Multi-Assignment Interacting Multiple Model for Tracking Microbubbles

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Abstract — **A novel multi-target tracking (MTT) method is developed in this paper, which is specifically designed to track microbubbles *in vitro*. The microbubbles, which frequently overlap and move erratically, are accurately tracked by a tailored multi-assignment (MA) algorithm combined with the interacting multiple model (IMM) estimator. The superiority of the proposed modified multi-assignment interacting multiple model (MA-IMM) tracking algorithm is demonstrated by way of a comparison to three other approaches.**

I. INTRODUCTION

Molecular imaging is involved in a number of disease states and is of great interest in various biomedical research areas [1-2]. Molecular imaging is based on the detection of molecular markers for the disease in blood vessels, usually with a contrast agent that is targeted to the markers. An ultrasound contrast agent, known as microbubbles, are tiny gas filled bubbles coated by targeting substance, which can be safely injected to blood vessels to enhance the ultrasound image by increasing the backscatter signal. The adhesion of targeted microbubbles has been assessed *in vitro* to determine whether a particular targeting substance could improve the adhesion. Accurately measuring the efficiency of adhesion requires tracking of all the observable microbubbles.

This paper puts forth a novel adaptation of a general multi-target tracking (MTT) method. There are two major challenges that the tracker must overcome in this application: 1) the fast-moving microbubbles frequently overlap the slow-moving ones as shown in Fig. 1, and 2) some microbubbles move intermittently, where a microbubble moves at a relatively high speed for some time before or after stopping, *i.e.*, the velocity of the microbubble changes dramatically. The intermittent movement of microbubbles and frequent overlapping make tracking them correctly extremely difficult. Our proposed MTT method is specifically adapted to robustly account for these problems.

II. SUMMARY OF TRACKING SOLUTION

A detailed description of all aspects of a general target tracking system can be found in [3]. The two key interrelated problems of MTT are data association and state estimation.

Data association is the decision process of determining which of the multiple measurements is chosen to update each track. Track refers to a symbolic representation of a target, which is the state estimate in the target tracking system. Measurement refers to the data or information obtained from the sensor. Many algorithms exist to address the competition among tracks for measurements [3-4], including multiple hypothesis tracking (MHT), joint probability data association (JPDA), and assignment algorithm. The multi-assignment (MA) algorithm, which provides superior performance than one-to-one (1-1) assignment for closely spaced objects [6], has been modified to improve the tracking performance.

State estimation is the process of evaluating the target state given the association result and measurement. The state contains all relevant information required to describe the system, usually including kinematic quantities and descriptive quantities in tracking problems. Typical state estimation techniques include the Kalman filter (KF), the interacting multiple model (IMM) estimator [7], and particle filters. The IMM estimator is selected as the estimator to cooperate with MA, as it is generally considered to provide better tracking performance than Kalman filter for the tracking of maneuvering targets [4].

Fig. 2 is the flow diagram of the proposed MTT algorithm. An explicit explanation of our implementation of the proposed multi-assignment interacting multiple model (MA-IMM) method will now be given [5]. A system with Markovian switching models is described by one of r hypothesized models $\{M^1, M^2, ..., M^r\}$ with a set of switching probabilities between the models [7]. Let M_k^j denote the event that model M^j is in effect at time k. The model switching is described by a finite state Markov chain with probabilities

$$p^{ij} = \Pr\{M_k^j \mid M_{k-1}^i\} \qquad (1)$$

which means the model switch from M^i at time $k-1$ to M^j at time k. The r^2 switching probabilities p^{ij} are assumed to be known a priori. For a linear system, the motion equation for M_k^j is given as

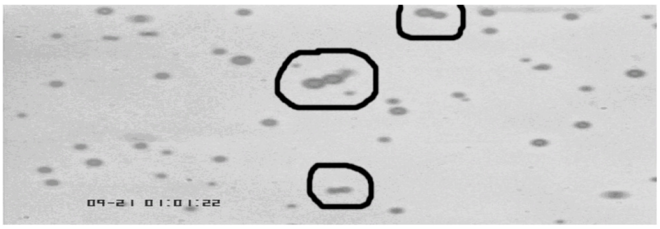

Fig. 1. Image of microbubbles, where overlapping cases are circled.

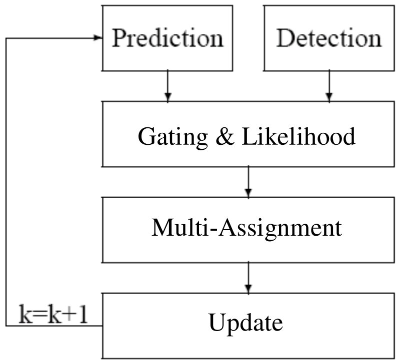

Fig. 2. The flow diagram of MA-IMM.

$$\mathbf{x}_k = \mathbf{F}_k^j \mathbf{x}_{k-1} + \mathbf{v}_k^j \qquad (2)$$

where $\mathbf{x}_k$ is the state vector at time *k*, $\mathbf{F}_k^j$ is a known matrix defining the linear motion model, and $\mathbf{v}_k^j$ is an independent and identically distributed (i.i.d.) zero mean Gaussian noise sequence with covariance matrix $\mathbf{Q}_k^j$. The measurement equation is given as

$$\mathbf{z}_k = \mathbf{H}_k^j \mathbf{x}_k + \mathbf{n}_k^j \qquad (3)$$

where $\mathbf{z}_k$ is the measurement vector, $\mathbf{H}_k^j$ is a known matrix defining the measurement model, and $\mathbf{n}_k^j$ is an i.i.d. zero mean Gaussian noise sequence with covariance matrix $\mathbf{R}_k^j$.

A. *Detection*

The detection method used to determine the existence and location of the microbubbles within a frame is based on grayscale morphology. After sequentially performing the grayscale morphological operations *open* and *close* on each frame of the video sequence, subtraction of the background yields microbubbles or unwanted artifacts. The microbubbles are easily distinguished from the unwanted artifacts by computing the appropriate features such as size and shape. Fig. 3 shows the result of our detection algorithm applied to Fig. 1.

B. *Prediction*

The prediction step is responsible for predicting the state estimates and model probabilities for the next time instance. Given the model-conditioned posterior state estimates $\left\{\hat{\mathbf{x}}_{k-1|k-1}^j\right\}_{j=1}^r$ with corresponding covariances $\left\{\mathbf{P}_{k-1|k-1}^j\right\}_{j=1}^r$ and model probabilities $\left\{\mu_{k-1|k-1}^j\right\}_{j=1}^r$ at time $k-1$, the model-conditioned prior state estimates $\left\{\hat{\mathbf{x}}_{k|k-1}^j\right\}_{j=1}^r$ with corresponding covariances $\left\{\mathbf{P}_{k|k-1}^j\right\}_{j=1}^r$ and prior model probabilities $\left\{\mu_{k|k-1}^j\right\}_{j=1}^r$ at time *k* can be calculated by

$$\hat{\mathbf{x}}_{k|k-1}^j = \mathbf{E}\left[\mathbf{x}_k \mid M_k^j, \mathbf{z}_{1:k-1}\right] = \mathbf{F}_k^j \hat{\mathbf{x}}_{k-1|k-1}^{0j} \qquad (4)$$

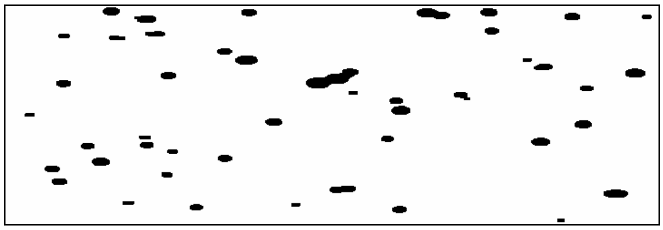

Fig. 3. Detection result from Fig.1.

$$\mathbf{P}_{k|k-1}^j = \mathbf{F}_k^j \mathbf{P}_{k-1|k-1}^{0j} \left[\mathbf{F}_k^j\right]^T + \mathbf{Q}_k^j \qquad (5)$$

$$\mu_{k|k-1}^j = \Pr\left\{M_k^j \mid \mathbf{z}_{1:k-1}\right\} = \sum_{i=1}^r p^{ij} \mu_{k-1|k-1}^i \qquad (6)$$

where $\left\{\hat{\mathbf{x}}_{k-1|k-1}^{0j}\right\}_{j=1}^r$ and $\left\{\mathbf{P}_{k-1|k-1}^{0j}\right\}_{j=1}^r$ are the interacted state estimates and the corresponding covariances, respectively [5, 7].

C. *Gating & Likelihood*

This step is responsible for preprocessing the data for MA to reduce the complexity. Gating is a technique to remove highly unlikely association candidates. The validation gate (VG) is the region in measurement space where the true measurement will lie with high probability, which is normally determined by the covariance of prior state estimate. We modify VG for better tracking performance by considering the prior knowledge and physical constraints. Since we know that the microbubbles move from left to right in horizontal direction, only the measurements which are on the right side of the target and are close to the target in vertical direction are most probably the measurement detected from the target. As illustrated in Fig. 4, the modified VG increases the reliability of MA by excluding unnecessary measurements.

The likelihood is the fundamental quantity for seeking the optimal assignment result. Let the set of tracks from time $k-1$ is denoted by $\left\{\mathcal{T}_{k-1}(n)\right\}_{n=1}^{N_{k-1}}$, the set of measurements from time k by $\left\{\mathbf{z}_k(m)\right\}_{m=1}^{M_k}$. Let $\Lambda_k(n,m)$ denote the combined likelihood of associating track $\mathcal{T}_{k-1}(n)$ to measurement $\mathbf{z}_k(m)$, which is given by

$$\Lambda_k(n,m) = \sum_{j=1}^r \Lambda_k^j(n,m)\, \mu_{k|k-1}^j(n) \qquad (7)$$

where $\Lambda_k^j(n,m)$ is the model-conditioned likelihood of associating measurement $\mathbf{z}_k(m)$ with track $\mathcal{T}_{k-1}(n)$ [4-5].

D. *Multi-Assignment*

The data association is formulated as an optimization problem subject to certain constraints in multi-assignment (MA) algorithm. The MA technique in our proposed method,

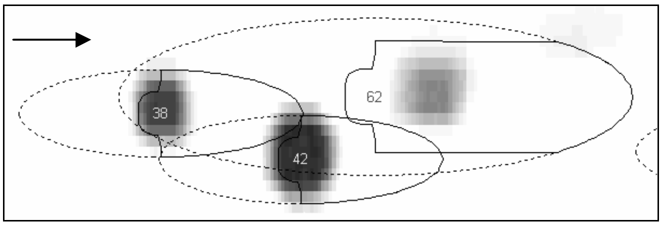

Fig. 4. The numbers indicate the prior estimate of different targets, while the dark objects are measurements. The standard VG and modified VG are shown as dash lines and solid lines, respectively. The modified VG of target 62 contains only one measurement, while the standard VG contains two unnecessary measurements on the left.

which is similar to the iterative one-to-one (1-1) assignment method in [6], only allows one measurement to be assigned to multiple tracks, while one track can only be assigned to one measurement. This modification is based on the fact that the microbubbles do not split into two in reality. The optimal association between the established tracks and the received measurements is obtained by minimizing the cost function proposed in [6], which could be solved by auction algorithm [8]. The cost function is highly related to all the likelihood calculated in the previous step.

E. Update

The assignment result and corresponding measurement $\mathbf{z}_k$ are used to update the model-conditioned prior estimates and model probabilities for each track to obtain the model-conditioned posterior estimates $\left\{\hat{\mathbf{x}}_{k|k}^j\right\}_{j=1}^r$ and the corresponding covariances $\left\{\mathbf{P}_{k|k}^j\right\}_{j=1}^r$, which are given by Kalman filter update equations

$$\mathbf{S}_k^j = \mathbf{H}_k^j \mathbf{P}_{k|k-1}^j \left[\mathbf{H}_k^j\right]^T + \mathbf{R}_k^j \tag{8}$$

$$\mathbf{K}_k^j = \mathbf{P}_{k|k-1}^j \left[\mathbf{H}_k^j\right]^T \left[\mathbf{S}_k^j\right]^{-1} \tag{9}$$

$$\hat{\mathbf{x}}_{k|k}^j = \hat{\mathbf{x}}_{k|k-1}^j + \mathbf{K}_k^j \left(\mathbf{z}_k - \mathbf{H}_k^j \hat{\mathbf{x}}_{k|k-1}^j\right) \tag{10}$$

$$\mathbf{P}_{k|k}^j = \mathbf{P}_{k|k-1}^j - \mathbf{K}_k^j \mathbf{H}_k^j \mathbf{P}_{k|k-1}^j \tag{11}$$

where $\mathbf{S}_k^j$ is the residual covariance, and $\mathbf{K}_k^j$ is the Kalman gain. The posterior model probabilities $\left\{\mu_{k|k}^j\right\}_{j=1}^r$ can be calculated by

$$\mu_{k|k}^j = \Pr\left\{M_k^j \mid \mathbf{z}_{1:k}\right\} = \frac{1}{c}\mu_{k|k-1}^j \Lambda_k^j \tag{12}$$

where c is the normalization factor

$$c = \sum_{j=1}^r \mu_{k|k-1}^j \Lambda_k^j \,, \tag{13}$$

and Λ_k^j denotes the likelihood of M_k^j given measurement $\mathbf{z}_k$, which can be computed as

$$\Lambda_k^j = \left|2\pi \mathbf{S}_k^j\right|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\left[\boldsymbol{\upsilon}_k^j\right]^T \left[\mathbf{S}_k^j\right]^{-1} \boldsymbol{\upsilon}_k^j\right\} \tag{14}$$

where $\boldsymbol{\upsilon}_k^j = \mathbf{z}_k - \mathbf{H}_k^j \hat{\mathbf{x}}_{k|k-1}^j$ is the residual.

Finally, by considering overall state estimates and model probabilities using (10)-(12), the combined posterior state estimate $\hat{\mathbf{x}}_{k|k}$ and the corresponding covariance $\mathbf{P}_{k|k}$ are given by

$$\hat{\mathbf{x}}_{k|k} = \sum_{j=1}^r \mu_{k|k}^j \hat{\mathbf{x}}_{k|k}^j \,, \tag{15}$$

$$\mathbf{P}_{k|k} = \sum_{j=1}^r \mu_{k|k}^j \left\{\mathbf{P}_{k|k}^j + \left[\hat{\mathbf{x}}_{k|k}^j - \hat{\mathbf{x}}_{k|k}\right]\left[\hat{\mathbf{x}}_{k|k}^j - \hat{\mathbf{x}}_{k|k}\right]^T\right\}. \tag{16}$$

III. RESULTS

Our experimental results show that the combination of MA and IMM, referred to as MA-IMM, provides a more accurate method to track microbubbles when compared to 1-1 assignment with KF (1-1 KF), MA with KF (MA KF), and 1-1 assignment with IMM (1-1 IMM). A two-model IMM estimator with random walk and constant velocity motion models was employed in the experiment, while constant velocity motion model was used in the KF. The switching probabilities for the IMM estimator were empirically set. The parameters used in the KF provided the best result from an extensive trial.

The six performance metrics we use to evaluate the four MTT algorithms are compiled in the following list with definitions and ideal values.

1) Track breakage (TB) is the total number of tracks minus the total number of actual targets. Ideally, the TB count would be zero.
2) Miscorrelation (MC) is defined as the number of false data associations, *i.e.*, a measurement is associated with a track that in the previous frame was updated by a measurement due to a different target. Ideally, the MC count would be zero.
3) A false positive (FP) is an incorrectly assigned measurement. A FP is caused by either clutter or MC. Ideally, the FP count would be zero.
4) A false negative (FN) occurs when an actual target is not assigned to a track. This quantity is also presented as a percentage by dividing with the track life lengths of all the targets. A FN is usually caused by missed measurement. Ideally, the FN count would be zero.
5) Total track life (TTL) of a target is the number of frames that the target is correctly tracked divided by the number of frames in which the target actually exists. Ideally, the TTL value would be 100%.
6) Relative complexity (RC) is the normalized computing time for each of the four MTT algorithms tested, which eliminates any hardware or system environment dependant issues. In this case, the computational expense is taken relative to the MA KF case.

Using a video sequence of 1826 frames of 125 microbubbles captured *in vitro* at a rate of 60 frames per second, the results of all six performance metrics from four MTT algorithms tested are given in Table I. It is evident from the comparison given in Table I that the MA-IMM algorithm yields less TB, MC, FP, and FN, as well as higher TTL value, than the other three MTT algorithms. The cost of these improvements is about 50% more computing time, reflected by the RC value. It should be noted that this is not a major drawback, since the MA-IMM algorithm can be easily implemented in a parallel processing environment.

IV. CONCLUSION

The multi-target tracking algorithm for tracking microbubbles *in vitro* must account for the intermittent movement and the overlapping behavior of the microbubbles, must be robust to noise and clutter, and must provide an accurate position estimate of each microbubble from frame to frame. In addition, the tracking method must be computationally feasible for practical implementation. The novel multi-target tracking method MA-IMM proposed in this paper provides a resourceful trade off between computational complexity and tracking performance by combining MA with IMM. The MA is accomplished by modifying the iterative 1-1 assignment described in [6].

This algorithm has been successfully employed for measuring the microbubble adhesion efficiency [9]. It can be easily extended to other applications in different research areas.

TABLE I
TRACKING RESULTS OF FOUR MTT ALGORITHMS

	TB	MC	FP	FN $\left(\frac{FN}{113522}\right)$	TTL	RC
1-1 KF	61	28	390	2447 (2.16%)	88.1%	1.01
MA KF	51	24	468	2294 (2.02%)	90.8%	1
1-1 IMM	28	4	29	2328 (2.05%)	89.1%	1.54
MA-IMM	23	2	9	1746 (1.54%)	94.8%	1.51

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