)]:

$$\begin{aligned} \Phi_\pi = \int_{t_0}^{t_1} \Bigg\{ & -\sum_{i=1}^{N=4} \pi_i(t) \ln \pi_i(t) + \beta \Bigg[\pi_1(t) x(t) \\ & + \alpha_2 \pi_2(t) \dot{x}(t) + \alpha_3 \pi_3(t) x(t) \dot{x}(t) + \alpha_4 \pi_4(t) \frac{\dot{x}(t)}{x(t)} \Bigg] \\ & + \gamma \left[\sum_{i=1}^{N=4} \pi_i(t) - 1 \right] \Bigg\} \mathrm{d}t \end{aligned} \tag{3}$$

where $x(t)$, $\dot{x}(t)$, $x(t)\dot{x}(t)$, and $\frac{\dot{x}(t)}{x(t)}$ – particular combinations for the corresponding effectiveness functions F_i, of the functional (2) with respect to the related alternatives; α_i – coefficients that consider the differences in the measurement units, $\alpha_1 = 1$.

The applied optimization (the subjective entropy extremization) principle by Professor V.A. Kasianov allows finding the extremals for the objective functionals (1,2,3) on conditions of meeting the systems of the corresponding Euler-Lagrange equations. Amongst the sought extremals, there are the preferences functions in the so-called canonical distributions view [[2], P. 115-135]. They are analogues to Jaynes' [10,11] and [12] derivated by Stratonovich. For instance, in the case of (3) [[4], P. 58, (4)]:

$$\pi_j = \frac{e^{\alpha_j \beta F_j}}{\sum_{i=1}^{N} e^{\alpha_i \beta F_i}} \tag{4}$$

By the methods of analytical research we can derivate a few dependences. And then, we conduct modeling and plot diagrams to visualize the hypothetical issues.

2.2. Derivations of Some Identities

For the general case expressed with the functional (2), except for "$-\beta$" instead of "$+\beta$", from the necessary conditions for the extremums to exist in the view of

$$\frac{\partial R^*}{\partial \pi_i} - \frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial R^*}{\partial \dot{\pi}_i} = 0$$

however

$$\frac{\partial R^*}{\partial \dot{\pi}_i} \equiv 0$$

therefore

$$\frac{\partial R^*}{\partial \pi_i} = 0 \tag{5}$$

where R^* – the under-integral function (integrand) of the functional (2), we obtain

$$-\ln \pi_i - 1 - \beta F_i + \gamma = 0 \tag{6}$$

Then, the equation (6) yields

$$\pi_i = \exp[\gamma - 1] \exp[-\beta F_i] \tag{7}$$

Form the normalizing condition we get

$$\sum_{i=1}^{N} \pi_i = \exp[\gamma - 1] \sum_{i=1}^{N} \exp[-\beta F_i] = 1 \tag{8}$$

Thus, from the expression (8)

$$\exp[\gamma - 1] = \frac{1}{\sum_{i=1}^{N} \exp[-\beta F_i]} \tag{9}$$

The equation (9) yields

$$\gamma = 1 - \ln \sum_{i=1}^{N} \exp[-\beta F_i] \tag{10}$$

On the other hand from the equation (6) we may express

$$\gamma = 1 + \ln \pi_j + \beta F_j \tag{11}$$

which should be satisfied for any alternative individual's preference function.

Then, equalizing expressions (10) and (11) we get

$$\gamma = 1 - \ln \sum_{i=1}^{N} \exp\left[-\beta F_i\right] = 1 + \ln \pi_j + \beta F_j$$

hence

$$\ln \sum_{i=1}^{N} \exp\left[-\beta F_i\right] + \ln \pi_j + \beta F_j = 0 \tag{12}$$

For example, in case of two alternatives the procedure (5-12) results in

$$\begin{aligned} &\ln \sum_{i=1}^{N=2} \exp\left[-\beta F_i\right] + \ln \pi_1 + \beta F_1 = 0 \\ &= \ln \sum_{i=1}^{N=2} \exp\left[-\beta F_i\right] + \ln \pi_2 + \beta F_2 \end{aligned} \tag{13}$$

and finally

$$\ln \frac{\pi_1}{\pi_2} + \beta\left(F_1 - F_2\right) = 0 \tag{14}$$

For the purposes of the optimal control theory problems formulation and modeling, sometimes, it is convenient to consider duality problems, like adjacent problems.

Let us consider a problem formulation with the objective functional

$$\Phi_{\tilde{F}} = \int_{t_0}^{t_1} \left\{ \begin{aligned} &-\sum_{i=1}^{N} \tilde{F}_i(t) \ln \tilde{F}_i(t) - \beta_{\tilde{F}} \sum_{i=1}^{N} \pi_i(t) \tilde{F}_i(t) \\ &+\gamma_{\tilde{F}} \left[\sum_{i=1}^{N} \tilde{F}_i(t) - 1 \right] \end{aligned} \right\} \mathrm{d}t \tag{15}$$

where

$$\tilde{F}_i(t) = \tilde{F}_i\left[\pi_1(t), \pi_2(t), \ldots, \pi_N(t)\right] \tag{16}$$

a corresponding dimensionless function; $\beta_{\tilde{F}}$ and $\gamma_{\tilde{F}}$ – analogues to structural parameters β, γ introduced before.

Extremizing the functional (15) on the principle of the entropy of the relative effectiveness functions (16) maximization by the methods similar to (5-14), in case of $N = 2$, we obtain the symmetrical to the identity (14) expression

$$\ln \frac{\tilde{F}_1}{\tilde{F}_2} = -\beta_{\tilde{F}}\left(\pi_1 - \pi_2\right) \tag{17}$$

Here, in the expression (17), there is the ideological concept of reversion (mirror reflection) correspondence to the law (14).

Now, let us consider the variant (2) with making allowance for " $-\beta$ " instead of " $+\beta$ ", the canonical distributions of preferences of the type of (4), and the simplest case of two achievable alternatives.

Then, calculating first partial derivatives of the equations of the sort of (4), where the corresponding α_i are introduced into the related with the alternative effectiveness functions F_i, we get

$$\frac{\partial \pi_1}{\partial F_1} = -\beta \pi_1 \pi_2 \ \textit{and}\ \frac{\partial \pi_1}{\partial F_2} = \beta \pi_1 \pi_2 \tag{18}$$

From the normalizing condition it follows

$$\frac{\partial \pi_1}{\partial F_1} = -\frac{\partial \pi_2}{\partial F_1} \tag{19}$$

Therefore, by comparison of the expressions (18,19) we can have

$$\frac{\partial \pi_1}{\partial F_1} = -\frac{\partial \pi_2}{\partial F_1} = -\frac{\partial \pi_1}{\partial F_2} \Rightarrow \frac{\partial \pi_2}{\partial F_1} = \frac{\partial \pi_1}{\partial F_2} \tag{20}$$

The expressions (20) represent the laws of subjective conservatism

$$\frac{\partial \pi_1}{\partial F_1} + \frac{\partial \pi_1}{\partial F_2} = 0 \ \textit{and}\ \frac{\partial \pi_1}{\partial F_2} - \frac{\partial \pi_2}{\partial F_1} = 0 \tag{21}$$

$$\frac{\partial \pi_2}{\partial F_1} + \frac{\partial \pi_2}{\partial F_2} = 0 \ \textit{and}\ \frac{\partial \pi_1}{\partial F_1} - \frac{\partial \pi_2}{\partial F_2} = 0 \tag{22}$$

For the case of some three alternatives the speculations similar to the formulae (18-22) yield the following expressions

$$\frac{\partial \pi_1}{\partial F_1} = -\beta \pi_1\left(\pi_2 + \pi_3\right), \frac{\partial \pi_1}{\partial F_2} = \beta \pi_1 \pi_2,$$

and

$$\frac{\partial \pi_1}{\partial F_3} = \beta \pi_1 \pi_3 \tag{23}$$

Thus

$$\frac{\partial \pi_1}{\partial F_1} + \frac{\partial \pi_1}{\partial F_2} + \frac{\partial \pi_1}{\partial F_3} = 0 \tag{24}$$

In an analogous way

$$\frac{\partial \pi_2}{\partial F_2} = -\beta \pi_2\left(\pi_1 + \pi_3\right), \frac{\partial \pi_2}{\partial F_1} = \beta \pi_1 \pi_2,$$

and

$$\frac{\partial \pi_2}{\partial F_3} = \beta \pi_2 \pi_3 \tag{25}$$

$$\frac{\partial \pi_3}{\partial F_3} = -\beta \pi_3\left(\pi_1 + \pi_2\right), \frac{\partial \pi_3}{\partial F_1} = \beta \pi_1 \pi_3,$$

and

$$\frac{\partial \pi_3}{\partial F_2} = \beta \pi_2 \pi_3 \tag{26}$$

$$\frac{\partial \pi_2}{\partial F_1} + \frac{\partial \pi_2}{\partial F_2} + \frac{\partial \pi_2}{\partial F_3} = 0$$

and

$$\frac{\partial \pi_3}{\partial F_1} + \frac{\partial \pi_3}{\partial F_2} + \frac{\partial \pi_3}{\partial F_3} = 0 \tag{27}$$

Equivalent to the expressions (23-27) derivations will give

$$\frac{\partial\pi_1}{\partial F_2}-\frac{\partial\pi_2}{\partial F_1}=0, \frac{\partial\pi_1}{\partial F_3}-\frac{\partial\pi_3}{\partial F_1}=0,$$

and

$$\frac{\partial\pi_2}{\partial F_3}-\frac{\partial\pi_3}{\partial F_2}=0 \tag{28}$$

At last

$$\begin{array}{l}\frac{\partial\pi_1}{\partial F_1}-\frac{\partial\pi_2}{\partial F_2}-\frac{\partial\pi_3}{\partial F_3}=2\beta\pi_2\pi_3\\ \frac{\partial\pi_1}{\partial F_1}-\frac{\partial\pi_2}{\partial F_2}+\frac{\partial\pi_3}{\partial F_3}=-2\beta\pi_1\pi_3\end{array},$$

and

$$\frac{\partial\pi_1}{\partial F_1}+\frac{\partial\pi_2}{\partial F_2}+\frac{\partial\pi_3}{\partial F_3}=-2\beta\left(\pi_1\pi_2+\pi_1\pi_3+\pi_2\pi_3\right) \tag{29}$$

From the derivations for the three alternatives represented with the expressions of (23-29) it is noticeable that the equalities (18-22) are the special cases of (23-29), and dependences (18-22) can be derived from the formulae (23-29) on conditions of application of the corresponding preferences functions as well as the functions of effectiveness. Going on with the number of alternatives we come to the following generalizations

$$\ln\frac{\pi_i}{\pi_k}+\beta\left(F_i-F_k\right)=0, \forall i,k\in\overline{1,N},$$

for any

$$j=\overline{1,\ N} \tag{30}$$

$$\frac{\partial\pi_i}{\partial F_k}-\frac{\partial\pi_k}{\partial F_i}=0, \forall i,k\in\overline{1,N}. \tag{31}$$

Modeling by the methods (1-31) gives the evidence that optimal control in active systems implies the existence of some values being conserved.

3. Results

Let us consider the having been conducted calculation experiment. Applicably to an active, let us say aviation transportation, system, for the case of two alternative modes of the aircraft operation, extremizing the objective functional of the kind of (2) as for the maximal distance of the horizontal flight, and having dependences

$$F_i=\frac{2\eta Q\rho v_i^2 S}{C_{x0}\left(\rho v_i^2 S\right)^2+b\left(2mg\right)^2} \tag{32}$$

where η – efficiency (coefficient of the useful action) of the propulsive complex, considered constant; Q – low calorific value of the fuel by its working mass; ρ – density of the air at the given altitude; v_i – function of the flight speed related to the reachable alternative; S – character square-area of the flying object; C_{x0} – value of the head resistance force coefficient at the value of the lifting force when it equaled zero which had been determined within the given diapason of speeds from the blowings in the wind tunnels (aerodynamic tubes); b – some stable value which had been determined in the analogous to C_{x0} way; m – mass of the flying apparatus; g – acceleration, stipulated by the gravitational force, which had been considered being constant and equaled to g = 9.81 m/s^2; the other assumed values: β = 0.045; η = 0.3; Q = 42,700·10^3 J/kg; ρ = 1 kg/m^3; S = 50 m^2; C_{x0} = 0.02; b = 0.045; m = 1·10^4 … 8·10^3 kg;

$$\begin{array}{l}v_1\left(m\right)=a_0m^3+a_1m^2+a_2m+a_3=m\mathbf{A};\\ v_2\left(m\right)=c_0m^3+c_1m^2+c_2m+c_3=m\mathbf{C};\end{array}$$

where

$$\mathbf{A}=\left\|\mathbf{A}\right\|=\left\|\begin{matrix}a_0\\a_1\\a_2\\a_3\end{matrix}\right\|=\left\|\begin{matrix}-2.397\cdot10^{-8}\\6.679\cdot10^{-4}\\-6.125\\1.859\cdot10^{4}\end{matrix}\right\|$$

$$\mathbf{C}=\left\|\mathbf{C}\right\|=\left\|\begin{matrix}c_0\\c_1\\c_2\\c_3\end{matrix}\right\|=\left\|\begin{matrix}3.974\cdot10^{-8}\\-1.005\cdot10^{-3}\\8.459\\-2.362\cdot10^{4}\end{matrix}\right\|; \tag{33}$$

we have got the law of subjective conservatism at the optimal control of the maximal distance horizontal flight in the view of (14,30), illustrated in Figure 1.

Modeling through (1-33) leads to the discussion on the topic of RP’s optimal controlling decisions making.

4. Discussion

The identity (14) is exactly the well known Weber-Fechner law in the view of [15]

$$p=k\ln\frac{S}{S_0} \tag{34}$$

where p – perception; k – a certain coefficient of proportionality; S – stimulus; S_0 – threshold of stimulus.

Or in the view of [16]

$$p-p_0=k\log\frac{S}{S_0} \tag{35}$$

where p_0 – threshold of perception.

It is obvious, for the interpretation of the identity (14) should be correct with the expressions (34,35), the corresponding values have to be considered: the preferences of the related alternatives π_1 and π_2 – as the stimuli, β is connected to k, and the effectiveness functions of F_1 and F_2 – as the perceptions.

Thus, there is a great analogy in these fundamental laws, and subjective preferences are interpreted as stimuli.

It is vice versa for the expression (17). The preferences are interpreted as perceptions.

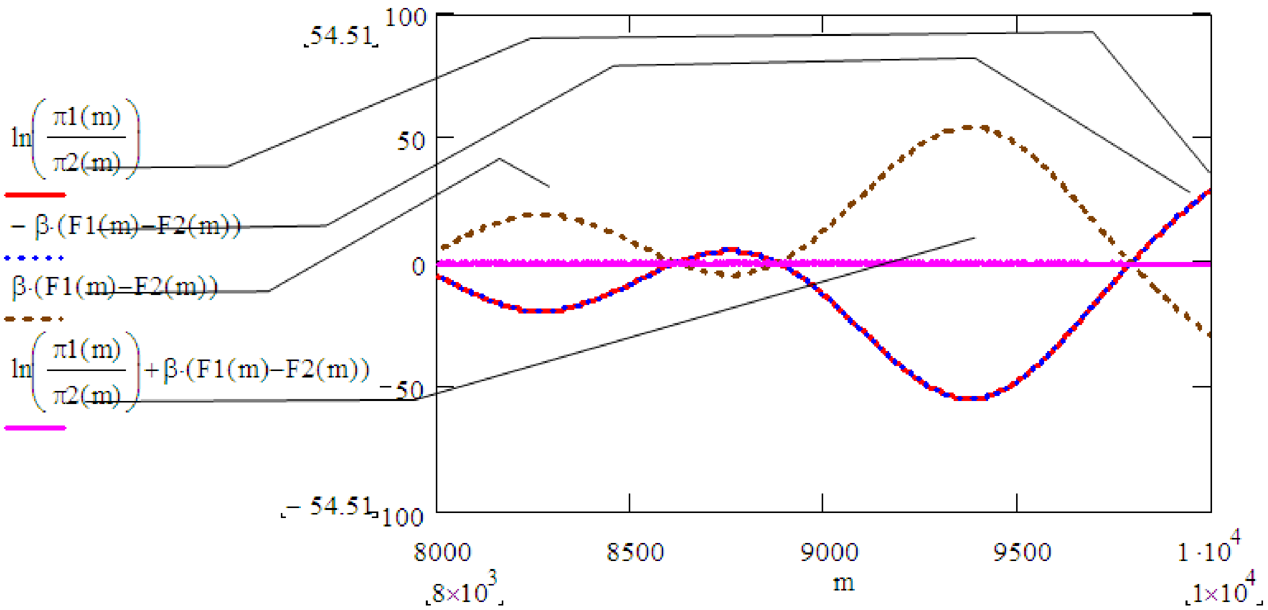

Figure 1. The law of subjective conservatism

Then, expressions analogous to the equality (14), namely (17), can also be interpreted in terms of the law of subjective value by Jakob Bernoulli, [17]:

$$Y = k \ln\left(\frac{X}{X_{\min}}\right) \tag{36}$$

where Y – subjective value of a good for a person; X – objective value of the good; X_{min} – minimal objective value.

In the expression (17) there is the concept correspondence to the law (36). The dependencies (17,36) are modeling subjective response to the objective alternative.

One of the possible applications of the principle, modeling, for example, in the case of the maximal distance horizontal flight with the functions of the effectiveness expressed in the view of the equation (32) with the conditions (33), shows that the optimal decision making (thus control) is realized on the basis of the subjective conservatism laws (14,30) obeying, see Figure 1.

5. Conclusions

The postulated in subjective analysis optimization principle (the author is Professor V.A. Kasianov) allows getting analytically the dependences revealed empirically for hundreds years ago. It evidently shows the justification of the postulates and makes a kind of a breakthrough in the theory of optimal control in active systems and applicable research problems modeling. The proposed approach uses the individual preferences of the RP at making decisions, gives a possibility of getting closure to more explainable and therefore precise solutions. The given concept makes it possible to derive laws of subjective conservatism (14,17-31). Moreover, the discovered by Ernst Heinrich Weber (1795–1878) in 1834 and developed by Gustav Theodor Fechner (1801–1887) in 1858 (for almost two hundred years ago) the main law of psychophysiology, the Weber-Fechner law (34,35), as well as the law of subjective value by Jakob Bernoulli (36) [15,16,17] have the explicit expressions in the terms of the subjective entropy extrimization principle (14,17,30). That is a good substantiation for such an approach of being correct.

Nevertheless, the applicability of the concept should be tested by further researches in the different scientific areas with other types of functionals and conditional restrictions. It seems prospective to combine the presented entropy paradigm with flight modelling and safety [18].

Statement of Competing Interests

The author has no competing interests.

List of Abbreviations

RP – responsible person.

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