

Graph Theory  
Origin and *Seven Bridges of Königsberg*  
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## **Graph Theory:**

**Graph theory** can be defined as the study of [\*graphs\*](#);

Graphs are mathematical structures used to model pair-wise relations between objects from a certain collection. **Graph** can be defined a set  $V$  of vertices and set of edges. Where,  $V$  is collection of  $|V| = n$  abstract data types. Vertices can be any abstract data types and can be presented with the points in the plane. These abstract data types are also called nodes. A line (line segment) connecting these nodes is called an edge. Again, more abstractly saying, edge can be an abstract data type that shows relation between the nodes (which again can be an abstract data types).

In this document, I would briefly go over through how and what led to the development of the graph theory which revolutionized the way many complicated problems were looked at and were solved.

**Leonhard Paul Euler (1707- 1783)** was a pioneering Swiss mathematician, who spent most of his life in Russia and Germany. Euler (pronounced as OILER) solved the first problem using graph theory and thereby led the foundation of very vast and important field of graph theory. He created first graph to simulate a real time place and situation to solve a problem which was then considered one of the toughest problems.



## Problem

The 'Königsberg bridge' problem originated in the city of Königsberg, formerly in Germany but, now known as Kaliningrad and part of Russia, located on the river Preger. The city had seven bridges, which connected two islands with the main-land via seven bridges. People staying there always wondered whether there was any way to walk over all the bridges once and only once. The below picture is the map of Königsberg during Euler's time showing the actual layout of the seven bridges, highlighting the river Preger and the bridges.

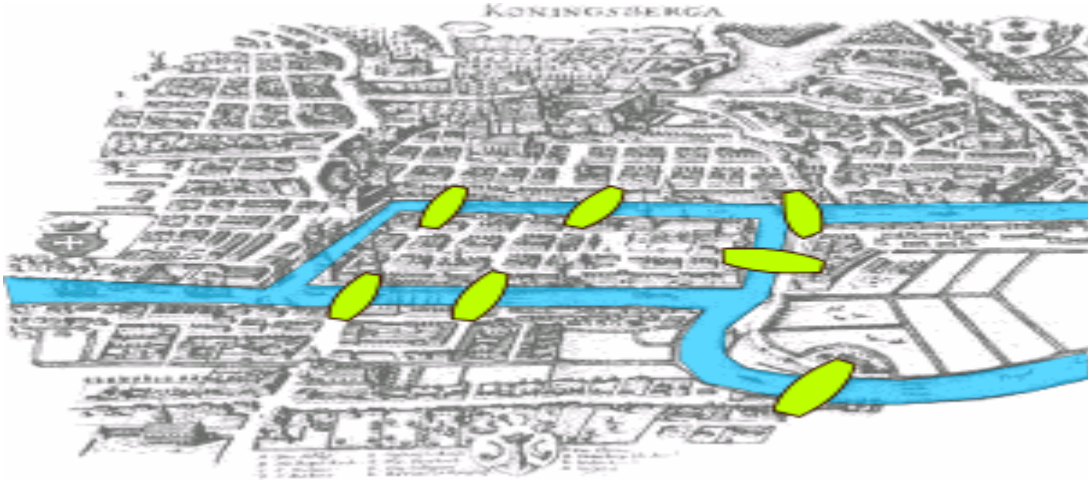


Fig. 1

In 1736 Euler came out with the solution in terms of graph theory. He proved that it was not possible to walk through the seven bridges exactly one time. In coming to this conclusion, Euler formulated the problem in terms of graph theory. He abstracted the case of Königsberg by eliminating all unnecessary features. He drew a *picture* consisting of "dots" that represented the landmasses and the line-segments representing the bridges that connected those land masses. The resulting picture might have looked somewhat similar to the figure shown below.

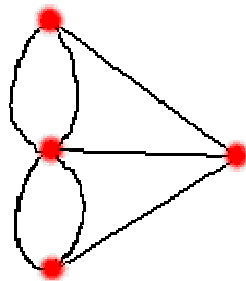


Fig. 2

This simplifies the problem to great extent. Now, the problem can be merely seen as the way of tracing the graph with a pencil without actually lifting it. One can try it in all possible ways, but you will soon figure out, it is not possible. But Euler not only proved that its not possible, but also explained why it is not and what should be the characteristic of the graphs, so that its edge could be traversed exactly once. He came out with the then new concept of degree of nodes. The **Degree of Node** can be defined as the number of edges touching a given node. Euler proposed that any given graph can be traversed with each edge traversed exactly once if and only if it had, zero or exactly two nodes with odd degrees. The graph following this condition is called, Eulerian circuit or path. We can easily infer this theorem. Exactly two nodes are, (and must be) starting and end of your trip. If it has even nodes than we can easily come and leave the node without repeating the edge twice or more.

In actual case of seven bridges of Königsberg, once the situation was presented in terms of graph, the case was simplified as the graph had just 4 nodes, with each node having odd degree. So, Euler concluded that these bridges cannot be traversed exactly once.

Using this theorem, we can create and solve number of problems. Suppose now, we want to make the graph created from bridges of Königsberg, a Euler's circuit. Now, as per Euler's theorem we need to introduce a path to make the degree of two nodes even. And other two nodes can be of odd degree out of which one has to be starting and other another the endpoint. Suppose we want to start our journey from blue node and end at the yellow node. So, the two nodes can have odd edges. But somehow we need to edit the actual graph by adding another edge to the graph such that the two other nodes have even degree. So, the resulting figure is shown below.

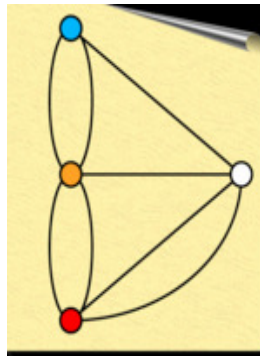


Fig. 3

[http://en.wikipedia.org/wiki/Seven\\_Bridges\\_of\\_K%C3%B6nigsberg](http://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg)

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