DOI 10.1515/aee-2016-0041

Algorithm for determining two-periodic steady-states in AC machines directly in time domain

TADEUSZ J. SOBCZYK

Institute on Electromechanical Energy Conversions, Cracow University of Technology
Warszawska 24, 31-155 Kraków, Poland
tel./fax: +48 12 628 2044
e-mail: tadeusz.sobczyk@pk.edu.pl

(Received: 17.11.2015, revised: 07.03.2016)

Abstract: This paper describes an algorithm for finding steady states in AC machines for the cases of their two-periodic nature. The algorithm enables to specify the steady-state solution identified directly in time domain despite of the fact that two-periodic waveforms are not repeated in any finite time interval. The basis for such an algorithm is a discrete differential operator that specifies the temporary values of the derivative of the two-periodic function in the selected set of points on the basis of the values of that function in the same set of points. It allows to develop algebraic equations defining the steady state solution reached in a chosen point set for the nonlinear differential equations describing the AC machines when electrical and mechanical equations should be solved together. That set of those values allows determining the steady state solution at any time instant up to infinity. The algorithm described in this paper is competitive with respect to the one known in literature an approach based on the harmonic balance method operated in frequency domain.

Key words: AC machines, steady state analysis, two-periodic solutions, analysis in time domain, discrete differential operator, iterative algorithms

1. Introduction

Determining the steady-states belong to the elementary problems in electrical engineering and methods for that are essential tools for exploring the properties of electrical systems. Steady-states in the electrical machines are of particular interest for engineers because are referred to their technical and economic parameters. The generic problem of determining such solutions for AC machines is complicated because of the need to find solutions of differential equations, such as

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathbf{L}(\mathbf{i},\varphi)\,\mathbf{i}\right)+\mathbf{R}\,\mathbf{i}=\mathbf{u}(t)\,, \tag{1a}$$

$$\mathrm{J}\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+\mathrm{D}\frac{\mathrm{d}\varphi}{\mathrm{d}t}=T_{\mathrm{em}}(i,\varphi)+T_{\mathrm{m}}(t,\varphi)\,. \tag{1b}$$

The first one (1a) describes electromagnetic phenomena in the machine, whereas the second (1b) describes the machine's motion. Together, they constitute a set of nonlinear differential equations, which cannot be directly solved using methods for determining the steady solutions of electrical systems. Often, the electro-magnetic and mechanical phenomena in the machine can be treated separately, i.e. when the angular velocity is constant $(\varphi=\Omega t+\varphi_0)$. For such cases the Equations (1a) become linear and the steady-state solutions can be found by solving the Equations (1a) by the harmonic balance method [1-4]. The solution is predicted in the form of the double Fourier series with two independent periods. The first one is related to the period of supplied voltages, and the second refers to the rotor's angular velocity determining the variations of inductances. It leads to the set of algebraic equations for Fourier series coefficients. In the simplest cases, for the symmetrically designed machines and at properly selected assumptions, the Equations (1a) can be transformed to the set of differential equations with constant coefficients, i.e. coefficients which are independent on the rotation angle φ. Such equations take the form

$$\mathbf{L}_{\mathrm{e}}\frac{\mathrm{d}\,\mathbf{i}_{\mathrm{e}}}{\mathrm{d}t}+\mathbf{R}_{\mathrm{e}}\mathbf{i}_{\mathrm{e}}+\frac{\mathrm{d}\varphi}{\mathrm{d}t}\mathbf{M}_{\mathrm{e}}\mathbf{i}_{\mathrm{e}}=\mathbf{u}_{\mathrm{e}}(t)\,,$$

and at constant speed ($\mathrm{d}\varphi/\mathrm{d}t=\Omega=\mathrm{const.}$) become linear with constant coefficients. For that class of equations it is rather easy to determine steady-states using the routine methods known in electrical engineering. This approach is widely used in engineering practice. However, in the cases when electromechanical interactions should be considered, i.e. if the angular velocity in the steady-state is also varied, the equations (1a) and (1b) have to be solved together. It can happen when the symmetry of machine is lost or perturbations of external mechanical load appear.

In [1, 2] an algorithm is presented for determining two-periodic solutions to the equations (1a) assuming periodicity of supplied voltages, constant angular velocity, linearity of the magnetic circuit and multi-harmonic dependence of the inductances on the angle of rotation. In [5] this algorithm is extended to a nonlinear magnetic circuit example. In [6] an algorithm is described to determine a steady-state solution for a symmetrical synchronous machine, described by "classical" equations (i.e. assuming magnetic linearity and mono-harmonic MMF of windings), subjecting to the periodic mechanical disturbance torque. For that case the Equations (1a) and (1b) are solved together. All these algorithms applied the harmonic balance method for periodic or two-periodic functions, so they operate in frequency domain. However, such algorithms complicate when solving the nonlinear differential equations because the required multiple calculations of nonlinear time functions appearing in the differential equations are based on the double Fourier series of solutions and vice versa – multiple calculations of the double Fourier series coefficients are based on those nonlinear time functions. This paper describes an algorithm which allows to directly calculate the time wave-forms of the two-periodic steady state solutions for equations (1a) and (1b), in spite of the fact that such waveforms are not repeated in any time interval. That algorithm omits difficulties appearing in algorithms based on the harmonic balance method.

2. The problem construct

From the mathematical point of view the problem in this paper focuses on to seeking solutions for the nonlinear differential equation system of the form

$$\frac{\mathrm{d}\,\mathbf{x}}{\mathrm{d}t} = \mathbf{f}(\mathbf{x},t)\,, \tag{2}$$

wherein $\mathbf{x}$ is a vector of sought solutions and $\mathbf{f}(\mathbf{x},t)$ is a vector of nonlinear functions with respect to the sought solutions, for which the steady state solutions can be predicted in the form of the double Fourier series of two-periodic functions

$$\mathbf{x}(t) = \sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\mathbf{X}_{r,s}\mathrm{e}^{\mathrm{j}r\Omega_{\mathrm{m}}t}\mathrm{e}^{\mathrm{j}s\Omega_{\mathrm{e}}t}\,. \tag{3}$$

In equations of AC machines (1a), (1b) the vector $\mathbf{x}$ can contain both electrical and mechanical variables and the steady state solutions in most cases are provided for the series (3) with two independent pulsations: Ω_{e} is a result of the periodicity of supplied voltages and Ω_{m} is a result of the rotor angular velocity of the machine. For further deliberations system (2) should be reduced to the form

$$\frac{\mathrm{d}\,\mathbf{x}}{\mathrm{d}t} = \mathbf{A}(\mathbf{x})\,\mathbf{x} + \mathbf{b}(t)\,, \tag{4}$$

by approximating the nonlinear functions in the $\mathbf{f}(\mathbf{x},t)$ vector by the Taylor series with respect to the variables $\mathbf{x}$. The vector $\mathbf{b}(t)$ in (3) has to contain two-periodic time functions for the solution to be doubly periodic.

In order to create an algorithm for determining the steady solution reached directly in the time domain a discrete differentiation operator of the two-periodic function will be defined. It determines the values of its derivative in the selected set of time instants on the basis of function values in the same set of time instants. Such operator using the magnitudes of steady-state solutions in the selected set of time instants allows to create the iterative algorithms leading to the solution of interest [10].

3. The discrete differentiation operator for two-periodic function in the time domain

A relatively simple differential operator can be specified for two-periodic functions in frequency domain. If one assumes that the sought two-periodic function has a first derivative, it follows that

$$\frac{\mathrm{d}\,\mathbf{x}}{\mathrm{d}t} = \sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} \mathrm{j}(r\Omega_\mathrm{m} + r\Omega_\mathrm{e})\mathbf{X}_{r,s}\mathrm{e}^{\mathrm{j}r\Omega_\mathrm{m}t}\mathrm{e}^{\mathrm{j}s\Omega_\mathrm{e}t} . \tag{5}$$

The relationship between the coefficients of the Fourier series of the function and of its first derivative can be written as

$$\boldsymbol{\mathcal{X}}' = \mathrm{j}\,\boldsymbol{\Omega}\,\boldsymbol{\mathcal{X}}. \tag{6}$$

In this formula $\boldsymbol{\mathcal{X}}'$ is a properly ordered hyper-vector of the Fourier coefficients of a two-periodic function derivative composed of vectors $\mathbf{X}'_r$

$$\boldsymbol{\mathcal{X}}' = \begin{bmatrix}\cdots & \mathbf{X}'_1 & \mathbf{X}'_0 & \mathbf{X}'_{-1} & \cdots\end{bmatrix}^T ,$$

of the form

$$\mathbf{X}'_r = \begin{bmatrix}\cdots & \mathbf{X}'_{r,1} & \mathbf{X}'_{r,0} & \mathbf{X}'_{r,-1} & \cdots\end{bmatrix}^T .$$

Similarly, $\boldsymbol{\mathcal{X}}$ is a properly ordered hyper-vector of the Fourier coefficients of a two-periodic function composed of vectors $\mathbf{X}_r$

$$\boldsymbol{\mathcal{X}}_r = \begin{bmatrix}\cdots & \mathbf{X}_1 & \mathbf{X}_0 & \mathbf{X}_{-1} & \cdots\end{bmatrix}^T ,$$

of the form

$$\mathbf{X}_r = \begin{bmatrix}\cdots & \mathbf{X}_{r,1} & \mathbf{X}_{r,0} & \mathbf{X}_{r,-1} & \cdots\end{bmatrix}^T .$$

The $\boldsymbol{\Omega}$ matrix is the differential operator for two-periodic functions in frequency domain, and is in the form of a hyper-diagonal matrix

$$\boldsymbol{\Omega} = \begin{bmatrix}\cdots & \boldsymbol{\Omega}_1 & \boldsymbol{\Omega}_0 & \boldsymbol{\Omega}_{-1} & \cdots\end{bmatrix},$$

in which the successive matrices $\boldsymbol{\Omega}_r$ are diagonal too and have the form

$$\boldsymbol{\Omega}_r = \begin{bmatrix}\cdots & \Omega_{r,1}\mathbf{E} & \Omega_{r,0}\mathbf{E} & \Omega_{r,-1}\mathbf{E} & \cdots\end{bmatrix},$$

where $\Omega_{r,s} = r\Omega_\mathrm{m} + s\Omega_\mathrm{e}$ and $\mathbf{E}$ is a unitary matrix.

To get the differential operator in time domain the relations between the values of a two-periodic function and its Fourier coefficients should be written. This can be done if the series (3) is limited to a finite number of terms $-\mathrm{R} \le r \le \mathrm{R}$ and $-\mathrm{S} \le s \le \mathrm{S}$. To do this, the expected solution (3) should be presented as a function of two variables [5, 10]

$$\mathbf{x}(\rho,\sigma) = \sum_{r=-\mathrm{R}}^{\mathrm{R}}\sum_{r=-\mathrm{S}}^{\mathrm{S}} \mathbf{X}_{r,s}\mathrm{e}^{\mathrm{j}r\rho}\mathrm{e}^{\mathrm{j}s\sigma} , \tag{7}$$

defining $\rho = \Omega_m t$ and $\sigma = \Omega_e t$ as independent variables. Such a function is periodic for each of these variables

$$\mathbf{x}_a(\rho,\sigma) = \mathbf{x}_a(\rho + 2\pi, \sigma) \text{ and } \mathbf{x}_a(\rho,\sigma) = \mathbf{x}_a(\rho, \sigma + 2\pi).$$

For such a function unique relations can be found between its values and the coefficients of the series (7) for selecting the following sets of points: $\rho_k = k\,\alpha$ for $-\mathrm{R} \le k \le \mathrm{R}$ where $\alpha = 2\pi/(2\mathrm{R}+1)$ and $\sigma_l = l\,\beta$ for $-\mathrm{S} \le l \le \mathrm{S}$ where $\beta = 2\pi/(2\mathrm{S}+1)$. After proper ordering this relation can be written as

$$\boldsymbol{x}_a = \boldsymbol{T}\,\boldsymbol{X}_a. \tag{8}$$

In this notation $\boldsymbol{x}_a$ is the hyper-vector containing values of the function (7) in the selected set of points, ordered according as follows

$$\boldsymbol{x}_a = \begin{bmatrix} \mathbf{x}_R^a & \cdots & \mathbf{x}_1^a & \mathbf{x}_0^a & \mathbf{x}_{-1}^a & \cdots & \mathbf{x}_{-R}^a \end{bmatrix}^T.$$

This hyper–vector is created with 2R + 1 vectors of the form

$$\mathbf{x}_k^a = \begin{bmatrix} \mathbf{x}_{k,S}^a & \cdots & \mathbf{x}_{k,1}^a & \mathbf{x}_{k,0}^a & \mathbf{x}_{k,-1}^a & \cdots & \mathbf{x}_{k,-S}^a \end{bmatrix}^T,$$

and contains the values of the function (6) at points (ρ_k, σ_l). The $\boldsymbol{X}_a$ hyper-vector is formed from the coefficients of the series (7) structured similarly. It is composed of 2R + 1 vectors $\mathbf{X}_k^a$

$$\boldsymbol{X}_a = \begin{bmatrix} \mathbf{X}_R^a & \cdots & \mathbf{X}_1^a & \mathbf{X}_0^a & \mathbf{X}_{-1}^a & \cdots & \mathbf{X}_{-R}^a \end{bmatrix}^T,$$

of in the form

$$\mathbf{X}_k^a = \begin{bmatrix} \mathbf{X}_{k,S}^a & \cdots & \mathbf{X}_{k,1}^a & \mathbf{X}_{k,0}^a & \mathbf{X}_{k,-1}^a & \cdots & \mathbf{X}_{k,-S}^a \end{bmatrix}^T,$$

and it contains the coefficients $\mathbf{X}_{k,l}^a$ of the series (7). The $\boldsymbol{T}$ hyper-matrix has the form

$$\boldsymbol{T} = \begin{bmatrix} a^{R^2}\mathbf{B} & & a^{R}\mathbf{B} & a^{0}\mathbf{B} & a^{-R}\mathbf{B} & & a^{-R^2}\mathbf{B} \\ & & & & & & \\ a^{R}\mathbf{B} & & a^{1}\mathbf{B} & a^{0}\mathbf{B} & a^{-1}\mathbf{B} & & a^{-R}\mathbf{B} \\ a^{0}\mathbf{B} & & a^{0}\mathbf{B} & a^{0}\mathbf{B} & a^{0}\mathbf{B} & & a^{0}\mathbf{B} \\ a^{-R}\mathbf{B} & & a^{-1}\mathbf{B} & a^{0}\mathbf{B} & a^{1}\mathbf{B} & & a^{R}\mathbf{B} \\ & & & & & & \\ a^{-R^2}\mathbf{B} & & a^{-R}\mathbf{B} & a^{0}\mathbf{B} & a^{R}\mathbf{B} & & a^{R^2}\mathbf{B} \end{bmatrix}; \quad a = e^{j\alpha},$$

$$\mathbf{B} = \begin{bmatrix} b^{S^2} & \cdots & b^{S} & b^{0} & b^{-S} & \cdots & b^{-S^2} \\ \vdots & & \vdots & \vdots & \vdots & & \\ b^{S} & \cdots & b^{1} & b^{0} & b^{-1} & \cdots & b^{-S} \\ b^{0} & \cdots & b^{0} & b^{0} & b^{0} & \cdots & b^{0} \\ b^{-S} & \cdots & b^{-1} & b^{0} & b^{1} & \cdots & b^{S} \\ & & & & & & \\ b^{-S^2} & \cdots & b^{-S} & b^{0} & b^{S} & \cdots & b^{S^2} \end{bmatrix}; \; b = e^{j\beta} .$$

The $\boldsymbol{T}$ matrix is the square one and has the dimensions (2R + 1)(2S + 1). It can be proved that it is the unitary matrix and satisfies the condition

$$(\boldsymbol{T})\ (\overset{*}{\boldsymbol{T}}{}^{T})\ = (2R+1)(2S+1)\,\boldsymbol{E}\ ,$$

where $\boldsymbol{E}$ is the unitary matrix of appropriate dimensions, so the inverse matrix has the following form

$$\boldsymbol{T}^{-1} = \frac{1}{(2R+1)(2S+1)}\ (\overset{*}{\boldsymbol{T}}{}^{T})\ .$$

It allows to write the relation between the coefficients of the double-series (7) and the values of function $\mathbf{x}_a(\rho,\sigma)$ at the points (ρ_k,σ_l) as

$$\boldsymbol{X}_a = \boldsymbol{T}^{-1}\,\boldsymbol{x}_a\ . \tag{9}$$

The compounds (8) and (9) allow to define the discrete differential operator in time domain on the basis of the operator (6) in frequency domain, reduced to the size resulting from (7). To do this, proceed as shown in the following equation

$$\boldsymbol{T}\,\boldsymbol{X}'_a = j\left(\boldsymbol{T}\ \boldsymbol{\Omega}_a\,\boldsymbol{T}^{-1}\right)(\boldsymbol{T}\,\boldsymbol{X}_a)\ .$$

This leads to a relationship between the values of the derivative and of the function itself at the point set (ρ_k,σ_l)

$$\boldsymbol{x}'_a = \boldsymbol{D}\ \boldsymbol{x}_a\ . \tag{10}$$

The $\boldsymbol{D}$ matrix is the sought discrete differential operator for the two-periodic function

$$\boldsymbol{D} = j\,(\boldsymbol{T}\ \boldsymbol{\Omega}_a\,\boldsymbol{T}^{-1})\ . \tag{11}$$

The $\boldsymbol{D}$ operator will be used to create an algorithm for determining the two-periodic steady-state solution for nonlinear differential equations of the form (4). The discrete differential operator for the periodic function is presented in [8, 9].

4. Determining algorithms for AC machine equations

To form equations determining the two-periodic solution directly in time domain the time derivative on the left-hand side in Equation (4) should be replaced by a discrete differential operator (11), whereas the functions on the right-hand of (4) should be substituted by their values at the points (ρ_k, σ_l), arranged as in the vector of values in the solution sought. This leads to a system of algebraic equations of the general form

$$\boldsymbol{D}\,\boldsymbol{x}_a = \boldsymbol{A}(\boldsymbol{x}_a)\,\boldsymbol{x}_a + \boldsymbol{b}_a \,. \tag{12}$$

The diagonal hyper-matrix $\boldsymbol{A}(\boldsymbol{x}_a)$ is formed from the values of the matrix $\mathbf{A}(\mathbf{x})$ in (4) for the point set (ρ_k, σ_l), arranged as in the vector $\boldsymbol{x}_a$, i.e.

$$\boldsymbol{A}(\boldsymbol{x}_a) = \mathrm{diag}\left[\mathbf{A}_R \quad \cdots \quad \mathbf{A}_1 \quad \mathbf{A}_0 \quad \mathbf{A}_{-1} \quad \cdots \quad \mathbf{A}_{-R}\right],$$

where $\mathbf{A}_k$ are the diagonal hyper-matrices of the form

$$\mathbf{A}_k = \mathrm{diag}\left[\mathbf{A}_{k,S} \quad \cdots \quad \mathbf{A}_{k,1} \quad \mathbf{A}_{k,0} \quad \mathbf{A}_{k,-1} \quad \cdots \quad \mathbf{A}_{k,-S}\right],$$

in which $\mathbf{A}_{k,l}$ are the diagonal matrices with the values of the $\mathbf{A}(\mathbf{x})$ matrix at the points (ρ_k, σ_l). The $\boldsymbol{b}_a$ vector is created based on the vector $\mathbf{b}(t)$ in (4), analogously to the hyper-vector $\boldsymbol{x}_a$.

The equation set (4) is nonlinear because the $\boldsymbol{A}(\boldsymbol{x}_a)$ matrix depends on the vector of solution $\boldsymbol{x}_a$, which is symbolically indicated in (12). So, the system of Equations (12) must be solved iteratively

$$\boldsymbol{x}_a^{i+1} = \left(\boldsymbol{D} - \boldsymbol{A}(\boldsymbol{x}_a^i)\right)^{-1} \boldsymbol{b}_a \,, \tag{13}$$

The Equation (13) is the basis for the iterative algorithm for determining the steady-state two-periodic solution to the nonlinear set of differential equations of the form (4). Such an iterative algorithm for the periodic steady state solution for the nonlinear differential equations has been developed in [9] and the first tests for a simple electromechanical converter have been presented in [10].

The discrete differential operator (11) can be applied directly to the machine equations in the form of (1a, b), as their reduction to the form (4) in some cases may be troublesome. It allows to create the algorithms based directly on the equations of AC machines (1a, b). Two cases should be distinguished:

1) when only the Equation (1a) is to be solved,
2) when the Equation (1a) and (1b) have to be solved together.

In the first case, the Equation (1a) describes the AC machines when the rotation speed of them is constant, but the winding inductances are arbitrary periodic functions of the rotary angle and the saturation of a machine magnetic circuit is considered. The algebraic equations, respective to (13) take the form

$$\left(\boldsymbol{D}\,\boldsymbol{L}(\boldsymbol{i}_a^i) + \boldsymbol{R}\right) \boldsymbol{i}_a^{i+1} = \boldsymbol{u}_a \,. \tag{14}$$

For the case 2, where it is necessary to solve the Equations (1a, b) together, they can be rewritten to the form

$$\mathbf{A}_2 \frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} + \frac{\mathrm{d}}{\mathrm{d}t}\big(\mathbf{A}_1(\mathbf{x})\,\mathbf{x}\big) + \mathbf{A}_0\mathbf{x} = \mathbf{b}(\mathbf{x},t)\,. \tag{15}$$

The corresponding to (13) equations determining the two-periodic steady-state solutions take the form

$$\left(\boldsymbol{A}_2\boldsymbol{D}^2 + \boldsymbol{D}\boldsymbol{A}_1(\boldsymbol{x}_\mathrm{a}^i) + \boldsymbol{A}_0\right)\,\boldsymbol{x}_\mathrm{a}^{i+1} = \boldsymbol{b}_\mathrm{a}\,. \tag{16}$$

All matrices and vectors in the Equations (14) and (16) are formed as for the Equations (13).

Equations (13), (14) and (16) allow to determine the solutions at the selected set of points for the two variables (ρ,σ) in the area

$$(-\pi \le \rho \le \pi\,;\ -\pi \le \sigma \le \pi\,). \tag{17}$$

In order to determine the solutions at any time instant from $t \in \{-\infty,\infty\}$, the values of substituted variables (ρ,σ) at the time instant "*t*" should be found (ρ_t,σ_t) and that point should be localized in the area (17). Next, the value of the solution at the time instant "*t*" could be approximated by the values of the solution at points (ρ_k,σ_l) closer to (ρ_t,σ_t). Another way is to determine, using the formula (9), the Fourier coefficients of the double Fourier series (7) based on the solutions at the points (ρ_k,σ_l) and to calculate values of the solution at any time instant based on the double Fourier series in time domain (3), limited to the finite number of terms.

6. Conclusions

The paper presents new ideas of algorithms for direct calculations in time domain of two-periodic steady-state solutions for AC machine equations where they are nonlinear due to many reasons. The algebraic nonlinear equations have been developed and have become the basis for those algorithms. They eliminate the need to use the Fourier series approach in frequency domain. The crucial element of these algorithms is a discrete differentiation operator in time domain for two-periodic functions, not repeated in any finite time interval.

References

[1] Sobczyk T., *Infinitely-dimensional linear and quadratic forms of electric machines,* Rozprawy Elektrotechniczne 29: 697-707 (1983).

[2] Sobczyk T., *A reinterpretation of the Floquet solution of the ordinary differential equation system with periodic coefficients as a problem of infinite matrix*, Compel 5: 1-22 (1986).

[3] Rusek J., *Computer analysis of induction machines by using the harmonic balance method*, Pub. University of Mining & Metallurgy (AGH), Cracow (2000) (in Polish).

[4] Sobczyk T., *Methodological aspects of mathematical modelling of induction machines*, WNT, Warsaw (2004) (in Polish).
[5] Sobczyk T., *Direct determination of two-periodic solution for nonlinear dynamic systems,* Compel 13: 509-529 (1994).
[6] Radzik M., *Algorithm for direct determination of steady-states in synchronous machines accounting for the equation of motion*, PhD Thesis, Cracow Univ. of Technology, Faculty of Electrical & Computer Eng. (2011) (in Polish).
[7] Radzik M., Sobczyk T.J., *Steady-state analysis of synchronous machines loaded by an angle depended torque*, Przegląd Elektrotechniczny 89(2b): 158-161 (2013).
[8] Sobczyk T., *Direct determination of periodic solution in time domain for differential equations*, Proc of Conf. "Selected Issues of Electrical Engineering & Electronics", Rzeszów-Czarna (2013) (in Polish).
[9] Sobczyk T.J., Radzik M., *Direct determination of periodic solution in the time domain for electromechanical converters*, Technical Transactions – Electrical Eng., Pub. Cracow University of Technology 112(2-E): 73-82 (2015).
[10] Sobczyk T.J., *An algorithm for determining steady-states for AC machines directly in time domain*, Electrical Machines – Transaction Journal 3(107): 1-5, 2015 (in Polish).