Mathematics through Art - Art through Mathematics

Mike Field
Department of Mathematics
University of Houston
mf@uh.edu
http://nothung.math.uh.edu/~mike/

Abstract

Computer software can be used for artistic design and as a means to facilitate the understanding of mathematical concepts. We discuss some of the issues that arise when we use computers in this way, and some of our own experiences teaching both art and mathematics students using software that can assist in the design and coloring of symmetric images.

1 Introduction

> Mathematicians have long since regarded it as demeaning to work on problems related to elementary geometry in two or three dimensions, in spite of the fact that it it precisely this sort of mathematics which is of practical value.
>
> Grünbaum & Shephard [1]

In this article, I want to address the question of the extent to which one can gain insight and skill in the fine arts and mathematics using computers. More specifically, I want to relate some of my own experiences (and prejudices) in this field[1].

I use the word prejudice advisedly. Although computers are now almost universally accepted in the mathematics community as a valuable *tool* for solving problems, there is certainly no consensus on whether or not computers are useful for teaching basic insight and understanding of mathematics. My own position, for example, is that (graphics) calculators should be treated the same way as alcohol in the USA - permission to use should not be granted until the age of 21, and then only in moderation[2]. Similarly, in the art community, there is considerable resistance to the idea that computer graphics, generated from say the Mandelbrot or Julia sets, can be art. For a professional *mathematician* to create artistic images based on mathematics is still, in some quarters, regarded as a betrayal.

While mathematics has long been intertwined with the arts, it is fairly recently that there has been an attempt to create art based on mathematical objects. In this article, however, I do not want to focus on the possibilities of creating significant art using computers. Rather, I want to consider whether mathematically based programs can be effectively used in the teaching of design – or the fine arts – and whether or not a program that has the potential for producing artistic images can be used effectively in the teaching of geometry and algebra.

In attempting a tentative, but positive, answer to this question, I will relate some of my own experiences using software I designed. I have included a number of images created using this software so that readers can judge whether or not this is a rich and worthwhile medium to be working with.

[1]The reader is cautioned that this article is not intended to describe, or in any sense evaluate, the many efforts that have been made by others in this area.

[2]This is serious – I do not allow the use of calculators in any of my tests or examinations.

1.1 History

About eleven years ago, I started to develop a computer program *prism* (PRogram for the Interactive Study of Maps). Initially, the program was used to design, create and color bounded symmetric images of the plane and square and hexagonal tilings. The images were created using methods based on chaotic dynamics and iterated function systems. In 1992, Marty Golubitsky and I published the book *Symmetry in Chaos* [2] which showed some of the designs we had created using *prism* as well as describing some of the underlying mathematics of 'symmetric chaos'. We refer the reader to [3, 4] for more information on symmetric chaos and its applications.

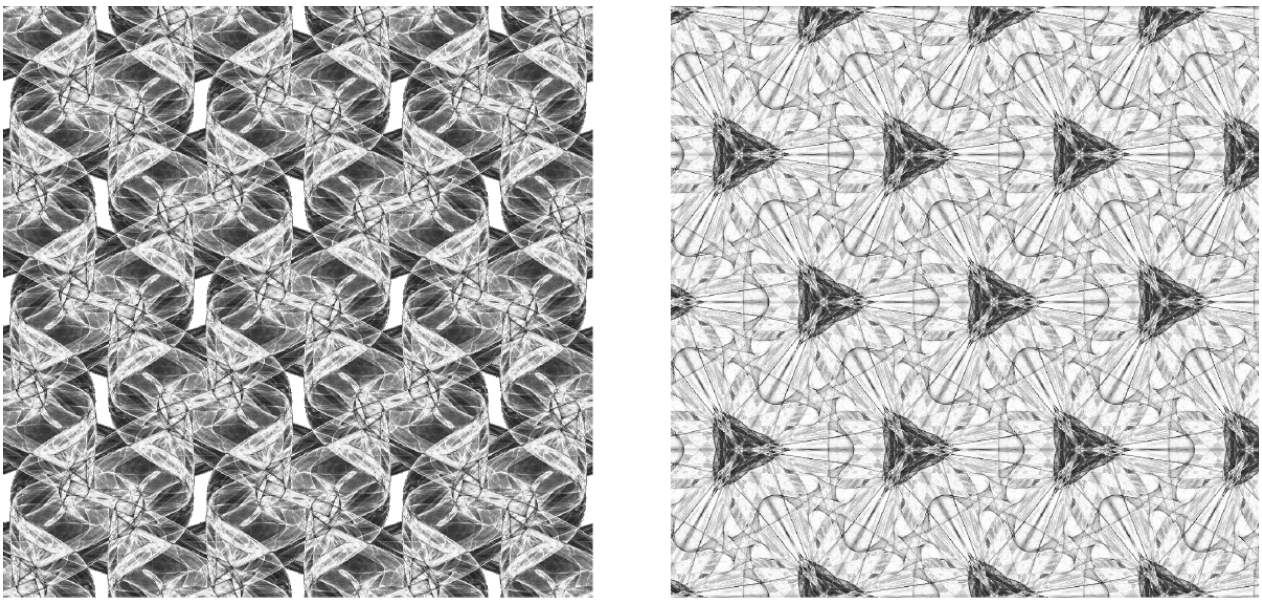

Figure 1: Wallpaper patterns of types **pgg** and **p31m**.

Over the years, *prism* has continued to develop and now includes a multiplicity of algorithms for all seventeen of the wallpaper patterns as well as the forty-six 2-color patterns. Examples of wallpaper patterns created using *prism* are shown in Figure 1 and (for two color wallpaper patterns) in Figure 4 at the end of the article. We refer to [5] for more details about *prism* and to the URL: nothung.math.uh./edu/˜mike for colored examples (a discussion of the theory underlying the creation of 2-color designs can be found in [6, 7]). Characteristically, the designs created using *prism* exhibit a rich complex and (of course!) symmetric structure. In Figure 2, we show the magnifications of small pieces of each of the quilts shown in Figure 1.

Successful coloring of the images produced by *prism* usually requires significant effort from the designer. In my opinion, one of the main attributes of *prism* is that, unlike some graphics and drawing software, it is not too hard to produce *ugly* or unsatisfying images (just as this is not difficult to do with paint and brush).

Although I developed *prism* for personal use in creating graphic designs, I have recently made extensive use of *prism* in a course I developed for art and design students at the University of Houston (UH). I have also used the program as the basis for a seminar run for (mainly) mathematics teachers in the Houston Independent School District (HISD). Using graphics software in this way raises some interesting, and controversial, questions about the potential for the creative use of computer software in the teaching of design and mathematics. In this article, I want to share some of my experiences teaching these Art-Math courses, address some of the issues that arise and, of course, give some illustrations of symmetric chaos.

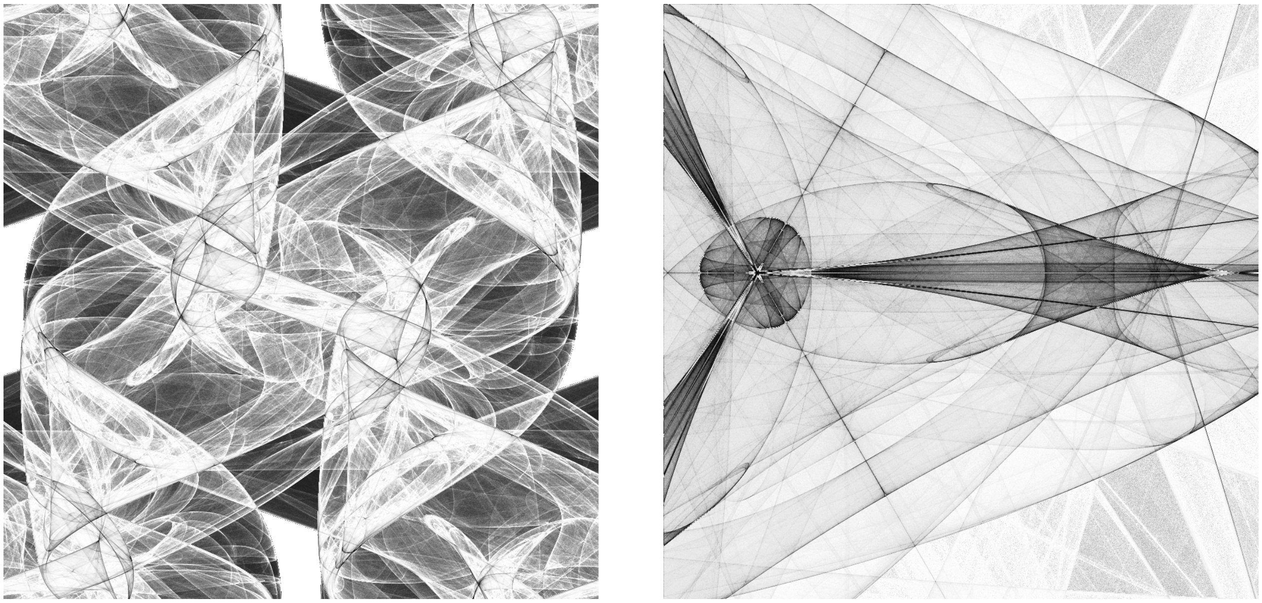

Figure 2: Fine detail of patterns shown in Figure 1.

2 Symmetry in design: Implementing a symmetry course for Art students

Mighty is geometry; joined with art, irresistible. Euripides.

2.1 Background

About four years ago, it was suggested by Angela Patton in the Department of Art at UH, that it would be worthwhile developing an interdisciplinary course on patterns, designs and symmetry based on *prism*. Although I had primarily developed the program for personal use – specifically, the design and creation of computer graphics – I was intrigued as to how art students would respond to an 'art' program, whose origins lay in mathematics, and I wanted to see what the students could do with the software. There was also the challenge of communicating some of the underlying mathematical ideas about geometry and symmetry to the 'math-unfriendly' students. In the event, I developed a new course 'Patterns, Designs and Symmetry' for Junior and Senior level students at UH. The course has so far been given twice and will become part of the regular course schedule in Fall 2000.

2.2 Implementation and results

One of the aims of the 'Patterns, Designs and Symmetry' class was to expose students to the mathematical concept of symmetry and thereby enhance their visual perception and design skills. This had to be done in a context where it was not acceptable to develop the mathematics in a formal didactic way. Indeed, many of the students were not only alarmed by the prospect of doing mathematics but were also nervous about the prospect of learning about and using computers. In brief, equations and formulae were banned from the class. Characters like 'X' and 'Z' were introduced for their symmetry properties rather than as symbols to be used in algebraic manipulation. Geometry was, however, emphasized. By the end of the course, students were expected to be able to find the glide reflection symmetries of a planar pattern, distinguish, for example, between wallpaper patterns of type **p3m1** and **p31m**, and identify the symmetries of a polyhedron (including the regular stellated polyhedra). The approach that was followed was a mix of theory and practice. Practice was vital. Students designed symmetric patterns, created informative posters and made solids. For example, some of the students made symmetrically colored models of the regular stellated dodecahedra. As a text we used the book by Washburn and Crowe [9] supplemented by my own notes and websites such as the geometry junkyard (URL: www.ics.uci.edu/˜eppstein/junkyard/).

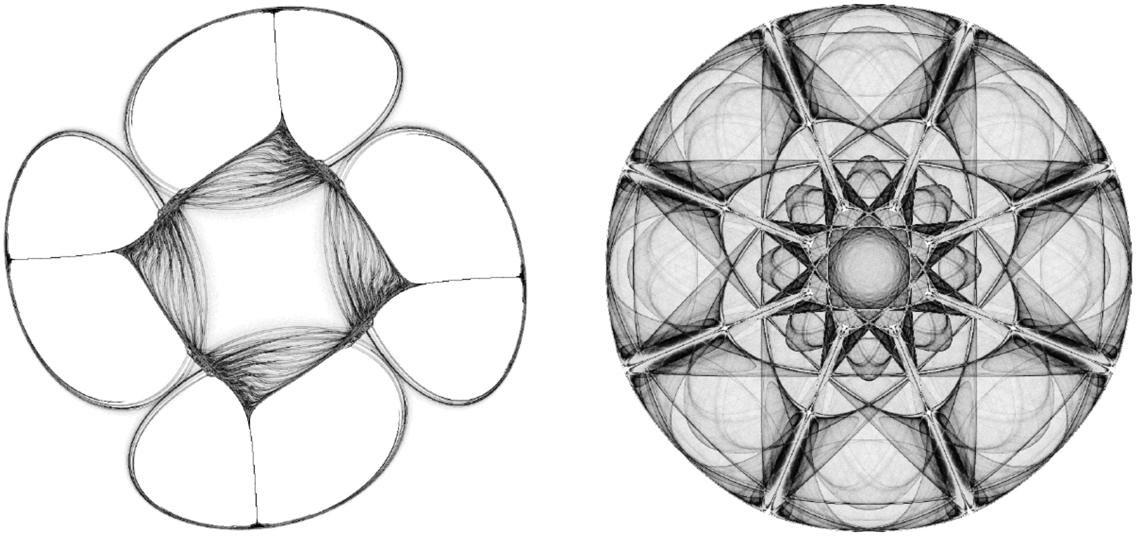

Figure 3: Bounded patterns with 4- and 8-fold symmetry

From the start, my aim was to sensitize the students to symmetry; to get the students obsessing about symmetry. In Houston, this was easily accomplished by initiating an active and ongoing study of the symmetry of (car) wheels. Using wheels as a model, we discussed the different types of (bounded) symmetry in the plane. In particular, the difference between rotational and reflectional symmetries – from a mathematical and design perspective. Symmetry was introduced as an operational property: an object is symmetric if you can pick it up, move it around and put it back in the same space as it was originally – but with a different orientation. Along the way, it was possible to surreptitiously introduce some ideas from algebra - to help with the composition of symmetries - and to point out how order of composition matters. Concurrent with the investigation of bounded symmetry, students were learning how to log on to a computer and use software packages. Their first project using *prism* was to design, and later color, a bounded symmetric image in the plane. In Figure 2.2, we show a pair of bounded symmetric images created using *prism*. This creation of a serious and attractive design was an essential, highly motivational, part of the learning process – a movement from theory (of symmetry) to producing a design. And the designs were often very attractive. (See the URL: nothung.math.uh.edu/˜patterns/indexart.html for some examples from the 1999 class.)

About 40% of the course was spent in a computer lab of about 30 Silicon Graphics machines. However, the refinement and coloring of designs was done on a small network of computers I had built that ran Linux. Perhaps it is surprising, but the students had little in the way of problems adapting to a Linux/Unix environment and overall the experience in the labs was very positive.

Was it worthwhile giving the course? Certainly, the students were happy (the course received very high evaluations in 1999). So also was the instructor. In my opinion the class left with a *positive* view of mathematics and what it could do. In particular, all of the students appreciated how symmetry was an integral part of design. The students also learnt about chaos, probability, mathematics and computers and, contrary to what one reads in some of the mainstream press, had rather little difficulty in handling a UNIX based system. In short, they learnt that many things that had supposed were beyond their intellectual reach were not - and were even fun. Even some of the mathematics.

3 Mathematics through Art?

The union of the mathematician with the poet, fervor with measure, passion with correctness, this surely is the ideal.
William James

3.1 Background

A year ago I ran a seminar on symmetry and patterns for the inaugural year of the *Houston Teacher's Institute* (HTI). The Houston Teachers Institute is part of a national project led by the Yale-New Haven Teachers Institute and supported by the DeWitt Wallace-Reader's Digest Foundation. (For information about the Yale-New Haven Teachers Institute, which was founded in 1977, see the URL: www.cis.yale.edu/ynhti.) Roughly speaking, the idea of the HTI is that a group of about twelve teachers (the 'Fellows') participate in a seminar led by a faculty member at UH with the goal of each Fellow producing a curriculum unit related to the topic of the seminar (see the HTI website, URL: www.uh.edu/hti, for more details).

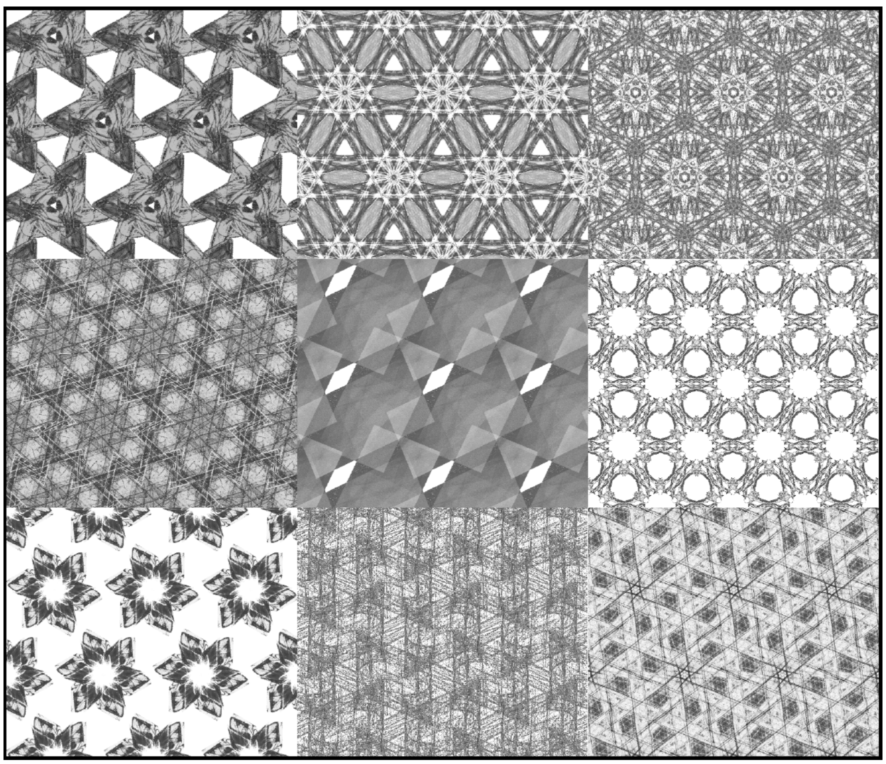

Figure 4: Quilts from quilt patterns

In the event I led a seminar on 'Symmetry, Patterns and Designs'. The majority of the Fellows in the seminar were mathematics teachers in local middle or high schools (the exceptions were one social studies, one art and a computer science teacher). In the seminar, I used *prism* as a tool to introduce the Fellows to ideas about symmetry and chaos. Part of the seminar involved each Fellow producing their own graphic. Some of these graphics were used as the basis for a quilt pattern which was used as the inside cover for the published set [8] of curriculum units for the entire 1999 Houston Teacher's Institute – see figure 4. (For the individual units see the HTI website.)

3.2 Implementation

While I was planning the seminar, I had to keep in mind that content had to be interesting and stimulating for the teachers. Practically speaking, that meant that what we discussed had to have the potential for practical application in a high or middle school classroom. In particular, it was essential that the material could be used to facilitate the understanding of geometry or algebra (if that were the teacher's specialty). In fact the situation for teaching general mathematics courses at college level is broadly similar. It is not simply a question of making mathematics appealing or user-friendly. The message has to have content – and that content must lead to an enhanced understanding of the world that the student perceives.

In the event, none of the teachers had any background in symmetry. This was an advantage as it meant there were no preconceptions to overcome. Just as for the art class, the topic of the symmetry of wheels was interesting to the teachers – but now because they knew that this was one of the topics teenagers could get excited about (we did not discuss the possible relation between symmetry, beauty and sexual attractiveness – though that came up in some of the curriculum units). Another area of great potential was the use of symmetry in textiles – again dress is important to the target group. Architecture and art provided other possibilities. One of the most interesting suggestions made was to combine art and mathematics classes as a way of allowing the students to use ideas about symmetry in their art – this for a group of students who were almost totally alienated from mathematics and who, on good days, spent their time in mathematics classes drawing (or doodling).

Underneath all this, serious progress was made. Symmetry (both in 2- and 3-dimensions) is very geometric. It is far from trivial and leads to interesting questions in geometry (for example, every rigid motion of the plane is either a rotation, or a glide reflection or a reflection or a translation). Understanding concepts such as transformations of the plane, glide reflection symmetries and the classification of wallpaper patterns is not easy. I would argue that developing this type of geometric understanding of the world around us is more valuable (and motivational) than knowing how to factorize $x^2 + 3x + 2$ or do long division of polynomials. Indeed, I cannot understand why we require college students to be adept at college algebra – as opposed, for example, to acquiring a decent knowledge of geometry or statistics. Other topics that arose from our discussions on chaos and symmetry were fractals (the 'chaos game') and probability. In Figure 3.2 we show two examples of symmetric fractals designed using *prism*.

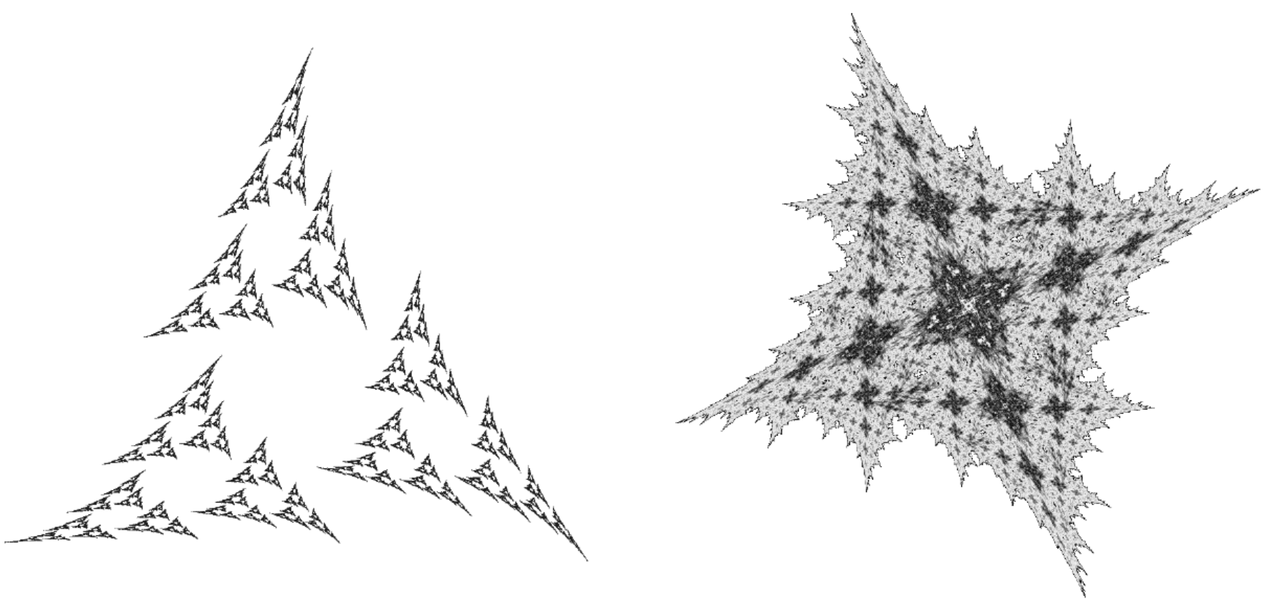

Figure 5: Symmetric fractals

A significant benefit of the seminar was the de-mystification of computers (which were at that time being widely introduced in HISD). Again the group was learning on UNIX machines and having fun creating *their* images.

3.3 The mystification of mathematics

There has been considerable controversy about the use of computers in mathematics teaching and the popularization of topics such as ‘chaos theory’. These questions have caused some polarization and division within the mathematics community. Roughly speaking, one group is concerned that mathematics needs to be presented in a more friendly and exciting way lest the whole subject become marginalized because of declining public interest. On the other hand, there is concern that popularization of mathematics is often at the expense of content and accuracy, a matter perhaps of selling one’s soul to the devil or reducing mathematics to ‘form without content’.

The widespread availability of modern computers and high performance graphics has led to a greater public awareness of some parts of mathematics, notably *chaos theory* (the ‘butterfly effect’) and *fractals* (the Mandelbrot and Julia sets). Computer users can routinely see some of the visual effects of chaos and fractals through their screen savers. On the other hand, at least in the USA, this awareness seems limited. In recent classes I have taken, none of the students had ever heard of the Mandelbrot set or ‘chaos theory’. While preparing this article, I viewed again the video *Fractals: the colors of infinity* [10], narrated by Arthur C Clark. Although the video was well received by members of my art class (and by a colleague in art), I found the video rather unsatisfying. A few quotations from the video will suffice: ‘The thumb-print of God’, ‘infinitely complex’, (referring to the Mandelbrot set), ‘This is how God created a system which gave us free will’, ‘collective unconscious’, ‘the mathematics offers new insight into the way the universe works’ and (referring to rose windows and Islamic art), ‘... many echos of the Mandelbrot set, centuries before it was discovered’.

These rather general statements carry the message that mathematics (which most people do not understand) can easily provide answers and insight – it is repeatedly emphasized in the video how simple it is to make the Mandelbrot set. Just as happened with catastrophe theory, fractals or chaos are presented as the ‘answer’. Indeed, as a simple and universal answer given by *deep* mathematics (which by implication, *you* cannot understand). Personally, I find this mathematics by intimidation. It is also often false or, at best, misleading. While the statement that ‘a butterfly flapping its wings in Beijing can cause a hurricane in Houston’ may be a good metaphor for sensitive dependence on initial conditions, it is meaningless from a scientific point of view. Specifically, it is untestable as a hypothesis and quite impossible to prove or disprove. Of course, all of this is mostly harmless when presented to adults on late night TV. It is another matter altogether if it presented to students as ‘mathematical reality’. The effect is just mystification, trivialization – and eventual alienation from mathematics.

The reality is that most of the good and interesting *questions* in mathematics (and life) are easy to state and explain. What is far from easy is the development of techniques to solve those problems:

> The paradox is now fully established that the utmost abstractions are the true weapons with which to control our thought of concrete facts.
>
> Alfred North Whitehead [11].

It is neither desirable nor necessary to shroud mathematics in a layer of magic and mystery. As in life, solutions to problems can be difficult and often only partial. Software designed for teaching needs to reflect these realities.

4 Conclusions

How should all of this affect the design of mathematical software for the (possibly) mathematically challenged student? I would argue that for the software package to be effective in teaching mathematics (or creating art), it must transcend the viewpoint of problem solving as equivalent to 'accurately keying in the problem'. A design philosophy that is predicated on the assumption that enjoyment, motivation and learning are *directly* proportional to the ease of use is a flawed philosophy. We get involved (and motivated) when we get stuck. In fact failure to solve a problem, after intense effort, is often the most effective way to learn. After one fails, one can really appreciate the instructor's solution. If we are designing software to help students understand and use mathematics, the software needs to be able to reflect the users' experience of trial and error in real life. That is, the interface needs to go beyond a correct (there is the solution), false (there is nothing) model. Sometimes the user should partly succeed or fail. Just as a program that does 'beautiful' Julia sets every time is in the end unsatisfying *precisely* because it always produces 'beautiful' Julia sets.

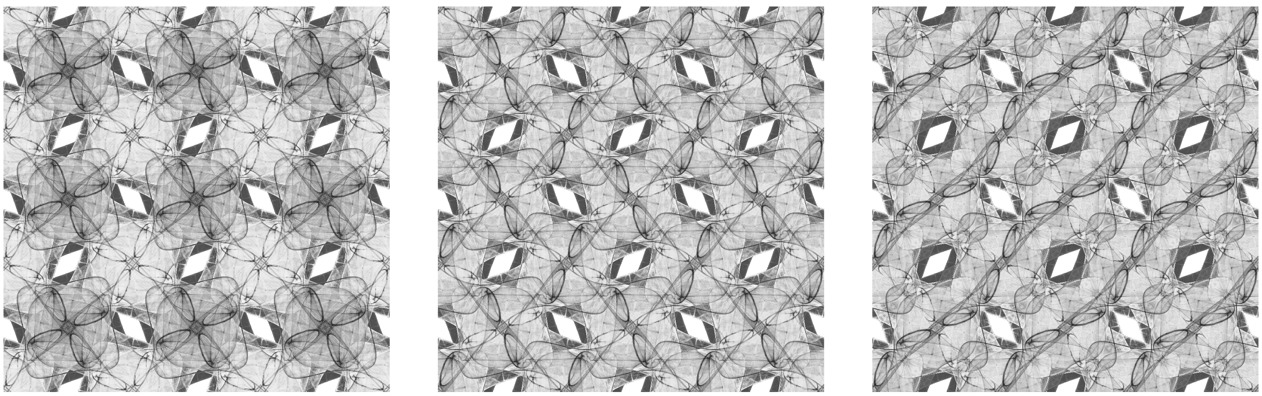

Figure 6: 2-colorings of a quilt of type **p4g**

In our final figure, we show three different 2-colorings of a quilt pattern of type **p4g**. If we follow the notation of Washburn & Crowe [9], these patterns are (reading from left to right) of types **p4g′m′**, **p4′gm′**, and **p4′g′m**. Each pattern consists of a pair of symmetrically related overlapping wallpaper patterns of types **p4**, **pgg**, and **cmm** respectively (see [6, 7]).

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