

Simulation of Three-mass Mechanical System using MATLAB Software

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Abstract The aim of the paper is the dynamic analysis of selected models of running gear by using MATLAB/Simulink. This paper concerns with numerical approach to a vehicle vertical vibration based on kinematic excitation. Kinematic variables calculation is demonstrated on the quarter three-mass dynamic model with additional shock absorber and spring. There is mathematical model that is numerically solved in MATLAB and is illustrated here together with the simulation in Simulink by block scheme. The results could be used for teaching purposes.

Keywords: MATLAB/Simulink, signal builder, mathematical model, block

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1. Introduction

Method of releasing is applied to drawing up mathematical model. Kinetic equations are drawn up by Newton's method. The result is kinematic quantities graphically represented and their numerical values. Selecting the way of solution is influenced by concern about mechanical system vibration problems [1]. MATLAB is efficient, interactive setting for scientific and engineering calculations. To be able to solve in Matlab it is necessary to modify system of differential equations of second order to system of differential equations of first order. Detailed calculation process is introduced in work [2].

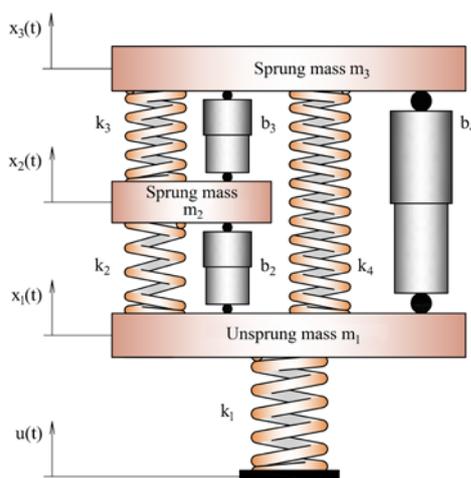


Figure 1. Quarter three-mass model

2. Drawing up Mathematical Model by Method of Releasing

Calculation of kinematic quantities is demonstrated on quarter three-mass dynamic model with additional damper and spring (Figure 1) [2,3]. The system is solved in MATLAB/Simulink.

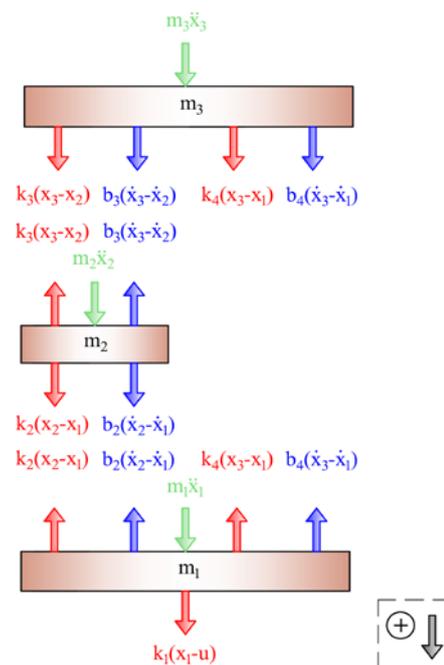


Figure 2. Released quarter model

Running gear is kinetically excited. Kinetic equations of mechanical system are made up by applying of releasing method (Figure 2).

Kinetic equations of damped three-mass mechanical system (Figure 2) are composed by using of releasing method and look like:

$$m_1 \ddot{x}_1 - b_2(\dot{x}_2 - \dot{x}_1) - b_4(\dot{x}_3 - \dot{x}_1) + k_1(x_1 - u) - k_2(x_2 - x_1) - k_4(x_3 - x_1) = 0, \quad (1)$$

$$m_2 \ddot{x}_2 + b_2(\dot{x}_2 - \dot{x}_1) - b_3(\dot{x}_3 - \dot{x}_2) + k_2(x_2 - x_1) - k_3(x_3 - x_2) = 0, \quad (2)$$

$$m_3 \ddot{x}_3 + b_3(\dot{x}_3 - \dot{x}_2) + b_4(\dot{x}_3 - \dot{x}_1) + k_3(x_3 - x_2) + k_4(x_3 - x_1) = 0. \quad (3)$$

We put the substitution:

$$\begin{aligned} x_1 &= x_1, \\ x_2 &= \dot{x}_1, \\ x_3 &= x_2, \\ x_4 &= \dot{x}_2, \\ x_5 &= x_3, \\ x_6 &= \dot{x}_3. \end{aligned} \quad (4)$$

By derivation equations (4) according to time we achieve:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= \frac{1}{m_1} [b_2(\dot{x}_2 - \dot{x}_1) + b_4(\dot{x}_3 - \dot{x}_1) - k_1(x_1 - u) + k_2(x_2 - x_1) + k_4(x_3 - x_1)], \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= \frac{1}{m_2} [-b_2(\dot{x}_2 - \dot{x}_1) + b_3(\dot{x}_3 - \dot{x}_2) - k_2(x_2 - x_1) + k_3(x_3 - x_2)], \\ \dot{x}_5 &= x_6, \\ \dot{x}_6 &= \frac{1}{m_3} [-b_3(\dot{x}_3 - \dot{x}_2) - b_4(\dot{x}_3 - \dot{x}_1) - k_3(x_3 - x_2) - k_4(x_3 - x_1)]. \end{aligned} \quad (5)$$

By repeated using of equations (4) and modifying we achieve six differential equations of first order:

$$\dot{x}_1 = x_2, \quad (6)$$

$$\dot{x}_2 = \frac{1}{m_1} [-(b_2 + b_4)x_2 + b_2x_4 + b_4x_6 - (k_1 + k_2 + k_4)x_1 + k_2x_3 + k_4x_5 + k_1u], \quad (7)$$

$$\dot{x}_3 = x_4, \quad (8)$$

$$\dot{x}_4 = \frac{1}{m_2} [b_2x_2 - (b_2 + b_3)x_4 + b_3x_6 + k_2x_1 - (k_2 + k_3)x_3 + k_3x_5], \quad (9)$$

$$\dot{x}_5 = x_6, \quad (10)$$

$$\dot{x}_6 = \frac{1}{m_3} [b_4x_2 + b_3x_4 - (b_3 + b_4)x_6 + k_4x_1 + k_3x_3 - (k_3 + k_4)x_5], \quad (11)$$

3. Solution of Mechanical System Vertical Vibration Using MATLAB/Simulink Program

At first the entry has to be defined – course of obstacle for solution in MATLAB. The function is

$$u(t) = \frac{A}{2} \left(1 - \cos \left(2\pi \frac{vt}{L_0} \right) \right), \quad (12)$$

where A is obstacle height, L_0 is its length and v is car speed [4].

The equations (6), (7), (8), (9), (10) and (11) need to be transcribed to m-file of MATLAB (Table 1). The name of file will be saved according to name of function. For numerical solution of differential equations MATLAB applies predefined functions that differ about method of integration. The time course depiction can be gained by standard function ode45 (Figure 3) [5].

Table 1. M-files of MATLAB

function3M.m	<pre>function dx=fun3M(t,x) m1=55; m2=400; m3=100; k1=230000; k2=30000; k3=50000; k4=0; b2=1500; b3=4000; b4=700; L0=5; v=15; A=0.03; u=(A/2)*(1-cos(2*pi*(v*t/L0))); % obstacle dx=[x(2); ... (-b2 + b4)*x(2)+b2*x(4)+b4*x(6) -(k1+k2+k4)*x(1)+k2*x(3)+k4*x(5)+k1*u)/m1; ... x(4); ... (b2*x(2)-(b2+b3)*x(4)+b3*x(6)+k2*x(1) -(k2+k3)*x(3)+k3*x(5))/m2; ... x(6); ... (b4*x(2)+b3*x(4)-(b3+b4)*x(6)+k4*x(1)+k3*x(3) -(k3+k4)*x(5))/m3];</pre>
course_3H.m	<pre>% course depiction – three-mass system [t,x]=ode45('fun3M',[0 5],[0; 0; 0; 0; 0; 0]); plot(t,x); grid on; % turning on and turning off the grid xlabel('t [s]'); ylabel('x1(t) [m], v1(t) [m/s], x2(t) [m], v2(t) [m/s], x3(t) [m], v3(t) [m/s]'); legend('x1(t)', 'v1(t)', 'x2(t)', 'v2(t)', 'x3(t)', 'v3(t)');</pre>

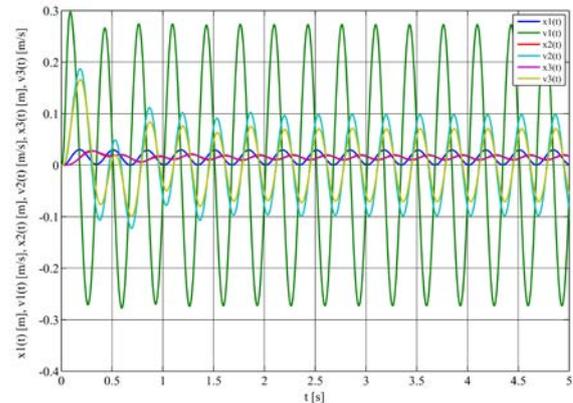


Figure 3. Displacements and velocities depiction

Modeling of mechanical system in Simulink helps us graphically represent the time progressions of kinetic and dynamic quantities. Modifying derived kinetic equations (1), (2) and (3) for three-mass model

$$\begin{aligned} \dot{x}_1 &= \frac{1}{m_1} [-(b_2 + b_4)\dot{x}_1 + b_2\dot{x}_2 + b_4\dot{x}_3 \\ &- (k_1 + k_2 + k_4)x_1 + k_2x_2 + k_4x_3 + k_1u], \end{aligned} \quad (13)$$

$$\ddot{x}_2 = \frac{1}{m_2} [b_2 \dot{x}_1 - (b_2 + b_3) \dot{x}_2 + b_3 \dot{x}_3 + k_2 x_1 - (k_2 + k_3) x_2 + k_3 x_3] \quad (14)$$

$$\ddot{x}_3 = \frac{1}{m_3} [b_4 \dot{x}_1 + b_3 \dot{x}_2 - (b_3 + b_4) \dot{x}_3 + k_4 x_1 + k_3 x_2 - (k_3 + k_4) x_3] \quad (15)$$

we make block diagram (Figure 4).

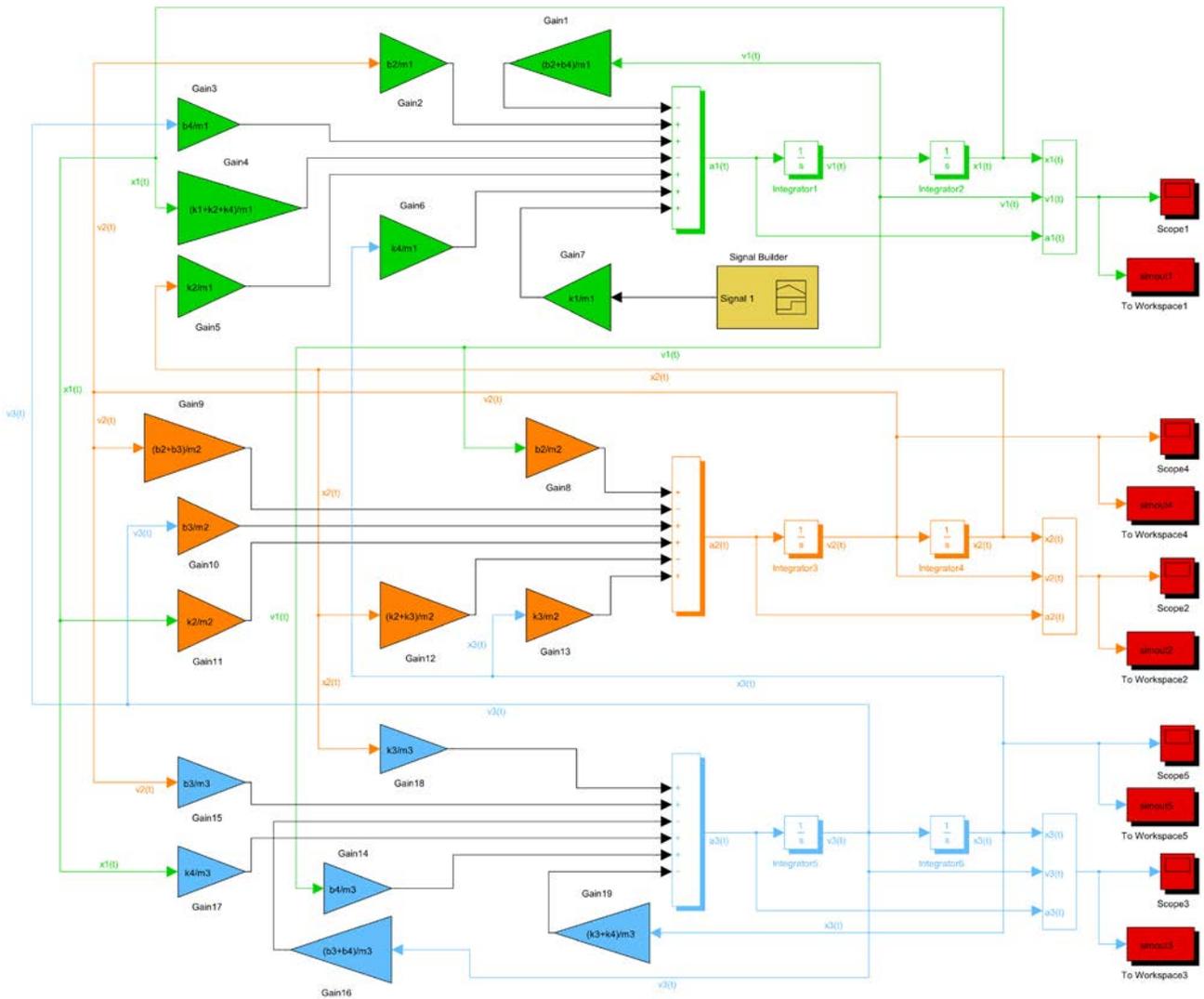


Figure 4. The block diagram of three-mass system in Simulink

Table 2 shows blocks overview from Figure 4, but the library in Simulink contains also other usable blocks for purposes of problem solving.

Table 2. Used blocks in program Simulink

Block	Library	Description
Gain	Math Operations	Multiplication of the input by a constant value
Sum	Math Operations	Sum element
Mux	Signal Routing	Combining into one output
To Workspace	Sinks	Saving the values into Workspace
Scope	Sinks	Diagram representation
Constant	Sources	Entering constant value
Integrator	Continuous	Lowering order
Signal Builder	Sources	Setting up a random excitation

There is a recommendation to change primal value $1e^{-3}$ to $1e^{-15}$ in main menu Simulation, Configuration Parameters before launching a simulation. There is also an option to set time lag of simulation.

The course of obstacle was made up by block Signal Builder. This course needs to be exported File, Export to Workspace and then Start simulation needs to be launched (Figure 5) [3].

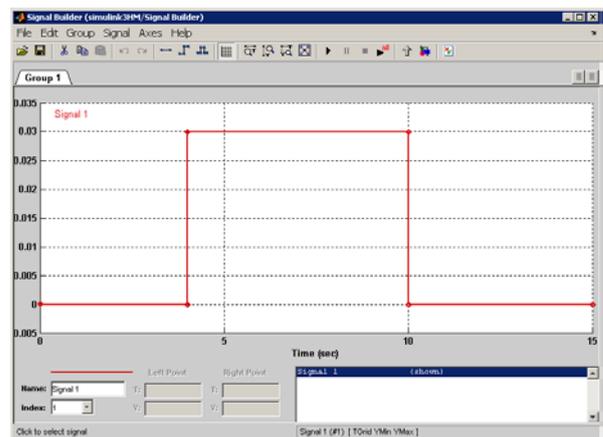


Figure 5. Block Signal Builder

In Figure 6 there is an illustration of defining random excitation in block Signal Builder.

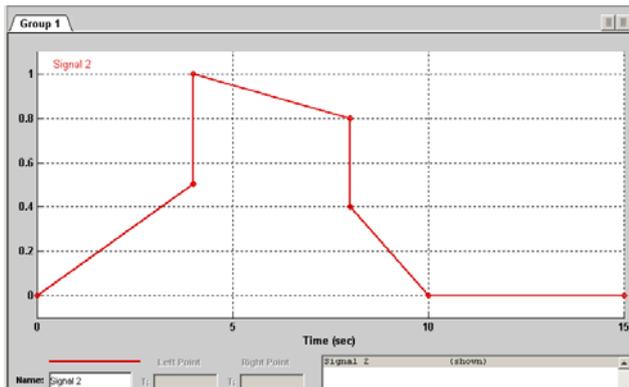


Figure 6. The illustration of defining random excitation in block Signal Builder

Table 3 shows the forms of input data recording to start a simulation in Simulink. These data will be saved into Workspace. It is recommended to use m-file for purposes of faster input editing.

Table 3. The forms of input data recording in Simulink

m-file	<pre>% input data - Simulink % m - [kg], k - [N/m], b - [N/(m/s)] m1=80; m2=300; m3=800; k1=230000; k2=40000; k3=55000; k4=3000; b2=1700; b3=4000; b4=1000;</pre>
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Graphical course of quantities is represented by m-file by subplot and plot (Figure 7, Figure 8). Function Plot is entered in Table 4.

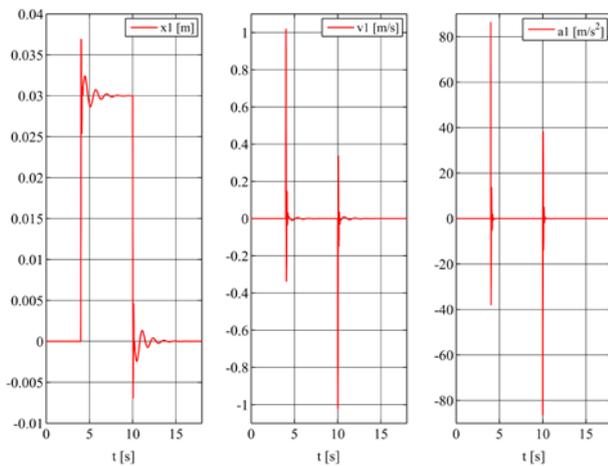


Figure 7. Displacement, velocity and acceleration course of mass 1 next to each other – simout1

Table 4. Function Plot

simulink_simout5_x3.m	<pre>% depiction simout5_x3 (mass "3" displacement only) figure(5) set(5,'Name','course simout5 in Simulink') plot(simout5.time, simout5.signals.values(:,1:1), 'LineWidth', 2.5) legend('x3=x3(t)') xlabel('t [s]') ylabel('x3 [m]') grid on</pre>
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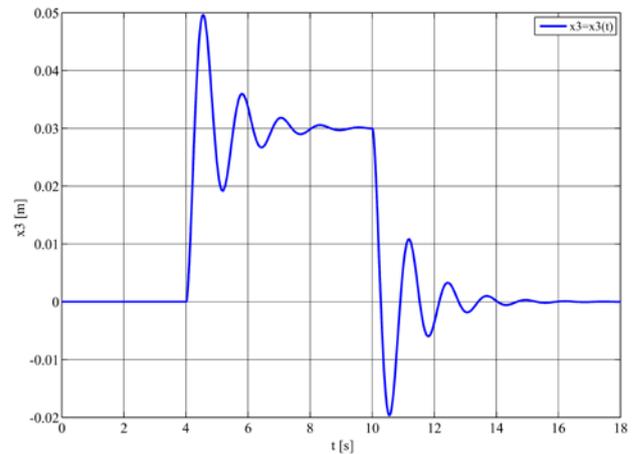


Figure 8. Displacement depiction of mass 3 only – simout5

4. Conclusion

The methodology of three-mass system calculation being solved by deriving of kinetic equations needed for numerical solution in MATLAB is illustrated here together with the simulation in Simulink by block schemes. First-order differential equations need to be assigned for solution in MATLAB and these equations are obtained from modification of second-order differential equations. Making up scheme in Simulink needs to proceed from second-order differential equation that describes system performance.

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