

Mathematical Formulation of Laminated Composite Thick Conical Shells

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Abstract

The mathematical formulation of thick conical shells using third order shear deformation of thick shell theory are presented. The equations of motion are obtained using Hamilton's principle. For present analysis, we consider shell's system transverse normal stress, rotary inertia and shear deformation.

Keywords: Conical Shell, free vibrations, third order shear deformation thick shell theory, Equations of Motion, Lamé' parameters.

1. Introduction

Many research articles investigated the theory of thick conical shells including (Qatu *et al.*, 2013; Qatu *et al.*, 2010). The vibration behavior of cantilevered laminated composite shallow conical shells were also explored recently (Korjakin *et al.*; Qatu *et al.*, 2010; Mukhopadhyay *et al.*, 2016). (Further, Reddy, 1984) explored vibrations of joined conical shells. Qatu (1994) carried out research about the steady-state torsional oscillations of multilayer truncated cones. The damping of the free vibrations of laminated composite conical shells was investigated by Qatu (1994) and while stiffened conical shells were studied by Reddy, (1984). A variable thickness of composite conical shells was discussed (Asadi & Qatu, 2012; Leissa & Qatu, 2011). Damping in multilayered conical shells was investigated by many researchers (Zannon and Qatu, 2014b). Pre-stressed conical shells were explored thoroughly by Qatu *et al.* (2014, 2015).

Towards this end, in this paper, we propose our contribution towards the mathematical theory of third order shear deformation thick conical shell theory (see Zannon *et al.* (TSDTZ); Qatu *et al.*, 2013) and its stress-strain deformation at the mid thick conical shell surface (Duc & Cong, 2015; Akbari *et al.*, 2015; Jam & Kiani, 2015, Viola *et al.*, 2016).

2. Mathematical Formulation of Conical Shell

The displacement components using the third-order shear deformation shell theory are given in (Asadi & Qatu, 2012; Leissa & Qatu, 2011). Conical shells are one form of engineering solids that are formed by revolving two non-parallel lines, mostly a line and axis of revolution. We are interested mainly in a particular type of shells which have a circular cross-section (Qatu, 1994).

A closed conical shell with circular sides (Figure 2.1) has a closed shape and the open conical shell can be obtained by cutting the sides of the solid between θ_1 and θ_2 (Roh *et al.*, 2008). An open conical shell with sides less than the half of the radius of the curvature then the solid is shallow (Qatu *et al.*, 2010; Dung *et al.*, 2014).

A typical fundamental equation of such solid can be written as (with the help of Lamé parameters) (Qatu *et al.*, 2010; Dung *et al.*, 2014; Jam & Kiani, 2015; Akbari *et al.*, 2015)

$$(ds)^2 = (d\alpha)^2 + \alpha^2 \sin^2(\varphi) d\theta,$$

$$A = 1; B = \alpha \sin(\varphi),$$

$$R_\alpha = \infty; R_\beta = \alpha \tan(\varphi).$$

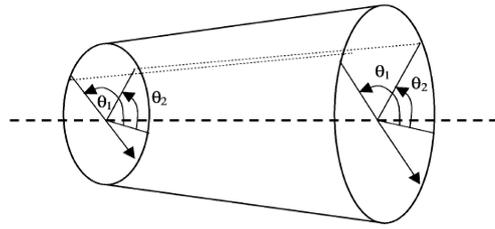


Figure 2.1 A closed conical shell (Qatu, 1994)

Consider Figure 2.2, which is a side view of the closed conical shell described in Figure 2.1.

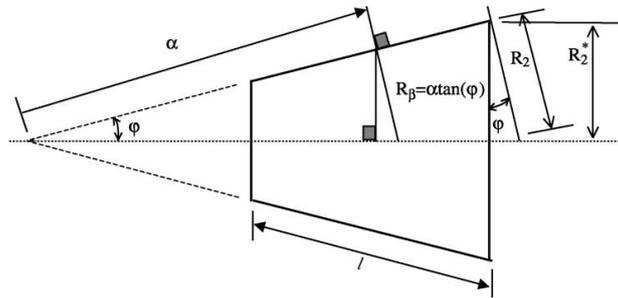


Figure 2.2 A a side view of the closed conical shell (Qatu, 1994)

3. Equilibrium Equations of Motion

A conical laminated shell with Lamé parameters is considered above. The Lamé parameters of middle surface are substituted in moment and force resultants (see Zannon et al, 2015) to formulate the conical shell equations for TSDTZ. Therefore, the strain-displacement equations and middle surface strains are obtained (Qatu et al., 2010; Dung et al., 2014; Jam & Kiani, 2015; Akbari et al., 2015)

$$\begin{aligned} \epsilon_\alpha &= \epsilon_{0\alpha} + z \kappa_\alpha^{(1)} + z^2 \kappa_\alpha^{(2)}, \\ \epsilon_\theta &= \frac{1}{(1 + z/\alpha \tan(\varphi))} (\epsilon_{0\theta} + z \kappa_\theta^{(1)} + z^2 \kappa_\theta^{(2)}), \\ \epsilon_z &= \psi_z(\alpha, \beta) \neq 0, \\ \epsilon_{\alpha\theta} &= \epsilon_{0\alpha\theta} + z \kappa_{\alpha\theta}^{(1)} + z^2 \kappa_{\alpha\theta}^{(2)}, \\ \epsilon_{\theta\alpha} &= \frac{1}{(1 + z/\alpha \tan(\varphi))} (\epsilon_{0\theta\alpha} + z \kappa_{\theta\alpha}^{(1)} + z^2 \kappa_{\theta\alpha}^{(2)}), \\ \gamma_{\alpha z} &= \gamma_{0\alpha z} + z G^{(1)} + z^2 G^{(2)}, \\ \gamma_{\theta z} &= \frac{1}{(1 + z/\alpha \tan(\varphi))} (\gamma_{0\theta z} + z E^{(1)} + z^2 E^{(2)}), \\ \epsilon_{0\alpha} &= \frac{\partial u_0}{\partial \alpha} \\ \epsilon_{0\theta} &= \frac{1}{\alpha \sin(\varphi)} \frac{\partial v_0}{\partial \theta} + \frac{u_0}{\alpha} + \frac{w_0}{\alpha \tan(\varphi)}, \\ \epsilon_{0\alpha\theta} &= \frac{\partial v_0}{\partial \alpha}, \\ \epsilon_{0\theta\alpha} &= \frac{1}{\alpha \sin(\varphi)} \frac{\partial u_0}{\partial \theta} - \frac{v_0}{\alpha}, \\ \gamma_{0\alpha z} &= \frac{\partial w_0}{\partial \alpha} + \psi_\alpha, \\ \gamma_{0\theta z} &= \frac{1}{\alpha \sin(\varphi)} \frac{\partial w_0}{\partial \theta} - \frac{v_0}{\alpha \tan(\varphi)} + \psi_\beta. \end{aligned}$$

The curvature and twist of the conical thick shells are considered

$$\begin{aligned} \kappa_{\alpha}^{(1)} &= \frac{\partial \psi_{\alpha}}{\partial \alpha}, \\ \kappa_{\theta}^{(1)} &= \frac{\partial \psi_{\beta}}{\partial \theta} + \frac{\psi_{\alpha}}{\alpha} + \frac{\psi_z}{\alpha \tan(\varphi)}, \\ \kappa_{\alpha\theta}^{(1)} &= \frac{\partial \psi_{\beta}}{\partial \alpha}, \\ \kappa_{\theta\alpha}^{(1)} &= \frac{1}{\alpha \sin(\varphi)} \frac{\partial \psi_{\alpha}}{\partial \theta} - \frac{\psi_{\beta}}{\alpha}. \\ \kappa_{\alpha}^{(2)} &= \frac{\partial \phi_{\alpha}}{\partial \alpha}, \\ \kappa_{\theta}^{(2)} &= \frac{\partial \phi_{\beta}}{\partial \theta} + \frac{\phi_{\alpha}}{\alpha}, \\ \kappa_{\alpha\theta}^{(2)} &= \frac{\partial \phi_{\beta}}{\partial \alpha}, \\ \kappa_{\theta\alpha}^{(2)} &= \frac{1}{\alpha \sin(\varphi)} \frac{\partial \phi_{\alpha}}{\partial \theta} - \frac{\phi_{\beta}}{\alpha}. \end{aligned}$$

Therefore, the equations of motion are (Qatu et al., 2013; Zannon & Qatu, 2014b; Qatu et al., 2010; Asadi & Qatu, 2012; Jam & Kiani, 2015; Duc & Cong, 2015; Akbari et al., 2015)

$$\begin{aligned} \frac{\partial}{\partial \alpha} (\alpha \sin(\varphi) N_{\alpha}) - \sin(\varphi) N_{\theta} + \frac{\partial}{\partial \theta} (N_{\theta\alpha}) + \alpha \sin(\varphi) q_{\alpha} &= \alpha \sin(\varphi) (\bar{I}_1 \ddot{u}_0 + \bar{I}_2 \ddot{\psi}_{\alpha}), \\ \frac{\partial}{\partial \alpha} (\alpha \sin(\varphi) N_{\alpha\theta}) + \sin(\varphi) N_{\theta\alpha} + \frac{\partial}{\partial \theta} (N_{\theta}) + \frac{\sin(\varphi)}{\tan(\varphi)} Q_{\theta} + \alpha \sin(\varphi) q_{\theta} &= \alpha \sin(\varphi) (\bar{I}_1 \ddot{v}_0 + \bar{I}_2 \ddot{\psi}_{\theta}), \\ \frac{\partial}{\partial \alpha} (\alpha \sin(\varphi) Q_{\alpha}) + \frac{\partial}{\partial \theta} Q_{\theta} - \alpha \sin(\varphi) \left(\frac{N_{\theta}}{\alpha \tan(\varphi)} \right) + \alpha \sin(\varphi) q_n &= \alpha \sin(\varphi) (\bar{I}_1 \ddot{w}_0), \\ \frac{\partial}{\partial \alpha} (\alpha \sin(\varphi) M_{\alpha}^{(1)}) - \sin(\varphi) M_{\theta}^{(1)} + \frac{\partial}{\partial \theta} (M_{\theta\alpha}^{(1)}) - \alpha \sin(\varphi) Q_{\alpha} + \alpha \sin(\varphi) m_{\alpha}^{(1)} &= \alpha \sin(\varphi) (\bar{I}_2 \ddot{u}_0 + \bar{I}_3 \ddot{\psi}_{\alpha}), \\ \frac{\partial}{\partial \theta} (M_{\theta}^{(1)}) + \frac{\partial}{\partial \alpha} (\alpha \sin(\varphi) M_{\alpha\theta}^{(1)}) + \sin(\varphi) M_{\theta\alpha}^{(1)} - \alpha \sin(\varphi) Q_{\theta} + \alpha \sin(\varphi) m_{\theta}^{(1)} &= \alpha \sin(\varphi) (\bar{I}_2 \ddot{v}_0 + \bar{I}_3 \ddot{\psi}_{\theta}), \\ \frac{\partial}{\partial \alpha} (\alpha \sin(\varphi) P_{\alpha}^{(1)}) + \frac{\partial}{\partial \theta} (P_{\theta}^{(1)}) - \alpha \sin(\varphi) \left(N_z + \frac{M_{\theta}^{(1)}}{\alpha \tan(\varphi)} \right) + \alpha \sin(\varphi) m_z &= \alpha \sin(\varphi) (\bar{I}_3 \ddot{y}_z), \\ \frac{\partial}{\partial \alpha} (\alpha \sin(\varphi) M_{\alpha}^{(2)}) - \sin(\varphi) M_{\theta}^{(2)} + \frac{\partial}{\partial \theta} (M_{\theta\alpha}^{(2)}) - 2\alpha \sin(\varphi) P_{\alpha}^{(1)} + \alpha \sin(\varphi) m_{\alpha}^{(2)} &= \alpha \sin(\varphi) (\bar{I}_3 \ddot{u}_0 + \bar{I}_4 \ddot{\phi}_{\alpha}), \\ \frac{\partial}{\partial \theta} (M_{\theta}^{(2)}) + \frac{\partial}{\partial \alpha} (\alpha \sin(\varphi) M_{\alpha\theta}^{(2)}) - \left(\frac{\sin(\varphi)}{\tan(\varphi)} P_{\theta}^{(2)} + 2\alpha \sin(\varphi) P_{\theta}^{(1)} \right) + \alpha \sin(\varphi) m_{\theta}^{(2)} &= \alpha \sin(\varphi) (\bar{I}_3 \ddot{v}_0 + \bar{I}_4 \ddot{\phi}_{\theta}). \end{aligned}$$

The boundary conditions are given (see Zannon et al., 2015; Qatu et al., 2013)

4. Mathematical Analysis

One cannot find an exact solution for a general lamination structure shell with general boundary conditions and/or lamination having series of sequence and layers (Qatu et al., 2013; Zannon et al., 2015; Qatu, 1994). Many researchers

talked about the vibration of shells as in Leissa & Qatu, (2011), she considered a thin plate in her paper "vibration of shells". One can be permitted to obtain a fundamental frequency with good accuracy as in Qatu et al., (2013) by using the classical thin plate (CPT), now using the shear deformation plate theories (SDPTs) can largely eliminate the inaccuracies. Later Qatu et al., (2010) and Reddy (1994) developed this subject, Leissa & Qatu (2011) studied the exact solutions "solutions which satisfy both the equations of motion, and boundary conditions" for simply supported cross-ply thick shell.

The partial differential equations of motions can be found from Qatu et al. (2013) and their solution forms can be found in many sources (Qatu et al., 2013; Zannon & Qatu, 2014b; Duc & Cong, 2015; Akbari et al., 2015). Substituting the solution forms in (Zannon & Qatu, 2014a; Qatu et al., 2010) we give a system of equations, rewrite the coefficient as an eigenvalue problem (Qatu et al., 2013; Qatu et al., 2010; Reddy, 1994), hence we get the form $([Z]-\lambda [N])\{\Delta\}=\{F(t)\}$ where $\lambda = \omega^2$, ω is the natural frequency and $\{\Delta\}$ is the displacement vector (Qatu et al., 2013). The structural stiffness parameters $\{Z_{ij}\}$ of the thick conical shell are following:

$$Z_{11} = -\bar{A}_{11} \cdot A^{*2} - \hat{A}_{66} \cdot B^{*2}, Z_{12} = -A_{12} \cdot A^* \cdot B^* - A_{66} \cdot A^* \cdot B^*,$$

$$Z_{13} = \frac{A_{12} \cdot A^*}{\alpha \tan(\varphi)}, Z_{14} = -\bar{B}_{11} \cdot A^{*2} - \hat{B}_{66} \cdot B^{*2},$$

$$Z_{15} = -B_{12} \cdot A^* \cdot B^* - B_{66} \cdot A^* \cdot B^*, Z_{16} = A_{13} \cdot A^* + \frac{B_{12} \cdot A^*}{\alpha \tan(\varphi)},$$

$$Z_{17} = -\bar{D}_{11} \cdot A^{*2} - \hat{D}_{66} \cdot B^{*2}, Z_{18} = -D_{12} \cdot A^* \cdot B^* - D_{66} \cdot A^* \cdot B^*,$$

$$Z_{21} = -A_{12} \cdot A^* \cdot B^* - A_{66} \cdot A^* \cdot B^*,$$

$$Z_{22} = -\frac{\hat{A}_{22} \cdot B^{*2}}{\alpha \tan(\varphi)} - \bar{A}_{66} \cdot A^{*2} + \frac{\hat{A}_{44}}{(\alpha \tan(\varphi))^2}, Z_{23} = \hat{A}_{22} \cdot B^*,$$

$$Z_{24} = -B_{12} \cdot A^* \cdot B^* - B_{66} \cdot A^* \cdot B^*,$$

$$Z_{25} = -\hat{B}_{22} \cdot B^{*2} - \bar{B}_{66} \cdot A^{*2} + \frac{\hat{A}_{44}}{\alpha \tan(\varphi)},$$

$$Z_{26} = A_{23} \cdot B^* + \frac{\hat{B}_{22} \cdot B^*}{\alpha \tan(\varphi)} + \frac{\hat{B}_{44} \cdot B^*}{\alpha \tan(\varphi)}, Z_{27} = -D_{12} \cdot A^* \cdot B^* - D_{66} \cdot A^* \cdot B^*,$$

$$Z_{28} = -\hat{D}_{22} \cdot B^{*2} - \bar{D}_{66} \cdot A^{*2} + \frac{2 \cdot \hat{B}_{44}}{\alpha \tan(\varphi)} + \frac{\hat{D}_{44}}{(\alpha \tan(\varphi))^2},$$

$$Z_{31} = \frac{A_{12} \cdot A^*}{\alpha \tan(\varphi)}, Z_{32} = \frac{\hat{A}_{44} \cdot B^*}{\alpha \tan(\varphi)}, Z_{33} = -\bar{A}_{55} \cdot A^{*2} - \hat{A}_{44} \cdot B^{*2},$$

$$Z_{34} = -\bar{A}_{55} \cdot A^* + \frac{B_{12} \cdot A^*}{\alpha \tan(\varphi)}, Z_{35} = -\hat{A}_{44} \cdot B^* + \frac{\hat{B}_{22} \cdot B^*}{\alpha \tan(\varphi)},$$

$$Z_{36} = -A^{*2} \cdot \bar{B}_{55} - B^{*2} \cdot \hat{B}_{44} - \frac{\hat{A}_{22}}{\alpha \tan(\varphi)} + \frac{\hat{B}_{22}}{(\alpha \tan(\varphi))^2}$$

$$Z_{37} = 2 \cdot A^* \cdot \bar{B}_{55} + \frac{D_{12} \cdot A^*}{\alpha \tan(\varphi)}, Z_{38} = -2 \cdot B^* \cdot \hat{B}_{44} + \frac{\hat{D}_{22} \cdot B^*}{\alpha \tan(\varphi)},$$

$$Z_{41} = Z_{14}, Z_{42} = Z_{24}, Z_{43} = Z_{34}$$

$$Z_{44} = -\bar{D}_{11} \cdot A^{*2} - \hat{D}_{66} \cdot B^{*2} - \bar{A}_{55},$$

$$Z_{45} = -D_{12} \cdot A^* \cdot B^* - D_{66} \cdot A^* \cdot B^*$$

$$Z_{46} = B_{13} \cdot A^* + \frac{D_{12} \cdot A^*}{\alpha \tan(\varphi)} - \bar{B}_{55} \cdot A^*,$$

$$Z_{47} = -\bar{E}_{11} \cdot A^* - \hat{E}_{66} \cdot B^{*2} - 2 \cdot \bar{B}_{55},$$

$$Z_{48} = -E_{12} \cdot A^* \cdot B^* - E_{66} \cdot A^* \cdot B^*, Z_{51} = Z_{15},$$

$$Z_{52} = Z_{25}, Z_{53} = Z_{35}, Z_{54} = Z_{45},$$

$$Z_{55} = -\hat{D}_{22} \cdot B^* - D_{66} \cdot A^{*2} - \hat{A}_{44},$$

$$Z_{56} = -\hat{B}_{44} \cdot B^*, Z_{57} = -E_{12} \cdot A^* \cdot B^* - E_{66} \cdot A^* \cdot B^*,$$

$$Z_{58} = -\hat{E}_{22} \cdot B^{*2} - \bar{E}_{66} \cdot A^{*2} - 2 \cdot \hat{B}_{44} - \frac{\hat{D}_{44}}{\alpha \tan(\varphi)},$$

$$Z_{61} = Z_{16}, Z_{62} = Z_{26}, Z_{63} = Z_{36}, Z_{64} = Z_{46}, Z_{65} = Z_{56}$$

$$Z_{66} = -\bar{D}_{55} \cdot A^{*2} - \hat{D}_{44} \cdot B^{*2} - A_{33} - \frac{B_{23}}{\alpha \tan(\varphi)} - \frac{\hat{D}_{22}}{(\alpha \tan(\varphi))^2},$$

$$Z_{67} = -2 \cdot \bar{D}_{55} \cdot A^* + \frac{E_{12} \cdot A^*}{\alpha \tan(\varphi)},$$

$$Z_{68} = -2 \cdot \hat{D}_{55} \cdot B^* + \frac{\hat{E}_{44} \cdot B^*}{R_\beta} + \frac{E_{12} \cdot B^*}{R_\alpha} + \frac{\hat{E}_{22} \cdot B^*}{R_\beta},$$

$$Z_{71} = Z_{17}, Z_{72} = Z_{27}, Z_{73} = Z_{37}, Z_{74} = Z_{47}, Z_{75} = Z_{57},$$

$$Z_{76} = Z_{67}, Z_{81} = Z_{18}, Z_{84} = Z_{48}, Z_{85} = Z_{58},$$

$$Z_{77} = -\bar{F}_{11} \cdot A^{*2} - \hat{F}_{66} \cdot B^{*2} - 4 \cdot \bar{D}_{55},$$

$$Z_{78} = -F_{12} \cdot A^* \cdot B^* - F_{66} \cdot A^* \cdot B^*,$$

$$Z_{82} = Z_{28}, Z_{83} = Z_{38}, Z_{86} = Z_{68}, Z_{87} = Z_{78},$$

$$Z_{88} = -\hat{F}_{22} \cdot B^{*2} - \bar{F}_{66} \cdot A^{*2} - \frac{\hat{E}_{44}}{\alpha \tan(\varphi)} - \frac{\hat{F}_{44}}{(\alpha \tan(\varphi))^2} - 4 \cdot \hat{D}_{44}.$$

The structural mass parameters $\{N_{ij}\}$ and the external applied load vector, as a function of time $\{F_{ij}\}$ are given (see Qatu et al., 2013; Zannon et al., 2015).

5. Conclusions

The mathematical analysis of the third order shear deformation theory (see Zannon et al., 2015; Qatu et al., 2013) are presented for simply supported with circular cross section of a thick conical shell. This solution will be used in further investigations to assess the results for free vibration analysis of the circular cross section thick shells.

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