Ontario

Geometry and Spatial Sense, Grades 4 to 6

A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6

2008

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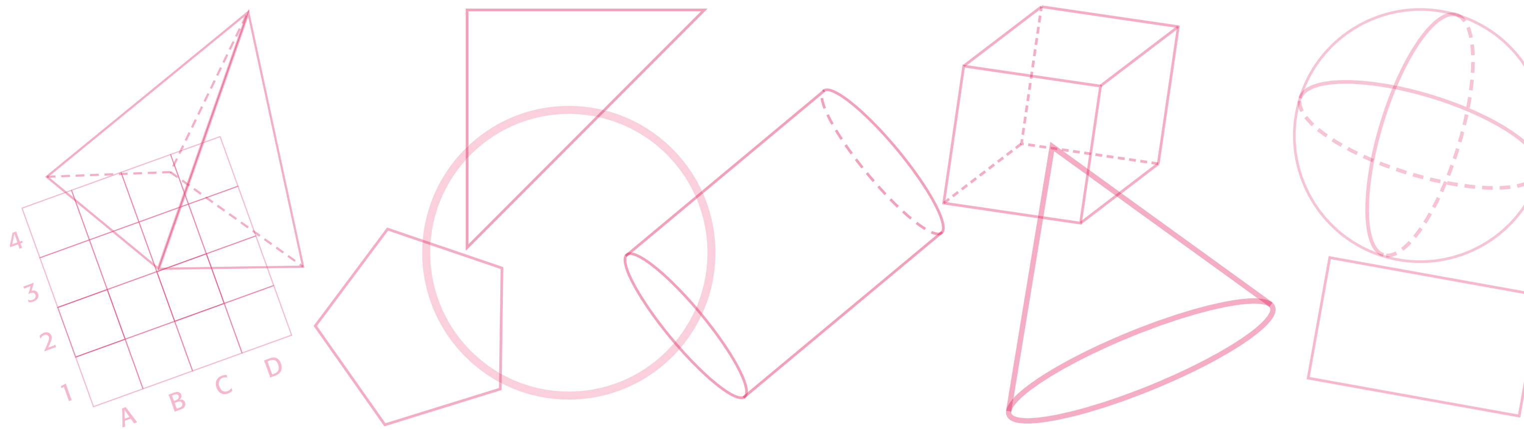
Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

CONTENTS



INTRODUCTION

Geometry and Spatial Sense, Grades 4 to 6 is a practical guide that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Geometry and Spatial Sense strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories that are elaborated in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

This guide provides:

- an overview of each of the three “big ideas”, or major mathematical themes, in the Geometry and Spatial Sense strand. The overview stresses the importance of focusing on the big ideas in mathematical instruction to achieve the goal of helping students gain a deeper understanding of the mathematical concepts;
- three sections that focus on the important curriculum topics of two-dimensional shapes, three-dimensional figures, and location and movement. Each of these sections provides a discussion of mathematical models and instructional strategies that have proved effective in helping students understand the mathematical concepts related to the topics;
- sample learning activities for Grades 4, 5, and 6. These learning activities illustrate how a learning activity can be designed to:
  – focus on an important curriculum topic;
  – involve students in applying the seven mathematical processes described in the mathematics curriculum document and reproduced on pages 10–11 of this document;
  – develop understanding of the big ideas in Geometry and Spatial Sense.

This guide also contains a list of the references cited throughout the guide. At the end of the guide is an appendix that discusses assessment strategies for teachers. There is also a glossary that includes mathematical and pedagogical terms used throughout the guide.

Working Towards Equitable Outcomes for Diverse Students

All students, whatever their socio-economic, ethnocultural, or linguistic background, must have opportunities to learn and to grow, both cognitively and socially. When students can make personal connections to their learning, and when they feel secure in their learning environment, their true capacity will be realized in their achievement. A commitment to equity and inclusive instruction in Ontario classrooms is therefore critical to enabling all students to succeed in school and, consequently, to become productive and contributing members of society.

To create effective conditions for learning, teachers must take care to avoid all forms of bias and stereotyping in resources and learning activities, which can quickly alienate students and limit their learning. Teachers should be aware of the need to provide a variety of experiences and to encourage multiple perspectives, so that the diversity of the class is recognized and all students feel respected and valued. Learning activities and resources for teaching mathematics should be inclusive, providing examples and illustrations and using approaches that recognize the range of experiences of students with diverse backgrounds, knowledge, skills, interests, and learning styles.

The following are some strategies for creating a learning environment that acknowledges and values the diversity of students and enables them to participate fully in the learning experience:

- providing mathematics problems with situations and contexts that are meaningful to all students (e.g., problems that reflect students' interests, home-life experiences, and cultural backgrounds and that arouse their curiosity and spirit of enquiry);
- using mathematics examples drawn from diverse cultures, including those of Aboriginal peoples;
- using children's literature that reflects various cultures and customs as a source of mathematical examples and situations;
- understanding and acknowledging customs and adjusting teaching strategies as necessary. For example, a student may come from a culture in which it is considered inappropriate for a child to ask for help, express opinions openly, or make direct eye contact with an adult;
- considering the appropriateness of references to holidays, celebrations, and traditions;
- providing clarification if the context of a learning activity is unfamiliar to students (e.g., describing or showing a food item that may be new to some students);
- evaluating the content of mathematics textbooks, children's literature, and supplementary materials for cultural or gender bias;
- designing learning and assessment activities that allow students with various learning styles (e.g., auditory, visual, tactile/kinaesthetic) to participate meaningfully;
- providing opportunities for students to work both independently and interdependently with others;

- providing opportunities for students to communicate orally and in writing in their home language (e.g., pairing English language learners with a first-language peer who also speaks English);
- using diagrams, pictures, manipulatives, sounds, and gestures to clarify mathematical vocabulary that may be new to English language learners.

For a full discussion of equity and diversity in the classroom, as well as a detailed checklist for providing inclusive mathematics instruction, see pages 34–40 in Volume 1 of *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

An important aspect of inclusive instruction is accommodating students with special education needs. The following section discusses accommodations and modifications as they relate to mathematics instruction.

Accommodations and Modifications

The learning activities in this guide have been designed for students with a range of learning needs. Instructional and assessment tasks are open-ended, allowing most students to participate fully in learning experiences. In some cases, individual students may require *accommodations* and/or *modifications*, in accordance with their Individual Education Plan (IEP), to support their participation in learning activities.

The term accommodations *is used to refer to the special teaching and assessment strategies, human supports, and/or individualized equipment required to enable a student to learn and to demonstrate learning. Accommodations do not alter the provincial curriculum expectations for the grade.*

Modifications *are changes made in the age-appropriate grade-level expectations for a subject . . . in order to meet a student's learning needs. These changes may involve developing expectations that reflect knowledge and skills required in the curriculum for a different grade level and/or increasing or decreasing the number and/or complexity of the regular grade-level curriculum expectations.*

(Ontario Ministry of Education, 2004, pp. 25–26)

PROVIDING ACCOMMODATIONS

Students may require accommodations, including special strategies, support, and/or equipment to allow them to participate in learning activities. There are three types of accommodations:

- *Instructional accommodations* are adjustments in teaching strategies, including styles of presentation, methods of organization, or the use of technology or multimedia.
- *Environmental accommodations* are supports or changes that the student may require in the physical environment of the classroom and/or the school, such as preferential seating or special lighting.
- *Assessment accommodations* are adjustments in assessment activities and methods that enable the student to demonstrate learning, such as allowing additional time to complete tasks or permitting oral responses to test questions.

Some of the ways in which teachers can provide accommodations with respect to mathematics learning activities are listed in the following chart.

Instructional Accommodations

- Vary instructional strategies, using different manipulatives, examples, and visuals (e.g., concrete materials, pictures, diagrams) as necessary to aid understanding.
- Rephrase information and instructions to make them simpler and clearer.
- Use non-verbal signals and gesture cues to convey information.
- Teach mathematical vocabulary explicitly.
- Have students work with a peer.
- Structure activities by breaking them into smaller steps.
- Model concepts using concrete materials, and encourage students to use them when learning concepts or working on problems.
- Have students use calculators and/or addition and multiplication grids for computations.
- Format worksheets so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Encourage students to use graphic organizers and graph paper to organize ideas and written work.
- Provide augmentative and alternative communications systems.
- Provide assistive technology, such as text-to-speech software.
- Provide time-management aids (e.g., checklists).
- Encourage students to verbalize as they work on mathematics problems.
- Provide access to computers.
- Reduce the number of tasks to be completed.
- Provide extra time to complete tasks.

Environmental Accommodations

- Provide an alternative workspace.
- Seat students strategically (e.g., near the front of the room; close to the teacher in group settings; with a classmate who can help them).
- Reduce visual distractions.
- Minimize background noise.
- Provide a quiet setting.
- Provide headphones to reduce audio distractions.
- Provide special lighting.
- Provide assistive devices or adaptive equipment.

Assessment Accommodations

- Have students demonstrate understanding using concrete materials or orally rather than in written form.
- Have students record oral responses on audiotape.
- Have students' responses on written tasks recorded by a scribe.
- Provide assistive technology, such as speech-to-text software.
- Provide an alternative setting.
- Provide assistive devices or adaptive equipment.
- Provide augmentative and alternative communications systems.

Assessment Accommodations

- Format tests so that they are easy to understand (e.g., use large-size font; an uncluttered layout; spatial cues, such as arrows; colour cues).
- Provide access to computers.
- Provide access to calculators and/or addition and multiplication grids.
- Provide visual cues (e.g., posters).
- Provide extra time to complete problems or tasks or answer questions.
- Reduce the number of tasks used to assess a concept or skill.

MODIFYING CURRICULUM EXPECTATIONS

Students who have an IEP may require modified expectations, which differ from the regular grade-level curriculum expectations. When developing modified expectations, teachers make important decisions regarding the concepts and skills that students need to learn.

Most of the learning activities in this document can be adapted for students who require modified expectations. The following chart provides examples of how a teacher could deliver learning activities that incorporate individual students' modified expectations.

Modified Program	What It Means	Example
Modified learning expectations, *same* activity, *same* materials	The student with modified expectations works on the same or a similar activity, using the same materials.	The learning activity involves sorting and classifying quadrilaterals (regular and irregular) by geometric properties related to symmetry, angles, and sides using a variety of tools (e.g., geoboards, protractors). Students with modified expectations identify and compare quadrilaterals (e.g., square, rectangle, rhombus, trapezoid) and sort and classify them by geometric properties (e.g., sides of equal length, parallel sides, right angles), using a variety of tools.
Modified learning expectations, *same* activity, *different* materials	The student with modified expectations engages in the same activity, but uses different materials that enable him/her to remain an equal participant in the activity.	The activity involves sketching different perspectives and views of three-dimensional figures. Students with modified expectations may build three-dimensional figures from a picture or model, using interlocking cubes. *(continued)*

Modified Program	What It Means	Example
Modified learning expectations, *different* activity, *different* materials	Students with modified expectations participate in different activities.	Students with modified expectations work on angle activities that reflect their learning expectations, using a variety of concrete materials.

(Adapted from *Education for All: The Report of the Expert Panel on Literacy and Numeracy Instruction for Students With Special Education Needs, Kindergarten to Grade 6*, p. 119.)

It is important to note that some students may require both accommodations and modified expectations.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- reasoning and proving
- reflecting
- selecting tools and computational strategies
- connecting
- representing
- communicating

The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or an inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;

- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this document provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The activities include questions that teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this document provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and to represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections between concrete, pictorial, and symbolic mathematical representations. Advice on helping students develop conceptual understanding is also provided. The problem-solving experience in many of the learning activities allows students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas by using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

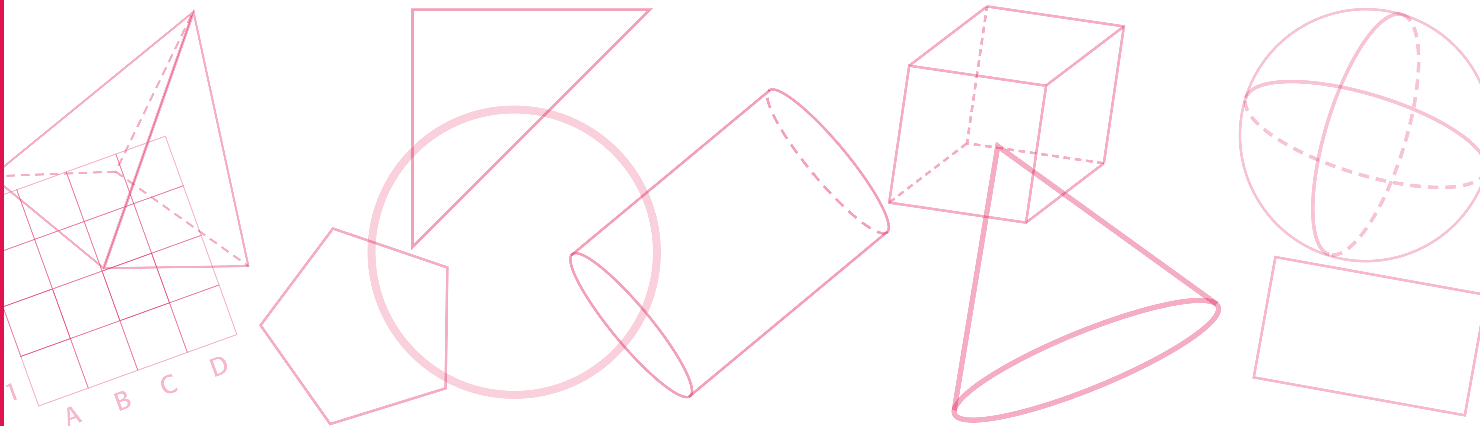
The following chart outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.

Characteristics of Junior Learners and Implications for Instruction

Area of Development	Characteristics of Junior Learners	Implications for Teaching Mathematics
Intellectual development	Generally, students in the junior grades: • prefer active learning experiences that allow them to interact with their peers; • are curious about the world around them; • are at a concrete, operational stage of development, and are often not ready to think abstractly; • enjoy and understand the subtleties of humour.	The mathematics program should provide: • learning experiences that allow students to actively explore and construct mathematical ideas; • learning situations that involve the use of concrete materials; • opportunities for students to see that mathematics is practical and important in their daily lives; • enjoyable activities that stimulate curiosity and interest; • tasks that challenge students to reason and think deeply about mathematical ideas.
Physical development	Generally, students in the junior grades: • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); • are concerned about body image; • are active and energetic; • display wide variations in physical development and maturity.	The mathematics program should provide: • opportunities for physical movement and hands-on learning; • a classroom that is safe and physically appealing. *(continued)*

Area of Development	Characteristics of Junior Learners	Implications for Teaching Mathematics
Psychological development	Generally, students in the junior grades: • are less reliant on praise, but still respond well to positive feedback; • accept greater responsibility for their actions and work; • are influenced by their peer groups.	The mathematics program should provide: • ongoing feedback on students' learning and progress; • an environment in which students can take risks without fear of ridicule; • opportunities for students to accept responsibilities for their work; • a classroom climate that supports diversity and encourages all members to work cooperatively.
Social development	Generally, students in the junior grades: • are less egocentric, yet require individual attention; • can be volatile and changeable in regard to friendship, yet want to be part of a social group; • can be talkative; • are more tentative and unsure of themselves; • mature socially at different rates.	The mathematics program should provide: • opportunities to work with others in a variety of groupings (pairs, small groups, large group); • opportunities to discuss mathematical ideas; • clear expectations of what is acceptable social behaviour; • learning activities that involve all students regardless of ability.
Moral and ethical development	Generally, students in the junior grades: • develop a strong sense of justice and fairness; • experiment with challenging the norm and ask "why" questions; • begin to consider others' points of view.	The mathematics program should provide: • learning experiences that provide equitable opportunities for participation by all students; • an environment in which all ideas are valued; • opportunities for students to share their own ideas and evaluate the ideas of others.

(Adapted, with permission, from *Making Math Happen in the Junior Years*. Elementary Teachers' Federation of Ontario, 2004.)



THE BIG IDEAS IN GEOMETRY AND SPATIAL SENSE

> *Geometry enables us to describe, analyze, and understand our physical world, so there is little wonder that it holds a central place in mathematics or that it should be a focus throughout the school mathematics curriculum.*
>
> (Gavin, Belkin, Spinelli, & St. Marie, 2001, p. 1)

Overview

The “big ideas” in the Geometry and Spatial Sense strand in Grades 4 to 6 are as follows:

- properties of two-dimensional shapes and three-dimensional figures
- geometric relationships
- location and movement

The curriculum expectations outlined in the Geometry and Spatial Sense strand for each grade in *The Ontario Curriculum, Grades 1–8: Mathematics, 2005* are organized around these big ideas.

In developing a mathematics program, it is important to concentrate on the big ideas and on the important knowledge and skills that relate to those big ideas. Programs that are organized around big ideas and focus on problem solving provide cohesive learning opportunities that allow students to explore mathematical concepts in depth. An emphasis on big ideas contributes to the main goal of mathematics instruction to help students gain a deeper understanding of mathematical concepts.

Teaching and Learning Mathematics: The Report of the Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004 outlines components of effective mathematics instruction, including a focus on big ideas in student learning:

> *When students construct a big idea, it is big because they make connections that allow them to use mathematics more effectively and powerfully. The big ideas are also critical leaps for students who are developing mathematical concepts and abilities.*
>
> (Expert Panel on Mathematics in Grades 4 to 6 in Ontario, 2004, p. 19)

Students are better able to see the connections in mathematics, and thus to learn mathematics, when it is organized in big, coherent "chunks". In organizing a mathematics program, teachers should concentrate on the big ideas in mathematics and view the expectations in the curriculum policy documents for Grades 4 to 6 as being clustered around those big ideas.

The clustering of expectations around big ideas provides a focus for student learning and for teacher professional development in mathematics. Teachers will find that investigating and discussing effective teaching strategies for a big idea is much more valuable than trying to determine specific strategies and approaches to help students achieve individual expectations. In fact, using big ideas as a focus helps teachers to see that the concepts presented in the curriculum expectations should not be taught as isolated bits of information but rather as a network of interrelated concepts.

In building a program, teachers need a sound understanding of the key mathematical concepts for their students' grade level, as well as an understanding of how those concepts connect with students' prior and future learning (Ma, 1999). They need to understand the "conceptual structure and basic attitudes of mathematics inherent in the elementary curriculum" (p. xxiv) and to know how best to teach the concepts to students. Concentrating on developing this knowledge will enhance effective teaching and provide teachers with the tools to differentiate instruction.

Focusing on the big ideas provides teachers with a global view of the concepts represented in the strand. The big ideas also act as a "lens" for:

- making instructional decisions (e.g., choosing an emphasis for a lesson or set of lessons);
- identifying prior learning;
- looking at students' thinking and understanding in relation to the mathematical concepts addressed in the curriculum (e.g., making note of the ways in which a student solves a problem in coordinate geometry);
- collecting observations and making anecdotal records;
- providing feedback to students;
- determining next steps;
- communicating concepts and providing feedback on students' achievement to parents[1] (e.g., in report card comments).

> *All learning, especially new learning, should be embedded in well-chosen contexts for learning – that is, contexts that are broad enough to allow students to investigate initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. Such rich contexts for learning open the door for students to see the "big ideas", or key principles, of mathematics, such as pattern or relationship.*
>
> (Ontario Ministry of Education, 2005, p. 25)

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1. In this document, *parent(s)* refers to parent(s) and guardian(s).

These big ideas are conceptually related and interdependent, and instructional experiences will often reflect more than one big idea. For example, when students create or analyse designs made by transforming a shape or shapes (location and movement), they demonstrate an understanding of congruence (geometric relationships), and of how congruence is connected to the properties of a shape (properties of two-dimensional shapes and three-dimensional figures).

Geometry can be thought of as the science of shapes and space, while spatial sense is "an intuitive feel for one's surroundings and the objects in them" (National Council of Teachers of Mathematics, 1989, p. 49). Geometry is an important area of mathematics because it provides students with a deeper appreciation for the world that surrounds them. Geometric forms can be found in the natural world as well as in virtually all areas of human creativity and ingenuity.

The skills and concepts developed through the study of Geometry and Spatial Sense play an important role in other areas of mathematics as well. For example, students use geometric models to help make sense of number concepts related to multiplication and fractions (e.g., use an array to show the distributive property of multiplication over addition); students apply spatial relationships to plots and graphs to organize and interpret data; and students develop an understanding of measurement concepts such as area and volume by manipulating geometric shapes and figures.

This section describes general principles of effective instruction in Geometry and Spatial Sense, and describes each of the big ideas in detail. Information about geometry topics, such as location, transformations, two-dimensional shapes and three-dimensional figures, and angles, is found in subsequent sections, along with sample grade-level lessons and activities.

General Principles of Instruction

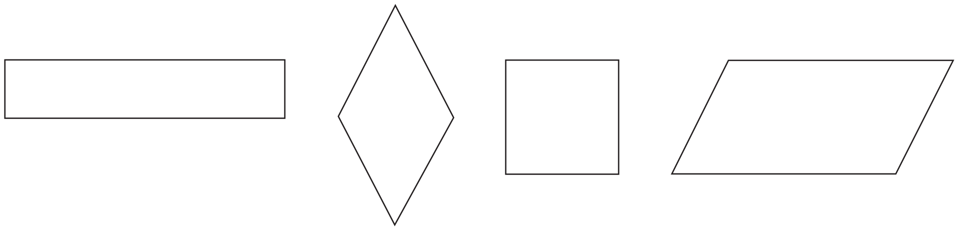
The following principles of instruction are relevant for teaching Geometry and Spatial Sense in the junior years:

- **Varied Lesson Types and Instructional Approaches:** Effective mathematics programs incorporate a variety of lesson types, such as problem-based lessons, games, and investigations. Within these different kinds of lessons, three approaches to mathematics instruction (guided, shared, and independent) should be used regularly to support student learning. A detailed discussion of these approaches may be found in Chapter 4: Instructional Approaches, in Volume 1 of *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.
- **Multiple Representations:** Geometric concepts such as symmetry, transformations, and angles can initially be difficult for junior students to understand. Providing a variety of representations of geometric concepts, including concrete materials that students can

manipulate, helps students relate new ideas to prior learnings. As students gain an understanding of geometric concepts through the manipulation of concrete materials, teachers can introduce other more abstract representations of the ideas (e.g., two-dimensional representations of three-dimensional figures and dynamic geometry modelling applications).

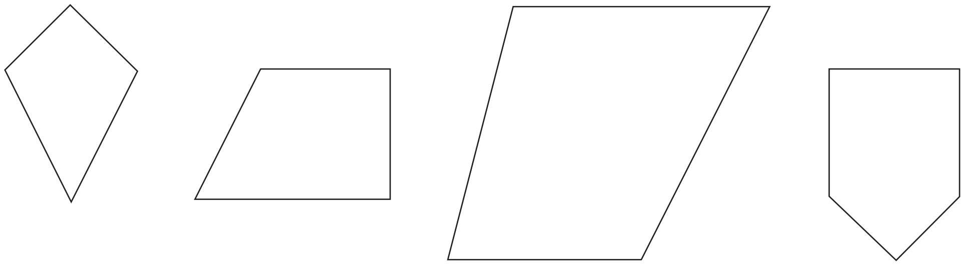
- **Use of Examples and Non-examples:** Students benefit from investigating two-dimensional shapes and three-dimensional figures in a variety of orientations and configurations. In addition to presenting numerous examples of specific shapes and figures, teachers should offer non-examples, which have been shown to help students eliminate irrelevant features and focus their attention on important common properties that define the different classes of shapes and figures.

Examples



These shapes are all parallelograms.

Non-examples



These shapes are not parallelograms.

- **Geometric Terminology:** As junior students begin to explore and describe more complex mathematical ideas and relationships, they become aware of the need for more precise terms and vocabulary. This "need" for more precise language encourages students to adopt appropriate geometric vocabulary. By the end of the junior years, students should use appropriate terminology (e.g., *parallel, perpendicular, equilateral triangle, face, edge, vertex*) to describe and justify their observations and conjectures about geometric shapes and figures. Students will also need to describe clearly the motions needed to transform two-dimensional shapes or three-dimensional figures (e.g., "I rotated the triangle 180° counterclockwise").

  In this guide, the term *plane shapes* will be used interchangeably with *two-dimensional shapes*, and *solid figures* will be used interchangeably with *three-dimensional figures*.

- **Technology:** Drawing programs, dynamic geometry computer applications (e.g., The Geometer's Sketchpad), and online applets are effective representational tools. These "virtual manipulatives" can assist students in learning important geometric concepts, allowing students to quickly and easily manipulate models of geometric shapes and figures in ways that are more difficult to demonstrate with concrete materials or hand-drawn representations. This flexibility allows students to focus on reasoning, reflecting, and the problem-solving process. For example, students can observe how changing one angle of a parallelogram to 90° affects the three other angles when opposite sides remain parallel. Through this type of dynamic representation, students can develop a deeper understanding of the relationship between parallelograms and rectangles.

Levels of Geometric Thought

Pierre van Hiele and Dina van Hiele-Geldof explored the development of geometric ideas in children and adults (Van de Walle & Folk, 2005). In their work, they proposed a five-level hierarchical model of geometric thinking that describes how individuals think and what types of geometric ideas they think about at each level of development. Advancement from one level to the next is more dependent on the amount of a student's experience with geometric thought than on the student's age and level of maturation.

- **Level 0: Visualization.** Students recognize and identify two-dimensional shapes and three-dimensional figures by their appearance as a whole. Students do not describe properties (defining characteristics) of shapes and figures. Level 0 represents the geometric thinking of many students in the early primary grades.
- **Level 1: Analysis.** Students recognize the properties of two-dimensional shapes and three-dimensional figures. They understand that all shapes or figures within a class share common properties (e.g., all rectangles have four sides, with opposite sides parallel and congruent). Level 1 represents the geometric thinking of many students in the later primary grades and the junior grades.
- **Level 2: Informal Deduction.** Students use informal, logical reasoning to deduce properties of two-dimensional shapes and three-dimensional figures (e.g., if one pair of opposite sides of a quadrilateral is parallel and congruent, then the other sides must be parallel and congruent). Students at this level can use this informal, logical reasoning to describe, explore, and test arguments about geometric forms and their properties. Level 2 represents the geometric thinking required in mathematics programs at the intermediate and secondary levels.
- **Level 3: Deduction.** As students continue to explore the relationships between and among the properties of geometric forms, they use deductive reasoning to make conclusions about abstract geometric principles. Level 3 represents the geometric thinking required in secondary and postsecondary mathematics courses.
- **Level 4: Rigour.** Students compare different geometric theories and hypotheses. Level 4 represents the geometric thinking required in advanced mathematics courses.

The levels described by the van Hieles are sequential, and success at one level depends on the development of geometric thinking at the preceding level. Typically, students at the primary level demonstrate characteristics of level 0 and are moving toward level 1 of the van Hieles' levels of geometric thought.

Students entering the junior grades are most likely functioning in the visualization and analysis levels (0 and 1) of geometric thought. The goal of the junior program is to provide instructional activities that will encourage children to develop the thinking and reasoning skills needed to move toward level 2 of the hierarchy, informal deduction.

Properties of Two-Dimensional Shapes and Three-Dimensional Figures

> *Both two- and three-dimensional shapes exist in great variety. There are many different ways to see and describe their similarities and differences. The more ways that one can classify and discriminate amongst these shapes, the better one understands them.*
>
> (Van de Walle & Folk, 2005, p. 324)

OVERVIEW

In the primary grades, mathematics instruction encourages students to focus on geometric features of two-dimensional shapes and three-dimensional figures. Instructional activities provide opportunities for students to manipulate, compare, sort, classify, compose, and decompose these geometric forms. These types of activities help students to identify and to informally describe some attributes and geometrical properties of two-dimensional shapes and three-dimensional figures.

In the junior grades, students continue to learn about the properties of two-dimensional shapes and three-dimensional figures through hands-on explorations and investigations.

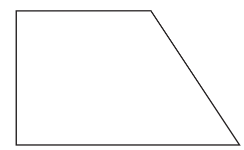
The following are key points that can be made about the properties of two-dimensional shapes and three-dimensional figures in the junior grades:

- Two-dimensional shapes and three-dimensional figures have properties that allow them to be identified, sorted, and classified.
- Angles are measures of turn, and can be classified by degree of rotation.
- An understanding of polygons and their properties allows students to explore and investigate concepts in geometry and measurement.
- An understanding of polyhedra and their properties helps develop an understanding of the solid world we live in, and helps make connections between two- and three-dimensional geometry.

IDENTIFYING, SORTING, AND CLASSIFYING SHAPES AND FIGURES ACCORDING TO THEIR PROPERTIES

In the primary grades, students explore geometric properties by sorting, comparing, identifying, and classifying two-dimensional shapes and three-dimensional figures. In the junior grades, these kinds of learning experiences help students develop a deeper understanding of geometric properties.

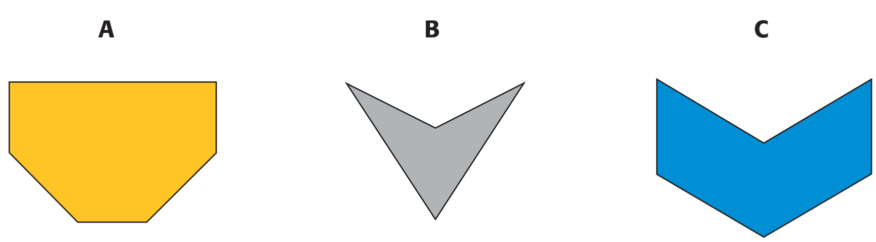
Identifying Shapes and Figures: Identifying shapes and figures involves more than looking at their appearance. Students must analyse the properties of the shape or figure (e.g., sides, angles, parallelism) to identify it accurately. If students have opportunities to view only traditional forms of shapes and figures, they will experience difficulties in recognizing non-traditional forms. For example, students who have observed only isosceles trapezoids may not identify the shape at the right as a trapezoid.



Students may also have difficulty understanding that shapes can be classified in more than one way (e.g., a square is a rectangle, parallelogram, and quadrilateral) if non-traditional examples of shapes are not explored.

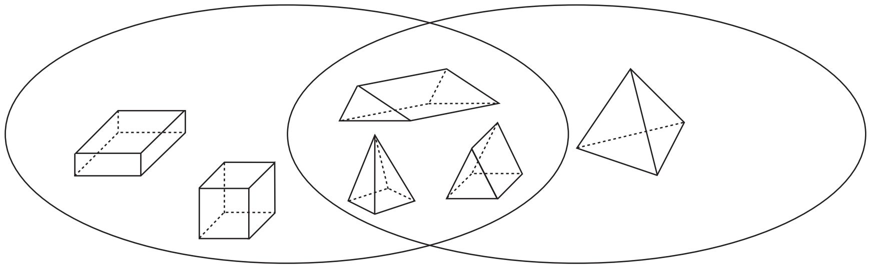
While it is important that students be able to correctly identify two-dimensional shapes and three-dimensional figures, it is more important that they be able to discuss the properties of a shape or figure and justify why it may or may not be identified with a classification or category. Reasoning and justification should always accompany identification.

Sorting Shapes and Figures: When students sort shapes and figures, they need to think about the relationships between the geometric forms they are sorting. Shapes may be related to one another in different ways, sharing a number of properties. Consider these three shapes:



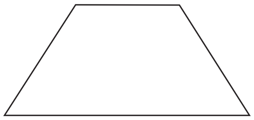
If asked to sort shape C with either shape A or shape B, students may reason differently. Some might suggest that C and A should be grouped together, since both are hexagons and have six sides. Another student might group shape C with shape B, since each is a non-convex or concave polygon (a polygon with at least one non-convex angle). Students' ability to communicate sorting rules effectively provides teachers with opportunities to assess understanding of how shapes and figures are related.

In the junior grades, sorting strategies and tools become increasingly complex. Venn diagrams are a useful tool for sorting shapes and figures into more than one category.

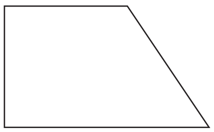


Students can create Venn diagrams, using sorting categories of their own choosing, or teachers can suggest sorting criteria. Sorting shapes and figures encourages students to think about their properties and about how a shape or figure relates to other shapes and figures.

Classifying Shapes and Figures: Classifying shapes and figures involves an understanding of the critical properties of a category (i.e., the properties that a shape or figure must possess in order to belong to a specific category). Teachers can encourage students to determine the critical properties of a category by asking them to describe the characteristics of a shape or figure and then to develop a definition. For example, students can describe these two trapezoids in many ways:



- four sides
- one pair of parallel sides
- one pair of non-parallel sides
- non-parallel sides congruent
- two pairs of congruent angles
- two acute angles, two obtuse angles



- four sides
- one pair of parallel sides
- one pair of non-parallel sides
- two right angles
- one acute angle, one obtuse angle

After comparing the two shapes, students might decide that the critical properties of trapezoids would be those common to both – a four-sided shape with one pair of parallel sides and one pair of non-parallel sides.

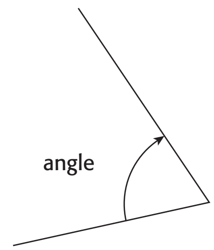
It is interesting to note that in the global mathematics community, there are two different ways to define a trapezoid – as a quadrilateral with **only** one pair of parallel sides, or a quadrilateral with **at least** one pair of parallel sides. Providing students with a third shape, such as a parallelogram, and having them discuss whether it is also a trapezoid encourages them to consider which trapezoid definition they find most suitable.

The study of geometry presents students with many opportunities for mathematical discussion and debate. Teachers should facilitate rich discussions by encouraging students to focus on specific properties and attributes of two-dimensional shapes and three-dimensional figures. The following sections describe the properties relevant to students in the junior years.

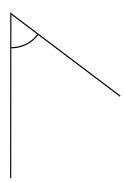
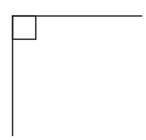
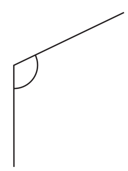
INVESTIGATING ANGLE PROPERTIES

Primary students are introduced to the concept of angles as they explore and describe the attributes and properties of two-dimensional shapes. They learn to identify right angles, and to describe other angles as smaller than or larger than the right angle. In the junior grades, students explore the concept of angle and angle properties in greater detail.

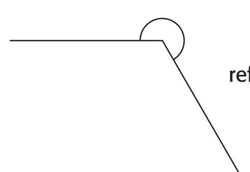
It is important for junior students to understand that measuring angles involves finding the amount of rotation between two lines, segments, or rays that meet at a common vertex.



Since angles are essentially a measure of rotation, there is a limit as to how they can be classified – the degree of the rotation. In this they are unlike polygons (e.g., triangles and quadrilaterals), which can be classified by different attributes – length of sides, measure of angles, and so forth.

Acute Angle	Right Angle	Obtuse Angle	Straight Angle
			
An angle less than 90° is an acute angle.	An angle of exactly 90° is a right angle.	An angle more than 90° but less than 180° is an obtuse angle.	An angle of exactly 180° is a straight angle.

A fifth classification of angles, known as a *reflex angle*, is an angle that measures more than 180° and less than 360°.



Although reflex angles are not specifically mentioned in the Ontario curriculum, students may wonder about the classification of angles larger than 180° in their explorations.

As students develop their understanding of the different classifications of angles, they will use angles as a property to classify different types of triangles (e.g., acute triangles, obtuse triangles, and right triangles) and quadrilaterals (e.g., the right angles of rectangles distinguish them as a subclass of parallelogram).

PROPERTIES OF POLYGONS

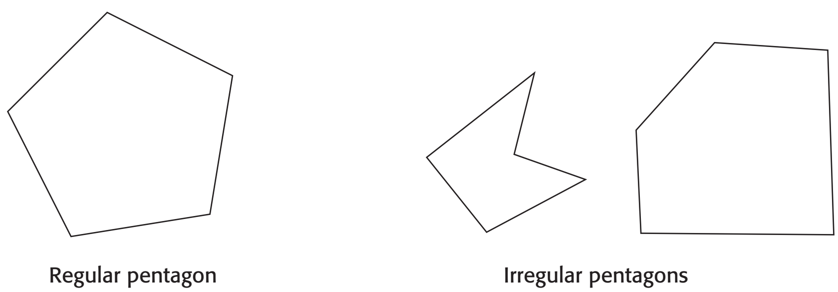
In the junior grades, students continue to develop their understanding of the properties of two-dimensional shapes. Students learn that a two-dimensional shape has length and width, but not depth. They also learn that shapes can have straight or curved sides.

Junior students focus on specific shapes called *polygons*. A *polygon* is a closed shape formed by three or more straight sides. Included among polygons are triangles, quadrilaterals, octagons, and so forth.

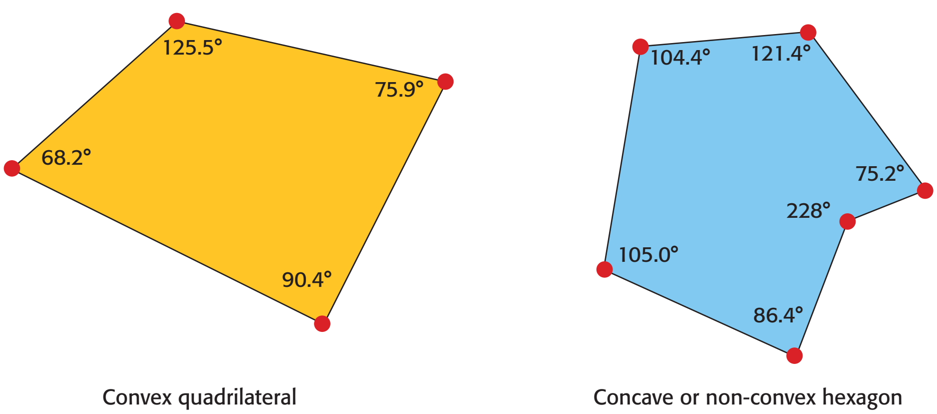
The properties of polygons examined at the junior level include number of sides, number of vertices, length of sides, size of angles, parallel lines, diagonals, and lines of symmetry. These properties are explored in more detail in the chapter "Learning About Two-Dimensional Shapes in the Junior Grades".

Exploring the properties of polygons allows students to identify and classify polygons in a number of ways:

- *Regular polygons*: all angles are equal, and all sides are the same length.
- *Irregular polygons*: not all angles are equal, and not all sides are the same length.



- *Convex polygons*: all interior angles are less than 180°.
- *Concave or non-convex polygons*: at least one interior angle is greater than 180°.



When students investigate and explore polygon properties rather than focus on recalling rigid definitions, they are more likely to develop analytical skills and geometric critical thinking skills.

PROPERTIES OF POLYHEDRA

In the primary years, students identify and sort three-dimensional figures according to their properties. Students' ability to use appropriate mathematical language in describing these properties increases as students continue to explore shapes and figures in the junior years. Students no longer simply say of figures that they "slide" or "roll", but rather that they have curved surfaces, faces, edges, and vertices. Students can use more complex mathematical concepts, such as parallelism, angle measures, and congruence, to sort and classify figures.

In the junior grades, students focus on specific figures called *polyhedra*. A *polyhedron* is a three-dimensional figure with faces made up of polygons. The polyhedra include prisms and pyramids.

The properties of polyhedra examined at the junior level include number and/or type of faces, number of edges, number of vertices, parallelism, and perpendicularity. These properties are explored in more detail in the chapter "Learning About Three-Dimensional Figures in the Junior Grades".

Once students have explored the various properties of polyhedra, they can begin using appropriate geometric vocabulary to describe the figures.

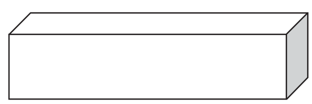


Figure 1

"Figure 1 has only rectangular faces. All of its edges have parallel, opposite edges, and all of its faces have congruent faces that are also parallel."

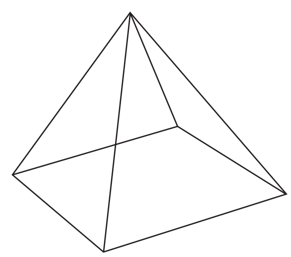


Figure 2

"Figure 2 has no faces parallel to other faces. It has both rectangular and triangular faces. Some edges of the figure run parallel to other edges; some edges meet up at one vertex."

As students explore three-dimensional figures using geometric vocabulary, they begin to recognize common defining properties among some of the solids.

Geometric Relationships

> *With well-designed activities, appropriate tools, and teachers' support, students can make and explore conjectures about geometry and can learn to reason carefully about geometric ideas from the earliest years of schooling. Geometry is more than definitions; it is about describing relationships and reasoning.*
>
> (NCTM, 2000, p. 41)

OVERVIEW

Students experience geometry in the world around them, both at school and at home. Personal experiences with shapes and solids begin at a very early age, and as students continue to develop their understanding of geometric concepts and their spatial sense in the junior years, they make connections between formal school geometry and the environmental geometry that they experience every day. Carefully planned activities will enable students to build on these connections and identify relationships between and among the various areas of geometry and spatial sense.

The following are key points that can be made about geometric relationships in the junior grades:

- Plane shapes and solid figures can be composed from or decomposed into other two-dimensional shapes and three-dimensional figures.
- Relationships exist between plane and solid geometry (e.g., the faces of a polyhedron are polygons; views of a solid figure can be represented in a two-dimensional drawing).
- Congruence is a special geometric relationship between two shapes or figures that have exactly the same size and shape.

COMPOSING AND DECOMPOSING TWO-DIMENSIONAL SHAPES AND THREE-DIMENSIONAL FIGURES

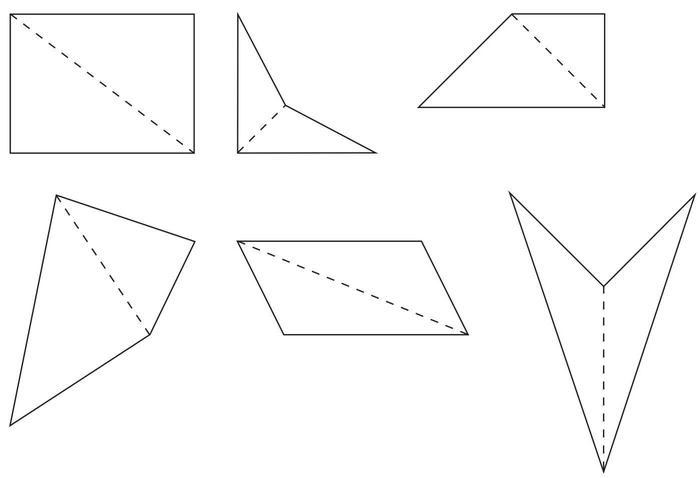
In the primary grades students develop the ability to identify and name common two-dimensional (plane) shapes and three-dimensional (solid) figures. The ability to identify these shapes and figures, and later name their properties, is indicative of student thinking in the first two van Hiele levels (visualization and analysis). The third van Hiele level, informal deduction, involves thinking about geometric properties without looking at a particular object or shape. Students see relationships between these properties, and draw conclusions from the relationships. For example, a student might think:

> *"If the quadrilateral has two pairs of parallel sides, it must be a parallelogram. If it is a rhombus, it also has two pairs of parallel sides, but they are all congruent. A rhombus must be a parallelogram."*

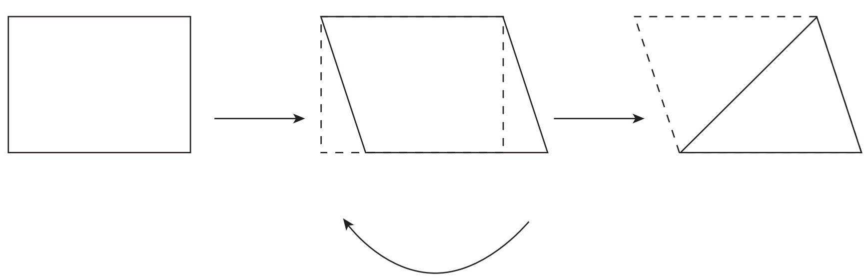
In order to prepare students for the informal deduction stage in the later grades, classroom instruction should focus on the physical relationships between shapes when they are combined (composed) or taken apart (decomposed). Teachers should use concrete materials and technology to help provide these experiences, in order to develop students' abilities to visualize.

In the primary grades, composing and decomposing plane shapes provides students with opportunities to develop spatial awareness and visualization skills. In the junior grades, these skills continue to develop, but composing and decomposing shapes is applied practically to the development of measurement concepts and formulas.

Consider the relationship between quadrilaterals and triangles. All quadrilaterals, both convex and concave, can be decomposed into two triangles.

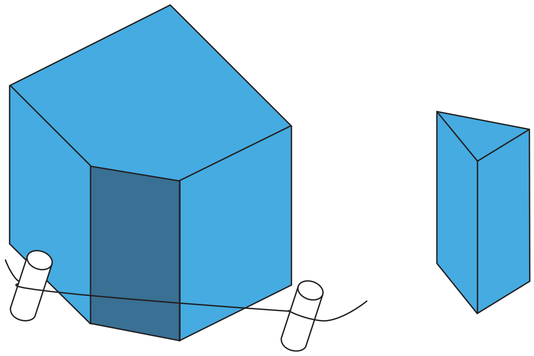


While this discovery alone may not be an important geometric concept, it does illustrate how students can build upon prior learnings or understandings in order to create new knowledge. By the end of Grade 6, students will be expected to develop strategies for finding the areas of rectangles, parallelograms, and triangles. These polygons are closely related, and the relationships between them can help students develop formulas for determining their areas.



This figure represents a continuum for developing area formulas of rectangles, parallelograms, and triangles. Students can use their knowledge of rectangular area to determine the area of a parallelogram by decomposing the parallelogram and recomposing it into a rectangle. Since all quadrilaterals can be decomposed into two triangles, it is also true that any triangle is half of a parallelogram. Understanding how these polygons are related to each other provides a starting point for the **development of area formulas**, and is much more powerful than simply memorizing the formulas.

Three-dimensional, or solid, figures can also be decomposed into two or more solids. Exploring this "slicing" of solid figures helps to reinforce the characteristics and properties of three-dimensional figures. For example, a teacher might present students with a cube made of modelling clay and ask, "How could you slice this solid to produce a triangular prism?"



Slicing the cube in this manner will produce a triangular prism, though it is not the only way of slicing the cube to produce a triangular prism.

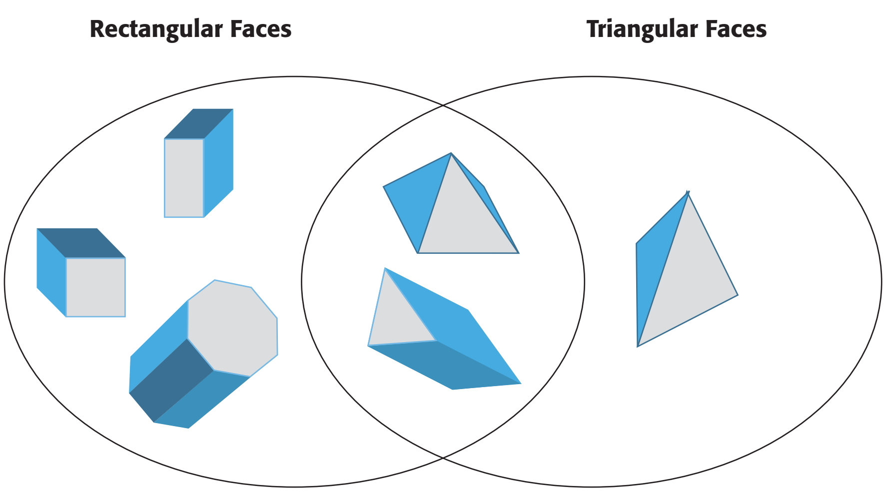
Just as composing and decomposing plane shapes helps students understand and develop area formulas, so knowing how three-dimensional figures can be decomposed and recomposed can help students better understand and develop volume concepts and formulas.

These types of recomposing and decomposing activities encourage students to begin making connections between plane shapes and solid figures. In the example above, the student must think of a "slice" that will produce a rectangular face "on the bottom", and triangular bases "on the ends".

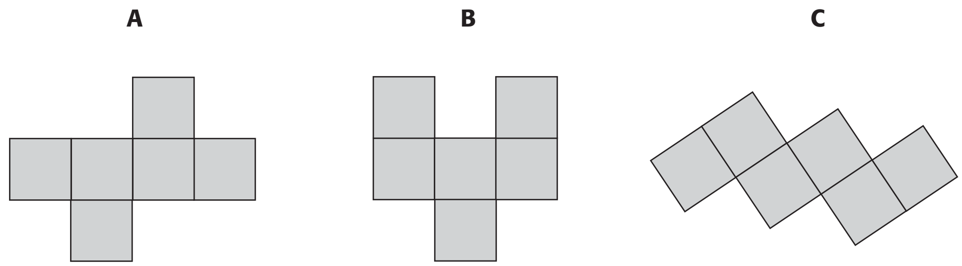
RELATIONSHIPS BETWEEN PLANE SHAPES AND SOLID FIGURES

Children experience many realistic situations that call for the two-dimensional representation of a three-dimensional object. Understanding how plane shapes relate to solid figures requires students to use *spatial sense* – to turn or manipulate objects in one's mind to obtain different views of solid objects.

In the primary grades, students experience using different face shapes to sort solids. For example, a student in Grade 3 might use a Venn diagram to sort solids by rectangular and triangular faces.

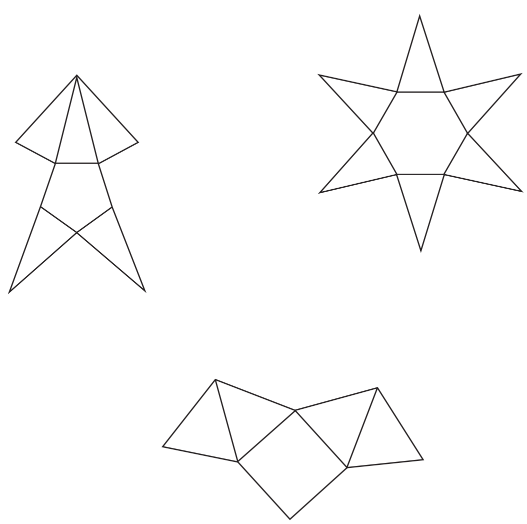


In the junior grades, students design and construct solids from nets and sketches, and begin to sketch two-dimensional representations of three-dimensional solids. In order to work flexibly with solids and their nets, students must not only identify the shapes of the faces of a solid, but also understand how the faces relate to each other. As an example, consider these three nets:



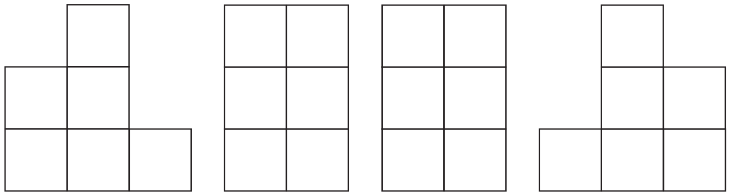
Which of the nets will form a cube? Students who simply count the number of faces might respond that all three will work, since a cube has six square faces. Net B, however, cannot be formed into a cube, and students might be surprised to find that net C *can* be formed into a cube.

Students develop spatial sense by visualizing the folding of nets into solids, but they require meaningful experiences with concrete materials (e.g. paper, Polydron pieces, Frameworks) to do so. Students can predict whether a net will form a particular solid, then use paper or Polydron pieces to construct the net and try it out. Teachers should help students make connections between nets and the properties of three-dimensional solids. For example, these are nets for pyramids. What common characteristics do they share?



Each of the nets is made of triangles and one other polygon. The number of triangles in the net is the same as the number of sides the polygon has.

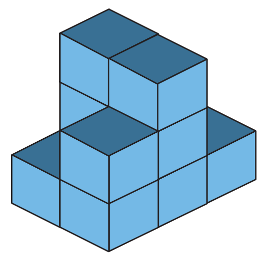
The relationships between two-dimensional representations of three-dimensional objects can also be explored using interlocking cubes. Teachers provide students with different views of a "building", and students must use the blocks to build the solid and sketch the three-dimensional view. Consider this example:



front left right rear

The solid that will produce these views would look like the one shown below right when drawn using isometric grid paper.

These types of activities can be modified in many ways:



- Students can be given the solid and asked to sketch the views.
- Students can be limited in the number of views they are given.
- Students can be given the isometric drawing and asked to provide sketches of the front, side, top, and rear views.

John Van de Walle (2005) suggests that an answer key can be made by drawing a top view, and indicating the number of blocks in each "vertical stack". The answer key for the example above would look like this:

1	3	1
2	3	1

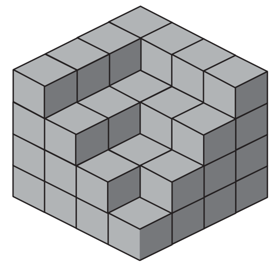
Top View

Students can make their own challenge cards with answer keys on the back and several views on the front.

Teachers should encourage students to make conjectures when working with two-dimensional representations of three-dimensional figures. Predicting and conjecturing activities can help students move out of the analysis level of geometric thought and into the informal deduction phase.

Another example of using interlocking cubes would be an activity in which students must look at an isometric drawing of a figure and determine the number of cubes needed to build the figure.

Estimate the number of cubes you would need to build this figure, and then determine the exact number. What strategy did you use to estimate? What strategy did you use to determine the exact number?



Although we live in a three-dimensional world, we are often required to represent aspects of it in two dimensions. It is important for students to see the relationships between plane and solid geometry in order to develop greater spatial sense and geometric thinking.

CONGRUENCE

Congruence is a special relationship shared by shapes and figures that are the same size and the same shape. Students develop an understanding of congruence in the primary grades. In the junior grades, they begin to apply that understanding to classify and categorize shapes and figures, build three-dimensional figures, and develop measurement formulas.

An important learning for students is that congruence depends on size and shape, but not orientation. The use of examples and non-examples helps students develop an understanding of congruence.

In the primary grades, students explore congruence by superimposing shapes on one another. In the junior grades, students can begin to use measurement as a tool for determining congruence. Although formal proofs are not suggested, students can measure the side lengths and angles of polygons and use these measures to discuss congruence.

Congruence offers opportunities for students to reason geometrically and think critically. Open-ended questions can allow for engaging student dialogue.

- Can two triangles share identical side lengths and not be congruent? Explain.
- Think of as many quadrilaterals as you can that have only one pair of congruent sides.
- Can a quadrilateral have only three congruent angles? Explain.
- Which solids have *only* congruent faces?

Location and Movement

> *Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Geometry helps us represent and describe objects and their interrelationships in space….*
>
> *Spatial sense is necessary for understanding and appreciating the many geometric aspects of our world. Insights and intuitions about the characteristics of two-dimensional shapes and three-dimensional figures, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Students develop their spatial sense by visualizing, drawing, and comparing shapes and figures in various positions.*
>
> (Ontario Ministry of Education, 2005, p. 9)

OVERVIEW

Spatial reasoning plays an important role in interpreting and understanding the world around us. As students develop spatial reasoning abilities, they can appreciate the important role geometry plays in art, science, and their everyday world. Spatial reasoning includes two important spatial abilities: spatial orientation and spatial visualization. Spatial orientation is the ability to locate and describe objects in space, and to carry out and describe transformations of objects. Spatial visualization is the ability to imagine, describe, and understand movements of two- and three-dimensional objects in space. These skills play an important role in representing and solving problems in all areas of mathematics and in real-world situations.

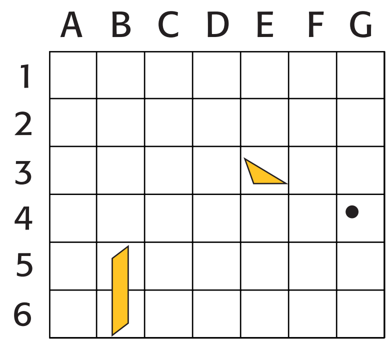
The following are key points that can be made about location and movement in the junior grades:

- A coordinate grid system can be used to describe the position of a plane shape or solid object.
- Different transformations can be used to describe the movement of a shape.

USING A COORDINATE GRID SYSTEM TO DESCRIBE POSITION

In the primary grades, students learn to use positional language and simple grid systems to describe the position of objects and their movement. In the junior grades, students develop flexibility in spatial reasoning by exploring a variety of coordinate systems.

Junior students extend their understanding of the simple grids introduced in the primary years to include a coordinate system commonly used for road maps and atlases. By convention, in this system a letter is assigned to each column of the grid and a number to each row. In this system the general location of an object within a specific contained area is described, not the exact location of the object.



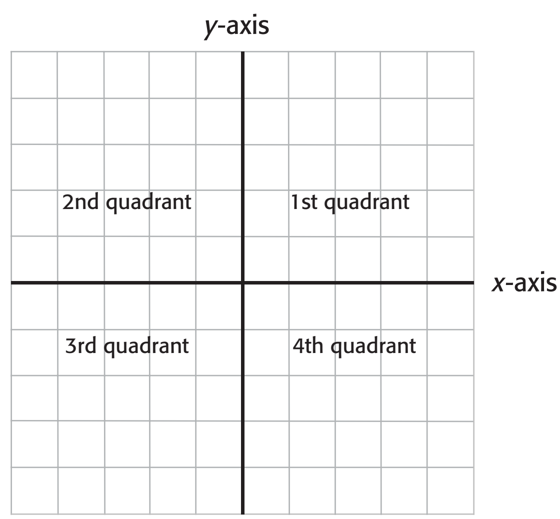
The triangle is located in E3.

The parallelogram is in B5 and B6.

The black circle is located in G4.

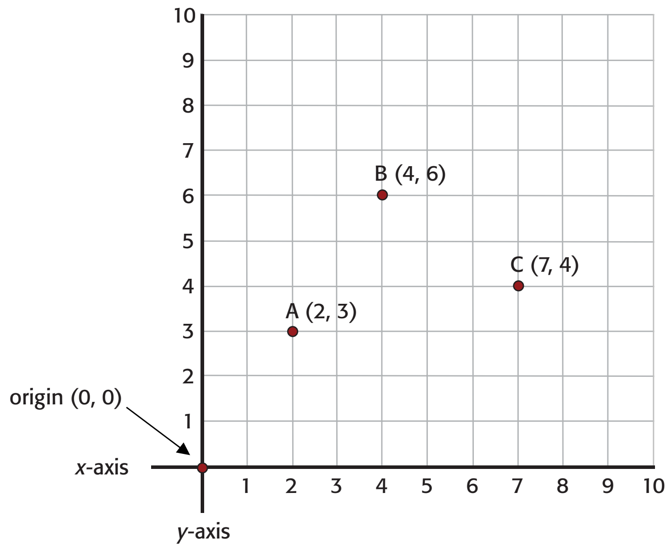
What is important for students to understand is that *two positional descriptors* are provided to describe the location of an object. One describes the location along a horizontal axis; the other, the location along a vertical axis. In this case, the horizontal descriptor is a letter, and the vertical descriptor is a number. These descriptors are sufficient to describe any location in two dimensions.

As students move into Grade 6, they extend their understanding of map grid systems to explore the **Cartesian coordinate plane**. The Cartesian plane, named after the French mathematician René Descartes (1596–1650), is also based on horizontal and vertical positional descriptors. In the Cartesian plane, both of these descriptors are numerical. In the Cartesian system, a pair of numbers is associated with each *point* in the plane rather than an *area* in the plane. This allows for a more exact description of the location of shapes or objects.



The Cartesian coordinate plane uses two axes along which numbers are plotted at regular intervals. The horizontal axis is known as the ***x*-axis**, and the vertical axis as the ***y*-axis**. The location of a point is described by an ordered pair of numbers representing the intersection of an *x* and a *y* “line”. The *x*- and *y*-axes divide the plane into four *quadrants* – the first, second, third, and fourth quadrants.

The First Quadrant of the Cartesian Coordinate Plane



In the labelling of points, the *x*-coordinate is always given first, then the *y*-coordinate. The coordinates are written in parenthesis and are separated by a comma (*x, y*). The **origin** is located at the intersection of the *x*- and *y*-axes, and is represented as the point (0, 0). The first quadrant of the coordinate plane is the quadrant that contains all the points with positive *x* and positive *y* coordinates.

As students explore locating points and objects on a Cartesian plane, teachers should provide learning opportunities that help students make connections with prior learnings about grid systems yet also focus on the differences between the systems. Examples include:

- using grids that have the grid lines drawn in (although in later years students will explore grids without grid lines, it is recommended that the lines be apparent in initial investigations);
- plotting points, and having students use ordered pairs to identify their locations;
- using ordered pairs to provide the location of points – the points can be joined to make a shape;
- using Battleship-like games to reinforce location.

In later grades, students will learn that the coordinate plane can be extended infinitely in four directions, and that four distinct quadrants are formed. In the junior grades, students should only be exploring the location of points in the first quadrant, where all integers are positive.

Students in the junior grades will also use the **cardinal directions**, or north, south, east, and west, to describe location. Students gain an understanding of these directions by exploring a variety of maps, and come to understand that on most maps, north is "up" and south is "down". East is "right" and west is "left". Teachers should help students understand why cardinal directions are more useful than directions based on left, right, forward, and backwards. For example, they might challenge students to describe a situation in which a left turn could send a person heading east.

USING GEOMETRIC TRANSFORMATIONS TO DESCRIBE THE MOVEMENT OF OBJECTS

> *Many people say they aren't very good with shape, or they have poor spatial sense. The typical belief is that you are either born with spatial sense or not. This simply is not true! We now know that when rich experiences with shape and spatial relationships are provided consistently over time, children can and do develop spatial sense.*
>
> (Van de Walle & Folk, 2005, p. 327)

Young children come to school with an understanding of how objects can be moved, which they have developed through their play experiences (e.g., assembling puzzles, building with blocks). In the primary years, students expand this understanding by investigating flips (reflections), slides (translations), and turns (rotations) through play, exploration, and problem-solving tasks. Primary learning experiences include kinaesthetic activities (e.g., movement games in the gym), and the manipulation of concrete materials (e.g., transparent mirrors, or Miras; tangrams; paper folding). With experience and modelling by the teacher, primary students begin to describe the results of the transformations in their own everyday language.

In the junior years, students predict the result of a transformation and describe what will happen to the object as the transformation is performed. As a result of guided investigations, students will eventually be able to look at the original orientation of an object and the result of a transformation and describe what transformation was performed without seeing the transformation occur.

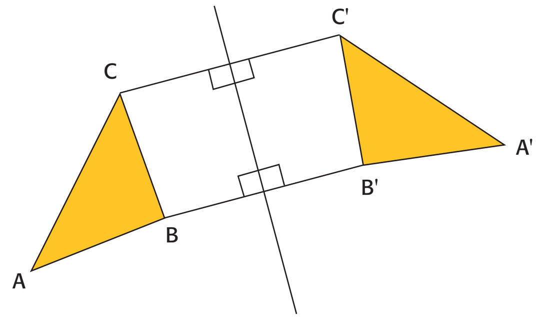
Junior students also use transformations (reflections, translations, and rotations) to demonstrate congruence and symmetry between pairs of shapes and within geometric patterns and designs.

In order to describe clearly the outcomes of their games and explorations, students need to develop precise mathematical language. Teachers should consistently model transformational vocabulary, including:

- *rotate*, *reflect*, *translate*
- *line of reflection*
- *point of rotation*
- directional language (*up*, *down*, *right*, *left*, *clockwise*, *counterclockwise*)
- benchmark turns (1/2, 1/4, 3/4)

REFLECTIONS

A reflection over a line is a transformation in which each point of the *original* shape has an *image* shape that is the same distance from the line of reflection as the original point, but is on the opposite side of the line.



In this example, the points A, B, and C are exactly the same distance from the line of reflection as points A', B' and C'.

In geometry, the prime (') symbol is used to label the vertices of an image of the original shape after a translation, rotation, or reflection. If a second transformation is described, a double prime (") symbol is used.

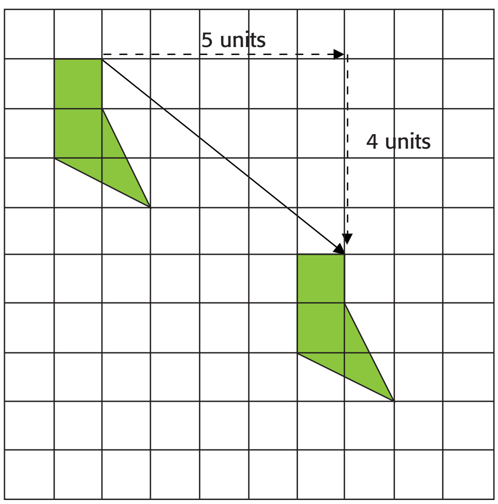
Although junior students are not concerned with formal proofs, they may notice that segments drawn to join corresponding points always meet the reflection line at right angles.

The reflection causes a change in the original position and orientation of a shape, but the reflected image is *congruent* to the original. In other words, the reflected image is the same size and shape, but it will be "facing" a different direction, and will be in another position. It is important to note that the reflection line can be drawn in any direction relative to the original shape – horizontally, vertically, or diagonally at any angle.

TRANSLATIONS

A translation can be described as a transformation that slides every point of a shape the same distance in the same direction. During a translation the orientation of the shape does not change and the *image* is congruent to the *original shape*. A translation can occur in any direction.

Translations can be described by the distance and direction of the movement. In the junior grades, translations can be described using a coordinate grid system. Directional language (e.g., *up*, *down*, *right*, and *left*; the cardinal directions) is combined with a number representing the magnitude of the movement.



The solid arrow represents the translation of 5 units to the right, and 4 units down. All of the points of the pentagon were translated the same distance in the same direction.

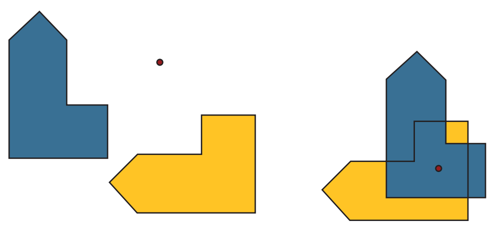
Although translations represent the movement of an object, Glass (2004) suggests that by focusing on the path that an object follows rather than on the relationship between the original shape and its image, students may develop misconceptions about transformations.

In the figure on page 35, the arrow represents the shortest path from the original shape to its image but does not represent the only path. The image might be a result of multiple translations. For example, it may first have been translated up 4 units and left 2 units, then down 8 units and right 7 units. Activities that encourage students to look at the translated image and describe possible paths it may have "taken" will help develop an understanding of the relationships between various transformations.

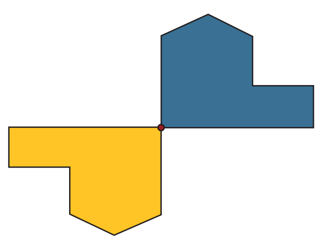
ROTATIONS

A rotation is a transformation that moves every point in a shape or figure around a fixed point, often called the *origin* or *point of rotation*. A rotation creates an *image* that is congruent to, and also preserves the orientation of, the *original shape*.

The point of rotation can be found anywhere on the plane, either outside the shape or within it.



When the point of rotation is found at a vertex of the shape, the original shape and its image will share the point.



Rotations are first described by students as fractions of turns. The yellow polygon in the example immediately above could be described as a rotation of 1/2 turn, either clockwise or counterclockwise. Students will recognise that every clockwise turn can also be described as a counterclockwise turn, and with rich explorations will discover that the sum of the two turns is always 1.

In the junior grades, students are expected to describe rotations in degrees (°). They will learn that a 1/2 turn can also be described as a 180° rotation, and that 1/4 turn can be described as a 90° rotation in either a clockwise or counterclockwise direction.

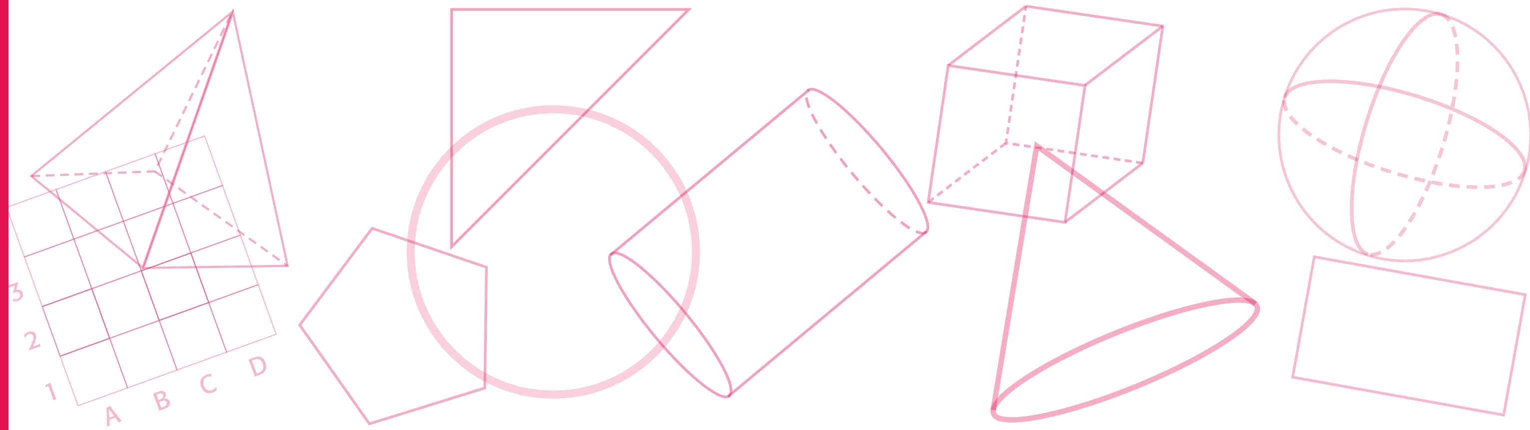
Relating Mathematics Topics to the Big Ideas

> *The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the "big ideas" of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.*
>
> (*The Ontario Curriculum, Grades 1–8: Mathematics, 2005*, p. 4)

In planning mathematics instruction, teachers generally develop learning opportunities related to curriculum topics, such as two-dimensional shapes and three-dimensional figures. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a lesson about three-dimensional figures, students can learn that the faces of three-dimensional figures are two-dimensional shapes, thereby deepening their understanding of the big idea of geometric relationships.

The learning activities in this guide do not address all topics in the Geometry and Spatial Sense strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Geometry and Spatial Sense. Teachers can use these models in developing other learning activities.



LEARNING ABOUT TWO-DIMENSIONAL SHAPES IN THE JUNIOR GRADES

Introduction

Developing an understanding of two-dimensional shapes and their properties is a gradual process – moving from experiential and visual learning to theoretical and inferential learning. Geometric thinking in the junior years begins to bridge the two kinds of learning.

PRIOR LEARNING

In the primary grades, students learn to recognize and describe geometric properties of two-dimensional shapes, such as the number of sides, number of vertices, length of sides, size of angles, and number of parallel lines. Some of these properties are introduced informally (e.g., “Parallel lines are two lines that run side by side in the same direction and stay the same distance apart”). Learning about geometric properties allows students to develop the concepts and language they need to analyse and describe two-dimensional shapes. Experiences in the primary classroom include identifying, comparing, sorting, and classifying shapes. Activities involving both examples and non-examples of shapes help to develop an understanding of the defining properties of various two-dimensional shapes.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

In the junior grades, students continue to identify, compare, sort, and classify two-dimensional shapes. These investigations include more complex shapes and introduce additional properties such as symmetry. Junior students become more specific when describing geometric properties in order to expand their geometric vocabulary and allow for more precision in classifying and identifying two-dimensional shapes. Experiences should include opportunities to analyse, describe, construct, and classify shapes while considering multiple properties. For example, students might be required to identify triangles from a group that includes both scalene and obtuse. Specifically, students in the junior grades explore triangles and quadrilaterals in depth, investigating side and angle properties as well as symmetry.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to two-dimensional geometry, listed in the following table:

Curriculum Expectations Related to Properties of Two-Dimensional Shapes, Grades 4, 5, and 6

By the end of Grade 4, students will:	By the end of Grade 5, students will:	By the end of Grade 6, students will:
Overall Expectation • identify quadrilaterals and three-dimensional figures and classify them by their geometric properties, and compare various angles to benchmarks. ***Specific Expectations*** • draw the lines of symmetry of two-dimensional shapes, through investigation using a variety of tools and strategies; • identify and compare different types of quadrilaterals (i.e., rectangle, square, trapezoid, parallelogram, rhombus) and sort and classify them by their geometric properties; • identify benchmark angles (i.e., straight angle, right angle, half a right angle), using a reference tool, and compare other angles to these benchmarks; • relate the names of the benchmark angles to their measures in degrees.	***Overall Expectation*** • identify and classify two-dimensional shapes by side and angle properties, and compare and sort three-dimensional figures. ***Specific Expectations*** • distinguish among polygons, regular polygons, and other two-dimensional shapes; • identify and classify acute, right, obtuse, and straight angles; • measure and construct angles up to 90°, using a protractor; • identify triangles (i.e., acute, right, obtuse, scalene, isosceles, equilateral), and classify them according to angle and side properties; • construct triangles, using a variety of tools, given acute or right angles and side measurements.	***Overall Expectation*** • classify and construct polygons and angles. ***Specific Expectations*** • sort and classify quadrilaterals by geometric properties related to symmetry, angles, and sides, through investigation using a variety of tools and strategies; • sort polygons according to the number of lines of symmetry and the order of rotational symmetry, through investigation using a variety of tools; • measure and construct angles up to 180° using a protractor, and classify them as acute, right, obtuse, or straight angles; • construct polygons using a variety of tools, given angle and side measurements.

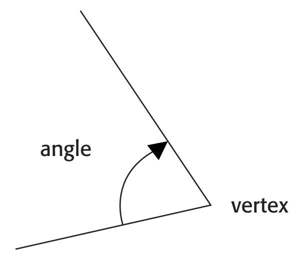
(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)

The sections that follow offer teachers strategies and content knowledge to address these expectations in the junior grades while helping students develop an understanding of two-dimensional geometry. Teachers can facilitate this understanding by helping students to:

- investigate angle properties using standard and non-standard tools and strategies;
- investigate the relationship of congruence;
- investigate polygon properties, including line and rotational symmetry, using concrete materials and technology;
- identify, compare, sort, classify, and construct polygons, including triangles and quadrilaterals, through investigations.

Investigating Angle Properties

It is important for junior students to understand that measuring angles involves finding the amount of rotation between two lines, segments, or rays that meet at a common vertex.



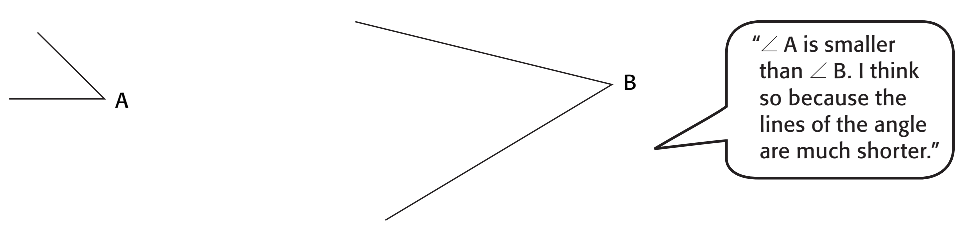
By the end of the junior grades, students should be comfortable with:

- comparing, classifying, measuring, and constructing angles;
- identifying and analysing angles as properties of geometric shapes.

Investigations should include angles in everyday objects and situations (e.g., angles formed where walls meet, angles of different golf club faces, angles formed when slicing a pizza).

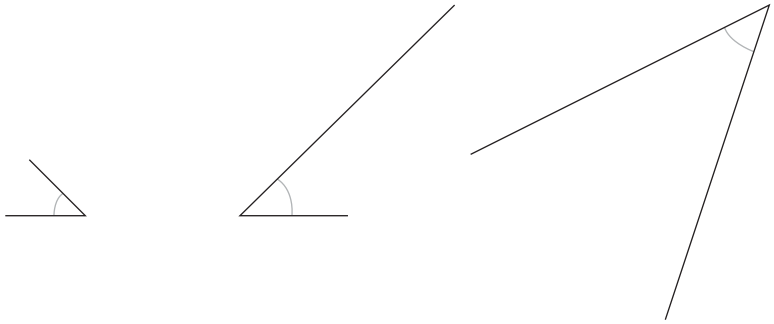
ANGLE MISCONCEPTIONS

In the study of angles, students may reveal certain misconceptions related to representation and visualization. Common misconceptions relate to various representations of angles and their measure or size. Students often associate the size of the angle with the length of the ray or line segment:

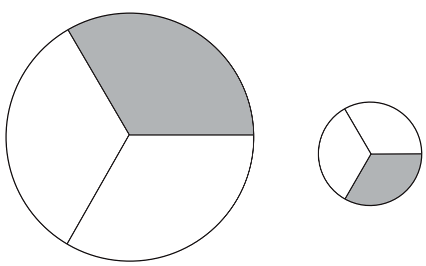


When students have genuine and meaningful experiences of measuring and comparing angles that are represented in a variety of ways, they are more likely to understand that an angle is essentially a measure of rotation.

The angles below are all equivalent; however, the line segments drawn for each representation are different. When students are able to compare angle sizes correctly, regardless of the line segment lengths, they are ready to begin measuring angles accurately.

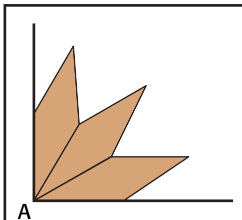


Probability and spinners offer an excellent opportunity for students to explore angles in context. Many students might reason that there is a greater probability of landing in the coloured region of Spinner A, since the area of that coloured sector is larger than the area of the coloured sector in Spinner B. Probability experiments would yield similar results for both spinners, however, and students would come to recognize that the measure of the *angle* for the coloured sector is the same for each spinner, and the length of the line segment (and therefore the size of the sector) has no effect on the measure of the angle.



BENCHMARK ANGLES

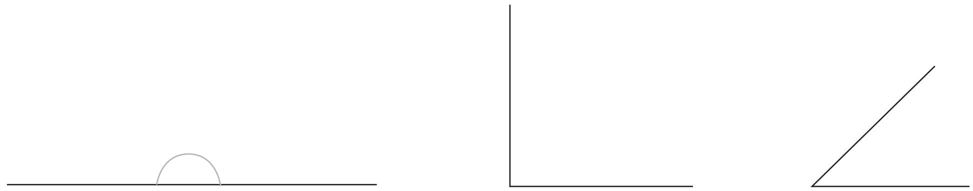
Just as benchmark numbers like 0, 1/2, and 1 help students develop a quantitative understanding of fractions, benchmark angles can help students identify and classify angle measures. Benchmark angles also help students develop an understanding of angle *size*. At first, students describe angles qualitatively (less than a right angle, greater than a straight angle); eventually they use a numerical value ("The angle is a little smaller than a right angle, so it's about 80°").



Students can measure angles using non-standard units, like the points of a rhombus. Angle A is "3 points".

Students should be encouraged to use non-standard tools to compare angles to benchmark angles. A colour tile or square pattern block can be used to describe angles as less than or greater than a right angle.

Important benchmark angles include:



Straight angle

Right angle

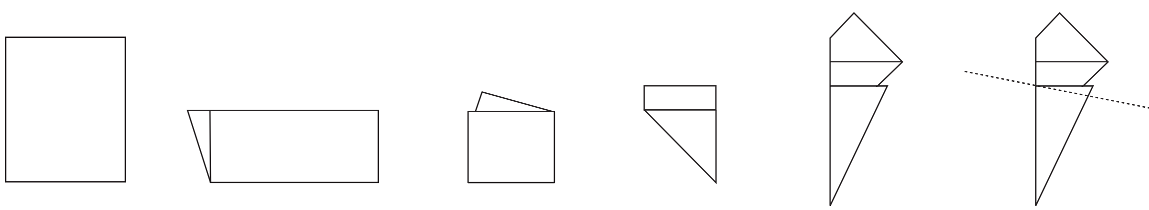
Half of a right angle (45°)

In the later junior grades, students begin using standard units and tools (e.g., protractors) to measure and construct angles. They also use specific angle measures to construct various polygons like quadrilaterals and triangles.

MEASURING ANGLES

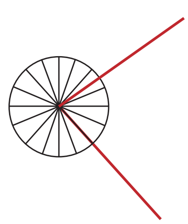
When students have had experiences identifying, comparing, and informally measuring angles using benchmarks, they can begin to measure angles with units and tools such as protractors. Standard protractors can be very confusing for junior students. There are no visible angles on the protractor. The unit markings representing angles on the protractor are very small and appear only on the edge. Most protractors contain two sets of numbers that run in both directions.

Students need experiences with more "informal" protractors containing larger unit angles to develop an understanding of how a protractor is used, and they can construct these informal protractors themselves. For example, they can fold a piece of waxed paper to make a transparent protractor.



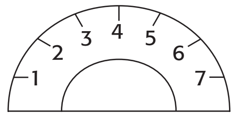
Waxed paper | 1st fold | 2nd fold | 3rd fold | 4th fold | Cut and unfold

This transparent protractor can be placed over angles or polygons, and students can see how to measure the smaller angles in those figures by fitting them within the larger unit angles of the waxed-paper protractor.



In this case the smaller angles could be referred to as wedges, and the angle above would measure approximately three and a half wedges.

Students could cut this full-circle protractor in half to resemble the standard protractor. Other informal protractors could be constructed from Bristol board, paper, and overhead transparencies that use outside markings and numbering that is similar to the numbering on a standard protractor.

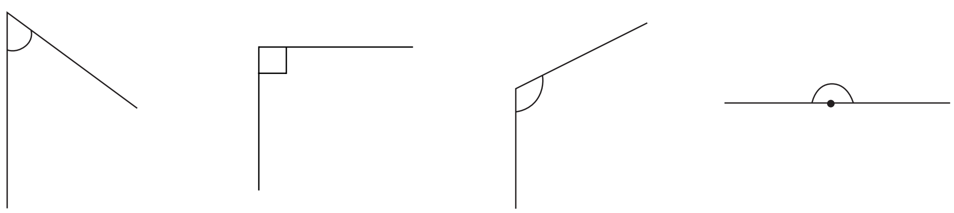


After these experiences, students can compare their informal protractors with the standard protractor to understand how this standard tool is used.

CLASSIFYING ANGLES

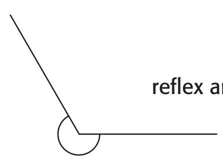
Since angles are essentially a measure of rotation, or the space between intersecting lines or segments, there is a limit as to how they can be classified – namely, the degree of the rotation. In contrast, polygons (e.g., triangles, quadrilaterals) can be classified by different attributes – length of sides, measure of angles, and so forth.

Acute Angle	**Right Angle**	**Obtuse Angle**	**Straight Angle**



An angle less than 90° is an acute angle.	An angle of exactly 90° is a right angle.	An angle of more than 90° but less than 180° is an obtuse angle.	An angle of exactly 180° is a straight angle.

A fifth classification of angles, known as a *reflex angle*, is an angle that measures more than 180° and less than 360°.



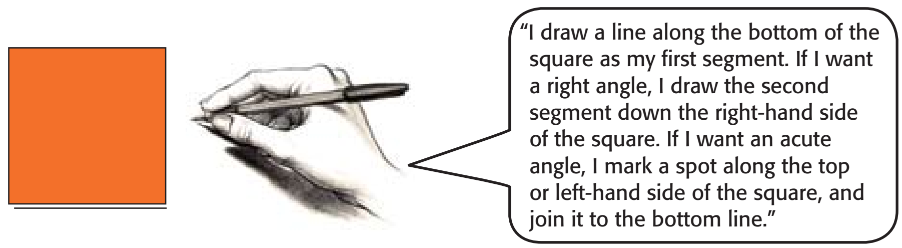
Although reflex angles are not specifically mentioned in the Ontario curriculum, students may wonder about the classification of angles larger than 180° in their explorations.

As students develop their understanding of the different classifications of angles, they will use angles as a property to classify different types of triangles (e.g., acute triangles, obtuse triangles, and right triangles) and different types of quadrilaterals (e.g., the right angles of rectangles distinguish them as a subclass of parallelograms).

CONSTRUCTING ANGLES

Students gain greater understanding of angle concepts by constructing angles. In their experiences of constructing angles, students should progress from using non-standard materials to using formal tools like protractors and compasses.

Pattern blocks that identify benchmark angles can be used to construct angles. For example, a square pattern block can be used to construct an acute or a right angle.

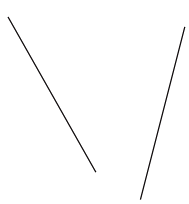


Other non-standard construction techniques include paper folding and using Miras. To construct angles of a specific measure (e.g., 64°, 125°), students need to be able to use a protractor or dynamic geometry software.

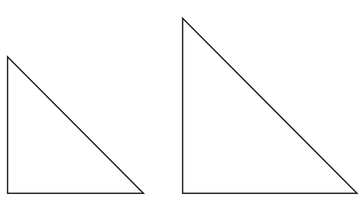
Investigating Congruence

Congruence is a special relationship between two-dimensional shapes that are the same size and same shape. In the primary grades students compare shapes and superimpose congruent shapes to show how one fits on top of the other. In the junior grades students continue these explorations and begin to use measurement as a tool for determining congruence.

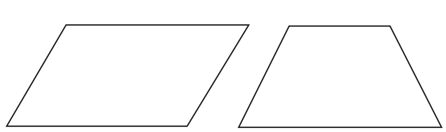
An important learning for students is that congruence depends on size and shape, but not orientation. The use of examples and non-examples helps develop an understanding of congruence.



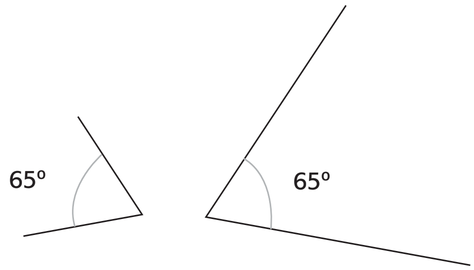
These two line segments are congruent, even though they are oriented differently, because they are the same length.



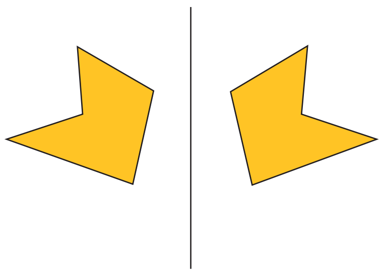
These triangles are not congruent. Although they are the same shape, they are not the same size.



A parallelogram has two pairs of congruent sides. An isosceles trapezoid has one pair of congruent sides.



These two angles are congruent because they are the same measure. The fact that the rays are different lengths and the angles “face” opposite directions does not affect congruence.



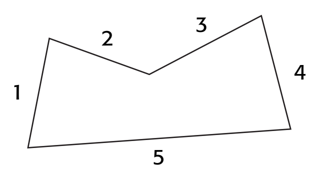
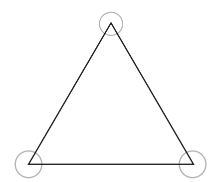
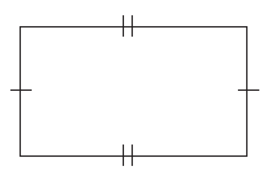
Shapes that are transformed by reflection, translation, or rotation exhibit congruence. The transformed shape is congruent to the original shape.

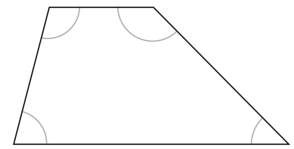
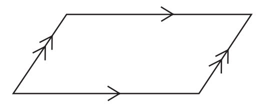
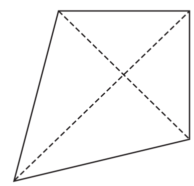
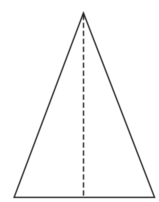
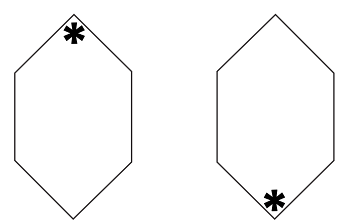
An understanding of congruence is beneficial when students encounter other geometric concepts such as transformations, tiling patterns, and symmetry.

Investigating Polygon Properties

In the junior grades, students continue to develop their understanding of the properties of two-dimensional shapes. They focus on specific shapes called *polygons*. A *polygon* is a closed shape formed by three or more straight sides. Polygons include triangles, quadrilaterals, octagons, and so forth.

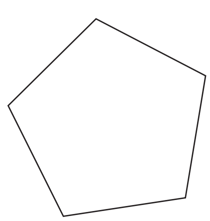
The properties of polygons are summarized in the following table:

Number of sides	One of the first properties students learn to consider is the number of sides a shape has. This information allows students to identify triangles, quadrilaterals, pentagons, hexagons, heptagons, octagons, and so forth.	
Number of vertices	Junior students should recognize that the point at which two lines meet is called a vertex. With experience, students will discover that the number of vertices in a polygon is the same as the number of sides.	
Length of sides	Students learn that the length of sides is an important property of many two-dimensional shapes. They recognize that the sides of squares are of equal length, and that pairs of opposite sides are equal for all other parallelograms.	

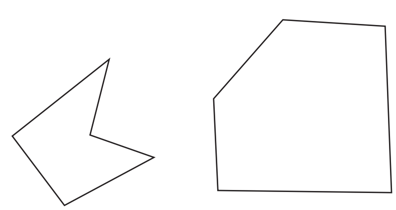
Size of angles	Junior students learn to consider angles as they identify classes and subclasses of two-dimensional shapes. For example, they can determine that an isosceles triangle has two identical angles and that an isosceles right triangle has one 90° angle and, therefore, two 45° angles. Both interior and exterior angles of polygons can be considered when identifying and classifying polygons.	
Parallel lines	Junior students will develop an understanding that parallel lines are always the same distance apart. They will recognize parallelism as an essential property in describing the sides of trapezoids and parallelograms.	
Diagonals	A diagonal is a line connecting two non-adjacent vertices. Junior students will recognize that a polygon can be classified by the number and nature of its diagonals.	
Lines of symmetry	Students can learn to identify or classify a polygon by the number of lines of symmetry it has. For example, squares have four lines of symmetry; isosceles triangles have one line of symmetry.	
Rotational symmetry	A shape has rotational symmetry if its position matches its original position after it has been rotated less than 360°.	 orignal position 1/2 turn

Exploring the properties of polygons allows students to identify and classify them in a number of ways:

- *Regular polygons:* all angles are equal and all sides are the same length.
- *Irregular polygons:* not all angles are equal, and not all sides are the same length.



Regular pentagon



Irregular pentagons

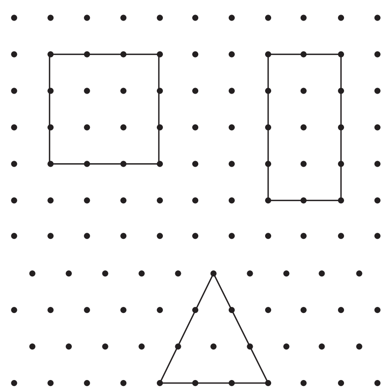
Junior students continue to become more precise in their description of two-dimensional properties and to become familiar with the property of symmetry.

SYMMETRY

In the primary grades students explore lines of symmetry, using paper folding, transparent tools, and drawings, and in Grade 3 students complete pictures and designs when given half of the image on one side of a line of symmetry. In the junior grades students expand their explorations and use lines of symmetry to classify two-dimensional shapes. In Grade 6 students explore the concept of rotational symmetry.

LINES OF SYMMETRY

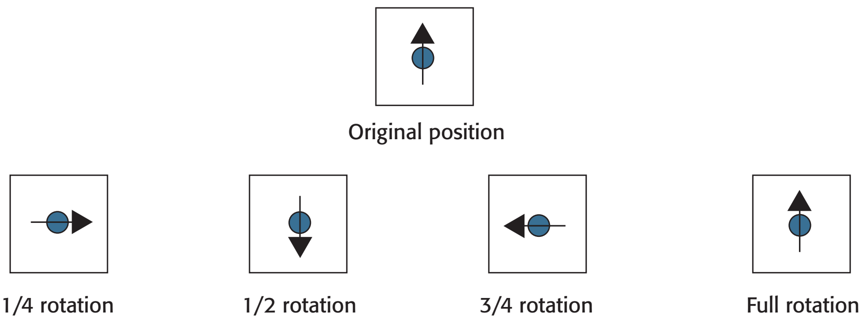
A line of symmetry divides a shape into two congruent parts that can be matched by folding the shape in half. Junior students should continue to investigate line symmetry in two-dimensional shapes, using a variety of tools including paper folding, the Mira, dot paper, and computer applications. Explorations should include using a variety of methods to determine the lines of symmetry in a variety of two-dimensional shapes. For example, students might be asked to find the number of lines of symmetry in the following shapes.



ROTATIONAL SYMMETRY

Rotational symmetry occurs when the position of a shape matches its original position after the shape has been rotated less than 360°. The *order of rotational symmetry* refers to the number of times the position of a shape matches its original position during a complete rotation about its centre.

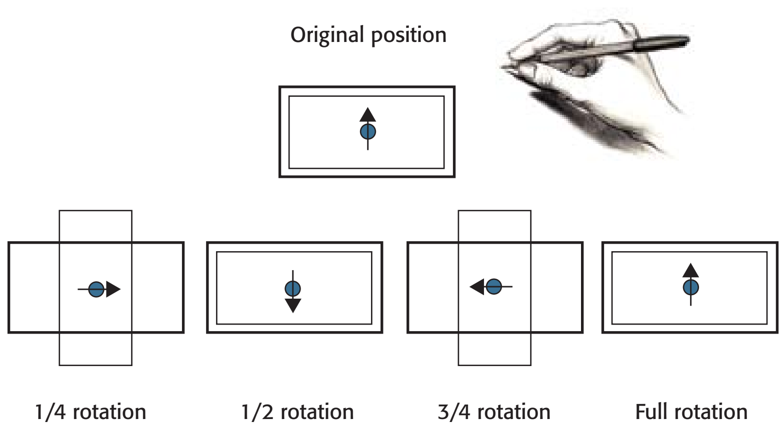
For example, a square has rotational symmetry.



When a square is rotated about its centre, its position matches its original position after a 1/4, 1/2, and 3/4 rotation: therefore, a square has rotational symmetry. It has rotational symmetry of order 4 because its position matches the original position four times during a complete rotation.

An important investigation involves exploring the relationship between the number of sides of the polygon and its order of rotational symmetry. All regular polygons have an order of symmetry equal to their number of sides – a square has an order of symmetry of 4, a regular hexagon has an order of rotational symmetry of 6, and so forth.

Junior students can determine rotational symmetry and the order of rotational symmetry of two-dimensional shapes by tracing a shape and then rotating that shape within the tracing to determine if it will fit more than one way.



Teachers should provide students with opportunities to determine rotational symmetry in a variety of two-dimensional shapes and have students attempt to create their own two-dimensional shapes, given the order of rotational symmetry.

Investigating Triangles and Quadrilaterals

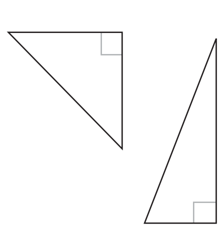
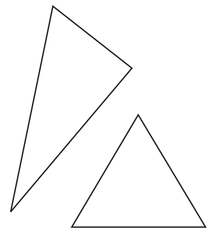
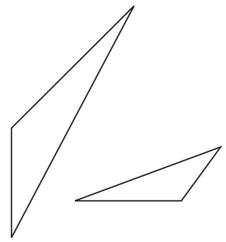
Junior students need to explore and investigate a variety of regular and irregular polygons. They will focus on two specific classes of polygons: triangles and quadrilaterals.

PROPERTIES OF TRIANGLES

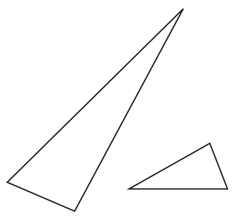
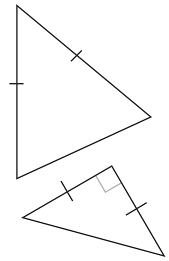
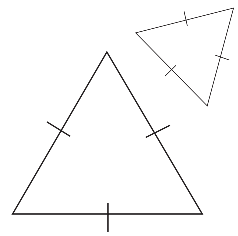
A triangle is a three-sided polygon. Every triangle has three sides and three angles. Through repeated opportunities to compare, describe, and construct triangles of various orientations and configurations, junior students will learn to identify important properties of subclasses of triangles.

Triangles can be classified according to some of their features:

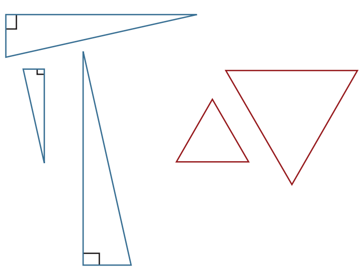
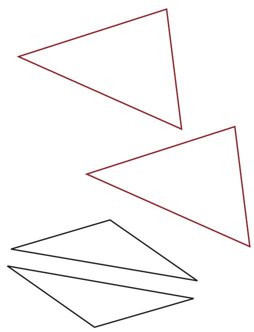
Angles:

Right Triangles	Acute Triangles	Obtuse Triangles
		
one right (90°) angle	all angles less than 90°	one angle greater than 90°

Sides:

Scalene Triangles	Isosceles Triangles	Equilateral Triangles
		
all sides different lengths	two congruent sides	all three sides congruent

Size:

Similar Triangles	Congruent Triangles
	
same shape, different size	same size and shape

Note that the concept of similarity of polygons is one that is beyond most junior-grade students and that similarity is not explicitly taught until Grade 7 in the Ontario curriculum. However, students may informally describe one triangle as having the same "shape" as another but being smaller or larger than the other. Such informal explorations should be encouraged, but formal proofs of similarity are developmentally more appropriate for older students.

It is important for junior students to explore triangle attributes (e.g., side length, angle measure) in combination in order to develop a greater understanding of the properties of triangles. Probing questions can help guide students in their exploration:

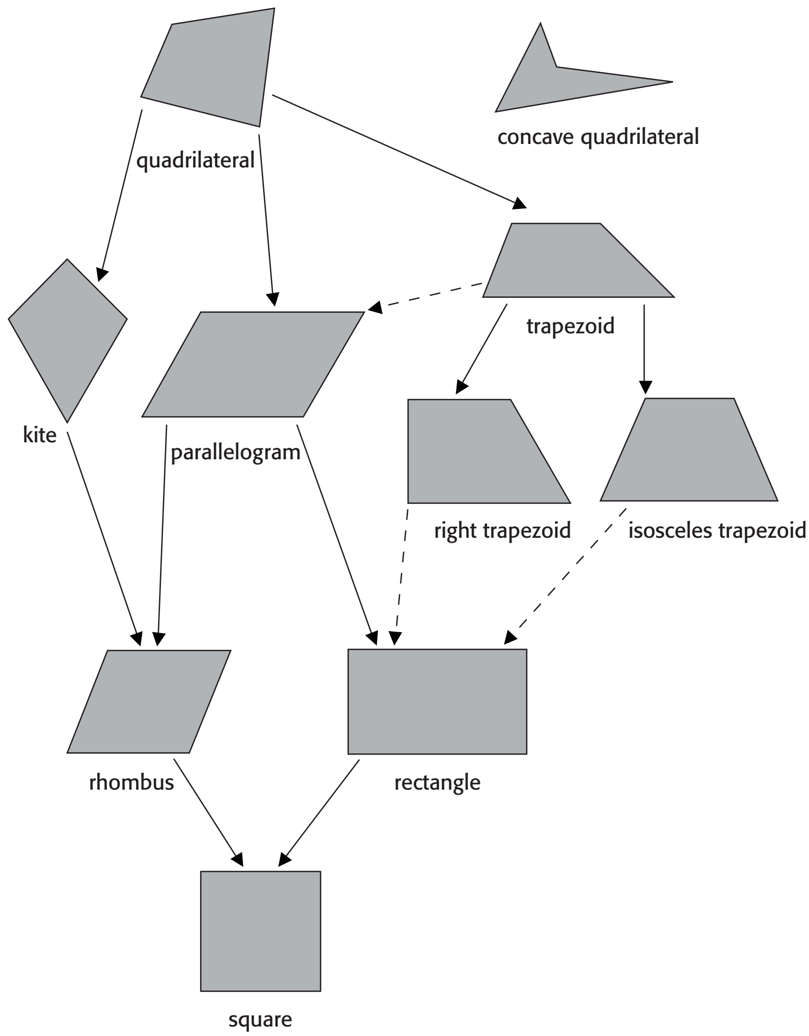
- "Can you construct a triangle that is both a right triangle and an isosceles triangle?"
- "Can you define/describe an equilateral triangle without using the word 'sides'?"
- "Can a triangle have more than one obtuse angle? Why or why not?"
- "Are all scalene triangles also acute triangles? Explain."
- "Can an equilateral triangle be obtuse?"

QUADRILATERAL PROPERTIES

A quadrilateral is a four-sided polygon. Every quadrilateral has four sides and four angles. There are many classifications of quadrilaterals, and students need to learn that quadrilaterals can belong to more than one category. A square, for example, can be classified as a parallelogram, a rectangle, and a rhombus. Students need opportunities to discuss examples and non-examples of classes of quadrilaterals in order to help identify defining properties.

The diagram on page 51 represents a classification of quadrilaterals that students in the junior grades should be familiar with. Each quadrilateral lower in the diagram represents a special case of a quadrilateral higher in the diagram.

Classification of Quadrilaterals

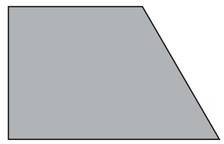


The preceding diagram illustrates that some quadrilaterals can be classified as more than one shape. For example, a rhombus is a parallelogram, as it has two sets of parallel sides. It is also a kite, because it has two pairs of congruent adjacent sides.

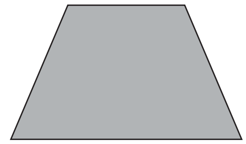
A NOTE ABOUT TRAPEZOIDS

Some mathematicians view the trapezoid as a special quadrilateral, and define it as *a quadrilateral having **exactly** one pair of parallel sides*. This definition is widely accepted in many countries and mathematics communities. Other mathematicians define a trapezoid as *a quadrilateral having at **least** one pair of parallel sides*. For this reason, the line in the diagram that joins the trapezoid to the parallelogram and the lines that join the right and isosceles trapezoids to the rectangle are drawn with *dotted lines*. Since both definitions are viewed as mathematically correct, what is important is the students' reasoning for including or excluding trapezoids in the hierarchy of quadrilaterals.

In the early junior grades, students are expected simply to identify trapezoids. In later grades, they will recognise that trapezoids can be classified like triangles – that is, according to various properties.

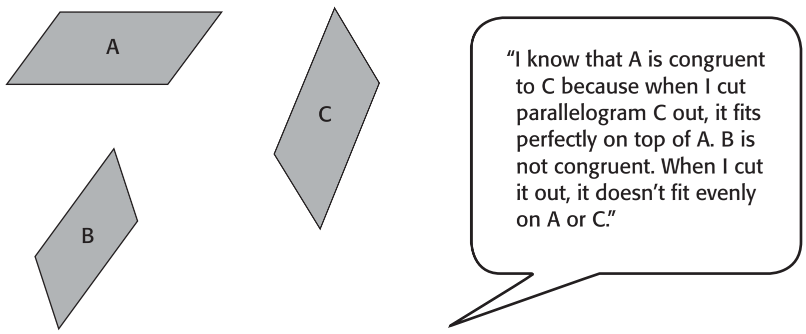


Right trapezoid



Isosceles trapezoid

Like triangles, quadrilaterals can be classified by their size. Quadrilaterals having the same shape and size are *congruent*; those whose shape is the same but whose size is different are *similar*. Formal proof of congruence or similarity involves careful examination of sides and angles. While students in the junior grades are not expected to conduct formal proofs, they can explore the congruence of quadrilaterals concretely:



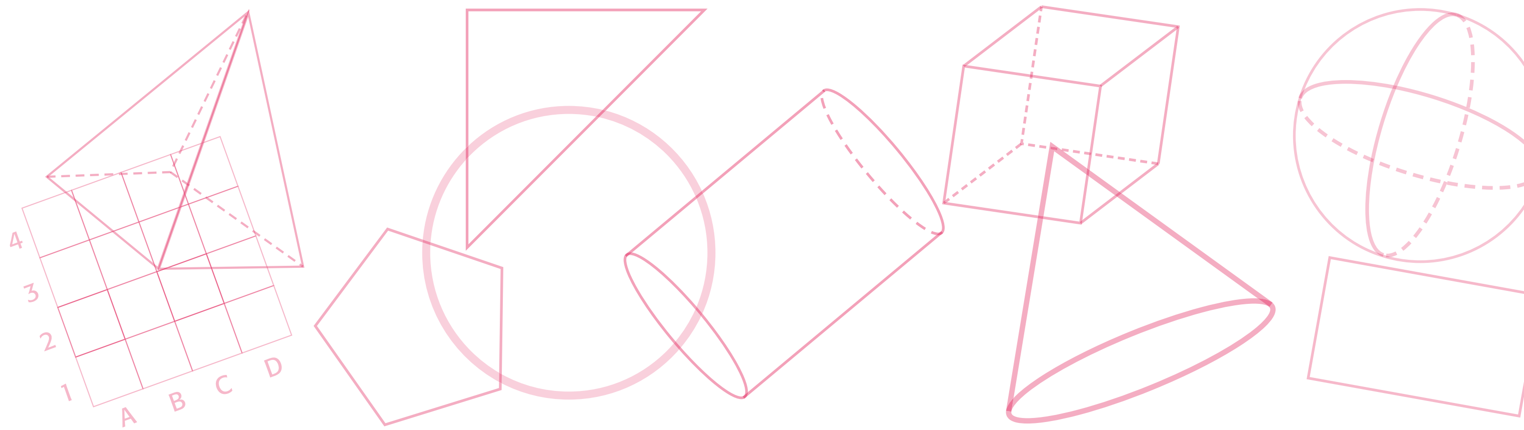
Teachers should use probing questions to encourage students to think about quadrilateral properties, with the goal of moving them from the analysis phase to the informal deduction phase. Some examples of questions include the following:

- "A quadrilateral has four congruent sides and at least one right angle. Can it be anything other than a square?"
- "Is a square a rhombus or a rectangle or both? Why do you think so?"
- "What can you say about the diagonals of a rectangle? How does this compare with what you can say about the diagonals of a parallelogram, which has no right angles?"

GENERAL INSTRUCTIONAL STRATEGIES

Students in the junior grades benefit from the following instructional strategies:

- having them investigate two-dimensional geometry using concrete materials like pattern blocks, geoboards, and Power Polygons;
- providing opportunities to analyse properties of shapes using examples and non-examples;
- requiring them to communicate solutions to problems related to two-dimensional shapes and angles using increasingly accurate terminology;
- providing experiences of determining properties and characteristics of geometric shapes using dynamic geometry software and other technologies.



LEARNING ABOUT THREE-DIMENSIONAL FIGURES IN THE JUNIOR GRADES

Introduction

The study of three-dimensional figures is closely connected with other mathematical topics and ideas, including two-dimensional geometry, measurement, and number. As students investigate the geometric properties and relationships of three-dimensional figures, their reasoning skills become more complex and their vocabulary more detailed.

PRIOR LEARNING

In the primary grades, students learn to recognize and describe some geometric properties of three-dimensional figures, such as the number and shape of faces and the number of edges or vertices. For the most part, they explore these properties concretely, using models of three-dimensional figures, though they also begin to investigate nets of rectangular prisms.

Learning about geometric properties allows students to develop the concepts and language they need to analyse and describe three-dimensional figures, and to discover relationships between two- and three-dimensional geometry. Experiences in the primary classroom include identifying, comparing, sorting, and classifying figures according to their basic properties, and making connections between two-dimensional shapes and three-dimensional figures.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

In the junior grades, students continue to identify, compare, sort, and classify three-dimensional figures, with a particular focus on pyramids and prisms. These investigations focus on making generalizations about a category of figures. As well, students in the junior grades develop spatial awareness by exploring two-dimensional representations of three-dimensional figures. These representations include nets; front, side, and top views; and isometric drawings.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to three-dimensional figures, listed in the table on the following page.

Curriculum Expectations Related to Properties of Three-Dimensional Figures, Grades 4, 5, and 6		
By the end of Grade 4, students will:	**By the end of Grade 5, students will:**	**By the end of Grade 6, students will:**
Overall Expectations • identify quadrilaterals and three-dimensional figures and classify them by their geometric properties, and compare various angles to benchmarks; • construct three-dimensional figures, using two-dimensional shapes. ***Specific Expectations*** • identify and describe prisms and pyramids, and classify them by their geometric properties; • construct a three-dimensional figure from a picture or model of the figure, using connecting cubes; • construct skeletons of three-dimensional figures, using a variety of tools, and sketch the skeletons; • draw and describe nets of rectangular and triangular prisms; • construct prisms and pyramids from given nets; • construct three-dimensional figures, using only congruent shapes.	***Overall Expectations*** • identify and classify two-dimensional shapes by side and angle properties, and compare and sort three-dimensional figures; • identify and construct nets of prisms and pyramids. ***Specific Expectations*** • distinguish among prisms, right prisms, pyramids, and other three-dimensional figures; • identify prisms and pyramids from their nets; • construct nets of prisms and pyramids using a variety of tools.	***Overall Expectation*** • sketch three-dimensional figures, and construct three-dimensional figures from drawings. ***Specific Expectations*** • build three-dimensional models using connecting cubes, given isometric sketches or different views; • sketch, using a variety of tools, isometric perspectives and different views of three-dimensional figures built with interlocking cubes.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)

The following sections explain content knowledge related to three-dimensional concepts in the junior grades, and provide instructional strategies that help students develop an understanding of three-dimensional figures. Teachers can facilitate this understanding by helping students to:

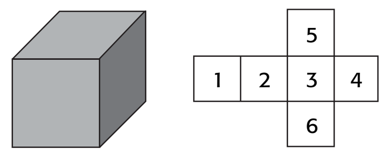
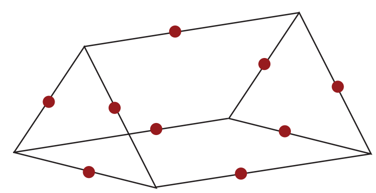
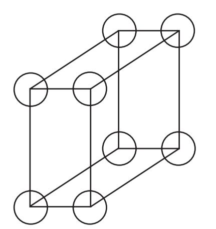
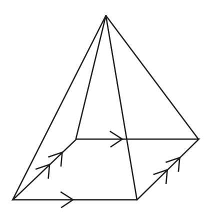
- investigate the properties of pyramids and prisms in meaningful ways;
- explore relationships between two- and three-dimensional geometry.

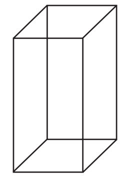
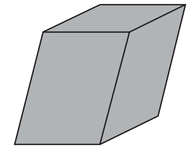
Properties of Prisms and Pyramids

In the primary years, students identify and sort three-dimensional figures according to their properties. As students continue to explore shapes and figures in the junior years, their ability to describe these properties increases, as does their ability to use appropriate mathematical language in so doing.

In the junior grades, students focus on specific figures called *polyhedra*. A *polyhedron* is a three-dimensional figure with faces made up of polygons. The polyhedra include prisms and pyramids.

The properties of polyhedra are summarized in the following table:

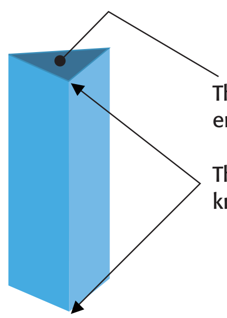
Number or type of faces	The face of a polyhedron is a flat surface. Students can describe polyhedra by the number or shape of their faces.	 A cube has 6 faces.
Number of edges	Junior students should recognize that the line where two faces meet is called an *edge*. Students should explore the relationships between the number of edges and faces in categories of polyhedra.	 A triangular prism has 9 edges.
Number of vertices	Students learn that a vertex is a point at which two or more edges meet. Students will explore the relationship between the number of edges, vertices, and faces of various polyhedra.	 A rectangular prism has 8 vertices.
Parallelism	Faces and edges of a figure can be described as being parallel or non-parallel to other faces and edges.	 Only the edges of the base are parallel in pyramids. *(continued)*

Perpendicularity	Perpendicularity is a concept that develops in the later junior grades. Edges and faces can often be described as "at right angles" to other faces and edges.	 The base of this rectan-gular prism is perpendicular to the vertical faces.	 The base of this prism is *not* perpendicular to its vertical faces.

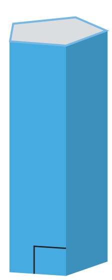
PRISM PROPERTIES

There are different formal and informal definitions of prisms. Despite the varying definitions, there is little disagreement as to what a prism is. Students recognise which solids are prisms and usually use common descriptions – for example, "a solid with two bases that are the same size and shape". Essentially, this definition clearly describes prisms; it is the role of the teacher to guide students to the richer descriptions that include more complex geometric concepts. An accepted definition of a prism is the following:

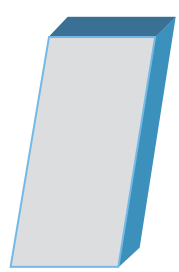
A *prism* is a solid geometric figure whose two ends are parallel and congruent polygons, called bases. Lines joining corresponding points on the bases are always parallel. The sides of prisms are always parallelograms.



If the vertical edges of the prism are perpendicular to the edges of the base, the prism is described as a **right prism**. When the edges do not run perpendicular to each other, the prism is said to be an **oblique prism**.



Right prism



Oblique prism

Prisms are named according to the shape of their bases. Some examples include rectangular prisms, triangular prisms, hexagonal prisms, and so forth. A special rectangular prism, with square faces congruent to the base, is known as a *cube*.

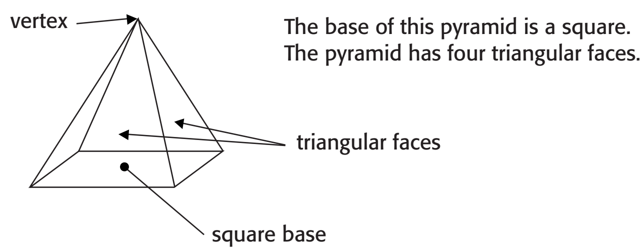
Students almost always wonder how cylinders are related to prisms, since they look and feel similar to prisms. Although they have been described as "circular prisms", they are not included in most accepted definitions of prisms, since their bases are not polygons. On the other hand, some mathematicians define prisms as special *cylinders* – cylinders are solids with two parallel bases connected by parallel elements. When the bases are polygons, the cylinders are prisms.

These differences in classifications underline the fact that definitions in geometry are conventions, and not all conventions are universally accepted. Student discussion and reasoning are more important in developing geometric thinking than is memorizing definitions.

PYRAMID PROPERTIES

Like prisms, pyramids are a special category of three-dimensional figures with common defining properties. The Ontario curriculum document formulates the meaning of *pyramid* as follows:

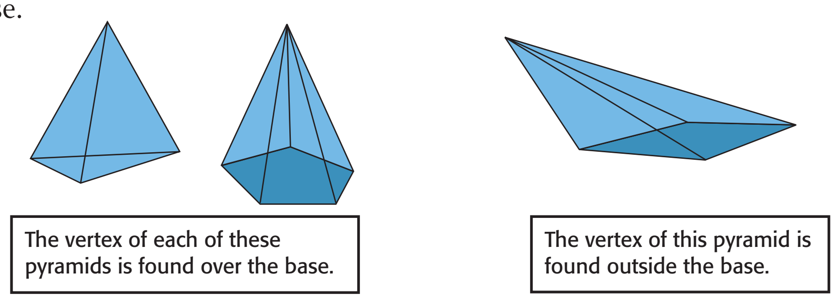
> A *pyramid* is a polyhedron whose base is a polygon and whose other faces are triangles that meet at a common vertex.



Like prisms, pyramids are named by the shape of their base. Examples of pyramids include square pyramids, hexagonal pyramids, and octagonal pyramids. A *tetrahedron* is a special type of triangular pyramid. All of the faces of a tetrahedron are congruent equilateral triangles.

John Van de Walle (2005) considers pyramids to be special cases of *cones*. He defines a cone as a solid with a base and a vertex that is not on the base. Edges join the vertex to the vertices of the base, and the base may be any shape at all. When the base is a polygon, the cone can be classified as a prism. Again, this conventional definition is not universally accepted. Some consider cones to be pyramids with circular bases and only one face.

Note that pyramids can have a common vertex that is directly over the base, or one that lies outside the base.

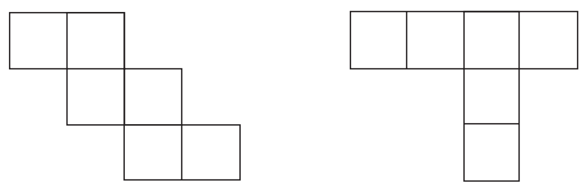


Representing Three-Dimensional Figures in Two Dimensions

In the junior grades students are expected to move flexibly between two- and three-dimensional representations of a figure. Two-dimensional representations can include nets, "rectangular" views, and isometric sketches.

A **net** is a pattern that can be folded into a three-dimensional figure. A net must include all of the two-dimensional faces of the figure. For example, a net of a cube must have six squares, a net for a triangular prism must have two triangles and three rectangles, and a net for a pentagonal pyramid must have one pentagon and five triangles.

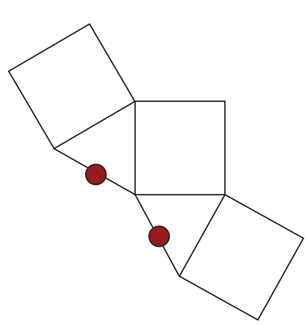
In addition to knowing the faces of a figure, students must develop a spatial awareness of how the faces "fit" together. Consider as an example these two patterns:



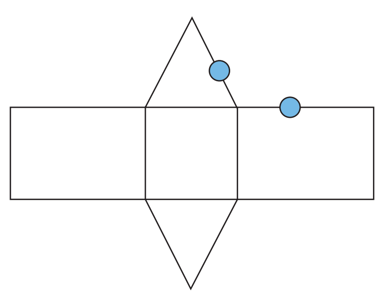
Although both of these patterns are composed of the correct number of faces to form a cube, only the one on the left can actually be folded to form a cube.

Initial experiences with nets should be concrete. Paper folding and Polydron pieces are examples of manipulatives that serve as an entry point for investigations involving nets of solids. Polydron shapes can be snapped together and taken apart easily, and folded and snapped together to form a solid. They are limited, however, in that the number of shapes is finite. Paper folding takes longer to prepare and is less "user-friendly", but it has no limits with respect to the type and size of shapes.

Student experiences with nets should be varied, and should include working from net to solid and from solid to net. When students are able to "unfold" a solid, they are more likely to understand the relationships between the faces. These relationships include shared edges and edge length. Similarly, students should be given examples and non-examples of nets to fold into solid figures. Consider these two nets for triangular prisms:



Net A



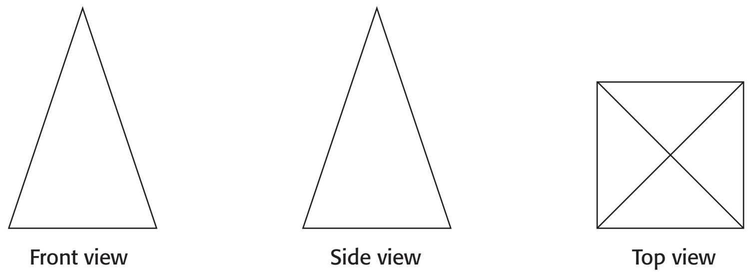
Net B

Although both nets have the correct number and type of faces found in a triangular prism, neither can be folded into one. In Net A, the faces are the correct size and shape, but do not have the correct shared edges. The sides with the red dots are an example of two sides that will be folded together to form a shared edge. In triangular prisms, the triangle faces are at opposite ends of the prism and do not share an edge.

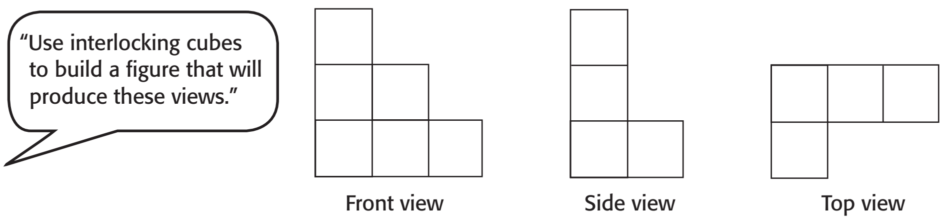
Net B also has the correct number and type of faces, but they are not all the correct size. The sides with the blue dots will be folded together to form an edge, but they are not the same length, so a shared edge will not be formed.

Explorations that focus on these types of relationships will help students make connections between two- and three-dimensional geometry and will help to develop their spatial awareness.

Students also use rectangular diagrams to represent solid figures in the junior grades. A **rectangular** view is a two-dimensional view of one side of a figure. The view can be from the front, back, side, top, or bottom, and is a shape or composition of shapes. Various views of a square pyramid are shown below:



Although it is useful for students to be able to identify solid figures from their various views, a greater sense of space develops when students explore views of composite figures. Interlocking cubes are an excellent tool for such explorations.



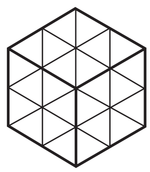
Again, students should work "both ways" when exploring rectangular views of solids – they should build the solid from different views, and should draw different views of a solid on grid paper when given a model of the solid.

As students become more confident in their explorations, investigations can focus on higher-level geometric thinking skills. For example:

- Can a pyramid have a side view that is a rectangle? Explain.
- What does the top view of a prism tell you about the number of rectangular faces it has?
- What is the fewest number of cubes needed to build a solid with these views? (Students are given three views of a solid made from interlocking cubes.)

When planning activities that involve working with rectangular views, it is important to keep in mind the second van Hiele level – analysis. The focus is on the properties of shapes and figures, and the ability to generalize from observations. For example, students might recognize that the number of sides of a prism base is the same as the number of rectangular faces of the prism. Students are likely to realize this generalization after taking part in a variety of carefully planned activities.

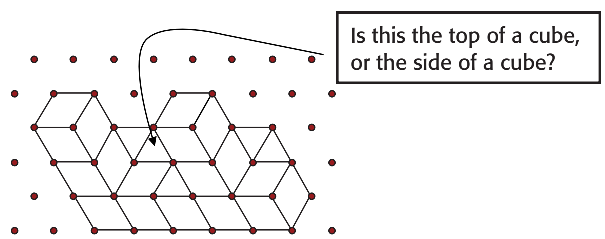
Isometric diagrams use an isometric grid. An isometric grid shows three axes instead of the two found in a rectangular grid. One axis runs vertically; the other two axes run "down" at 30° angles to the left and right.



This cube has been drawn using isometric grid paper. Note the directions of the three axes – one is vertical; two run down diagonally left and right.

Note that the perimeter of the two-dimensional drawing is a perfect hexagon, and all the thick lines are of equal length.

Isometric diagrams encourage students to visualize pieces or sections of solids that cannot be seen. When working from an isometric view and building a solid from interlocking cubes, students may be required to make assumptions about cubes they may or may not be able to view directly. For example, in the diagram below, students would have to justify placing a cube in the space indicated by the arrow.



LEARNING ABOUT LOCATION AND MOVEMENT IN THE JUNIOR GRADES

Introduction

Spatial sense can be described as the intuitive awareness of one's surroundings and of the objects in them. Having spatial sense enables students to accurately describe the location of objects in their surroundings and of shapes in an abstract plane. Understanding and being able to visualize the various ways in which shapes and objects can move also contribute to the development of spatial reasoning skills.

PRIOR LEARNING

In the primary grades, students learn to describe the location of an object using expressions such as "in front of", "beside" and "to the left of". They describe transformations as slides, flips, and turns, and discuss the results of these transformations by comparing the transformed image with the original shape. In investigating lines of symmetry, primary students use concrete materials and tools to develop a greater understanding of reflections.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

Instructional strategies for teaching location and movement in the junior grades should focus on an understanding of *spatial relationships*. The location of a point is related to both a horizontal and a vertical position. Transformations are related to one another – for example, the position of an image after a rotation can often be replicated by a combination of other transformations. Although location and movement can be treated as separate components of a junior curriculum, when they are taught in combination with a focus on relationships, students are able to make important mathematical connections.

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to location and movement, listed in the table on the following page:

Curriculum Expectations Related to Location and Movement, Grades 4, 5, and 6		
By the end of Grade 4, students will:	**By the end of Grade 5, students will:**	**By the end of Grade 6, students will:**
Overall Expectation • identify and describe the location of an object, using a grid map, and reflect two-dimensional shapes. ***Specific Expectations*** • identify and describe the general location of an object using a grid system; • identify, perform, and describe reflections using a variety of tools; • create and analyse symmetrical designs by reflecting a shape, or shapes, using a variety of tools and identify the congruent shapes in the designs.	***Overall Expectation*** • identify and describe the location of an object, using the cardinal directions, and translate two-dimensional shapes. ***Specific Expectations*** • locate an object using the cardinal directions and a coordinate system; • compare grid systems commonly used on maps; • identify, perform, and describe translations, using a variety of tools; • create and analyse designs by translating and/or reflecting a shape, or shapes, using a variety of tools.	***Overall Expectation*** • describe location in the first quadrant of a coordinate system, and rotate two-dimensional shapes. ***Specific Expectations*** • explain how a coordinate system represents location, and plot points in the first quadrant of a Cartesian coordinate plane; • identify, perform, and describe, through investigation using a variety of tools, rotations of 180° and clockwise and counter-clockwise rotations of 90°, with the centre of rotation inside or outside the shape; • create and analyse designs made by reflecting, translating, and/or rotating a shape, or shapes, by 90° or 180°.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005)

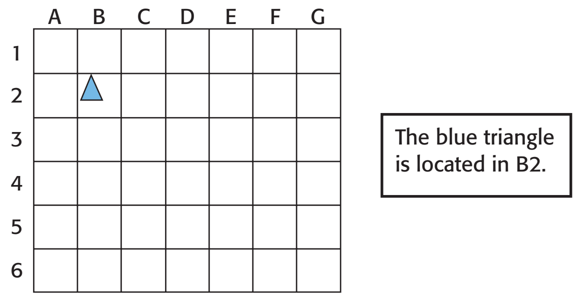
The following sections explain content knowledge related to location and movement concepts in the junior grades, and provide instructional strategies that help students develop an understanding of location and movement. Teachers can facilitate this understanding by helping students to:

- connect prior learnings about location with formal coordinate geometry;
- explore relationships in the movement of shapes and objects;
- make connections between transformational geometry and coordinate geometry.

Grid and Coordinate Systems

An understanding of the coordinate systems, including the Cartesian plane, is essential to later learnings, as it is a fundamental aspect of both geometry and algebra. It is important that junior teachers make connections with prior learnings about location when teaching about grid or coordinate geometry.

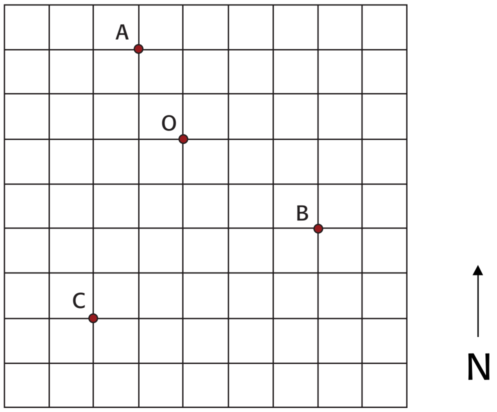
Students' first experiences with coordinate geometry in the junior grades come in the form of grids, commonly used in maps. A **grid system** uses a combination of letters and numbers to describe a general location of a shape or an object. Because the grid system identifies an *area* rather than a point, precise locations cannot be described.



Students will bring to the junior grades an understanding of grids that comes from both classroom experiences and non-classroom experiences. The hundreds chart, social studies maps, data management activities, and games are some examples of students' exposure to grids. Teachers should try to make connections with those learnings when introducing grids. Some examples include:

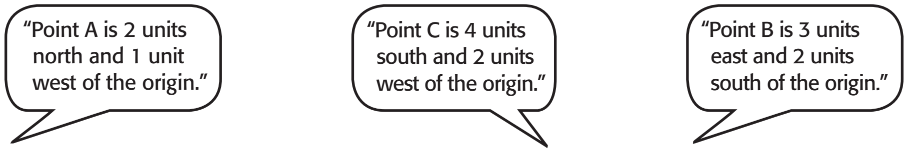
- exploring location on non-labelled grids. Students may use phrases like "third from the left and two down" to identify a square. These phrases can later be connected with labels;
- predicting new locations on the basis of a particular movement. For example, if you started in square B1, and kept moving to the right, how would the label of each new square change? What if you started in B1 and kept moving down?;
- exploring local and provincial maps and atlases that use similar grid identification systems;
- playing games that require the use of a grid. For example, students might play games similar to Battleship, in which hidden shapes must be found by guessing grid positions.

Coordinate systems differ from grid systems in that they identify a *point* rather than an *area*. In a coordinate system, the *lines* are labelled, rather than the *area bounded by the lines*. One way to describe or label these lines is to use the cardinal directions with a numbering system to describe the location of an object in relation to a point.



In the grid on page 63, point O represents the origin, or starting point. A teacher might ask:

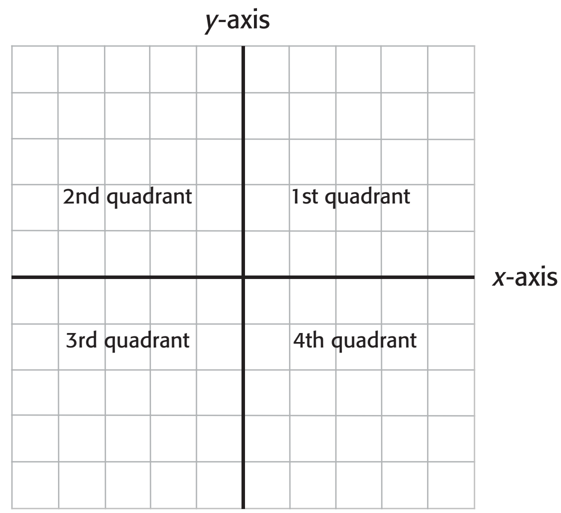
> *"Describe the position of points A, B, and C in relation to point O. Use cardinal directions (e.g., north, south) rather than conventional directions (e.g., up, down, right)."*



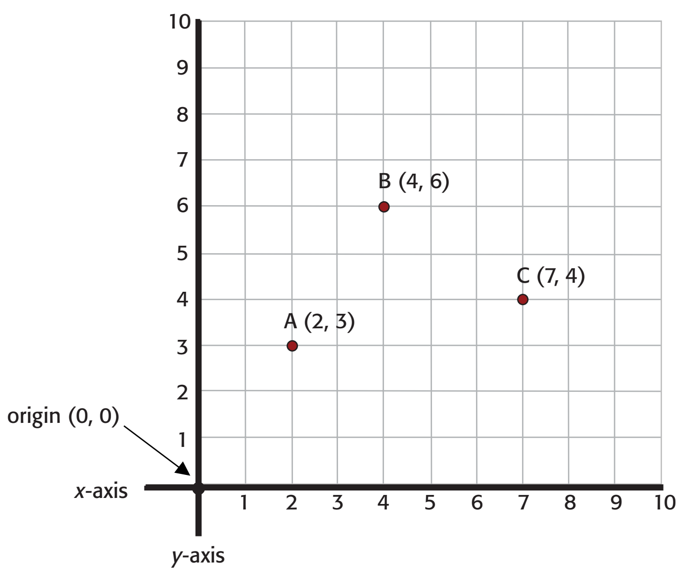
Many activities can be developed to offer students meaningful first experiences with a coordinate system. Students will develop fundamental ideas about grids and strategies for navigating them in these types of activities. These ideas and strategies are an important component of the mathematics they will learn in later grades (NCTM, 2000).

In the later junior grades, students are formally introduced to the Cartesian plane and learn to use ordered pairs to identify points.

The **Cartesian coordinate plane** uses two axes along which numbers are plotted at regular intervals. The horizontal axis is known as the ***x*-axis**, and the vertical axis as the ***y*-axis**. The location of a point is described by an ordered pair of numbers representing the intersection of an *x* and a *y* "line". The *x*- and *y*-axes divide the plane into four *quadrants* – the first, second, third, and fourth quadrants.



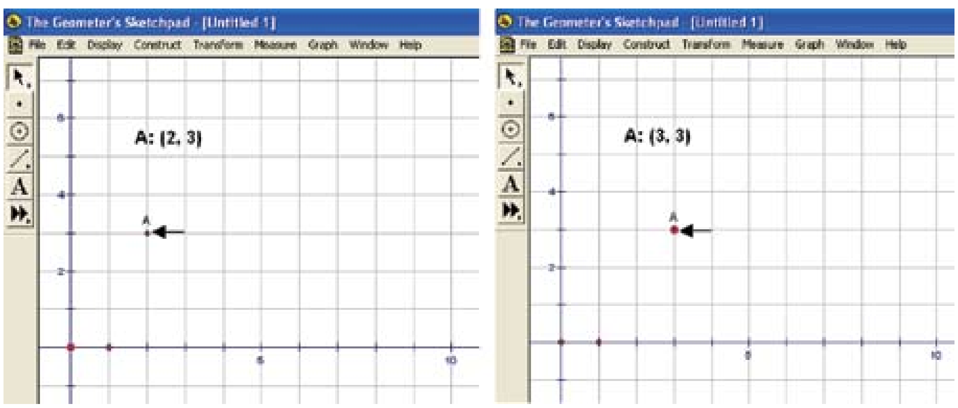
The First Quadrant of the Cartesian Coordinate Plane



The first quadrant of the coordinate plane is the quadrant that contains all the points with positive *x* and positive *y* coordinates. In labelling points, the *x*-coordinate is always given first, then the *y*-coordinate. The coordinates are written in parentheses and are separated by a comma (*x*, *y*). The **origin** is located at the intersection of the *x*- and *y*-axes, and is represented as the point (0, 0).

A common misconception students experience with initial investigations of the Cartesian plane is confusing the order of the numbers of a point. While some students may rely on phrases or other "tricks" to help remember which axes the numbers represent (e.g., "You go in the door first and then up the stairs"), recent developments in dynamic geometry software offer students more meaningful opportunities to develop this understanding.

Consider as an example The Geometer's Sketchpad. This program allows students to call up a point on a grid, label it, and display its coordinates. When the point is "dragged" or moved about the grid, the coordinates change as the point moves.



A sample exploration in which students use The Geometer's Sketchpad might include questions like the following:

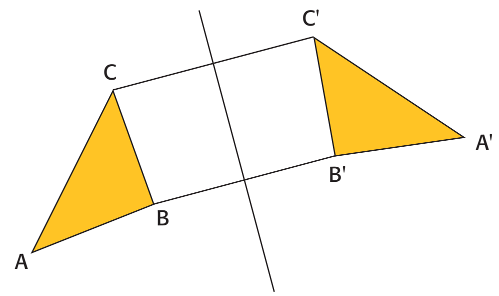
- Drag point A horizontally along a single line. What do you notice about how the coordinates change each time the point is moved along the line?
- Drag point A vertically along a single line. How do the coordinates change this time?

When students drag a point and see the coordinates change, they may better understand which ordered pair corresponds with which axis. As a point is moved horizontally but its vertical position remains unchanged, the second number in the ordered pair remains unchanged. So that number must represent the vertical (or *y*-axis) position. "Seeing and doing" with dynamic software is powerful and can make a greater impact than simply plotting points and listing coordinates. Rich explorations with grid and coordinate systems can help students develop greater spatial awareness and deepen their understanding of spatial relationships, while providing foundational knowledge important to mathematics in the later grades.

Relationships in Transformational Geometry

Junior students identify, describe, and perform three different types of transformations in geometry and spatial sense – reflections, translations, and rotations. Although students can study each transformation independently, it is when they explore the relationships between transformations that they begin to deepen their understanding and make connections.

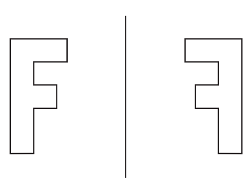
A **reflection** over a line is a transformation in which each point of the *original* shape, sometimes called the *pre-image*, has an *image* shape that is the same distance from the line of reflection as the original point, but is on the opposite side of the line.



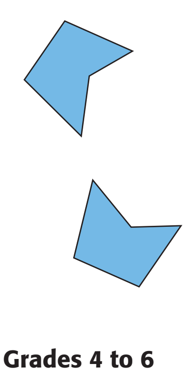
In this example, the points A, B, and C are exactly the same distance from the mirror line as are points A', B' and C.'

Students' initial experiences with reflections should be grounded in concrete materials. Students can physically flip objects across a line of reflection created by folding a piece of paper, or by using physical shapes like pattern blocks, plastic polygons, or cutouts. Technology like The Geometer's Sketchpad allows students to explore reflections dynamically once they are comfortable with the basics of the software.

Teachers should select shapes that are not symmetrical for initial explorations with reflections. Students will more readily see the change of orientation in the reflected image when shapes are not symmetrical. Block letters work well and will engage students, as they may choose letters in their name.

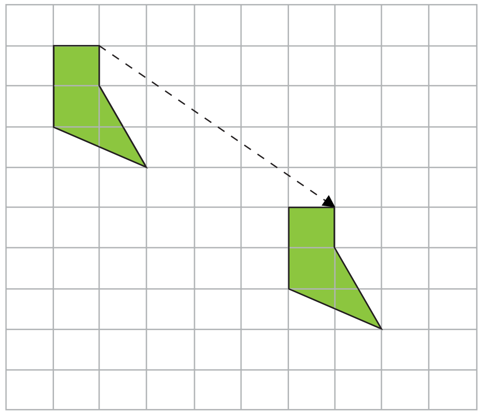


Reflection activities should not simply focus on students' performing the transformation; they should also require students to determine and describe the transformation that has taken place. An example of a rich activity is having students determine the location of the line of reflection, given the original shape and its image.



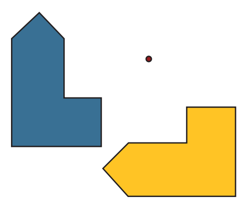
Draw the mirror line that will produce this reflection.

A **translation** can be described as a transformation that slides every point of a shape the same distance in the same direction.



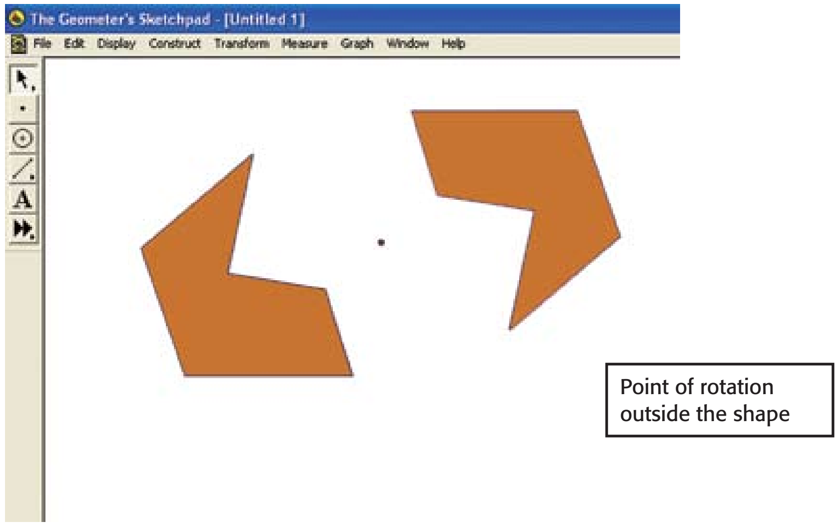
It is useful for students to describe the translation of a shape in multiple ways. In the diagram above, the pentagon is shown to be directly translated "4 down and 5 to the right", but the image may also have resulted from multiple translations. For example, the pentagon may have moved 5 units right and 3 units up, followed by a second translation of 7 units down. Activities that allow students to show multiple ways of arriving at a solution help them develop flexibility in their mathematical thinking and can deepen their understanding of spatial relationships.

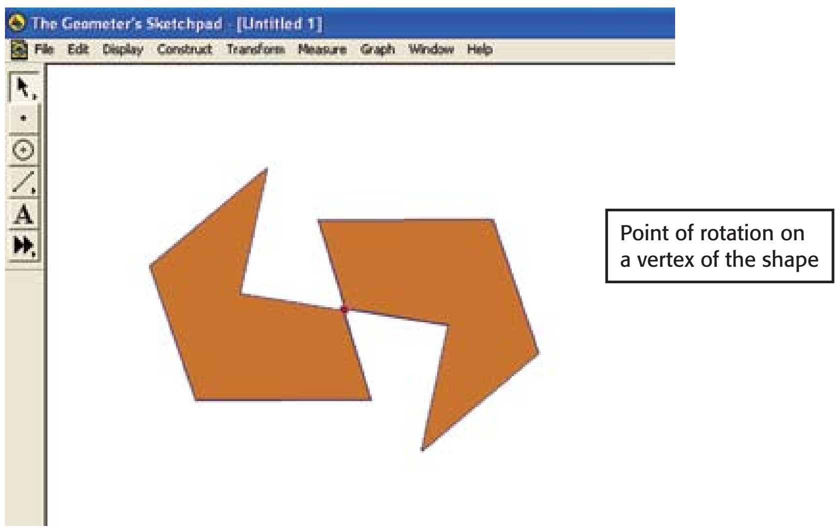
A rotation is a transformation that moves every point in a shape or figure around a fixed point, often called the *origin* or *point of rotation*.

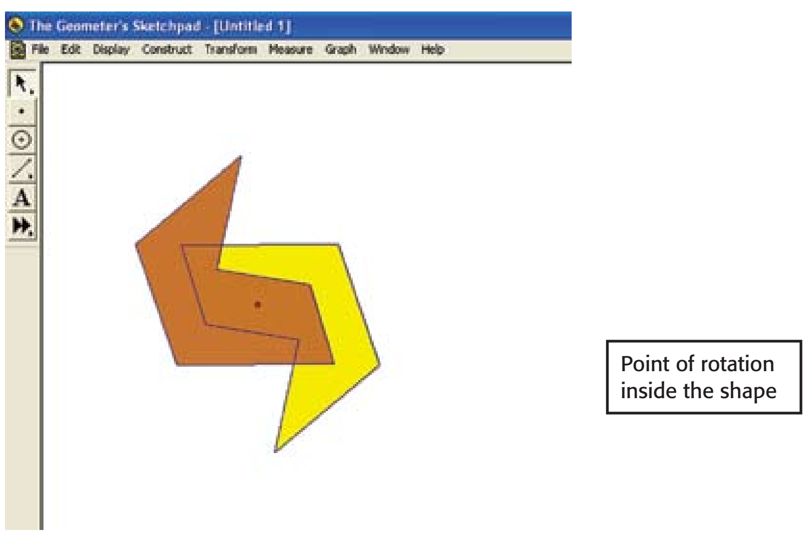


Students should use a number of different tools and resources to explore rotations. Simple tools, like a shape on a piece of paper and a pencil to hold it down at a rotation point, are excellent for initial investigations. As students begin to describe rotations by *degree of rotation* (i.e., fractions or degrees) more sophisticated tools, such as grids or dynamic geometry software, provide opportunities for deeper learning.

The Geometer's Sketchpad, for example, allows students to move the point of rotation and provides immediate feedback about the effect on the rotated image of moving the point of rotation.







The software also enables students to perform rotations of a specific measure (e.g., 45°). For example, in the following problem students predict and then determine how many rotations of a certain degree will be needed to transform an image back to the original shape.

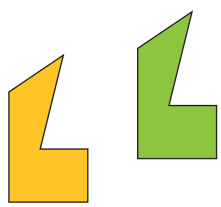
- Construct any shape and place a point of rotation outside the shape. Predict how many 45° rotations it will take before a rotated image will return to its original position.
- Perform repeated 45° rotations until the image returns to the original position. How many rotations did it take? Why do you think that is? How would the answer change if the rotations were 60°?

Congruence, Orientation, and Distance

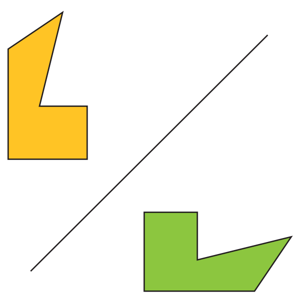
All three transformations (reflection, translation, rotation) have common characteristics. An original shape or object that is sometimes called the *pre-image* is transformed by an action or series of actions applied to the shape, and a resulting *image* is created. The image is directly related to the pre-image with respect to *congruence, orientation,* and *distance*.

Congruent objects are the same size and shape, and *orientation* can be thought of as an object's position or alignment relative to points of the compass or other specific directions.

Each of the three transformations preserves congruency – the original pre-image is always congruent to the image. Translations also preserve orientation between the original shape and its image, but rotations and reflections change the orientation of the original shape.



The yellow hexagon has been translated up and to the right. The resulting image is congruent to the original and has also maintained orientation – the "L" faces right, and the sharp point is on the top.



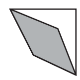
Here the hexagon has been reflected over a line. The resulting image is still congruent to the original shape, but its orientation has changed. The "L" is now facing up, and the sharp point of the hexagon points to the right.

Another important concept that comes from exploring the relationships between transformations is the idea of *distance*. Consider these probing questions:

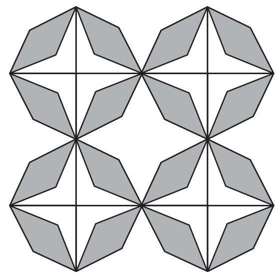
- "Measure the distance of points in the original shape and its reflected image to the mirror line. What do you notice?"
- "Compare the distances between the corresponding points of ΔCDE and its translated image. What can you tell me about the distances?"
- "Measure the distance from point A and from point A' to the point of rotation. How does this compare with other original and image points?"

Once students fundamentally understand the process of applying a transformation to a shape, they should begin to explore relationships between the transformations while considering orientation, congruency, and distance.

One way to explore these relationships is to create, analyse, and describe designs created by multiple transformations. Look at the design below and the original shape used to create the design. What transformations occurred in the creation of the design?



Original shape



Design

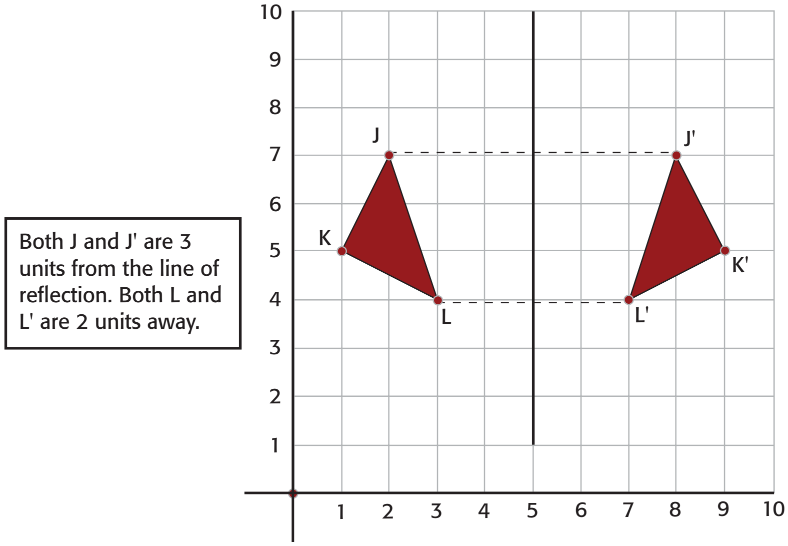
The generation of multiple responses allows for an examination of relationships between transformations. Providing students with investigations and explorations that allow for multiple solutions will also enable students with diverse learning needs and styles to be successful. An example would be to provide students with an original shape and its image, and ask them to identify two or more transformations that may have been applied to the pre-image to produce the resulting image. The number of transformations, as well as the sophistication of student responses, allows teachers to identify individual learning needs. By allowing students to share their responses, teachers provide a forum for students to construct their own learning.

Relationships Between Transformational and Coordinate Geometry

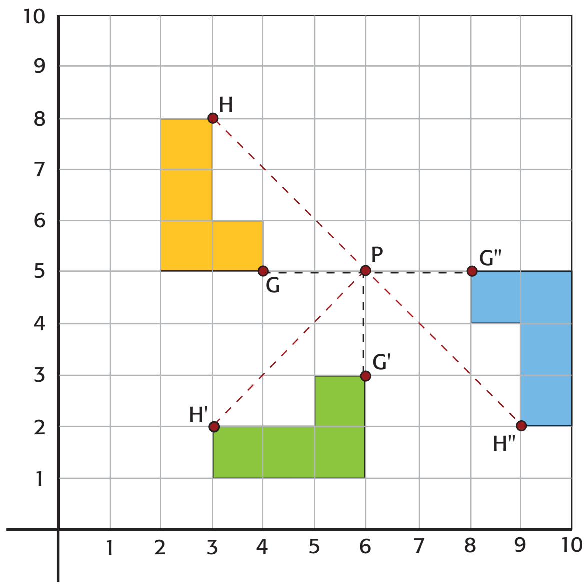
Although geometric learnings of location and movement are separate and distinct, it is important to help students discover connections between the two areas of geometry. Well-developed spatial sense, which is based on an awareness of the location of objects in one's surroundings, also encompasses knowledge of how objects can be moved about the space to create and organize space.

Using grid systems and coordinate systems as tools for exploring transformations enables students to make connections between location and movement, and provides a vehicle for describing resultant images. For example, students can use grid squares to describe a translation that has occurred, or can use grid squares to perform a translation and describe the position of the translated image. Using a coordinate plane, students can translate the vertices of a shape to create an image. Investigations can focus on translating shapes, identifying the translation that has taken place, and determining whether a translation has been accurately performed.

Coordinate systems can help students better understand how distance is preserved in transformations. When a point is reflected, the original point and its image are the same distance from the line of reflection.



Investigating the concept of distance in rotations is difficult to do unless a coordinate system is used. When a shape is rotated about a point, there is a direct relationship between the points of the pre-image and the points of the rotated image – corresponding points are an equal distance from the point of rotation.



In the figure above, the yellow hexagon was rotated first by a 1/4 turn to the left, resulting in the green hexagon. It was then rotated 1/2 turn to the right, resulting in the blue hexagon.

In all three shapes, the point G and its rotated image points G' and G" are 2 units away from the point of rotation P. Points H, H', and H" are 3 diagonal units away from the point of rotation P.

An effective means of investigating this property of distance is to provide students with a pre-image and its rotated image, and ask them to find the point of rotation.

When students use what they know about location to investigate the movement of shapes, they are likely to develop a greater understanding of both concepts. Although each topic can be learned separately, exploring the relationships between the two can help students develop greater spatial sense and awareness.

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Learning Activities

Introduction to the Learning Activities

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to Geometry and Spatial Sense. The learning activities also support students in developing their understanding of the big ideas outlined in the first part of this guide. Learning activities are provided for the following topics: two-dimensional geometry, three-dimensional geometry, location, and movement.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or task.

WORKING ON IT: In this part, students work on the mathematical task, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students' understanding of mathematical concepts.

HOME CONNECTION: This section is addressed to parents or guardians, and includes a task for students to do at home that is connected to the mathematical focus of the learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the activities and learning connections.

Grade 4 Learning Activity Two-Dimensional Shapes: Comparing Angles

This activity is adapted, with permission, from John A. Van de Walle, *Teaching Student-Centered Mathematics, Grades 3–5*. Toronto: Pearson: 2006, p. 254. The activity is also found in the Grades 5–8 book in the series.

OVERVIEW

In this learning activity, students use benchmark angles, pattern blocks, and unit angles as a reference to find angles smaller than, equal to, and larger than 90°. The focus is on gaining a visual reference for important benchmark angles.

BIG IDEAS

This learning activity focuses on the following big idea:

Properties of two-dimensional shapes and three-dimensional figures: Students develop a sense of the size of benchmark angles through investigation. Concrete materials and the classroom environment provide measuring tools for comparing angles to benchmarks.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- identify benchmark angles (e.g., straight angle, right angle, half a right angle), using a reference tool (e.g., paper and fasteners, pattern blocks, straws), and compare other angles to these benchmarks (e.g., "The angle the door makes with the wall is smaller than a right angle but greater than half a right angle.");
- relate the names of the benchmark angles to their measures in degrees (e.g., a right angle is 90°).

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- identify quadrilaterals and three-dimensional figures and classify them by their geometric properties, and compare various angles to benchmarks.

TIME: approximately 60 minutes

ABOUT THE LEARNING ACTIVITY

MATERIALS

- pattern blocks (1 set per each pair of students)
- overhead projector
- tagboard or index cards (1 piece per student)
- scissors (1 per small group of students)
- **2D4.BLM1: Pattern Block Angles Chart** (1 per student)
- **2D4.BLM2: Angle Search** (1 per student)

MATH LANGUAGE

- ray
- angle
- right angle
- obtuse angle
- acute angle
- unit angle

INSTRUCTIONAL GROUPING: whole class, pairs

INSTRUCTIONAL SEQUENCING

This learning activity provides for students an opportunity to reinforce their understanding of angle concepts and to develop an understanding of benchmark angles. It is intended as an introductory lesson, and students should have been introduced to the concept of angle prior to the lesson.

ABOUT THE MATH

Angles can be a difficult geometric concept for students, often because too much emphasis is placed on measuring angles and not enough on developing an understanding of angles. Comparing angles to benchmark angles enables students to focus on the angles themselves and not the measuring of angles.

One common misconception about angles is that degrees are needed to measure angles. A degree is no more than a small angle. Just as smaller units of length are used to measure a length or distance, the unit used for measuring an angle must also be an angle.

A physical model of an informal unit (such as a wedge of tagboard or pattern block) can help students visualize benchmark angles, as well as introduce them to the skills required in measuring angles. Using informal units makes it easier to focus directly on the attribute rather than the measuring procedure. It is important when measuring angles to begin with informal units and then move to degrees and formal measuring tools like protractors.

GETTING STARTED

An angle can be thought of as one line segment rotating away from the other. Use two strips of tagboard or cardboard joined with a paper fastener to form a vertex to demonstrate this idea. As you rotate one strip of the tagboard, the size of the angle is seen to get larger. Ask students what the space in between the two line segments is called. Explain to students that the distance between the two line segments is called an “angle” and that today they will be looking at different angles found in polygons and in the classroom.

- Show students an orange square pattern block, and then place it on the overhead projector. Trace the square. Ask a student to identify one angle. Trace the angle, drawing an arc to indicate the angle between the two line segments. Ask students what they know about this kind of angle. Define a square corner as a **right angle**. Elicit from the class the fact that all rectangles (including squares) have four right angles.
- Show students a green triangle pattern block, and then place it on the overhead projector. Trace the triangle and ask a student to identify one angle. Trace the angle, drawing an arc to indicate the angle between the two line segments. Repeat with the other two angles. Ask students how these angles compare to those found in the square. Define an angle that is smaller than a right angle as an **acute angle**.
- Show students a tan rhombus pattern block, and then place it on the overhead projector. Trace the rhombus. Ask a student to identify an angle larger than a right angle in the rhombus. Trace the angle, drawing an arc to indicate the angle between the two line segments. Define an angle that is larger than a right angle as an **obtuse angle**.

Create an anchor chart on the board or flip chart from the pattern block examples. Draw a right angle, an acute angle, and an obtuse angle. Invite students to make observations and name each angle.

WORKING ON IT

Ask the students, working as partners, to identify the number and types of angles found in the remaining three pattern blocks (trapezoid, blue rhombus, and hexagon). Ask them to use the angles they have found in the square, triangle, and tan rhombus as tools to help them.

Students are to communicate their findings in **2D4.BLM1: Pattern Block Angles Chart**. The chart includes:

- the pattern block
- the number and type of each angle found in the pattern block
- how they determined each angle

SAMPLE STUDENT STRATEGY

I used the square pattern block to help me find the angles in the trapezoid. The two top angles were outside the square, so they are bigger than 90°, and are obtuse. The two bottom angles are inside the square, so they are less than 90°, and are acute.

Next, give each student an index card or a small piece of tagboard. Have students use the acute angle from the tan rhombus pattern block to draw a narrow angle on the tagboard or the card and then cut it out. The resulting wedge can be used as a unit angle. Students will use this unit angle to find and identify three other angles found anywhere in the classroom. They will record their findings in the final three rows of the chart.

Students may use the pattern blocks to help them find relationships between the unit angle and other angles. For example, students may discover that it takes three unit angles to make a right angle.

REFLECTING AND CONNECTING

This activity results in an understanding of benchmark angles – in particular, the benchmark angle of 90°. Students also begin to develop an understanding of the measurement process – iterating a unit angle to measure a larger angle.

Ask students to explain their findings. Use probing questions like the following:

- "How did you decide what types of angles there are in each pattern block?"
- "What did you use as a comparison or benchmark angle?"
- "Did you have to do a comparison for every angle in each pattern block? Why or why not?"
- "Could you have used a different benchmark angle? Would that have made things easier or more difficult?"

Students should realize that using the square, or right angle, as a benchmark angle is the most efficient way of categorizing angles as acute, right, or obtuse.

Have students share their explorations of environmental angles and their explanations of how they used the tagboard unit angle to measure or compare angles. Ask questions like the following:

- "How did you use your unit angle?"
- "Did you have any difficulties using it to measure other angles?"
- "How could you describe the size of the angle you measured?"
- "How did you know if the angle you measured was obtuse, right, or acute?"
- "Did anyone use his or her unit angle in a different way?"
- "What were the most common types of angles you could find? Why do you think that is?"

Use the students' examples to emphasize the focus of the lesson – angles can be classified using a right angle as a benchmark. Angles larger than 90° are obtuse, and angles smaller than 90° are acute. In later grades, students will learn that a straight angle is 180°, but at Grade 4 it is enough to generalize about angles larger than 90°.

A unit angle that is iterated a number of times can help students visualize more exact benchmarks. Students may find that it takes about one and one-half of their unit angles to make a "half of a right angle", or that one unit angle is a third of a right angle. Even if they do not come to recognize the fractional representation, or to realize that the actual measure of the unit angle is 30°, it is important for them to be aware that it takes three unit angles to make a right angle.

ADAPTATIONS/EXTENSIONS

Students who struggle making comparisons may need to use a model of a right angle (made of tagboard) instead of the acute angle from the tan rhombus. Students can immediately see whether an angle is right, obtuse, or acute if they use a right angle as a benchmark.

As an extension, ask students to compare the kinds of angles they see most frequently. Encourage students to discuss why there are so many right angles in the classroom and environment.

ASSESSMENT

Observe students as they complete the chart, and assess how well they:

- identify benchmark angles (i.e., acute, right, obtuse) using a reference tool;
- compare angles to benchmark angles;
- classify and sort angles according to benchmarks;
- relate the names of the benchmark angles to their measures in degrees (e.g., a right angle is 90°).

The charts can be collected and assessed for students' use of appropriate terminology in communication.

HOME CONNECTION

Send home **2D4.BLM2: Angle Search**. In this task, students are asked to look for angles in their environment and to record where each angle was found, what kind of angle it is, and why the angle is important. Make sure students take their unit angles home for this activity, and read through the questions with them to clarify the task.

LEARNING CONNECTION 1
Quadrilateral Twister

MATERIALS

- **2D4.BLM3: Quadrilateral Twister Floor Mat**
- **2D4.BLM4: Quadrilateral Twister Cards**
- scissors

Prepare a floor mat, using **2D4.BLM3: Quadrilateral Twister Floor Mat** as a template. Prepare cards from **2D4.BLM4: Quadrilateral Twister Cards**, or have students prepare them.

Students work in groups of four. Each student in the group has a role:

- Card Manager
- Card Reader
- Player 1 and Player 2

The Card Manager shuffles the cards and holds them. The Card Reader draws a card and reads the properties to one of the players. Player 1 and Player 2 select the quadrilateral on the mat to match the card. The Card Reader verifies that each player has chosen the correct shape (the names of the shapes are on the bottoms of the cards). A player keeps the designated hand or foot on the selected quadrilateral until a card is drawn with instructions for that hand or foot to move elsewhere.

The game continues until one of the players can no longer reach a quadrilateral that matches his or her card. The pairs then change roles and a new game begins. The game may also be played with the players alternating cards, instead of each player trying to perform the same move.

LEARNING CONNECTION 2
Quadrilaterals Rule!

MATERIALS

- **2D4.BLM5: Quadrilateral Shape Cards**
- card stock
- paper cut into strips
- **2D4.BLM6: Quadrilaterals Rule Sorting Chart**

Students work in pairs for this activity.

This activity provides students with the opportunity to sort quadrilaterals and then discover sorting rules. The activity will focus the students' attention on the properties of two-dimensional shapes and the relationships between the shapes. Copy the shapes from the **2D4.BLM5: Quadrilateral Shape Cards** onto card stock and cut them out. Use strips of paper to make these statements:

- No right angles
- One or more right angles
- One or more acute (smaller than 90°) angles
- One or more obtuse (larger than 90°) angles
- All sides the same length
- Two pairs of adjacent sides congruent
- One set of parallel sides

Make blank strips available for students who want to write their own rule. One student sorts some of the shapes into the two spaces in the first column of **2D4.BLM6: Quadrilaterals Rule Sorting Chart**. The other student uses the sorting rule strips to determine the rule(s). Students take turns sorting and guessing.

LEARNING CONNECTION 3
Angle Add Up

MATERIALS

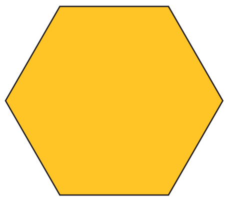
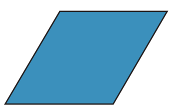
- 10 straws per group
- paper and pencil to record the scores

Students work in groups of two or three for this game. They use straws to create and identify straight, acute, obtuse, and right angles.

The first student holds ten straws, with the bottom of the straws touching the desk or table top. When the student opens his or her hand, the straws fall to create angles. Each straight angle (180°) is worth one point; obtuse angles (more than 90°) and acute angles (less than 90°) are both worth two points; and right angles (90°) are worth three points. Students take turns and record their scores for each round. The first student to gain 20 points wins the game.

The straws must be touching to form an angle – close doesn't count!

Pattern Block Angles Chart

Object Measured	Number and Type of Angles	How Do You Know?
		
		
		

Angle Search

Dear Parent/Guardian:

In math this week, we have been learning about angles. Your child has been measuring different angles in class, using a “unit” angle to determine if the angles are acute, right, or obtuse angles.

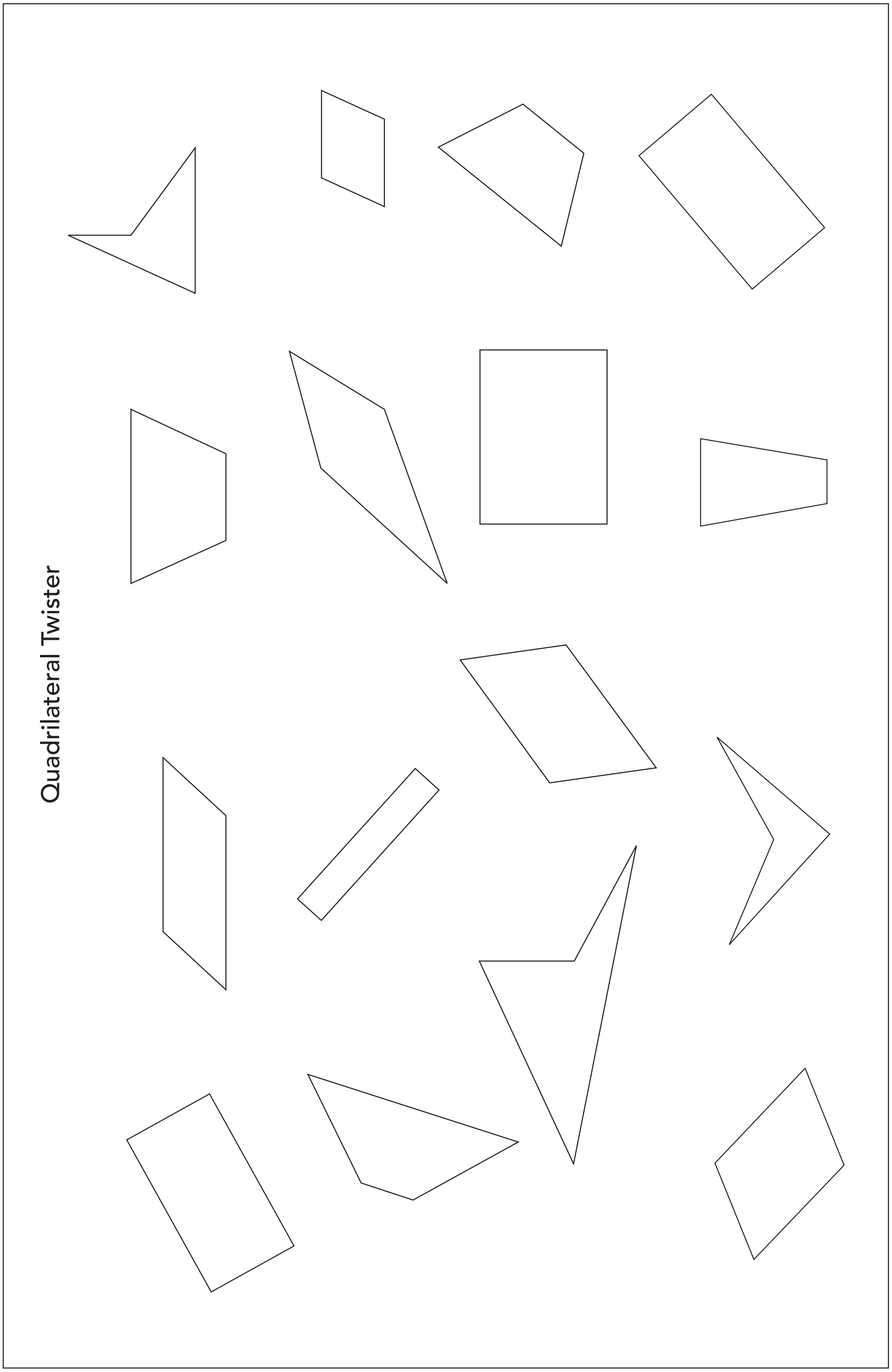
Here is an opportunity for your child to demonstrate what he or she has learned.

Ask your child to find different angles in your home. Using the chart below, describe where the angle was found, what kind of angle it is, and why the angle is important for that object. Try to find at least one example of each kind of angle (acute, right, obtuse).

Where was the angle found?	What kind of angle is it?	Why is the angle important?

Thank you for your assistance with this activity.

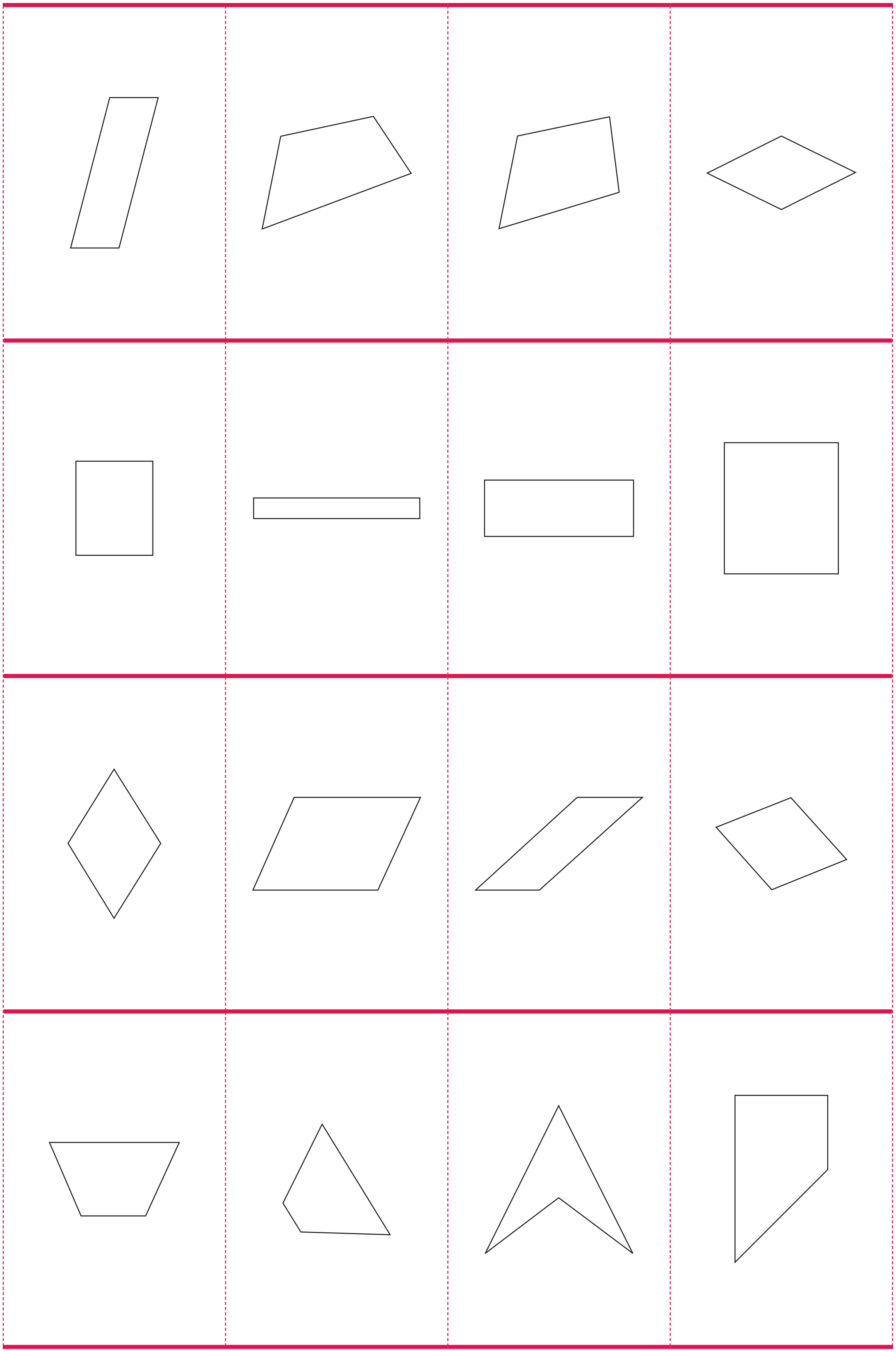
Quadrilateral Twister Floor Mat



Quadrilateral Twister Cards

Right hand on a shape that has: • two pairs of congruent adjacent sides Shape: kite	Right hand on a shape that has: • only one pair of parallel sides Shape: trapezoid	Right hand on a shape that has: • two pairs of parallel sides • four right angles Shape: rectangle	Right hand on a shape that has: • two pairs of parallel sides Shape: parallelogram
Left hand on a shape that has: • two pairs of congruent adjacent sides Shape: kite	Left hand on a shape that has: • only one pair of parallel sides Shape: trapezoid	Left hand on a shape that has: • two pairs of parallel sides • four right angles Shape: rectangle	Left hand on a shape that has: • four sides • no congruent sides Shape: quadrilateral
Right foot on a shape that has: • two pairs of congruent adjacent sides Shape: kite	Right foot on a shape that has: • only one pair of parallel sides Shape: trapezoid	Right foot on a shape that has: • two pairs of parallel sides • four right angles Shape: rectangle	Right foot on a shape that has: • two pairs of parallel sides Shape: parallelogram
Left foot on a shape that has: • two pairs of congruent adjacent sides Shape: kite	Left foot on a shape that has: • only one pair of parallel sides Shape: trapezoid	Left foot on a shape that has: • two pairs of parallel sides • four right angles Shape: rectangle	Left foot on a shape that has: • four sides • no congruent sides Shape: quadrilateral

QUADRILATERAL SHAPE CARDS



QUADRILATERALS RULE SORTING CHART

Quadrilaterals Rule!	
Sorting Space:	I think the rule is/rules are:
Sorting Space:	I think the rule is/rules are:

Grade 4 Learning Activity Three-Dimensional Figures: Construction Challenge

OVERVIEW

In this task, students work with Polydron pieces that represent the two-dimensional faces of three-dimensional figures. Using a given set of these shapes, students try to construct and name as many three-dimensional figures as they can. This task also provides students with opportunities to describe and classify prisms and pyramids by their geometric properties as they sort the three-dimensional figures constructed by each group.

BIG IDEAS

This learning activity focuses on the following big ideas:

Properties of two-dimensional shapes and three-dimensional figures: Students develop an understanding of the properties of prisms and pyramids through the use of two-dimensional shapes.

Geometric relationships: Students investigate the relationships between two-dimensional shapes and three-dimensional figures. They also develop an understanding of how different properties of three-dimensional figures (e.g., edges and faces) are related.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- identify and describe prisms and pyramids, and classify them by their geometric properties (i.e., shape of faces, number of edges, number of vertices), using concrete materials;
- construct three-dimensional figures (e.g., cube, tetrahedron), using only congruent shapes.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- construct three-dimensional figures, using two-dimensional shapes.

ABOUT THE LEARNING ACTIVITY

TIME: approximately 60 minutes

MATERIALS

- triangular prism and square-based pyramid solids (for teacher demonstration)
- 2 demonstration Polydron sets consisting of 4 equilateral triangles and 3 squares
- Polydron sets (for this activity, 1 set consists of: 6 equilateral triangles, 6 squares, 2 hexagons, and 2 pentagons) (1 set per group)

- **3D4.BLM1: 3-D Construction Challenge** (1 per pair of students)
- chart paper (1 per group of students)
- **3D4.BLM2a–b: What Figures Can You Construct?** (1 per student)

MATH LANGUAGE

- three-dimensional figure
- two-dimensional shape
- triangular prism
- parallel
- congruent
- square-based pyramid
- triangular pyramid
- cube
- rectangular prism
- pentagonal prism
- pentagonal pyramid
- hexagonal prism
- face
- edge
- vertex
- vertices

INSTRUCTIONAL GROUPING: whole class, small groups

INSTRUCTIONAL SEQUENCING

This lesson is intended to be delivered in the middle of a geometry unit on three-dimensional figures. Students should be familiar with the names of three-dimensional figures but do not need to have had significant experience with their properties. In this activity students investigate many of the properties of prisms and pyramids.

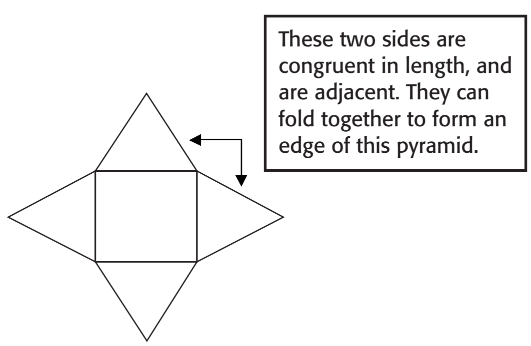
ABOUT THE MATH

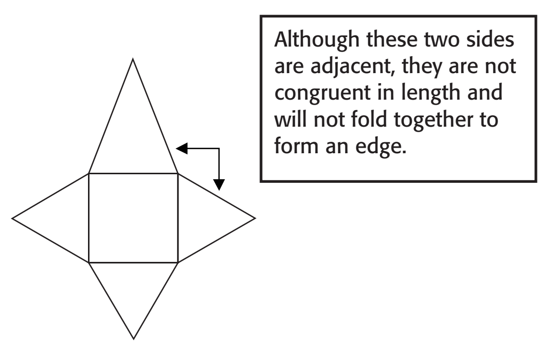
In Grade 4, students practise constructing three-dimensional figures from two-dimensional shapes. They build on their experience with prisms and investigate pyramids.

The focus of this lesson is identifying the similarities and differences among and between prisms and pyramids. Included in these are the following:

- All prisms have congruent parallel bases that are joined by rectangular faces.
- All pyramids have one base with triangular faces that meet at one common vertex (called an apex).
- The names of both prisms and pyramids are determined by the two-dimensional shapes that form their bases.

Working with two-dimensional shapes also develops an understanding of how various properties (e.g., edges and faces) are related. For example, two faces can share an edge only if the sides are adjacent and congruent in length.

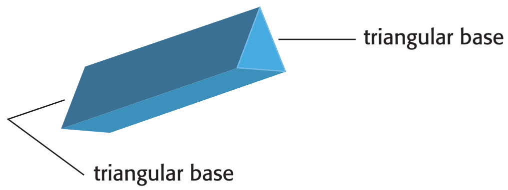




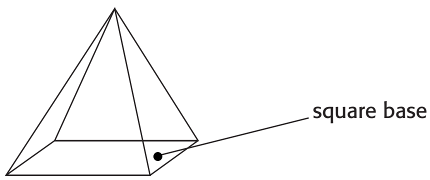
Students have opportunities to represent their ideas physically, share their thinking verbally, and record their ideas in writing. While they work in their groups, students apply problem-solving strategies to build as many different three-dimensional figures as they can and to test whether they have found all of the figures that they believe are possible.

GETTING STARTED

Show students a triangular prism. Ask them to name the three-dimensional figure and explain why it is called a triangular prism. Recall or explain that prisms are named according to their congruent parallel bases.



Show students a square pyramid. Again, ask students to name the figure and explain why it is called a square pyramid.



Allow students a few minutes to discuss with their elbow partner how prisms and pyramids are named. Elicit responses from the class, reiterating that the shape of the base determines the name of the pyramid or prism.

Introduce the following scenario:

> "A company that makes two-dimensional shapes is trying to sell a small set of the shapes as a new building toy. The two-dimensional shapes look like these Polydron pieces. (Show the following seven Polydron pieces: four equilateral triangles and three squares.) The company wants to be able to tell people about all the different three-dimensional figures that can be constructed with their new building set. The company has asked for your help in constructing all the three-dimensional figures that are possible from the two-dimensional shapes in the set, and asks you to record your findings."

Ask students to consider the seven Polydron pieces that you just showed them. "What three-dimensional figure can be constructed from these two-dimensional shapes?" Invite a student to come up to build 1 three-dimensional figure from some of the seven Polydron pieces and name it.

Ask: "Is it possible to build another three-dimensional figure from the same seven pieces?" Invite another student to build and name a second three-dimensional figure using the second set of the same seven Polydron pieces.

Ask the students to compare the 2 three-dimensional figures. "How are the two figures alike? How are they different?" (There are three possible figures that students could construct: a triangular prism, a triangular pyramid, or a square-based pyramid.) Encourage students to consider the properties of three-dimensional figures when comparing the two figures (i.e., number and shapes of faces, number of edges, number of vertices).

Students may recognize that a third three-dimensional figure can be constructed from the same seven Polydron pieces. Assure them that they will have an opportunity to construct any other figures that they believe consist of some of these seven faces when they begin working in their groups.

WORKING ON IT

Arrange students in groups of four. Distribute one copy of **3D4.BLM1: 3-D Construction Challenge** to each pair of students and one set of Polydron pieces (six each of equilateral triangles and squares, and two each of pentagons and hexagons) for each group.

Instruct students to work with a partner within their groups to build 1 three-dimensional figure from the two-dimensional shapes. Once both pairs have built one figure, they record the name of their figure on **3D4.BLM1: 3-D Construction Challenge** and share it with the other pair in their group. The two pairs then discuss and record the similarities of and differences between the 2 three-dimensional figures that they have just constructed.

Note: It is possible to build seven standard three-dimensional figures from the given Polydron set (triangular pyramid, triangular prism, square-based pyramid, cube, pentagonal prism, hexagonal prism, and triangular dipyramid made from six equilateral triangles). A pentagonal pyramid is possible in theory, but the Polydron edges will not fold far enough to construct a pentagonal pyramid without breaking apart. A hexagonal pyramid is not possible with equilateral triangles, because the angles of the six triangles that meet total 360° – the triangular faces would lie flat on the hexagonal base.

Assessment Opportunity

Circulate to assess students' problem-solving strategies as they try to build as many three-dimensional figures as possible from the set of two-dimensional shapes. If students believe that they have found all of the possible figures, ask, "How do you know?", to obtain further information about their problem-solving strategies.

Students continue to build and compare different three-dimensional figures (taking the two figures apart after each comparison and beginning again) until the group members believe that they have constructed all the three-dimensional figures possible, given the two-dimensional shapes provided.

Students may also construct non-standard three-dimensional figures by combining figures. For example, students may combine a triangular prism and a square-based pyramid by removing the square base of the pyramid and using one of the square faces of the prism as the base. The resulting three-dimensional figure is an octahedron (named for the number of faces – eight). Several such combinations can be constructed from the given set of Polydron shapes. Students should be encouraged to name these figures according to the number of faces in the figure.

REFLECTING AND CONNECTING

Select a few groups to choose 2 three-dimensional figures from those they have constructed to rebuild and share with the class. Ask group members to be prepared to share how the 2 three-dimensional figures they have chosen are alike and how they are different.

Have each group share its 2 three-dimensional figures and explain the similarities and differences. You may wish to do this orally as a large group, or have groups prepare a poster outlining the similarities and differences.

Display on the board ledge the three-dimensional figures constructed by each group. After the members of the second group have shared their figures, ask the students how they can sort the four figures now on the ledge. Record the sorting rule on the board, and have the students place each figure in a group according to the rule. (Students may create more than two groups, depending on what sorting rule is determined.)

> **Assessment Opportunity**
> *As students present the similarities and differences between their two shapes, assess their ability to express (represent) their thinking using the models and appropriate mathematical language.*

As each new group of students shares, have its members sort their two figures into the groups on the board ledge and explain why each figure belongs in the particular group chosen. Allow students to question the sorting if they disagree or do not understand the group members' reasoning. Allow groups to suggest a new sorting rule if it better emphasizes the properties of three-dimensional figures.

After all the groups have shared, review the sorting rule and the figures in each group. Ask:

- "Were any of the three-dimensional figures that you constructed not included in these groups?"
- "What is the three-dimensional figure and in which group does it belong? Why?"
- "What part of this investigation did you find easy? What part was challenging?"

Connect the activity with the original challenge, asking students if there are enough pieces in the set to create an appropriate number of three-dimensional figures. Ask students what pieces they might add to the set, and why.

ADAPTATIONS/EXTENSIONS

Some students may have difficulty finding the similarities of and differences between the 2 three-dimensional figures, especially if both figures are prisms or pyramids. Encourage these students to count the number of faces, edges, and vertices, and/or compare the shapes of the two-dimensional faces in each figure. You may wish to create/include an anchor chart that can serve as a starting point for students. List the above strategies – count the number of faces, and so forth.

Students requiring an extension can be challenged to construct two or more three-dimensional figures with a specified number of faces, edges, or vertices (e.g., construct two or more three-dimensional figures with 8 edges).

ASSESSMENT

Observe students to assess how well they:

- construct three-dimensional figures from two-dimensional shapes;
- identify and describe prisms and pyramids;
- classify prisms and pyramids by geometric properties

As students are constructing and comparing their figures, circulate around the room and ask students to describe the features of one figure and tell how it is similar to or different from another figure. Record anecdotal observations about the accuracy of their responses and their ability to communicate effectively.

HOME CONNECTION

Send home **3D4.BLM2a–b: What Figures Can You Construct?** In this task, students are given a set of two-dimensional shapes and asked to name 4 three-dimensional figures that can be constructed from those shapes. Suggest to students that they may wish to cut out the shapes and tape them together if it helps them visualize the figures.

LEARNING CONNECTION 1
Constructing Skeletons

MATERIALS

- **3D4.BLM3: Constructing Skeletons**
- round toothpicks (16 per pair of students)
- modelling clay (small block per pair of students)

Have students work in pairs to construct and name as many different three-dimensional skeletons as possible with the toothpicks and modelling clay provided.

Have students record the names of their constructions as well as the number of toothpicks and pieces of connective modelling clay required for each figure on **3D4.BLM3: Constructing Skeletons**.

LEARNING CONNECTION 2
Teach an Alien

MATERIALS

- rectangular prism (for teacher demonstration)
- **3D4.BLM4: Teach an Alien** (1 per student)
- three-dimensional solid figures (cube, triangular pyramid, triangular prism, square pyramid, pentagonal prism) for student reference

Begin by showing students a rectangular prism. Ask them to describe the three-dimensional figure in such a way that someone who has never seen one (e.g., a reading buddy, younger sibling) would know what it looks like. Encourage students to use appropriate mathematical language and to describe the geometric properties of the prism.

Distribute to each student a copy of **3D4.BLM4: Teach an Alien**. Have students select one net from the five shown and identify the three-dimensional figure that can be constructed from that net. Then, have students write a description of the figure for an alien from another planet who has never seen that figure before. Student pairs will exchange descriptions and try to build the figure from each other's descriptions.

LEARNING CONNECTION 3
Cube Structures

MATERIALS

- overhead projector
- transparency of **3D4.BLM5: 6-Cube Rectangular Prism**
- interlocking cubes (8 per student)
- **3D4.BLM6: More Rectangular Prisms** (1 per pair of students)

Begin by showing students the first prism, **3D4.BLM5: 6-Cube Rectangular Prism**, on the overhead.

Distribute 8 interlocking cubes to each student. Have students try to construct the 6-cube rectangular prism shown on the overhead.

Distribute **3D4.BLM6: More Rectangular Prisms** to each pair of students. Have students each construct the first prism and compare it with his or her partner's prism. When both students agree that their prisms are congruent and match the prism in the picture, they move on to construct the next rectangular prism.

Once they have accurately constructed all four of the rectangular prisms pictured, students are asked to work with their partners to construct as many different 12-cube rectangular prisms as they can.

LEARNING CONNECTION 4
Identify the Nets

MATERIALS

- **3D4.BLM7a–b: Identify the Nets** (1 per student)
- Polydron shapes

Begin by showing students a net for a triangular prism made from Polydron shapes. Ask students to identify the three-dimensional figure that will be created by folding the net. Next, show students an arrangement of the same Polydron shapes that will not fold to make a triangular prism. Ask students whether or not the arrangement is a net, and ask them to explain why or why not. When students have recognized (through visualization or folding) that the arrangement is not a net for a triangular prism, ask them how they could change it to make a net. Invite a student to come up to do so.

Distribute a copy of **3D4.BLM7: Identify the Nets** to each student. Have students work with a partner to determine which arrangements of two-dimensional shapes are nets (i.e., will fold to make a three-dimensional figure). Have Polydron shapes available for students for the activity. Alternatively, enlarge the page on a photocopier for those students who might need to cut out the arrangements and try folding them to determine whether or not they make three-dimensional figures.

If students determine that an arrangement is a net, have them identify the three-dimensional figure that the net will make. Then, have students redraw the arrangements that are not nets so that they become nets for rectangular or triangular prisms.

3-D Construction Challenge

Names: ____________________

Name of Your Figure	Name of Other Figure	Similarities	Differences

What Figures Can You Construct?

Dear Parent/Guardian:

In math this week, we have been learning about constructing three-dimensional figures from two-dimensional shapes. Your child has been using two-dimensional Polydron shapes in class to build three-dimensional models of figures.

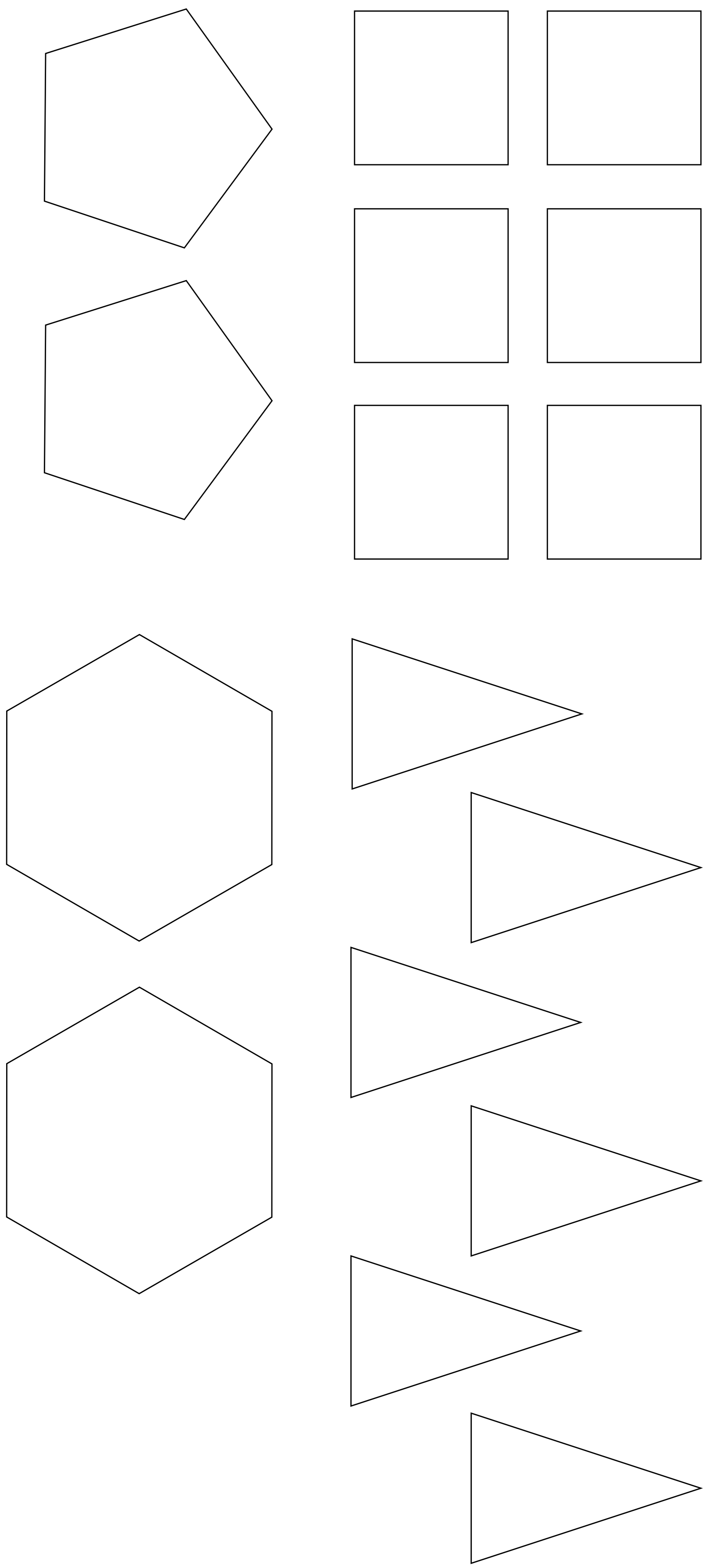
Here is an opportunity for your child to demonstrate what he or she has learned.

Ask your child to find at least 4 three-dimensional figures that can be constructed from the set of shapes pictured on the next page. The shapes may be cut out and taped together if it helps your child visualize the figures.

To verify the answers, ask your child to list all of the two-dimensional shapes that form the faces of each figure that he or she has named.

Thank you for your assistance.

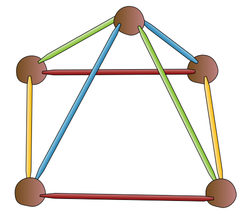
Name of three-dimensional figure	Numbers and shapes of all faces
cube	6 squares



Constructing Skeletons

Work with a partner.

Using a maximum of 16 toothpicks and 10 pieces of modelling clay for each skeleton, construct as many skeletons of three-dimensional figures as you can.



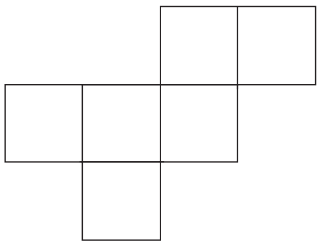
(You will need to take each skeleton apart after you have recorded the information about it in the table below.)

Name of Figure	Number of Edges (toothpicks)	Number of Vertices (pieces of modelling clay)

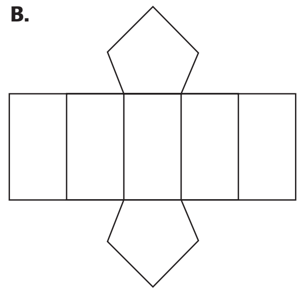
Teach an Alien

1. Choose one of these nets of three-dimensional figures.

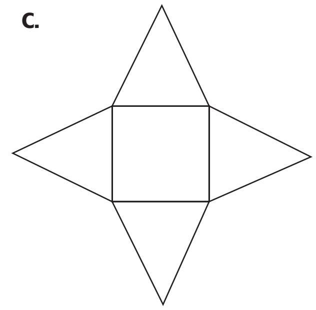
A.



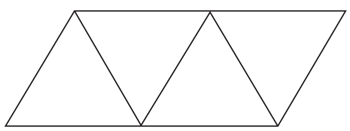
B.



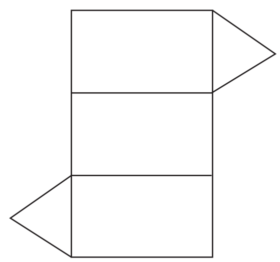
C.



D.



E.



2. Identify the figure.

   Net _____ will fold to make a ____.

3. Write a description of the figure for an alien from another planet who has never seen the figure before.

____

____

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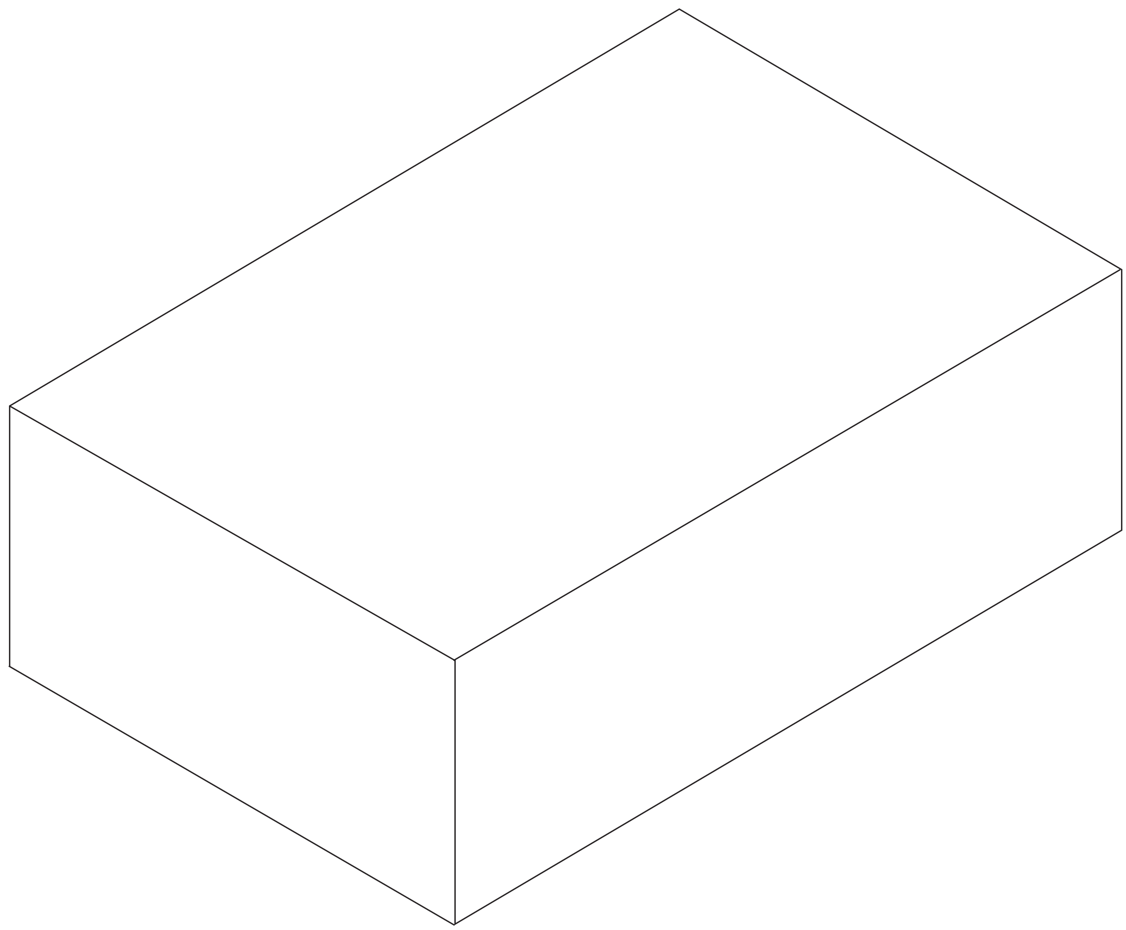
6-Cube Rectangular Prism

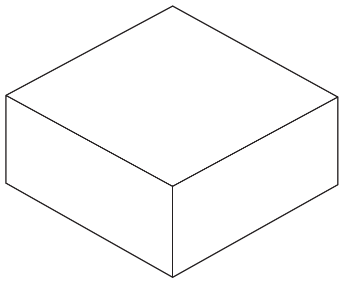
Use 6 connecting cubes to construct a rectangular prism.



More Rectangular Prisms

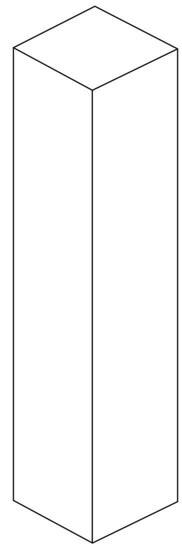
Construct each of the following rectangular prisms, using the number of connecting cubes indicated. Compare your completed prism with your partner's prism. Once you and your partner agree that the two prisms are congruent and match the prism in the picture, build the next prism.

A.



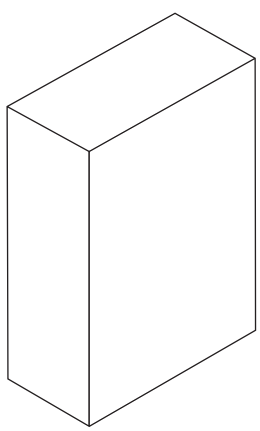
Use 4 cubes.

B.



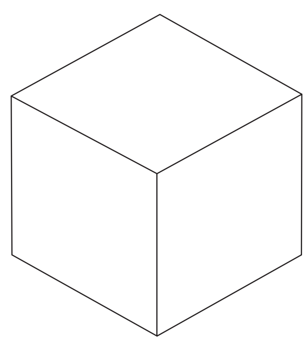
Use 5 cubes.

C.



Use 6 cubes.

D.



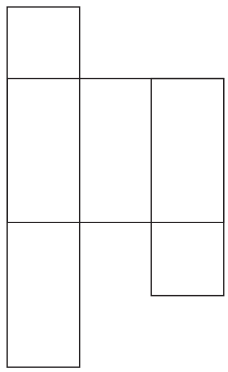
Use 8 cubes.

Work with your partner to construct as many different 12-cube rectangular prisms as you can.

Identify the Nets

Determine whether each of the arrangements of two-dimensional shapes below is a net. If it is a net, write the name of the three-dimensional figure that can be made through folding.

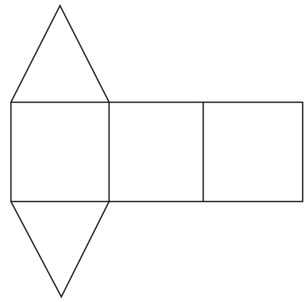
A.



❑ Net ❑ Not a net

3-D Figure: __________

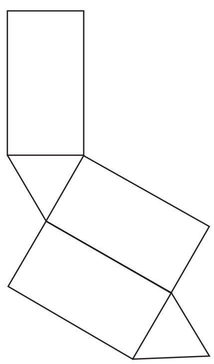
B.



❑ Net ❑ Not a net

3-D Figure: __________

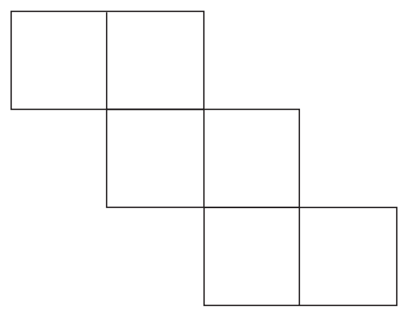
C.



❑ Net ❑ Not a net

3-D Figure: __________

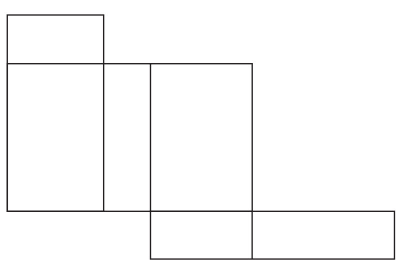
D.



❑ Net ❑ Not a net

3-D Figure: __________

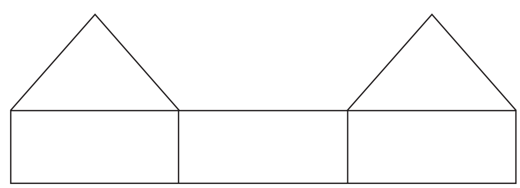
E.



❑ Net ❑ Not a net

3-D Figure: __________

F.



❑ Net ❑ Not a net

3-D Figure: __________

For the arrangements that you checked as "Not a net", redraw the shapes below so that the arrangements are nets for rectangular or triangular prisms.

Grade 4 Learning Activity Location: Check Mate

OVERVIEW

In this learning activity, students play a game in which they determine the location of four rectangles on a coordinate grid. The game provides an opportunity for students to identify the location of objects using a grid system.

BIG IDEAS

This learning activity focuses on the following big idea:

Location and movement: Students describe the position of an object, using a coordinate grid system.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.

Students will:

- identify and describe the general location of an object using a grid system (e.g., "The library is located at A3 on the map.")

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- identify and describe the location of an object, using a grid map, and reflect two-dimensional shapes

ABOUT THE LEARNING ACTIVITY

TIME: approximately 60 minutes

MATERIALS

- Transparency of the "Your Shapes" grid from **Loc4.BLM1a–b: Game Boards**
- overhead projector
- large copy on chart paper or the board of the "Your Opponents' Shapes" grid from **Loc4.BLM1a–b: Game Boards**,or a learning carpet or 10 × 10 grid on a carpet and craft sticks
- **Loc4.BLM2: Game Instructions** (1 per student)
- **Loc4.BLM1a–b: Game Boards** (1 per student)
- sheets of chart paper or large newsprint (1 per pair of students)
- markers (1 per student)
- **Loc4.BLM3: Playing Check Mate** (1 per student)

INSTRUCTIONAL GROUPING: pairs

MATH LANGUAGE

- grid
- cell
- location
- inside
- outside
- rectangle
- square
- hexagon
- trapezoid
- triangle

INSTRUCTIONAL SEQUENCING

This learning activity provides an introduction to the use of a grid system. Students locate objects within cells on a grid. Before starting this activity students should review the different types of quadrilaterals and their properties.

ABOUT THE MATH

The ability to use a grid system in describing the location of an object is an important component of spatial reasoning. This activity builds on prior knowledge developed throughout the location and movement activities established in the Geometry and Spatial Sense document for the primary grades, and prepares students for the more formal identification of points using a coordinate grid system.

GETTING STARTED

Prior to the lesson, mark four rectangles on an overhead transparency of the "Your Shapes" grid from **Loc4.BLM1a–b: Game Boards**, making at least one of them a square. Students will attempt to locate the four rectangles while you keep them hidden.

On a large sheet of paper or on the board, make a copy of the "Your Opponents' Shapes" grid from **Loc4.BLM1a–b: Game Boards** as a reference for your students. Review how to read across the bottom of the grid (naming a letter) before reading up the grid (naming a number) to identify a cell.

Tell your students that on your overhead grid you have hidden four rectangles for them to find. Select one student to be the "marker" on the large grid. The other students take turns to call out locations, at each turn using a letter followed by a number. After a student has identified a cell on the grid, call out "Check" if any part of a rectangle occupies that cell or "Miss" if the cell lies outside the rectangles. The marker will then place on the grid a check mark for a hit or an X for a miss. When the students have located all of the check marks for a particular rectangle, say, "Check Mate".

Alternatively, the checks and X's could be recorded on a learning carpet or 10 × 10 grid on a carpet, with craft sticks used for making the marks.

Reveal your overhead transparency, indicating how the check marks and X's match those on the large grid. One of the students may tell you that you have drawn a square rather than a rectangle. It is an ideal opportunity for a class review of the square as a special rectangle.

Provide each student with a copy of **Loc4.BLM2: Game Instructions**, and review the instructions for playing the game.

WORKING ON IT

Have the class work in groups of four (two versus two) to allow for discussion and collaboration while playing the game. Provide each student with a copy of **Loc4.BLM1a–b: Game Boards**. As students play the game, observe the various strategies they use. Pose questions to help students think about their strategies:

- "What strategy are you using to hide your rectangles?"
- "What strategy are you using to search for your opponents' rectangles?"
- "What strategy do you use once you get a 'check'?"
- "Did you change any of your strategies? Why?"

When students have had a chance to play the game a number of times, provide each pair of students with markers and a sheet of chart paper or large newsprint. Ask students to record their game-playing strategies on the paper to show how they hid their rectangles and found their opponents' rectangles.

Make a note of pairs who might share their strategies during the Reflecting and Connecting part of the lesson. Include groups whose methods varied in their degree of efficiency (e.g., hiding rectangles at random, hiding rectangles in specific locations and orientations, making guesses at random, using a specific pattern to guess).

REFLECTING AND CONNECTING

Once students have had several opportunities to play Check Mate, bring them together to share their ideas and strategies. Try to order the presentations so that students observe inefficient strategies (picking cells at random) first, followed by more efficient methods.

As students explain their ideas, ask questions to help them to describe their strategies:

- "How does your strategy work?"
- "Did you change your original strategy?"
- "Did you know when you would get a second check mark?"
- "Would you change your strategy for hiding rectangles next time?"

Following the presentations, ask students to observe the work that has been posted and to consider the efficiency of the various strategies. Ask:

- "Which strategy, in your opinion, is an efficient strategy for hiding rectangles?"
- "Which strategy, in your opinion, is an efficient strategy for finding rectangles?"
- "How would you explain the strategies to someone who has never used them?"

Avoid commenting that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students' work to emphasize geometric ideas:

- A grid system can be helpful in locating specific objects.
- The properties of a two-dimensional shape can help you predict where the shape will be on a grid (e.g., the number of sides in a rectangle).

Provide students with an opportunity to play additional or related games to try new strategies.

ADAPTATIONS/EXTENSIONS

For those students having difficulty with the concept, working on a smaller grid (e.g., 5 × 5), with only one rectangle and one square on the grids may be helpful.

Students needing a challenge could use the hexagon, trapezoid, triangle, and square pattern blocks. They draw around these shapes on their grid paper, making sure that they only draw around full squares or half squares.

The challenge is greater than in "Check Mate" because when a student "hits" a half square, the person with the drawing must say, "Top left", "Top right", "Bottom left", or "Bottom right" to indicate that a half square has a check mark placed in it.

ASSESSMENT

Observe students as they play the game and assess how well they:

- locate and mark specific cells on the grid;
- apply appropriate strategies to play the game;
- explain their strategy;
- judge the efficiency of various strategies;
- modify or change strategies to find more efficient ways to solve the problem.

HOME CONNECTION

Send home **Loc4.BLM3: Playing Check Mate**. In the letter, parents are told about the game played in class and asked to have their child teach them how to play the game. Photocopy **Loc4.BLM2: Game Instructions** onto the back of this letter. Attach two copies of **Loc4.BLM1a–b: Game Boards**, so that parent and child have their own playing sheets.

LEARNING CONNECTION 1
Hot, Warm, Cold

MATERIALS

- Overhead transparency of the "Your Shapes" grid from **Loc4.BLM4: Instructions and Game Boards for Hot, Warm, Cold**
- Large copy of the "Your Opponents' Shapes" grid from **Loc4.BLM4: Instructions and Game Boards for Hot, Warm, Cold**
- **Loc4.BLM4: Instructions and Game Boards for Hot, Warm, Cold** (1 per student or pair of students)

This game builds on the concept of locating cells on a grid and reviews the names of the five quadrilaterals in the Grade 4 geometry expectations. The aim of the game is to be the first to locate all five quadrilaterals "hidden" by the other team. Students will benefit from a review of the quadrilaterals and their properties before beginning this activity.

Prior to the lesson, on an overhead transparency of the "Your Shapes" grid from **Loc4.BLM4: Instructions and Game Boards for Hot, Warm, Cold**, follow the directions for marking the five different quadrilaterals.

On a large sheet of paper, or on the board, make a copy of the "Your Opponents' Shapes" grid from **Loc4.BLM4: Instructions and Game Boards for Hot, Warm, Cold**.

Play a practice game. Select one student to be the "marker" on the large grid. The other students take turns calling out a location on the grid (e.g., D3). When the students miss a quadrilateral, the marker places an X in the cell named, and you provide clues (as indicated on **Loc4.BLM4: Instructions and Game Boards for Hot, Warm, Cold**). When the students have a hit, say the name of the quadrilateral, and have the marker draw that shape in the cell named.

Mark on the concealed overhead transparency each of the locations that the students name, saying and drawing the shape of the quadrilateral when that shape is hit, and putting an X in a cell for a miss. The overhead transparency can then be compared with the large grid at the end of the game.

Review the instructions on the blackline master for playing the game.

Have the class play the game in two's or four's. Pairing the students will allow for discussion and collaboration. (When students are playing the game for the first time, it might be worth-while to pair a weaker student with a stronger one.) Each individual or pair receives a copy of **Loc4.BLM4: Instructions and Game Boards for Hot, Warm, Cold** (a "Your Shapes" grid for drawing their quadrilaterals and recording the hits and misses of the other team, and a "Your Opponents' Shapes" grid for recording their own hits and misses).

LEARNING CONNECTION 2
Know Your Neighbourhood

MATERIALS

- gridless map of the school neighbourhood or of a town or city (1 per student)

Have students find landmarks on the map and describe the location of each. Lead a discussion about the locations of the landmarks on the map and the difficulties encountered in describing those locations to others.

Challenge the students to think of ways in which they could make describing locations on the map easier. Have them record their ideas on their map, or on separate pieces of paper.

Observe students as they solve the problem, and make note of the different methods that they use. For example, students might:

- create a legend to designate landmarks;
- circle landmarks on the map;
- create a grid system to identify cells on the map.

Ask several students to explain their strategies to the class. Include a variety of strategies. After a variety of strategies have been presented, ask the following questions to help students evaluate the different methods.

- "Which strategies were easy to use and efficient? Why?"
- "Which strategies are similar? How are they alike?"
- "What strategy would you use if you were to solve this problem again? Why?"

Loc4.BLM1a

Game Boards

Your Shapes

10										
9										
8										
7										
6										
5										
4										
3										
2										
1										
	A	B	C	D	E	F	G	H	I	J

Your Opponents' Shapes

	A	B	C	D	E	F	G	H	I	J
10										
9										
8										
7										
6										
5										
4										
3										
2										
1										

Game Instructions

Instructions for Playing "Check Mate"

1. Making your own game board

Each person draws four rectangles on his or her "Your Shapes" grid, making at least one of them a square. Your team needs to make sure that your rectangles are in the same place.

Rectangles must not overlap and need to be traced over the grid lines. Teams must not see their opponents' grids.

2. Playing the game

On each team, each person has a copy of the game boards to keep track of the guesses in the game.

Each team takes turns naming a location where they think their opponents may have drawn a rectangle. Remember: **name the letter before the number**. Each person records hits (√) or misses (X) on the "Your Opponents' Shapes" grid to keep track of the team's guesses.

Each person marks on the "Your Shapes" game board the hits and misses made by the opponents. If your opponents name a position inside one of your team's rectangles, say, "Check", and make a check mark inside that cell. If your opponents name a position outside your team's rectangles, say, "Miss", and make an X inside that cell. When the other team completely fills one of your team's rectangles, say, "Check Mate". Be sure to mark all hits and misses, so that your team and your opponents' team can compare game boards at the end of the game.

3. Ending the game

When one team has check marks inside all of the other team's rectangles, the game is over.

The teams review each other's sheets to make sure that no mistakes have been made.

Playing Check Mate

Dear Parent/Guardian:

In class we have been playing a game called Check Mate. A game sheet and instructions for playing the game are included, but please have your child explain the game to you, so that you know how well your child understands the game and the geometry concepts that are necessary for playing the game well.

When you were younger, you might have played games similar to this – for example, Battleship. If you and your child enjoy Check Mate, similar games are available in stores and on the Internet.

Have fun playing the game, and please contact me with any questions.

Thank you for your help.

GAME INSTRUCTIONS

1. Each player needs two grids, one titled "Your Shapes" and the other titled "Your Opponents' Shapes".
2. Each player needs to draw four rectangles on the "Your Shapes" grid. Rectangles must not overlap and need to be traced on the grid lines. Do not show the location of these rectangles to your opponent.
3. The players take turns naming locations where they think their opponent may have drawn a rectangle. Remember: **name the letter before the number**. Each player records a hit or a miss on the "Your Opponents' Shapes" grid. This will help you remember your guesses.
4. If your opponent names a position inside one of your rectangles, say "Check", and mark the location (√) on your "Your Shapes" grid. Your opponent will place a check in that location on his or her "Your Opponents' Shapes" grid. If your opponent names a position outside your rectangles, say "Miss", and mark an X in that location on your "Your Shapes" grid. Your opponent will mark an X in that cell on his or her "Your Opponents' Shapes" grid. When the other player completely fills one of your rectangles, say, "Check Mate".
5. Take turns naming locations on each other's grids. When one player has check marks inside all of the other player's rectangles, the game is over. Review each other's sheets to make sure that no mistakes have been made.

Instructions and Game Boards for Hot, Warm, Cold

This game may be played by individuals or pairs.

Draw five quadrilaterals on your grid: a rectangle, a trapezoid, a parallelogram, a rhombus, and a square. Make sure that each quadrilateral fits inside one cell.

The pair (or person) playing against you will call out a location, such as D3.

If they hit one of your shapes, say, "Hit", and tell them the name of the shape.

If they miss one of your shapes, say, "Miss", but provide a clue for the nearest shape – for example:

- If they miss by **one square**, say something like, "Hot, go to the right" or "Hot, go down".
- If they miss by **two squares**, say something like, "Warm, go to the left" or "Warm, go up ".
- If they are **three or more squares** away from any shape, say, "Cold".

Then it is your turn to call out a location and receive clues.

The first pair (or person) to hit all five of the other pairs' quadrilaterals is the winner.

Your Shapes

6						
5						
4						
3						
2						
1						
	A	B	C	D	E	F

Your Opponents' Shapes

6						
5						
4						
3						
2						
1						
	A	B	C	D	E	F

Grade 4 Learning Activity Movement: Hit the Target

OVERVIEW

In this learning activity, students play a game in which they determine the line of reflection needed to reflect a shape onto a target. The game provides an opportunity for students to identify and perform reflections and make predictions, and reinforces the concept of using a grid system to describe the location of an object.

BIG IDEAS

This learning activity focuses on the following big idea:

Location and movement: Students develop an understanding of how to use different transformations in describing the movement of an object.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- identify and describe the general location of an object using a grid system (e.g., "The library is located at A3 on the map.");
- identify, perform, and describe reflections using a variety of tools (e.g., Mira, dot paper, technology).

These specific expectations contributes to the development of the following **overall expectation**.

Students will:

- identify and describe the location of an object, using a grid map, and reflect two-dimensional shapes.

ABOUT THE LEARNING ACTIVITY

TIME: approximately 60 minutes

MATERIALS

- Miras (1 per pair of students)
- rulers (1 per pair of students)
- **Mov4.BLM1: Design Reflection** (1 per student)
- **Mov4.BLM2: Hit the Target** (1 per pair of students)
- sheets of chart paper or large newsprint (1 per pair of students)
- markers (1 per pair of students)
- **Mov4.BLM3: Scavenger Hunt** (1 per student)

MATH LANGUAGE

- reflection
- congruent
- symmetry
- predictions
- Mira
- triangle
- square
- rectangle
- rhombus
- hexagon
- circle
- pentagon
- parallelogram

INSTRUCTIONAL GROUPING: pairs

INSTRUCTIONAL SEQUENCING

This learning activity provides an introduction to the term *symmetry* and the use of a line of reflection to create symmetrical designs. Before beginning this activity, students should review the meaning of *congruence*.

ABOUT THE MATH

Rich and appropriate experiences help children connect mathematics with their everyday world and develop the spatial reasoning skills necessary to be successful in and out of school. This activity provides opportunities for students to represent and "see" the mathematical ideas associated with reflections.

GETTING STARTED

Have students draw a design to the left or the right of the line of reflection on **Mov4.BLM1: Design Reflection**. Tell students that the design must touch the line in some way. When they have finished, they should make a reflection drawing on the opposite side of the line. They should use a Mira or fold the paper to check their work. Lead a brief class discussion to consider the following questions:

- "What strategies did you use to create the reflection?"
- "How can you tell if your second drawing is a reflection?"
- "What is symmetry?"
- "Would you do anything different the next time?"

Introduce **Mov4.BLM2: Hit the Target** and explain to the students the rules on it.

WORKING ON IT

Have students play the game in groups of four (two versus two) to allow for discussion and collaboration. Provide each student pair with a copy of **Mov4.BLM2: Hit the Target**, a ruler, and a Mira. As students play the game, observe the various strategies they use. Pose questions to help students think about their strategies:

- “What strategy did you use to decide where the line would go?”
- “Did you change your strategy? Why?”
- “Did you use the grid as part of your strategy?”

When students have had a chance to play the game, provide each pair with markers and a sheet of chart paper or large newsprint. Ask students to record their strategies on the paper to show how they chose the location for their line of reflection.

Make a note of pairs who might share their strategies during the Reflecting and Connecting part of the lesson. Include groups whose methods varied in their degree of efficiency (e.g., using guess and check, finding a cell halfway between the edge of the shape and the target, finding the cell halfway between the centre of the shape and centre of the target).

REFLECTING AND CONNECTING

Once students have had the opportunity to play the game and record their strategies, bring them together to share their ideas. Try to order the presentations so that students observe inefficient strategies (e.g., picking at random) first, followed by more efficient methods.

As students explain their ideas, ask questions to help them to describe their strategies:

- "How does your strategy work?”
- “Did you change your original strategy? Why?”
- "Did you use the grid in your strategy? How?”

Following the presentations, ask students to observe the work that has been posted and to consider the efficiency of the various strategies. Ask:

- “Which strategy, in your opinion, is an efficient strategy for finding the line of reflection?”
- “How would you explain the strategy to someone who has never used it?”

Avoid commenting that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students’ work to emphasize geometric ideas:

- A grid system can be helpful in locating specific objects.
- A reflection is the same distance away from the line of reflection as the original object.

Provide for students an opportunity to play additional or related games to try new strategies.

ADAPTATIONS/EXTENSIONS

Those students having difficulty with the concept may benefit from working on an enlarged copy of the game showing one shape and the target.

Students could use plain grids, pattern blocks instead of shapes, and a piece of string as the target. They could reflect the pattern blocks onto the target from different locations.

ASSESSMENT

Observe students as they play the game and assess how well they:

- locate and mark lines of reflection on the grid;
- apply appropriate strategies to play the game;
- explain their strategy;
- judge the efficiency of various strategies;
- modify or change strategies to find more efficient ways to solve the problem.

HOME CONNECTION

Send home **Mov4:BLM3: Scavenger Hunt**. This Home Connection task provides an opportunity for parents and students to discuss reflections and symmetry.

LEARNING CONNECTION 1
Symmetry in the Real World

MATERIALS

- **Mov4.BLM4: Real-World Symmetry** (1 per student)
- overhead transparency of **Mov4.BLM4: Real-World Symmetry**
- Miras (1 per pair of students)
- scissors (1 per pair of students)

Explain that real-world shapes rarely have perfect symmetry and that in this activity being close is fine. The discussion about symmetry, rather than the correct answer, is the important component of this learning connection.

Provide each student with a copy of **Mov4.BLM4: Real-World Symmetry** and have each student write down the number of lines of symmetry that he or she predicts is in each representation of a real-world shape pictured in the blackline master. The students should make their predictions by looking at the shapes and visualizing the lines of symmetry. They should not use Miras, paper folding, or any other means to assist their predictions.

Use an overhead transparency of **Mov4.BLM4: Real-World Symmetry**, and ask students to share their predictions for each shape and their reasons for making them. Be sure to elicit whether there is more than one prediction for any given shape, and ask the reasons for alternative predictions. Have the students refine their predictions on the basis of feedback from their peers, and take a "thumbs up" vote to decide on the number of lines of symmetry for each real-world shape. Next to each of the shapes, record the number of lines of symmetry that the majority of students voted for.

Discuss ways to check for lines of symmetry. Then hand out the Miras. Ask the students to check for lines of symmetry by using the Miras and also by cutting out the shapes and folding them.

After the students have verified the lines of symmetry by various means, have a discussion about their answers. Deal with the shapes that created the most controversy regarding the number of their lines of symmetry.

LEARNING CONNECTION 2
Carmen's Carpets

MATERIALS

- sample symmetrical tile, carpet, or wallpaper design
- overhead projector
- overhead transparency of **Mov4.BLM5: Carmen's Carpets**
- **Mov4.BLM5: Carmen's Carpets** (1 per student)

Have a picture or an example of a symmetrical tile, carpet, or wallpaper design. Lead a brief class discussion about the use of symmetry in wallpaper, tile, and carpet designs. Display the overhead transparency of **Mov4.BLM5: Carmen's Carpets**. Discuss the problem presented on the overhead transparency.

Provide each student with a copy of **Mov4.BLM5: Carmen's Carpets**, and have students work on the solution. Observe them as they solve the problem, and make note of the different methods they use to determine the number of lines of symmetry in the shapes. For example, students might use:

- a ruler to measure
- paper folding
- a Mira
- a pattern or rule to determine the number of lines

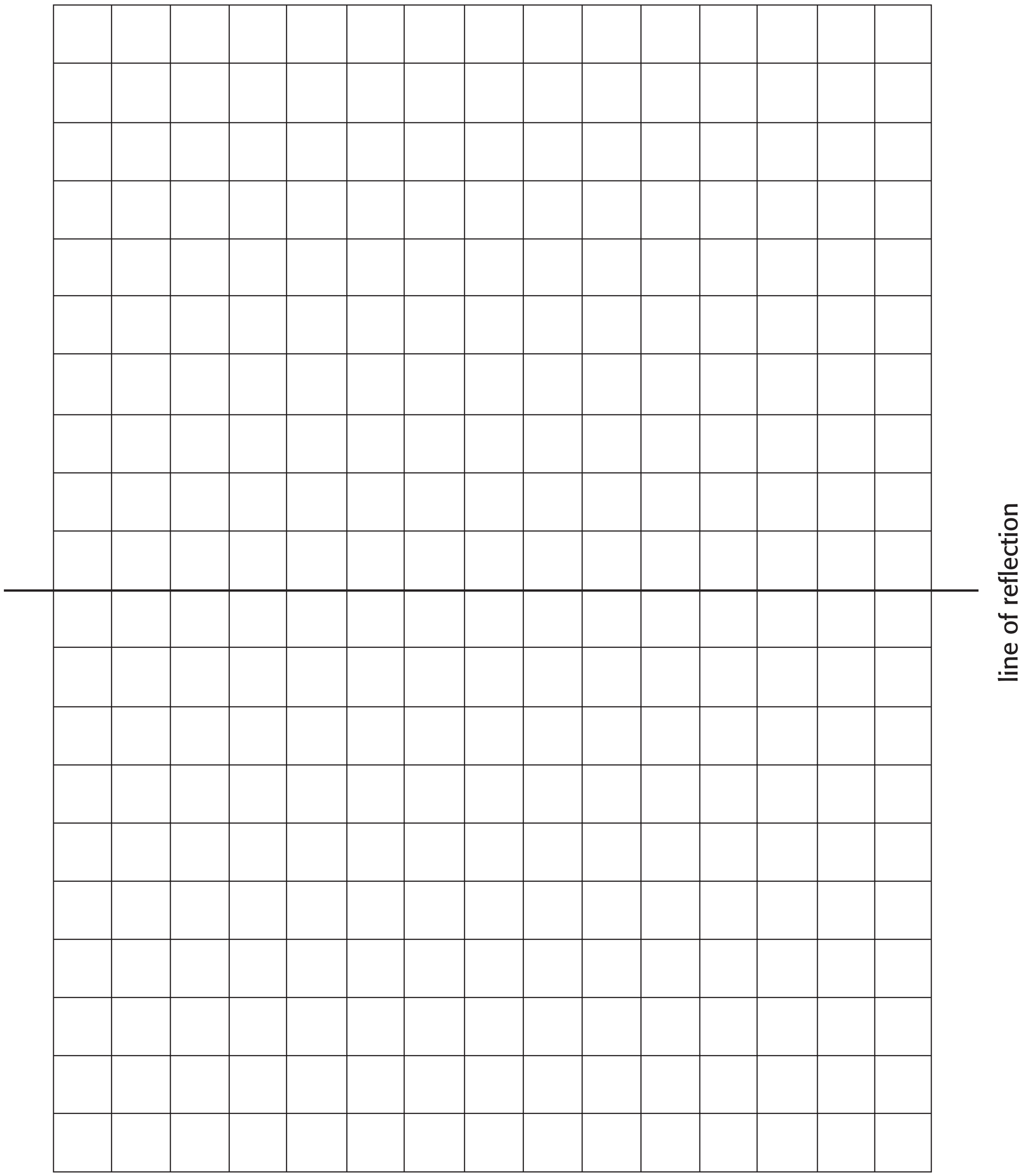
Some students may jump to the conclusion that the number of sides is equal to the number of lines of symmetry. Ask other students if they feel this rule is true for all the shapes. Through discussion and further investigation, students may discover that this is true only of regular polygons, such as the square, the equilateral triangle, the regular pentagon, and the regular hexagon.

Ask several students to explain their strategies to the class. Include a variety of strategies – for example, using paper folding, using a Mira, and using a pattern if applicable.

After a variety of strategies have been presented, have students evaluate the different methods by asking the following questions:

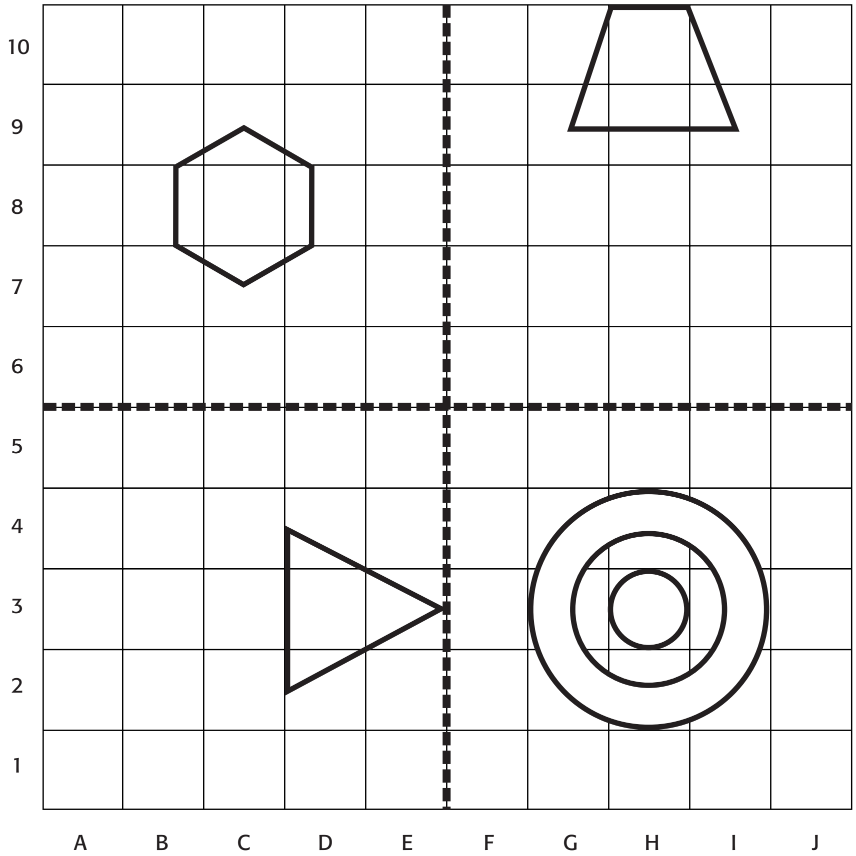
- "Which strategies were efficient and easy to use? Why?"
- "Which strategies are similar? How are they alike?"
- "What strategy would you use if you were to solve this problem again? Why?"

Design Reflection



Hit the Target

Start with the triangle. Each team, at the same time, draws a line on its own grid with its ruler so that when a Mira is placed on that line, the triangle is reflected onto the target. Each team is allowed three attempts. Each team checks the other team's score after each attempt. Your team puts a check mark in the box below for its highest score of the three attempts.



If your team's reflection is:	First try	Second try	Third try
completely inside the target	10	8	6
touching the inner circle	8	6	4
touching the middle circle	6	4	2
touching the outside circle	4	2	1

Next, each team draws a line on its own grid with its ruler so that when a Mira is placed on that line, the trapezoid is reflected onto the target. Scoring is the same as before.

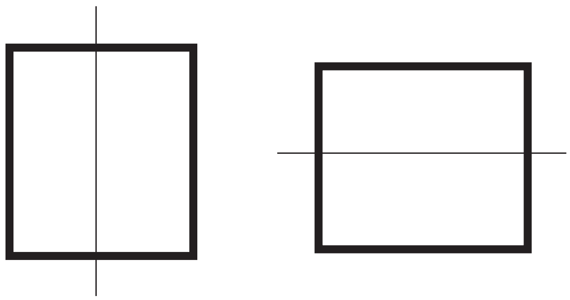
Finally, each team draws a line on its own grid with its ruler so that when a Mira is placed on that line, the hexagon is reflected onto the target. Scoring is the same as before.

Your team should now have three scores or checks to add up for a total score.

Scavenger Hunt

Dear Parent/Guardian:

We have been learning about reflections and symmetry. Please help your child conduct a scavenger hunt to look for objects that have symmetry. For example:



Picture frames

Water bottle

Before you begin, ask your child to explain the terms *line of reflection* and *symmetry*, so that you know how well he or she understands the geometry concepts that are necessary for conducting the scavenger hunt.

Objects found in our home that could have a line of reflection are:

1. ______________________________

2. ______________________________

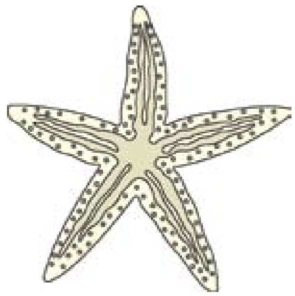
3. ______________________________

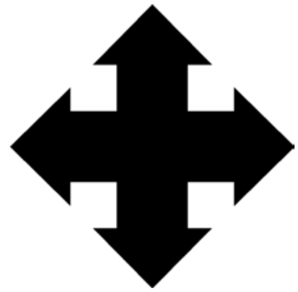
4. ______________________________

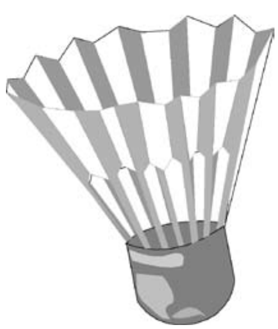
5. ______________________________

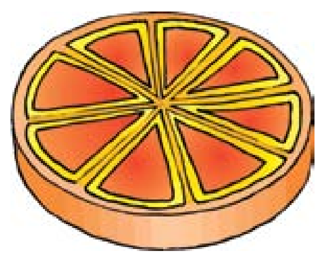
Real-World Symmetry

Decide on the number of lines of symmetry for each of these items. Write your answer next to each item.



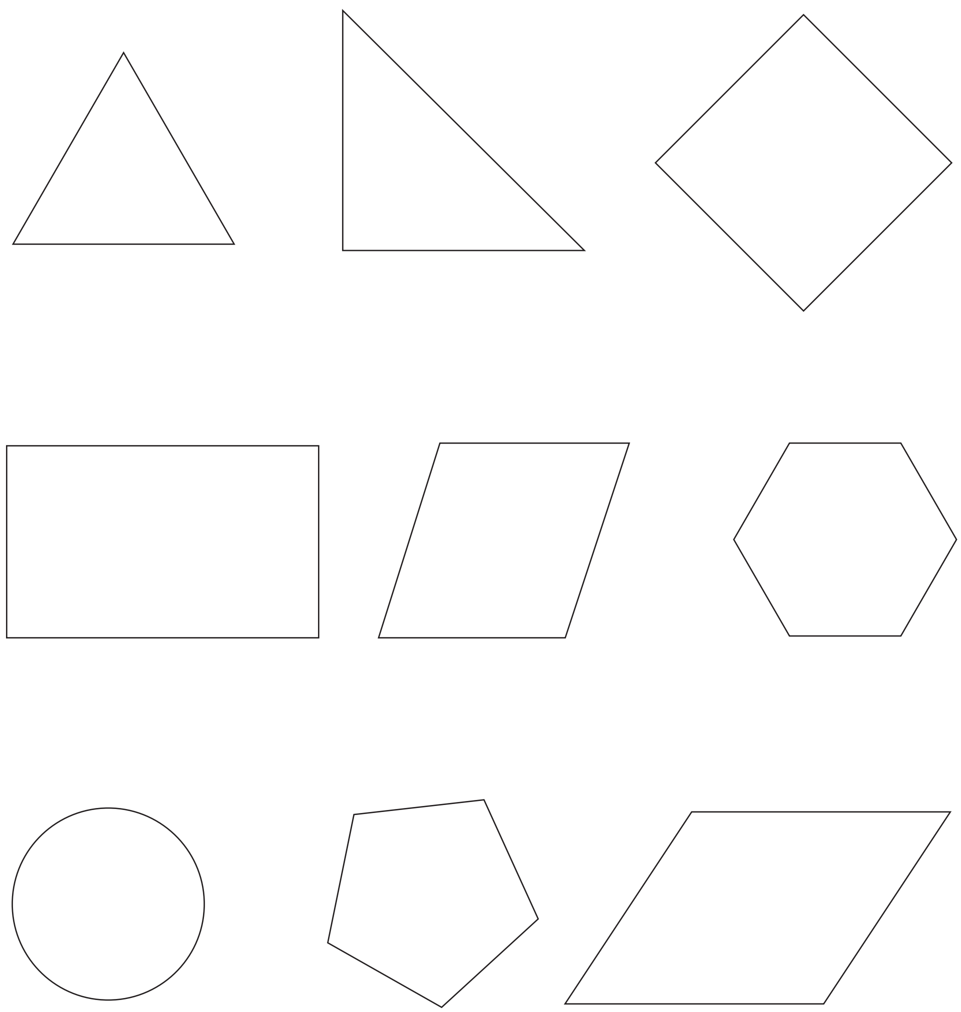






Carmen's Carpets

Carmen is a carpet designer. She likes to design carpets that contain the most symmetrical designs, because these carpets sell the best. She would like to use the following shapes in the carpets, and she wants to know which of the shapes have the most lines of symmetry.



Grade 5 Learning Activity Two-Dimensional Shapes: Triangle Sort

This activity is adapted, with permission, from John A. Van de Walle, *Teaching Student-Centered Mathematics: Grades 3–5*. Toronto: Pearson, 2006, p. 225. The activity is also found in the Grades 5–8 book of the series.

OVERVIEW

In this learning activity, students sort and classify triangles into three groups so that no triangle belongs to more than one group. This problem-solving lesson provides an opportunity for students to explore the properties of triangles and develop definitions of the different types of triangles.

BIG IDEAS

This learning activity focuses on the following big ideas:

Properties of two-dimensional shapes and three-dimensional figures: Students use the properties of triangles to develop definitions for different types of triangles.

Geometric relationships: Students explore the relationships between various triangle properties (e.g., the relationship between side length and angle measure in an isosceles triangle).

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.

Students will:

- identify triangles (i.e., acute, right, obtuse, scalene, isosceles, equilateral), and classify them according to angle and side properties.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- identify and classify two-dimensional shapes by side and angle properties, and compare and sort three-dimensional figures.

TIME: approximately 60 minutes

ABOUT THE LEARNING ACTIVITY

MATERIALS

- overhead projector
- pictures of triangles
- scissors (a pair per student)
- **2D5.BLM1: Assorted Triangles** (1 per pair of students)
- **2D5.BLM2: Triangle Sketching Chart** (1 per pair of students)
- **2D5.BLM3: Triangles in the Environment** (1 per student)

MATH LANGUAGE

- right angle (square corner)
- acute angle
- obtuse angle
- line segment
- vertex
- isosceles
- equilateral
- scalene

INSTRUCTIONAL GROUPING: partners

INSTRUCTIONAL SEQUENCING

This learning activity provides students with an opportunity to explore triangle properties and relationships in order to develop definitions for the different types of triangles. Prior to this activity, students should have an understanding of angles in relation to 90° – for example, larger than 90° is an obtuse angle, smaller than 90° is an acute angle, and 90° is a right angle (square corner). The activity builds on this knowledge of angles.

ABOUT THE MATH

Sorting and classifying triangles allows students to develop their own definitions of the different types of triangles on the basis of their observations and understandings. The activity of sorting and classifying can lead to a discussion of the *defining* properties of a subclass of triangles (e.g., an equilateral triangle is literally translated as an "equal-sided" triangle, but one of its properties is also that it has three equal angles). As students explore triangles and determine groups and subclasses, they connect the defining properties with their experiences, rather than receive them handed down as definitions.

Students will recognize that triangles can be classified by two of their properties – their side lengths (equilateral, isosceles, and scalene) or their angle measures (acute, obtuse, or right). Investigating which triangles can be classified in more than one way (e.g., a right isosceles triangle) and which cannot (e.g., a right equilateral triangle) helps deepen students' understanding of triangle properties.

GETTING STARTED

Place a picture of a triangle on the board or an overhead projector. Invite students to share what they observe about the triangle with an elbow partner. Open the discussion to the larger group. Record the students' observations on the board as they share their discussions. Place a second (different) triangle beside the first and ask students to share their observations of this triangle with an elbow partner. Open the discussion to the larger group and record their observations. Invite students to compare and contrast the two triangles. Explain to students that in this lesson they will be working with a partner to examine and sort a variety of triangles.

WORKING ON IT

Students will work with a partner for this activity. Provide each pair of students with a copy of **2D5.BLM1: Assorted Triangles** and scissors with which to cut out the triangles on the blackline master. Explain to students that their task is to sort the entire collection of triangles into three groups so that *no triangle belongs to two groups*. When they have sorted the triangles, they must record a name for each group, as well as a description that defines the group of triangles.

When students have created their groups and developed a definition for each, encourage them to repeat the activity but to use a different sorting criterion. Again, students should name each group and write a description that defines the group.

As students work, observe how they discuss the properties of the triangles. Do they notice angles, and if so, how do they describe the angles? Do they notice congruent sides? What kind of language do they use for recording the descriptions of the groupings? If a student pair is stuck on only one property (e.g., side length), you may have to hint at the other property (e.g., angle measure). However, it is best if students struggle somewhat with the activity before you provide them with hints. Look for student pairs to share their findings during the reflection part of the lesson.

REFLECTING AND CONNECTING

Bring the students back to the larger group. Ask various pairs to share one of the ways in which they sorted the triangles (either angles measures or side lengths). You may have a student pair read out the triangle letters in a grouping, and then you may challenge the class to look at the triangles in that grouping and predict the description before the pair shares it. Ask the class questions like the following:

- "Which types of triangles did you look for first?"
- "What was the first thing you noticed about some of the triangles?"
- "When you found all the triangles for one group, how did you decide what the next group would be?"

Have student pairs read their definitions for the different groups, encouraging them to use appropriate geometric terminology. Ask other pairs if they sorted their triangles in a similar manner. Model the appropriate terms (e.g., *scalene*, *isosceles*, *right*), and record terms and descriptions in an anchor chart for future reference. Alternatively, you might wish to have students create posters for each type of triangle.

Repeat the process for the sorting of the second property (either angles or side lengths). Ask questions like the following:

- "Was it difficult to find another way to sort the triangles?"
- "Which property, angles or side lengths, was easier for you to use to find differences in the triangles? Why?"
- "Can a triangle belong to more than one category?"
- "Did you need to use any tools to help you sort the triangles?"

Note that this last question helps focus in on the properties that are used to categorize triangles – a ruler measures length, and a protractor measures angles.

ADAPTATIONS/EXTENSIONS

Students who are having difficulty with the task should be encouraged to sort according to only one property. You might also provide them with a ruler as a hint about how to get started. This activity is also one in which homogeneous grouping could benefit both struggling students and those who are able to grasp the concepts quickly. There are many triangles to be sorted, and a dominant student might quickly move them around, preventing a partner from analysing them and deciding where they should go.

As an extension, provide students with a copy of **2D5.BLM2: Triangle Sketching Chart**. Ask students to sketch a triangle for each of the nine cells. The triangle must match the characteristics of both the column header and the row header. Note that it is impossible to draw two of the triangles (a right equilateral and an obtuse equilateral). This discovery can lead to an interesting discussion about the measures of angles in a triangle.

ASSESSMENT

Observe students as they solve the problem, and assess how well they:

- identify triangles (i.e., acute, right, obtuse, scalene, isosceles, equilateral);
- classify triangles according to angle and side properties;
- identify and classify two-dimensional shapes by side and angle properties;
- explain sorting criteria and description criteria.

This lesson may be used appropriately to assess students' use of geometric terminology, particularly after the Reflecting and Connecting part of the lesson. It is more important that students use the terms in context rather than recall a definition by rote.

HOME CONNECTION

In this Home Connection, students look for triangles in their environment. Send home **2D5.BLM3: Triangles in the Environment**. Students will use it to record examples of the types of triangles they see on their way home from school and in their homes.

LEARNING CONNECTION 1
Measuring and Constructing Angles in The Geometer's Sketchpad

MATERIALS

- The Geometer's Sketchpad (1 computer per person/per pair)
- **2D5.BLM4: GSP Angles Instructions**
- math journals

This activity is designed to provide students with the opportunity to learn some of the features of The Geometer's Sketchpad while investigating the properties of angles. It is intended that students work with partners, or individually if there are enough computers.

Provide each student/pair with a copy of **2D5.BLM4: GSP Angles Instructions**. Students follow the instructions on the page to complete the task.

Have a whole class discussion after the activity. Ask students questions like the following:

- "What aspects of the task did you find challenging?"
- "Up to now, we've talked about an angle as being the measure of rotation between two rays or line segments. Does working with this program change your ideas about that definition?"
- "What is required to measure an angle?" (This will raise an interesting discussion, for in GSP, only three points are required to form an angle.)

Although this activity is designed to be fairly procedural, it will generate rich discussion about the properties of angles and the tools required to construct them.

LEARNING CONNECTION 2
Spinning for Triangles

MATERIALS

- **2D5.BLM5: Triangle Construction Spinners** (1 per pair of students)
- pencil (1 per pair of students)
- paper clip (1 per pair of students)
- rulers (1 per pair of students)
- protractors (1 per pair of students)

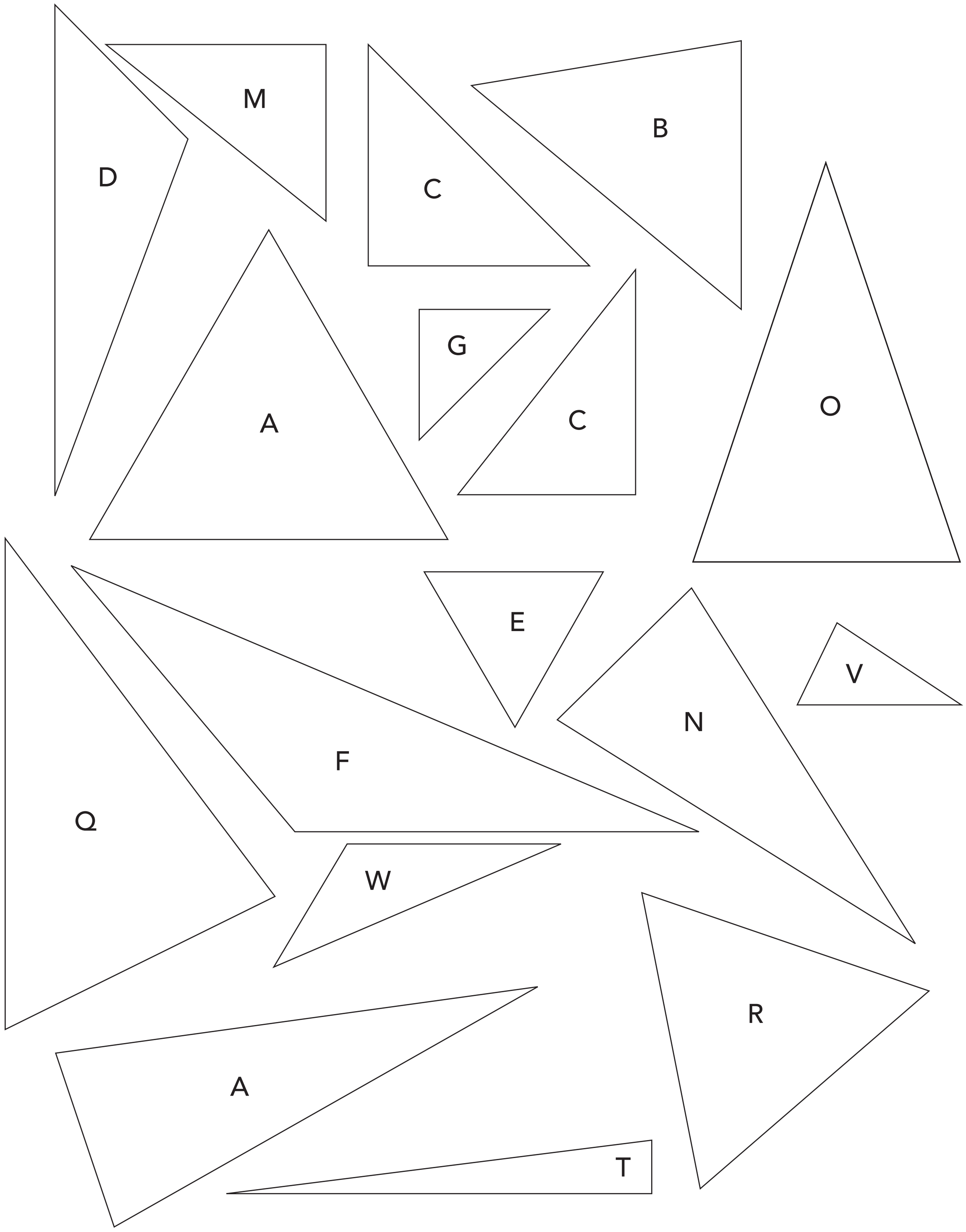
This activity is a game to be played by two players. Each pair of players makes a spinner with a copy of **2D5.BLM5: Triangle Construction Spinners**, a paper clip, and a pencil.

Players take turns. Player 1 spins on both of the spinners, then constructs a triangle that satisfies the criteria shown on the spinners (e.g., a triangle with an angle of 120° and no sides equal). Player 2 checks to see that the construction is accurate. If he or she agrees, Player 1 scores a point. Player 2 takes a turn.

If a player is unable to construct a triangle, the other player may attempt it to steal a point. If no one is able to construct the triangle, no points are given, and it is the other player's turn.

The game continues until one player scores 5 points, or the game can be timed.

Assorted Triangles



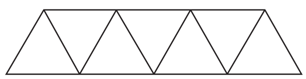
Triangle Sketching Chart

	Equilateral	Isosceles	Scalene
Right			
Acute			
Obtuse			

Triangles in the Environment

Dear Parent/Guardian:

In math this week, we have been learning about different types of triangles. Your child has been sorting and classifying different types of triangles in class. Here is an opportunity for your child to demonstrate what he or she has learned. Ask your child to find different triangles in and around your home and between your home and the school. Then ask your child to describe the type of triangle, create a sketch, and describe the purpose of the triangle.

Type of Triangle	Sketch	Purpose
Example: Equilateral – it has sides with the same length		– part of a bridge structure – must provide strength to the bridge and support the beams

Thank you for your assistance with this activity.

GSP Angles Instructions

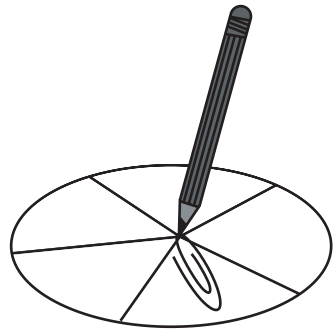
Follow the instructions on this page, and answer any questions in your math journal.

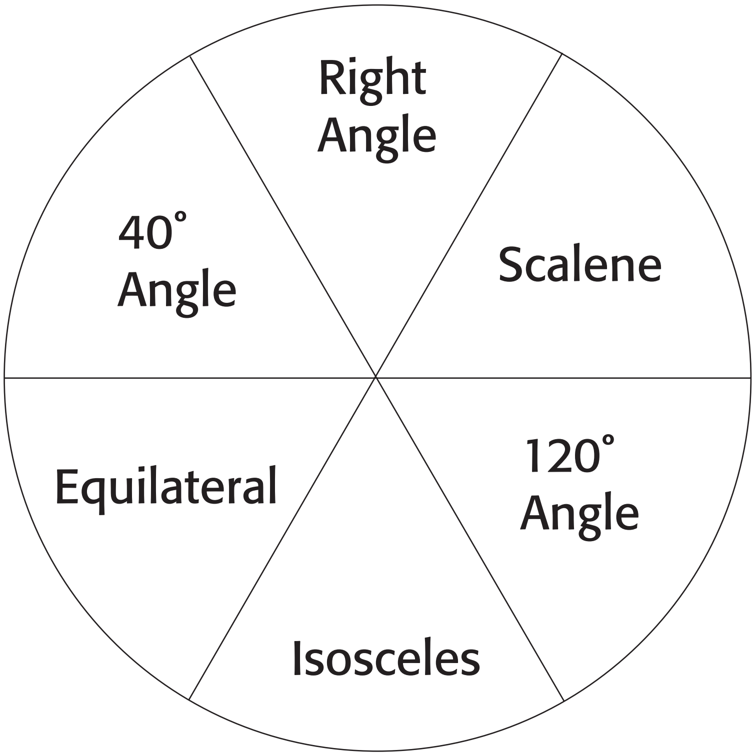
1. Open The Geometer's Sketchpad.
2. From the **Edit** menu, choose **Preferences**.
3. Change the **Precision for Angle** from hundredths to **units**.
4. Use the **Point tool** to place three points on the work area.
5. Click on the **Straightedge tool**, and join the segments to form two line segments joined at a vertex.
6. Select the **Text tool**, and click on each point to label it.
7. Click on the **Select tool**, and click on the white area to deselect everything. **Then click on the three points** in order – A, B, C.
8. From the **Measure** pull-down menu, choose **Angle**. What happens?
9. Drag point A around the plane. What happens to the angle measure?
10. Can you move point A so that the angle measure stays the same?
11. Use the **Tools** and the **Measure** menu to create a right angle. Describe in your math journal how you did so.

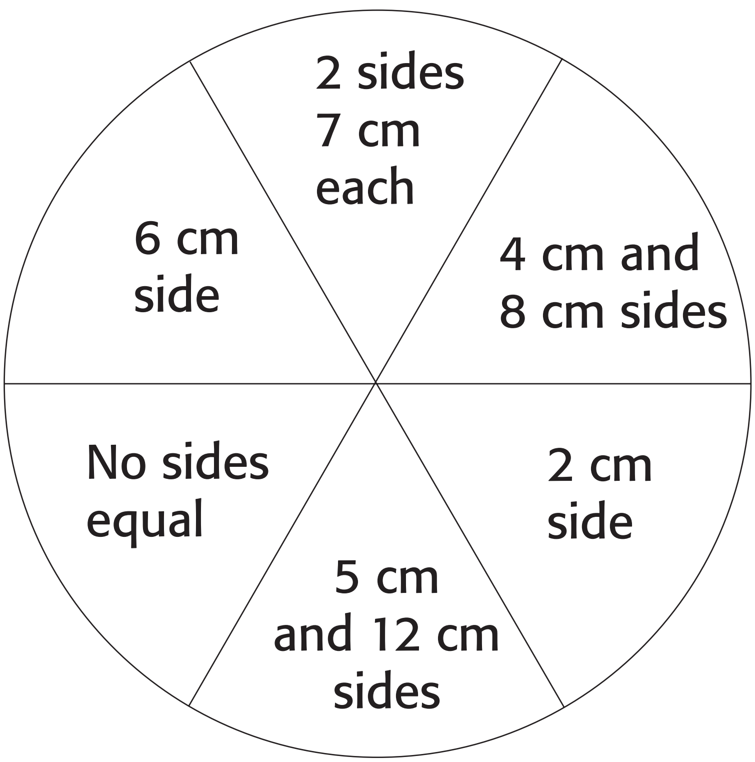
CHALLENGE: Can you use the **compass tool** and Animate features to create a constantly changing angle?

Triangle Construction Spinners

Make two spinners, using this page, a paper clip, and a pencil.







Grade 5 Learning Activity Three-Dimensional Figures: Package Possibilities

OVERVIEW

In this investigation, students gain experience identifying pyramids from nets and classifying these three-dimensional figures by their geometric properties. Students have opportunities to represent their understanding of three-dimensional figures physically, through drawings, and verbally, using appropriate mathematical language.

BIG IDEAS

This learning activity focuses on the following big ideas:

Properties of two-dimensional shapes and three-dimensional figures: Students develop an understanding of the properties of pyramids, specifically looking at faces, edges, and vertices in two-dimensional representations.

Geometric relationships: Students must be able to use two-dimensional nets to recognize and represent three-dimensional figures. This activity calls for students to focus on the relationship between the two-dimensional faces of an object and its three-dimensional construction.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- identify prisms and pyramids from their nets;
- construct nets of prisms and pyramids, using a variety of tools (e.g., grid paper, isometric dot paper, Polydrons, computer application).

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- identify and construct nets of prisms and pyramids.

ABOUT THE LEARNING ACTIVITY

TIME: approximately 60 minutes

MATERIALS

- net of a triangular pyramid made from Polydron pieces
- overhead projector and overhead transparency
- sample of a box to represent any three-dimensional figure (e.g., tissue box)

- transparency of isometric dot paper
- **3D5.BLM1: Pyramid Packages** (1 per student)
- Polydron set
- solid square-based pyramid
- sheets of isometric dot paper for recording nets (2 or 3 per pair of students)
- blank overhead transparencies of isometric dot paper (1 per pair of students)
- overhead markers (1 per pair of students)
- large sheet of blank chart paper
- tape
- **3D5.BLM2: Find the Nets** (1 per student)
- **3D5.BLM3: Six Nets of a Square Pyramid** for teacher reference

MATH LANGUAGE

- net
- three-dimensional figure
- two-dimensional shape
- equilateral triangular faces
- triangular pyramid
- tetrahedron
- isometric dot paper
- square-based pyramid
- isosceles triangles
- reflection
- rotation

INSTRUCTIONAL SEQUENCING

This lesson can serve as an introduction to a unit on three-dimensional geometry or can be used after students have had some experiences sorting and classifying three-dimensional figures by their properties. Students should be able to recognize many three-dimensional figures but do not need to have many prior experiences with nets.

INSTRUCTIONAL GROUPING: individual, pairs

ABOUT THE MATH

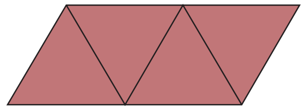
In Grade 5, students are expected to construct nets of prisms and pyramids using a variety of tools.

In this task, students will predict the number of nets possible for a square-based pyramid and then use two-dimensional Polydron shapes to investigate all of the different nets. After constructing a square-based pyramid, students will explore different nets by unfolding and refolding the Polydron faces. If it is assumed that two nets are the same if they can be rotated or flipped to overlap each other, there are six possible nets for a square-based pyramid. It is important to give students the time to have a rich discussion about what qualifies as *different*, rather than stipulate before the lesson the idea that rotated or flipped nets are not unique.

This task also provides opportunities for students to develop their reasoning, proving, and reflecting skills as they predict the number of possible nets for the square-based pyramid, construct the nets, adjust their strategies, and then confirm or revise their predictions.

GETTING STARTED

Show students the following net for a triangular pyramid constructed from Polydron pieces.



Ask students what an arrangement of two-dimensional faces is called (net). Ask students to predict the three-dimensional figure that will be created by folding the net. "How do you know?"

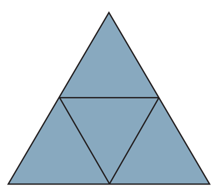
Try to elicit properties of the three-dimensional figure that are evident in the net – that is, the number and shapes of the faces. Ask students to be specific in identifying the two-dimensional faces – that is, equilateral triangular faces. Invite a student to test the prediction by folding the net.

Discuss the two names we use for this particular three-dimensional figure – triangular pyramid and tetrahedron. Discuss why the figure is named in both of these ways (pyramids are named for their base; polyhedra in general are named for the number of polygonal faces – *tetra* means "four").

Unfold the net to form the same net you originally showed to the students. On an overhead transparency, draw the net for the tetrahedron, and display it.

Then ask: "Is it possible to arrange the faces differently and still end up with the same three-dimensional figure?" Invite another student to unfold the triangular pyramid to create a net that is different from the original one.

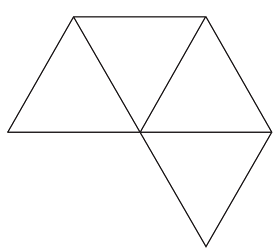
The student should construct this net:



If the student unfolds the figure to create the same net, only reflected and/or rotated, ask the class whether the net is, in fact, a different net. If necessary, rotate the net in your hands to allow the students to see that the net is actually the same one you started the activity with.

Draw the new net (pictured above) on the same overhead transparency.

Next, draw the following arrangement of triangular faces on the transparency:



Ask: "What about this arrangement of four triangular faces? Will it fold to make a triangular pyramid? Explain why it is or is not a net for a tetrahedron." (It is not a net, because two of the triangles will overlap when the arrangement is folded, and there will be no base.)

Explain to students that there are sometimes many different nets for a given three-dimensional figure. The triangular pyramid, or tetrahedron, has only the two shown above. Students will now explore different nets of another pyramid.

WORKING ON IT

Introduce the following scenario:

> "An electronics company is designing a new video gaming system and wants it to have a unique shape. The company is considering a square-based pyramid design, but wants to know what the packaging layout will be in order to determine costs. The company has asked for your help in determining all of the possible nets for packaging this square-based pyramid gaming system."

Show students a package (e.g., cereal box, facial tissue box) and explain that the electronics company wants the box for their gaming system to be the same shape as the system itself – a square-based pyramid.

Give each student a copy of **3D5.BLM1: Pyramid Packages**. Ask that each student predict the total number of different nets possible for a square-based pyramid and record both his or her prediction and reasoning on the handout. (A student may say, for example, that a square-based pyramid will have three different nets because the triangular pyramid had two nets and the square pyramid has one more face than the triangular pyramid.) Allow time for students to think individually, and then have them discuss their predictions with an elbow partner.

Arrange students in pairs. Explain that students will begin by constructing a square-based pyramid from Polydron pieces.

Have each pair of students determine which Polydron pieces they will need to construct a square-based pyramid. Display a solid square-based pyramid for students who may need to refer to it. Students will require a square and four triangles. To add to the difficulty level, allow students to choose their own Polydron pieces. Provide choices for students who may require additional support. Most will choose four equilateral or four isosceles triangles, but two right-angled triangles, one equilateral triangle, and one isosceles triangle will also make a square-based pyramid. Note that **only** the combination of a square and four equilateral triangles will allow for the creation of all six nets (see **3D5.BLM3: Six Nets of a Square Pyramid**). Invite one of the partners to collect the necessary pieces from a bin of Polydron pieces.

Assessment Opportunity

Circulate to assess students' reasoning and reflecting by listening to pairs' discussions and asking probing questions. For example: "Do you still think your prediction of the number of possible nets for the square-based pyramid is accurate? Why or why not? How would you revise your prediction now? Explain your reasoning."

Instruct pairs of students to construct their square-based pyramid. Circulating to distribute several sheets of isometric dot paper to each pair allows you to ensure that all pairs have constructed the correct three-dimensional figure. Allow students to exchange or add Polydron pieces if they have discovered that they are missing some. Ask students to then unfold their figure to create a net. Have students draw the net on the isometric dot paper.

Have students refold their net to form the square-based pyramid again. Ask students to continue unfolding and refolding to find as many different nets for the square-based pyramid as they can. Each different net that they can find is to be recorded on their paper.

As students work, circulate with sheets of isometric dot overhead transparencies. Ask each pair of students to use an overhead marker to record one of their nets – selected by you – on the transparency. Try to include as many different nets as possible, but also look for nets that are simply reflections or rotations of each other. These will enhance the discussion during the Reflecting and Connecting part of the lesson.

REFLECTING AND CONNECTING

Bring the class together. Ask pairs of students how many nets they found. (Answers will differ, depending on whether students found all six unique nets and whether they have reflections and/or rotations of the same net.)

Ask each pair of students in turn to come up to the overhead projector to share with the class the net that they recorded on the acetate. Have them explain the geometric properties of the square-based pyramid and tell how their net can be folded to form that figure (i.e., which sides of the two-dimensional faces meet to form the edges of the three-dimensional figure). Tape each net to a large white piece of chart paper after the net has been shared, so that all of the shared nets can be viewed at once.

Assessment Opportunity

As students present their net, assess their ability to communicate how the net would be folded to form a solid. Look for appropriate and accurate terminology, like edge, face, *and* vertex.

As each new pair shares a net, ask students to consider whether the net is truly unique or whether it can be reflected or rotated to match a net already presented (and displayed on the chart paper).

Ask: "Are any of the nets you found not represented on the chart?" Allow students to share their drawings of any new nets with the class. If the class determines that a net is unique, have the student draw the net on a transparency of isometric dot paper and add it to the chart.

Post the completed chart in the classroom.

Elicit student strategies for finding the unique nets. Ask questions like the following:

- "Did you use any particular strategy for finding all the nets?"
- "How many groups randomly tried different combinations?"
- "Did anyone use a pattern or similar strategy to try to find all of the nets?"

Try to generate a discussion that looks at a systematic method for finding the nets. For example, some students may have started with the net having one triangle on each of the sides of the square, and then moved to one triangle on three sides of the square, then two on each side, and so forth. Ask if this strategy could be applied to predict the number of nets for other types of polyhedra.

Make a connection with the original scenario; for example, "How might this information help the toy company?" Ask about the considerations a company might take into account when designing a package.

ADAPTATIONS/EXTENSIONS

Some students may experience difficulty in drawing their nets on the blank paper. Provide them with graph paper or isometric dot paper to help them with their sketches. They may also trace the Polydron nets onto a blank sheet of paper if they are unable to create their own.

Challenge students to construct a pentagonal prism and find all of its nets by unfolding and refolding. Have them record all of the nets they find.

ASSESSMENT

Observe students to see how well they:

- identify and construct nets of pyramids using the Polydron pieces.

This lesson focuses on skills and reasoning more than content knowledge. It is appropriate to assess process skills, like reasoning and communication, during the lesson. Look to see how well students use accurate and appropriate terms to communicate their findings, and as you circulate around the room, observe students as they struggle to find additional nets. What is their reasoning? Do they use a system or strategy, or do they randomly piece the polygon faces together?

HOME CONNECTION

Send home **3D5.BLM2: Find the Nets**. In this homework task, students identify nets of prisms and pyramids from several arrangements of two-dimensional shapes through visualization or folding, and then explain why the other arrangements are not nets.

Inform students that they will cut out the nets in class the next day to see if their reasoning was correct.

LEARNING CONNECTION 1
Nets in AppleWorks

MATERIALS

- **3D5.BLM4: Drawing Nets in AppleWorks** (1 per student)
- computers with AppleWorks (1 per student or pair of students)

Activate students' prior experience of using the draw tools in AppleWorks by discussing with the class the various functions of each tool. If students' experience is limited, they may need some time to review the different tools in AppleWorks and how they are used.

Distribute a copy of **3D5.BLM4: Drawing Nets in AppleWorks** to each student. Following the instructions provided, students use AppleWorks to create nets for the square-based pyramid and triangular prism.

Once students have created their nets, they may save or print them for assessment or use them to create figures.

LEARNING CONNECTION 2
Net Characters

MATERIALS

- **3D5.BLM5a–b: Net Characters** (1 per student)
- Polydron sets (1 set per group of 4 students)

Arrange students in groups of four. Distribute **3D5.BLM5a–b: Net Characters** to each student.

Begin by asking students to predict whether the first character is a net for a prism, a pyramid, or another three-dimensional figure, and have them name the three-dimensional figure that can be formed by folding the net.

Next, invite a student to come up to test the prediction with the Polydron shapes. Once the correct figure has been confirmed, instruct all students to record the name of the figure on their handouts. (square-based prism).

Students continue to identify the nets of pyramids and prisms from several character nets on the handout. Allow enough time for each group of students to try to identify each net.

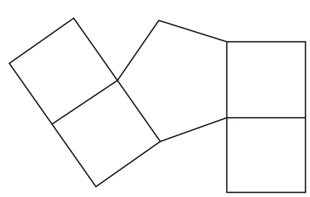
Distribute Polydron sets to each group. Instruct groups to test their predictions for each net by constructing the net and folding it to create a three-dimensional figure.

LEARNING CONNECTION 3
Complete the Nets

MATERIALS

- a partially completed net constructed from Polydron shapes, as shown
- one additional pentagon and one additional square Polydron shape (hidden)
- **3D5.BLM6a–b: Complete the Nets** (1 per student)
- rulers (1 per student)
- Polydron pieces

Begin by showing students the partially completed net constructed from Polydron shapes below. Tell students that the net is missing one or two faces that you have hidden. Ask them to predict what the three-dimensional figure is and to tell you what faces are missing. Ask: "How do you know?" (It is a net for a pentagonal prism with one pentagon and one square face missing.)



Unveil the missing faces and invite a student to come up to add the missing faces to the net. Ask why the student chose to put the faces where he or she did. Ask the student to then "test" his or her placement of the faces. If the placement is incorrect, can the students analyse the mistake and use it to learn more about nets and the placement of faces?

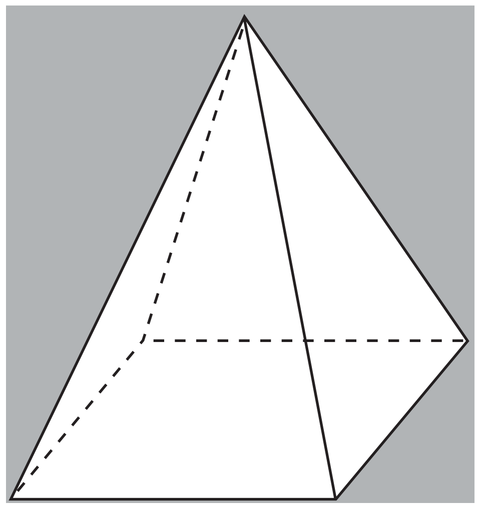
Explain to students that they will be given a handout with drawings of six partial nets. They are to complete the nets (by adding one or two missing faces to each), identify the three-dimensional figure that each net makes when completed, and draw a different net for each figure.

Distribute one copy of **3D5.BLM6a–b: Complete the Nets** to each student.

Allow students to work with a partner or in small groups to complete the handout. As a scaffold for students who struggle with this skill, have Polydron shapes or another appropriate material available to aid in the "testing" of face placement.

Pyramid Packages

An electronics company is designing a new video gaming system and wants it to have a unique shape. The company is considering a square-based pyramid design, but would like to know all of the packaging layouts for this figure in order to determine packaging costs. The marketing department manager has asked for your help in determining all of the possible nets for packaging their square-based pyramid gaming system.



1. Predict the total number of different nets possible for constructing a square-based pyramid.

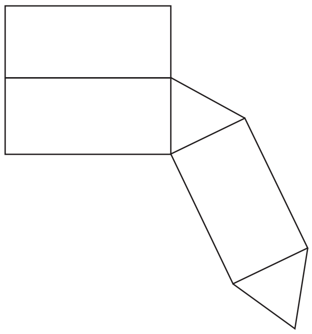
   I think there are ______ different nets possible for constructing a square-based pyramid.

2. Test your prediction by constructing square-based pyramids from Polydron pieces and unfolding them to create different nets. Draw each net on isometric dot paper.

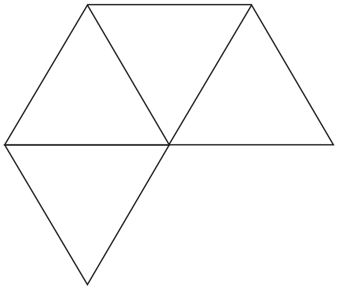
Find the Nets

Only some of the following configurations of shapes are nets for three-dimensional figures. Determine whether each configuration is a net. If it is, name the three-dimensional figure that the net will form when folded. If a configuration is not a net, explain why it is not a net.

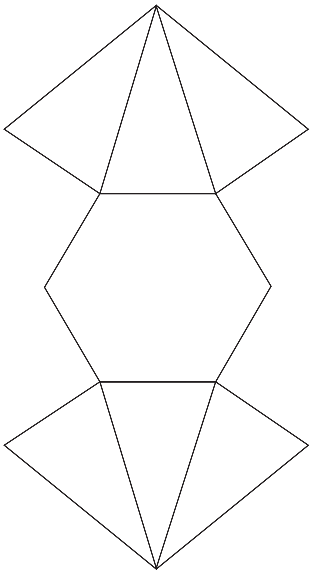
A.



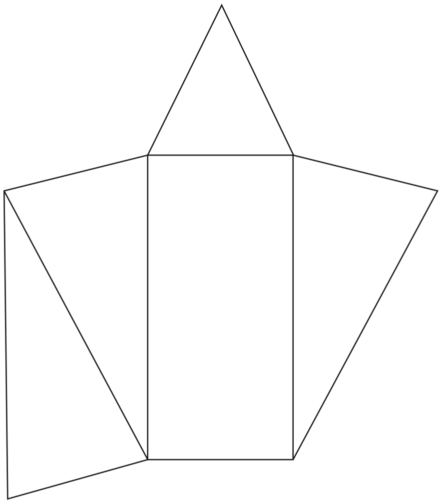
B.



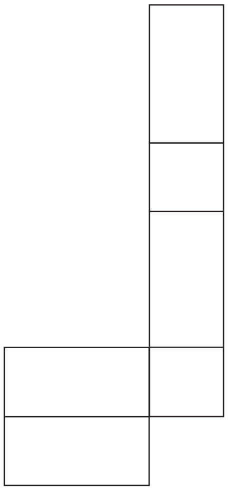
C.



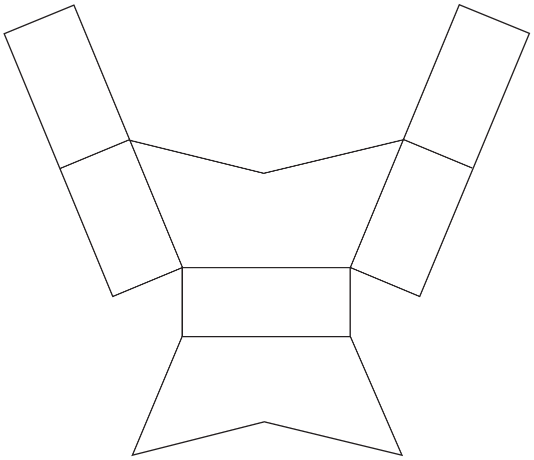
D.



E.

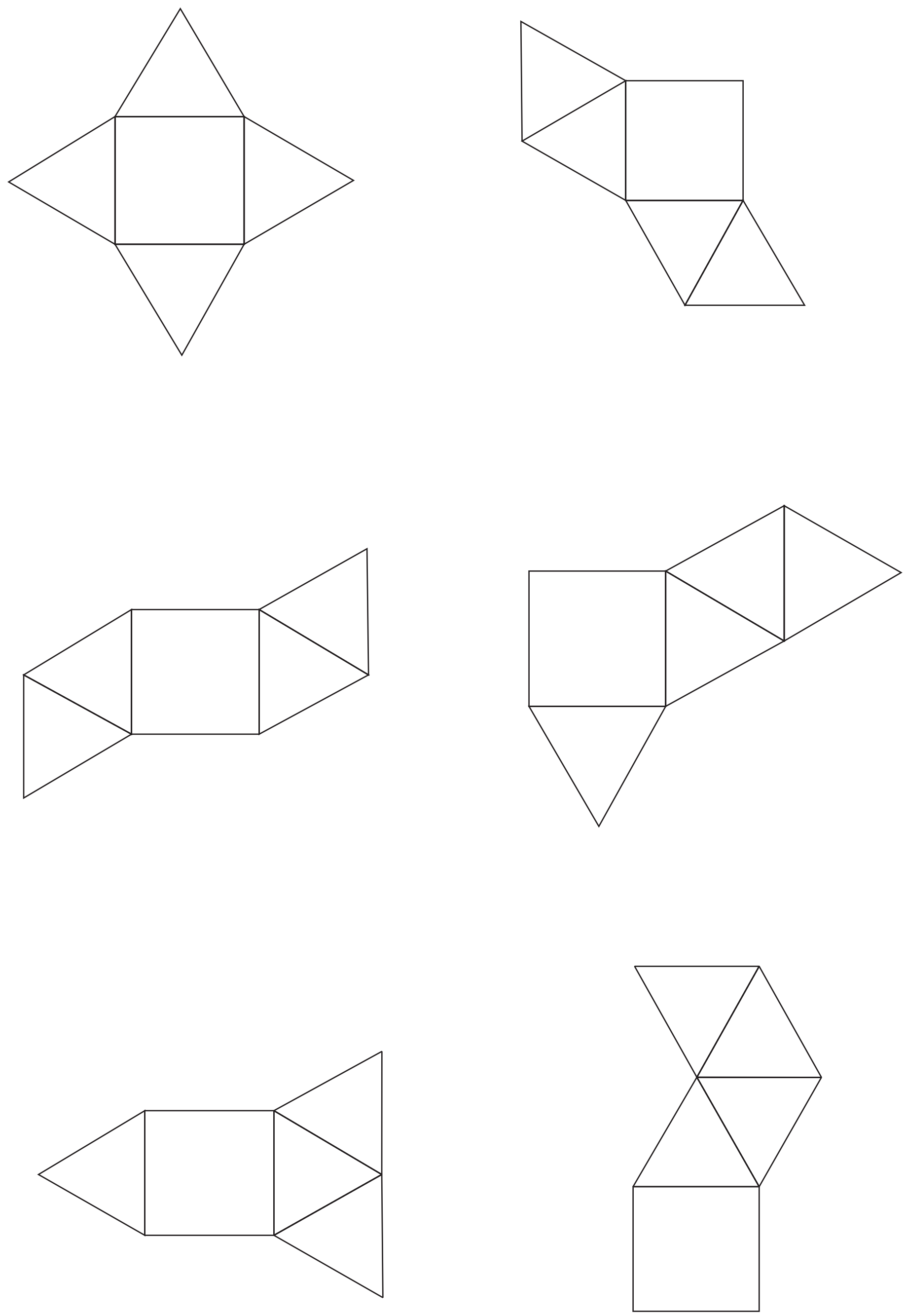


F.



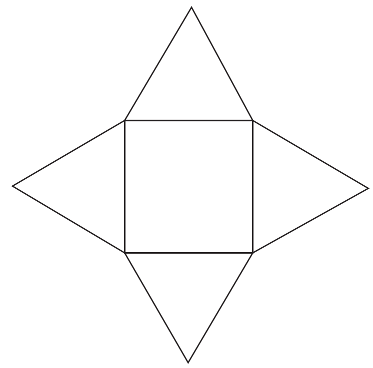
(*Hint*: This **is** a net. What three-dimensional figure will it form?)

Six Nets of a Square Pyramid



Drawing Nets in AppleWorks

1. Follow these steps to create a net for a square-based pyramid:



- Open a new drawing in AppleWorks.
- From the Format menu, choose Rulers – Ruler Settings. In the new window, click on Graphics under Ruler Type; click on Centimetres under Units; and type in 5 for Divisions.
- Click on the rectangle tool, ▭ , from the Tool Bar at the left of the screen. Draw a square that is 2 cm by 2 cm near the centre of the page. This is the base of the pyramid.
- Click on the regular polygon tool, △. Click on Edit – Polygon sides to make sure the number of sides selected is 3. Click OK.
- Draw a triangle along one side of the square so that one whole side of the triangle meets one whole side of the square.
- To make a congruent triangle, choose Edit – Duplicate.
- Next, choose Arrange – Rotate 90° to rotate the new triangle so that it can meet the square along one whole side. Click and drag the new triangle so that it shares a full side with the square.
- Duplicate the triangle once again. Choose Arrange – Rotate 90° to rotate the new triangle and drag it so that it also shares a full side with the square.
- Duplicate, rotate, and drag a triangle one more time to complete the net for a square-based pyramid.
- To move the net, hold down the Shift key and click on each of the five faces. Then choose Arrange – Group. Now move the net closer to the top of the page.

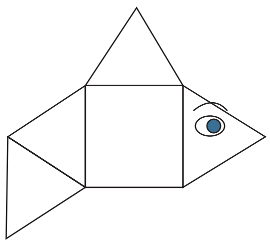
2. On the bottom half of the page, draw a different net for a square-based pyramid, using what you have learned about drawing nets in AppleWorks. Save your work as “Square-based Pyramid Nets”.

3. Open a new AppleWorks drawing (File – New – Drawing). Draw two nets for a triangular prism. Save your work as “Triangular Prism Nets”.

Net Characters

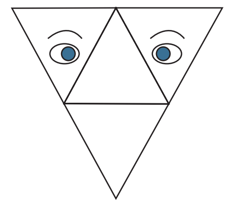
Most of the characters below are nets for prisms and pyramids. Check the appropriate box below each net character to indicate whether it is a net for a prism, a pyramid, or another three-dimensional figure.

A.



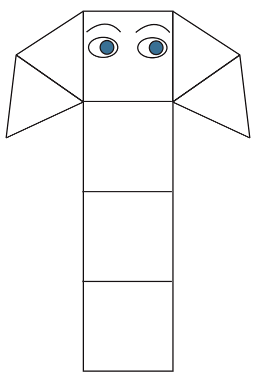
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

B.



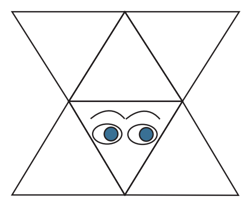
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

C.



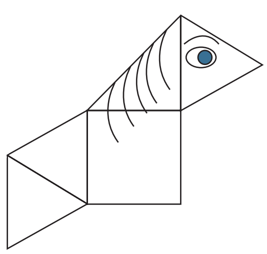
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

D.



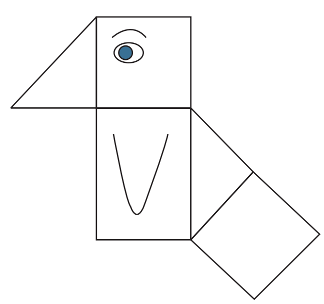
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

E.



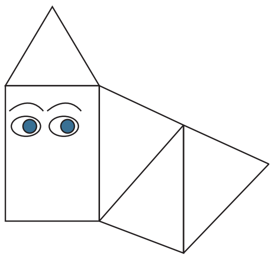
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

F.



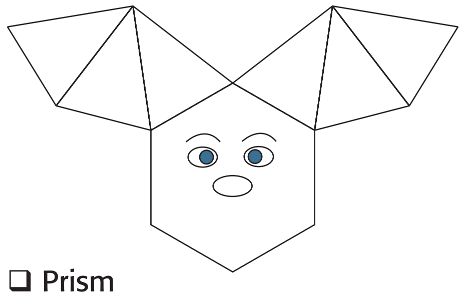
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

G.



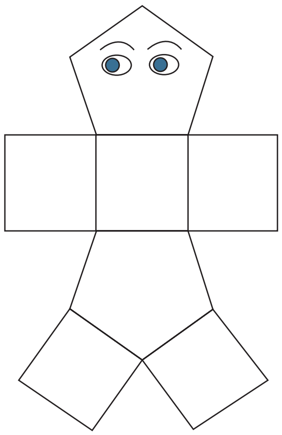
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

H.



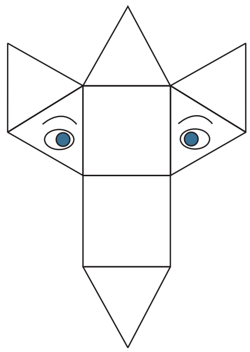
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

I.



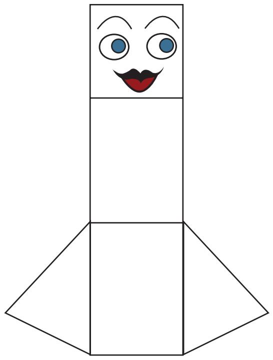
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

J.



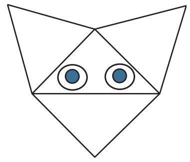
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

K.



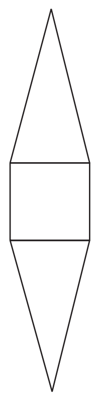
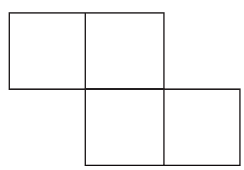
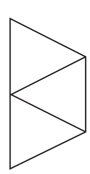
- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

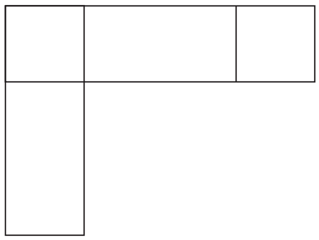
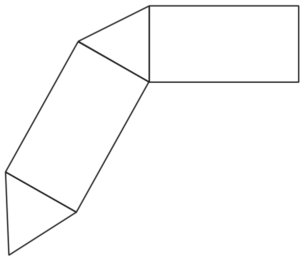
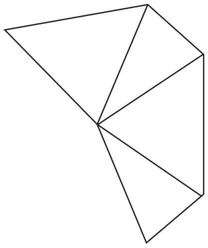
L.



- ❑ Prism
- ❑ Pyramid
- ❑ Other 3-D figure

Complete the Nets

Complete the partial nets by adding up to two missing faces.	Draw a different net for the same figure.
1.  Name of figure:	
2.  Name of figure:	
3.  Name of figure:	

Complete the partial nets by adding up to two missing faces.	Draw a different net for the same figure.
4.  Name of figure:	
5.  Name of figure:	
6.  Name of figure:	

Grade 5 Learning Activity
Location: City Treasure Hunt

OVERVIEW

In this learning activity, students use a coordinate grid system to describe a location on the intersection of lines. This problem-solving lesson provides students with an opportunity to explore the concept of describing precise locations.

BIG IDEAS

This learning activity focuses on the following big idea:

Location and movement: The position of an object can be described using a coordinate grid system. In Grade 5, students move from describing a location on the basis of spaces (using numbers and letters to identify an area) to describing a location on the basis of the intersection of lines (using a coordinate system based on the cardinal directions to describe a specific point).

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.

Students will:

- locate an object using the cardinal directions (i.e., north, south, east, west) and a coordinate system (e.g., "If I walk 5 steps north and 3 steps east, I will arrive at the apple tree.").

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- identify and describe the location of an object, using the cardinal directions, and translate two-dimensional shapes.

ABOUT THE LEARNING ACTIVITY

TIME: approximately 90 minutes

MATERIALS

- markers (1 per pair of students)
- overhead projector
- overhead transparency of **Loc5.BLM1: City Treasure Hunt Map**
- **Loc5.BLM2: City Treasure Hunt Planning Sheet** (1 per student)
- **Loc5.BLM3: City Treasure Hunt Answer Sheet** (1 per student)
- **Loc5.BLM4: Planning a Trip** (1 per student)
- sheets of chart paper (1 per pair of students)

MATH LANGUAGE

- cardinal directions
- grid
- grid lines
- location
- intersection
- specific
- north
- south
- east
- west
- axis
- axes

INSTRUCTIONAL SEQUENCING

This learning activity provides an introduction to the grid system as a means of identifying specific locations using cardinal directions and intersecting lines. Before beginning this activity students should review cardinal directions and the grid system by exploring the accompanying learning connections or similar activities.

INSTRUCTIONAL GROUPING: pairs

ABOUT THE MATH

The key new idea for Grade 5 is location based on grid lines, rather then location based on spaces. After playing the game Drawing Directions (Learning Connection 2 or a similar activity), the students should be comfortable with identifying a specific location on a grid. In this activity they will be required to give the east-west directions before the north-south directions to prepare students for the use of ordered pairs in Grade 6.

GETTING STARTED

Discuss how some cities in North America are designed on a grid system, with avenues often going east-west, and streets often running north-south.

Use a transparency or poster of **Loc5.BLM1: City Treasure Hunt Map** to show the streets of a pretend city. Explain that the students will be required to find five treasures in the city by following your directions.

Read the instructions one at a time, each time asking a volunteer to come up to label the location and mark the route that must be travelled. Each volunteer should use a different-coloured marker, so that the routes can be distinguished. Students are to calculate the number of blocks travelled for each of the instructions, as well as the total distance travelled.

INSTRUCTIONS

Answers are in brackets.

Start at the intersection of 10th Street and 2nd Avenue. Label the location S.

1) (first student) Travel 3 blocks west and 4 blocks north. You will find Aunt Agatha's artwork. Label it A. Mark your route, the location of the treasure (7th St and 6th Av), and the number of blocks you have travelled (7).

2) (next student) Travel 2 blocks west and 4 blocks south. You will find a beautiful bronze bear. Label it B. Mark your route, the location of the treasure (5th St and 2nd Av), and the number of blocks you have travelled (6).

3) (next student) Travel 4 blocks east and 2 blocks north. You will find a cherished china cup. Label it C. Mark your route, the location of the treasure (9th St and 4th Av), and the number of blocks you have travelled (6).

4) (next student) Travel 6 blocks west and 3 blocks north. You will find a delicate and delightful diamond. Label it D. Mark your route, the location of the treasure (3rd St and 7th Av), and the number of blocks you have travelled (9).

5) (next student) Travel 3 blocks east and 3 blocks south. You will find an empty but enticing envelope. Label it E. Mark your route, the location of the treasure (6th St and 4th Av), and the number of blocks you have travelled (6).

6) (next student) Travel 4 blocks west and 5 blocks north to the finish. Label it F. Mark your route, the location of the finish (2nd St and 9th Av), and the number of blocks you have travelled (9).

7) Ask, "What distance has been travelled in total?" (43 blocks).

WORKING ON IT

Tell the students that you want them to work in pairs to plan a treasure hunt. Hand out **Loc5.BLM2: City Treasure Hunt Planning Sheet** and **Loc5.BLM3: City Treasure Hunt Answer Sheet**, one copy of each per student. Review the instructions with the students. As students plan their treasure hunts, observe the various strategies they use. Pose questions to help students think about their strategies.

- "What strategy are you using to make your treasure hunt between 40 and 50 blocks long?"
- "What strategy are you using to make sure that the shortest possible route to all the treasures is less than 30 blocks?"
- "Have you modified your strategies? Why?"

If time allows, once students have had a chance to create their treasure hunts, have them copy their directions without the number of blocks and then trade with another pair and use the answer sheet to complete the treasure hunt.

After students have had time to complete their treasure hunts, provide each pair with markers and a sheet of chart paper. Ask students to record the strategies they used to design their treasure hunt.

Make a note of groups that might share their strategies and solutions during the Reflecting and Connecting part of the lesson. Include groups whose methods varied in their sophistication.

REFLECTING AND CONNECTING

Reconvene the class. Ask a few groups to share their strategies and post their work. Try to order the presentations so that students observe inefficient strategies first, followed by more efficient methods.

As students explain their work, ask questions that probe their thinking:

- "How did you make sure that your treasure hunt would be between 40 and 50 blocks long?"
- "Why did you use that strategy?"
- "How did you make sure that the shortest route to all your treasures would be less than 30 blocks long?"
- "Would you use the same strategies next time? Why or why not?"
- "How would you change your strategy the next time?"
- "Is your strategy similar to another strategy? Why or why not?"

Following the presentations, ask students to observe the work that has been posted and to consider the efficiency of the various strategies. Ask:

- "Which strategy, in your opinion, is an efficient strategy?"
- "How would you explain this strategy to someone who has never used it?"

Avoid commenting that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students' work to emphasize important ideas about the location of objects:

- Using intersecting lines on a grid allows us to pinpoint specific locations.
- Cardinal directions (north, south, east, west) can orient people in an intended direction.

ADAPTATIONS/EXTENSIONS

For students experiencing difficulty, reduce the grid to 5 × 5 and reduce the number of treasures and blocks.

For more of a challenge, ask students to create a symbol or design with their treasure hunt path.

ASSESSMENT

Observe students as they play the game and assess how well they:

- use intersecting lines to locate objects on a coordinate grid;
- apply appropriate strategies to play the game;
- explain their strategy;
- judge the efficiency of various strategies;
- modify or change strategies to find more efficient strategies.

HOME CONNECTION

Send home **Loc5.BLM4: Planning a Trip**, in which parents are asked to use life experiences to assist their child's understanding of location in the real world.

LEARNING CONNECTION 1
What's on My Mind?

MATERIALS

- **Loc5.BLM5: What's on My Mind?** (1 per student)
- Transparency of the **Loc5.BLM5: What's on My Mind?** game board

To assess prior knowledge, use the overhead transparency of **Loc5.BLM5: What's on My Mind?** Start at one of the four shapes (hexagon, square, trapezoid, triangle) and provide the directions – for example, for the first letter of a selected name (e.g., south 1 and east 3). Write down the directions on the overhead transparency or board, and verbalize the directions. Encourage students to write down the letters privately to keep track. Continue until someone guesses the correct name. Review the written directions with the class, writing in the correct letter next to each of the clues.

Let the student with the correct answer be the next caller. Provide other categories, such as cities, countries, animals, plants, colours.

Each student receives a copy of **Loc5.BLM5: What's on My Mind?** game board and instructions. Students play in pairs.

Before starting, each pair selects its own category. Each player selects a name in that category and writes down the directions for each letter of the name, as was previously demonstrated on the overhead projector or board. For example:

M – From the square, west 3, south 3

A – From the triangle, east 1, north 5

R – From the trapezoid, south 3, east 4

Y – From the hexagon, north 4, west 3

LEARNING CONNECTION 2
Drawing Directions

MATERIALS

- **Loc5.BLM6: Drawing Directions**
- rulers

This investigation builds on the idea that a coordinate grid system can be used to describe the position of an object. The focus changes from a location based on cells to identify an area to a location based on the intersection of lines to identify a specific point.

Organize the students in pairs for this activity. Hand out **Loc5.BLM6: Drawing Directions** (one copy per student). Review the instructions for the activity.

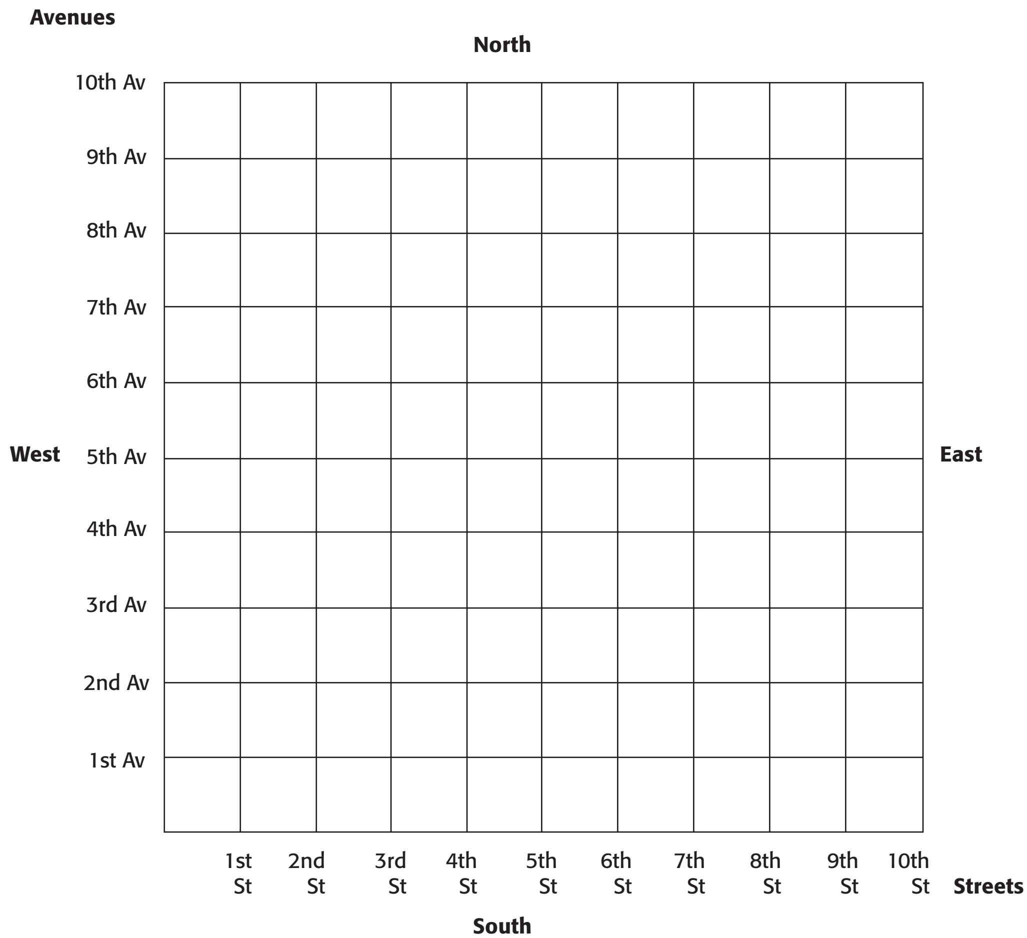
Observe the students as they work on this activity, and make note of the different methods they use (e.g., drawing a rectangle with vertices inside cells, measuring the rectangle with a ruler, drawing a rectangle with vertices on the intersection of lines).

You may need to use leading questions to encourage some students to label their grids or use the intersection of lines instead of cells. Choose some groups of students to present their strategies to the class.

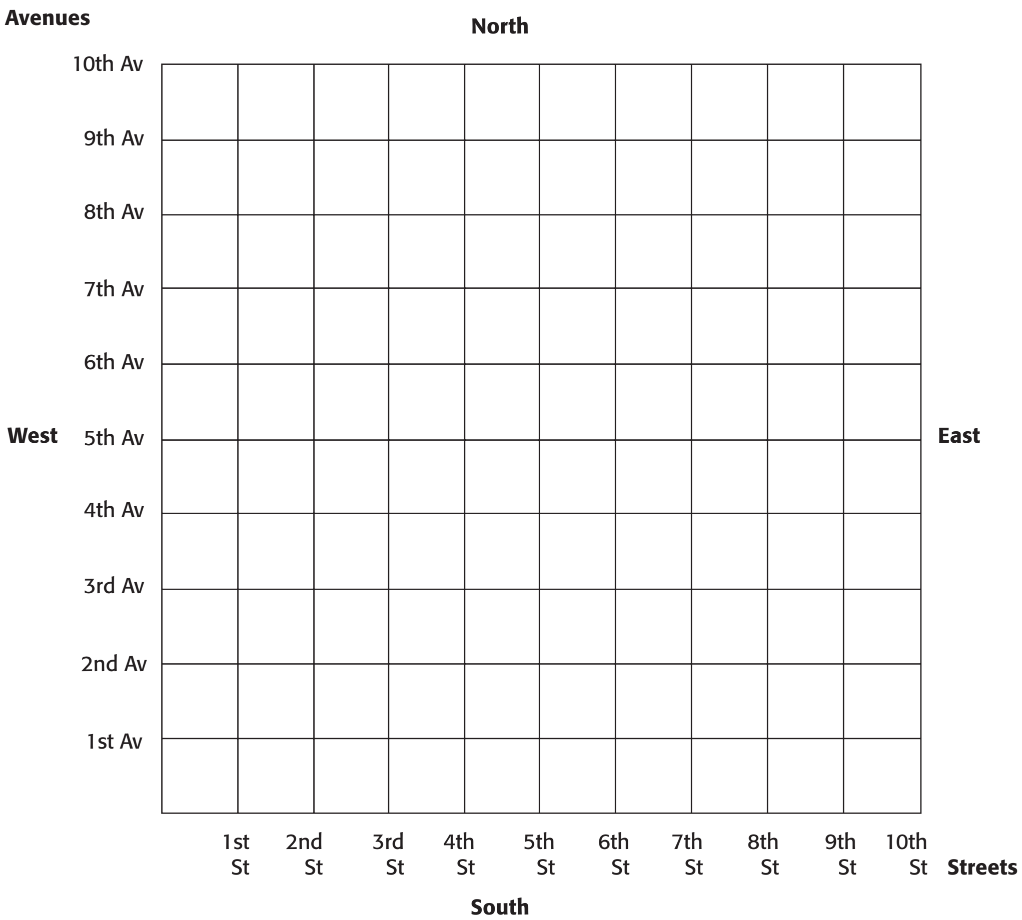
After the students have finished, ask selected students to explain their strategies to the class. Include a variety of strategies. After a variety of strategies have been presented, ask the following questions to help students evaluate the different methods:

- "What strategies did you use to create a shape?"
- "Which strategies were easy to use and efficient? Why?"
- "Which strategies are similar? How are they alike? How are they different?"
- "What strategy would you use if you were to solve the problem again? Why?"

City Treasure Hunt Map



City Treasure Hunt Planning Sheet



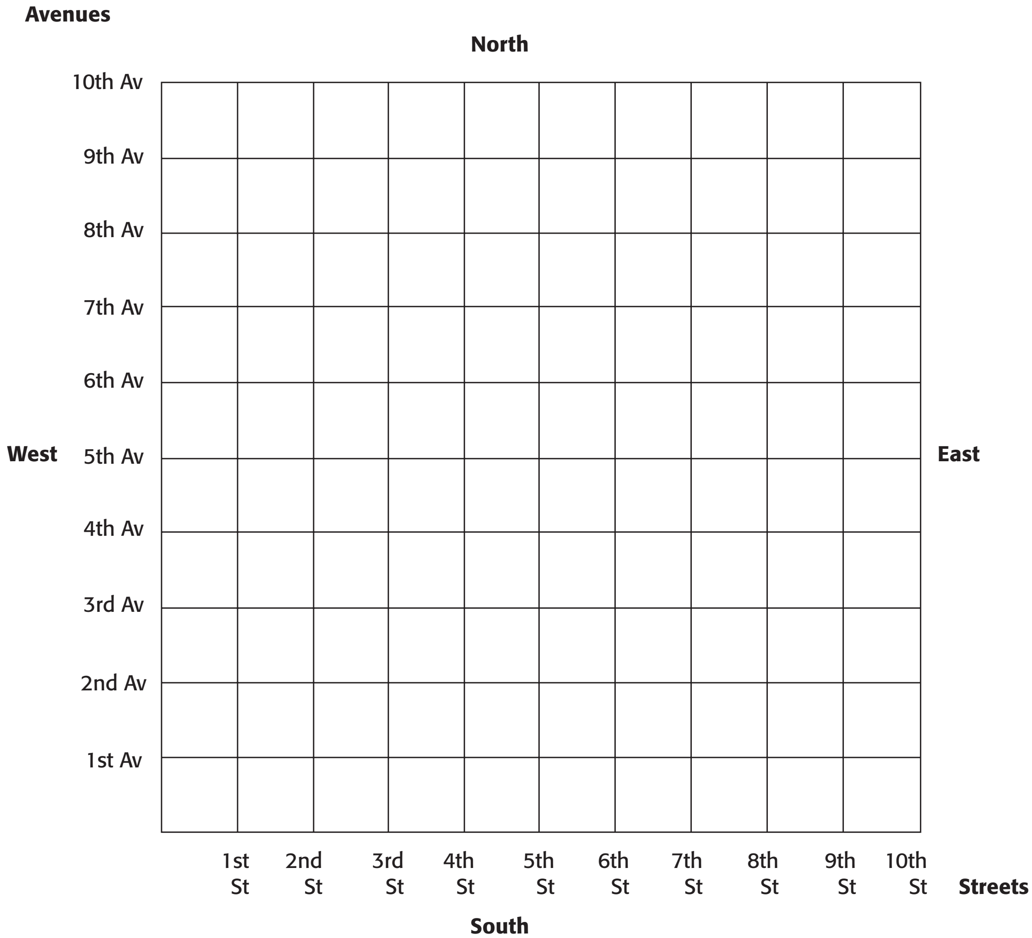
In your City Treasure Hunt you need to provide for:

- a total travel distance of between 40 and 50 blocks when treasures are collected in order;
- a total travel distance of less than 30 blocks when treasures are collected in the shortest order;
- five treasure points;
- directions that send the treasure hunter east or west by streets, and then north or south by avenues (e.g., "Travel 3 blocks east and 2 blocks north").

Start at the intersection of _______ and _______ and label it S.

Distance and direction	Treasure	Label	# of blocks
1. Travel _______________ and ______________.	You will find _____________ .	Label it _____ .	_____
2. Travel _______________ and ______________.	You will find _____________ .	Label it _____.	_____
3. Travel _______________ and ______________.	You will find _____________ .	Label it _____.	_____
4. Travel _______________ and ______________.	You will find _____________ .	Label it _____.	_____
5. Travel _______________ and ______________.	You will find _____________ .	Label it _____.	_____
6. Travel _______________ and ______________.	You will be at the finish. Label it F.		

City Treasure Hunt Answer Sheet



Label the start **S** on your grid.

1. Mark a letter ___ on your grid for the location of the first clue.

How many blocks did you travel? ___

2. Mark a letter ___ on your grid for the location of the second clue.

How many blocks did you travel? ___

3. Mark a letter ___ on your grid for the location of the third clue.

How many blocks did you travel? ___

4. Mark a letter ___ on your grid for the location of the fourth clue.

How many blocks did you travel? ___

5. Mark a letter ___ on your grid for the location of the fifth clue.

How many blocks did you travel? ___

6. Label the finish F on your grid. How many blocks did you travel? __

Name the total number of blocks travelled. ___

Collect the treasures in any order. What is the least number of blocks from start to finish? ___

Planning a Trip

Dear Parent/Guardian:

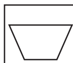
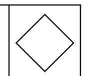
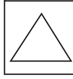
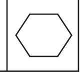
In geometry we have been working on how locations can be found in cities and on maps. The next time you take a trip, perhaps you could ask your child to plan the route, taking into consideration directions such as north, south, east, and west. Your child could calculate the number of city blocks for the trip, or the number of kilometres for a longer trip.

Having your child find specific places on the basis of your directions will also give him or her a good understanding of distance as well as direction.

Thank you for helping to show that studying location in geometry is important, and helping your child to make links between school math and real-world math.

What's on My Mind?

North

			X			
	A	G	W	Q	K	
	V	B	Y	I	Z	
	E	T	M	R	O	
	P	D	N	S	L	
	U	F	C	H	J	
						

West — East

South

INSTRUCTIONS FOR WHAT'S ON MY MIND?

Students play in pairs or groups of three.

The students select a category – for example, names of people, countries, or animals.

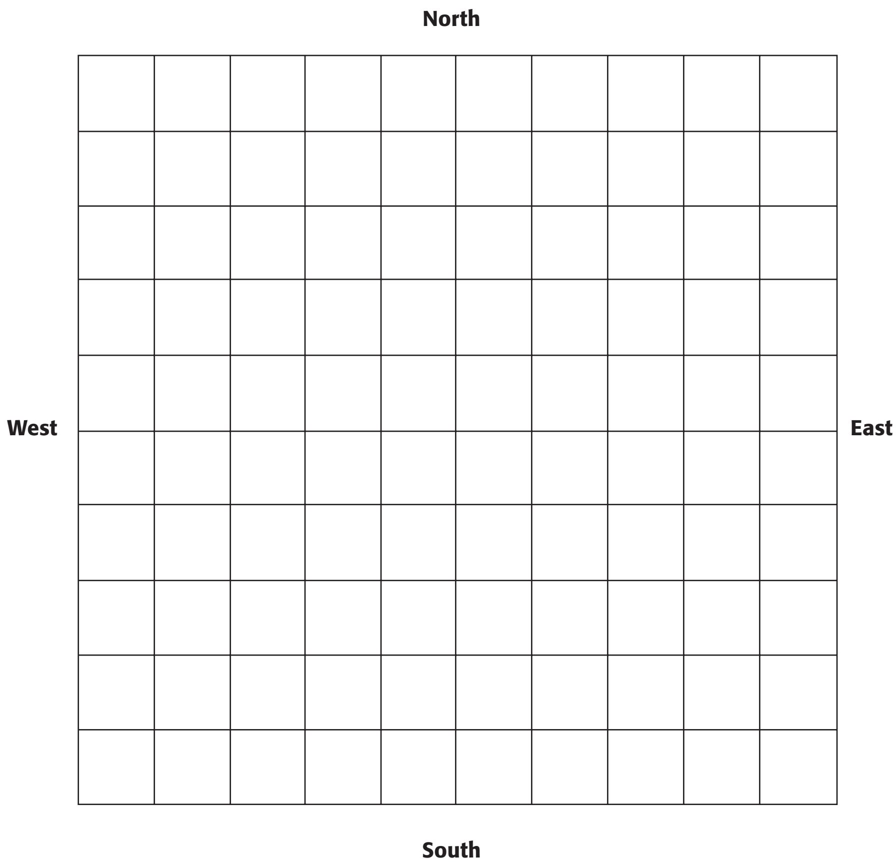
Each student secretly writes down a word that he or she is thinking about in that category, and writes the sequence of directions needed to reach the letters of the name. The letters can be scrambled or out of order to add a challenge.

Getting to each new letter in the secret word should start at a shape, followed by the directions from that shape, for example:

- **S** – From the triangle, east 4 and north 2
- **A** – From the square, west 5 and south 1
- **M** – From the hexagon, west 3 and north 3

The students decide who reads his or her clues first. That student keeps providing directions for new letters until someone else in the group guesses the name. Each of the other students in the group is allowed only one guess, so students should not be too quick to say a name. The person who guesses the name first is the next person to call out the clues.

Drawing Directions



INSTRUCTIONS FOR DRAWING DIRECTIONS

Work with a partner.

Draw a shape or simple design on the grid.

Write directions for drawing the shape or design so that someone following the directions using a blank grid could make the exact same shape or design in the exact same location without seeing your shape or design.

Grade 5 Learning Activity Movement: Drawing Designs

OVERVIEW

In this learning activity, students create a picture puzzle with directions containing translations. The activity provides an opportunity for students to identify and perform translations and to make predictions; it reinforces the concept of using a grid system to describe the movements of an object.

BIG IDEAS

This learning activity focuses on the following big idea:

Location and movement: Students investigate using different transformations to describe the movement of an object.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.

Students will:

- identify, perform, and describe translations, using a variety of tools (e.g., geoboard, dot paper, computer program).

This specific expectation contributes to the development of the following **overall expectation**:

Students will:

- identify and describe the location of an object, using the cardinal directions, and translate two-dimensional shapes.

ABOUT THE LEARNING ACTIVITY

TIME: approximately two 60-minute periods

MATERIALS

- overhead projector
- transparency of **Mov5.BLM1: Translations**
- clear piece of acetate
- **Mov5.BLM2: What Am I?** (1 per student)
- sheets of tracing paper (1 per student)
- **Mov5.BLM3: Design Instructions** (1 per student)
- sheets of grid chart paper or large newsprint (2 per student)
- **Mov5.BLM4: Translations in the Home** (1 per student)
- markers
- ruler or metre stick (1 per pair of students)

MATH LANGUAGE

- triangle
- trapezoid
- pentagon
- hexagon
- circle
- translation
- congruent
- orientation
- position
- size
- shape
- prove

INSTRUCTIONAL SEQUENCING

This learning activity provides an introduction to combining translations with a grid system to locate and move objects to specific locations. Before starting this activity, students should review cardinal directions and the use of a grid – for example, by doing Learning Activity 1 or a similar activity.

INSTRUCTIONAL GROUPING: pairs

ABOUT THE MATH

It would be beneficial for students to have experiences with some of the activities in the Location portion of this guide before attempting this activity. This Drawing Designs learning activity reinforces the use of a coordinate grid, and provides opportunities for exploring congruency as it relates to the translation of shapes.

GETTING STARTED

Have a brief discussion of the term *slide* and about where slides might be seen in the world (e.g., playground slide, hockey player on ice).

Introduce the term *translation (slide)*, using an overhead transparency of **Mov5.BLM1: Translations**.

Place a clear piece of acetate on top of the transparency, and use a coloured overhead marker to trace around the triangle. Mark an arrow in the direction in which you are going to translate the triangle. Translate (slide) the triangle along the path of the marked arrow.

Ask the students: "What changes?" and "What stays the same?" Discuss why the size and shape of the triangle stay the same, as does the orientation, but the position of the triangle changes. Explain that in math we use the term *congruent* when two shapes have the same size and shape. Translate the triangle along other paths, asking similar questions.

Ask students if there is a way to describe the direction and length of the translation (number of squares, cardinal directions).

Distribute copies of **Mov5.BLM2: What Am I?** and tracing paper. Review the instructions with the students. Have the students complete the activity in pairs. Observe the various strategies that students use during the activity (e.g., using an edge of a shape to count the squares, numbering the grid). After students have completed the activity, lead a class discussion, asking questions such as the following:

- "What strategies did you use to translate the shapes?"
- "How did you know the shape was in the correct location?"
- "What strategy saved time during the task?"
- "What strategy would you use next time?"

WORKING ON IT

Organize students into pairs for this activity.

Hand out a copy of **Mov5.BLM3: Design Instructions** and one sheet of grid chart paper to each student. Inform students that they will be using pattern blocks or their own shapes to create their own design instructions. When they have finished the activity, the shapes should be "mixed up" on the page and the instructions they write should translate the shapes into their design.

As students work on the activity, observe the various strategies that they use. Pose questions to help students think about their strategies:

- "What strategy did you use to create your design?"
- "How did you come up with your instructions?"
- "Did you change any of your strategies? Why?"

When students have completed their designs, ask them to record their strategies on another piece of grid chart paper or large newsprint. Make a note of pairs who might share their strategies during the Reflecting and Connecting part of the lesson. Include groups who used various methods that range in their degree of efficiency (e.g., using shapes that do not fit easily on the grid, making the design first and working backwards for the instructions, giving diagonal instructions, labelling the grid with numbers).

REFLECTING AND CONNECTING

Once students have had an opportunity to complete the activity and record their strategies, bring them together to share their ideas. Try to order the presentations so that students observe inefficient strategies first, followed by more efficient methods.

As students explain their ideas, ask questions to help them to describe their strategies:

- "How did you choose your shapes?"
- "What strategy did you use to make your design?"
- "How would you explain your strategy to someone who hasn't done this?"

Avoid commenting that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students' work to emphasize geometric ideas:

- The movement of an object can be described using translation.
- A translation does not affect the size, shape, or orientation of an object.
- A coordinate grid is helpful for locating objects and describing movement.

ADAPTATIONS/EXTENSIONS

Students experiencing difficulties with this activity may use geoboards (and/or dot paper) to construct (draw) shapes according to specific oral directions from the teacher or another group member. They translate these figures according to specific oral directions (e.g., two spaces to the right and four up). Students can compare their geoboard (dot paper) constructions and translations with those of their peers to check for accuracy, and can discuss discrepancies.

ASSESSMENT

Observe students as they complete the activity and assess how well they:

- apply appropriate strategies for creating a design and the instructions for it;
- translate shapes with accuracy;
- describe the translations (length, direction);
- explain their strategy;
- judge the efficiency of various strategies.

HOME CONNECTION

Send home **Mov5.BLM4: Translations in the Home**. In this home connection students are asked to look at patterns and designs in their home and/or garden where translations have been used, and list the two most interesting translations they can find.

LEARNING CONNECTION 1
What Stays the Same?

MATERIALS

- large outside 10 × 10 grid or tiled floor
- string

Place four students on a large 10 ×10 grid or learning carpet to make a shape (e.g., a rectangle). Each student stands on the grid at a vertex of the rectangle holding the end of a piece of string. Connect the student "vertices" with one another by string "edges" to make the rectangle.

Give directions. For example: "Everyone move north three units, then everyone move west two units." After each move ask the students, "What remains the same?" and "What changes?" They will find that shape and size remain the same (congruency is preserved), as does orientation. The location changes. If the shape or size does change, talk about strategies to prevent that from happening (e.g., use a grid, use equal steps).

Use different students, change the shape to a triangle or hexagon, and repeat the procedure.

Working in groups of six or eight, students can take turns giving directions and checking on what remains the same and what changes.

LEARNING CONNECTION 2
How Many Ways?

MATERIALS

- **Mov5.BLM5: How Many Ways?** (1 per student)
- overhead transparency of **Mov5.BLM5: How Many Ways?**
- tracing paper
- Mira

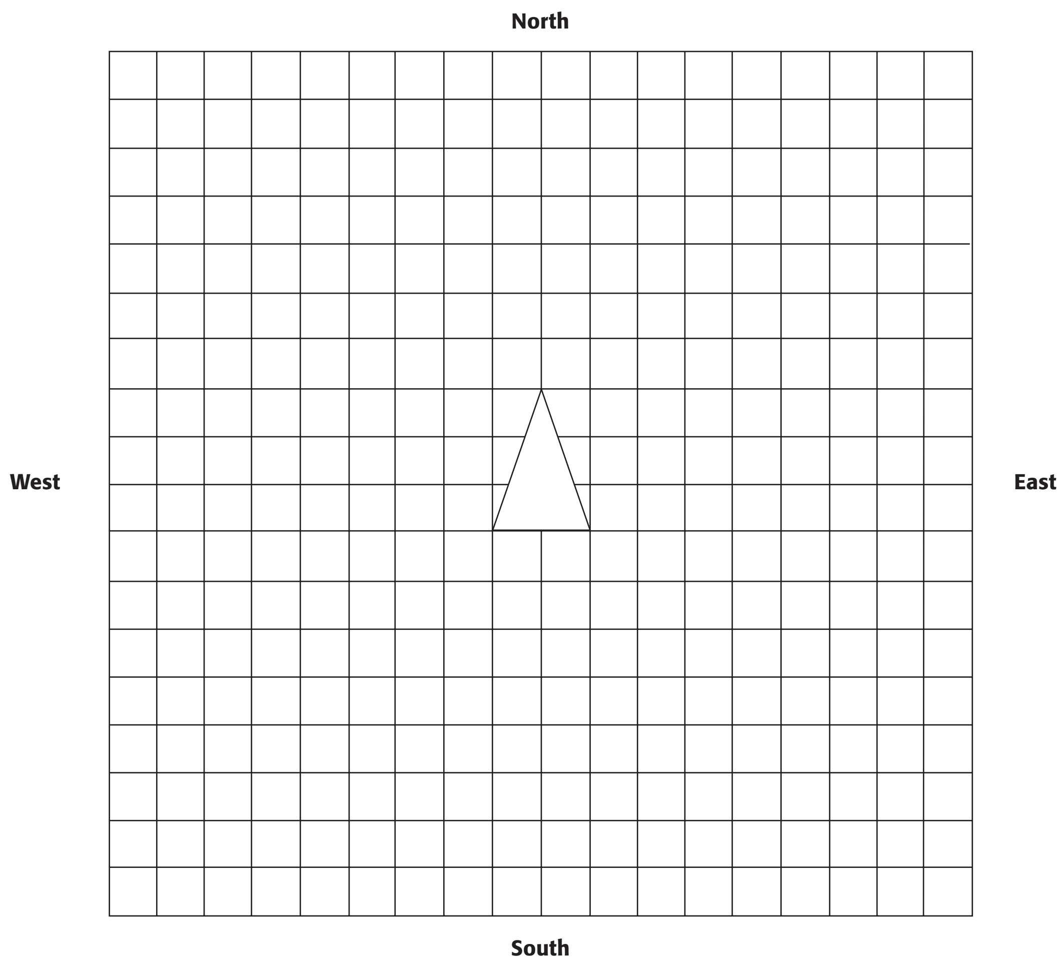
Before beginning the activity, review the following with students:

- For translations, we travel east or west before travelling north or south.
- For reflections, we can talk about a horizontal reflection over an east-west line (e.g., 6th Avenue), or a vertical reflection over a north-south line (e.g., 3rd Street).

Have students work in pairs for this activity. Hand out **Mov5.BLM5: How Many Ways?** and review the instructions. Have tracing paper and Miras available for student use.

Take up the activity with the whole class, using an overhead transparency of **Mov5.BLM5: How Many Ways?** or a large grid of the blackline master drawn on the board or chart paper. Students volunteer to show different ways of moving the shape from position to position. Discuss what remains the same and what changes during a reflection and during a translation.

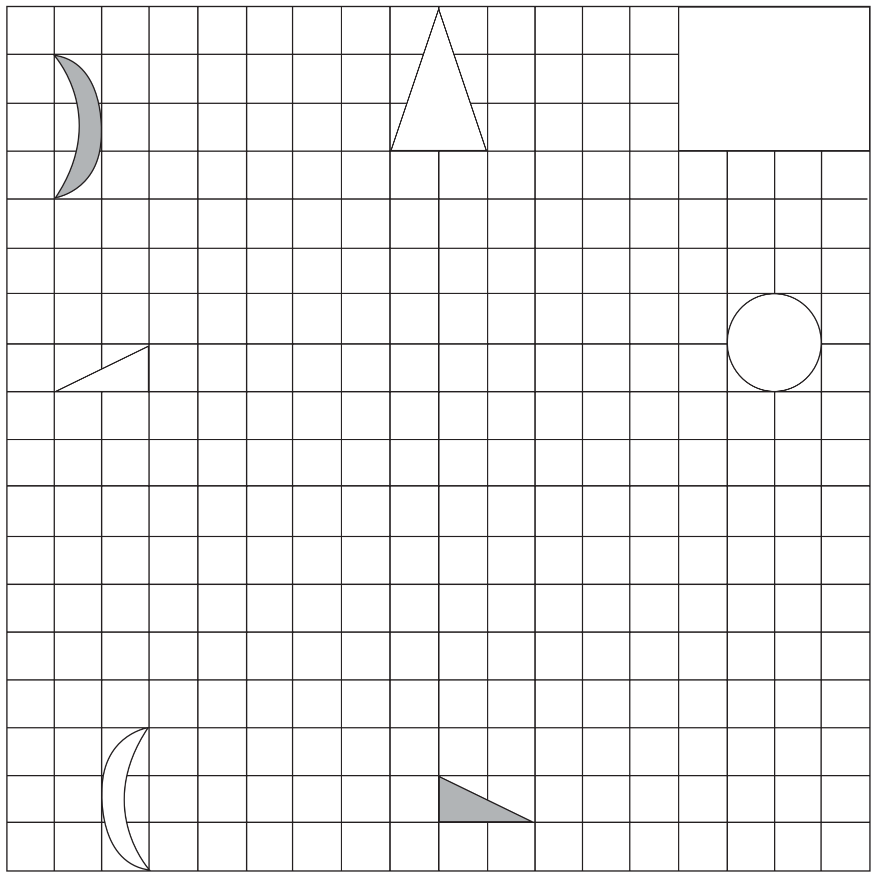
Translations



What Am I?

North

West



East

South

Use your tracing paper to trace the shapes. Then translate them, using the instructions below, and sketch them in the new location.

1. Translate the circle 7 blocks west.
2. Translate the shaded half-moon 9 blocks east and 10 blocks south.
3. Translate the shaded right-angled triangle 4 blocks west and 8 blocks north.
4. Translate the white right-angled triangle 10 blocks east and 1 block south.
5. Translate the white half-moon 5 blocks east and 4 blocks north.
6. Translate the rectangle 7 blocks west and 8 blocks south.
7. Translate the isosceles triangle 3 blocks south.

Design Instructions

MATERIALS

- pattern blocks or other shapes
- grid chart paper
- markers
- ruler or metre stick

Use a ruler or metre stick and a marker to make a line that divides your grid chart paper in half.

On the top half of the grid chart paper:

– Create a design, using pattern blocks or your own shapes. Trace your design lightly in pencil to show the location.
– Move the pattern blocks or shapes to different locations on the grid. Use a marker to trace each new location.

On the bottom half of the grid chart paper:

– Create a set of instructions that will translate the shapes to make your design.
– Start each instruction with "translate" and use directions and numbers.
For example: Translate the triangle 4 blocks north and 6 blocks east.

Erase the pencil tracing of your design.

Translations in the Home

Dear Parent/Guardian:

At the moment we are studying geometry in math. Today your child was introduced to a new geometry word, *translation*.

The word *translation* in geometry has a very different meaning from the one used in everyday language, when we talk about a translation from one language into another. In math, translation refers to sliding a shape or an object along a straight path. For example:

In geometry, people say that the triangle on the left has been translated to the right.

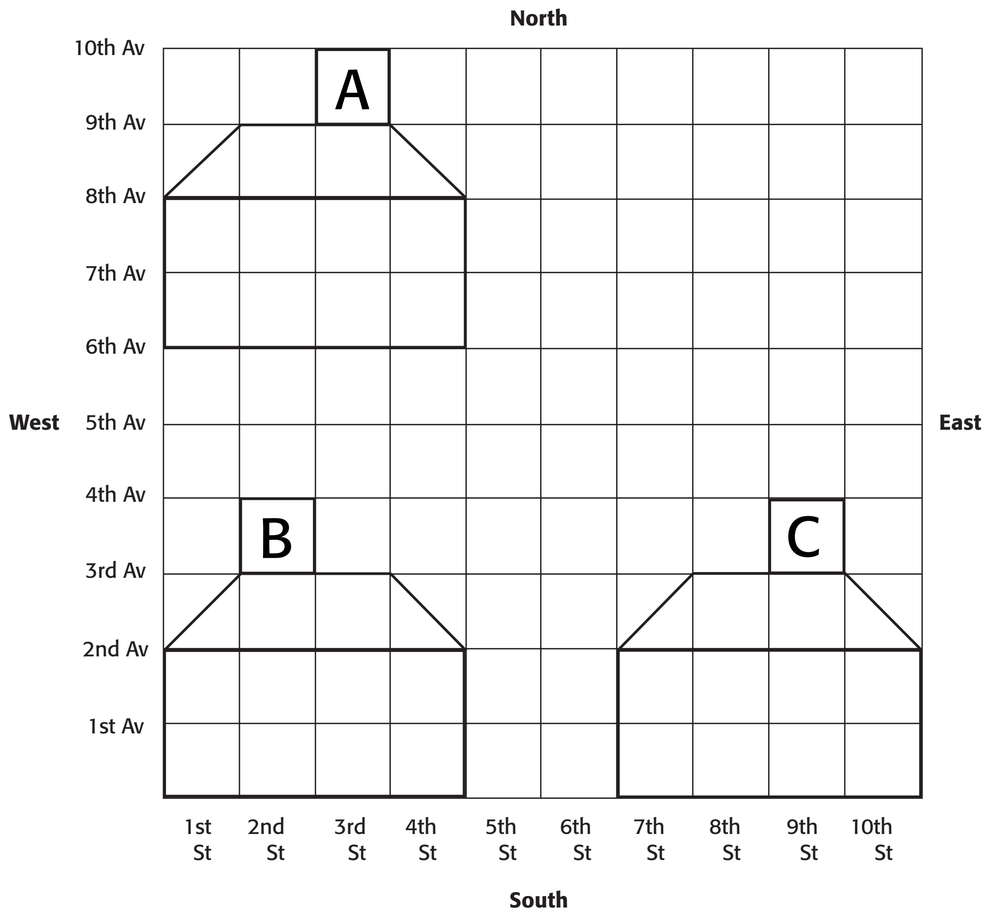
Ask your child to find some designs in the home or garden in which translations (slides) have been used to make patterns (e.g., wallpaper, designs on clothes, rows in the vegetable garden). Your child is also encouraged to look through magazines or newspapers for translations.

On the back of this letter, or on a separate piece of paper, ask your child to draw the two most interesting translations found in your home or garden, describe where they were found, and bring the answers to class tomorrow.

Any time you can give to this activity and to talking with your child about the idea of translation will benefit your child's mathematical growth and development.

Thank you for any help you have the time to provide.

How Many Ways?



INSTRUCTIONS

List all the ways in which you can move the house from:

Position A to Position B

Position B to Position C

Position A to Position C

Grade 6 Learning Activity Two-Dimensional Shapes: Connect the Dots

OVERVIEW

In this learning activity, students use dynamic geometry software to construct and measure angles. They develop their understanding of benchmark angles, and they construct polygons, given side length and angle restrictions.

BIG IDEAS

This learning activity focuses on the following big ideas:

Properties of two-dimensional shapes and three-dimensional figures: Students develop a sense of the size of benchmark angles through investigation. Concrete materials and the classroom environment provide measuring tools with which to compare angles to benchmarks.

Geometric relationships: Students explore the relationships between various properties of polygons, and of quadrilaterals in particular. For example, if a polygon has all congruent sides, it also has congruent angles.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- measure and construct angles up to 180° using a protractor, and classify them as acute, right, obtuse, or straight angles;
- construct polygons using a variety of tools, given angle and side measurements.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- classify and construct polygons and angles.

TIME: approximately 80 minutes

ABOUT THE LEARNING ACTIVITY

MATERIALS

- computer and data projector
- The Geometer's Sketchpad dynamic geometry software
- Gr6AngleDemo.gsp file (downloadable from www.edu.gov.on.ca/eng/studentsuccess/lms/library.html)

- number cubes (dice) (1 per pair of students)
- **2D6.BLM1: Angle Cards** (1 per pair of students) to be cut out, shuffled, and placed face down as a deck
- **2D6.BLM2: Shape Cards** (1 per pair of students) to be cut out, shuffled, and placed face down as a deck
- protractors, rulers, and pencils if needed
- **2D6.BLM3: Measuring Angles and Lengths in GSP** (as needed)
- **2D6.BLM4: Quadrilateral Angle Properties** (1 per student)

MATH LANGUAGE

- point
- line segment
- angle
- ray
- vertex
- protractor
- polygon
- triangle
- rectangle
- square
- rhombus
- trapezoid
- parallelogram
- quadrilateral
- kite

INSTRUCTIONAL GROUPING: pairs

INSTRUCTIONAL SEQUENCING

Students should have had significant experiences constructing and measuring angles up to 90°, using pencil, paper, ruler, and protractors. They should also have had experience using The Geometer's Sketchpad (GSP) software and be familiar with the basic functions of the program. For GSP-based tutorials to help students learn to use The Geometer's Sketchpad, refer to the Curriculum Services Canada website for TIPS resources at http://www.curriculum.org/csc/library/tips/section7details.shtml#sketchpad.

ABOUT THE MATH

In Grade 6 students continue to sort and classify quadrilaterals according to geometric properties. Students construct polygons, using geoboards, dynamic geometry software, and protractors. The construction of angles is extended to include angles from 0° to 180°. As students investigate and construct angles, they begin to apply their knowledge of angles to the properties of polygons.

In geometry, there is an important distinction between *drawing* and *constructing*. The Geometer's Sketchpad helps to illustrate this distinction. A constructed shape or geometric situation will remain unchanged, but a drawn one may not. For example, students may construct an equilateral triangle in GSP and can manipulate it to change its size. However, it will always remain equilateral, regardless of side length. Students may draw an equilateral triangle in GSP, using the angle measure or length measure function, but when they move a point on the triangle, the type of triangle changes. Equal angles and side lengths are not preserved.

In Getting Started, students see some of the features of Sketchpad, but more importantly, they build on their knowledge of benchmark angles to include 45° and 135° angles – half of a right angle, and one and a half right angles.

GETTING STARTED

This part of the lesson can be presented to the whole class as a teacher demonstration, or students can work with a partner at a computer and work through the activity. If students are to work on the file, it must be placed on a shared drive to which students have access. This part of the lesson is presented here as a teacher demonstration.

Using a computer and a data projector, open The Geometer's Sketchpad. Activate students' prior use of the program by asking:

- "Who remembers what we've used this program for in the past?"
- "Can you tell me what some of the tools on the left are called, and what they do?"
- "What are some of the reasons why we use The Geometer's Sketchpad in addition to pencil-and-paper geometry?"

Open the demonstration file called "Gr6AngleDemo.gsp file". This file is available as a download from www.edu.gov.on.ca/eng/studentsuccess/lms/library.html.

The file shows two segments joined at a vertex to form an angle, ∠ABC. Ask students to identify the geometric features they see – points, vertex, segments, and angle. Explain that the action buttons perform the actions described (e.g., clicking on the "Rotate segment AB in a clockwise direction" button starts the rotation, and clicking again on the button stops it; clicking on the "Reset" button returns the sketch to its original appearance). Ask the following questions:

- "What do you think the measure of ∠ABC is?" (45° – show the angle measure, then hide it again)
- "What type of angle is it?" (an acute angle)
- "I'm going to rotate segment AB counterclockwise. I want you to tell me to stop when you think ∠ABC is a right angle."

Stop the rotation when the students tell you to, and show the measure of the angle. Hide the angle again, and rotate segment AB counterclockwise to approximate one and a half right angles, or 135°.

Continue manipulating ∠ABC, using the action buttons and asking students to predict the measures before revealing them to the class. Future useful benchmark angles include 30°, 60°, and 120°.

Consider manipulating ∠ABC by dragging points instead of using the buttons, to illustrate how you can:

- create angles of different sizes (by dragging a point);
- make the lengths of the line segments different but have the angle remain the same (e.g., by dragging point C to make segment BC longer);
- rotate the angle (by dragging point C and point A to create a 45° angle in which segment BC is not horizontal).

Before pairing students off for Working on It, demonstrate some examples of polygon constructions by clicking on the "Show a triangle with this interior angle" and "Show a quadrilateral with this interior angle" buttons. By clicking on the "More" button, students can attempt to use ∠ABC to construct their own polygons. When you close the sketch or exit The Geometer's Sketchpad, a prompt may appear asking whether you want to save any changes to the sketch. Select "Don't Save" in order to keep the sketch in its original form.

WORKING ON IT

Students are to be paired up on a single computer for this activity. Each pair needs a number cube (die), **2D6.BLM1: Angle Cards**, and **2D6.BLM2: Shape Cards**.

Students roll the number cube to determine the length of one side of the polygon. Next, students select an angle card from the shuffled deck to determine the angle, and a shape card to determine the shape. The challenge is to use the angle and side length at least once in constructing a polygon with software, or if there are not enough computers, with a protractor, ruler, and pencil. Students may require **2D6.BLM3: Measuring Angles and Lengths in GSP** to help with some of the features of The Geometer's Sketchpad.

As students investigate the construction of each polygon, they will begin to discover that some shapes cannot be created, given the particular side and angle components.

After the students have spent some time on the activity, have them separate the quadrilateral cards from the other shape cards and use only quadrilaterals as they continue with the activity. Students can then reflect on the challenges they encounter in creating quadrilaterals.

REFLECTING AND CONNECTING

Ask the following questions to help students think about the task:

- "What properties of a polygon/quadrilateral do you need to know in order to construct it?"
- "What polygons/quadrilaterals were easiest to construct?"
- "What did you discover about the polygons/quadrilaterals you tried to construct?"
- "What surprised you about some of the constructions?"

Ask students to identify some of the tasks that were impossible to complete. For example, if students drew a right-angled triangle card and an angle measure card of 135°, they would find it impossible to construct the polygon. For each instance, ask students to provide an explanation of why they were unable to do the construction.

Talk with the students about some of the properties of specific quadrilaterals. Refer to side lengths, interior angles, and the location of lines with respect to one another (e.g., parallel, adjacent). Ask:

- "What can you tell me about all of the quadrilaterals you constructed?"
- "How were they the same and how were they different?"
- "Were there properties that were related? For example, does knowing something about side length tell you something about the angles?"

If this is the first time the students have used Sketchpad, spend some time discussing the program with them, identifying the advantages and disadvantages of the program over traditional pencil-and-paper constructions.

ADAPTIONS/EXTENSIONS

During the Working on It some students may benefit from using pencils, rulers, and protractors for the activities rather than the software. Also, they may need to work with polygons that have already been partially constructed. For example, they could be given two side lengths and the included angle and asked to construct a parallelogram.

Challenge students to create polygon features cards (angle measures, side lengths, polygon names) to create a game that can be played by two or more students.

ASSESSMENT

Observe students to assess how well they:

- measure, construct, and sort acute, right, obtuse, or straight angles;
- describe the construction of polygons with a focus on properties (angle measurement, length and number of sides).

During the reflection, note how well students make connections to the properties of quadrilaterals after constructing quadrilaterals. Students will be able to perform the construction task without thinking too deeply about properties and their relationship to each other. Probing questions can help assess the connections that they make.

- "Why were you unable to construct some of the shapes on the cards?"
- "What could you have changed to be able to make the construction?"
- "Given the length and angle measure, what possible quadrilaterals could you construct?"

HOME CONNECTION

Send home **2D6.BLM4: Quadrilateral Angle Properties**. Students are to name each quadrilateral, measure the angles in it, and write a definition of it using angle properties.

Students will need to take protractors home to complete the task. Remind students to be specific when naming the quadrilaterals (e.g., right trapezoid, isosceles trapezoid).

LEARNING CONNECTION 1
Rotating Polygons

MATERIALS

- **2D6.BLM5: Rotating Polygons** (1 per pair of students)
- tracing paper, cut into eighths (12 pieces per pair of students)
- math journals

Have students work with a partner. Provide each pair with a copy of **2D6.BLM5: Rotating Polygons** and 12 pieces of the tracing paper.

Students are to place one of the small pieces of tracing paper over one of the polygons, and use a ruler to trace it. They then rotate the traced shape over the original polygon, noting how many times the traced shape matches the orientation of the original polygon.

The partner records in his or her math journal the name of the shape and the number of times the tracing matched the original. Partners take turns recording and tracing/rotating until they have completed the process with all of the polygons.

Bring the whole class together for a discussion. Ask which shapes matched the original position only once during a complete rotation. Explain that those polygons do not have rotational symmetry, because in one rotation, they matched the original position one time.

Identify with the class the polygons with an order of rotational symmetry greater than 1. Give the student pairs time to discuss those polygons and see if they can find any patterns or commonalities between them. Students should discover that regular polygons have an order of rotational symmetry equal to their number of sides.

As an extension, challenge students to create polygons that have rotational symmetry. For example, ask them to draw a hexagon that is not regular and has rotational symmetry.

LEARNING CONNECTION 2
What's My Line

MATERIALS

- **2D6.BLM6: Quadrilateral Cards** (1 per group of students)
- **2D6.BLM7: What's My Line Place Mat** (1 per group of students)
- Miras (1 per group of students)
- poster paper

Students work in groups of two or three. Provide each group with a copy of **2D6.BLM6: Quadrilateral Cards** and **2D6.BLM7: What's My Line Place Mat**. Students are to cut out the cards and place each card in the appropriate quadrant of the place mat, depending on the number and type of lines of symmetry the quadrilateral has. Quadrilaterals with no line of symmetry can be left off the mat.

Once the quadrilaterals have been sorted, students use Miras to see if they have been placed correctly in the place mat. Students can also use the Mira to help with sorting.

Have students write a summary on poster paper of their discoveries during the activity. Have them identify any common characteristics of quadrilaterals with the same number of lines of symmetry and then write a description for that group (e.g., all rectangles that are not squares have two lines of symmetry – a horizontal line and a vertical line).

Initiate a whole-class discussion about the sorting of the quadrilaterals. Ask:

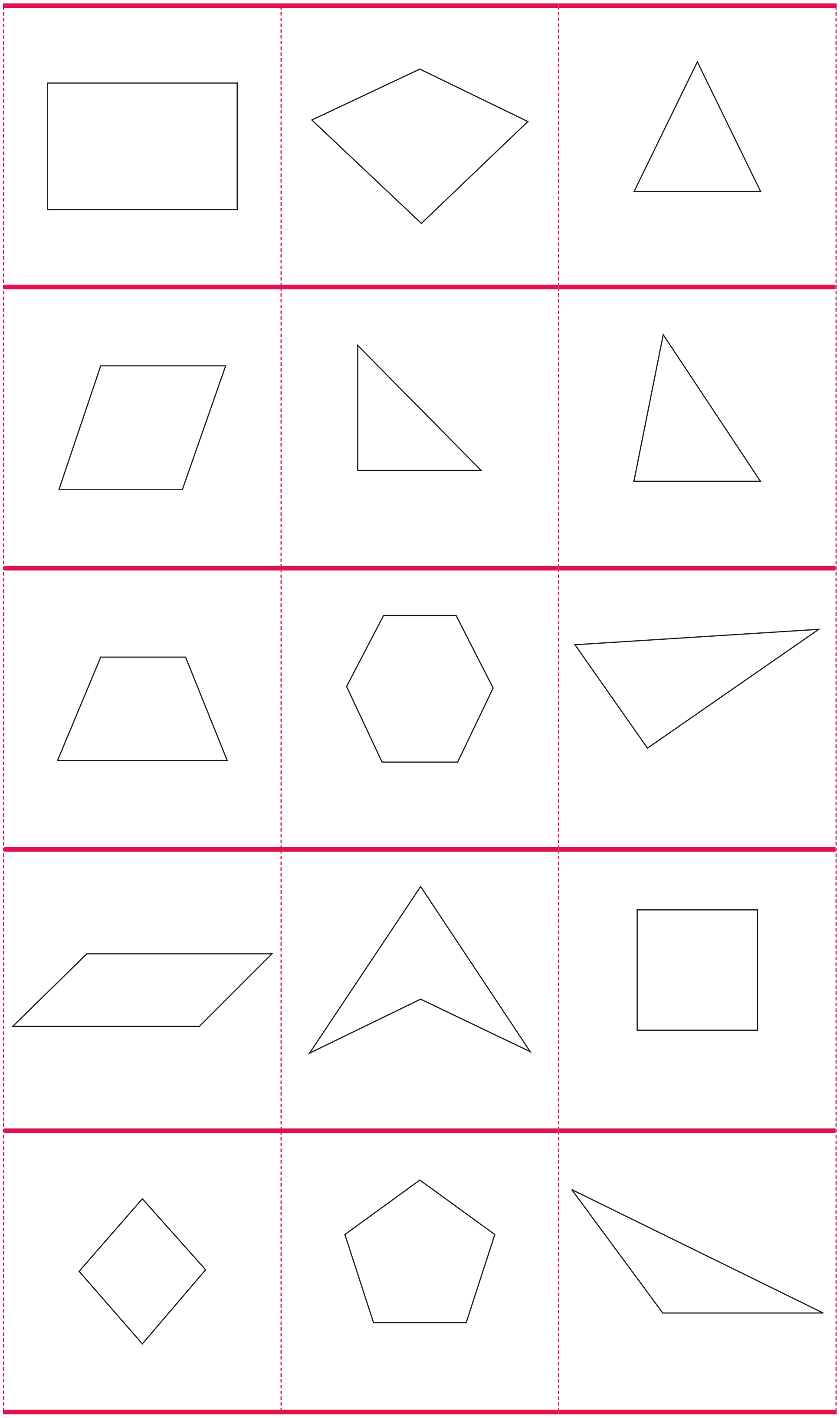
- "Do you think your rule will hold true for all ____________________ (rectangles, kites, etc.)?"
- "Which quadrilaterals have no line of symmetry? Can you make a generalization about those?"
- "How specific do you need to be when making your rules? For example, can you talk about all trapezoids, or just specific types?"

Students should recognize that using lines of symmetry is an alternative method of classifying quadrilaterals.

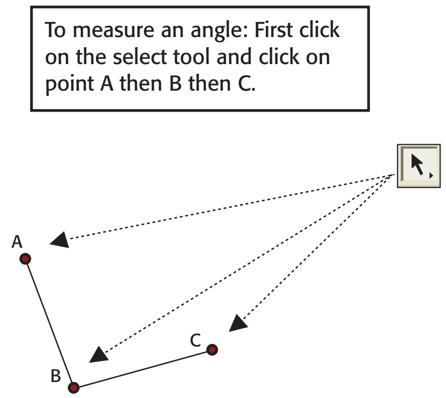
Angle Cards

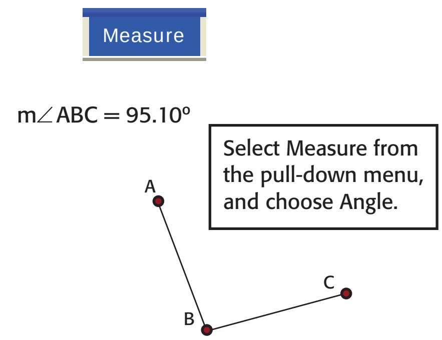
45°	90°	25°
30°	120°	60°
135°	15°	55°
115°	180°	25°
35°	100°	40°

Shape Cards

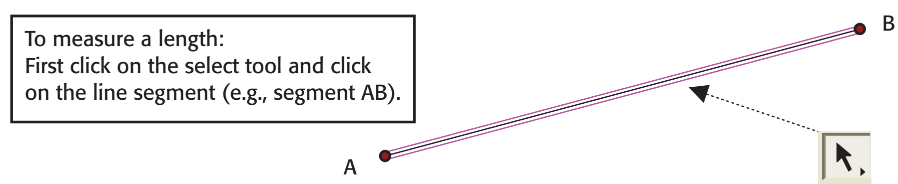


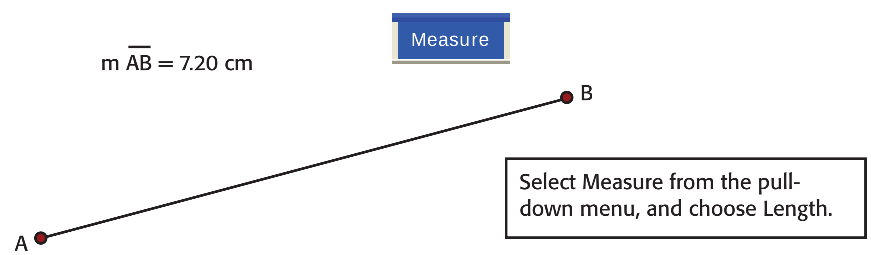
Measuring Angles and Lengths in GSP





Note that you must click on the points in the proper order – the point that is the vertex must be selected second, or you will not get the Angle option from the pull-down Measure menu.

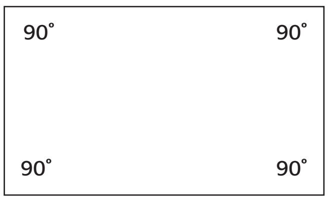
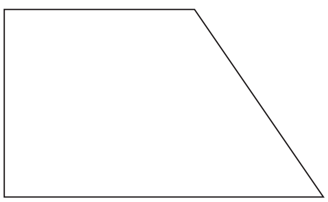
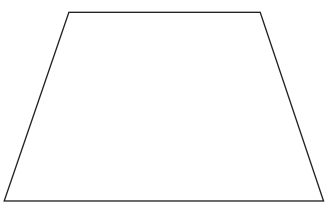
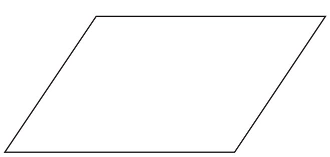
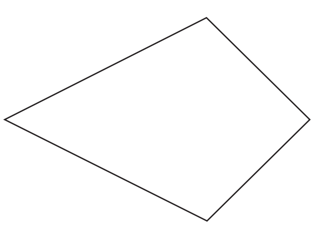




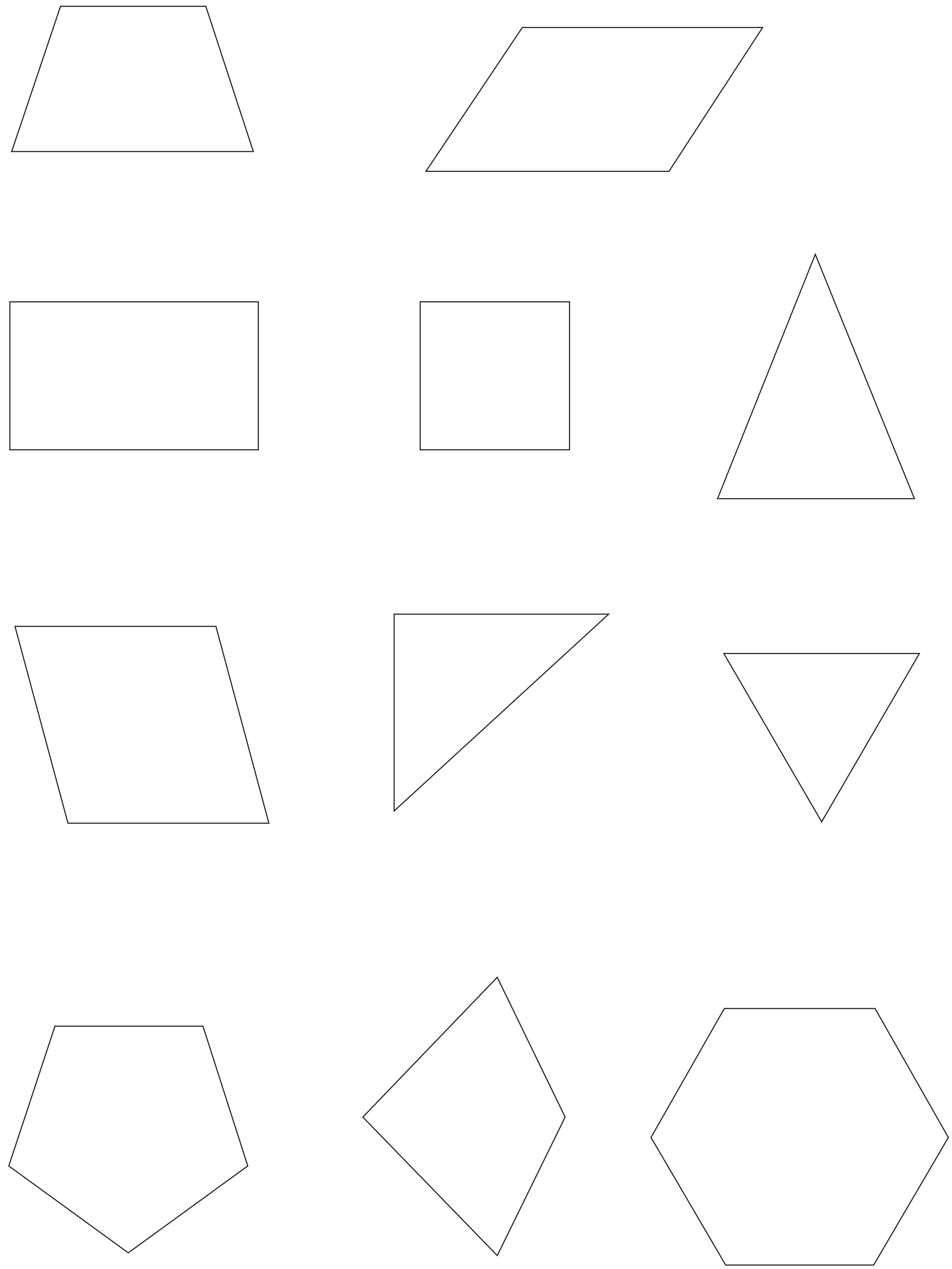
Alternatively, you can right-click on the segment, and choose Length from the menu that appears.

Quadrilateral Angle Properties

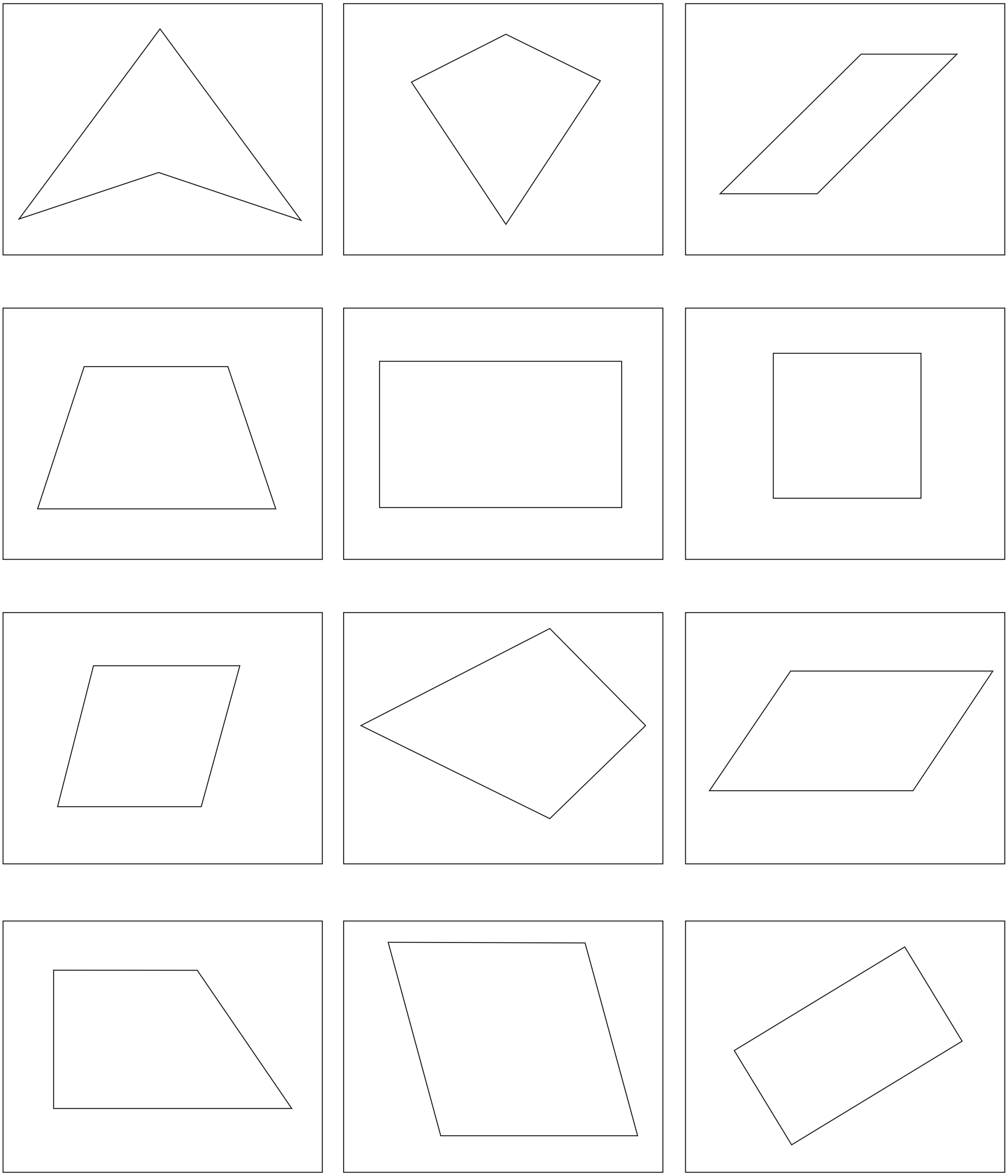
Complete the following chart by naming each quadrilateral, measuring and recording the angles in it, and writing a definition of it using its angle properties. Be specific when you name each quadrilateral. An example is done for you.

	Rectangle	A rectangle is a quadrilateral with four 90° angles.
		
		
		
		

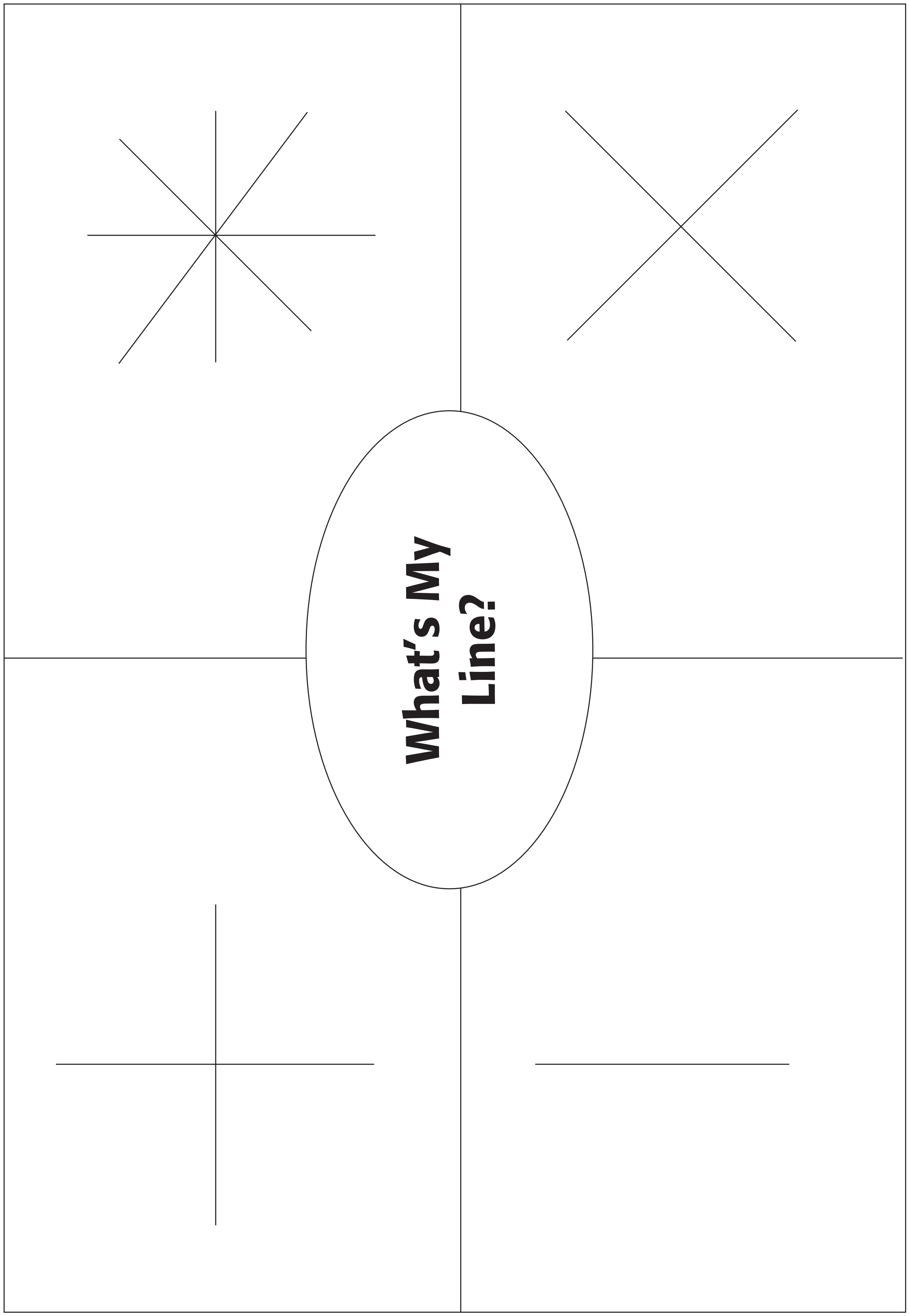
Rotating Polygons



Quadrilateral Cards



What's My Line Place Mat



Grade 6 Learning Activity Three-Dimensional Figures: Sketching Climbing Structures

OVERVIEW

In this task, students build a three-dimensional model, given an isometric sketch. They sketch the different views of the structure and complete an isometric sketch of the same structure from a different perspective.

BIG IDEAS

This learning activity focuses on the following big ideas:

Properties of two-dimensional shapes and three-dimensional figures: Students develop their spatial sense through investigating the properties of three-dimensional figures, specifically looking at faces, edges, and vertices from different angles.

Geometric relationships: Students must be able to flexibly represent three-dimensional figures as concrete objects, but also in two-dimensional sketches or drawings. This activity draws upon the relationship between the two-dimensional faces of an object and its three-dimensional construction.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- build three-dimensional models using connecting cubes, given isometric sketches or different views (i.e., top, side, front) of the structure;
- sketch, using a variety of tools (e.g., isometric dot paper, dynamic geometry software), isometric perspectives and different views (i.e., top, side, front) of three-dimensional figures built with interlocking cubes.

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- sketch three-dimensional figures, and construct three-dimensional figures from drawings.

TIME: approximately 60 minutes

ABOUT THE LEARNING ACTIVITY

MATERIALS

- interlocking cubes (3 per student)
- **3D6.BLM1: 2 cm Grid Paper** (2 sheets per student)
- **3D6.BLM2: Isometric Dot Paper**, cut into half sheets (2 half sheets per student)
- overhead projector
- **3D6.BLM3: Sketching Climbing Structures** (1 per student)
- **3D6.BLM4: Sketching Isometric Perspectives** (1 per student)
- **3D6.BLM5a–b: Teach an Adult to Sketch** (1 per student)

MATH LANGUAGE

- congruent
- orthographic sketches or views (top, front, side)
- rotate
- mirror image
- two-dimensional representation
- three-dimensional figure
- isometric dot paper
- isometric perspective or view
- vertical line segment
- diagonal line segment
- horizontal line segment
- edge

INSTRUCTIONAL GROUPING: individuals, pairs

INSTRUCTIONAL SEQUENCING

This learning activity focuses on the relationships between two-dimensional drawings and three-dimension figures. Prior to this lesson, students should be familiar with building three-dimensional solids from top, front, and side views.

ABOUT THE MATH

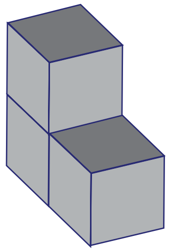
In Grade 6, students learn to draw different views (i.e., top, front, side) and sketch isometric perspectives of three-dimensional figures. They also require opportunities to build three-dimensional models, given isometric sketches or different views of a structure.

These experiences will help students develop an understanding of the relationship between three-dimensional figures and their two-dimensional representations. Students have opportunities to represent their thinking concretely and through two different types of drawings. They also have opportunities to make connections between two-dimensional sketches of three-dimensional figures in the classroom and the use of these types of two-dimensional sketches in the real world (e.g., in assembly instructions for toys, furniture, model cars or planes).

Isometric diagrams use an isometric grid. An isometric grid shows three axes instead of the two found in a rectangular grid. One axis runs vertically; the other two axes run "down" at 30° angles to the left and right.

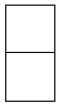
GETTING STARTED

Distribute three interlocking cubes and one sheet of **3D6.BLM1: 2 cm Grid Paper** to each student. Show students the following L-shaped structure built from interlocking cubes:



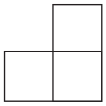
Ask students to build a congruent structure with their three interlocking cubes. Have them place the structure on the 2 cm grid paper so that the single cube is facing them. Instruct students to label the front, back, left side, and right side of the structure on the paper. (Circulate as they do so to ensure that all students have the structure oriented in the same way.)

Ask students what they see when they look at the structure from the front view. (Suggest that students look from the level of their desks to get the complete front view.) Students should see a vertical rectangle divided in half:



Have students draw the front view next to the "Front" label on their page.

Next, have students rotate their page 90° clockwise so they are looking at the right side view. Again, ask what they see (mirror image of an "L"). Have students draw this view next to the "Right Side" label on their page:



Continue having the students rotate their pages 90° clockwise to record the back and left side views. Finally, have them look down on the structure to see the top view. They should notice that the top view could be drawn by tracing around the base of the structure.

Explain that the views they have drawn are called *orthographic* views and that they are two-dimensional representations of the three-dimensional structure. Ask students to write "Orthographic Sketches" at the top of the page on which they have just completed the views.

Explain that orthographic sketches are only one type of two-dimensional representation of a three-dimensional figure. Isometric sketches, or perspective drawings, are also two-dimensional representations of three-dimensional figures. Make connections to students' previous experiences with orthographic and/or isometric sketches/representations. Ask, "When might you see/use this? How might this kind of sketch be used in the real world?" Provide examples like floor plans and elevations that architects and designers would use. You may wish to have some examples for the students to view on the overhead projector.

Distribute one half-sheet of **3D6.BLM2: Isometric Dot Paper** to each student. Discuss any prior experiences students have had of using isometric dot paper (drawing two-dimensional shapes, drawing nets). Inform students that they are now going to try to sketch an isometric view, or a three-dimensional view, of the L-shaped structure for which they have already completed orthographic sketches. Ask students to write "Isometric Sketch" at the top of their half-page of isometric dot paper.

Have students arrange the structure on their page with the orthographic views so that the front view is once again facing them. Now have them rotate the page 45° counterclockwise so that they are seeing the front-left view of the structure.

Ensure that all students have the structure oriented in the same way, and then instruct them to use the following steps to sketch the isometric perspective. Model the steps on the overhead projector as you go through them.

Step 1: Connect two dots vertically to represent the closest vertical edge of the structure (edge of single cube).

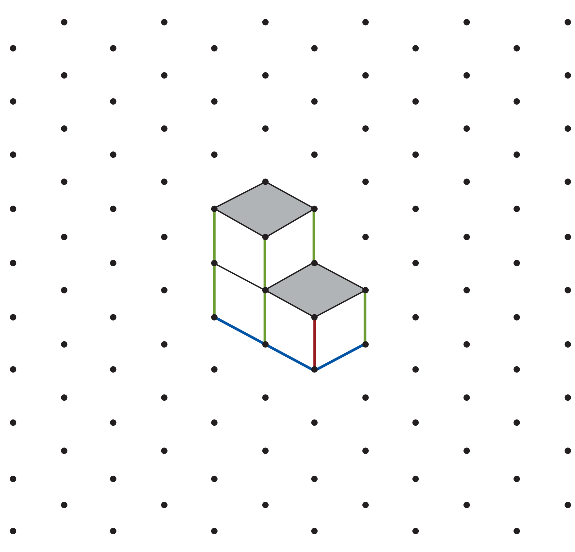
Step 2: Draw diagonal line segments to connect three dots up to the left and two dots up to the right to represent the bottom horizontal edges of the structure. Note that the line segment that you drew up to the left is twice as long as the one that you drew up to the right because it represents an edge that is two cubes long.

Step 3: Draw vertical line segments for the remaining four vertical edges.

Step 4: Complete the diagonal line segments to represent the remaining horizontal edges that are parallel to the bottom edges.

Step 5: Shade the faces of the structure that are visible in the top view only.

The finished drawing should look like this:



WORKING ON IT

Have students work in pairs. Describe the following scenario to the class:

> "A company that manufactures climbing equipment for school playgrounds is introducing new climbing blocks that can be combined to create a variety of different play structures. The designer of these blocks has sent a sketch of one structure that can be built using six of the climbing blocks. The company has asked for your help in creating top, front, and side views of the structure as well as a different isometric perspective to be included in the assembly instructions for the new climbing blocks."

Distribute to each student a copy of **3D6.BLM3: Sketching Climbing Structures**, 1 sheet of **3D6.BLM1: 2cm Grid Paper**, and a half sheet of **3D6.BLM2: Isometric Dot Paper**.

Working with a partner, students are to combine their cubes and use the Designer's Sketch shown on **3D6.BLM3: Sketching Climbing Structures** to build a model of the climbing structure. Then have each student sketch top, front, and side views and draw a different isometric perspective of the same arrangement of climbing blocks.

Assessment Opportunity

Circulate to assess students' ability to connect the two-dimensional faces of the three-dimensional figure with the orthographic views of the figure.

REFLECTING AND CONNECTING

Bring the class together. Refer back to the five views of the L-shaped structure on the overhead. Ask:

- "What similarities are there between the sketches of the five views and the isometric sketch? What differences are there?"
- "Which sketches did you find easier to draw? Why?"
- "What did you find easiest about sketching the climbing structure? What did you find most challenging?"
- "Which type of sketch do you think will be most helpful for assisting people in building the climbing structure? Why?"
- "Where else are orthographic and isometric drawings used?" (toy/furniture assembly instructions, building plans, model cars/planes, installation instructions)

Ask students to make connections between the two types of sketches – how do the shape and number of faces of the orthographic sketches relate to the isometric drawing?

ADAPTATIONS/EXTENSIONS

Some students may have difficulty drawing the isometric perspective of the climbing structure. Encourage them to follow the steps used in drawing the L-shaped structure. Try enlarging the isometric dot paper on a photocopier so that the distance between diagonal dots is equal to the length of one toothpick and have these students place the toothpicks on the dot paper until the sketch "looks right" before drawing the lines in pencil. Provide struggling students with **3D6.BLM4: Sketching Isometric Perspectives** for step-by-step instructions on how the designer's sketch was created. Suggest that they use a similar method for creating a different perspective.

Ask students who finish early or require a challenge to create a different climbing structure using an additional climbing block (for a total of seven) and to sketch orthographic and isometric views of the new structure.

ASSESSMENT

Observe students to see how well they:

- build a three-dimensional model, given an isometric sketch of the structure;
- sketch isometric perspectives and different views of three-dimensional figures built with interlocking cubes.

This lesson is particularly focused on skills, rather than content knowledge. It would be appropriate to focus on the process expectations of representing, communicating, and connecting. While there are many practical applications of both isometric and orthographic sketching, they are not always readily evident to the junior learner. Students must stretch their thinking to make connections with the skills in this lesson.

HOME CONNECTION

Send home **3D6.BLM5a–b: Teach an Adult to Sketch**. In this Home Connection activity, students find a simple polyhedron-shaped object at home and teach a parent or guardian how to sketch two different isometric perspectives of the object using isometric dot paper.

LEARNING CONNECTION 1
Drawing Views in The Geometer's Sketchpad

MATERIALS

- **3D6.BLM6a–b: Drawing Views in The Geometer's Sketchpad** (1 per student)
- computers with The Geometer's Sketchpad software (1 per student or pair of students)
- interlocking cubes (6 different-coloured cubes per student or pair of students)

Although it is possible for students with no experience in using The Geometer's Sketchpad (GSP) to follow the instructions provided to create top, side, and front views of a three-dimensional structure, it is recommended that students have a minimum of one period to explore GSP. For GSP-based tutorials to help students learn to use The Geometer's Sketchpad, refer to the Curriculum Services Canada website for TIPS resources (http://www.curriculum.org/csc/library/tips/section7details.shtml#sketchpad).

Distribute one copy of **3D6.BLM6a–b: Drawing Views in The Geometer's Sketchpad** to each student.

Arrange students in pairs, either to work together on one computer or to work side by side on individual computers. (Consider pairing students of low reading ability with students who have stronger reading skills.)

Students begin by using six different-coloured interlocking cubes to build a structure from an isometric sketch provided. Students then follow instructions to draw top, left side, right side, and front views of the structure using The Geometer's Sketchpad.

LEARNING CONNECTION 2
The Great Imitator

MATERIALS

- a three-dimensional structure made from 6 interlocking cubes (for teacher demonstration – hidden from students' view)
- **3D6.BLM7: Imitator Demo Structure and Views**
- interlocking cubes (10 per student)

Begin by telling students that you have a three-dimensional figure built from six interlocking cubes hidden behind a book and that you are going to describe the structure to the class. Students are to try to build a structure congruent to yours on the basis of your description. Students may not ask questions.

Distribute ten interlocking cubes to each student (students will be using up to ten to build their own structures later on).

Describe the top, side, and front views of your figure in turn. Use precise mathematical language. Ask students to build the figure on the basis of your description. You may wish to use **3D6.BLM7: Imitator Demo Structure and Views** or design your own figure.

Have students first share what they built with a partner. Then students share with the whole class to see whether any of their constructions are congruent to your structure.

Give a round of applause to any students who were able to imitate your structure. Discuss with students the kind of language and/or description that was useful in helping them create the structure. Ask students what was challenging about this task.

Explain that students will now have an opportunity to play a game similar to what the class just did. The game is called The Great Imitator.

The Great Imitator is played as follows:

Students work with a partner. In each pair, one student (the Creator) builds a structure, using six to ten interlocking cubes, and hides it from his or her partner (the Imitator). The Creator then describes the front, top, and side views of the structure, using mathematical language, while the Imitator tries to recreate the structure. The Imitator may not ask questions. Students then compare the two structures to see whether they are congruent. Points may be awarded to students who build a figure that is congruent to the Creator's figure.

Partners then switch roles. Play the game several times.

LEARNING CONNECTION 3
Sketches and Views Cards

MATERIALS

- interlocking cubes (5 per student)
- small index cards (2 per student)
- **3D6.BLM1: 2 cm Grid Paper**, cut into smaller pieces for gluing onto an index card (1 smaller piece per student)
- **3D6.BLM2: Isometric Dot Paper**, cut into smaller pieces for gluing onto an index card (1 smaller piece per student)
- glue sticks

Explain to students that they are going to create cards for a game that they will then have an opportunity to play with a partner.

Each student begins by using five interlocking cubes to build a structure. As students build, distribute two small index cards to each student.

Students then draw three orthographic views (top, side, and front) on grid paper, and the isometric view on a small piece of isometric dot paper. Finally, they cut out the views and glue each to an index card. The orthographic views go on one card, and the isometric view on another.

Once the students have finished, all of the orthographic views cards and all of the isometric sketch cards are collected and shuffled separately. (If some students finish well before others, have them switch cards with a partner and have the partners try to build each other's structures from the orthographic views or the isometric sketch.)

To play the game, all students draw a card from the views card pile, build the structure, and then try to find the matching sketch card for their structure. Alternatively, students draw a card from the sketch card pile, build the structure, and then try to find the matching views card.

LEARNING CONNECTION 4
Construction Challenge

MATERIALS

- interlocking cubes (40 per group of students)
- **3D6.BLM8: The 8 Four-Cube Structures** (answer page)
- **3D6.BLM9a–c: Construction Challenge** (1 per student)

Arrange students into groups of four. Distribute forty interlocking cubes to each group. Challenge students to build as many different four-cube structures as possible. Remind students to test for congruent structures by rotating each structure and viewing it from different perspectives to make sure it is unique. (There are 8 unique four-cube structures. It is important to allow students time to discover this on their own. They will need the experience of visualizing the 8 four-cube structures in different orientations in order to be successful in the activity that follows.)

Once groups of students have discovered what they believe to be all of the possible four-cube structures, have each group present one structure to the class. Continue having each group present a structure, making sure to encourage students to test for congruence, until all 8 unique four-cube structures have been shared. Display the 8 structures and make sure each group has all 8 built. (It may help students to visualize the structures if each of the four-cube structures is constructed from different-coloured interlocking cubes.)

Distribute to each student one copy of **3D6.BLM9a–c: Construction Challenge**.

Have students work in groups to combine two or more of their four-cube structures (as indicated on the handout) to form other structures, given the isometric sketches of the other structures.

Note: This activity is far more challenging than it first appears. However, the experience is well worth it for students as they continue to develop their spatial sense and visualization skills. Encourage groups to share as they successfully construct a figure – have them decompose the figure into its four-cube parts for their classmates. This will help students to see and copy how the structure can be formed and provide motivation for students to continue to work on the other challenges.

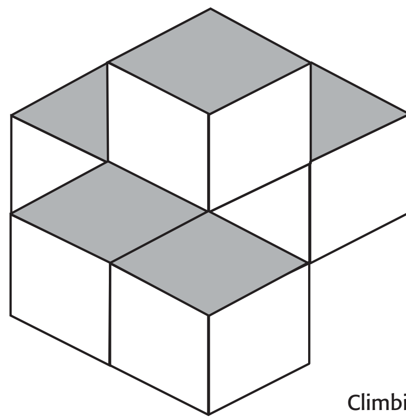
2 cm Grid Paper

Isometric Dot Paper

Sketching Climbing Structures

A company that manufactures climbing equipment for playgrounds is introducing new climbing blocks that can be combined to create a variety of different play structures. The designer of these blocks has forwarded a sketch of one structure that can be constructed using six of the climbing blocks. The company has asked for your help in creating top, front, and side views of the structure as well as a different isometric perspective to be included in the assembly instructions for the new climbing blocks.

Designer's Sketch



Climbing Structure A

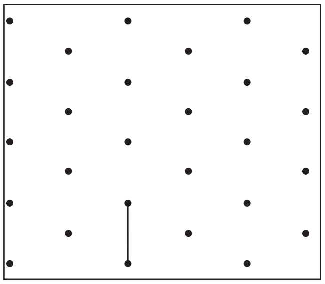
1. Build a model of Climbing Structure A, using six interlocking cubes.

2. Draw and label the top, front, and side views of the structure on grid paper.

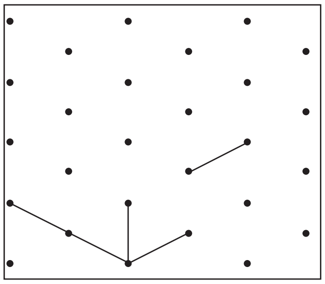
3. Sketch a different isometric perspective of the same structure on isometric dot paper.

Sketching Isometric Perspectives

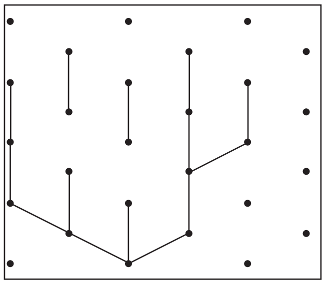
1. Draw a line segment to represent the closest vertical edge of the structure. (Connect two vertical dots.)



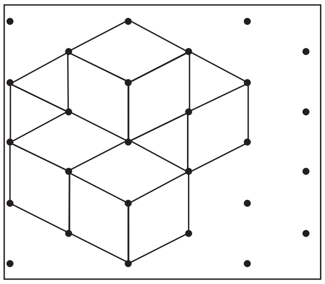
2. Draw diagonal line segments up to the left and up to the right to represent the bottom horizontal edges of the structure.



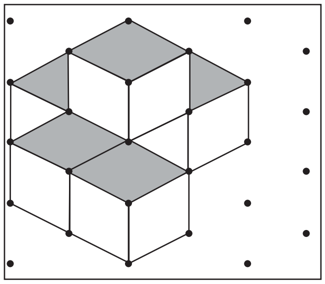
3. Draw vertical line segments for the remaining vertical edges.



4. Complete the sketch by drawing diagonal line segments to represent the remaining horizontal edges that are parallel to the bottom edges.



5. Shade the faces that are visible in the top view.



Teach an Adult to Sketch

Dear Parent/Guardian:

We have been learning how to sketch isometric perspectives of three-dimensional figures in math. Your child has had opportunities to sketch polyhedra (three-dimensional figures with polygon faces) built from interlocking cubes using special paper called isometric dot paper.

In this activity, your child will teach you or another adult at home how to draw two different isometric perspectives of a polyhedron.

Please help your child find a simple polyhedron-shaped object at home (e.g., a facial tissue box, a cereal box).

Ask your child to show you two different isometric perspectives of this object and teach you how to draw them using the isometric dot paper provided.

In trying to teach you how to draw the isometric perspectives, your child will be reinforcing the learning that occurred in the classroom and gaining confidence in his or her ability to sketch three-dimensional figures.

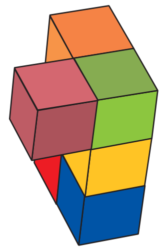
Thank you for your time and support of your child's learning.

Name of Object:

Name of Object:

Drawing Views in The Geometer's Sketchpad

Build this three-dimensional structure, using six different-coloured interlocking cubes.



Follow the instructions below to draw and label top, left side, right side, and front views of your structure.

1. Open The Geometer's Sketchpad.

2. Prepare the screen:

- To create a grid, click on **Graph – Define Coordinate System**.
- Next, click on the two blue lines (axes) and two red dots. Choose **Display – Hide Objects** to hide these objects.
- To create dotted grid lines, click on one point of intersection of any two grid lines. The lines are now highlighted pink. Choose **Display – Line Width – Dotted**. Click anywhere on the screen other than a dot to deselect the dots.

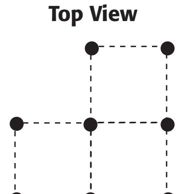
3. Create a label:

- To create a label for your drawing, choose the **Text Tool** from the tool bar at the left of the screen. Click on a dot near the top left corner of the screen and drag to create a text box. Type: "Top View" (or "Left Side View", "Right Side View", "Front View" for subsequent drawings).

4. Draw the view:

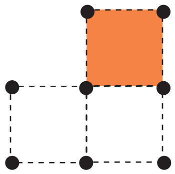
- From the **Graph** menu, choose **Snap Points**.
- Next, choose the **Straightedge Tool** from the tool bar. Click on a dot on the screen and drag to draw one line segment in the view. Continue drawing line segments by clicking one dot and dragging to another dot until you have completed the squares in the view.

Top View

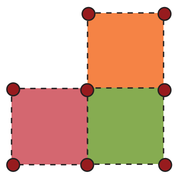


5. Add colour:

- Select the **Selection Arrow Tool** from the tool bar. Click the four corners of one square in your view in clockwise order. Select **Construct – Quadrilateral Interior** to fill the square. Choose **Display – Color** to select a colour to match the cube in the view.



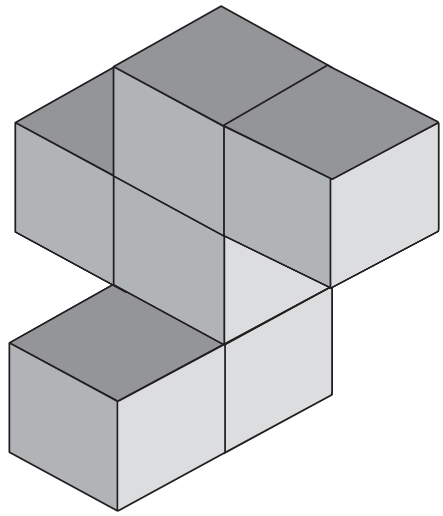
- Add colour to the remaining squares in the view according to the coloured cubes in your structure.



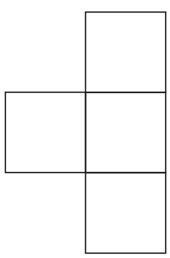
6. Follow steps 2–4 to complete the three other views (left side, right side, front) of your structure.

7. Save your work as "Orthographic Views of a 6-Cube Structure".

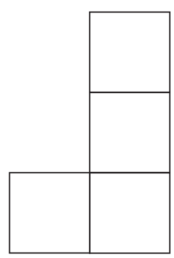
Imitator Demo Structure and Views



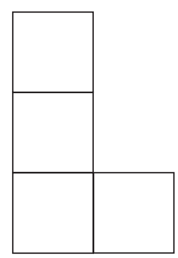
top view



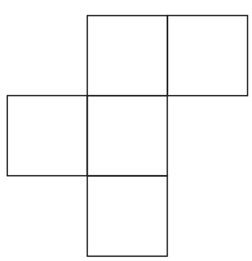
front view



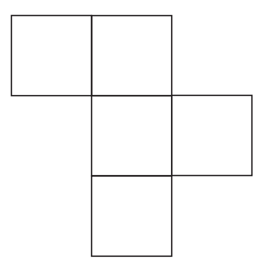
rear view



left side view

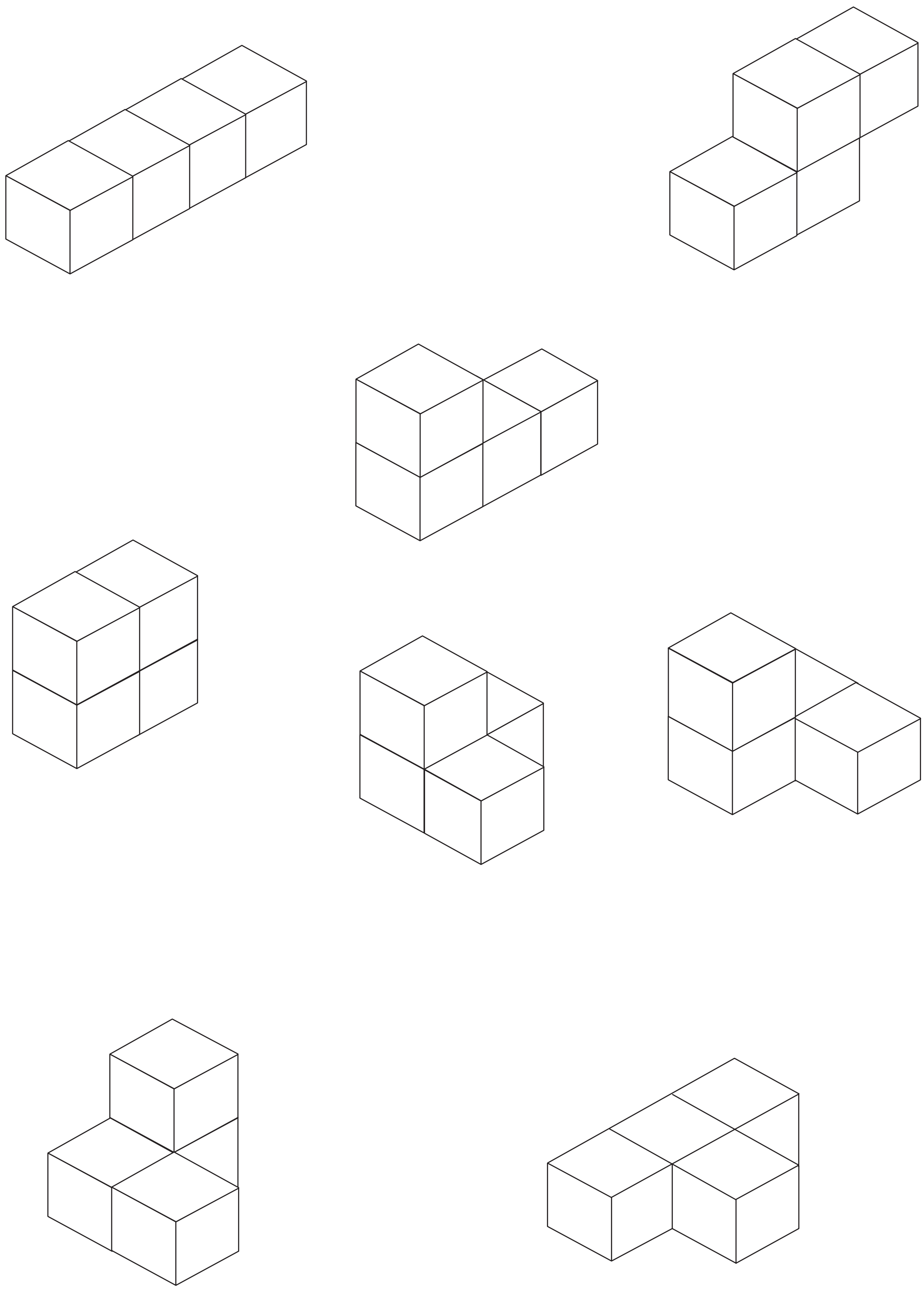


right side view



The 8 Four-Cube Structures

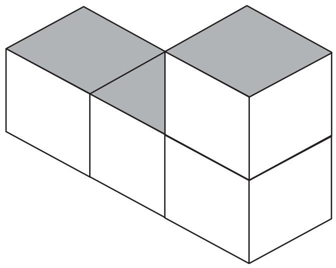
Here are the eight possible structures that can be constructed from four interlocking cubes:



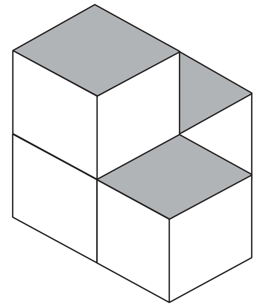
Construction Challenge

Use the four-cube structures indicated to construct the new structures shown in each question.

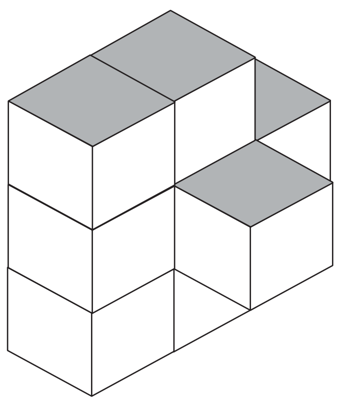
1. Use

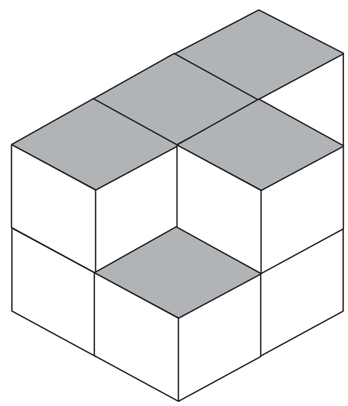


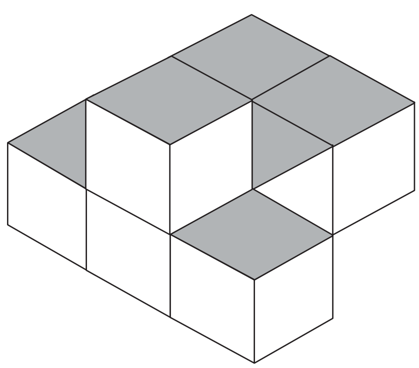
and



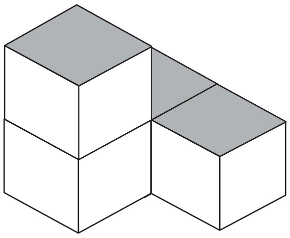
to construct these:



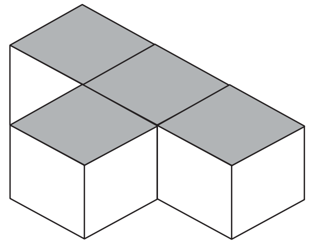




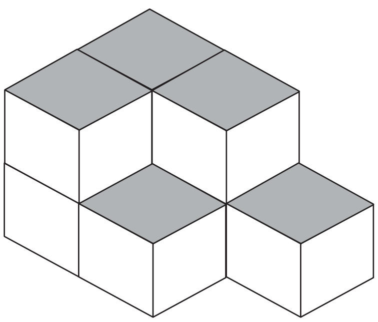
2. Use

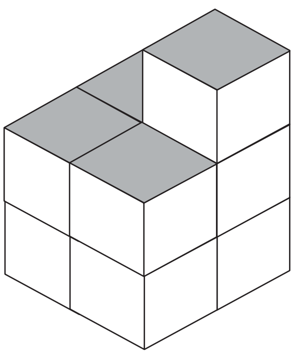


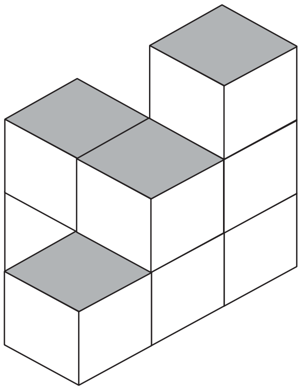
and



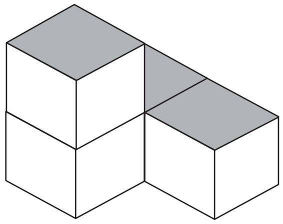
to construct these:



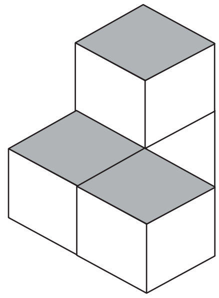




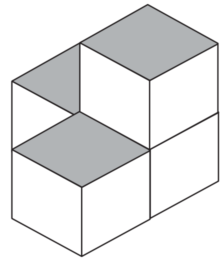
3. Use



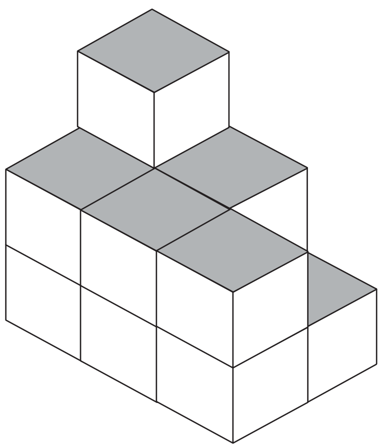
and

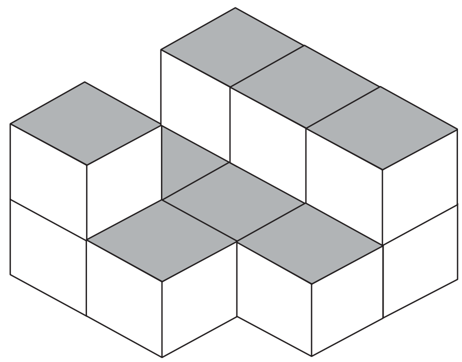


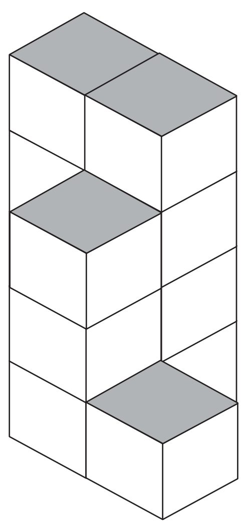
and



to construct these:







Grade 6 Learning Activity
Location: Name My Shapes

OVERVIEW

In this learning activity, students create a shape puzzle, using a coordinate grid, and describe precise locations, using ordered pairs.

BIG IDEAS

This learning activity focuses on the following big idea:

Location and movement: Students investigate using a coordinate grid system to describe the position of an object.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.

Students will:

- explain how a coordinate system represents location, and plot points in the first quadrant of a Cartesian coordinate plane.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- describe location in the first quadrant of a coordinate system, and rotate two-dimensional shapes.

ABOUT THE LEARNING ACTIVITY

TIME: approximately 60 minutes

MATERIALS

- overhead transparency of **Loc6.BLM1: Blank 10 × 10 Grid**
- overhead projector
- **Loc6.BLM1: Blank 10 × 10 Grid** (1 per student)
- **Loc6.BLM2: Name My Shapes Instructions** (1 per student)
- **Loc6.BLM3: Name My Shapes** (2 per student)
- sheets of chart paper or large newsprint (1 per pair of students)
- markers (1 per pair of students)
- **Loc6.BLM4: Introduction to Name My Shapes** (1 per student)

MATH LANGUAGE

- first quadrant
- coordinate grid
- location
- vertical axis
- horizontal axis
- axes
- ordered pair
- origin
- triangle
- rectangle
- isosceles triangle
- equilateral triangle
- quadrilateral
- pentagon
- hexagon
- vertex
- vertices

INSTRUCTIONAL GROUPING: pairs

INSTRUCTIONAL SEQUENCING

This learning activity provides an introduction to the use of ordered pairs and a grid system in locating precise points in the first quadrant of a coordinate plane. Students may benefit from a review of two-dimensional shapes and their properties before the activity.

ABOUT THE MATH

Using a coordinate system to represent the location of two-dimensional shapes assists students in recognizing important properties of these shapes while they learn how to plot points in the first quadrant of a Cartesian coordinate plane. Recognizing and analysing patterns in the coordinate plane will be particularly important in later grades when students examine graphical representations of various algebraic functions.

GETTING STARTED

Have an enlarged version of **Loc6.BLM1: Blank 10 × 10 Grid** on the board, or make a transparency for use on the overhead projector. Ask students how a specific location is found on a coordinate grid (i.e., counting along the horizontal axis, then up the vertical axis – this concept should be a review of Grade 5 location skills). Introduce the idea of using an ordered pair to designate a specific location. Demonstrate how the notation (6, 2) means 6 along the horizontal axis, followed by 2 up the vertical axis. Ask, "What does (2, 6) mean?" Discuss the similarities of and differences between ordered pairs that have their numbers in reverse order. Also, discuss the point (0, 0) – the origin.

Once the students are comfortable locating points on the coordinate grid, provide each student with a copy of **Loc6.BLM1: Blank 10 × 10 Grid**, and write the following ordered pairs on the board or the overhead projector:

(5, 7) (7, 5) (5, 3) (3, 5)

Students plot these points on their grids, and join the four points in order with a ruler. They record the name of the shape drawn on their grid.

Ask volunteers to plot the four points on the board or the overhead projector.

Ask other members of the class to justify why the locations are correct or incorrect.

Once the four points have been plotted and the points connected in order with a ruler, have students name the shape.

WORKING ON IT

Provide each student with one copy of **Loc6.BLM2: Name My Shapes Instructions** and two copies of **Loc6.BLM3: Name My Shapes**. Explain the rules of the game, and have the students play the game. As students play the game, observe the various strategies they use. Pose questions to help students think about their strategies:

- "Did you find a strategy to locate the vertices of your shapes quickly?"
- "What strategy did you use to name the shapes?"
- "Were any of you able to predict what shape it might be just from the ordered pairs?"
- "Did you change any of your strategies? Why?"

When students have had a chance to play the game, provide each pair with markers and a sheet of chart paper or large newsprint. Ask students to record their strategies or any problems they encountered while playing the game.

Make a note of pairs who might share their strategies during the Reflecting and Connecting part of the lesson. Include pairs who used different strategies or who disagreed about a shape name.

REFLECTING AND CONNECTING

Once students have had the opportunity to play the game and record their strategies, bring them together to share their ideas. Try to order the presentations so that students observe a variety of strategies and any unsolved solutions regarding the names of shapes that the whole class could attempt to resolve.

Following the presentations, ask students to observe the work that has been posted and to consider the efficiency of the different strategies. Ask:

- "Which strategy, in your opinion, is an efficient strategy for locating the vertices of shapes?"
- "How would you explain the strategy to someone who has never used it?"

Avoid commenting that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students' work to emphasize geometric ideas:

- A coordinate grid system is helpful for locating and plotting specific points.
- Using ordered pairs is an efficient way to identify points on a coordinate grid.

ADAPTATIONS/EXTENSIONS

Groups of four can select and refine a drawing for presentation to the class.

Students could create a computer version of this activity, using The Geometer's Sketchpad.

ASSESSMENT

Observe students as they play the game and assess how well they:

- locate and mark the vertices of shapes on the coordinate grid;
- create ordered pairs to represent the location of the vertices;
- plot points on a grid from ordered pairs;
- apply appropriate strategies to play the game;
- judge the efficiency of various strategies.

HOME CONNECTION

Send home one copy of **Loc6.BLM4: Introduction to Name My Shapes**, two copies of **Loc6.BLM2: Name My Shapes Instructions**, and a suitable number of copies of **Loc6.BLM3: Name My Shapes**. Parents have their child explain the game Name My Shapes and play the game once or twice with their child. Parents are asked to complete a short questionnaire at the bottom of the letter.

LEARNING CONNECTION 1
Find the Missing Point

MATERIALS

- **Loc6.BLM1: Blank 10 × 10 Grid**
- Overhead transparency of **Loc6.BLM1: Blank 10 × 10 Grid**

Provide two points of a triangle and tell the students that the shape you have started to plot is an isosceles triangle. Have the students write down the coordinates of the third point. (There are several answers.)

Have a volunteer place his or her answer on an overhead transparency of **Loc6.BLM1: Blank 10 × 10 Grid** or a large grid drawn on the board, and explain why it is correct.

Ask if anyone has a different answer that he or she thinks is correct, and repeat the previous directions. Continue until no more solutions are provided.

Offer different challenges – for example: two points of a square, three points of a parallelogram, four points of a regular hexagon.

Students can develop their own challenges, to be solved on grid paper or on The Geometer's Sketchpad.

LEARNING CONNECTION 2
Coordinate Challenge

MATERIALS

- **Loc6.BLM5: Coordinate Challenge** (1 per student)
- **Loc6.BLM1: Blank 10 × 10 Grid** (1 per student)
- overhead transparency or chart-sized copy of **Loc6.BLM1: Blank 10 × 10 Grid**

Hand out **Loc6BLM1: Blank 10 × 10 Grid** (one per student). Ask students to plot the following points: (1, 9) (5, 2) (5, 9) (1, 2). Ask the students to join the points and name the shape.

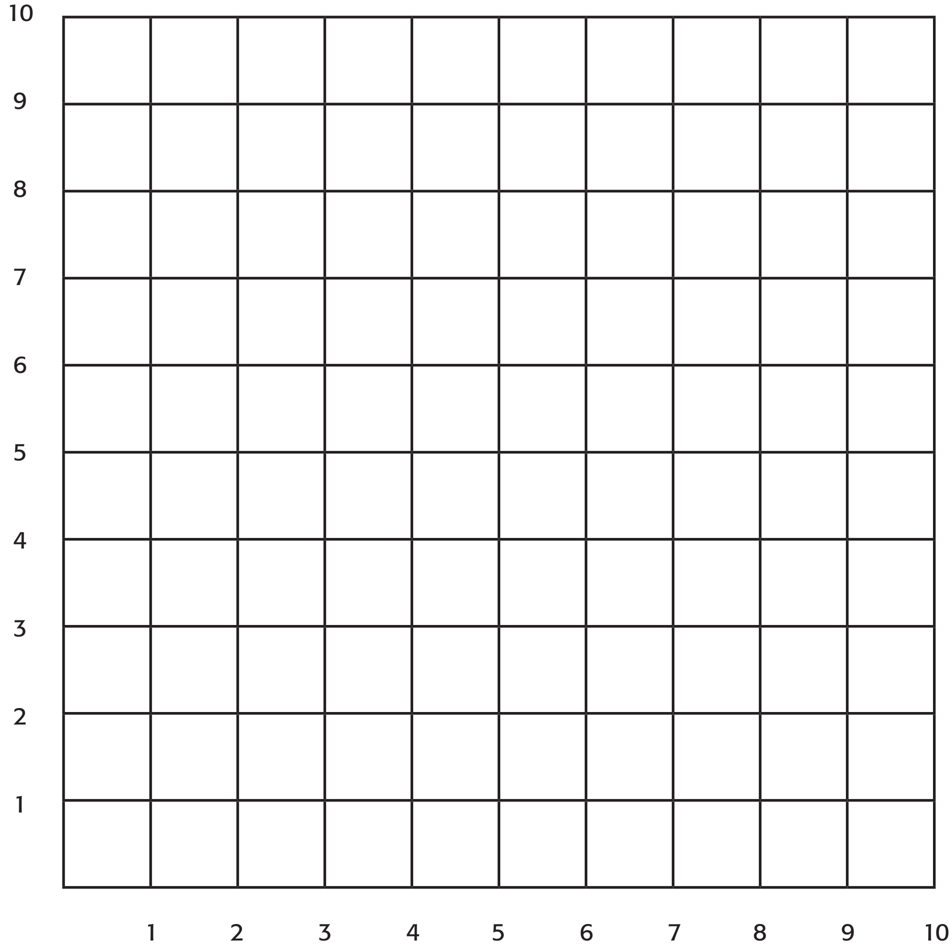
Have a volunteer place his or her answer on the overhead transparency of **Loc6.BLM1: Blank 10 × 10 Grid**, or on a large grid drawn on the board, and explain why it is correct.

Ask the students to find and record the perimeter and area of the shape on the grid. Ask for a volunteer to record the perimeter of the shape on the overhead grid and explain why it is correct. Ask for another volunteer to record the area of the shape on the overhead grid and explain why it is correct. During the explanations allow time for discussion and clarification.

Distribute **Loc6.BLM5: Coordinate Challenge** (one per student). Review the instructions, and have the students complete the challenge.

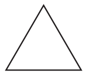
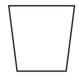
Have volunteers record their answers on an overhead transparency, and allow for discussions of each answer.

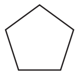
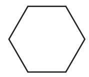
Blank 10 × 10 Grid



Name My Shapes Instructions

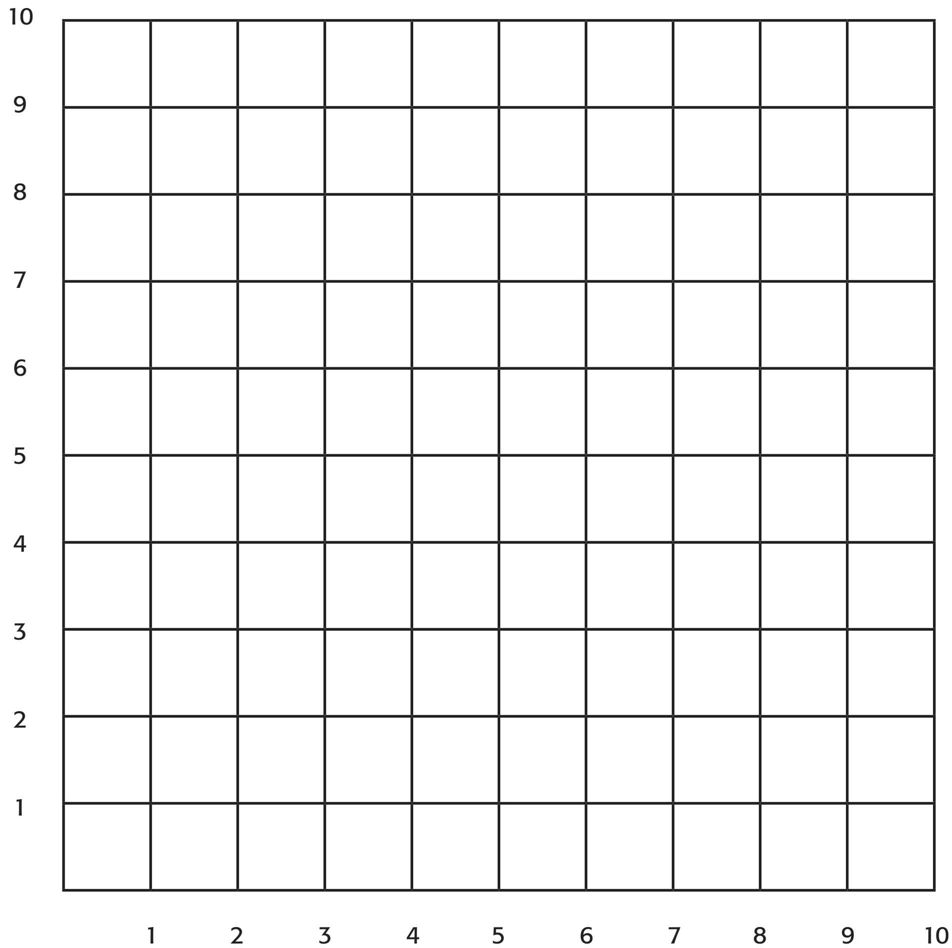
1. Work with a partner as a team of two.
2. Draw a design together, using a **total of four shapes**. The four shapes can be regular or irregular but need to be chosen from the following:

   Triangle    Quadrilateral 

   Pentagon    Hexagon 

   You may use the same shape more than once, but your design may have a total of only four shapes. Your shapes can overlap if you want them to.
3. Under your design, use ordered pairs to write the locations of the vertices for each of the shapes. Then write the name of the shape, being as accurate as you can (e.g., irregular pentagon, isosceles triangle).
4. When you have finished drawing your design, find another team to challenge. Do not show the other team your design.
5. Choose one team to start reading the points for their design. That team read the ordered pairs for the first shape while the other team plot and connect the points, and name the shape on their blank game sheets. Then switch and have the other team read the ordered pairs for their first shape while the first team draw and name the shape on their blank game sheets. Keep going until both teams have finished all four shapes.
6. After you have finished, compare your drawings and shape names with those of the other team. Make note of any shape names you could not agree on.

Name My Shapes



Shape one

Coordinates: ______________________________

Name: ______________________________

Shape two

Coordinates: ______________________________

Name: ______________________________

Shape three

Coordinates: ______________________________

Name: ______________________________

Shape four

Coordinates: ______________________________

Name: ______________________________

Introduction to Name My Shapes

Dear Parent/Guardian:

In class we have been playing a game called Name My Shapes. (Two copies of the game board and instructions are attached to this letter.)

The game is designed to help your child plot ordered pairs and review two-dimensional shapes, such as hexagons, parallelograms, and trapezoids.

Over the next few days, please find a time to let your child explain how to play the game and then play a game or two with you. Please fill out the questionnaire below, to let me know how well your child can explain the mathematical ideas in this game.

Thank you.

QUESTIONNAIRE

Name of person completing the questionnaire ______________________________

Please circle one answer below each question:

1. __________________________ was able to explain the game.

very well   well   somewhat well   not very well

2. __________________________ understood how to plot ordered pairs on the grid, and could tell me how to plot them.

very well   well   somewhat well   not very well

3. __________________________ knew the names of most of the shapes we used.

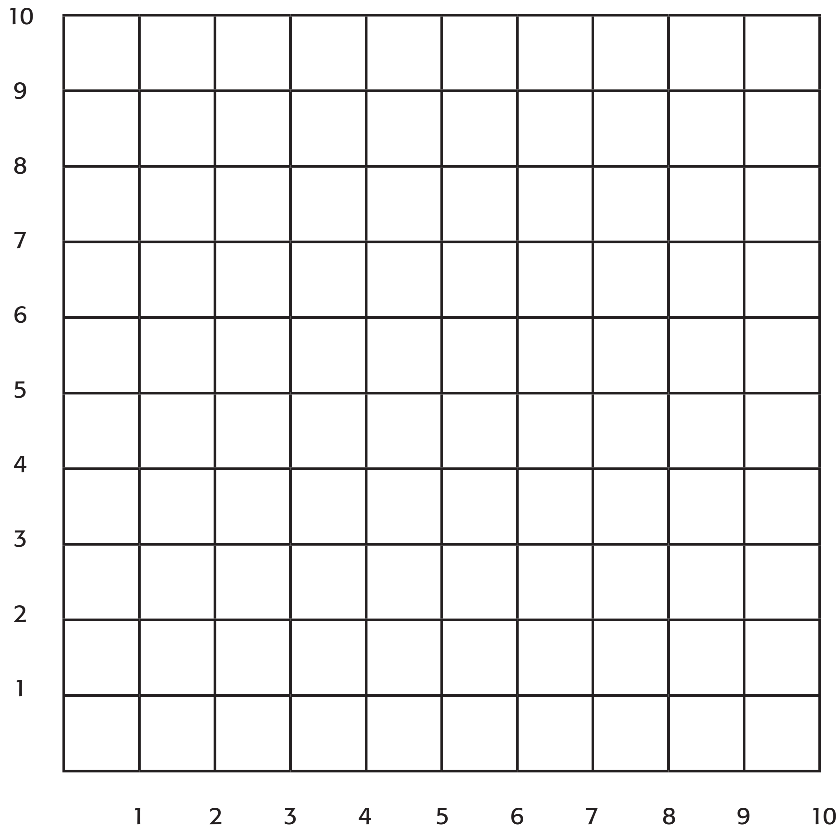
very well   well   somewhat well   not very well

4. Some directions and /or shapes __________________________ was not too sure about are:

____

Comments:

Coordinate Challenge



Draw your solutions to the following questions on the grid above, and label each shape as instructed. List the coordinates for the vertices of each shape, using ordered pairs.

1. Draw a rectangle with an area of 9 cm^2. Label it ABCD. The vertices are located at: A (   ) B (   ) C (   ) D (   )

2. Draw a right-angled isosceles triangle. Label it EFG. The vertices are located at:

E (   ) F (   ) G (   )

3. Draw a square with a perimeter of 12 cm. Label it HIJK. The vertices are located at: H (   ) I (   ) J (   ) K (   )

4. Draw a hexagon with one vertex at (1, 1). Label it, using letters of your choice, and list where all the vertices are located:

5. Draw an obtuse-angled triangle with a perimeter of less than 15 cm. Label it XYZ. The vertices are located at: X (   ) Y (   ) Z (   )

Grade 6 Learning Activity Movement: Logo Search and Design

OVERVIEW

In this learning activity, students analyse and design logos, using different transformations and kinds of symmetry.

BIG IDEAS

This learning activity focuses on the following big idea:

Location and movement: Students can use different transformations to describe the movement of an object.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- identify, perform, and describe, through investigation using a variety of tools (e.g., grid paper, tissue paper, protractor, computer technology), rotations of 180° and clockwise and counterclockwise rotations of 90°, with the centre of rotation inside or outside the shape.
- create and analyse designs made by reflecting, translating, and/or rotating a shape, or shapes, by 90° or 180°;

These specific expectations contribute to the development of the following **overall expectation**.

Students will:

- describe location in the first quadrant of a coordinate system, and rotate two-dimensional shapes.

ABOUT THE LEARNING ACTIVITY

TIME: approximately two 60-minute periods

Materials

- logo samples
- sheets of chart paper or large newsprint (1 per small group of students; 1 per pair of students)
- **Mov6.BLM1a–b: Logos** (1 per student)
- **Mov6.BLM2: Logo Design** (1 per pair of students)
- geoboards, Miras, pattern blocks
- markers or crayons (1 per pair of students)

MATH LANGUAGE

- symmetry
- reflection
- rotation
- rotational symmetry
- shapes
- properties

INSTRUCTIONAL GROUPING: individual and pairs or groups of three

INSTRUCTIONAL SEQUENCING

This learning activity provides an opportunity to explore the use of rotational and line symmetry in logo design. It is important for students to have completed the Learning Connection "Alphabet Rotations" or some other rotation activities, before completing this main activity, so that students have some prior knowledge of rotational symmetry.

ABOUT THE MATH

It is important to offer students real-world experiences that provide opportunities for them to visualize rotations and rotational symmetry. Students experience more difficulty in visualizing the rotation of objects than they do in reflecting similar objects about a line of symmetry.

GETTING STARTED

Before the lesson, collect some examples of logos containing rotational and/or line symmetry. Using several of the examples, have a brief whole-class discussion with your students about logos. Have other examples ready for students to examine in small groups. Distribute the logos and ask students to jot down on chart paper or large newsprint ideas of what makes an effective logo.

As students are working, you may want to ask:

- "What do you like about the logo your group is examining?"
- "Why do you think the company chose this logo?"
- "What are some of the characteristics or properties of the logo?"

Select some students to present their observations about the logos they were examining. Try to choose students who will provide a variety of observations, especially those who notice symmetry in the logo.

Discuss how many logos use some form of symmetry through the use of rotations and reflections, and show examples of symmetry.

Provide each student with a copy of **Mov6.BLM1a–b: Logos**. Tell the students that they will have homework tonight but they should ask parents/guardians to help if they can (the parent letter explains the assignment). Explain that tonight's homework is to search through magazines, newspapers, Internet sites, and so forth, to find a few logos that are visually appealing and contain examples of symmetry, one of which must be of rotational symmetry. Ask students to cut out logos (with their parents' permission) or copy them as accurately as possible. Then students should fill out the questions on the back of the parent letter and bring the answers and logos back to school.

WORKING ON IT

On another day ask a few volunteers to show their most interesting logo to the class, and have them explain the mathematical symmetry and the shapes found in the logo. You may want to create a Logo bulletin board with the logos students have brought in.

Organize the students into pairs for the next activity. Hand out copies of **Mov6.BLM2: Logo Design**, and go over the instructions with the students.

As the students are working, observe the various strategies that they use. Pose questions to help students think about their strategies:

- "Have you thought of a strategy to find out if an object or shape has symmetry?"
- "Did you change your strategy? Why?"
- "Did you use any tools in the classroom as part of your strategy?"

As students are completing their logos, provide each pair with chart paper or large newsprint and markers to record the strategies that they used to create and check their logos.

Make note of pairs who might share their strategies during the Reflecting and Connecting portion of the lesson. Include groups who used different strategies for creating designs with line and rotational symmetry (e.g., guess and check, rotating their design, using a Mira, using dot paper and a line of reflection).

REFLECTING AND CONNECTING

After students have completed their designs and recorded their strategies, bring them together to share their ideas. Try to order the presentations so that the students observe strategies varying in efficiency.

As students explain their ideas, ask questions to help them to describe their strategies:

- "How did you select the shapes in your logo?"
- "Where is the line and rotational symmetry in this logo?"
- "What strategies did you use to include symmetry in your logo?"
- "Why do you think many logos have symmetry?"

Avoid commenting that some logos and/or strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students' work to emphasize geometric ideas:

- Designs can be made by reflecting or rotating shapes.
- Real-world objects contain line and rotational symmetry.
- Reflections and rotations can create symmetrical designs.

A "gallery walk" at the conclusion of this activity provides all students with an opportunity to display their work, see the logos produced by their peers, and explain their designs to other class members.

ADAPTATIONS/EXTENSIONS

Groups can present their design to the school administration and/or team/club members, explaining why they believe their logo should be adopted.

ASSESSMENT

Observe students as they work on and explain their logo designs, and assess how well they:

- explain why they selected the mathematical shapes and tools to design their logos;
- show their understanding of the symmetries associated with their logos;
- explain the strategies they used to create line and rotational symmetry in their logos;
- judge the efficiency of various strategies.

Student reflections on the reverse of **Mov6.BLM6 Logos** can be used for diagnostic purposes, to provide information on students' strengths and difficulties.

HOME CONNECTION

See **Mov6.BLM1a–b: Logos** for a letter to be sent home to parents as part of the Getting Started portion of the lesson.

LEARNING CONNECTION 1
Regular Shapes and Rotational Symmetry

MATERIALS

- **Mov6.BLM3a–d: Rotating Regular Shapes** (1 per student)
- 2 large demonstration copies of each shape on **Mov6.BLM3a–d: Rotating Regular Shapes**
- pieces of tracing paper (4 per student)
- paper fasteners (1 per student)
- rulers (1 per pair of students)
- protractors (1 per pair of students)

Distribute copies of **Mov6.BLM3a–d: Rotating Regular Shapes** (one copy per student). Ask, "Why are these two-dimensional shapes called *regular*?" Allow students to examine the shapes in pairs to determine possible patterns. Facilitate a discussion through which students discover that all the sides and all the angles must be congruent for a shape to be called regular.

Have each student trace the shapes from **Mov6.BLM3a–d: Rotating Regular Shapes**. Students cut out the shapes from the four pages of the blackline master and the shapes from their tracing paper. Discuss how a centre of rotation allows you to turn a regular polygon onto itself exactly. There can be no overlapping parts when this occurs. Demonstrate with the large equilateral triangles.

Ask students to work with a partner to see if they can find a way to mark the centre of rotation for the equilateral triangle. Allow time for experimentation, and ask for volunteers to share their findings. (The centre of rotation can be found by bisecting one angle of the triangle through folding, repeating this process with a second angle, and finding where the two fold lines meet. Bisecting a third angle will verify that the centre of rotation is correct. Demonstrate with a large equilateral triangle if none of the students discover the strategy.)

Label the centre of rotation for each of the large equilateral triangles and place one triangle exactly on top of the other, using a paper fastener to hold the triangles in place. Label the angles A, B, and C. Slowly rotate the top copy until it lies exactly over the bottom triangle. Have the students tell you when each overlying occurs. Keep rotating the triangle until it returns to its original position. Ask, "How many times does one equilateral triangle lie exactly over the other?" (3)

Instruct the students to work in pairs or groups of three, repeating the process with the square, the pentagon, and the regular hexagon. Students should record their findings and look for patterns.

Have a discussion to see which patterns the students have found. You want them to recognize that the number of rotations corresponds to the number of sides. However, the students might discover other interesting ideas or patterns.

Ask if their patterns or ideas would be true for other regular polygons. What about irregular polygons? Have the students cut out other shapes to test their conjectures.

Teachers with access to The Geometer's Sketchpad might want to let their students test their conjectures in that environment, exploring more complex regular polygons.

LEARNING CONNECTION 2
Alphabet Rotations

MATERIALS

- **Mov6.BLM4: Rotating the Alphabet** (1 per student)
- tracing paper (1 piece per student)

Students use **Mov6.BLM4: Rotating the Alphabet** to discover which of the capital letters of the alphabet have rotational symmetry. Students work individually to decide on the letters that they believe have rotational symmetry. At this time they are not permitted to use any aids and must base their conjectures on perception.

Students then work in pairs, checking their conjectures with tracing paper to prove which of the letters have rotational symmetry.

Students could also discover, and then check, which of the numbers and the lower case letters of the alphabet have rotational symmetry.

LEARNING CONNECTION 3
Quadrilaterals, Questions, and Symmetry

MATERIALS

- **Mov6.BLM5: Quadrilaterals, Questions, and Symmetry** (1 per student)
- Mira (1 for every two students)
- sheets of tracing paper (1 per student)
- Miras (1 per pair of students)

Select a shape that is not on the sheet, such as a hexagon, to explain how the worksheet is to be completed. The emphasis is on reasoning, followed by proving the various symmetries of the quadrilaterals.

Students work individually to complete the chart on **Mov6.BLM5: Quadrilaterals, Questions, and Symmetry**. At this point, they are NOT to use any tools, such as Miras or tracing paper.

Students then work in pairs, using tracing paper and Miras to verify their conjectures.

During a whole-class discussion, students reflect on what they discovered. Those who are interested might explore the symmetries of other two-dimensional shapes.

Logos

Dear Parent/Guardian:

In geometry we are exploring the different forms of symmetry that large companies use when designing their logos.

Students are being asked to search through magazines, newspapers, the Internet, and so forth, to find a few logos that are visually appealing, are not offensive, and contain examples of symmetry (especially rotational symmetry).

Please ask your child to explain the terms *rotational symmetry* and *line symmetry* before he or she begins to look for logos.

If your child finds logos in magazines or newspapers, he or she can either cut them out or copy them. Logos that your child finds through an Internet search can be printed.

To search the Internet, at home or at the public library, your child can go into a search engine like Google and type “most famous company logos”.

After finding the logos, your child should complete the questions on the back of this letter.

Thank you for helping your child to make connections between geometry in school and geometry in the real world.

Logos (Reverse side)

STUDENT REFLECTIONS

Please complete the following questions about the two or three most interesting logos that you found.

1. What I found most interesting about these logos is:

____

2. The line symmetry used in one of the logos is ____________________ (horizontal, vertical, diagonal).

3. Something I found interesting/unusual about the line symmetry is:

____

4. The rotational symmetry used in one of the logos is ____________ (90°, 180°, other).

5. Something I found interesting/unusual about the rotational symmetry is:

____

6. The transformations used in logo #1 are:

____

7. The transformations used in logo #2 are:

____

8. Some mathematical shapes I found in my logos are:

____

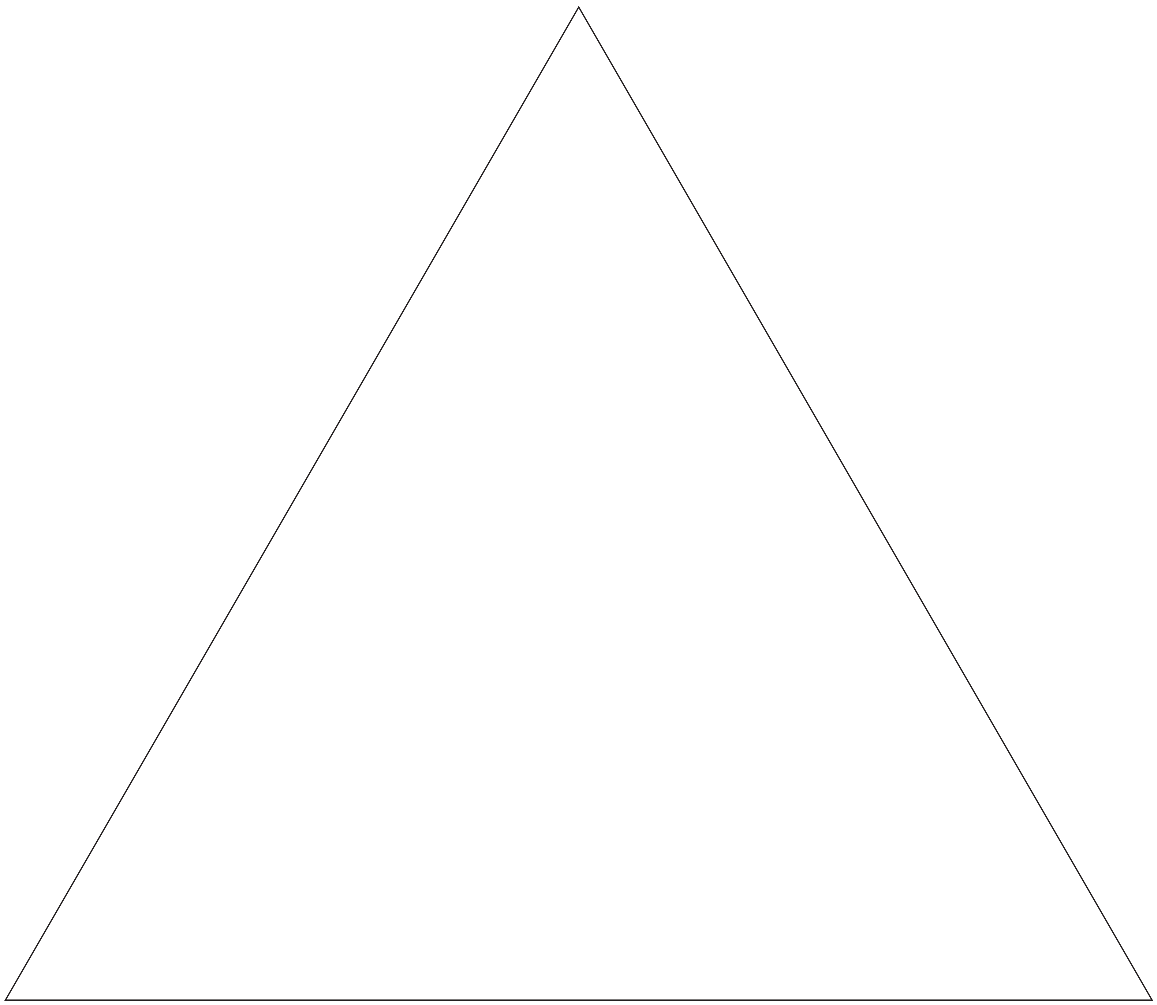
9. Something I still don't understand about symmetry or transformations is:

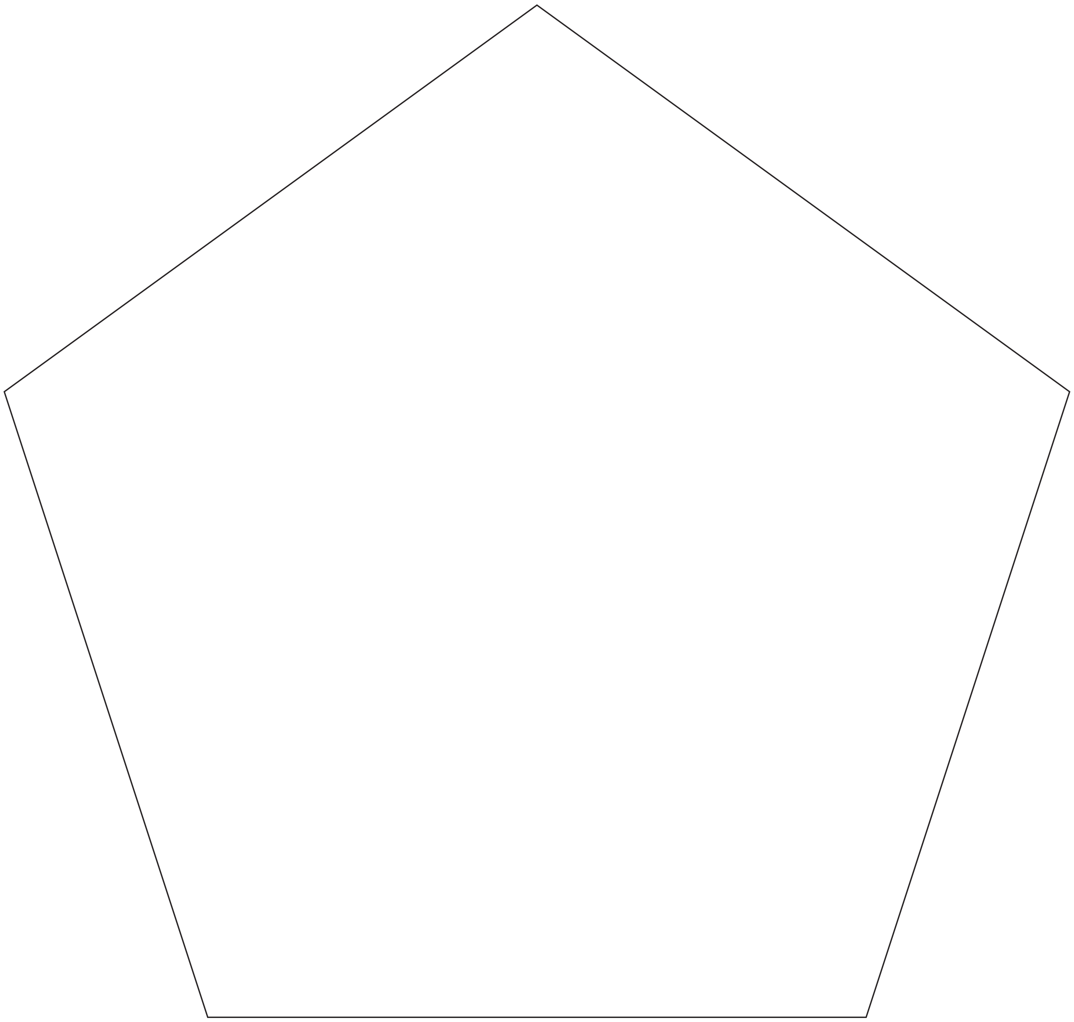
____

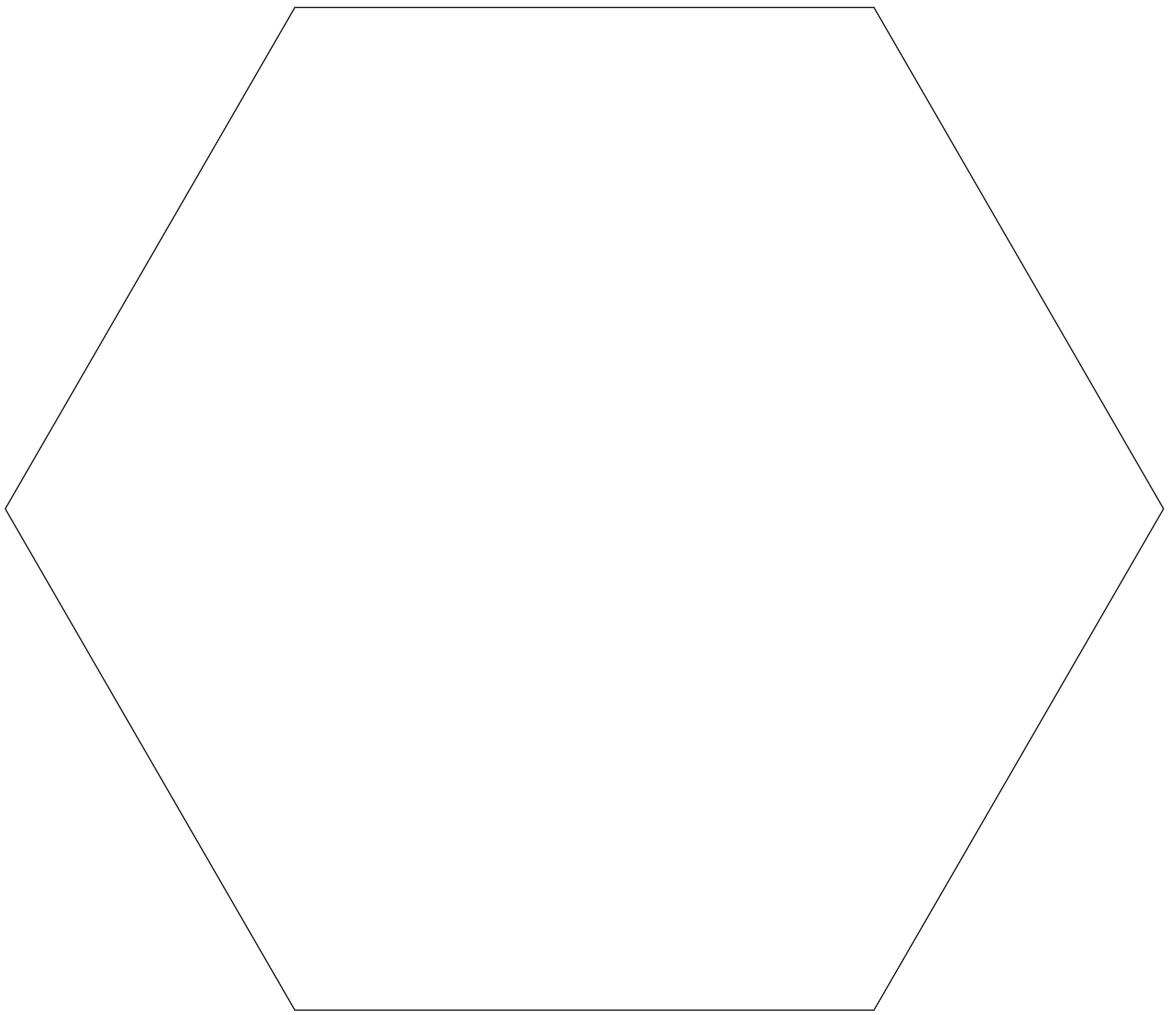
Logo Design

1. Work with a partner.
2. Design a logo for our classroom or school.
3. Your logo must contain an example of line symmetry and an example of rotational symmetry.
4. You may use any of the manipulatives and/or paper provided.
5. You need to work as a team and be able to explain the strategies you used to design and create your logo, and you need to be able to show that your logo contains an example of line symmetry and an example of rotational symmetry.

Rotating Regular Shapes







Rotating the Alphabet

Which capital letters have rotational symmetry?

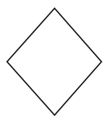
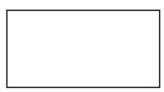
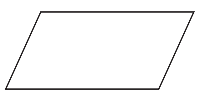
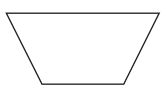
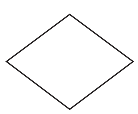
A	B	C	D	
E	F	G	H	I
J	K	L	M	N
O	P	Q	R	
S	T	U	V	
W	X	Y	Z	

Quadrilaterals, Questions, and Symmetry

Some quadrilaterals have lines of symmetry; others do not. Some quadrilaterals have rotational symmetry; others do not.

Your task is to complete the chart below. For each of the quadrilaterals, provide the most precise name you know, and write the name in the first column. Then use your imagination to speculate whether the shape has lines of symmetry or rotational symmetry.

In the last row, draw a different quadrilateral, name it, and complete the last two columns.

Name	Shape	Lines of symmetry	Rotational symmetry
		Yes __ How many? ____ No __	Yes ___ No ___
		Yes __ How many? ____ No __	Yes ___ No ___
		Yes __ How many? ____ No __	Yes ___ No ___
		Yes __ How many? ____ No __	Yes ___ No ___
		Yes __ How many? ____ No __	Yes ___ No ___
		Yes __ How many? ____ No __	Yes ___ No ___

Once you have completed the chart, ask for some tracing paper and a Mira.

Work with a partner to verify your predictions. If you disagree with each other, take turns explaining why and try to come to an agreement.

APPENDIX: GUIDELINES FOR ASSESSMENT

There are three types of assessment: assessment *for* learning, assessment *as* learning, and assessment *of* learning.

Assessment *for* learning involves teachers observing the knowledge, skills, experience, and interests their students demonstrate, and using those observations to tailor instruction to meet identified student needs and to provide detailed feedback to students to help them improve their learning.

Assessment *as* learning is a process of developing and supporting students' metacognitive skills. Students develop these skills as they monitor their own learning, adapt their thinking, and let the ideas of others (peers and teachers) influence their learning. Assessment *as* learning helps students achieve deeper understanding.

Assessment *of* learning is summative. It includes cumulative observations of learning and involves the use of the achievement chart to make judgements about how the student has done with respect to the standards. Assessment *of* learning confirms what students know and are able to do, and involves reporting on whether and how well they have achieved the curriculum expectations.

Teachers use assessment data, gathered throughout the instruction–assessment–instruction cycle, to monitor students' progress, inform teaching, and provide feedback to improve student learning. Effective teachers view instruction and assessment as integrated and simultaneous processes. Successful assessment strategies – those that help to improve student learning – are thought out and defined ahead of time in an **assessment plan**.

An **assessment profile**, developed by the teacher for each student, can be an effective way of organizing assessment data to track student progress. Students can also maintain their own **portfolios**, in which they collect samples of their work that show growth over time.

Creating an Assessment Plan

To ensure fair and consistent assessment throughout the learning process, teachers should work collaboratively with colleagues to create assessment and instructional plans. Ideally, such planning should start with learning goals and work backwards to identify the assessment and instructional strategies that will help students achieve those goals.

Guiding Questions	Planning Activities
"What do I want students to learn?"	Teachers begin planning by identifying the overall and specific expectations from the Ontario curriculum that will be the focus of learning in a given period. The expectations may need to be broken down into specific, incremental learning goals. These goals need to be shared with students, before and during instruction, in clear, age-appropriate language.
"How will I know they have learned it?"	Teachers determine how students' learning will be assessed and evaluated. Both the methods of assessment and the criteria for judging the level of performance need to be shared with students.
"How will I structure the learning?"	Teachers identify scaffolded instructional strategies that will help students achieve the learning goals and that integrate instruction with ongoing assessment and feedback.

An assessment plan should include:

- clear learning goals and criteria for success;
- ideas for incorporating both assessment *for* learning and assessment *as* learning into each series of lessons, before, during, and after teaching and learning;
- a variety of assessment strategies and tools linked carefully to each instructional activity;
- information about how the assessment profiles will be organized;
- information about how the students' assessment portfolios will be maintained.

Feedback

When conducting assessment *for* learning, teachers continuously provide timely, descriptive, and specific feedback to students to help them improve their learning. At the outset of instruction, the teacher shares and clarifies the learning goals and assessment criteria with the students. Effective feedback focuses the student on his or her progress towards the learning goals. When providing effective feedback, teachers indicate:

- what good work looks like and what the student is doing well;
- what the student needs to do to improve the work;
- what specific strategies the student can use to make those improvements.

Feedback is provided during the learning process in a variety of ways – for example, through written comments, oral feedback, and modelling. A record of such feedback can be maintained in an assessment profile.

Assessment Profile

An assessment profile is a collection of key assessment evidence, gathered by the teacher over time, about a student's progress and levels of achievement. The information contained in the profile helps the teacher plan instruction to meet the student's specific needs. An extensive collection of student work and assessment information helps the teacher document the student's progress and evaluate and report on his or her achievement at a specific point in time.

The assessment profile also informs the teacher's conversations with students and parents about the students' progress. Maintaining an assessment profile facilitates a planned, systematic approach to the management of assessment information.

Assessment profiles may include:

- assessments conducted after teaching, and significant assessments made during teaching;
- samples of student work done in the classroom;
- samples of student work that demonstrates the achievement of expectations;
- teacher observation and assessment notes, conference notes;
- EQAO results;
- results from board-level assessments;
- interest inventories;
- notes on instructional strategies that worked well for the student.

Student Assessment Portfolio

A portfolio is a collection of work selected by the student that represents his or her improvement in learning. It is maintained by the student, with the teacher's support. Assembling the portfolio enables students to engage actively in assessment *as* learning, as they reflect on their progress. At times, the teacher may guide students in the selection of samples that show how well they have accomplished a task, that illustrate their improvement over a period of time, or that provide a rationale for the teacher's assessment decisions. Selections are made on the basis of previously agreed assessment criteria.

Student assessment portfolios can also be useful during student/teacher and parent/teacher conferences. A portfolio may contain:

- work samples that the student feels reflect growth;
- the student's personal reflections;
- self-assessment checklists;
- information from peer assessments;
- tracking sheets of completed tasks.

Students should not be required to assign marks, either to their own work or to the work of their peers. Marking is part of the evaluation of student work (i.e., judging the quality of the work and assigning a mark) and is the responsibility of the teacher.

Assessment Before, During, and After Learning

Teachers assess students' achievement at all stages of the instructional and assessment cycle.

Assessment *before* new instruction identifies students' prior knowledge, skills, strengths, and needs and helps teachers plan instruction.

Effective assessment *during* new instruction determines how well students are progressing and helps teachers plan required additional instruction. The teacher uses a variety of assessment strategies, such as focused observations, student performance tasks, and student self- and peer assessment, all based on shared learning goals and assessment criteria. As noted earlier, the teacher provides students with feedback on an ongoing basis during learning to help them improve.

During an instructional period, the teacher often spends part of the time working with small groups to provide additional support, as needed. The rest of the time can be used to monitor and assess students' work as they practise the strategies being learned. The teacher's notes from his or her observations of students as they practise the new learning can be used to provide timely feedback, to develop students' assessment profiles, and to plan future lessons. Students can also take the opportunity during this time to get feedback from other students.

Assessment *after* new learning has a summative purpose. As assessment *of* learning, it involves collecting evidence on which to base the evaluation of student achievement, develop teaching practice, and report progress to parents and students. After new learning, teachers assess students' understanding, observing whether and how the students incorporate feedback into their performance of an existing task or how they complete a new task related to the same learning goals. The assessment information gathered at this point, based on the identified curriculum expectations and the criteria and descriptors in the achievement chart, contributes to the evaluation that will be shared with students and their parents during conferences and by means of the grade assigned and the comments provided on the report card.

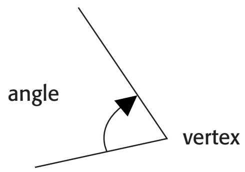
Glossary

acute angle. An angle whose measure is between 0° and 90°.

acute triangle. A triangle whose angles all measure less than 90°.

adjacent angle. One of two angles that have a common vertex and common side.

angle. A shape formed by two rays or two line segments with a common endpoint. Measuring angles involves finding the amount of rotation between two rays or line segments with a common endpoint or vertex.



attribute. A quantitative or qualitative characteristic of a shape, an object, or an occurrence; for example, colour, size, thickness, or number of sides. An attribute may or may not be a property. *See also* **property (geometric)**.

axis. A reference line used in a graph or coordinate system.

base. In three-dimensional figures, the face that is usually seen as the bottom (e.g., the square face of a square-based pyramid). In prisms, the two congruent and parallel faces are called bases (e.g., the triangular faces of a triangular prism).

big ideas. In mathematics, the important concepts or major underlying principles. For example, in this document, the big ideas that have been identified for Grades 4 to 6 in the Geometry and Spatial Sense strand of the Ontario curriculum are properties of two-dimensional shapes and three-dimensional figures, geometric relationships, and location and movement.

cardinal directions. The four main points of the compass: north, east, south, and west.

circle. A two-dimensional shape with a curved side. All points on the side of a circle are equidistant from its centre.

Cartesian coordinate plane. A plane that contains an *x*-axis (horizontal) and a *y*-axis (vertical), which are used to describe the location of a point. Also called *coordinate plane*.

compose. Order or arrange parts to form a whole. In geometry, two-dimensional shapes and three-dimensional figures can compose larger shapes and figures. *See also* **decompose**.

concave polygon. A polygon containing at least one interior angle greater than 180°.

cone. A three-dimensional figure with a circular base and a curved surface that tapers to a common point.

congruent. Having the same size and shape.

conjecture. An unproven mathematical theorem; a conclusion based on incomplete information.

convex polygon. A polygon whose interior angles are all less than 180°.

coordinate grid. A plane that contains horizontal and vertical lines that intersect to form squares or rectangles. In a coordinate grid system (for example, a road map), the representation of an object within the squares or rectangles describes the location of the object.

coordinate plane. *See* **Cartesian coordinate plane**.

coordinates. *See* **ordered pair**.

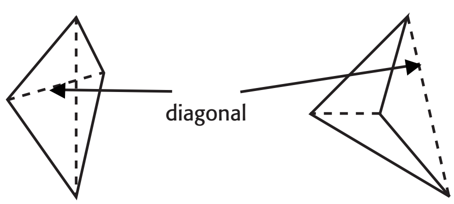
coordinate system. A system that identifies a point rather than an area. In a coordinate system, the lines are labelled, rather than the area bounded by the lines.

cube. A right rectangular prism with six congruent square faces. A cube is one of the Platonic solids. Also called *hexahedron*.

cylinder. A three-dimensional figure with two parallel and congruent circular faces and a curved surface.

decompose. Separate a whole into parts. In geometry, two-dimensional shapes and three-dimensional figures can be decomposed into smaller shapes and figures. *See also* **compose**.

diagonal. A line segment joining two vertices of a polygon that are not next to each other (i.e., that are not joined by one side).



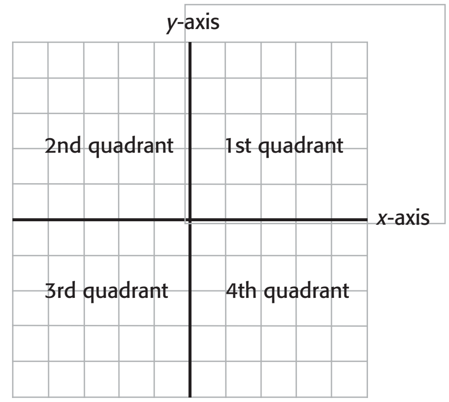
edge. The intersection of a pair of faces in a three-dimensional figure.

equilateral triangle. A triangle with three equal sides.

face. One of the polygons that make up a polyhedron.

figure. *See* **three-dimensional figure**.

first quadrant. In the coordinate plane, the quadrant that contains all the points with positive *x* and positive *y* coordinates.



flip. *See* **reflection**.

Frameworks. Commercially produced learning tools that help students learn about geometric figures. Frameworks is a system of interlocking frames for geometric constructions. The frames in this system match the dimensions, constructions, and features of Polydrons. *See also* **Polydrons**.

geometry. The study of mathematics that deals with the spatial relationships, properties, movement, and location of two-dimensional shapes and three-dimensional figures. The name comes from two Greek words meaning *earth* and *measure*.

grid. A network of regularly spaced lines that cross one another at right angles to form squares or rectangles.

grid system. A system that uses a combination of a grid and letters and numbers to describe a general location of a shape or an object. Because the grid system identifies an area rather than a point, it cannot be used to describe precise locations. *See also* **grid**.

hexagon. A polygon with six sides.

integer. Any one of the numbers …, –4, –3, –2, –1, 0, 1, 2, 3, 4,….

irregular polygon. A polygon that does not have all sides and all angles equal. *See also* **regular polygon**.

isometric drawing. A perspective drawing of a three-dimensional figure.

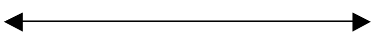
isometric dot paper. Dot paper used for creating isometric drawings. The dots are formed by the vertices of equilateral triangles. Also called *triangular dot paper* or *triangle dot paper*.

isosceles trapezoid. A trapezoid that has two sides of equal length.

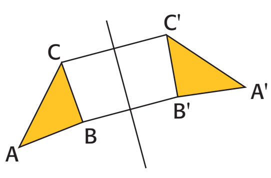
isosceles triangle. A triangle that has two sides of equal length.

kite. A quadrilateral that is not a parallelogram, but has two pairs of equal sides that are adjacent to each other.

line. A geometric figure that has no thickness but its length goes on infinitely in two directions.



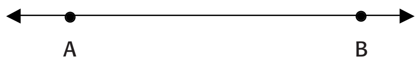
line of reflection. A line that acts as a mirror in the form of a perpendicular bisector so that corresponding points are the same distance from the line.



In this example, the points A, B, and C are exactly the same distance from the mirror line as are points A', B' and C'.

line of symmetry. A line that divides a shape into two congruent parts that can be matched by folding the shape in half. *See also* **symmetry**.

line segment. The part of a line between two points on a line.

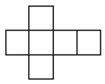


magnitude. An attribute relating to size or quantity.

manipulative. An object that students handle and use in constructing their own understanding of mathematical concepts and skills and in illustrating their understanding. Some examples are geoboards, geometric solids, and pattern blocks.

Mira. A transparent mirror used to locate reflection lines, reflection images, and lines of symmetry, and to determine congruency and line symmetry.

net. A pattern that can be folded to make a three-dimensional figure.



A net of a cube

obtuse angle. An angle that measures more than 90° and less than 180°.

obtuse triangle. A triangle with one angle that measures more than 90° and less than 180°.

octagon. A polygon with eight sides.

order of rotational symmetry. The number of times the position of a shape coincides with its original position during one complete rotation about its centre. For example, a square has rotational symmetry of order 4. *See also* **rotational symmetry**.

ordered pair. Two numbers, in order, that are used to describe the location of a point on a plane, relative to a point of origin (0, 0); for example, (2, 6). On a coordinate plane, the first number is the horizontal coordinate of a point, and the second is the vertical coordinate of a point. *See also* **coordinates**.

orientation. The relative physical position or direction of something. The orientation of a shape may change following a rotation or reflection. The orientation of a shape does not change following a translation.

origin. The point of intersection of the vertical and horizontal axes of a Cartesian plane. The coordinates of the origin are (0, 0).

parallel. Extending in the same direction, remaining the same distance apart. Parallel lines or parallel shapes never meet, because they are always the same distance apart.

parallelism. The state of being parallel.

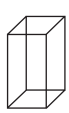
parallel lines. Lines in the same plane that do not intersect.

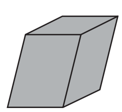
parallelogram. A quadrilateral whose opposite sides are parallel.

parallel sides. On a shape, sides that would never intersect.

pentagon. A polygon with five sides.

perpendicularity. The state of being perpendicular or having right angles.


The base of this rectangular prism is perpendicular to its vertical faces.


The base of this prism is *not* perpendicular to its vertical faces.

plane shape. *See* **two-dimensional shape**.

point of intersection. The point at which two or more lines intersect.

point of rotation. The point about which a shape is rotated.

Polydron. Commercially produced learning tools that help students learn about geometric figures. Polydron is a system of interlocking shapes for geometric constructions. *See also* **Frameworks**.

polygon. A closed shape formed by three or more line segments; for example, triangle, quadrilateral, pentagon, octagon.

polyhedron. A three-dimensional figure that has polygons as faces.

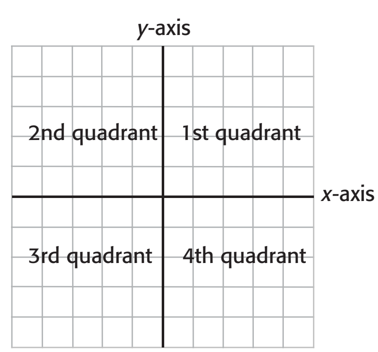
positional descriptor. A symbol that represents the position of an object on a grid or a coordinate system.

prism. A three-dimensional figure with two bases that are parallel and congruent. A prism is named by the shape of its bases; for example, rectangular prism, triangular prism.

property. An attribute that remains the same for a class of objects or shapes. A property of any parallelogram, for example, is that its opposite sides are congruent. *See also* **attribute**.

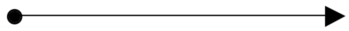
pyramid. A three-dimensional figure whose base is a polygon and whose other faces are triangles that meet at a common vertex. A pyramid is named by the shape of its base; for example, square-based pyramid, triangle-based pyramid.

quadrant. One of the four regions formed by the intersection of the *x*-axis and the *y*-axis in a coordinate plane.



quadrilateral. A polygon with four sides.

ray. A line that has a starting point but no endpoint.



rectangle. A quadrilateral in which opposite sides are equal, and all interior angles are right angles.

rectangular prism. A three-dimensional figure with two parallel and congruent rectangular faces. The four other faces are also rectangular.

reflection. A transformation that flips a shape over an axis to form a congruent shape. A *reflection image* is the mirror image that results from a reflection. Also called *flip*.

regular polygon. A closed figure in which all sides are equal and all angles are equal. *See also* **irregular polygon**.

reflex angle. An angle that measures more than 180° and less than 360°.

rhombus. A parallelogram with equal sides. Sometimes called *diamond*.

right angle. An angle that measures 90°.

right prism. A prism whose rectangular faces are perpendicular to its congruent bases.

right trapezoid. A trapezoid with at least one right angle.

right triangle. A triangle with exactly one right angle.

rotation. A transformation that turns a shape about a fixed point to form a congruent shape. A *rotation image* is the result of a rotation. Also called *turn*.

rotational symmetry. A geometric property of a shape whose position coincides with its original position after a rotation of less than 360° about its centre. For example, the position of a square coincides with its original position after a 1/4 turn, a 1/2 turn, and a 3/4 turn, so a square has rotational symmetry. *See also* **symmetry** and **order of rotational symmetry**.

slide. *See* **translation**.

solid figure. *See* **three-dimensional figure**.

spatial sense. An intuitive awareness of one's surroundings and the objects in them.

sphere. A three-dimensional figure with a curved surface. All points on the surface of a sphere are equidistant from its centre. A sphere looks like a ball.

square. A rectangle with four equal sides and four right angles.

square-based pyramid. A three-dimensional figure with a base that is square and four triangular faces.

straight angle. An angle that measures 180°.

symmetry. In a two-dimensional shape, the property of having two parts that match exactly, either when one half is a mirror image of the other half (line symmetry) or when one part can take the place of another if the shape is rotated (rotational symmetry). *See also* **line symmetry** and **rotational symmetry**.

three-dimensional figure. A figure that has length, width, and depth; a figure that is three-dimensional. Also called *figure* or *solid figure*.

transformation. A change in a figure that results in a different position, orientation, or size. The transformations include the *translation* (slide), *reflection* (flip), and *rotation* (turn). *See also* **translation**, **reflection**, **rotation**.

translation. A transformation that moves every point on a shape the same distance, in the same direction, to form a congruent shape. A *translation image* is the result of a translation. Also called *slide*.

trapezoid. A quadrilateral with one pair of parallel sides, or a quadrilateral with at least one pair of parallel sides.

triangle. A polygon with three sides.

triangle-based pyramid. A three-dimensional figure with a triangular base and three triangular faces.

turn. *See* **rotation**.

two-dimensional shape. A shape that has length and width but no depth.

Venn diagram. A diagram consisting of overlapping and/or nested shapes used to show what two or more sets have and do not have in common.

vertex. The common endpoint of the two line segments or rays of an angle. *See also* **angle**.

***x*-axis.** The horizontal number line on the Cartesian coordinate plane.

***y*-axis.** The vertical number line on the Cartesian coordinate plane.

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