Operational Risk Capital Allocation and Integration of Risks

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Abstract

Modelling and measuring operational risk have become topics of intensive discussion in the financial services industry due to the new Basel Accord proposal requiring internationally active banks to set aside capital against these risks. To set the context a short review of the recommended options for calculation of such a capital charge for operational risk is first presented. Critical comments from the industry and the obvious lack of clarity in the suggested methodology show that much more research is needed in the short time left before this new proposal is supposed to be accepted. In this paper we present a risk capital framework which is based upon the assumption that for a well managed bank market and credit risk management yield sufficient capital provision against these risks and give a threshold for the identification of the extreme losses being characterized as operational from the regulators' viewpoint. Our capital allocation rule links operational with market and credit risks and provides a risk measure for the tails of loss distributions at both the firm-wide and business unit levels.

1. Introduction

In September 1998 in a special address to the Credit Risk Modeling Conference held in London, W. McDonough, Basel Committee chairman and chief executive of the Federal Reserve Bank of New York, turned the attention of delegates to operational risk, reminding them about the events at Barings and Daiwa banks. At that moment, known to only a very few of the conference participants, the biggest bail-out of our time was being discussed in New York, that of Long Term Capital Management (LTCM). The consequences for financial stability of its subsequent near failure raised the concern of regulators at the international level.

As a reaction to that event and to many other highly publicized events, all of which have involved ‘mismanagement’ such as fraud, unauthorized trading or mistaken long term views, the New Basel Accord [1] specifically defines *operational risk*. For the first time banks will be required to reserve capital against risks other than credit and market. Three approaches are proposed by the Basel Committee for calculating operational risk capital, but all three are lacking specifics and require the availability of appropriate -- increasingly detailed -- data for quantification of operational risks. All three approaches follow a simple actuarial methodology similar to the risk-based capital rules of the 1988 Basel Capital Accord. Whether these rules will seriously underestimate or overestimate the losses caused by criminals or technological failures remains an open question. But a more serious question must be asked: Does the proposed approach have serious flaws in its formulation and application which will limit current and all future innovation for credit and market risk management?

2. The new operational risk charge -- regulatory response to major risk mis-management

In what follows a brief review of the current Basel proposal is given, followed by a summary of the criticism contained in available comments [2]. Probably the strongest is that of the American Banking Association:

'The inclusion of an operational risk capital charge … is arbitrary, undeveloped and not capable of being implemented.'

The need for clarification of the new Basel alternative operational risk quantification is obvious. The complexity of the originating causes of operational risk, the 'rare event' nature of *significant* losses and the desire to integrate operational risk capital provision with that for market and credit risks all lead us to capital allocation rules based on results from *extreme value theory* (EVT)[1]. Application of EVT to operational risk modeling serves as the principal objective of regulation:

'A real concern of supervisors is the low-probability, high-severity event that can produce losses large enough to threaten a financial institution's health' [5].

The theory of extreme events tells us that expected severity is a linearly increasing function of a specified threshold (see, (7) in Section 4). Therefore regulatory operational risk capital will have to be increased to higher and higher levels and this is why extreme operational risk should be monitored and only *partially* covered by economic capital within the discretion of an institution. The lender of last resort can be considered to grant a form of put option on the uncovered extreme risk whose premium in effect is regulatory capital.

The fact is that no data now exists for evaluation of operational risk events similar to Barings, Daiwa or LTCM. The possibility of effectively pooling such data across institutions seems unrealistic for many years to come and is statistically invalid without much further research. What is required by the new proposal would be equivalent to

[1] For the basic theory and an extensive list of references, see [3]. For a current overview of EVT applications in finance see [4].

benchmarking such operational risks as ‘standard fraud’ or ‘average technology breakdown’.

In our method only internal data of trading and banking books should be used for operational risk assessment. The presence of ‘extremes’ will indicate that some operational risks exist since the ‘normal’ markets assumption (i.e. losses being not far too from their mean) is then violated. The question of the precise choice of appropriate level of capital allocation -- economic or regulatory -- is one left for the industry and regulators to decide in the future. We attempt here only to provide an appropriate methodology.

The banking industry responded to the 1988 Basel Accord by investing in research and development of internal risk models for market and credit risks based on the *value at risk* (VaR) paradigm. The 1996 amendment allowing the banks to use their own internal VaR models for market risk management and the current acceptance of elements of internal credit models is an admission that carefully specified VaR models can deliver a more accurate measure of risk. It has also lead to practical reductions in the capital charges of leading institutions. In new proposal the Basel committee has countered by imposing a regulatory capital charge related to operational risk.

Operational risk definitions are based on the identification of causes whose consequences are often not measurable. The new Accord definition of operational risk replaces earlier long lists of everything that could go wrong [6] with the shorter classification summary:
‘The risk of direct or indirect loss resulting from inadequate or failed internal processes, people and systems or from external events.’ [New Basel Capital Accord definition]

Thus at present the earlier debate on the definition of operational risk has evolved to the current debate on the amount of regulatory capital required to cover it. Nevertheless differences in defining risk may result in irreconcilable differences between the results of quantitative models.

Overall the operational risk charge will represent 20% of the minimum regulatory capital bank capital ratio[2]. A bank's total capital ratio -- minimum 8% -- will be measured by [Overview of new Accord, p.12, point 63]:

$$\frac{Total\ Capital}{Credit\ Risk + 12.5(Market\ Risk + Operational\ Risk)}$$

Equivalently, total regulatory capital must exceed the sum of the assessments of 8% of credit risk, all of market risk and all of operational risk, i.e.

$$Total\ Capital\ \geq 0.08\ Credit\ Risk + Market\ Risk + Operational\ Risk\ .$$

Three options have been proposed for calculation of the operational risk charge for an institution. In all three options the value of regulatory operational risk capital is proportional to some *exposure indicator* (EI) which is an accounting measure of bank activity.

Option 1 -- The basic indicator approach

Gross income is proposed as an exposure indicator and is measured by a rolling 3 year average. The charge or *operational risk capital* is equal to a fixed proportion α (approximately 30%) of the gross income, i.e.

The charge equals EI multiplied by α*.*

Option 2 -- The standardised approach

A bank is divided into standard business lines. Regulators for each business line specify an exposure indicator serving as a proxy for the area's activity (see Table 1).

The charge for each business line equals a standard risk indicator or *exposure indicator* (EI) of business line multiplied by an *individual factor* β_i . The level of the factors

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[2] This will be reduced due to criticism from industry [Update on the New Basel Capital Accord 25.7.01]

β_i will be calculated to reflect the different weightings of business lines (from a given broad range of standard weightings) and the institution's EI values.

The total charge equals the sum of business line charges.

Business Line	**Exposure Indicator**
Corporate Finance	Gross Income
Trading and Sales	Gross Income
Retail Banking	Annual Average Assets
Commercial Banking	Annual Average Assets
Payment and Settlement	Annual Settlement Throughput
Asset Management	Total Funds Under Management
Retail Brokerage	Gross Income

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Table 1. *The standardised approach: division of an average bank into business lines*

Option 3 -- The internal measurement approach

This approach involves a more detailed view of operational losses by considering a number of operational risk types for each business line. The classification in Table 2 presents the current view of risk types, and business lines and exposure indicators. For each business line/risk type, a bank provides an exposure indicator (EI) which is a proxy for the size of the risk exposure, an (expected) frequency of loss events given by the *probability of a loss event* (PE) and an (expected) severity of loss given by a *loss given event* (LGE) value.

Expected loss (EL) by business line and loss type is a product of EI, PE and LGE, i.e.

$$EL = EI \times PE \times LGE. \qquad (1)$$

The charge by business line and loss type equals EL multiplied by an individual factor γ, which will be determined by supervisors on the basis of industry-wide data. The industry-wide loss distribution and the regulatory specified gamma term are supposed to capture the differences in the risk profiles of individual banks.

<table>
<tr><td rowspan="2"></td><td colspan="4">Event Type Category</td></tr>
<tr><td colspan="2">Write-downs due to theft, fraud, unauthorized activity; Loss of recourse; Restitution; Regulatory and compliance penalties; Legal liabilities</td><td>Employment Practices and Workplace Safety</td><td>Damage to Physical Assets</td></tr>
<tr><td>Business Line</td><td>EI -- Financial Statement-Based</td><td>EI -- Transactional Value-Based</td><td></td><td></td></tr>
<tr><td>Corporate Finance</td><td>Gross Income</td><td>Value of Deals</td><td rowspan="8">Total Compensation OR Total Number of Employees</td><td rowspan="8">Book Value of Physical Assets</td></tr>
<tr><td>Trading & Sales</td><td>Gross Income</td><td>Value of Trades</td></tr>
<tr><td>Retail Banking</td><td>Gross Income / Total Assets</td><td>Value of Retail Transactions</td></tr>
<tr><td>Commercial Banking</td><td>Gross Income / Total Assets</td><td>Value of Com. Bank. Trans.</td></tr>
<tr><td>Payment & Settlement</td><td>Gross Income</td><td>Value of Trans. Settled & Payments Made</td></tr>
<tr><td>Agency Services</td><td>Gross Income/ Assets u. Man.</td><td>N/A</td></tr>
<tr><td>Asset Management</td><td>Gross Income/ Funds u. Man.</td><td>N/A</td></tr>
<tr><td>Retail Brokerage</td><td>Gross Income</td><td>Value of Transactions</td></tr>
</table>

Table 2. *Business lines, loss types and exposure indicators*

Each option of the new proposal is viewed as a ‘progressive step’ in the management of operational risk at ‘increasing levels of sophistication’. Yet, Option 3 lacks any clarity in problem formulation. In spite of a vague proposal (γ factor) to calibrate the operational risk charge based on expected and unexpected losses, the charge is proportional to *expected* loss.

Let us try to define an expected operational loss for Option 3 and compare this definition with an expected credit loss. Assume long term stationarity of the environment of an institution and – using discrete modeling for conceptual simplicity -- define

losses $l_k \in L,\ \mathrm{k} = 1,\ldots,K,$ occurring with probability $P(l_k)$ and
risk type events $e_{ij} \in E,\ i = 1,\ldots,n,$ j=1,...,N, occuring with probability $P(e_{ij})$ or *PE*.
Then the expected loss is given by

$$E(\boldsymbol{L}) = \sum_k P(l_k)\, l_k$$
$$= E(E(\boldsymbol{L} \mid \boldsymbol{e})) = \sum_{i,j} P(e_{ij}) E(\boldsymbol{L} \mid e_{ij}) = \sum_{ij} \sum_k l_k P(l_k \mid e_{ij}) P(e_{ij}). \qquad (2)$$

The loss given event (LGE) occurs with the conditional probability of the loss ***L*** being at level *l* given the realization of the event *e*. Such operational risk events belong to the set *E* and are indexed by the numbers of business lines and risk types. In our previous work [9] we proposed to collect data across business lines and risk types in the cells of Table 3 below.

Event Type Category 1,..,N	Technology failure 1	…	Fraud j	…	External event N	**Total**
Business Unit 1,…n						
1	L_1^1		L_1^j		L_1^N	$L_1^1, L_1^2, \ldots, L_1^N$
2	L_2^1		L_2^j	…	L_2^N	$L_2^1, L_2^2, \ldots, L_2^N$
…						
i	L_i^1		L_i^j		L^N	$L_i^1, L_i^2, \ldots, L_i^N$
…		…		…		…
n	L_n^1		L_n^j		L_n^N	$L_n^1, L_n^2, \ldots, L_n^N$
Firm-wide	$L_1^1, L_2^1, \ldots, L_N^1$	…	$L_1^j, L_2^j, \ldots, L_n^j$	…	$L_1^N, L_2^N, \ldots, L_n^N$	$L^1, L^2, \ldots, L^N$

Table 3 *Firm-wide matrix of operational losses*

A fundamental concept of actuarial modeling is the distinction between unconditional and conditional event probabilities. Assuming that the *unconditional* probability of the (i, j)th event $P(e_{ij})$, is its expected frequency (empirical or subjective), the *conditional* probability $P(e_{ij} \mid l_k)$ of the event is its probability if we knew what the realized value l_k of the loss would be. The unconditional probability of the event e_i is the average value of its conditional probabilities across *all* realizations of losses possible from this event

$$P(e_{ij}) = \sum_k P(e_{ij} \mid l_k) P(l_k).$$

By Bayes theorem the probability of loss given event e_i is

$$P(l_k \mid e_{ij}) = \sum_k \frac{P(e_{ij} \mid l_k) P(l_k)}{\sum_{i,j} P(e_{ij} \mid l_k) P(l_k)},$$

where P(l_k) is the unconditional probability of a loss of severity l_k from *any* source, and the expected loss, LGE given event e_{ij} is given by

$$E(L \mid e_{ij}) = \sum_k l_k P(l_k \mid e_{ij}), \qquad (3)$$

the inner sum of (2).

Thus the simplification of (2) embodied in (1) is to specify a *single* (representative?) expected loss severity for each risk type and business line of the event occurring with probability PE .

Given the lack of operational risk data it is nontrivial to reconcile calculation of *total expected losses* across all business lines and risk types for an individual bank. The final step, adjusting by *individual* factors γ and exposure factors EI in order to define total *unexpected loss* -- indicating that the previous considerations are *not* institution specific -- is doubtful, due to the limited of availability of operational risk data for the cells of Tables 2 and 3 and their current limited relevance across the industry.

The expression for the calculation of operational expected loss EL borrows notation from expected loss calculations in credit models (see, for example, Chapter 4 of [7]).

Expected loss = exposure x loss given default x probability of default.

For most purposes a credit loss arises only in event of obligor default, thus *loss given default*. But for default events the conditional probabilities are driven by systematic risk factors.

'The conditional default probability is defined across all possible realizations of some systematic risk factors X which are identified with some specific observable quantities such as macroeconomic variables or industrial sector performance indicators, or may be left abstract. Regardless of their identity, it is assumed that all correlation in credit events is due to common sensitivity to these factors' [8].

-- Do systematic risk factors driving operational risks exist and, if so, what is an *industry wide* loss distribution?

Perhaps an even more important question is the following.

-- How does an operational risk charge (based on such factors or not) relate to market and credit risk management?

If any of the options proposed by Basel are used, the unfortunate answer is that with gross income, or any other size related exposure indicator, a potential increase in gross income through successful market and credit risk management would be *penalized* by the operational risk charge.

The industry's dissatisfaction with the proposed operational risk capital allocation may be summarized by the following extract from the BBA's comments [2]:

'Using proxies for the size of operational risk is an admission that measurement of operational risk does not lend itself to the approaches which have been developed for market and credit risk. Indeed, the proxy proposed is simpler than the use of risk weighted assets in the current credit regime or the use of the market value of positions in the standard market risk evaluations.'

Overwhelming criticism of the new Accord proposals has been heard from the financial services industry and hopefully other options for evaluation of operational risk will be considered. We attempt to propose some alternatives here.

3. Our risk capital framework

Industry hue and cry surrounding the operational risk charge is equally matched by the confused state of operational risk modeling. Everything from scorecards to fuzzy logic, Bayesian networks, neural networks, extreme value theory or a hybrids of the above have been proposed for application to operational risk management. The new Basel Proposal has been mostly influenced by actuarial models, probably because its operational risk definition is based on lists of loss events. But since it is not clear where the boundary between market, credit and operational risk lies and what indeed a meaningful industry-wide operational loss distribution is, it is difficult to compare or evaluate most of proposed methods.

By definition, operational risk management for an individual bank or business unit in a given situation depends on the correct identification of the risk factors causing operational losses. Since external and internal causal factors are included in the definition, not only the bank's own *operations* may lead to operational risk but also all financial information received by banks is inherently related to causal operational risk factors. Some of these, concerning out-of-the- ordinary events, contribute to *significant losses*. Examples of such events include natural disasters as well as major social or political events, and all may be considered to be *rare events.* Statistical analysis of data, which includes the effects of such rare events, requires special techniques which lie *outside* the assumption of *normality*. We term the related risk factors *external*. Processing all incoming information and taking decisions at different levels of the bank may lead to further losses caused by *internal* factors reflected in increased business costs (i.e. *operations* risks). Some such causes are human or technological errors, lack of control to prevent unauthorized or inappropriate transactions being made, fraud and faulty reporting. The relation between those two classes of causal factors and their importance for a particular business unit should be reflected in any strategic view of the risks involved.

Statistical patterns of *loss data* attributed to these external and internal types of causal factors can be very different. For example:

- Mistakes in accounting, transaction errors and other human errors generate loss distributions which are usually normal
- Natural disasters lead to distributions which have ’fat’ or ‘long’ tails.
- Fraudulent activity, which can be observed in trading data subject to *market* risk, also leads to heavy tails of the trading P&L distribution.
- Similar P&L distributions emerge from trading futures and government bonds in emerging markets through times of political crisis.

In general losses may be classified into two categories:

(1) *low* value but *frequently* occurring

(2) *significant* in value but *rare*.

Modeling each category of losses requires specific techniques, but more important from the view point of data collection is to identify losses which have already been accounted for by the existing risk management process.

With the view that control procedures can be developed for illumination of the first type of low value/frequent losses and that the cost of such control procedures will be accounted for in the operations budget, we assume that only losses of large magnitude are considered for operational risk capital provision. The aim of operational risk capital provision is to insure the capacity of a bank to continue to operate -- through the availability of sufficient economic capital -- in an adverse environment or when its internal operations have caused large unexpected losses,.

Operational risk could be hidden in a number of different accounts for balance sheet reporting, both in banking and trading books. To ensure an appropriate control environment the first step in operational risk management should be a careful analysis of all available data to identify the statistical patterns of losses related to identifiable risk factors as a part of an institution's financial surveillance system.

4. The model

Inclusion of operational risk into the regulatory framework requires a revision of accepted market and credit risk management practice. Recall that VaR provides a measure of the market risk due to adverse market movements under *normal* market conditions, with back-testing performed to assess the accuracy of the implemented VaR models over time (usually a year). Similarly, credit provision corresponds to *normal* credit conditions with an indicative worst-case portfolio credit loss at some confidence level, calculated over a (one year) time horizon. One might thus naturally ask how the definition of "normality" relates to operational risk and to the problem of internal bank controls and external supervision. These questions are critical, particularly regarding extreme losses, since market, credit and operational risks become entangled at the time of occurrence of very large losses. *Double counting* is potentially the most serious problem for all major business units involved in trading, investment and lending. Reporting *integrated* market and credit value at risk can rectify such a problem. But how can operational risk capital charges be differentiated from market and credit allocation while keeping an integrated view of risk management?

Recall a simple definition of operational risk which has been adopted in practice:

All risks that are not market or credit risk are operational risks.

This definition of operational risk as complementary to market and credit risk allows us to derive a capital charge for operational risk from internal operational risk measurement. To make it consistent with existing credit and market risk models, a starting point is the construction of an historical *profit and loss* (P&L) distribution for the level of the organization of interest. *Profit* data must be included in the analysis, as it positions the actual P&L distribution by defining its mean and median. The presence of large or extreme losses over a period specified by the data collection process will indicate that something has been (and may again be) wrong.

Ideally, statistical analysis of profits and losses would form part of the normal financial surveillance system of the bank. Further identification of the causes of specific losses may use some additional qualitative analysis. The important point is that this surveillance is concerned with the identification of the “normality” of business processes.
Quantification of operational risk starts with the identification of market and credit unexpected loss thresholds obtained from VaR models. We assume that credit VaR and Market VaR are known from internal risk models as a part of financial reporting. From the modelling viewpoint the reporting process must verify the assumptions of internal market and credit models. Losses within the limits of market and credit value at risk can be accommodated by market and credit economic capital. (The reasons for those losses may be further assessed through supervision and control.)
Only losses of larger magnitude need be considered for operational risk capital provision. Hence, as noted above, we here adopt the accepted practice of defining operational risk as ‘everything which is not market or credit risk’ and *assume operational losses to be in the category of losses which are larger than those due to market or credit risks under normal market conditions.*

All forms of risk are reflected in financial reports, with market risk concentrating in the trading book and credit risk in the banking books. Current practice is for each business unit to have its own specialised risk management. Still all business units are exposed to operational risks. Pillar 2 – the *Supervisory Review Process* –

‘is intended to ensure that each bank has a sound internal process in place to assess the adequacy of its capital based on a thorough evaluation of its risks.’

Thus at the strategic level capital allocation for market, credit and operational risks must be assessed for the institution at least once a year. Risk management reporting is already available in the form of market or credit VaRs and the corresponding profit and loss distributions supporting these calculations. Therefore, at the conceptual level, an *integrated profit and loss distribution* at the highest level of the organisation may be constructed with a threshold loss level[3] obtained from market and credit risk models.

- The level of loss due to market risk, which is exceeded with probability π, is denoted by u_{VaR} .
- The level of loss due to both credit and market risks, which is exceeded probability $\rho \le \pi$, is denoted u_{CVaR} . It is assumed that $u_{CVaR} \le u_{VaR}$.
- Losses beyond the u_{CVaR} level are *unexpected losses* and are *defined* to belong to the operational risk category. Thus *operational risks* are defined as *excesses over normal both market and credit unexpected losses* in the P&L distribution as shown in Figure 1.

Operational risk capital measures may be now derived from descriptive statistics of the empirical distribution or from the parameters of an appropriate approximating distribution. Relations between the thresholds for market and credit risk should be re-examined with respect to the overall implementation of risk management procedures with a view to the definitions of 'expected' and 'unexpected' losses. For the purpose of operational risk management the *unexpected loss threshold* u should also be consistent with statistical assumptions required for the asymptotic behaviour of extremes. From the view point of *integrated* risk management the choice of such a threshold should be such that the u_{CVaR} level approximately equals the statistically derived threshold *u*.

[3] Adjustment of market VaR value to include credit VaR or reconciliation of profit and loss from the relevant operations to obtained a consolidated VaR is a separate problem which depends on implementation issues which are institution specific.

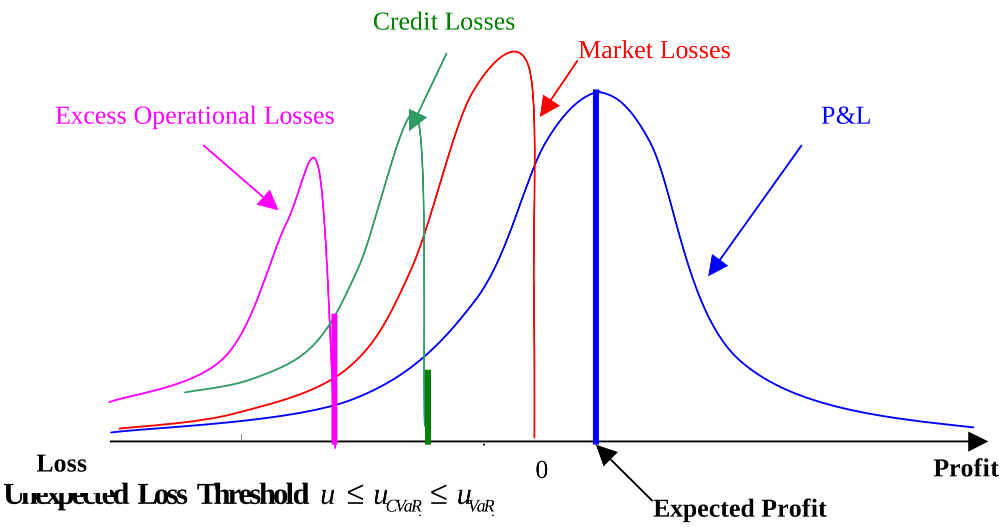

Figure 1 *Decomposition of the loss-tail of a Profit & Loss distribution into its three loss-types (market, credit and operational losses) and definition of the threshold for extreme operational losses.*

Our capital allocation for operational risks is thus based on results from extreme value theory. The operational risk capital will be derived from the parameters of an *asymptotic* distribution of *extremes* of profit and loss. Required theoretical results and procedures for parameter estimation are given in our recent papers [9, 10].

In the case of extreme losses (i.e. heavy or long tailed underlying P&L distributions) the modelling involves a few levels of approximation. First, one must verify that the underlying P&L distribution belongs to the class of the *max-stable* distributions, i.e. the behaviour of the tail of the P&L distribution may be explained by that of its maximum term. The *generalised extreme value* (GEV) distribution $H_{\xi;\mu,\sigma}$ provides a representation for the limiting distribution of the maximum with *shape parameter* ξ and normalised by the *location* parameter μ and the *scale* parameter σ. More formally, let $\boldsymbol{X}_1,\ldots,\boldsymbol{X}_n$ represent *independent identically distributed* (i.i.d.) random variables (losses, here considered as positive) with distribution function *F* and denote their *maximum* by $\boldsymbol{M}_n = \max(\boldsymbol{X}_1, \boldsymbol{X}_2, ..., \boldsymbol{X}_n)$. Then

$$\mathrm{H}_{\xi;\mu,\sigma}(x) = \begin{cases} \exp\left[-\left(1+\xi\dfrac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right] & \text{if } \xi \neq 0, \quad 1+\xi\dfrac{x-\mu}{\sigma} > 0 \\ \exp\left[-\exp\left(-\dfrac{x-\mu}{\sigma}\right)\right] & \text{if } \xi = 0 \quad . \end{cases} \tag{4}$$

Second, given a high threshold level u, the *distribution of excesses* **Y**:= **X**-u is given by the *conditional distribution* function in terms of the tail of the underlying distribution *F* as

$$F_u(y) := \mathrm{P}(\boldsymbol{X} - u \leq y \mid \boldsymbol{X} > u) = \frac{F(u+y) - F(u)}{1 - F(u)} \quad \text{for } 0 \leq y < \infty. \tag{5}$$

The limiting distribution $G_{\xi,\beta}$ (*y*) of excesses as $u \to \infty$ is known as the *generalised Pareto distribution* (GPD) with *shape parameter* ξ and *scale parameter* $\beta = \sigma + \xi(u - \mu)$

$$G_{\xi,\beta}(y) = \begin{cases} 1-\left(1+\xi\dfrac{y}{\beta}\right)^{-1/\xi} & \xi \neq 0 \\ 1-\exp\left(-\dfrac{y}{\beta}\right) & \xi = 0 \end{cases} \qquad \text{where} \quad y \in \begin{cases} [0,\infty] & \xi \geq 0 \\ [0,-\dfrac{\beta}{\xi}] & \xi < 0. \end{cases} \tag{6}$$

The GPD is an approximation of F_u, i.e. $\lim_{u \to x_F} \sup_{0 \leq y \leq y_F} | F_u(y) - G_{\xi,\beta(u)}(y) | = 0$,

where x_F (possibly infinite) is the right end point of the support of the distribution given by *F* and $y_F := x_F - u$, for some positive (measurable) function of the threshold *u* given by *β(u)*, provided that the distribution *F* is in the max-domain of attraction of the GEV distribution. This approximation is only 'good' in the asymptotic sense (i.e. as the threshold $u \to \infty$). Thus the choice of threshold must satisfy the asymptotic convergence conditions, i.e. be large enough for a valid approximation; but when *u* is too high classical parameter estimators for ξ and β_u may have too high a variance due to the small number of exceedances of such a threshold. In the literature [3, 11, 12,] various techniques have been proposed for a statistically reliable choice of threshold.

Third, we must model operational losses *over time*. The number of exceedances $\boldsymbol{N}_u$ over a threshold u and the exceedance times may be represented as a point process which converges[4] weakly to a limiting *Poisson process* with *intensity* λ_u. The resulting asymptotic model is known as the *peaks over threshold* (POT) model [13, 14] with intensity λ_u . This intensity must be measured in the same time units as the given underlying profit and loss data (e.g. daily, weekly, monthly, etc.).

The threshold value *u* chosen according to the above steps (1-3) is defined as the *unexpected loss threshold.* In [9, 10] we proposed operational risk measures and a rule for calculating an excess operational risk charge which are summarised below:

- *Severity* of the losses is modelled by the GPD. The expectation of the excess loss distribution, i.e. *expected severity,* is our coherent risk measure given by

$$E(\boldsymbol{X}-u \mid \boldsymbol{X}>u)=\frac{\beta_u+\xi u}{1-\xi} \quad \text{with } \beta := \sigma+\xi(u-\mu). \tag{7}$$

- The number of exceedances $\boldsymbol{N}_u$ over the threshold u and the corresponding exceedance times are modelled by a Poisson point process with intensity (*frequency* per unit time) given by

$$\lambda_u := \left(1+\xi\,\frac{(u-\mu)}{\sigma}\right)^{-\frac{1}{\xi}} \quad . \tag{8}$$

- *Extra* capital provision for operational risk over the *unexpected loss threshold u* is estimated as the *expectation of the excess loss* distribution (expected severity) scaled by the *intensity* λ_u of the Poisson process, *viz.*

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[4] Convergence to a Poisson process requires more assumptions, see Chapter 5 of [3] for details.

$$\lambda_u \;\mathrm{E}(\boldsymbol{X}-u \mid \boldsymbol{X}>u)=\lambda_u \;\frac{\beta_u+\xi u}{1-\xi}, \tag{9}$$

where u, β, ξ and λ are the parameters of the POT model and time is measured in the same units as data collection frequency, e.g. hours, days, weeks, etc. (Note that β_u and λ_u may be expressed in terms of μ and σ.)

- The *total* amount of *capital* provided against extreme operational risks for a time period of length T will then be calculated by

$$u_T+\lambda_u \; T \; E(\boldsymbol{X}-u \mid \boldsymbol{X}>u)=u_T+\lambda_u \; T \; \frac{\beta+\xi u}{1-\xi}, \tag{10}$$

where u_T represents the total capital provision for market and credit risk, which may in the first instance be considered to be equal to u under the assumption of max-stability.

For operational risk the accuracy of economic the capital allocation (10) depends of course on both the correct choice of threshold and accurate estimates of the GPD parameters.

For accurate estimation of the GPD parameters a sufficient amount of data is required. On the another hand, for a valid GPD approximation the threshold should be sufficiently high. Unfortunately, higher thresholds provide less data. However *hierarchical Bayesian simulation* methods [15, 16] for parameter estimation allow one to overcome the problems associated with lack of data through intensive computation. Our computational procedures accounting for extreme event dependencies across business lines/risk types are described in [10]. Operational loss data is organised into a matrix according to loss type and business units as in Table 3. In the current implementation the parameters for individual business units are estimated from business unit data pooled by risk type. The example in [10] analyses the (external) operational risks caused by the Russian default of 1998 for four business units of a trading group. Alternatively the procedure could be applied to one business unit across different loss types. Conceptually, both loss factor and business unit dimensions can be simultaneously accommodated, but at the cost of increased complexity and computation.

For overall capital allocation at the top level of the bank, we would hope to reduce the overall assessed capital allocation due to portfolio diversification effects and to identify the high-risk factors for specific business units of the firm – both achieved in the limited context of the example of [10].

5. Integration of risk management

Success in operational risk management is dependent on the ability of the industry to handle an increasing amount of information processing related to the control and management of an institution's performance. For large international banks involved in all types of activities the task of capital provision planning is enormous.

Various risk-adjusted performance measures have been proposed for the optimization of capital allocation *within* the firm (see, [17] for a specification of different risk-adjusted performance measures and the regulatory capital framework's return incentives). Derived from capital pricing theory, the risk-adjusted return calculation is a *single time period* optimization applied to the combined credit, market and operational risks of a business unit and implicitly dictates a credit risk time horizon which is much longer than that of market risk. The choice of optimization period and the reconciliation of the time horizon in the models related to different risk types are challenging topics for research.

Traditional accounting and regulatory reporting processes require banks to submit their reports to the banking supervisors once every year. Aggregation of threshold levels – and specifically adjusting with respect to a common time horizon -- requires considerable cooperation between the various specialised risk management groups.

The time-dependent evolution of credit risk involves multi-year horizons. An initial one-year transition matrix used in calculations of the migration of credit ratings is usually derived from rating agency data from longer-term transition matrices which impose the assumption of a steady-state for the credit portfolio distribution.

Comparison of our threshold derived from EVT analysis with the credit VaR threshold will need the results of an internal credit model and a detailed description of the credit portfolio. Over what time such assumptions are valid and what is an appropriate procedure to identify such a time interval are questions which remain to be answered.

Unlike credit risk, market risk management is performed daily (DEaR) and the evaluation of market risk at longer time horizons becomes increasingly dependent on distributional assumptions for the underlying profit and loss distribution. A regulatory multiplier used in connection with internal market risk models guarantees that there are no violations of internal VaR limits. The introduction of an operational risk charge means that that this market risk multiplier should be bounded. Our operational risk charge is proportional to the intensity of the process of exceedances. In our example [10], the 5% threshold level gives a satisfactory allocation (a 46.9% margin of safety compared with actual losses) which is only slowly changing (a 2% improvement) with nearly a two fold increase (1.6) in threshold level.

6. Conclusion

By allowing banks to use their own internal models for their trading book and with the current move towards a model-driven *internal ratings-based* (IRB) approach for the banking book, regulators are starting to use economic capital in lieu of regulatory capital. Yet, the operational risk regulatory charge was motivated by the worst recent financial failures and is insensitive to the risk management of an institution. Should operational risk be modelled without a clearly stated relation between risks of different types?

Models for market, and to a lesser extent for credit, risk are accepted and tested. Their outputs determine capital provisions for market and credit risks. The risk capital framework proposed here allows quantification of operational risk losses based on an integrated view of risks and a high level control chart philosophy in which extreme losses exceeding a fixed unexpected level are used annually (say) to estimate the required excess capital provision. Such a combined allocation of economic capital for market, credit and operational risks reinforces a risk sensitive management corresponding to the firm's mix of business, performance and level of capitalisation.

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